PYTHAGOREAN NEUTROSOPHIC b-OPEN & semi-OPEN SETS in PYTHAGOREAN NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract: **In this paper we define the notion of Pythagorean neutrosophic b-open sets (resp. b-closed) and Pythagorean neutrosophic semi-open sets (resp. preopen and α -open) . Their properties are investigated.**

Keywords **: PNS-Pythagorean neutrosophic set, PNTS-Pythagorean Neutrosophic Topological space, PNsOS-Pythagorean Neutrosophic semi-open set, PNpOS-Pythagorean Neutrosophic pre open set, PNbOS- Pythagorean Neutrosophic b open set, PNbCS- Pythagorean Neutrosophic b closed set, PNbcl(U)- Pythagorean Neutrosophic b closure of U, PNbint(U)- Pythagorean Neutrosophic b interior of U.**

I. INTRODUCTION AND PRELIMINARIES

1.1 *INTRODUCTION*

The fuzzy set was introduced by Zadeh [16] in 1965. In 1968, Chang [4] defined the concept of fuzzy topological space and generalized some basic notions of topology. Intuitionistic fuzzy set was introduced by Atanassov [2,3] in 1983.Joung Kon Jeon et al.[8] introduced and studied the notions of Intuitionistic fuzzy α -continuity and pre-continuity.The concept of Neutrosophic set was introduced by Smarandache [10] and Wang et al [12] introduced the notion of interval neutrosophic set theory. The concept of crisp set and neutrosophic crisp set topological spaces were introduced by A.A. Salama and S.A. Albowi [9].In 2013, Yager introduced the concept of Pythagorean membership grades in multicriteria decision making [15]. Later, Yager, Zahand and Xu [13] gave some basic operations for Pythagorean fuzzy number.Iswarya et.al.[7] studied the concept of neutrosophic semi-open sets[NSO] and neutrosophic semi-closed sets[NSC].In 2017, Imran et.al.[6] introduced neutrosophic semi-α open sets and studied their fundamental properities. Arockiarani et.al.[1] defined neutrosophic semi-open (resp. pre-open and α -open) functions and investigated their relations. Rao et.al.[11] introduced neutrosophic pre-open sets. P.Evanzalin Ebenanjar, H Jude Immaculate and C Bazil Wilfred [5] defined neutrosophic b-open sets in neutrosophic topological space and investigated their properities.

Through this paper, we introduce the concept of Pythagorean neutrosophic b-open (resp. b-closed), semi-open sets (resp. pre-open, α -open) and their properties are investigated.

1.2 *PRELIMINARIES*

Definition1: [16] A fuzzy set $A=\{\langle x,\mu_A(x)\rangle \mid x\in X\}$ in a universe of discourse X is characterized by a membership function, μ_A, as follows: $\mu_A : X\rightarrow[0,1]$.

Definition 2: [2, 3] Let X be a non empty set. Then A is called an Intuitionistic fuzzy set (in short, IFS) of X, if it is an object having the form $A=\{\langle x,\mu_A,\gamma_A\rangle x\in X\}$ where the function $\mu_A : X\rightarrow[0,1]$ and $\gamma_A : X\rightarrow[0,1]$ denote the degree of membership $\mu_A(x)$ and degree of non membership $\gamma_A(x)$ of each element $x\in X$ to the set A and satisfies the condition that, $0\le\mu_A(x)+\gamma_A(x)\le 1$.

Definition 3: [10] Let X be a non empty set. Then A is called an neutrosophic set (in short,NS) of X, if it is an object having the form $A=\{\langle x,\mu_A,\sigma_A,\gamma_A\rangle : x\in X\}$ where the function $\mu_A : X\rightarrow[0,1], \sigma_A : X\rightarrow[0,1]$ and $\gamma_A : X\rightarrow[0,1]$ denote the degree of membership (namely $\mu_A(x)$),degree of indeterminacy (namely $\sigma_A(x)$) and degree of non membership (namely $\gamma_A(x)$)of each element $x\in X$ to the set A and satisfies the condition that, $0\le\mu_A(x)+\sigma_A(x)+\gamma_A(x)\le 3$.

