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Relationships among several similarity measures of single-valued neutrosophic sets

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Abstract. Single-valued neutrosophic set (SVNS) handling the uncertainties characterized by truth, indeterminacy, and falsity membership degrees, is an extension of fuzzy set and intuitionistic fuzzy set. It provides a more flexible way to capture uncertainty. This paper is devoted to the disscussion of relationships among several existing similarity measures of SVNSs. In addition, it is shown that some existing similarity measures are equivalent. The comparative results of this study provide convenience for applying similarity measures of SVNSs to practical problems.

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1. Introduction

Uncertainty, incomplete, and inconsistent information can be found in many real-life systems and may enter some problems in a much more complex ways. The theory of fuzzy set(FS) proposed by Zadeh [23] in 1965 has achieved a great success in various real applications to handle uncertainty. Subsequently, several new concepts of high-order fuzzy sets have been presented. Among them, The intuitionistic fuzzy set(IFS) on a universe X introduced by Atanassov [1] is a typical generalization of fuzzy set. An IFS consists of a membership function and a non-membership function of the universe and provides a flexible mathematical framework to incomplete and uncertain information processing. Smarandache [13] originally introduced the concept of neutrosophic set(NS) in 1998 which is a generalization of fuzzy set and intuitionistic fuzzy set [15]. A neutrosophic set A in a universal set X is characterized independently by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, and neutrosophy [14] is a branch of philosophy and a mathematical tool for studying the origin, nature, and scope of neutralities. On the other hand, the original neutrosophic set is mainly used for philosophical applications, especially when distinction is required between absolute and relative truth (falsity, indeterminacy). In order to easily use the neutrosophic set in real scientific and engineering fields, Smarandache [13] in 1998 and Wang et al. [17] in 2010 proposed the concept of single-valued neutrosophic set(SVNS), which is an instance of neutrosophic set, and also introduced the set-theoretic operations and a series of properties of single-valued neutrosophic sets. The single-valued neutrosophic set theory has been proven to be useful in many scientific fields, such as multi-attribute decision making, machine learning, fault diagnosis, and so on. Dimple and Harish [12] proposed the subtraction and division operations on interval neutrosophic set. The deficiencies of the existing operations are validated through some counter-examples. Harish and Nancy [5] presented some new operational laws called logarithm operational laws for the single-valued neutrosophic numbers (SVNN). Various desirable properties of the proposed operational laws are contemplated. Further, based on these laws, different weighted averaging and geometric aggregation operators are developed. In addition, the related results have been extended to linguistic single-valued neutrosophic sets [6], a multi-criteria decision making method based on prioritized muirhead mean aggregation operator under neutrosophic set environment is presented [7]. Harish and Nancy [10] proposed an improved score function for ranking the single as well as interval-valued neutrosophic sets by incorporating the idea of hesitation degree between the truth and false degrees. Moreover, a decision-making method is presented based on proposed function.

The study of similarity measure is of particular importance because, in many practical situations, we need to compare two objects in order to determine whether they are identical or approximately identical or at least to what degree they are identical. Up to now, a lot of research has been done about these information measures with applications in the field of neutrosophic set theory. Harish and Nancy [8] developed a nonlinear programming (NP) model based on the technique of TOPSIS. A likelihood-based comparison relation for interval neutrosophic numbers (INNs) is proposed to determine the ranking of considered alternatives and the related decision making method is presented. In addition, some new biparametric distance measures on single-valued neutrosophic sets are presented [4]. The application of these measures to pattern recognition and medical diagnosis are surveyed. Nancy and Harish [11] proposed an axiomatic definition of divergence measure for single-valued neutrosophic sets (SVNSs). The properties of the proposed divergence measure have been studied. Broumi and Smarandache [2] presented a method to calculate the distance between SVNSs on the basis of Hausdorff distance and proposed some similarity measures based on the distance and matching function to calculate the similarity degree between SVNSs. Majumdar and Samanta [9] presented several similarity measures for SVNSs based on the Hamming (Euclidian) distance and normalized Hamming (Euclidian) distance between two SVNSs. Majumdar and Samanta [9] presented a similarity measure of SVNSs based on the min and max operators and Ye [22] proposed another new similarity measure of SVNSs based on the min and max operators. The vector similarity measure is one of important tools for the degree of similarity between objects. The Jaccard, Dice, and cosine similarity measures ([18, 19, 20]) are often used for this purpose. Therefore, Ye [21] proposed three vector similarity measures for SVNSs based on the extension of the Jaccard, Dice, and cosine similarity measures between vectors and applied them to multi-criteria decision-making problems with simplified neutrosophic information.

The similarity measures of SVNSs have been extensively studied. Based on different application background, researchers have proposed many kinds of similarity measures for SVNSs and applied them to decision making, pattern recognition, medical diagnosis and so on. We note that the relationships among these similarity measures have not been systematically investigated. The main purpose of this paper is to make comparative analysis of some existing similarity measures for SVNSs. It will enrich the theory and application of similarity measures and provide the thread for constructing general similarity measures for SVNSs. The rest of this manuscript is organized as follows. In Section 2, we recall some basic concepts and similarity measures of SVNSs. In Section 3, we put forward the definition of equivalence for similarity measures, and investigate the relationships among some similarity measures of SVNSs, such as distance based similarity measures, the min and max operators based similarity measures, and vector similarity measures in terms of inequality and equivalence. Some examples are presented to illustrate the equivalence. Finally, some conclusions and future research possibilities are provided in Sect. 4.

