Recent Neutrosophic Models for KRP Systems

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***Abstract*—Knowledge Representation and Processing (KRP) plays an important role in the development of expert systems as engines for accelerating the processes of economic and social life development. This paper considers neutrosophic models in order to support soft computing modelling and processing of data collected under imprecision hypothesis. Two algebraic computational models are described related to basic operations and ranking. Graph based representations are considered and the basic ideas on obtaind KRP architectures are detailed.**

***Index Terms*—imprecision modelling, fuzzy, intuitionistic fuzzy, neutrosophy, abI - format, TIF - format, KRP systems**

I. Introduction

Dealing with imprecision during knowledge acquisition, representation and processing is one of the most challenging tasks when designing Knowledge Representation and Processing (KRP) systems [19]. From numerical point of view the following approaches were proposed by researchers starting from 1950: interval mathematics based methods [20], fuzzy numbers and their arithmetics [26], intuitionistic fuzzy numbers and the associated algorithms [3], [13] and neutrosophic numbers [23].

According to Moore (1966) [20], in order to work with an uncertain number x, is better to work with an interval $[a, b]$ containing x. Also, given a function f, the value $f(x)$ will belong to an interval $[u, v]$ being also imprecise.

Following [20], if $\diamond$ is one generic operator representing addition, subtraction, multiplication or division of real numbers, that corresponds to the operator in interval arithmetic, denoted by $\diamondsuit$, then the following definition can be used: $[a, b] \diamondsuit [c, d] = [\min(a\diamond c, a\diamond d, b\diamond c, b\diamond d), \max(a\diamond c, a\diamond d, b\diamond c, b\diamond d)]$, if and only if $x \diamond y$ is defined for $x \in [a, b]$, and $y \in [c, d]$.

In the case of division, a rule taking into account the number zero is necessary:

$1/[c, 0] = [-\infty, 1/c]$, and
$1/[0, d] = [1/d, \infty]$.

Combining numbers with intervals is easy because any number μ is the same as interval $[\mu, \mu]$.

Let X be a nonempty set representing the universe of discourse. A fuzzy set A of X is defined by all elements belonging to X according to a membership degree f_A defined for every element of X taking values in $[0, 1]$. In this case, let us denote by A the following set: $A = (x, f_A(x)), x \in X$. This kind of set was introduced by Zadeh [26], who defined the non-membership degree as $1 - f_A(x)$, for every $x \in X$ (see also [27]). When changing the membership degree function, a new fuzzy set is obtained.

If, for every $x \in X$, a non-membership degree function g_A is defined ($g_A : X \to [0, 1]$), with $f_A(x) + g_A(x)$ being less or equal 1), then the Atanassov intuitionistic fuzzy sets theory is applied. Other characteristics are possibly to be introduced [3]–[5], [13]:

- the *degree of indeterminacy* $h_A(x) = 1 - f_A(x) - g_A(x)$,
- the *degree of favor* of x like $m_A(x) = f_A(x)(1+h_A(x))$, and the
- *degree of against* of x like $n_A(x) = g_A(x)(1 + h_A(x))$.

For fuzzy sets, there is no indeterminacy.

Two intuitionistic fuzzy sets of X, denoted by A, and B, are *similarly* if there is $x \in X$ such that $f_A(x) = f_B(x)$, and $g_A(x) = g_B(x)$. If these properties apply for all $x \in X$, then A and B are *equal*, or *comparable*. Also, two intuitionistic fuzzy sets A and B are *equivalent* if and only if a bijection h is identified in order to have:

$$f_B(x) = h(f_A(x)),$$

and

$$g_B(x) = h(g_A(x)),$$

for every $x \in X$. Hence, $f_A(x) = h^{-1}(f_B(x))$ and $g_A(x) = h^{-1}(g_B(x))$.

The concepts of "subset", respectively "proper subset" is defined by the direct order relation between membership degrees and the inverse order relation between non-membership degrees, namely:

1) $A \subseteq B$ if and only if $f_A(x) \leq f_B(x)$, and $g_A(x) \geq g_B(x)$, and
2) $A \subset B$ if and only if $f_A(x) < f_B(x)$, and $g_A(x) > g_B(x)$, respectively.

