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Row-products of soft matrices with applications in multiple-disjoint decision making

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ABSTRACT

In this paper, we firstly introduce row-products of the soft matrices and investigate their properties and algebraic structures in detail. We aim to show that these row-products can be used in handling decision making problems. We therefore propose two new methods called a soft max-row decision making method and a multi-soft distributive max–min decision making method employing these operations. These methods are utilized to obtain an optimum choice when the decision makers evaluate the objects of disjoint universe sets according to the parameters during decision making. Also, we argue that the first of them can be employed to solve the decision problems handled in [6,16]. The second method that we propose to solve the decision problems involving multi-disjoint universe sets is a generalization of the soft decision method presented in [9]. By constructing them, we pioneer the idea that the soft matrices can be used to deal with decision making involving the multi-disjoint universe sets, which it is shortly called a multiple-disjoint decision making. Moreover, we present the outstanding examples to verify the practicality and effectiveness of the emerging methods. Finally, we give Scilab codes for each step of our methods and put forward that these codes make the process of decision making faster and easier.

1. Introduction

Most of the existing generic principles for modeling, computation and reasoning usually depend on certain and precise data. However, almost all of practical problems within fields such as economics, environment,engineering, medical science, social science involve vague, uncertain, imprecise or unknown data. We cannot successfully utilize these principles because of various types of uncertainties existing in these problems. To deal with such problems, special mathematical principles such as fuzzy set theory [49], rough set theory [36], vague set theory [20] and interval mathematics [22] are developed. Moreover, there are a great amount of research and applications concerning these mathematical principles. Recently, Molodtsov [34] initiated the concept of the soft set as an effective mathematical principle to cope the difficulties based on the uncertainties. Especially, the soft set theory and its applications have attracted the interest of many researchers in handling problems mixed all kinds of vagueness. The interest in this theory is still growing rapidly. As reviewed in [34], a wide range of applications of soft set have been developed in various fields including the smoothness of functions, operations research, Riemann integration, Perron integration, game theory, measurement theory, probability theory and etc. In the following, we briefly review the mainly studies on the soft set theory in existing literatures.

After Molodtsov [34] and then Biswas et al. [31] pioneered the studies on the operations of the soft sets, many researchers explored thoroughly the operations, properties and applications of soft set theory. Ali et al. [5] introduced some operations of the soft sets such as the restricted union, restricted intersection, extended intersection and restricted difference, and also improved the notion of complement of a soft set. Sezgin and Atagün [41] derived the properties of some operations on soft sets, and proved that De Morgan's law valid for the different operations. Jiang et al. [28] defined the concept of extended soft set employing the notion of Description Logics to act the parameters of soft sets, and then described some operations of this concept. In [38], the definition of soft equality and some related properties were introduced. Majumdar and Samanta [33] studied on the similarity of soft sets, and presented two types of similarity measure between the soft sets. Konkov et al. [30] focused the soft set theory based on optimization. In [37,45], the notion of soft information based on the

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soft set theory was developed. Mushrif et al. [35] proposed a novel algorithm based on the concept of soft set for classification of the natural textures. Xiao et al. [47] presented a synthetically evaluating method based on the soft set theory to overcome the deficiency of traditional method deal with problem containing uncertainty. Also, algebraic and topological structures of the soft sets have been studied increasingly in recent years. Aktaş and Çağman [3,4] first introduced the concept of soft group and discussed several properties employing Molodtsov's definition of the soft set. Atagün and Aygün [8] argued that the set of all soft sets on the universe set is an abelian group under the each operations named "the inverse group of soft sets" and "the characteristic group of soft sets". Feng et al. [17] gave the definitions of soft semirings, soft subsemirings, soft ideals, idealistic soft semirings and soft semiring homomorphisms. Acar et al. [1] initiated the soft ring. Atagün and Sezgin [10] developed the soft subrings and soft ideals of a ring, the soft subfields of a field and the soft submodule of a left R-module. Also, they investigated basic properties about soft substructures of the rings, the fields and the modules. Hazra et al. [24] introduced the concept of soft topology on a soft set, and thus initiated the foundations of theory of soft topological spaces. In [42], the definitions of soft closed sets, soft open sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms were given in detail. Shi and Pang [43] noted that there are some errors in [42], and they also discussed the properties of soft separation axioms and soft regular spaces.

