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Pyhthagorean neutorsophic b-open sets in pythagorean nutorsophic topological spaces

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Abstract The purpose of this paper is to introduce and study the notion of Pythagorean neutrosophic b-open sets by using the notion of Pythagorean neutrosophic open set. Besides, we define the concepts of Pythagorean neutosophic b-open function, Pythagorean neutrosophic b-continuous function and Pythagorean neutrosophic bhomeomorphism. Moreover, some of their properties are proved.

Key Words Pythagorean neutrosophic b-open sets, Pythagorean neutosophic b-open function,

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1 Introduction

The notion of fuzzy set was introduced by Zadeh [12] and then this notion has been studied by many mathematicians in different fields of the general topology (see [6, 4]). In 1968, Chang [5] introduced the notion of fuzzy topological spaces, as well as, some basics concepts in general topology. Besides, Atanassov [2, 3] in 1983 defined the concept of intuitionistic fuzzy set. Furthermore, the notion of neutrosophic set was introduced by Smarandache [9] and so Wang et.al. studied some of its properties on interval neutrosophic set. Moreover, the notion of neutrosophic topological space was defined by Salama and Albowi [8]. By using the notions mentioned above, Yager [11] in 2013 introduced the concept of Pythagorean membership grades, later Yager, Zahand and Xu [10] proved some properties on Pythagorean fuzzy set. On the other hand, in 2017 Arockiarani [1] introduced and studied the notion of neutrosophic pre-open set, Besides, Shena and Nirmala [7] introduced the notion of Pythagorean neutrosophic open sets and showed some properties on Pythagorean neutrosophic α -open set. In this paper, we used the notion of Pythagorean neutrosophic open set to introduce and study the concept of Pythagorean neutrosophic b-open set, besides we show some of its properties. We also define the concepts of Pythagorean neutosophic b-open function, Pythagorean neutrosophic b-continuous function and Pythagorean neutrosophic b-homeomorphism. Moreover, some of their properties are proved.

Throughout this paper, (X, τ), (Y, σ) and (Z, ω) are topological spaces on which no separation axioms are assumed unless otherwise mentioned. Furthermore, we sometimes write X, Y or Z instead of

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$(X,\tau),(Y,\sigma)$ or (Z,ω), respectively. Now, we show some Definitions which are useful for the developing of this paper.

Definition 1.1. [12] A fuzzy set $A = \{hx, \mu A(x)i : x \in X\}$ is a universe of discourse X is characterized by a membership function μA as $\mu A : X \to [0,1]$.

Definition 1.2. [2, 3] Let X be a non-empty set. Then, A is said to be an intuitionistic fuzzy set of X if there is a $A = \{hx, \mu A, \gamma Ai : x \in X\}$ where the function $\mu A : X \to [0,1]$ and $\gamma A : X \to [0,1]$ denote the degree of membership $\mu A(x)$ and degree of non-membership γA of every element $x \in X$ to the set A and satisfies the condition $0 \leqslant \mu A(x) + \gamma A(x) \leqslant 2$.

Definition 1.3. [9] Let X be a non-empty set. Then, A is said to be a neutrosophic set of X if there is a $A = hx, \mu A, \sigma A, \gamma Ai : x \in X$ where the function $\mu A : X \to [0,1], \sigma A : X \to [0,1]$ and $\gamma A : X \to [0,1]$ denote the degree of membership (namely $\mu A(x)$), degree of indeterminacy (namely $\sigma A(x)$) and degree of non-membership (namely $\gamma A(x)$) of each element $x \in X$ to the set A and satisfies the condition $0 \leqslant \mu A(x) + \sigma A(x) + \gamma A(x) \leqslant 3$.

Definition 1.4. [11] Let X be a universal set. Then, a Pythagorean fuzzy set A, which is a set of ordered pairs on X and it is defined by $A = \{hx, \mu A(x), \gamma A(x)i : x \in X\}$ where the function $\mu A : X \to [0,1]$ and $\gamma A : X \to [0,1]$ define the degree of membership and the degree of nonmembership respectively , of the element $x \in X$ to A, which is subsets in X and for every $x \in X : 0 \leqslant (\mu A(x))2 + (\gamma A(x))2 \leqslant 1$. Assuming that $0 \leqslant (\mu A(x))2 + (\gamma A(x))2 \leqslant 1$, there is a degree of indeterminacy of $x \in X$ to A defined by $\Pi A(x) = p(\mu A(x))2 + (\gamma A(x))2$ and $\Pi A(x) \in [0,1]$.