A new extension to a three-valued representation was considered by Smarandache [23]. If $T_A(x)$ is the degree of membership, $F_A(x)$ is the degree of non-membership, and $I_A(x)$ is the degree of indeterminacy, the neutrosophic representation of a set A consists of three functions (T_A, I_A, F_A), such as $\forall x \in X$, $0 \leq T_A(x), F_A(x), I_A(x) \leq 1$, with ${}^-0 \leq T_A(x) + F_A(x) + I_A(x) \leq 3^+$, where for any real number ${}^-a$, and a^+ are a sets of hyper-real numbers in non-standard analysis [22]:

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- ${}^{-}a = \inf\{a - \epsilon, \epsilon \in R^*, \epsilon \text{ infinitesimal}\}$, and
- $a^+ = \sup\{a + \epsilon, \epsilon \in R^*, \epsilon \text{ infinitesimal}\}$.

Due to the dificulty of the non-standard interval interpretation $]^{-}0, 1^{+}[$ for real engineering and scientific problems, the standard unit interval $[0, 1]$ is used. An element x of the universe of discourse is called *significant* with respect to neutrosophic set A of X if the following degrees are significant: the *degree of truth-membership* or *falsity - membership* or *indeterminacy - membership* value, i.e., $T_A(x)$ or $I_A(x)$ or $F_A(x)$ is greater than 0.5.

An intuitionistic neutrosophic set is defined by

$$\tilde{A} =< x, T_A(x), I_A(x), F_A(x) >,$$

where, $\forall x \in X$,

$$\min\{T_A(x), F_A(x)\} \leq 0.5,$$

$$\min\{T_A(x), I_A(x)\} \leq 0.5,$$

and

$$\min\{F_A(x), I_A(x)\} \leq 0.5,$$

when

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 2.$$

Current computational models for neutrosophic information were described in [1] and [2]. In the following, we are interested in processing neutrosophic data according to algebraic and graph based models.

II. ALGEBRAIC NEUTROSOPHIC MODELS

Representing knowledge by neutrosophic numbers assumes an indeterminacy component to be added. The following models of a neutrosophic number are presented: abI – format and TIF – format.

Formally, a neutrosophic number in abI – format can be written as a + bI, where a (the *determinate part*) and b (the *indeterminate part*) are real or complex numbers, and I is the *indeterminacy operator* with $I^2 = I$, $I+I = 2I$, and $I-I = 0$. [16].

Definition 2.1 *A neutrosophic complex number has the form:* $a+bi+cI+diI$, where a, b, c, and d are reals, $I^2 = I$, and $i^2 = -1$.

Remark 2.1 If n is any positive integer then $I^n = I$. $xI + yI = (x+y)I$, $0I = 0$, and both $1/I$, and I/I are undefined.

Definition 2.2 (*Basic arithmetic operations with abI – neutrosophic numbers*) If $x = a + bI$, and $y = c + dI$ are two abI – neutrosophic numbers, then:

1) $x + y = (a + c) + (b + d)I$,
2) $x - y = (a - c) + (b - d)I$,
3) $\alpha x = \alpha a + \alpha bI$ (with α be real or complex number),
4) $xy = ac + (ad + bc + bd)I$,
5) $x/y = u + vI$, when defined, with $u = \dfrac{a}{c}$, and $v = \dfrac{bc - ad}{c(c+d)}$.

Remark 2.2 $(a + bI)^2 = a^2 + (2ab + b^2)I$.

Remark 2.3 If $a \geq 0$, and $a+b \geq 0$, then $\sqrt{a + bI} = u+vI$, where $u + vI \in \{s_1, s_2, s_3, s_4\}$, such as:

- $s_1 = (\sqrt{a}, -\sqrt{a} + \sqrt{a+b})$,
- $s_2 = (\sqrt{a}, -\sqrt{a} - \sqrt{a+b})$,
- $s_3 = (-\sqrt{a}, \sqrt{a} + \sqrt{a+b})$,
- $s_4 = (-\sqrt{a}, \sqrt{a} - \sqrt{a+b})$.

The TIF – format is used for single value neutrosophic numbers. If x is a TIF – neutrosophic number [25], then $x = (t, i, f)$, where t, i, f are real numbers belonging to $[0, 1]$. According to [25], the ”TIF – Neutrosophic Structures” (based on the components T = *truth*, I = *indeterminacy*, F = *falsehood*) are different from the ”Neutrosophic Algebraic Structures” (based on neutrosophic numbers of the form $a+bI$, as given by Definition 2.1 and Definition 2.2).