2. Some concepts and similarity measures of SVNSs

In this section, we review some basic concepts and existing similarity measures related to SVNSs, which will be used in the rest of the paper .

2.1. Basic Definitions.

Definition 2.1 ([13])**.** Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, where $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$ such that $T_A(x) : X \to]^-0, 1^+[$, $I_A(x) : X \to]^-0, 1^+[$ and $F_A(x) : X \to]^-0, 1^+[$, and the sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$ satisfies the condition $^-0 \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3^+$.

In order to easily apply neutrosophic set theory to science and engineering, Smarandache [13] and Wang et al. [17] presented the concept of single-valued neutrosophic set(SVNS) as follows.

Definition 2.2 ([17])**.** Let X be a space of points (objects), with a generic element in X denoted by x. A single-valued neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. A single-valued neutrosophic set A can be denoted by

$$A = \{(x, T_A(x), I_A(x), F_A(x))|x \in X\},$$

where $T_A(x), I_A(x), F_A(x) \in [0, 1]$ for each $x \in X$, and the sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$ satisfies the condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

In this paper, a single-valued neutrosophic set A in X is also denoted by

$$A = \{(x, A(x))|x \in X\},$$

where $A(x) = (T_A(x), I_A(x), F_A(x))$ and $T_A(x), I_A(x), F_A(x) \in [0,1]$, for each $x \in X$. We use the symbol $SVNS(X)$ to denote the set of all single-valued neutrosophic sets in X.

Two single-valued neutrosophic sets A and B are equal, written as $A = B$, if and only if $T_A(x) = T_B(x), I_A(x) = I_B(x)$, and $F_A(x) = F_B(x)$ for any $x \in X$. There are three types of inclusion relation for single-valued neutrosophic sets ([24, 25]). In this paper, we consider the widely used definition proposed by Smarandache ([9, 13]).

Definition 2.3 ([13, 16])**.** Let X be a finite set and $A, B \in SVNS(X)$. A is contained in B, denoted by $A \subseteq B$, if $T_A(x) \le T_B(x)$,$I_A(x) \ge I_B(x)$, and $F_A(x) \ge F_B(x)$, for any $x \in X$.

For two $SVNSsA = \{(x, T_A(x), I_A(x), F_A(x))|x \in X\}$ and $B = \{(x, T_B(x), I_B(x), F_B(x))|x \in X\}$, there are the following operations [14]:

(i) Complement

$$A^c = \{(x, F_A(x), 1 - I_A(x), T_A(x))|x \in X\},$$

(ii) Union

$$A \cup B = \{(x, T_A(x) \bigvee T_B(x), I_A(x) \bigwedge I_B(x), F_A(x) \bigwedge F_B(x))|x \in X\},$$

(iii) Intersection

$$A \cap B = \{(x, T_A(x) \bigwedge T_B(x), I_A(x) \bigvee I_B(x), F_A(x) \bigvee F_B(x))|x \in X\}.$$

2.2. **Existing similarity measures.**

Definition 2.4 ([3])**.** A real function $d : SVNS(X) \times SVNS(X) \to [0,1]$ is called a distance measure, where d satisfies the following axioms for $A, B, C \in SVNS(X)$:

(P1) $0 \le d(A, B) \le 1$,
(P2) $d(A, B) = 0$ iff $A = B$,
(P3) $d(A, B) = d(B, A)$,
(P4) if $A \subseteq B \subseteq C$, then $d(A, C) \ge d(A, B)$ and $d(A, C) \ge d(B, C)$.

Definition 2.5 ([2, 9])**.** Let X be a finite set of objects. A function $S : SVNS(x) \times SVNS(x) \to [0,1]$ is called a similarity measure for single-valued neutrosophic sets in X, if it satisfy the following properties:

(S1) $S(A, B) = 1$ if and only if $A = B$,
(S2) $S(A, B) = S(B, A)$,
(S3) $S(A, C) \le S(A, B), S(A, C) \le S(B, C)$, if $A \subseteq B \subseteq C$,
(S4) $S(A, B) = 0$ if and only if $|T_A(x) - T_B(x)| = 1, |I_A(x) - I_B(x)| = 1$ and $|F_A(x) - F_B(x)| = 1$, for any $x \in X$.

Several researchers have addressed the various types of distance and similarity measures. We introduce some existing distance measures, similarity measures based on the min and max operators, and vector similarity measures for SVNSs. Let $X = \{x_1, x_2, \cdots, x_n\}$, $A, B \in SVNS(X)$ and $A = \{(x_\imath, T_A(x_\imath), I_A(x_\imath), F_A(x_\imath))|x_\imath \in X\}$, $B = \{(x_\imath, T_B(x_\imath), I_B(x_\imath), F_B(x_\imath))|x_\imath \in X\}$.