Definition 1.5. [7] Let X be a non-empty set. Then, A is said to be a Pythagorean neutrosophic set (or simply, P N) of X if there is a $A = \{hx, \mu A, \sigma A, \gamma Ai : x \in X\}$ where the function $\mu A : X \to [0,1], \sigma A : X \to [0,1]$ and $\gamma A : X \to [0,1]$ denote the degree of membership (namely $\mu A(x)$), degree of indeterminacy (namely $\sigma A(x)$) and degree of non-membership (namely $\gamma A(x)$) of each element $x \in X$ to the set A and satisfies the condition $0 \leqslant \mu A(x)2 + \sigma A(x)2 + \gamma A(x)2 \leqslant 1$.

Definition 1.6. [7] A Pythagorean neutrosophic topology (or simply, P NT) on a non-empty set X is a family of τ of Pythagorean neutrosophic sets in X satisfying the following conditions: (1) $0, 1 \in \tau$.

(2) $G1 \cap G2 \in \tau$, for any $G1, G2 \in \tau$.

(3) $SGi \in \tau$, for any arbitrary family $\{Gi : Gi \in \tau, i \in I\}$. In this case, the pair (X,τ) is said to be a Pythagorean neutrosophic topological spaces, besides any Pythagorean neutrosophic set in τ is known as Pythagorean neutrosophic neutrosophic open set in X.

Definition 1.7. For a Pythagorean neutrosophic set A in a Pythagorean neutrosophic topological space (X,τ) is said to be Pythagorean neutrosophic α -open set [7] if $A \subseteq PNInt((PNCl(PNInt(A)))$.

Theorem 1.8. [7] Every Pythagorean neutrosophic open set is Pythagorean neutrosophic α -open set.

Definition 1.9. [7] Let $f : (X,\tau) \to (Y,\sigma)$ be a function where (X,τ) and (Y,σ) are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic if $f1(V)$ is a Pythagorean neutrosophic in X for every Pythagorean neutrosophic open set V in Y .

Definition 1.10. [7] Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function where (X, τ) and (Y, σ) are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic α -continuous if $f1(V)$ is a Pythagorean neutrosophic α -open in X for every Pythagorean neutrosophic open set V in Y .

Definition 1.11. [7] Let f :(X, τ)$\rightarrow$ (Y, σ) be a function where (X, tau) and (Y, σ) are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic α -open if $f(A)$ is Pythagorean neutrosophic α-open set in Y for every Pythagorean neutrosophic open set A in X.

2 Pythagorean neutrosophic b-open sets

In this section we introduce and study the notion of Pythagorean neutrosophic b-open set and we show some of its properties.

Definition 2.1. Let X be a non-empty set. If a, b, c are real standard or non standard subsets of $]0, 1+[$, then the Pythagorean neutrosophic set $x_{a,b,c}$ is said to be Pythagorean neutrosophic point (or simply, P NP) in X and it is given by: $x_{a,b,c}(x_p) = (a, b, c)$ if $x = x_p(0, 0, 1)$ if $x = x_p$ For each $x_p \in X$ is said to be the support of $x_{a,b,c}$, where a denotes the degree of membership value, b denotes the degree of indeterminacy and c is the degree of non-membership value of $x_{a,b,c}$.

Definition 2.2. For a Pythagorean neutrosophic set A in a Pythagorean neutrosophic topological space (X, τ) is said to be Pythagorean neutrosophic b-open set (or simply, P N bOS) if $A \subseteq PNInt((PNCl(A)) \cup PNCl(PNInt(A))$. The complement of a Pythagorean neutrosophic b-open set is called Pythagorean neutrosophic b-closed set.

Remark 1. The collection of all Pythagorean neutrosophic b-open sets and Pythagorean neutrosophic b-closed sets are denoted by P N bOS(X, τ) and P N bCS(X, τ), respectively.

Proposition 2.3. Let (X, τ) be a Pythagorean neutrosophic topological space and $A \subseteq X$. Then, If A is a Pythagorean neutrosophic α -open set, then A is Pythagorean neutrosophic b-open set. Proof: Let A be a Pythagorean neutrosophic α -open, then by the Definition 1, $A \subseteq PNInt(PNCl(PNInt(A)))$, since $Int(A) \subseteq A$, this implies that $A \subseteq PNInt(PNCl(A)) \cup PNCl(PNInt(A))$. Therefore, A is Pythagorean neutrosophic b-open.