Definition 2.3 (*Basic arithmetic operations with TIF – neutrosophic numbers*) Let $A = (t_1, i_1, f_1)$ and $B = (t_2, i_2, f_2)$ be two single-valued neutrosophic numbers, where t_1, i_1, f_1, t_2, i_2, $f_2 \in [0, 1]$, with $0 \leq t_1 + i_1 + f_1 \leq 3$ and $0 \leq t_2 + i_2 + f_2 \leq 3$. Using $''\vee''$ as the ”union/or” fuzzy operator, and $''\wedge''$ as the ”intersection/and” fuzzy operator [27], the basic arithmetic operations can be defined by:

1) Neutrosophic addition:

$$A \oplus_1 B = (t_1 \vee t_2, i_1 \wedge i_2, f_1 \wedge f_2),$$

$$A \oplus_2 B = (t_1 \vee t_2, i_1 \vee i_2, f_1 \vee f_2);$$

2) Neutrosophic multiplication:

$$A \otimes_1 B = (t_1 \wedge t_2, i_1 \vee i_2, f_1 \vee f_2),$$

$$A \otimes_2 B = (t_1 \wedge t_2, i_1 \wedge i_2, f_1 \vee f_2).$$

Many researchers (see [25]) already defined neutrosophic scalar multiplication, neutrosophic power, neutrosophic subtraction, and neutrosophic division when TIF – neutrosophic numbers are used for computing under imprecision.

Definition 2.4 (*Neutrosophic scalar multiplication* and *Neutrosophic power*) If A is a TIF – neutrosophic number, $A = (t, i, f)$, and λ is a positive real number, then

$$\lambda A = (1 - (1 - t)^\lambda, i^\lambda, f^\lambda),$$

and

$$A^\lambda = (t^\lambda, 1 - (1 - i)^\lambda, 1 - (1 - f)^\lambda).$$

Proposition 2.1 (*TIF – Neutrosophic subtraction*) Let $A = (t_1, i_1, f_1)$ and $B = (t_2, i_2, f_2)$ be two single-valued neutrosophic numbers, where t_1, i_1, f_1, t_2, i_2, $f_2 \in [0, 1]$, with $0 \leq t_1 + i_1 + f_1 \leq 3$ and $0 \leq t_2 + i_2 + f_2 \leq 3$. If $t_2 \neq 1$, $i_2 \neq 0$, and $f_2 \neq 0$, then [25]:

$$A \ominus B = (t_3, i_3, f_3),$$

where

$$t_3 = \frac{t_1 - t_2}{1 - t_2}, i_3 = \frac{i_1}{i_2}, f_3 = \frac{f_1}{f_2}.$$

Proof. It is easy to verify that $(t_3, i_3, f_3) \oplus (t_2, i_2, f_2) = (t_1, i_1, f_1)$, according to the rule: $(t_1, i_1, f_1) \oplus (t_2, i_2, f_2) = (t_1 + t_2 - t_1t_2, i_1i_2, f_1f_2)$.

Remark 2.4 The $\ominus$ operation is the opposite of the $\oplus$ operation, when specific constraints apply.

Proposition 2.2 (*TIF – Neutrosophic division*) Let $A = (t_1, i_1, f_1)$ and $B = (t_2, i_2, f_2)$ be two single-valued neutrosophic numbers, where $t_1,\ i_1,\ f_1,\ t_2,\ i_2,\ f_2 \in [0,1]$, with $0 \leq t_1 + i_1 + f_1 \leq 3$ and $0 \leq t_2 + i_2 + f_2 \leq 3$. If $t_2 \neq 0$, $i_2 \neq 1$, and $f_2 \neq 1$, then [25]:

$$A \oslash B = (t_3, i_3, f_3),$$

where

$$t_3 = \frac{t_1}{t_2}, i_3 = \frac{i_1 - i_2}{1 - i_2}, f_3 = \frac{f_1 - f_2}{1 - f_2}.$$

Proof. It is easy to verify that $(t_3, i_3, f_3) \otimes (t_2, i_2, f_2) = (t_1, i_1, f_1)$, according to the rule: $(t_1, i_1, f_1) \otimes (t_2, i_2, f_2) = (t_1 t_2, i_1 + i_2 - i_1 i_2, f_1 + f_2 - f_1 f_2)$.