The extended Hausdorff distance [2]:

$$D_H(A,B) = \frac{1}{n}\sum_{i=1}^{n} max[|T_A(x_i) - T_B(x_i)|, |I_A(x_i) - I_B(x_i)|, |F_A(x_i) - F_B(x_i)|].$$

The normalized Hamming distance [9]:

$$D_{NH}(A,B) = \frac{1}{3n}\sum_{i=1}^{n}[|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|].$$

The normalized Euclidean distance [9]:

$$D_{NE}(A,B) = \{\frac{1}{3n}\sum_{i=1}^{n}[(T_A(x_i) - T_B(x_i))^2 + (I_A(x_i) - I_B(x_i))^2 + (F_A(x_i) - F_B(x_i))^2]\}^{1/2}.$$

Majumdar and Samanta have introduced the following similarity measure based on the min and max operators [9]:

$$S_1(A,B) = \frac{\sum_{i=1}^{n}[\min(T_A(x_i),T_B(x_i)) + \min(I_A(x_i),I_B(x_i)) + \min(F_A(x_i),F_B(x_i))]}{\sum_{i=1}^{n}[\max(T_A(x_i),T_B(x_i)) + \max(I_A(x_i),I_B(x_i)) + \max(F_A(x_i),F_B(x_i))]}.$$

Ye proposed other new similarity measures based on the min and max operators [22]:

$$S_2(A,B) = \frac{1}{n}\sum_{i=1}^{n}\frac{\min(T_A(x_i),T_B(x_i)) + \min(I_A(x_i),I_B(x_i)) + \min(F_A(x_i),F_B(x_i))}{\max(T_A(x_i),T_B(x_i)) + \max(I_A(x_i),I_B(x_i)) + \max(F_A(x_i),F_B(x_i))},$$

$$S_3(A,B) = \frac{\sum_{i=1}^{n}\omega_i[\min(T_A(x_i),T_B(x_i)) + \min(I_A(x_i),I_B(x_i)) + \min(F_A(x_i),F_B(x_i))]}{\sum_{i=1}^{n}\omega_i[\max(T_A(x_i),T_B(x_i)) + \max(I_A(x_i),I_B(x_i)) + \max(F_A(x_i),F_B(x_i))]},$$

$$S_4(A,B) = \sum_{i=1}^{n}\omega_i\frac{\min(T_A(x_i),T_B(x_i)) + \min(I_A(x_i),I_B(x_i)) + \min(F_A(x_i),F_B(x_i))}{\max(T_A(x_i),T_B(x_i)) + \max(I_A(x_i),I_B(x_i)) + \max(F_A(x_i),F_B(x_i))}.$$

The weight of an element x_i is $\omega_i (i = 1, 2, \dots n)$ with $\omega_i \in [0,1]$,and $\sum_{i=1}^{n}\omega_i = 1$.

Ye proposed three vector similarity measures based on the extension of the Jaccard, Dice, and cosine similarity measures in vector space [21]:

$$S_J(A,B) = \frac{1}{n}\sum_{i=1}^{n}\frac{T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i)}{[M - N]}.$$

There, we let $M = (T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)) + (T_B^2(x_i) + I_B^2(x_i) + F_B^2(x_i))$, $N = (T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i))$.

$$S_D(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{2(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i))}{(T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i))+(T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i))},$$

$$S_C(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i)}{\sqrt{T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i)}\sqrt{T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i)}},$$

$$WS_J(A,B)=\sum_{i=1}^{n}\omega_i\frac{T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i)}{[M-N]}.$$

There, we let $M=(T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i))+(T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i))$, $N=(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i))$.

$$WS_D(A,B)=\sum_{i=1}^{n}\omega_i\frac{2(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i))}{(T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i))+(T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i))},$$

$$WS_C(A,B)=\sum_{i=1}^{n}\omega_i\frac{T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i)}{\sqrt{T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i)}\sqrt{T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i)}}.$$

The weight of an element x_i is $\omega_i(i=1,2,\dots n)$ with $\omega_i\in[0,1]$,and $\sum_{i=1}^{n}\omega_i=1$.

3. The relationships between similarity measures of SVNSs

In this section, we make a comparative study of some existing similarity measures for SVNSs to reveal the relationships among them. The results obtained are beneficial for users to select an appropriate similarity measure for meeting their requirements. In addition, it will provide the thread for constructing general similarity measures for SVNSs.

3.1. The inequality for similarity measures of SVNSs.

Proposition 3.1. *The distance $D_H(A,B)$, $D_{NH}(A,B)$, and $D_{NE}(A,B)$ satisfy the inequality for any two SVNSs A and B: $D_{NE}^2(A,B)\le D_{NH}(A,B)\le D_H(A,B)$. The similarity measures based on distance satisfy $1-D_H(A,B)\le 1-D_{NH}(A,B)\le 1-D_{NE}^2(A,B)$.*

Proof. For two SVNSs A and B, we have $T_A(x),I_A(x),F_A(x)\in[0,1]$, $T_B(x),I_B(x)$, $F_B(x)\in[0,1]$, for each $x\in X$. Then $|T_A(x_i)-T_B(x_i)|\le 1$, $|I_A(x_i)-I_B(x_i)|\le 1$, $|F_A(x_i)-F_B(x_i)|\le 1$. Thus

$|T_A(x_i)-T_B(x_i)|\bigvee|I_A(x_i)-I_B(x_i)|\bigvee|F_A(x_i)-F_B(x_i)|$

$\geq \frac{1}{3}(|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|)$.
So we have

$$\begin{aligned} D_{NH}(A,B) &= \frac{1}{3n}\sum_{i=1}^{n}[|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|] \\ &= \frac{1}{n}\sum_{i=1}^{n}[\frac{1}{3}(|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|)] \\ &\leq \frac{1}{n}\sum_{i=1}^{n} max[|T_A(x_i) - T_B(x_i)|, |I_A(x_i) - I_B(x_i)|, |F_A(x_i) - F_B(x_i)|] \\ &= D_H(A,B). \end{aligned}$$

