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Reducts in single valued neutrosophic ß-covering approximation spaces

Reducts in single valued neutrosophic β-covering approximation spaces

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Abstract. The problem of reducts is an interesting issue in all rough set models. In this paper, we propose the concept of the reduct in a single valued neutrosophic β-covering approximation space (SVN-β-CAS). Moreover, reducts in SVN-β-CASs are investigated while adding and removing some objects of the universe, respectively. Firstly, the notion of the reduct in a SVN-β-CAS is presented. It can be seen as the generalization of the reduct in covering and fuzzy β-covering approximation spaces. Then, two new SVN-β-CASs are presented while adding and removing some objects of the original universe. Finally, some properties of reducts of SVN β-coverings are investigated while adding and removing some objects, respectively.

1. Introduction

Ma [4] generalized fuzzy covering approximation spaces [1, 2, 3] to fuzzy β-covering approximation spaces by through replacing the value 1 with a parameter β. Inspired by Ma's work, D'eer et al. [5, 6] studied fuzzy neighborhood operators and other work [7, 8]. Especially, Yang and Hu [9], [10], [11] established some fuzzy covering-based rough set models and presented the notion of reduct in fuzzy β-covering approximation space. Then, Huang et al. [12] proposed a matrix approach for computing the reduct of a fuzzy β-covering.

After extending fuzzy β-covering approximation spaces to SVN sets [13], [14], Wang and Zhang. [15], [16] presented SVN-β-CASs. Inspired by the work of Yang and Hu [9], [10], [11] and Huang et al. [12], we study the problem of reduct in SVN-β-CASs in this paper. On one hand, the notion of the reduct in a SVN-β-CAS, as the generalization of the reduct in covering and fuzzy β-covering approximation spaces, is presented. Some properties of the reduct are proposed. On the one hand, two new SVN-β-CASs are presented while adding and removing some objects of the original universe. Some properties of reducts of SVN β-coverings are investigated while adding and removing some objects, respectively. Moreover, some relationships between the original SVN-β-CAS and two new SVN-β-CASs are investigated, respectively.

The rest of this paper is organized as follows. Section II reviews some fundamental definitions about SVN sets and SVN-β-CASs. In Section III, the notion of the reduct in a SVN-β-CAS is presented, as well as its properties. In Section IV, we present some definitions and properties for updating the reduct in the SVN-β-CAS while adding and removing objects. Section V is the conclusion and the further work.

2. Basic definitions

This section recalls some fundamental definitions related to SVN sets and SVN covering-based rough sets. We call U is a universe which is a nonempty and finite set.

Definition 1. (SVN set [14]) Let U be a nonempty fixed set. A SVN set A in U is defined as an object of the following form:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in U\}, \quad (1)$$

where T_A: $U \to [0, 1]$, I_A: $U \to [0, 1]$ and $F_A(x)$: $U \to [0, 1]$ are called the degree of truth, indeterminacy, and falsity memberships of the element $x \in U$ to A, respectively. They satisfy $0 \le T_A(x)+I_A(x)+F_A(x) \le 3$ for all $x \in U$. SVN(U) is the family of all SVN sets in U.

Specially, for two SVN numbers $\alpha = <a, b, c>$ and $\beta = <d, e, f>$, $\alpha \le \beta \Leftrightarrow a \le d$, $b \ge e$ and $c \ge f$. Some operations on SVN(U) are listed as follows [14]: for any $A, B \in$ SVN(U),

(1). $A \subseteq B \Leftrightarrow T_A(x) \le T_B(x)$, $I_B(x) \le I_A(x)$ and $F_B(x) \le F_A(x)$ for all $x \in U$;

(2). $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$;

(3). $A \cap B = \{\langle x, T_A(x) \wedge T_B(x), I_A(x) \vee I_B(x), F_A(x) \vee F_B(x)\rangle: x \in U\}$;

(4). $A \cup B = \{\langle x, T_A(x) \vee T_B(x), I_A(x) \wedge I_B(x), F_A(x) \wedge F_B(x)\rangle: x \in U\}$;

Then, Wang and Zhang [15] presented the definition of SVN-β-CAS.