Remark 2.5 The $\oslash$ operation is the opposite of the $\otimes$ operation, when specific constraints apply.

Another format uses mathematical functions to describe the three-degrees functions of the neutrosophic set associated to the number. Usually, triangular, or trapezoidal functions are used to define every component of the TIF representation [10].

Definition 2.5 (*Ranking abI – real neutrosophic numbers*)

1) $a + bI \equiv c + dI$ if and only if $a = c$ and $b = d$.
2) If $a < c$ then $a + bI \preceq c + dI$.
3) If $a = c$ and $b \leq d$ then $a + bI \preceq c + dI$.

Proposition 2.3. The relation $\preceq$ introduces a partial order on the real neutrosophic numbers set.

Definition 2.6 (*Ranking TIF – neutrosophic numbers*) Let $x = (t_1, i_1, f_1)$ and $y = (t_2, i_2, f_2)$ be be two single-valued neutrosophic numbers, where $t_1,\ i_1,\ f_1,\ t_2,\ i_2,\ f_2 \in [0,1]$, with $0 \leq t_1 + i_1 + f_1 \leq 3$ and $0 \leq t_2 + i_2 + f_2 \leq 3$. $x \lhd y$ if one of the following statements are true.

1) $t_1 < t_2$.
2) $t_1 = t_2$ and $f_2 > f_1$.
3) $t_1 = t_2$, $f_2 = f_1$ and $i_2 < i_1$.

$x \equiv y$ if and only if $t_1 = t_2$, $i_1 = i_2$ and $f_1 = f_2$.

Proposition 2.4. The relation $\lhd$ introduces a partial order on the real neutrosophic numbers set.

The above computational models make possible the usage of neutrosophic numbers in real world applications where indeterminacy can be accepted and a more realistic interpretation of results is obtained. Following techniques from [15] it is easy to show that abI format can be useful to derive many properties of some linear algebra neutrosophic models like matrices, determinants etc.

III. Graph based neutrosophic models

The basic terminology for knowledge representation and processing by graphs is based on [7]–[9], [21]. Neutrosophic graphs was introduced firstly in [16]. Graphs have vertices (nodes) which are connected by arcs (oriented graphs) or edges (non oriented graphs). A special class of graphs used in KRP development is represented by conceptual graphs [6].

Firstly, let us consider abI – graphs: every connection is marked by an abI – neutrosophic number. Considering the neutrosophic matrix

$$M_N = \begin{pmatrix} 0 & 3 & 0 & 2I & 0 \\ 3+I & 0 & 0 & 7+2I & 0 \\ 1 & 0 & 0 & 2+I & 4I \\ 0 & 7 & 2 & 0 & 5 \\ 0 & 7 & 0 & 5+I & 0 \end{pmatrix}$$

as providing adjacency information on a graph having five vertices $\{A, B, C, D, E\}$, it is easy to find well determined connections (AB, CA, DB, DC, DE, EB), pure indeterminate connections (AD, CE), and imprecise weighted connections (BA, BD, CD, ED). However, this matrix can be viewed in an interval-based representation framework, like:

$$M_I = \begin{pmatrix} [0,0] & [3,3] & [0,0] & [0,2] & [0,0] \\ [3,4] & [0,0] & [0,0] & [7,9] & [0,0] \\ [1,1] & [0,0] & [0,0] & [2,3] & [0,4] \\ [0,0] & [7,7] & [2,2] & [0,0] & [5,5] \\ [0,0] & [7,7] & [0,0] & [5,6] & [0,0] \end{pmatrix}.$$

Definition 3.1 (Neutrosophic walking) A neutrosophic walk in the graph $G = (V, E)$ is a finite sequence of vertices and edges as in the statement: from v_{i_0} by e_{j_1} to v_{i_1} by e_{j_2} to ... to $v_{i_{k-1}}$ by e_{j_k} to v_{i_k}. The length of the walk is k.