Hence $D_{NH}(A,B) \leq D_H(A,B)$. On the other hand,

$$\begin{aligned} D^2_{NE}(A,B) &= \frac{1}{3n}\sum_{i=1}^{n}[(T_A(x_i) - T_B(x_i))^2 + (I_A(x_i) - I_B(x_i))^2 + (F_A(x_i) - F_B(x_i))^2] \\ &\leq \frac{1}{3n}\sum_{i=1}^{n}[|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|] \\ &= D_{NH}(A,B). \end{aligned}$$

Then $D^2_{NE}(A,B) \leq D_{NH}(A,B)$. Thus we obtained that

$$D^2_{NE}(A,B) \leq D_{NH}(A,B) \leq D_H(A,B).$$

So similarity measures based on distance satisfy the following inequalities:

$$1 - D_H(A,B) \leq 1 - D_{NH}(A,B) \leq 1 - D^2_{NE}(A,B).$$

□

Proposition 3.2. *For any two SVNSs A and B, similarity measures $S_J(A,B)$, $S_D(A,B)$, $S_C(A,B)$, $WS_J(A,B)$, $WS_D(A,B)$, and $WS_C(A,B)$ satisfy the inequality:*

(1) $S_D(A,B) \leq S_C(A,B), S_D(A,B) \leq 2S_J(A,B)$;
(2) $WS_D(A,B) \leq WS_C(A,B), WS_D(A,B) \leq 2WS_J(A,B)$.

Proof. For two SVNSs A and B, we have $T_A(x), I_A(x), F_A(x) \in [0,1]$, $T_B(x), I_B(x)$, $F_B(x) \in [0,1]$, for each $x \in X$.

(1) Based on the fundamental inequality, we have
$[(T^2_A(x_i) + I^2_A(x_i) + F^2_A(x_i)) + (T^2_B(x_i) + I^2_B(x_i) + F^2_B(x_i))]$
$\geq 2\sqrt{T^2_A(x_i) + I^2_A(x_i) + F^2_A(x_i)}\sqrt{T^2_B(x_i) + I^2_B(x_i) + F^2_B(x_i)}$,

$$\begin{aligned} S_D(A,B) &= \frac{1}{n}\sum_{i=1}^{n}\frac{2(T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i))}{(T^2_A(x_i) + I^2_A(x_i) + F^2_A(x_i)) + (T^2_B(x_i) + I^2_B(x_i) + F^2_B(x_i))} \\ &\leq \frac{1}{n}\sum_{i=1}^{n}\frac{T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i)}{\sqrt{T^2_A(x_i) + I^2_A(x_i) + F^2_A(x_i)}\sqrt{T^2_B(x_i) + I^2_B(x_i) + F^2_B(x_i)}} \\ &= S_C(A,B). \end{aligned}$$

Then $S_D(A,B) \leq S_C(A,B)$. On the other hand,

$$\begin{aligned} 2S_J(A,B) &= \frac{1}{n}\sum_{i=1}^{n} \frac{2(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i))}{[M-N]} \\ &\geq \frac{1}{n}\sum_{i=1}^{n} \frac{2(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i))}{(T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i))+(T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i))} \\ &= S_D(A,B). \end{aligned}$$

There, we let $M = (T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i))+(T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i))$, $N = (T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i))$. Then $S_D(A,B) \leq 2S_J(A,B)$.

(2) Usually, one takes the weight of each element x_i for $x_i \in X$ into account. Assume that the weight of an element x_i is ω_i $(i = 1, 2, \ldots, n)$ with $\omega_i \in [0,1]$ and $\sum_{i=1}^{n}\omega_i = 1$. Then

$$\begin{aligned} WS_D(A,B) &= \sum_{i=1}^{n} \omega_i \frac{2(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i))}{(T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i))+(T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i))} \\ &\leq \sum_{i=1}^{n} \omega_i \frac{T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i)}{\sqrt{T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i)}\sqrt{T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i)}} \\ &= WS_C(A,B). \end{aligned}$$

Thus $WS_D(A,B) \leq WS_C(A,B)$. On the other hand,

$$\begin{aligned} 2WS_J(A,B) &= \sum_{i=1}^{n} \omega_i \frac{2(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i))}{[M-N]} \\ &\geq \sum_{i=1}^{n} \omega_i \frac{2(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i))}{(T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i))+(T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i))} \\ &= WS_D(A,B). \end{aligned}$$

There, we let $M = (T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i))+(T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i))$, $N = (T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i))$. So $WS_D(A,B) \leq 2WS_J(A,B)$. □

3.2. **The equivalence for similarity measures of SVNSs.** The equivalence for similarity measures of $SVNSs$ can be used to judgement decision results accurately. In this section, we proposed the notion of equivalence for similarity measures of $SVNSs$, which is proposed based on the ordering of objects of the $SVNSs$ has not changed.

Definition 3.3. Supposed that $S_1(A,B)$, $S_2(A,B)$ are similarity measures of SVNSs, $S_1(A,B)$ is equivalent to $S_2(A,B)$, defined $S_1(A,B) \sim S_2(A,B)$, if for any four SVNSs A, B, A', B' in $X = \{x_1, x_2, \ldots, x_n\}$, the similarity measure $S_1(A,B)$ and $S_2(A,B)$ satisfy:

$$S_1(A,B) \leq S_1(A',B') \Leftrightarrow S_2(A,B) \leq S_2(A',B')$$

that is, the ordering of objects of the SVNSs has not changed.