Definition 2. ([15]) Let U be a universe and SVN(U) be the SVN power set of U. For a SVN number $\beta = <d, e, f>$, we call $\boldsymbol{C} = \{C_1, C_2, \cdots, C_m\}$, with $C_i \in$ SVN(U)(i = 1, 2, ..., m), a SVN β-covering of U, if for all $x \in U$, $C_i \in \boldsymbol{C}$ exists such that $C_i(x) \ge_1 \beta$. We also call (U, $\boldsymbol{C}$) a SVN-β-CAS.

3. Reducts in SVN-β-CASs

To solve the problem of under which conditions two coverings (or fuzzy β-coverings) generate the same rough upper and lower approximation operators, Zhu et al. [21] and Yang et al. [10] present the concept of reducible elements in different covering models. Inspired by their work, we propose the concept of the reduct in a SVN-β-CAS. Firstly, we present the concept of reducible elements in a SVN β-covering.

Definition 3. Let (U, $\boldsymbol{C}$) be a SVN-β-CAS and $C \in \boldsymbol{C}$. If C can be expressed as a union of some SVN sets in $\boldsymbol{C}-\{C\}$, C is called a SVN reducible element (SVNRE) in $\boldsymbol{C}$; otherwise C is called a SVN irreducible element (SVNIE) in $\boldsymbol{C}$.

Example 1. Let $U = \{x_1, x_2, x_3, x_4, x_5\}$, $\boldsymbol{C} = \{C_1, C_2, C_3, C_4\}$ and $\beta = <0.5, 0.3, 0.4>$, where

$$C_1 = \frac{(0.6,0.2,0.2)}{x_1} + \frac{(0.3,0.2,0.6)}{x_2} + \frac{(0.2,0.3,0.4)}{x_3} + \frac{(0.4,0.5,0.3)}{x_4},\ C_2 = \frac{(0.6,0.3,0.2)}{x_1} + \frac{(0.3,0.4,0.6)}{x_2} + \frac{(0.1,0.3,0.4)}{x_3} + \frac{(0.4,0.5,0.3)}{x_4},$$

$$C_3 = \frac{(0.4,0.2,0.7)}{x_1} + \frac{(0.3,0.2,0.8)}{x_2} + \frac{(0.2,0.3,0.4)}{x_3} + \frac{(0.3,0.6,0.5)}{x_4},\ C_4 = \frac{(0.1,0.5,0.5)}{x_1} + \frac{(0.6,0.1,0.3)}{x_2} + \frac{(0.6,0.3,0.2)}{x_3} + \frac{(0.5,0.3,0.4)}{x_4}.$$

We can see that $\boldsymbol{C}$ is a SVN β-covering of U. Then $C_1 = C_2 \cup C_3$. Hence C_1 is a SVNRE in $\boldsymbol{C}$, and $\boldsymbol{C} - \{C_1\}$ is also a SVN $\langle 0.5, 0.3, 0.4\rangle$-covering of U.

Then Properties 1 and 2 give some characterizations about removing several SVNREs in an SVN-β-CAS.

Proposition 1. Let (U,$\boldsymbol{C}$) be a SVN-β-CAS and $C \in \boldsymbol{C}$. If C is a SVNRE in $\boldsymbol{C}$, then $\boldsymbol{C}-\{C\}$ is also a SVN β-covering of U.

Proof. Suppose β = $\langle a,b,c\rangle$ and $C = \{C, C_1, C_2, ..., C_m\}$, where C, $C_i \in$ SVN(U) (i = 1, 2, ..., m). Since C is a SVNRE of $\boldsymbol{C}$, $\bigcup_{i=1}^{m} C_i \cup C = \bigcup_{i=1}^{m} C_i$. Hence $T_{\bigcup_{i=1}^{m} C_i}(x) = T_{\bigcup_{i=1}^{m} C_i \cup C}(x) \ge a$, $I_{\bigcup_{i=1}^{m} C_i}(x) = I_{\bigcup_{i=1}^{m} C_i \cup C}(x) \le b$ and $F_{\bigcup_{i=1}^{m} C_i}(x) = F_{\bigcup_{i=1}^{m} C_i \cup C}(x) \le c$ for any $x \in U$. Therefore, $\boldsymbol{C}-\{C\}$ is also a SVN β-covering of U.