Remark 3.1 If $N(e_{j_s})$ is the neutrosophic weight of the edge e_{j_s}, $s = 1, 2, \cdots, k$, then the distance from the initial vertex v_{i_0} to the terminal vertex v_{i_k} is the sum of the neutrosphic weights:

$$d_k = \sum_{s=1}^{k} N(e_{j_s}).$$

Example 3.1 Let one consider the walks $w_1 = (ABDE)$, $w_2 = (ABDCE)$, and $w_3 = (EBDCA)$, then

1) $d_{w_1} = 3 + (7 + 2I) + 5 = 15 + 2I \in [15, 17]$,
2) $d_{w_2} = 3 + (7 + 2I) + 2 + (4I) = 12 + 6I \in [12, 18]$,
3) $d_{w_3} = 7 + (7 + 2I) + 2 + 1 = 17 + 2I \in [17, 19]$.

However, even, formally, the second walk was considered, as an alternative to the first walk, the connection (CE) is pure indeterminate, which not guarantee the possibility of the walk. For real life applications, a possible walk cover only edges having weights $a + bI$, with $a \neq 0$.

Remark 3.2 Following the procedure of ranking abI - numbers, and one algorithm to find minimum distance walk, the corresponding version for abI-graphs is obtained.

Let us consider TIF – graphs: every connection is marked by a TIF – single value neutrosophic number. Considering TIF matrix obtained from M_N by converting the existence of neutrosophic weights in TIF format:

$$M_{TIF} = \begin{pmatrix} (0,0,1) & (1,0,0) & (0,0,1) & (0,1,0) & (0,0,1) \\ (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) & (0,0,1) \\ (1,0,0) & (0,0,1) & (0,0,1) & (1,1,0) & (0,1,0) \\ (0,0,1) & (1,0,0) & (1,0,0) & (0,0,1) & (1,0,0) \\ (0,0,1) & (1,0,0) & (0,0,1) & (1,1,0) & (0,0,1) \end{pmatrix}$$

Remark 3.3 Using the computational rules based on definition 2.3, the existence of walks of any length can be study by a variant of Roy-Warshal algorithm.

IV. Improving KRP systems

In this section we are interesed in KRP systems that support algebraic and graph-based algorithms. Firstly, let us remind the basic structure of KRP systems using fuzzy, intuitionistic fuzzy and neutrosophic computational models [19].

Both fuzzy numbers and intuitionistic fuzzy numbers can be used in the context of KRP systems to support imprecision. The most used categories of IF-numbers are represented by triangular, respective trapezoidal intuitionistic fuzzy numbers. As presented above, intuitionistic fuzzy representation is a particular case of neutrosophic representation with the sum of all three components equal to unity.

Therefore, one Intuitionistic Fuzzy KRP system is composed by:

- Input module (crisp data),
- Intuitionistic Fuzzification Unit (IFU) oriented to IF-processing,
- Intuitionistic Fuzzy Knowledge Base (IF-rules),
- Intuitionistic Fuzzy Inference Engine (intuitionistic fuzzy logic),
- Intuitionistic Defuzzification Unit (IDU), responsible for converting IF to crisp,
- Output module (processing crisp data to visualize the results).

Based on [27], the following intuitionistic fuzzy functions can be used to convert crisp value or to represent IF-numbers or IF-linguistic variables by the IFU module: Triangular (_/_), Trapezoidal ($/^{--}\backslash$), Left-shape ($/^{-}$), Right-shape ($^{-}\backslash$), Sigmoid ($-/^{-}$), S-shape ($\_/^{-}$), and Gaussian functions ($\_/^{\frown}\backslash\_$). These models are the most suitable for a large class of real world applications.

Example 4.1 Let a be a number to be modelled as an IF entity by a gaussian function. Then f_a (resp. g_a) function can be given by

$$f_a(x) = \exp\left(-\frac{(x-a)^2)}{\delta}\right) - \alpha$$

$$(\text{resp. } g_a(x) = 1 - \exp\left(-\frac{(x-a)^2)}{\delta}\right)),$$

where α is the degree of indeterminacy, and δ is a dispersion parameter.

Example 4.2 Let m and λ be two positive values (indicating a location parameter, and a steepness factor). The sigmoid intuitionistic fuzzy model is given by:

$$f_{m,\lambda}(x) = \frac{1}{1+exp\,(-\lambda(x-m))} - \alpha$$

$$(\text{resp. } g_{m,\lambda}(x) = 1 - \frac{1}{1+exp(-\lambda(x-m))}),$$

where α is also the degree of indeterminacy.