Definition 2.4. A Pythagorean neutrosophic set V in a Pythagorean neutrosophic topological space (X, τ) is said to be Pythagorean neutrosophic bclosed (or simply, P N bCS)) if $V \subseteq PNInt(PNCl(V)) \cup PNCl(PNInt(V))$.

Definition 2.5. Let (X, τ) be a Pythagorean neutrosophic topological space and V be a Pythagorean neutrosophic set on X. Then we define the Pythagorean neutrosophic b-interior and Pythagorean neutrosophic bclosure of V as:

(1) Pythagorean neutrosophic b-interior of V (or simply, P NBINT(V)) as the union of all Pythagorean neutrosophic b-open sets of X contained in V . It means that $PNBINT(V) = S\{A : A$ is a P N bOS in X and $A \subseteq V\}$.

(2) Pythagorean neutrosophic b-closure of V (or simply, P NBCL(V)) as the intersection of all Pythagorean neutrosophic b-closed set of X containing V . It means that $PNBCL(V) = T\{B : B$ is a P N bCS in X and $V \subseteq B\}$.

Remark 2. By the Definition 2, we can see that P NBCL(V) is the smallest Pythagorean neutrosophic b-closed set of X which contains V . Besides, P NBINT(V) is the largest Pythagorean neutrosophic b-open set of X which is contained in V .

Proposition 2.6. Let V be a Pythagorean neutrosophic set in a Pythagorean neutrosophic topological space (X, τ). Then, the following statements hold:

(1) If V is Pythagorean neutrosophic b-open set, then Cl(V) is is a Pythagorean neutrosophic b-closed set.

(2) If V is Pythagorean neutrosophic b-closed set, then Cl(V) is is a Pythagorean neutrosophic b-open set. Proof: The proof is followed by the Definitions 2, 2 and 2.

Theorem 2.7. Let V be a Pythagorean neutrosophic set in a Pythagorean neutrosophic topological space (X, τ). Then, the following statements hold:

(1) Cl(P NBINT(V)) = P NBCL(Cl(V)).

(2) Cl(P NBCL(V)) = P NBINT(Cl(V)).

Proof: We begin proving (1): Let V be a Pythagorean neutrosophic set. Now, by the Definition 2 part (1), P NBINT(V) = $S\{A : A$ is a P N bOS in X and $A \subseteq V\}$, this implies that Cl(P NBINT(V)) = Cl(S$\{A : A$ is a P N bOS in X and A$\subseteq V\}$)= $T\{Cl(A) : Cl(A)$ is a P N bCS in X and $Cl(V) \subseteq Cl(A)\}$. Now, we will replace Cl(A) by B, then we have that $Cl(PNBINT(V) = T\{B : B$is a P N bCS in X and Cl(V)$\subseteq B\}$, and so Cl(P NBINT(V)) = P NBCL(Cl(V)). The proof of (2) is similar to (1).

Theorem 2.8. For a Pythagorean neutrosophic topological space (X, τ) and $A, B \subseteq X$. The following statements hold:

(1) Every Pythagorean neutrosophic set is Pythagorean neutrosophic b-open set.

(2) P NBINT(P NBINT(A)) = P NBINT(A).

(3) P NBCL(P NBCL((A)) = P NBCL(A).

(4) Let A, B be two Pythagorean neutrosophic b-open sets, then $PNbOS(A) \cup PNbOS(B) = PNbOS(A \cup B)$.

(5) Let A, B be two Pythagorean neutrosophic b-closed sets, then P N $bCS(A) \cap PNbCS(B) = PNbCS(A \cap B)$.

(6) For any two sets A, B, P$NBINT(A) \cap PNBINT(B) = PNBInt(A \cap B)$.

(7) For any two sets $A, B, PNBCL(A) \cup PNBCL(B) = PNBCL(A \cup B)$.

(8) If A is $PNbOS(X, \tau), then A = PNBINT(A)$.

(9) If $A \subseteq B$, then $PNBINT(A) \subseteq PNBINT(B)$.

(10) For any two sets A, B, P $NBINT(A) \cup PNBINT(B) \subseteq PNBINT(A \cup B)$.