Proposition 2. Let (U,$\boldsymbol{C}$) be a SVN-β-CAS, C be a SVNRE in $\boldsymbol{C}$ and $C_1 \in \boldsymbol{C}-\{C\}$. Then C_1 is a SVNRE in $\boldsymbol{C}$ SVN $\Leftrightarrow$ SVN C_1 is a SVNRE in $\boldsymbol{C}-\{C\}$.

Proof. Suppose $C=\{C,C_1,C_2,...,C_m\}$, where C, C_i $\in$SVN(U) (i = 1, 2, ..., m). Since C is a SVNRE in $\boldsymbol{C}$, $\exists$ C_{i1}, C_{i2}, ..., C_{it} $\in$ $\boldsymbol{C}-\{C\}$ ($1 < t \leq m$) such that $C=\bigcup_{k=1}^{t}C_{ik}$.

($\Rightarrow$): Since C_1 is a SVNRE in $\boldsymbol{C}$, $\exists$ C_{j1}, C_{j2}, ..., C_{js} $\in$ $\boldsymbol{C}-\{C_1\}$ ($1 < s \leq m$) such that $C_1=\bigcup_{p=1}^{s}C_{jp}$. Consider the following two cases:

• Case 1: SVN $C\notin\{C_{j1},C_{j2},...,C_{js}\}$, then C_1 is a SVNRE in $\boldsymbol{C}-\{C\}$;

• Case 2: Otherwise $C\in\{C_{j1},C_{j2},...,C_{js}\}$, then $\exists$ $r\in\{i=1,2,...,s\}$, such that C = C_{jr}. Hence $C_1=\bigcup_{((p=1)\wedge(p\neq r))}^{s}C_{jp}\cup C=\bigcup_{((p=1)\wedge(p\neq r))}^{s}C_{jp}\cup(\bigcup_{k=1}^{t}C_{ik})$. Therefore, C_1 is a SVNRE in $\boldsymbol{C}-\{C\}$.

($\Leftarrow$) It is immediate.

Inspired by Properties 1 and 2, we present the notion of the reduct in a SVN-β-CAS.

Definition 4. Let (U, $\boldsymbol{C}$) be a SVN-β-CAS and $\boldsymbol{D}\subseteq\boldsymbol{C}$. SVN $\boldsymbol{D}$ is the set of all SVNIEs, then $\boldsymbol{D}$ is called the reduct of $\boldsymbol{C}$ and denoted by $\Gamma(\boldsymbol{C})$.

Example 2. (Continued from Example 1) $\Gamma(\boldsymbol{C})$ = $\{C_2, C_3, C_4\}$.

4. Reducts in SVN-β-CASs while adding and deleting some objects

This section presents two new SVN-β-CASs while adding and removing some objects of the original universe. Moreover, some properties of reducts of SVN β-coverings are studies while adding and removing some objects. In this section, t denotes an integer which is more than 1.

4.1 Reducts in SVN-β-CASs while adding some objects

Firstly, we propose some new properties about reducts of SVN β-coverings while adding some objects of a universe. The notion of the increasing SVN-β-CAS is presented as follows.

Definition 5. Let (U, $\boldsymbol{C}$) be a SVN-β-CAS of U, where $U=\{x_1,x_2,...,x_n\}$ and $\boldsymbol{C}=\{C_1,C_2,...,C_m\}$. ($U^+$, $\boldsymbol{C}^+$) is called an increasing SVN-β-CAS from (U, $\boldsymbol{C}$), where U^+ = $\{x_1, x_2, \cdots, x_n, x_{n+1}, \cdots, x_{n+t}\}$, $\boldsymbol{C}^+$ = $\{C_1^+, C_2^+, \cdots, C_m^+\}$, and for any $1 \leq j \leq m$,

$$\begin{cases} C_j^+(x_i)=C_j(x_i), 1\leq i\leq n; \\ (\bigcup_{j=1}^{m}C_j^+)(x_i)\geq\beta, n+1\leq i\leq n+t. \end{cases} \tag{2}$$

The following proposition shows that an increasing SVN-β-CAS from the original SVN-β-CAS is also a SVN-β-CAS.