The IDU module can be implemented in two steps:

1) Obtain an indicator function χ_A, based on f_A, g_A, and two scalar values α and β, describing the importance of the two components:

$$\chi_A(x) = \alpha f_A(x) + \beta(1 - g_A(x)) \in [0, 1].$$

2) Use any defuzzyfication method [27] to identify a crisp value.

Neutrosophication is recommended to be used for particular applications when fuzzy and intuitionistic fuzzy techniques, used to establish the strength of causal relationships among states, goals, events and orther concepts used in systems modelling, need improvements to cover indeterminacy aspects. In the following, a special adjancency matrix is used for particular graphs: the neutrosophic cognitive maps and neutrosophic relational maps which are generalizations of fuzzy cognitive maps and respectively fuzzy relational maps [11], [12].

A Neutrosophic Cognitive Map (NCM) represents the causal relationship between concepts using weighted arcs described by neutrosophic numbers. According to [14], as a particular case, the corresponding neutrosophic adjacency matrix related to a neutrosophic cognitive map has values belonging to the set {0, 1, -1, I}, where 0, 1, respective I are used to describe a missing connection, a direct proportionally connection, respective an indeterminate conection, and -1 describes an inverse proportionally connection. abI and TIF formats can also be used when describing the degrees of influence in NCMs.

In a particular neutrosophic KRP based on NCMs, the weights are assigned by experts. However, in time, the dynamic evolution of system under consideration may ask for updating the weights, or the value associated to some concepts according to some transitional formulas, including feedback factors. In this case the computational framework presented above is useful to produce the new values/weights.

Let consider that at the moment of time t, the concept C_i is characterized by a neutrosophic number $V_i^{(t)}$, and is influenced by m concepts, denoted by C_{j_1}, C_{j_2}, $\cdots$, C_{j_m}, with values $V_{j_1}^{(t)}$, $V_{j_2}^{(t)}$, $\cdots$, $V_{j_m}^{(t)}$, and the weights W_{i,j_1}, W_{i,j_2}, $\cdots$, W_{i,j_m}. The updated value of $V_i^{(t+1)}$ can be obtained by a mechanism such as: $V_i^{(t+1)} = f(\sum_{k=1}^{m} V_{j_k}^{(t)} W_{i,j_k} + V_i^{(t)})$, where f is a threshold function to be defined according to some requirements based on the problem to be solved.

Taking into account, both abI and TIF formats for representing neutrosophic numbers, the framework of neutrosophic KRP systems should consider the following components:

- Input module (crisp data),
- Neutrosophication Unit (to convert crisp data to neutrosophic representation: abI or TIF neutrosophic numbers),
- Neutrosophic Knowledge Base (facts/rules and their associated degrees),
- Neutrosophic Inference Engine (based on neutrosophic logic),
- Deneutrosophication Unit (DU) used to convert from neutrosophic representation to crisp data),
- Output module (crisp data visualization).

Moreover, the neutrosophication unit preprocesses crisp input data to identify valid cases, invalid cases, ambiguous cases. Every item in the knowledge base is described by three components to be used for inferential operations on the content.

Example 4.3 Let us consider the TIF representation for a set of objects U related to a property P, when $(T_P(u), I_P(u), F_P(u)), \forall u \in U$ are obtained by a preliminary research. One function $f(u)$ (where $f \in \{T, I, F\}$) can be given as single value or by a linguistic variable to be modelled by some fuzzy, intutionistic fuzzy or neutrosophic numbers. The neutrosophication unit for crisp numbers will associate to the crisp value a, the TIF-representation $(1, 0, 0)$. However, the linguistic expression "greater than a" will be encoded by neutrosophic functions associated to the components T, I and F, following the relevant aspects/rules obtained by experts in the field of the application to be implemented.

Once identified neutrosophic operators for logical connectors, the neutrosophic rule base can be used as in classical framework of expert systems implemented by logic programming or production rules.

The Neutrosophic Inference Engine (NIE) works like any Logical Inference Engine, but using specific operators according to the framework under consideration. The NIE component is an extension of all already known inference engines.

The Deneutrosophication Unit is responsible with filtering membership/validity information in order to provide a center of gravity, or a particular mean of data.