Definition 4: [14] Let X be a universal set. Then, a Pythagorean fuzzy set A, which is a set of ordered pairs over X, is defined by $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$ where the functions $\mu_A : X\rightarrow[0,1]$ and $\gamma_A : X\rightarrow[0,1]$ define the degree of membership and the degree of nonmembership, respectively, of the element $x\in X$ to A , which is a subset of X, and for every $x\in X : 0\le(\mu_A(x))^2+(\gamma_A(x))^2\le 1$. Supposing $0\le(\mu_A(x))^2+(\gamma_A(x))^2\le 1$, then there is a degree of indeterminacy of $x\in X$ to A defined by $\pi_A(x)=\sqrt{[(\mu_A(x))^2+(\gamma_A(x))^2]}$ and $\pi_A(x)\in[0,1]$

Definition 5: [1] A Neutrosophic set *A* in a neutrosophic topological space *(X, T)* is called

1) A neutrosophic semi open set (briefly NsOS) if $A\subseteq$ *Ncl (Nint(A)).*

2) A Neutrosophic α -open set (briefly Nα OS) if $A\subseteq$ *Nint(Ncl (Nint(A))).*

3) A Neutrosophic preopen set (briefly NpOS) if $A\subseteq$ *Nint(Ncl(A)).*

A Neutrosophic set is called neutrosophic semi closed (resp. neutrosophic α -closed, neutrosophic pre closed) if the complement of A is a neutrosophic semi-open (resp.neutrosophic α -open, neutrosophic preopen).

Definition 6: [5] A NS U in a NTS Z is called

(i)Neutrosophic b-open (NbO) set iff $U \subseteq Nint(N\,cl(U)) \cup Ncl(Nint(U))$

(ii)Neutrosophic b-closed (NbC) set iff $U \supseteq Nint(N\,cl(U)) \cap Ncl(Nint(U))$

It is obvious that $NpO(Z) \cup NsO(Z) \subseteq NbO(Z)$. The inclusion cannot be replaced with equalities.

II. PYTHAGOREAN NEUTROSOPHIC SEMIOPEN, α -OPEN and PREOPEN SETS

Definition2.1

Let X be a non - empty set.If r, t, s be real standard or non standard subsets of $]0^-,1^+[$ then the Pythagorean neutrosophic set $x_{r,t,s}$ is called a Pythagorean neutrosophic point (in short PNP) in X give by

$$x_{r,t,s}(x_p) = \begin{cases} (r,t,s), & \text{if } x = x_p \\ (0,0,1), & \text{if } x \neq x_p \end{cases}$$

for $x_p \in X$ is called the support of $x_{r,t,s}$, where r denotes the degree of membership value, t denotes the degree of indeterminancy and s is the degree of non - membership value of $x_{r,t,s}$.

Definition2.2

A Pythagorean Neutrosophic set A in a Pythagorean neutrosophic topological space (X, T) is called

1) A Pythagorean neutrosophic semi open set (briefly PNsOS) if $A \subseteq PNcl\,(PNint(A))$.

2) A Pythagorean Neutrosophic α -open set (briefly PNα OS) if $A \subseteq PNint(PNcl\,(PNint(A)))$.

3) A Pythagorean neutrosophic preopen set (briefly PNpOS) if $A \subseteq PNint(PNcl(A))$.

A Pythagorean Neutrosophic set is called Pythagorean neutrosophic semi closed (resp. Pythagorean neutrosophic α -closed, Pythagorean neutrosophic pre closed) if the complement of A is a Pythagorean neutrosophic semi-open (resp. Pythagorean neutrosophic α -open, Pythagorean neutrosophic preopen).

Proposition: 2.3

Let (X,T) be a Pythagorean neutrosophic topological space. If A is a Pythagorean neutrosophic α-open set then it is a Pythagorean neutrosophic semi-open set.

Proposition2.4

Let (X,T) be a Pythagorean neutrosophic topological space. If A is a Pythagorean neutrosophic α-open set then it is a Pythagorean neutrosophic pre-open set.