Proposition 3.4. *The distance $D_{NH}(A,B)$, and $D_{NE}(A,B)$ satisfy the property for any two SVNSs A, B that $D_{NH}(A,B)$, and $D_{NE}(A,B)$ are equivalent. The similarity measures based on distance $1-D_{NH}(A,B)$ and $1-D_{NE}(A,B)$ are equivalent.*

Proof. For any four SVNSs A, B, A', B' in $X=\{x_1,x_2,\dots,x_n\}$, we have $T_A(x), I_A(x), F_A(x)\in[0,1]$, $T_B(x), I_B(x), F_B(x)\in[0,1]$, for each $x\in X$. Then

$$|T_A(x_i)-T_B(x_i)|\le 1,\ |I_A(x_i)-I_B(x_i)|\le 1, |F_A(x_i)-F_B(x_i)|\le 1.$$

Necessity: If we have $D_{NH}(A,B)\le D_{NH}(A',B')$, that is to say,

$$\begin{aligned}D_{NH}(A,B)&=\frac{1}{3n}\sum_{i=1}^{n}[|T_A(x_i)-T_B(x_i)|+|I_A(x_i)-I_B(x_i)|+|F_A(x_i)-F_B(x_i)|]\\&\le\frac{1}{3n}\sum_{i=1}^{n}[|T_{A'}(x_i)-T_{B'}(x_i)|+|I_{A'}(x_i)-I_{B'}(x_i)|+|F_{A'}(x_i)-F_{B'}(x_i)|]\\&=D_{NH}(A',B').\end{aligned}$$

Then $|T_A(x_i)-T_B(x_i)|+|I_A(x_i)-I_B(x_i)|+|F_A(x_i)-F_B(x_i)|$
$\le|T_{A'}(x_i)-T_{B'}(x_i)|+|I_{A'}(x_i)-I_{B'}(x_i)|+|F_{A'}(x_i)-F_{B'}(x_i)|$.
Thus

$$\begin{aligned}D_{NE}(A,B)&=\{\frac{1}{3n}\sum_{i=1}^{n}[(T_A(x_i)-T_B(x_i))^2+(I_A(x_i)-I_B(x_i))^2+(F_A(x_i)-F_B(x_i))^2]\}^{1/2}\\&\le\{\frac{1}{3n}\sum_{i=1}^{n}[(T_{A'}(x_i)-T_{B'}(x_i))^2+(I_{A'}(x_i)-I_{B'}(x_i))^2+(F_{A'}(x_i)-F_{B'}(x_i))^2]\}^{1/2}\\&=D_{NE}(A',B').\end{aligned}$$

Sufficiency: If we have $D_{NE}(A,B)\le D_{NE}(A',B')$, that is to say,

$$\begin{aligned}D_{NE}(A,B)&=\{\frac{1}{3n}\sum_{i=1}^{n}[(T_A(x_i)-T_B(x_i))^2+(I_A(x_i)-I_B(x_i))^2+(F_A(x_i)-F_B(x_i))^2]\}^{1/2}\\&\le\{\frac{1}{3n}\sum_{i=1}^{n}[(T_{A'}(x_i)-T_{B'}(x_i))^2+(I_{A'}(x_i)-I_{B'}(x_i))^2+(F_{A'}(x_i)-F_{B'}(x_i))^2]\}^{1/2}\\&=D_{NE}(A',B').\end{aligned}$$

Then $(T_A(x_i)-T_B(x_i))^2+(I_A(x_i)-I_B(x_i))^2+(F_A(x_i)-F_B(x_i))^2$
$\le(T_{A'}(x_i)-T_{B'}(x_i))^2+(I_A(x_{i'})-I_{B'}(x_i))^2+(F_{A'}(x_i)-F_{B'}(x_i))$,
$|T_A(x_i)-T_B(x_i)|+|I_A(x_i)-I_B(x_i)|+|F_A(x_i)-F_B(x_i)|$
$\le|T_{A'}(x_i)-T_{B'}(x_i)|+|I_{A'}(x_i)-I_{B'}(x_i)|+|F_{A'}(x_i)-F_{B'}(x_i)|$.
Thus

$$\begin{aligned}D_{NH}(A,B)&=\frac{1}{3n}\sum_{i=1}^{n}[|T_A(x_i)-T_B(x_i)|+|I_A(x_i)-I_B(x_i)|+|F_A(x_i)-F_B(x_i)|]\\&\le\frac{1}{3n}\sum_{i=1}^{n}[|T_{A'}(x_i)-T_{B'}(x_i)|+|I_{A'}(x_i)-I_{B'}(x_i)|+|F_{A'}(x_i)-F_{B'}(x_i)|]\\&=D_{NH}(A',B').\end{aligned}$$

So $D_{NH}(A,B) \sim D_{NE}(A,B)$ and $1-D_{NH}(A,B) \sim 1-D_{NE}(A,B)$. □

Proposition 3.5. *For two SVNSs A and B, similarity measures $S_1(A,B)$, $S_2(A,B)$, $S_3(A,B)$, and $S_4(A,B)$, satisfy the property: $S_1(A,B)$, $S_2(A,B)$, $S_3(A,B)$, and $S_4(A,B)$ are equivalent.*

Proof. For any four SVNSs A, B, A', B' in $X=\{x_1,x_2,\ldots,x_n\}$, we have $T_A(x), I_A(x), F_A(x) \in [0,1], T_B(x), I_B(x), F_B(x) \in [0,1]$, for each $x \in X$.