(11) If A is P N $bCS(X, \tau), then A = PNBCL(A)$.

(12) If $A \subseteq B$, then $PNBCL(A) \subseteq PNBCL(B)$.

(13) For any two sets A, B, $PNBCL(A \cap B) \subseteq PNBCL(A) \cap PNBCL(B)$.

Proof: The proofs of (1), (2), (3), (4), (5), (9), (11) and (12) are followed by the Definitions 2 and 2. The proofs of (6), (7) and (8) are followed by the Definition 2 and the proofs of (10) and (13) are followed by the Definition 2 and parts (9) and (12) of this Theorem. The following example shows that the intersection of two Pythagorean neutrosophic b-open sets need not be a Pythagorean neutrosophic b-open set.

Example 1. Let $X = q, w$, $A = h(0.1, 0.3, 0.5), (0.3, 0.5, 0.7)i$, $B = h(0.1, 0.1, 0.4)$, $(0.7, 0.5, 0.3)i, C = h(0.4, 0.6, 0.9), (0.6, 03, 0.3)i$ and D = h(0.3, 0.5, 0.3),(0.9, 0.5, 0.9)i. Then, τ is a Pythagorean neutrosophic topological space. Now, choose A1 = h(0.3, 0.5, 0.3),(1.0, 0.1, 0.1)i and A2 = h(1.0, 1.0, 0.4),(0.9, 0.4, 0.6)i. We can see that $A1 \cap A2$ is not a Pythagorean neutrosophic b-open set of (X, τ). The following example shows that the union of two Pythagorean neutrosophic b-closed sets need not be a Pythagorean neutrosophic b-closed set. Example 2. By the example 2, we can imply that $Ac1 \cup Ac2$ is not a Pythagorean neutrosophic b-closed set of (X, τ).

Proposition 2.9. Let A be a Pythagorean neutrosophic set in Pythagorean neutrosophic topological space (X, τ). If B is a Pythagorean neutrosophic b-open set and $B \subseteq A \subseteq PNInt(PNCl(A)) \cup PNCl(PNInt(A))$, then A is a Pythagorean neutrosophic b-open set.

Proof: Let B be a Pythagorean neutrosophic b-open set, then by the Definition 2, $B \subseteq PNInt(PNCl(B)) \cup PNCl(PNInt(B))$, and so $B \subseteq A \subseteq PNInt(PNCl(B)) \cup$ P NCl(P NInt(B))⊆ P NInt(P NCl(A))∪ P NCl(P NInt(A)). In consequence, A is a Pythagorean neutrosophic b-open set Theorem 2.10. Arbitrary union of Pythagorean neutrosophic b-open sets is a Pythagorean neutrosophic b-open set. Proof: Let A1, A2, ...An be a collection of Pythagorean neutrosophic bopen sets, then by the Definition 2, A1⊆ P NInt(P NCl(A1))$\cup PNCl(PNInt(A1))$, $A2 \subseteq PNInt(PNCl(A2)) \cup PNCl(PNInt(A2))$, ..., $An \subseteq$ P NInt(P NCl(An))∪ P NCl(P NInt(An)). Now, A1 $\cup A2 \cup ... \cup An \subseteq$(P NInt(P NCl(A1)) ∪ P NCl(P NInt(A1))) ∪ (P NInt(P NCl(A2)) ∪ P NCl(P NInt(A2))) ∪...∪ (P NInt(P NCl(An))∪ P NCl(P NInt(AN))), by the Theorem 2.2 parts (7) and (10), A1 $\cup A2 \cup ... \cup An \subseteq$ P NInt(P NCl(A1 $\cup A2 \cup ... \cup An)) \cup P$ NCl(P NInt(A1 $\cup A2\cup$...∪ An)). This proofs that A1 $\cup A2\cup$...∪ An is a Pythagorean neutrosophic b-open set. Remark 3. By the Example 2, the arbitrary intersection of Pythagorean neotrosophic b-open sets need not be a Pythagorean neotrosophic b-open set.

Proposition 2.11. Arbitrary intersection of Pythagorean neutrosophic bclosed sets is a Pythagorean neutrosophic b-closed set. Proof: The proof is followed by the Theorem 2.3 and parts (6) and (13) of the Theorem 2.2. Remark 4. By the Example 2, the arbitrary union of Pythagorean neotrosophic b-closed sets need not be a Pythagorean neotrosophic b-closed set.