Proposition 2.5

Let A be a Pythagorean neutrosophic set in a Pythagorean neutrosophic topological space (X,T). If B is a Pythagorean neutrosophic semi-open set such that $B \subseteq A \subseteq PN\,\text{int}(PNcl(B))$, Then A is a Pythagorean neutrosophic α-open set.

Proof:

Since B is a Pythagorean Neutrosophic semi-open set, we have $B \subseteq PNcl(PN\,\text{int}(B))$.Thus,

$$A \subseteq PN\,\text{int}(PNcl(B)) \subseteq PN\,\text{int}(PNcl(PNcl(PN\,\text{int}(B)))) = PN\,\text{int}(PNcl(PN\,\text{int}(B)))$$
$$\subseteq PN\,\text{int}(PNcl(PN\,\text{int}(A)))$$

And so A is a Pythagorean neutrosophic α-open set.

Lemma2.6

Any union of PN α-open sets (resp. Pythagorean neutrosophic pre-open sets) is a PN α-open sets. (resp., PNpOS).

Proof:

Let A be a PNαOS and B be another PNαOS. This implies $A \subseteq PN\,\text{int}(PNcl(A)), B \subseteq PN\,\text{int}(PNcl(B))$.Union of these two sets gives, $PN\,\text{int}(PNcl(A)) \cup PN\,\text{int}(PNcl(B)) \Rightarrow PN\,\text{int}(PNcl(A \cup B))$. Thus $A \cup B$ is also PN α-open set.

Proposition2.7

A Pythagorean neutrosophic set A in a Pythagorean neutrosophic topological space X is Pythagorean neutrosophic α-open(resp.Pythagorean neutrosophic pre-open) iff for every Pythagorean neutrosophic point $x_{r,t,s} \in A$ there exists a Pythagorean neutrosophic α-open(resp.Pythagorean neutrosophic pre-open) $B_{x_{r,t,s}}$ such that $x_{r,t,s} \in B_{x_{r,t,s}} \subseteq A$.

Proof:

If A is a Pythagorean neutrosophic α - open set (resp.Pythagorean neutrosophic pre - open set), then we may take $B_{x_{r,t,s}} = A$ for every $x_{r,t,s} \in A$.Conversely assume that for every Pythagorean neutrosophic point $x_{r,t,s} \in A$, there exists a neutrosophic α - open set (resp., Pythagorean neutrosophic pre - open set), $B_{x_{r,t,s}}$ such that $x_{r,t,s} \in B_{x_{r,t,s}} \subseteq A$.Then, $A = \bigcup\{x_{r,t,s} | x_{r,t,s} \in A\} \subseteq \{B_{x_{r,t,s}} | x_{r,t,s} \in A\} \subseteq A$, *and so* $A = \bigcup\{B_{x_{r,t,s}} | x_{r,t,s} \in A\}$, which is a Pythagorean neutrosophic α - open set (resp., Pythagorean neutrosophic pre - open set) by Lemma 2.6

Definition2.8

Let f be a function from a Pythagorean neutrosophic topological spaces (X,τ) and (Y,S). Then *f* is called

(i) a Pythagorean Neutrosophic open function if *f (A)* is a Pythagorean neutrosophic open set in Y for every Pythagorean neutrosophic open set *A* in *X.*

(ii) a Pythagorean neutrosophic α - open function if *f (A)* is a Pythagorean Neutrosophic α -open set in Y for every Pythagorean neutrosophic open set *A* in *X*.

(iii) a Pythagorean neutrosophic preopen function if *f(A)* is a Pythagorean neutrosophic preopen set in Y for every Pythagorean neutrosophic open set *A* in *X*.

(iv) a Pythagorean semi-open function if *f (A)* is Pythagorean semi-open set in *Y* for every Pythagorean neutrosophic open set *A* in *X*.

Proposition2.9

Let (X,T),(Y,S) and (Z,R) be three Pythagorean neutrosophic topological spaces, let $f:(X,T)\to(Y,S)$ and $g:(Y,S)\to(Z,R)$ be functions. If *f* is Pythagorean neutrosophic α -open and *g* is Pythagorean neutrosophic α -open (resp.,Pythagorean neutrosophic preopen), then $g \circ f$ is Pythagorean neutrosophic α -open (resp.Pythagorean Neutrosophic preopen).