Proposition 3. Let (U, $\boldsymbol{C}$) be a SVN-β-CAS of U, where U = $\{x_1, x_2, \cdots, x_n\}$ and $\boldsymbol{C}$ = $\{C_1, C_2, \cdots, C_m\}$. Then ($U^+$, $\boldsymbol{C}^+$) is also a SVN-β-CAS of U^+.

Proof. According to Definition 5, $(\bigcup_{j=1}^{m}C_j^+)(x_i)=(\bigcup_{j=1}^{m}C_j)(x_i)\geq\beta$ for each i $\in$ {1, 2, $\cdots$, n}, and $(\bigcup_{j=1}^{m}C_j^+)(x_i)\geq\beta$ for any i $\in$ $\{n + 1, \cdots, n + t\}$. Hence, (U^+, $\boldsymbol{C}^+$) is also a SVN-β-CAS of U^+ by Definition 2.

Example 3. (Continued from Example 1) From Example 1, $\boldsymbol{C}$ is a SVN β-covering of U ($\beta\leq\langle0.5,0.3,0.4\rangle$), where U = $\{x_1, x_2, x_3, x_4\}$ and $\boldsymbol{C}$ = $\{C_1, C_2, C_3, C_4\}$. Suppose U^+ = $\{x_1, x_2, x_3, x_4, x_5\}$ and $\boldsymbol{C}$ = $\{C_1^+, C_2^+, C_3^+, C_4^+\}$, where

$$C_1^+=\frac{(0.6,0.2,0.2)}{x_1}+\frac{(0.3,0.2,0.6)}{x_2}+\frac{(0.2,0.3,0.4)}{x_3}+\frac{(0.4,0.5,0.3)}{x_4}+\frac{(0.7,0.3,0.2)}{x_5},\ C_2^+=\frac{(0.6,0.3,0.2)}{x_1}+\frac{(0.3,0.4,0.6)}{x_2}+\frac{(0.1,0.3,0.4)}{x_3}+\frac{(0.4,0.5,0.3)}{x_4}+\frac{(0.5,0.6,0.7)}{x_5},$$

$$C_3^+=\frac{(0.4,0.2,0.7)}{x_1}+\frac{(0.3,0.2,0.8)}{x_2}+\frac{(0.2,0.3,0.4)}{x_3}+\frac{(0.3,0.6,0.5)}{x_4}+\frac{(0.8,0.3,0.2)}{x_5},\ C_4^+=\frac{(0.1,0.5,0.5)}{x_1}+\frac{(0.6,0.1,0.3)}{x_2}+\frac{(0.6,0.3,0.2)}{x_3}+\frac{(0.5,0.3,0.4)}{x_4}+\frac{(0.9,0.1,0.8)}{x_5}.$$

According to Definitions 2 and 5, we know $\boldsymbol{C}^+$ is a SVN $\langle0.5,0.3,0.4\rangle$-covering of U^+.

The following proposition indicates the containing relation between a SVN-β-CAS and its increasing SVN-β-CAS.

Proposition 4. Let (U, $\boldsymbol{C}$) and (U^+, $\boldsymbol{C}^+$) be two SVN-β-CASs, where $\boldsymbol{C} = \{C_1, C_2, \cdots, C_m\}$ and $\boldsymbol{C}^+ = \{C_1^+, C_2^+, \cdots, C_m^+\}$. SVN $C_i^+ \subseteq C_j^+$ for any $i, j \in \{1, 2, \cdots, m\}$, then $C_i \subseteq C_j$.

Proof. For any $i, j \in \{1, 2, \cdots, m\}$,

$$C_i^+ \subseteq C_j^+ \Rightarrow C_i^+(x) \leq C_j^+(x), \forall x \in U \Rightarrow C_i(x) \leq C_j(x), \forall x \in U \Rightarrow C_i \subseteq C_j.$$

We find a relationship about SVNREs between a SVN-β-CAS and its increasing SVN-β-CAS.

Proposition 5. Let (U, $\boldsymbol{C}$) and (U^+, $\boldsymbol{C}^+$) be two SVN-β-CASs, where $\boldsymbol{C} = \{C_1, C_2, \cdots, C_m\}$ and $\boldsymbol{C}^+ = \{C_1^+, C_2^+, \cdots, C_m^+\}$. SVN C_i^+ is a SVNRE in $\boldsymbol{C}^+$, then C_i is a SVNRE in $\boldsymbol{C}$ for any $i \in \{1, 2, \cdots, m\}$.