Theorem 2.12. A Pythagorean neutrosophic set A in a Pythagorean neutrosophic topological space (X, τ) is Pythagorean neutrosophic b-open if and only for every Pythagorean neutrosophic point $x_{a,b,c} \in$ A there exits a Pythagorean neutrosophic b-open $Bx_{a,b,c}$ such that $x_{a,b,c} \in Bx_{a,b,c} \subseteq$A.

Proof: Necessary: Let A be a Pythagorean neutrosophic b-open set. Then, we have that $Bx_{a,b,c} =$ A for each $x_{a,b,c}$. Sufficiency: Suppose that for every Pythagorean neutrosophic point $x_{a,b,c} \in A$, there exits a neutrosophic b-open set $Bx_{a,b,c}$ such that $x_{a,b,c} \in Bx_{a,b,c} \subseteq A$. Thus, $A = Sx_{a,b,c} : x_{a,b,c} \in A \subseteq$

$Bx_{a,b,c} : x_{a,b,c} \in A \subseteq A$ and then, $A = S\{Bx_{a,b,c} : x_{a,b,c} \in A\}$. Therefore, by the Theorem 2.3, it is a Pythagorean neutrosophic b-open set.

Definition 2.13. Let f : $(X, \tau) \to (Y, \sigma)$ be a function where (X, τ) and (Y, σ) are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic b-open if f(A) is Pythagorean neutrosophic b-open set in Y for every Pythagorean neutrosophic open set A in X.

Proposition 2.14. Let f : $(X, \tau) \to (Y, \sigma)$ be a function where (X, τ) and (Y, σ) are Pythagorean neutrosophic topological spaces. If f is Pythagorean neutrosophic α -open, then f is Pythagorean neutrosophic b-open. Proof: Let f be a Pythagorean neutrosophic α-open and A be a Pythagorean neutrosophic open set in X. Then, by hypothesis f(A) is a Pythagorean neutrosophic α-open set in Y , by the Proposition 2, f(A) is a Pythagorean neutrosophic b-open set in X. Therefore, f is a Pythagorean neutrosophic b-open function.

3 Pythagorean neutrosophic b-continuous functions

In this section we used the notion of Pythagorean neutrosophic b-open set to introduce and study the concepts of Pythagorean neutrosophic bcontinuous function and Pythagorean neutrosophic b-homeomorphism, as well as, some of their properties are shown.

Definition 3.1. Let $f : (X, \tau) \to (Y, \sigma)$ be a function where (X, τ) and (Y, σ) are Pythagorean neutrosophic topological spaces. Then, f is saidto be Pythagorean neutrosophic b-continuous if $ff1(V)$ is a Pythagorean neutrosophic b-open set in X for every Pythagorean neutrosophic open set V in Y .

Proposition 3.2. Every Pythagorean neutrosophic continuous function is Pythagorean neutrosophic b-continuous function. Proof: The proof is followed by the Definition 1.1 and Proposition 2.

Definition 3.3. Let $x_{a,b,c}$ be a Pythagorean neutrosophic point of a Pythagorean neutrosophic topological space (X, τ). A Pythagorean neutrosophic set D of X is said to be Pythagorean neutrosophic neighbourhood of $x_{a,b,c}$ if there exits a Pythagorean neutrosophic open set V in X such that $x_{a,b,c} \in V \subseteq D$

Proposition 3.4. Let f : $(X, \tau) \to (Y, \sigma)$ be a function where (X, τ) and (Y, σ) are Pythagorean neutrosophic topological spaces. Then, the following statements are equivalent:

(1) f is a Pythagorean neutrosophic b-continuous function.

(2) For each Pythagorean neutrosophic point $x_{a,b,c}$ and every Pythagorean neutrosophic A of $f(x_{a,b,c})$, there exits a Pythagorean neutrosophic b-open set B of X such that $x_{a,b,c} \in B \subseteq f1(A)$.

(3) For each Pythagorean neutrosophic point $x_{a,b,c} \in X$ and every Pythagorean neutrosophic neighbourhood A of $f(x_{a,b,c})$, there exits a Pythagorean neutrosophic b-open set B of X such that $x_{a,b,c} \in Bandf(B) \subseteq A$.