Proof: The proof is straightforward.

Proposition:2.10

Let $(X,T)$and(Y,S) are Pythagorean neutrosophic topological spaces. If $f:(X,T)\to(Y,S)$ is Pythagorean neutrosophic α -open then it is Pythagorean neutrosophic semi open.

Proof:

Assume that *f* is Pythagorean neutrosophic α-open and let *A* be a Pythagorean neutrosophic open set in X. Then, *f (A)* is a Pythagorean neutrosophic α-open set in Y. It follows from (prop 2.1) that *f (A)* is a Pythagorean neutrosophic semi open set so that *f* is a Pythagorean neutrosophic semi open function.

III. PYTHAGOREAN NEUTROSOPHIC b-OPEN and b-CLOSED SETS

Definition3.1

A PNS U in a PNTS Z is called

(i)Pythagorean neutrosophic b-open (PNbO) set iff $U \subseteq PNint(PN\,cl(U)) \cup PNcl(PNint(U))$

(ii)Pythagorean neutrosophic b-closed (PNbC) set iff $U \supseteq PNint(PN\,cl(U)) \cap PNcl(PNint(U))$

It is obvious that $PNpO(Z) \cup PNsO(Z) \subseteq PNbO(Z)$. The inclusion cannot be replaced with equalities.

Theorem3.2

For a PNS U in a PNTS Z

(i)U is a PNbO set iff $\overline{U}$ is a PNbC set.

(ii) U is a PNbC set iff $\overline{U}$ is a PNbO set.

Proof: Obvious from the definition.

Definition: 3.3

Let (Z, τ) be a PNTS and U be a PNS over Z.

(i)Pythagorean Neutrosophic b-interior of U briefly *[PNbint(U)]* is the union of all Pythagorean neutrosophic b-open sets of Z contained in U.That is, $PNb\text{int}(U) = \bigcup\{G : G \text{ is a PNbO set in } Z \text{ and } G \subseteq U\}$

(ii) Pythagorean neutrosophic b-closure of U briefly *[PNbcl(U)]* is the intersection of all Pythagorean neutrosophic b-closed sets of Z contained in U. That is, $PNbcl(U) = \bigcap\{H : h \text{ is a PNbC set in } z \text{ and } K \supseteq U\}$.

Clearly *PNbcl(U)* is the smallest neutrosophic b-cloosed set over Z which contains U and *PNbint(U)* is the largest neutrosophic b-open set over Z which is contained in U.

Theorem3.4

Let U be a PNS in a PNTS Z. Then,

$(i)\left(\overline{PNb\operatorname{int}(U)}\right) = PNbcl(\overline{U})$
$(ii)\left(\overline{PNbcl(U)}\right) = PNb\operatorname{int}(\overline{U})$

(i)Let U be PNS in PNTS. Now $PNb\operatorname{int}(U) = \bigcup\{D:D \text{ is a PNbO set in } Z \text{ and } D \subseteq U\}$

Then $\left(\overline{PNb\operatorname{int}(U)}\right) = \left[\overline{\bigcup\{D:D \text{ is a PNbO set in } Z \text{ and } D \subseteq U\}}\right] = \bigcap\{\overline{D}:\overline{D} \text{ is a PNbC set in } Z \text{ and } (\overline{U}) \subseteq \overline{D}\}$. Replacing $\overline{D}$ by M, we get $\left(\overline{PNb\operatorname{int}(U)}\right) = \bigcap\{\overline{D}:\overline{D} \text{ is a PNbC set in } Z \text{ and } (\overline{U}) \subseteq \overline{D}\}$. Replacing $\overline{D}$ by M, we get $\left(\overline{\text{PNbint(U)}}\right) = \bigcap\{M : M \text{ is a PNbC set in Z and } M \supseteq (\overline{U})\}$, $\left(\overline{PNb\operatorname{int}(U)}\right) = PNbcl\left((\overline{U})\right)$. This proves (i). Analogously (ii) can be proved.

Theorem3.5

In a Pythagorean neutrosophic topological space Z

(i)Every Pythagorean neutrosophic pre-open set is a Pythagorean neutrosophic b-open set.