Proof. It is immediate according to Definitions 3 and 5.

By Proposition 4, we find that the converse of it does not hold, i.e., "If C_i is a SVNRE in $\boldsymbol{C}$, then C_i^+ is a SVNRE in $\boldsymbol{C}^+$ for any $i \in \{1, 2, \cdots, m\}$." is not true. For example, since $C_1 = C_2 \cup C_3$ in Example 3, C_1 is a SVNRE in $\boldsymbol{C}$. However, C_1^+ is not a SVNRE in $\boldsymbol{C}^+$. Then, we give the following corollary by Proposition 4.

Corollary 1. Let (U, $\boldsymbol{C}$) and (U^+, $\boldsymbol{C}^+$) be two SVN-β-CASs, where $\boldsymbol{C} = \{C_1, C_2, \cdots, C_m\}$ and $\boldsymbol{C}^+ = \{C_1^+, C_2^+, \cdots, C_m^+\}$. If C_i is an SVNIE in $\boldsymbol{C}$, then C_i^+ is an SVNIE in $\boldsymbol{C}^+$ for any $i \in \{1, 2, \cdots, m\}$.

Proof. It is immediate by Proposition 4.

Example 4. (Continued from Example 3) C_2, C_3 and C_4 are SVNIEs in $\boldsymbol{C}$. C_2^+, C_3^+ and C_4^+ are SVNIEs in $\boldsymbol{C}^+$.

By Corollary 1, the converse of it does not hold, i.e., "If C_i^+ is an SVNIE in $\boldsymbol{C}^+$, then C_i is an SVNIE in $\boldsymbol{C}$ for any $i \in \{1, 2, \cdots, m\}$." is not true. For example, C_1^+ is an SVNIE in $\boldsymbol{C}^+$ in Example 3. However, C_1 is a SVNRE in $\boldsymbol{C}$. Based on Corollary 1, we propose Theorem 1.

Theorem 1. Let (U, $\boldsymbol{C}$) and (U^+, $\boldsymbol{C}^+$) be two SVN-β-CASs. Then

$$|\Gamma(\boldsymbol{C})| \leq |\Gamma(\boldsymbol{C}^+)|. \tag{3}$$

Proof. By Definition 5, $\Gamma(\boldsymbol{C})$ and $\Gamma(\boldsymbol{C}^+)$ are families of all SVNIEs of $\boldsymbol{C}$ and $\boldsymbol{C}^+$, respectively. Hence, it is immediate by Corollary 1.

Note that $|\Gamma(\boldsymbol{C})|$ and $|\Gamma(\boldsymbol{C}^+)|$ denote the cardinality of $\Gamma(\boldsymbol{C})$ and $\Gamma(\boldsymbol{C}^+)$, respectively.

Example 5. (Continued from Example 3) $\Gamma(\boldsymbol{C}) = \{C_2, C_3, C_4\}$, $\Gamma(\boldsymbol{C}^+) = \{C_1^+, C_2^+, C_3^+, C_4^+\}$. Hence, $|\Gamma(\boldsymbol{C})| = 3$ and $|\Gamma(\boldsymbol{C}^+)| = 4$, i.e., $|\Gamma(\boldsymbol{C})| \leq |\Gamma(\boldsymbol{C}^+)|$.

4.2 Reducts in SVN-β-CASs while removing some objects

We present the concept of declining SVN-β-CAS in the following definition.

Definition 6. Let (U, $\boldsymbol{C}$) be a SVN-β-CAS of U, where $U = \{x_1, x_2, ..., x_n\}$ and $\boldsymbol{C} = \{C_1, C_2, ..., C_m\}$. We call ($U^-$, $\boldsymbol{C}^-$) a declining SVN-β-CAS from (U, $\boldsymbol{C}$), where $U^- = \{x_1, x_2, \cdots, x_{n-t}\}$, $\boldsymbol{C}^- = \{C_1^-, C_2^-, \cdots, C_m^-\}$, and $C_j^-(x_i) = C_j(x_i)$ for any $1 \leq i \leq n-t$, $1 \leq j \leq m$.