Necessity: If we have $S_1(A,B) \le S_1(A',B')$, that is to say,

$$\begin{aligned} S_1(A,B) &= \frac{\sum_{i=1}^{n}[\min(T_A(x_i),T_B(x_i))+\min(I_A(x_i),I_B(x_i))+\min(F_A(x_i),F_B(x_i))]}{\sum_{i=1}^{n}[\max(T_A(x_i),T_B(x_i))+\max(I_A(x_i),I_B(x_i))+\max(F_A(x_i),F_B(x_i))]} \\ &\le \frac{\sum_{i=1}^{n}[\min(T_{A'}(x_i),T_{B'}(x_i))+\min(I_{A'}(x_i),I_{B'}(x_i))+\min(F_{A'}(x_i),F_{B'}(x_i))]}{\sum_{i=1}^{n}[\max(T_{A'}(x_i),T_{B'}(x_i))+\max(I_{A'}(x_i),I_{B'}(x_i))+\max(F_{A'}(x_i),F_{B'}(x_i))]} \\ &= S_1(A',B'). \end{aligned}$$

Then $\min(T_A(x_i),T_B(x_i))+\min(I_A(x_i),I_B(x_i))+\min(F_A(x_i),F_B(x_i))$
$\le \min(T_{A'}(x_i),T_{B'}(x_i))+\min(I_{A'}(x_i),I_{B'}(x_i))+\min(F_{A'}(x_i),F_{B'}(x_i))$,
$\max(T_A(x_i),T_B(x_i))+\max(I_A(x_i),I_B(x_i))+\max(F_A(x_i),F_B(x_i))$
$\ge \max(T_{A'}(x_i),T_{B'}(x_i))+\max(I_{A'}(x_i),I_{B'}(x_i))+\max(F_{A'}(x_i),F_{B'}(x_i))$.
Thus

$$\begin{aligned} S_2(A,B) &= \frac{1}{n}\sum_{i=1}^{n}\frac{\min(T_A(x_i),T_B(x_i))+\min(I_A(x_i),I_B(x_i))+\min(F_A(x_i),F_B(x_i))}{\max(T_A(x_i),T_B(x_i))+\max(I_A(x_i),I_B(x_i))+\max(F_A(x_i),F_B(x_i))} \\ &\le \frac{1}{n}\sum_{i=1}^{n}\frac{\min(T_{A'}(x_i),T_{B'}(x_i))+\min(I_{A'}(x_i),I_{B'}(x_i))+\min(F_{A'}(x_i),F_{B'}(x_i))}{\max(T_{A'}(x_i),T_{B'}(x_i))+\max(I_{A'}(x_i),I_{B'}(x_i))+\max(F_{A'}(x_i),F_{B'}(x_i))} \\ &= S_2(A',B'). \end{aligned}$$

Sufficiency: If we have $S_2(A,B) \le S_2(A',B')$, that is to say,

$$\begin{aligned} S_2(A,B) &= \frac{1}{n}\sum_{i=1}^{n}\frac{\min(T_A(x_i),T_B(x_i))+\min(I_A(x_i),I_B(x_i))+\min(F_A(x_i),F_B(x_i))}{\max(T_A(x_i),T_B(x_i))+\max(I_A(x_i),I_B(x_i))+\max(F_A(x_i),F_B(x_i))} \\ &\le \frac{1}{n}\sum_{i=1}^{n}\frac{\min(T_{A'}(x_i),T_{B'}(x_i))+\min(I_{A'}(x_i),I_{B'}(x_i))+\min(F_{A'}(x_i),F_{B'}(x_i))}{\max(T_{A'}(x_i),T_{B'}(x_i))+\max(I_{A'}(x_i),I_{B'}(x_i))+\max(F_{A'}(x_i),F_{B'}(x_i))} \\ &= S_2(A',B'). \end{aligned}$$

Then $\min(T_A(x_i),T_B(x_i))+\min(I_A(x_i),I_B(x_i))+\min(F_A(x_i),F_B(x_i))$
$\le \min(T_{A'}(x_i),T_{B'}(x_i))+\min(I_{A'}(x_i),I_{B'}(x_i))+\min(F_{A'}(x_i),F_{B'}(x_i))$,
$\max(T_A(x_i),T_B(x_i))+\max(I_A(x_i),I_B(x_i))+\max(F_A(x_i),F_B(x_i))$

$\geq \max(T_{A'}(x_i), T_{B'}(x_i)) + \max(I_{A'}(x_i), I_{B'}(x_i)) + \max(F_{A'}(x_i), F_{B'}(x_i))$.
Thus

$$\begin{aligned} S_1(A,B) &= \frac{\sum_{i=1}^{n}[\min(T_A(x_i),T_B(x_i)) + \min(I_A(x_i),I_B(x_i)) + \min(F_A(x_i),F_B(x_i))]}{\sum_{i=1}^{n}[\max(T_A(x_i),T_B(x_i)) + \max(I_A(x_i),I_B(x_i)) + \max(F_A(x_i),F_B(x_i))]} \\ &\leq \frac{\sum_{i=1}^{n}[\min(T_{A'}(x_i),T_{B'}(x_i)) + \min(I_{A'}(x_i),I_{B'}(x_i)) + \min(F_{A'}(x_i),F_{B'}(x_i))]}{\sum_{i=1}^{n}[\max(T_{A'}(x_i),T_{B'}(x_i)) + \max(I_{A'}(x_i),I_{B'}(x_i)) + \max(F_{A'}(x_i),F_{B'}(x_i))]} \\ &= S_1(A',B'). \end{aligned}$$

So $S_1(A,B) \sim S_2(A,B)$.