(ii)Every Pythagorean semi-open set is a Pythagorean neutrosophic b-open set.

Proof:

Let U br a PNpO set in a PNTS Z. Then $U \subseteq PN\operatorname{int} PNcl(U)$ which implies $U \subseteq PN\operatorname{int} PNcl(U) \bigcup PN\operatorname{int} U$ $\subseteq PN\operatorname{int} PNclU \bigcup PNclPN\operatorname{int} U$. Thus U is PNbO set.
(ii)Let U be a PNsO set in a PNTS Z. Then $U \subseteq PNclPN\operatorname{int}(U)$ whivh implies $U \subseteq PNclPN\operatorname{int}(U) \bigcup PN\operatorname{int} U$ $\subseteq PNclPN\operatorname{int} U \bigcup PN\operatorname{int} PNclU$. Thus U is A PNbO set.

IV. PROPERTIES OF THE PYTHAGOREAN NEUTROSOPHIC B-INTERIOR AND B-CLOSURE OPERATOR WITH OTHER OPERATORS

Theorem4.1

Let U be a PNS in PNTS Z. Then
$(i) PNsclU = U \bigcup PN\operatorname{int} PNclU \text{ and } PNs\operatorname{int} U = U \bigcap PNclPN\operatorname{int} U$
$(ii) PNpclU = U \bigcup PNclPN\operatorname{int} U \text{ and } PNp\operatorname{int} U = U \bigcap PN\operatorname{int} PNclU$

Proof:

$(i) PNsclU \supseteq PN\text{int}\, PNclPNsclU \supseteq PN\text{int}\, PNclU$

$U \bigcup PNsclU = PNsclU \supseteq U \bigcup PN\text{int}\, PNclU$

$So\ U \bigcup PN\text{int}\, PNclU \subseteq PNsclU$---------(1)

$Also\ U \subseteq PNsclU,\ PN\text{int}\, PNclU \subseteq PN\text{int}\, PNclPNsclU \subseteq PNsclU$

$U \bigcup PN\text{int}\, PNclU \subseteq PNsclU \bigcup U \subseteq PNsclU$-----------(2) **T**

From (1) and (2), $PNsclU = U \bigcup PN\text{int}\, PNclU$

$PN\sin tU = U \bigcap PNclPN\text{int}\, U$ can be proved by taking the complement of $PNsclU = U \bigcup PN\text{int}\, PNclU$

This proves (i)

The proof for (ii) is analogous.

Theorem4.2

Let U be a PNS in PNTS. Then

(i)$PNbclU = PNsclU \bigcap PNpclU$

(ii)$PNbintU = PNsintU \bigcup PNpintU$

Proof:

(i)Since PNbclU is a PNbO set.

We have $PNbclU \supseteq PNintPNcl(PNbclU) \bigcap PNclPNint(PNbclU) \supseteq PNintPNclU \bigcap PNclPNintU$ and also

$PNbclU \supseteq U \bigcup PNintPNclU \bigcap PNclPNintU = PNsclU \bigcap PNpclU$

The reverse inclusion is clear. Therefore $PNbclU = PNsclU \bigcap PNpclU$

Analogously (ii) can be proved.

Theorem4.3

Let U be a PNS in PNTS. Then

$(i) PNsclPNsintU = PN\sin tU \bigcup PN\text{int}\, PNclPN\text{int}\, U$

$(ii) PNsintPNsclU = PNsclU \bigcap PNclPN\text{int}\, PNclU$

Proof:

We have $PNsclPNsintU = PNsintU \bigcup PN\text{int}\, PNcl(PNsintU) = PN\sin tU \bigcup PN\text{int}\, (PNcl[U \bigcap PNclPN\text{int}\, U])$

$\subseteq PNsintU \bigcup PN\text{int}\, [PNclU \bigcap PNcl(PN\text{int}\, U)] = PNsintU \bigcup PN\text{int}\, [PNcl(PN\text{int}\, U)]$

To establish the opposite inclusion we observe that,

$PNscl(PNsintU) = PNsintU \bigcup PN\text{int}\, PNcl(PNsintU) \supseteq PNsintU \bigcup PN\text{int}\, PNcl(PN\text{int}\, U)$

Therefore we have $PNsclPNsintU = PNsintU \bigcup PN\text{int}\, PNclPN\text{int}\, U$.