The following proposition shows that the removing SVN-β-CAS is also a SVN-β-CAS.

Proposition 6. Let (U, $\boldsymbol{C}$) be a SVN-β-CAS of U, where $U = \{x_1, x_2, \cdots, x_n\}$ and $\boldsymbol{C} = \{C_1, C_2, \cdots, C_m\}$. Then ($U^-$, $\boldsymbol{C}^-$) is also a SVN-β-CAS of U^-.

Proof. Suppose $U = \{C_1, C_2, \cdots, C_{m-t}\}$. By Definition 6, $(\cup_{j=1}^{m} C_j^-)(x_i) = (\cup_{j=1}^{m} C_j)(x_i) \geq \beta$ for any $i \in \{1, 2, \cdots, n-t\}$. Hence, ($U^-$, $\boldsymbol{C}^-$) is also a SVN-β-CAS of U^- by Definition 2.

Example 6. Let $U = \{x_1, x_2, x_3, x_4, x_5\}$, $\boldsymbol{C} = \{C_1, C_2, C_3, C_4\}$ and β = $\langle 0.6, 0.3, 0.4 \rangle$), where

$$C_1 = \frac{(0.7,0.3,0.2)}{x_1} + \frac{(0.3,0.2,0.6)}{x_2} + \frac{(0.2,0.3,0.4)}{x_3} + \frac{(0.4,0.5,0.3)}{x_4} + \frac{(0.7,0.3,0.2)}{x_5}, \quad C_2 = \frac{(0.7,0.3,0.2)}{x_1} + \frac{(0.3,0.4,0.6)}{x_2} + \frac{(0.1,0.3,0.4)}{x_3} + \frac{(0.4,0.5,0.3)}{x_4} + \frac{(0.5,0.6,0.7)}{x_5},$$

$$C_3 = \frac{(0.4,0.2,0.7)}{x_1} + \frac{(0.3,0.2,0.8)}{x_2} + \frac{(0.2,0.3,0.4)}{x_3} + \frac{(0.3,0.6,0.5)}{x_4} + \frac{(0.8,0.3,0.2)}{x_5}, \quad C_4 = \frac{(0.1,0.5,0.5)}{x_1} + \frac{(0.6,0.1,0.3)}{x_2} + \frac{(0.6,0.3,0.2)}{x_3} + \frac{(0.8,0.3,0.4)}{x_4} + \frac{(0.6,0.1,0.8)}{x_5}.$$

According to Definition 2, we know $\boldsymbol{C}$ is a SVN β-covering of U. Let $U^- = \{x_1, x_2, x_3, x_4\}$ and $\boldsymbol{C}^- = \{C_1^-, C_2^-, C_3^-, C_4^-\}$, where

$$C_1^- = \frac{(0.7,0.2,0.2)}{x_1} + \frac{(0.3,0.2,0.6)}{x_2} + \frac{(0.2,0.3,0.4)}{x_3} + \frac{(0.4,0.5,0.3)}{x_4},\ C_2^- = \frac{(0.7,0.3,0.2)}{x_1} + \frac{(0.3,0.4,0.6)}{x_2} + \frac{(0.1,0.3,0.4)}{x_3} + \frac{(0.4,0.5,0.3)}{x_4},$$

$$C_3^- = \frac{(0.4,0.2,0.7)}{x_1} + \frac{(0.3,0.2,0.8)}{x_2} + \frac{(0.2,0.3,0.4)}{x_3} + \frac{(0.3,0.6,0.5)}{x_4},\ C_4^- = \frac{(0.1,0.5,0.5)}{x_1} + \frac{(0.6,0.1,0.3)}{x_2} + \frac{(0.6,0.3,0.2)}{x_3} + \frac{(0.8,0.3,0.4)}{x_4}.$$

According to Definitions 2 and 6, we know $\boldsymbol{C}^-$ is a SVN $\langle 0.6,0.3,0.4 \rangle$-covering of U^-.

Then, Proposition 7 shows the containing relation between a SVN-β-CAS and its declining SVN-β-CAS.