Similarly, we also can proof that $S_1(A,B) \sim S_3(A,B)$, $S_2(A,B) \sim S_4(A,B)$. Hence we can get $S_1(A,B)$, $S_2(A,B)$, $S_3(A,B)$, and $S_4(A,B)$ are equivalent. □

Proposition 3.6. *Similarity measures $S_J(A,B)$, $S_D(A,B)$, $WS_J(A,B)$, and $WS_D(A,B)$ satisfy the property for any two SVNSs A, B, $S_J(A,B)$, $S_D(A,B)$, $WS_J(A,B)$, and $WS_D(A,B)$ are equivalent.*

Proof. For any four SVNSs A, B, A', B' in $X = \{x_1, x_2, \ldots, x_n\}$, we have $T_A(x), I_A(x), F_A(x) \in [0,1], T_B(x), I_B(x), F_B(x) \in [0,1]$ for each $x \in X$.

Necessity: If we have $S_J(A,B) \leq S_J(A',B')$, that is to say,

$$\begin{aligned} S_J(A,B) &= \frac{1}{n}\sum_{i=1}^{n} \frac{T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i)}{[M-N]} \\ &\leq \frac{1}{n}\sum_{i=1}^{n} \frac{T_{A'}(x_i)T_{B'}(x_i) + I_{A'}(x_i)I_{B'}(x_i) + F_{A'}(x_i)F_{B'}(x_i)}{[M'-N']} \\ &= S_J(A',B'). \end{aligned}$$

There, $M = (T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)) + (T_B^2(x_i) + I_B^2(x_i) + F_B^2(x_i))$,
$N = (T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i))$,
$M' = (T_{A'}^2(x_i) + I_{A'}^2(x_i) + F_{A'}^2(x_i)) + (T_{B'}^2(x_i) + I_{B'}^2(x_i) + F_{B'}^2(x_i))$,
$N' = (T_{A'}(x_i)T_{B'}(x_i) + I_{A'}(x_i)I_{B'}(x_i) + F_{A'}(x_i)F_{B'}(x_i))$.
Then $T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i)$
$\leq T_{A'}(x_i)T_{B'}(x_i) + I_{A'}(x_i)I_{B'}(x_i) + F_{A'}(x_i)F_{B'}(x_i)$,
$(T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)) + (T_B^2(x_i) + I_B^2(x_i) + F_B^2(x_i))$
$\geq (T_{A'}^2(x_i) + I_{A'}^2(x_i) + F_{A'}^2(x_i)) + (T_{B'}^2(x_i) + I_{B'}^2(x_i) + F_{B'}^2(x_i))$.
Thus

$$\begin{aligned} S_D(A,B) &= \frac{1}{n}\sum_{i=1}^{n} \frac{2(T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i))}{(T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)) + (T_B^2(x_i) + I_B^2(x_i) + F_B^2(x_i))} \\ &\leq \frac{1}{n}\sum_{i=1}^{n} \frac{2(T_{A'}(x_i)T_{B'}(x_i) + I_{A'}(x_i)I_{B'}(x_i) + F_{A'}(x_i)F_{B'}(x_i))}{(T_{A'}^2(x_i) + I_{A'}^2(x_i) + F_{A'}^2(x_i)) + (T_{B'}^2(x_i) + I_{B'}^2(x_i) + F_{B'}^2(x_i))} \\ &= S_D(A',B'). \end{aligned}$$

Sufficiency: If we have $S_D(A,B) \le S_D(A',B')$, that is to say,

$$\begin{aligned} S_D(A,B) &= \frac{1}{n}\sum_{i=1}^{n} \frac{2(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i))}{(T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i))+(T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i))} \\ &\le \frac{1}{n}\sum_{i=1}^{n} \frac{2(T_{A'}(x_i)T_{B'}(x_i)+I_{A'}(x_i)I_{B'}(x_i)+F_{A'}(x_i)F_{B'}(x_i))}{(T_{A'}^2(x_i)+I_{A'}^2(x_i)+F_{A'}^2(x_i))+(T_{B'}^2(x_i)+I_{B'}^2(x_i)+F_{B'}^2(x_i))} \\ &= S_D(A',B'). \end{aligned}$$

Then

$$\begin{aligned} & T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i) \\ &\le T_{A'}(x_i)T_{B'}(x_i)+I_{A'}(x_i)I_{B'}(x_i)+F_{A'}(x_i)F_{B'}(x_i), \\ & (T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i))+(T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i)) \\ &\ge (T_{A'}^2(x_i)+I_{A'}^2(x_i)+F_{A'}^2(x_i))+(T_{B'}^2(x_i)+I_{B'}^2(x_i)+F_{B'}^2(x_i)), \\ & [(T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i))+(T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i)) \\ & \quad -(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i))] \\ &\ge [(T_{A'}^2(x_i)I_{A'}^2(x_i)+F_{A'}^2(x_i))+(T_{B'}^2(x_i)+I_{B'}^2(x_i)+F_{B'}^2(x_i)) \\ & \quad -(T_{A'}(x_i)T_{B'}(x_i)+I_{A'}(x_i)I_{B'}(x_i)+F_{A'}(x_i)F_{B'}(x_i))]. \end{aligned}$$

Thus

$$\begin{aligned} S_J(A,B) &= \frac{1}{n}\sum_{i=1}^{n} \frac{T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i)}{[M-N]} \\ &\le \frac{1}{n}\sum_{i=1}^{n} \frac{T_{A'}(x_i)T_{B'}(x_i)+I_{A'}(x_i)I_{B'}(x_i)+F_{A'}(x_i)F_{B'}(x_i)}{[M'-N']} \\ &= S_J(A',B'). \end{aligned}$$