Following the above presentation, the DU module can be implemented, also, in two steps:

1) Obtain an indicator function χ_A, based on T_A, I_A, F_A and three scalar values α, β, and γ describing the importance of the three components:
$$\chi_A(x) = \alpha T_A(x) + \beta I_A(x) + \gamma(1 - F_A(x)) \in [0, 1].$$
2) Use any defuzzyfication method [27] to identify a crisp value.

Example 4.4 Let A be a neutrosophic set given by:

$$T_A(x) = \exp\left(-\frac{(x-2)^2)}{2}\right) - 0.01,$$

$$F_A(x) = 1 - \exp\left(-\frac{(x-2)^2)}{2}\right),$$

and

$$I_A(x) = \exp\left(-(x-2.05)^2\right),$$

having the shapes in figures 1 to 3. In Fig. 4, it is shown that in a neutrosophic framework the sum of the three degrees (membership, non-membership, indeterminate) can be over 1.00. Let $\alpha = 0.7$, $\beta = 0.1$, and $\gamma = 0.2$, corresponding to the main importance of membership (degree of truth). The indicator function is given in Fig. 5.

The final result can be given by cutting strategies to obtain an interval, or by center of area or center of gravity to obtain olny a crisp value [27].

Therefore, implementing a neutrosophic framework for KRP systems asks for various techniques, many of them belonging to interval based computing, and fuzzy / intuitionistic fuzzy inference engine development.

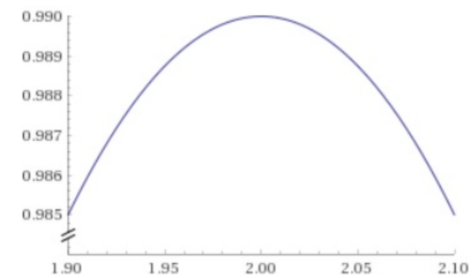

Fig. 1. The membership function T_A.

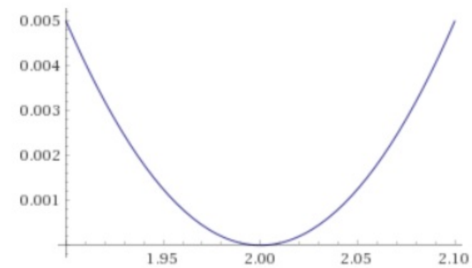

Fig. 2. The non-membership function F_A.

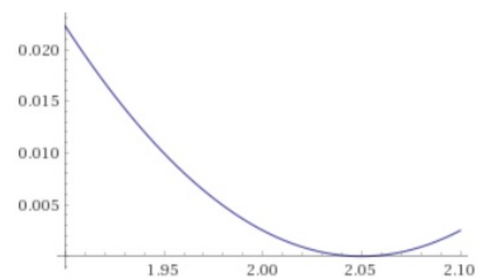

Fig. 3. The indeterminate function I_A.

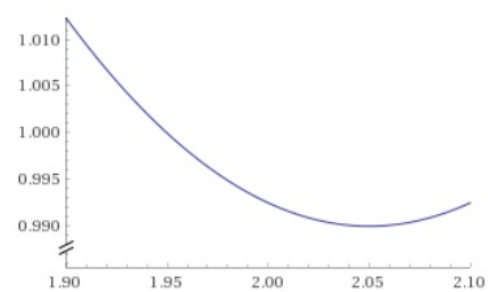

Fig. 4. The function $T_A + F_A + I_A$ having also value over 1.00.

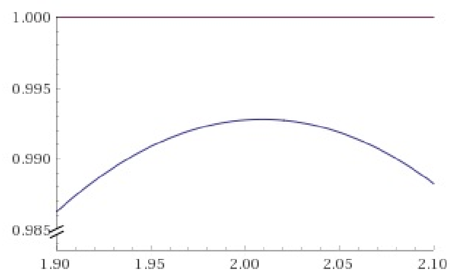

Fig. 5. The function χ_A.

V. Conclusions

This paper had presented some recent neutrosophic models suitable to process data in abI and TIF format. An overview on Zadeh's fuzzy models, Atanassov's intuitionistic fuzzy propositions, and Smarandache's neutrosophic extensions is given in introduction. Algebraic models and neutrosophic graph representations and algorithmic considerations were described next. Combining previous work [19] and the above models, an extended structure of a Neutrosophic KRP system is obtained in the fourth section.

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