Proposition 7. Let (U, $\boldsymbol{C}$) and (U^-, $\boldsymbol{C}^-$) be two SVN-β-CASs, where $\boldsymbol{C} = \{C_1, C_2, \cdots, C_m\}$ and $\boldsymbol{C}^- = \{C_1^-, C_2^-, \cdots, C_m^-\}$. If $C_i \subseteq C_j$ for any $i, j \in \{1, 2, \cdots, m\}$, then $C_i^- \subseteq C_j^-$.

Proof. For any $i, j \in \{1, 2, \cdots, m\}$,

$$C_i \subseteq C_j \Rightarrow C_i(x) \leq C_j(x), \forall x \in U \Rightarrow C_i^-(x) \leq C_j^-(x), \forall x \in U \Rightarrow C_i^- \subseteq C_j^-.$$

Proposition 8 shows a relationship about SVNREs between a SVN-β-CAS and it's declining SVN-β-CAS as follows.

Proposition 8. Let (U, $\boldsymbol{C}$) and (U^-, $\boldsymbol{C}^-$) be two SVN-β-CASs, where $\boldsymbol{C} = \{C_1, C_2, \cdots, C_m\}$ and $\boldsymbol{C}^- = \{C_1^-, C_2^-, \cdots, C_m^-\}$. If C_i is a SVNRE in $\boldsymbol{C}$, then C_i^- is a SVNRE in $\boldsymbol{C}^-$ for any $i \in \{1, 2, \cdots, m\}$.

Proof. It is immediate by Definitions 3 and 6.

Inspired by Proposition 8, we find that the converse of it does not hold, i.e., "SVN C_i^- is a SVNRE in $\boldsymbol{C}$, then C_i is a SVNRE in $\boldsymbol{C}$ for any $i \in \{1, 2, \cdots, m\}$." is not true. For example, since $C_1^- = C_2^- \cup C_3^-$ in Example 6, C_1^- is a SVNRE in $\boldsymbol{C}^-$. However, C_1 is not a SVNRE in $\boldsymbol{C}$. Based on Proposition 8, we present the following corollary.

Corollary 2. Let (U, $\boldsymbol{C}$) and (U^-, $\boldsymbol{C}^-$) be two SVN-β-CASs, where $\boldsymbol{C} = \{C_1, C_2, \cdots, C_m\}$ and $\boldsymbol{C}^- = \{C_1^-, C_2^-, \cdots, C_m^-\}$. If C_i^- is an SVNIE in $\boldsymbol{C}^-$, then C_i is an SVNIE in $\boldsymbol{C}$ for any $i \in \{1, 2, \cdots, m\}$.

Proof. It is immediate by Proposition 8.

Example 7. (Continued from Example 6) C_2^-, C_3^- and C_4^- are SVNIEs in $\boldsymbol{C}^-$. C_2, C_3 and C_4 are SVNIEs in $\boldsymbol{C}$.

Based on Corollary 2, we give the following theorem.

Theorem 2. Let (U, $\boldsymbol{C}$) and (U^-, $\boldsymbol{C}^-$) be two SVN-β-CASs. Then

$$|\Gamma(\mathcal{C}^-)| \leq |\Gamma(\mathcal{C})|. \tag{3}$$

Proof. By Definition 5, $\Gamma(\boldsymbol{C})$ and $\Gamma(\boldsymbol{C}^-)$ are families of all SVNIEs of $\boldsymbol{C}$ and $\boldsymbol{C}^-$, respectively. Hence, it is immediate by Corollary 2.

Example 8. (Continued from Example 6) $\Gamma(\boldsymbol{C}) = \{C_1, C_2, C_3, C_4\}$, $\Gamma(\boldsymbol{C}^-) = \{C_2^-, C_3^-, C_4^-\}$. Hence, $|\Gamma(\boldsymbol{C})| = 4$ and $|\Gamma(\boldsymbol{C}^-)| = 3$. That is to say, $|\Gamma(\boldsymbol{C}^-)| \leq |\Gamma(\boldsymbol{C})|$.

5. Conclusion

In this paper, we propose the concept of the reduct in a SVN-β-CAS. It will be helpful to solve other problems in SVN covering-based rough set models. Moreover, reducts in SVN-β-CASs are investigated while adding and removing some objects of the universe. In future, updating the reduct while adding and deleting objects at the same time will be done.

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