This proves (i).

The proof for (ii) is analogous.

Theorem4.4

Let U be a PNS in PNTS. Then
$(i) PNpclPNp\text{int} U = PNp\text{int} U \cup PNclPN\text{int} U$
$(ii) PNp\text{int} PNpclU = PNpclU \cap PN\text{int} PNclU$

Proof:

This proves that (i)
Analogously (ii) can be proved.

V. PYTHAGOREAN NEUTROSOPHIC CONTINUITY

Definition 5.1

Let f be a function from a Pythagorean neutrosophic topological space (X,T) to a Pythagorean neutrosophic topological space (Y,S).Then f is called a Pyhtagorean neutrosophic pre - continuous function if $f^{-1}(B)$ is a Pythagorean neutrosophic preopen set in X for every Pythagorean neutrosophic open set B in Y.

Proposition5.2

For a function f from a Pythagorean neutrosophic topological spaces (X,T) to an (Y,S), the following are equivaent
(i) f is Pythagorean neutrosophic pre - continuous.
(ii) $f^{-1}(B)$ is a Pythagorean neutrosophic preclosed set in X for every Pythagorean neutrosophic closed set B in Y.
(iii) $PNcl(PN\text{int}(f^{-1}(A))) \subseteq f^{-1}(PNcl(A))$ for every Pythagorean neutrosophic set A in Y.

Proof:

$(i) \Rightarrow (ii)$ The Proof is straightforward.
(ii) $\Rightarrow (iii)$ Let A be a Pythagorean neutrosophic set in Y.Then $PNcl(A)$ is Pythagorean neutrosophic closed.
It follows from (ii) that $f^{-1}(PNcl(A))$ is a Pythagorean neutrosophic preclosed set in X so that
$PNcl(PN\text{int}(f^{-1}(A))) \subseteq PNcl(PN\text{int}(f^{-1}(PNcl(A)))) \subseteq f^{-1}(PNcl(A))$.
$(iii) \Rightarrow (i)$ Let A be a Pythagorean neutrosophic open set in Y. Then $\overline{A}$ is a Pythagorean neutrosophic closed set in Y, ans so $PNcl(PN\text{int}(f^{-1}(\overline{A}))) \subseteq f^{-1}(PNcl(\overline{A})) = f^{-1}(A)$. This implies that $\overline{PN\text{int}(PNcl(f^{-1}(A)))} = f^{-1}(A)$.
This implies that $\overline{PN\text{int}(PNcl(f^{-1}(A)))} = PNcl\overline{(PNcl(f^{-1}(A)))} = PNcl(PN\text{int}(\overline{f^{-1}(A)}))$
$= PNcl(PN\text{int}(f^{-1}(\overline{A}))) \subseteq f^{-1}(\overline{A}) = \overline{f^{-1}(A)}$, and thus $f^{-1}(A) \subseteq PN\text{int}(PNcl(f^{-1}(A)))$. Hence $f^{-1}(A)$ is a Pythagorean neutrosophic preopen set in X, and f is a Pythagorean neutrosophic precontinuous.

Definition5.3

Let $x_{r,t,s}$ be a Pythagorean neutrosophic point of a Pythagorean neutrosophic topological space (X,T). A Pythagorean neutrosophic set A of X is called Pythagorean neutrosophic neighbourhood of $x_{r,t,s}$ if there exists a Pythagorean neutrosophic open set B in X such that $x_{r,t,s} \in B \subseteq A$.

Proposition5.4

Let f be a function from a Pythagorean neutrosophic topological space (X,T) to a Pythagorean neutrosophic topological space (Y,S).Then the following assertions are equivalent.

(i) f is a Pythagorean neutrosophic pre - continuous function.

(ii)For each Pythagorean neutrosophic point $x_{r,t,s} \in X$ and every Pythagorean neutrosophic neighbourhood A of $f(x_{r,t,s})$,there exists a Pythagorean neutrosophic preopen set B in X such that $x_{r,t,s} \in B \subseteq f^{-1}(A)$.