Proof: (1) $\Rightarrow$ (2): Let $x_{a,b,c}$ be a Pythagorean neutrosophic point of X and let A be a Pythagorean neutrosophic neighbourhood of $f(x_{a,b,c})$. Then, there exits a Pythagorean neutrosophic open set B of Y such that $f(x_{a,b,c}) \in B \subseteq A$. Now, since f is a Pythagorean neutrosophic b-continuous function, we have that f 1(B) is a Pythagorean neutrosophic b-open set of X and $x_{a,b,c} \in f1(f(x_{a,b,c})) \subseteq f1(B) \subseteq f1(A)$ and this ends the proof.

(2) $\Rightarrow$ (3): Let $x_{a,b,c}$ be a Pythagorean neutrosophic point of X and let A be a Pythagorean neutrosophic neighbourhood of $f(x_{a,b,c})$. By hypothesis, there exits a Pythagorean neutrosophic b-open set B of X such that $x_{a,b,c} \in B \subseteq f1(A)$ and then $x_{a,b,c} \in B$ of X such that $f(B) \subseteq f(f1(A)) \subseteq A$ and this ends the proof.

(3) $\Rightarrow$ (1): Let B be a Pythagorean neutrosophic open set of Y and let $x_{a,b,c} \in f1(B)$ and so $f(x_{a,b,c}) \in B$ and then B is a Pythagorean neutrosophic neighbourhood of $f(x_{a,b,c})$. Now, since B is a Pythagorean neutrosophic open set and by hypothesis, there exits a Pythagorean neutrosophic b-open set A of X such that $x_{a,b,c} \in A$ and $f(A) \subseteq B$. Indeed, $x_{a,b,c} \in A \subseteq f1(f(A)) \subseteq f1(B)$ and this implies that f 1(B) is aPythagorean neutrosophic b-open set of X. Therefore, f is a Pythagorean neutrosophic b-open continuous function.

Proposition 3.5. Let f : $(X, \tau) \to (Y, \sigma)$ be a function where (X, τ) and (Y, σ) are Pythagorean neutrosophic topological spaces. If f is a Pythagorean neutrosophic α -continuous function, then f is a Pythagorean neutrosophic b-open function.

Proof: The proof is followed by the Definitions 1, 3 and Proposition 2.

Definition 3.6. Let $f : (X, \tau) \to (Y, \sigma)$ be a bijection function where (X, τ) and (Y, σ) are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean nuetrosophic b-homeomorphism if f and f 1 are Pythagorean neutrosophic b-continuous functions.

Example 3. Let $X = \{q, w\}$ and $Y = \{e, r\}$. Then, $\tau = \{0N, U1, U2, 1N\}$ and $\sigma = \{0N, V, 1N\}$ are Pythagorean neutrosophic topological spaces on X and Y respectively, where U1 = hx,(0.2, 0.4, 0.7),(0.4, 0.4, 0.4)i, U2 = hx,(0.3, 0.5, 0.6),(0.5, 0.4, 0.6)i and V = hy,(0.3, 0.5, 0.6),(0.5, 0.2, 0.7)i. Then, we define the function $f : (X, \tau) \to (Y, \sigma)$ as f(q) = e and f(w) = w. We can see that f and f 1 are Pythagorean neutrosophic b-continuous and then f is Pythagorean neutrosophic b-homeomorphism.

Definition 3.7. Let $f : (X, \tau) \to (Y, \sigma)$ be a bijection function where (X, τ) and (Y, σ) are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean nuetrosophic homeomorphism if f and f 1 are Pythagorean neutrosophic continuous functions.

Theorem 3.8. Each Pythagorean neutrosophic homeomorphism is Pythagorean neustrosophic b-homeomorphism.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be a bijection and Pythagorean neutrosophic homeomorphism function in which f and f 1 are Pythagorean neutrosophic continuous functions. Since that every Pythagorean neutrosophic continuous function is Pythagorean neutrosophic b-continuous, this implies that f and f 1 are Pythagorean neutrosophic b-continuous functions. Therefore, f is a Pythagorean neutrosophic b-homeomorphism. Proof: The following example shows that the converse of the above Theorem need not be true. Example 4. Let $X = \{q, w\}$ and $Y = \{e, r\}$. Then, $\tau = \{0N, U1, U2, 1N\}$ and $\sigma = \{0N, V, 1N\}$ are Pythagorean neutrosophic topological spaces on X and Y respectively, where U1 = hx,(0.3, 0.5, 0.8),(0.4, 0.4, 0.4)i, U2 = hx,(0.1, 0.3, 0.8),(0.1, 0.5, 0.8)i and V = hy,(0.4, 0.5, 0.6),(0.1, 0.3, 0.6)i. Then, we define the function f : $(X, \tau) \rightarrow$ (Y, σ) as f(q) = e and f(w) = w. We can see that f is a Pythagorean neutrosophic b-homeomorphism, but it is not a Pythagorean neutrosophic homeomorphism.