There,

$$\begin{aligned} M &= (T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i))+(T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i)), \\ N &= (T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i)), \\ M' &= (T_{A'}^2(x_i)+I_{A'}^2(x_i)+F_{A'}^2(x_i))+(T_{B'}^2(x_i)+I_{B'}^2(x_i)+F_{B'}^2(x_i)), \\ N' &= (T_{A'}(x_i)T_{B'}(x_i)+I_{A'}(x_i)I_{B'}(x_i)+F_{A'}(x_i)F_{B'}(x_i)). \end{aligned}$$

So $S_J(A,B) \sim S_D(A,B)$. Similarly, we also can proof that $S_J(A,B) \sim WS_J(A,B)$, $S_D(A,B) \sim WS_D(A,B)$. Hence we can get $S_J(A,B)$, $S_D(A,B)$, $WS_J(A,B)$, and $WS_D(A,B)$ are equivalent. □

Example 3.7. We consider four SVNSs A, B, A' and B' in X and compare their similarity measures that if $1-D_H(A,B)$ is equivalent to $1-D_{NH}(A,B)$. Assume that there are four SVNSs in X, $A=\{\langle x,0.0,0.1,0.9\rangle|x\in X\}$, $B=\{\langle x,0.4,0.5,0.6\rangle|x\in X\}$, $A'=\{\langle x,0.7,0.3,0.0\rangle|x\in X\}$ and $B'=\{\langle x,0.3,0.2,0.5\rangle|x\in X\}$.

For any four $SVNSs$ A, B, A', B' in $X=\{x\}$, we have
$|T_A(x_i)-T_B(x_i)|=0.4$, $|I_A(x_i)-I_B(x_i)|=0.4$, $|F_A(x_i)-F_B(x_i)|=0.3$,
$|T_{A'}(x_i)-T_{B'}(x_i)|=0.4$, $|I_{A'}(x_i)-I_{B'}(x_i)|=0.1$, $|F_{A'}(x_i)-F_{B'}(x_i)|=0.5$.

Then, we know that

$$\begin{aligned} D_H(A,B) &= \frac{1}{n}\sum_{i=1}^{n} max[|T_A(x_i)-T_B(x_i)|,|I_A(x_i)-I_B(x_i)|,|F_A(x_i)-F_B(x_i)|] \\ &= max[0.4,0.4,0.3] \\ &= 0.4, \end{aligned}$$

$$\begin{aligned} D_H(A', B') &= \frac{1}{n}\sum_{i=1}^{n} max[|T_{A'}(x_i) - T_{B'}(x_i)|, |I_{A'}(x_i) - I_{B'}(x_i)|, |F_{A'}(x_i) - F_{B'}(x_i)|] \\ &= max[0.4, 0.1, 0.5] \\ &= 0.5. \end{aligned}$$

Thus $D_H(A, B) \leq D_H(A', B')$.

$$\begin{aligned} D_{NH}(A, B) &= \frac{1}{3n}\sum_{i=1}^{n}[|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|] \\ &= \frac{1}{3}[0.4 + 0.4 + 0.3] \\ &= \frac{11}{30}, \end{aligned}$$

$$\begin{aligned} D_{NH}(A', B') &= \frac{1}{3n}\sum_{i=1}^{n}[|T_{A'}(x_i) - T_{B'}(x_i)| + |I_{A'}(x_i) - I_{B'}(x_i)| + |F_{A'}(x_i) - F_{B'}(x_i)|] \\ &= \frac{1}{3}[0.4 + 0.1 + 0.5] \\ &= \frac{1}{3}. \end{aligned}$$

So $D_{NH}(A, B) \geq D_{NH}(A', B')$.
When $D_H(A, B) \leq D_H(A', B')$, we find that $D_{NH}(A, B) \geq D_{NH}(A', B')$. Hence $D_H(A, B)$ and $D_{NH}(A, B)$ are not equivalent, similarity measures $1 - D_H(A, B)$ and $1 - D_{NH}(A, B)$ are not equivalent.

4. Conclusions

SVNSs are applied to problems with imprecise, uncertain, incomplete and inconsistent information existing in the real world. Although the similarity measures of SVNSs are proposed and applied to decision making, pattern recognition, and medical diagnosis, the relationships among these similarity measures have not been systematically investigated. In this paper, we propose the definition of equivalence of similarity measures on the basis of the ordering of objects of the SVNSs has not changed. Then, we investigate the relationships among some similarity measures of SVNSs, such as distance based similarity measures, the min and max operators based similarity measures, and vector similarity measures in terms of inequality and equivalence in detail. We prove that the distance $D_{NH}(A, B)$, and $D_{NE}(A, B)$ are equivalent; the min and max operators based similarity measures $S_1(A, B)$, $S_2(A, B)$, $S_3(A, B)$, and $S_4(A, B)$ are equivalent; vector similarity measures $S_J(A, B)$, $S_D(A, B)$, $WS_J(A, B)$, and $WS_D(A, B)$ are equivalent. Finally we demonstrate the effectiveness of the equivalence by a example.

In future work, we will discuss the applications of equivalence of similarity measures in other areas such as multi-attribute decision making, medical diagnosis, fault diagnosis and so on. At the same time, it is necessary and meaningful to study the relationships among similarity measures of different types because of our method to compare these similarity measures is limited in the same types. At the end of this paper, we hope that these conclusions can bring some new enlightenments to the related research.

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