(iii)For each Pythagorean neutrosophic point $x_{r,t,s} \in X$ and every Pythagorean neutrosophic neighbourhood A of $f(x_{r,t,s})$, there exists a Pythagorean neutrosophic preopen set B in X such that $x_{r,t,s} \in B$ and $f(B) \subseteq A$.

Proof:

$(i) \Rightarrow (ii)$Let $x_{r,t,s}$ be a Pythagorean neutrosophic point in X and let A be a Pythagorean neutrosophic neighbourhood of $f(x_{r,t,s})$.Then there exists a Pythagorean neutrosophic open set B in Y such that $f(x_{r,t,s}) \in B \subseteq A$.Since f is a Pythagorean neutrosophic pre - continuous function, we know that $f^{-1}(B)$ is a Pythagorean neutrosophic preopen set in X and $x_{r,t,s} \in f^{-1}(f(x_{r,t,s})) \subseteq f^{-1}(B) \subseteq f^{-1}(A)$.

Consequently (ii) is valid.

(ii) $\Rightarrow$ (iii) Let $x_{r,t,s}$ be a Pythagorean neutrosophic point in X and let A be a Pythagorean neutrosophic neighbouhood of $f(x_{r,t,s})$.The condition (ii) implies that there exists a Pythagorean neutrosophic preopen set B in X such that $x_{r,t,s} \in B \subseteq f^{-1}(A)$ so that $x_{r,t,s} \in B$ and in X such that $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.

Hence (iii) is true.

(iii) $\Rightarrow$ (i)Let B be a Pythagorean neutrosophic open set in Y and let $x_{r,t,s} \in f^{-1}(B)$.Then $f(x_{r,t,s}) \in B$, and so B is a Pythagorean neutrosophic neighbourhood of $f(x_{r,t,s})$ since B is a Pythagorean neutrosophic open set.It followsfrom(iii) that there exists a Pythagorean neutrosophic pre - opwn set A in X such that $x_{r,t,s} \in A$ and $f(A) \subseteq B$ so that $x_{r,t,s} \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$.Applying Proposition 2.4 induces that $f^{-1}(B)$ is a Pythagorean neutrosophic preopen set in X.Therefore, f is a Pythagorean neutrosophic pre - continuous function.

Definition: 5.5

Let f be a function from a Pythagorean neutrosophic topological space (X, T) to a Pythagorean neutrosophic topological space (Y, S). Then f is called a Pythagorean neutrosophic α - continuous funcion if $f^{-1}(B)$ is a Pythagorean neutrosophic α - open set in X for every Pythagorean neutrosophic open set B in Y.

Proposition5.6

Let f be a function from a Pythagorean neutrosophioc topological space (X,T) to a Pythagorean neutrosophic topological space (Y,S) that satisfies $PNcl(PNint(PNcl(f^{-1}(B)))) \subseteq f^{-1}(PNcl(B))$ for every Pythagorean neutrosophic set B in Y.Then f ia Pythagorean neutrosophic α - continuous function.

Proof:

Let B be a Pythagorean neutrosophic open set in Y. Then $\overline{B}$ is a Pythagorean neutrosophic closed set in Y, which implies that from hypothesis that $PNcl(PNint(PNcl(f^{-1}(\overline{B})))) \subseteq f^{-1}(PNcl(\overline{B})) = f^{-1}(\overline{B})$. It follows that $\overline{PNint(PNcl(PNint(f^{-1}(B))))} = PNcl(\overline{PNcl(PNint(f^{-1}(B)))})$

$$= PNcl(PNint(\overline{PNint(f^{-1}(B))}))$$

$$= PNcl(PNint(PNcl(\overline{f^{-1}(B)})))$$

$$= PNcl(PNint(PNcl(f^{-1}(\overline{B})))) \subseteq f^{-1}(\overline{B})$$

$$= \overline{f^{-1}(B)}$$

so that $f^{-1}(B) \subseteq PNint(PNcl(PNint(f^{-1}(B))))$

This shows that $f^{-1}(B)$ is a Pythagorean neutrosophic α - open set in X.