Theorem 3.9. Let f : $(X, \tau) \rightarrow (Y, \sigma)$ be a bijection function where (X, τ) and (Y, σ) are Pythagorean neutrosophic topological spaces. Then, the following statements hold:

(1) f is Pythagorean neutrosophic b-closed.

(2) f is Pythagorean neutrosophic b-open.

(3) f is Pythagorean neutrosophic b-homeomorphism.

Proof: (1) ⇒(2) : Let f be a bijection Pythagorean neutrosophic b-closed function. Then, f 1 is Pythagorean neutrosophic b-continuous function. Now, since every Pythagorean neutrosophic open set of (X, τ) is a Pythagorean neutrosophic b-open set of (X, τ), this implies that f is a Pythagorean neutrosophic b-open function.

(2) ⇒ (3) : Let f be a bijective Pythagorean neutrosophic b-open function. Then, f 1 is a Pythagorean neutrosophic b-continuous function. Indeed, f and f 1 are Pythagorean neutrosophic b-continuous functions. Therefore, f is a Pythagorean neutrosophic b-homeomorphism.

(3) ⇒ (1) : Let f be a Pythagorean neutrosophic b-homeomorphism. Then, f and f 1 are Pythagorean neutrosophic b-continuous functions. Since every Pythagorean neutrosophic closed set of (X, τ) is a Pythagorean neutrosophic b-closed set of (X, τ), this implies that f is a Pythagorean neutrosophic b-closed function. The following example shows that the composition of two Pythagorean neutrosophic b-homeomorphisms need not be a Pythagorean neutrosophic b-homeomorphism.

Example 5. Let $X = \{q, w\}, Y = \{e, r\}$ and $Z = \{t, y\}$. Then, $\tau = \{0N, U, 1N\}$, $\sigma = \{0N, V, 1N\}$ and $\omega = \{0N, W, 1N\}$ are Pythagorean neutrosophic topological spaces on X, Y and Z respectively, where U = hx,(0.1, 0.3, 0.5),(0.3, 0.5, 0.7)i, V = hy,(0.2, 0.7, 0.9),(0.3, 0.6, 0.7)i and W = hz,(0.7, 0.5, 0.2),(0.7, 0.7, 0.2)i. We define the function $f : (X, \tau) \rightarrow (Y, \sigma)$ as $f(q) = e$ and $f(w) = r$. Besides, we define the function $g : (Y, \sigma) \rightarrow (Z, \omega)$ as $g(e) = t$ and $g(r) = y$. We can see that f and g are Pythagorean neutrosophic b-homeomorphism, but $g \diamond f$ is not a Pythagorean neutrosophic b-homeomorphism.

Definition 3.10. Let f : $(X, \tau) \rightarrow (Y, \sigma)$ be a function where (X, τ) and (Y, σ) are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic b-irresolute if f 1(V) is a Pythagorean neutrosophic b-open set in X for every Pythagorean neutrosophic b-open set V in Y .

Definition 3.11. Let f : $(X, \tau) \rightarrow (Y, \sigma)$ be a bijection function where (X, τ) and (Y, σ) are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic bi-homeomorphism if f and f 1 are Pythagorean neutrosophic b-irresolute functions.

Theorem 3.12. Every Pythagorean neutrosophic bi-homeomorphism is a Pythagorean neutrosophic b-homeomorphism.