Hence,f is a Pythagorean neutrosophic α - continuous function.

Proposition5.7

Let f be a function from a Pythgorean neutrosophic topological space (X,T) to a Pythagorean neutrosophic space (Y,S).Then the following assertions are equivalent.

(i)f ia Pythagorean neutrosophic α - continuous.

(ii)For each Pythagorean neutrosophic point $x_{r,t,s} \in X$ and every Pythagorean neutrosophic neighbourhood A of $f(x_{r,t,s})$, there exists a Pythagorean neutrosophic α - open set B in X such that $x_{r,t,s} \in B \subseteq f^{-1}(A)$.

(*iii*)For each Pythagorean neutrosophic point $x_{r,t,s} \in X$ and every Pythagorean neutrosophic neighbourhood A of $f(x_{r,t,s})$, there exists a Pythagorean neutrosophic α - open set B in X such that $x_{r,t,s} \in B$ and $f(B) \subseteq A$.

Proof

$(i) \Rightarrow (ii)$Let $x_{r,t,s}$ be a Pythagorean neutrosophic point in X and let A be a Pythagorean neutrosophic neighbourhood of $f(x_{r,t,s})$.Then there exists a neutrosophic open set B in Y suchb that $f(x_{r,t,s}) \in B \subseteq A$. Since f is Pythagorean neutrosophic α - continuous, we know that $f^{-1}(B)$ is a Pythagorean neutrosophic α - continuous, we know that $f^{-1}(B)$ is a Pythagorean neutrosophic α - open set in X and $x_{r,t,s} \in f^{-1}(f(x_{r,t,s}))$ $\subseteq f^{-1}(B) \subseteq f^{-1}(A)$.Consequently (ii) is valid.

(ii) $\Rightarrow$ (iii)Let $x_{r,t,s}$ be a Pythagorean neutrosophic point in X and let A be a Pythagorean neutrosophic neighbourhoodof $f(x_{r,t,s})$.The condition (ii) implies that there exists a Pythagorean neutrosophic α - open set B in X such that $x_{r,t,s} \in B \subseteq f^{-1}(A)$ so that $x_{r,t,s} \in B$ and $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.Hence (iii) is true.

(iii) $\Rightarrow$ (i)Let B be a Pythagorean neutrosophic open set in Y and let $x_{r,t,s} \in f^{-1}(B)$.Then $f(x_{r,t,s}) \in B$, and so B is a Pythagorean neutrosophic neighbourhood of $f(x_{r,t,s})$ since B is Pythagorean neutrosophic open set. It follows from (iii) that there exists a Pythagorean neutrosophic α - open set A in X such that $x_{r,t,s} \in A$ and $f(A) \subseteq B$ so that $x_{r,t,s} \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$.Applying Proposition 2.4 induces that $f^{-1}(B)$ is a Pythagorean neutrosophic α - contiuous function.

Proposition5.8

Let f be a function from a Pythagorean neutrosophic topological space (X, T) to a Pythagorean neutrosophic topological space (Y, S).If f is Pythagorean neutrosophic α - continuous, then it is Pythagorean neutrosophic semi - continuous.

Proof:

Let B be a Pythagorean neutrosophic openset in Y.Since f is Pythagorean neutrosophic α - continuous, f^{-1}(B) is a Pythagorean neutrosophic semiopen set in X.It follows from Prop 2.1 that f^{-1}(B) is a Pythagorean neutrosophic semiopen set in X so that f is a Pythagorean neutrosophic semi - continuous function.

Proposition5.9

Let f be a function from a Pythagorean neutrosophic toplogical space (X, T) to aPythagorean neutrosophic topological space (Y, S).If f is Pythagorean neutrosophic α - continuous, then it is Pythagorean neutrosophic pre - continuous.

VI.CONCLUSION

In this paper we have introduced Pythagorean b-open (resp.-closed) sets and also semi-open (resp. pre-open, alpha-open) sets. We have discussed Pythagorean Neutrosophic continuity and we also compared the properties of Pythagorean neutrosophic b-interior and b-closure operator with other operators.

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