Proof: Let f : $(X, \tau) \rightarrow (Y, \sigma)$ be a bijection and Pythagorean neutrosophic bi-homeomorphism function. Suppose that B is a Pythagorean neutrosophic closed set of (Y, σ), this implies that B is a Pythagorean neutrosophic b-closed set of (Y, σ). Now, since f is Pythagorean neutrosophic irresolute, f 1(B) is a Pythagorean neutrosophic b-closed set of (X, τ) . Indeed, f is a Pythagorean neutrosophic b-continuous function. therefore, f and f 1 are Pythagorean neutrosophi b-continuous functions and then f is Pythagorean neutrosophic b-homeomorphism. The following example shows that the converse of the above Theorem need not be true. Example 6. Let $X = \{q, w\}$ and $Y = \{e, r\}$. Then, $\tau = \{0N, U1, U2, 1N\}$ and $\sigma = \{0N, V, 1N\}$ are Pythagorean neutrosophic topological spaces on X and Y respectively, where U1 = hx,(0.2, 0.4, 0.6),(0.3, 0.3, 0.3)i, U2 = hx,(0.4, 0.7, 0.9),(0.1, 0.1, 0.3)i and V = hy,(0.4, 0.7, 0.9),(0.1, 0.2, 0.3)i. Then, we define the function f : $(X, \tau) \to (Y, \sigma)$ as f(q) = e and f(w) = w. We can see that f is a Pythagorean neutrosophic b-homeomorphism, but it is not a Pythagorean neutrosophic bi-homeomorphism.

Theorem 3.13. If f : $(X, \tau) \to (Y, \sigma)$ and g : $(Y, \sigma) \to (Z, \omega)$ are Pythagorean neutrosophic bi-homeomorphisms, then $g \diamond f$: $(X, \tau) \to (Z, \omega)$ is a Pythagorean neutrosophic bi-homeomorphism.

Proof: Let f and g be two Pythagorean neutrosophic b-homeomorphisms. Now, suppose that B is a Pythagorean neutrosophic b-closed set of (Z, ω), then g 1(B) is a Pythagorean neutrosophic b-closed set of (Y, σ). Then by hypothesis, f 1(g 1(B)) is a Pythagorean neutrosophic b-closed set of (X, τ) . Therefore, g $\diamond$ f is a Pythagorean neutrosophic b-irresolute function Now, let β be a Pythagorean neutrosophic b-closed set of (X, τ) . By assumption, $f(\beta)$ is a Pythagorean neutrosophic b-closed set of (Y, σ). Then, by hypothesis, $g(f(\beta))$ is a Pythagorean neutrosophic b-closed set of (Z, ω). This implies that g $\diamond$ f is a Pythagorean neutrosophic b-irresolute function and then g $\diamond$ f is a Pythagorean neutrosophic bi-homeomorphism.

References

1 Arockiarani, I., Dhavaseelan, R., Jafari, S. and Parimala, M.: On some notations and functions in neutrosophic topological spaces, Neutrosophic sets and Systems. Vol. 16 (2017): 16 19.

2 Atannasov, K.: Intuitionistic fuzzy sets, Fuzzy sets and Systems. Vol. 20(1) (1965): 87 96.

3 Atannasov, K.: Intuitionistic fuzzy sets, Springer Physica-Verlag, Heidelberg. (1999).

4 Banu, V. and Chandrasekar, S.: Neutrosophic α gs Continuity And Neutrosophic α gs Irresolute Maps, Neutrosophic sets and Systems. Vo. 27 (2019): 163 170.

5 Chang, C.: Fuzzy topological spaces,J. Math. Anal Appl. Vol. 24(1) (1968): 182 190.

6 Jansi, R., Mohana, K. and Smarandache, F.: Correlation Measure for Pythagorean Neutrosophic Sets with T and F as Dependent Neutrosophic Components, Neutrosophic sets and Systems. Vol 30 (2019): 202 212.

7 Sneha, T. and Nirmala, F.: Pythagorean neutrosophic b-open and semi-open sets in Pythagorean neutrosophic topological spaces, Infokara Research. Vol. 9(1) (2020): 860 872.

8 Salama, A. and Albowi, S.: Neutrosophic set and Nutrosophic topological spaces, IOSR Jouranl of Mathematics. Vol. 3(4) (2012): 31 35.

9 Smarandache, F.: A unifying field in logics-neotrusophic: Neutrosophic probability, set and logic, Rehoboth: American Research Press. (1999).

10 Xu, Z. and Yagar, R.: Some geometric aggregation operations based on intuitionistic fuzzy sets , Int. J. Gen. Syst. Vol. 35 (2006): 417 433.

11 Yager, R. and Abbasov, A.: Pythagorean membership grades, complex numbers and decision making, Int. J. Intell. Syst. Vol. 28 (2013): 436 452.

12 Zadeh, L.: Fuzzy sets, Inform and control. Vol. 8 (1965).