Up to the present, there has been many practical application of the soft set theory, especially the use of soft set in decision making. Maji et al. [32] introduced representations in the form of binary information table of the soft sets. They showed that these representations can be used to find optimal object when a decision maker evaluates the objects of a universe set under the specified criteria. Thus, they pioneered the soft set theory based on the decision making. Zhou and Xiao [50] presented data analysis approaches of the soft set under incomplete information, and then calculated the decision value of an object with incomplete information for the standard soft sets. In [15,29], the parameter reduction algorithm eliminating some of the parameters of a soft set was constructed. In [25], an approach for attribute reduction in multi-valued information system under the soft set theory was proposed. In [48], they introduced the parameterization reduction of the fuzzy soft set. Also, all of these researchers dealing with parameter reduction argued that this notion facilitates the process of decision making. Gong et al. [21] defined the concept of bijective soft set and analyzed the availability of the bijective soft set in decision making problems. Xiao et al. [46] developed the exclusive disjunctive soft set which is a special form of the bijective soft set, and posited that it can be applied in information systems. Çağman and Enginoğlu [13] derived some products of the soft sets. By employing these products, they proposed $uni-\mathrm{int}$ decision making algorithm which can be used to choose the optimal object when two decision makers evaluate the objects of a universe set under the described parameters. Feng et al. [18] constructed novel decision making schemes named the $uni-\mathrm{int}^k$, $uni-\mathrm{int}_s^t$ and $\mathrm{int}^m-\mathrm{int}^n$ improving the $uni-\mathrm{int}$ decision making method. In [14], Çağman and Enginoğlu defined the concept of soft matrix which is a matrix representation of the soft set, and then investigated some related operations such as intersection, union, and product, or product, and-not product and or-not product. Also, they established a new algorithm called soft max–min decision making algorithm which is proposed to deal with the decision making problem mentioned in [13]. Atagün et al. [9] generalized the products derived in [14] for the different types of soft matrices and proposed a soft distributive max–min decision algorithm improving the algorithm presented in [14]. Basu et al. [12] developed the different types of matrices in the soft set theory and suggested a novel efficient procedure to solve the decision making problems that may contain more than two decision makers. Feng and Zhou [19] introduced the concepts of soft discernibility matrix and weighted soft discernibility matrix, and offered a new decision algorithm in handling decision making. Also, the efficiency of the method was tested to find the optimum choice of a decision maker analyzing the objects of a universe set according to his own parameters. Moreover, multi-criteria decision making approaches were studied and their applications were presented [23,27,39,40]. All of the above-mentioned decision methods aimed to solve the decision problems which focus on finding the optimal object according to the parameters from a universe set.

In [7], the notion of soft multiset theory as a generalization of the soft set theory was introduced. Afterwards, Alkhazaleh and Salleh [6] developed the fuzzy soft multiset as a combination of the fuzzy set and the soft multiset, and Deli et al. [16] defined the concept of neutrosophic soft multiset. They proposed the decision procedures employing the fuzzy soft multiset and the neutrosophic soft multiset to deal with the decision making problems which focus on finding the optimum choice with respect to the observations of experts under the choice parameters from each of three universe sets. It is worthwhile to mention that the proposed procedures pioneered to solve the decision making problems involving the multi-disjoint universe sets.

Until now, it was not possible to solve this kind of the decision problems by using the standard soft set and soft matrix. The goal of this paper is to present the decision algorithms based on the soft matrices which provide both efficiency and speed in computations to solve these decision problems. Relatedly, we propose two new decision algorithms in this paper. One of them focuses to solve the decision making problems tackled in [6,16]. The other method that we propose to deal with decision making problems involving the multi disjoint universe sets is a generalization of the soft distributive max–min decision making method [9] which is the improved version of the decision method presented in [14].

This work is organized as follows. Section 2 is established on the basic information about the soft set theory and soft matrix theory. In Section 3, we derive the row-products of soft matrices such as And, And-Not, Or, Or-Not and then discuss their related properties and algebraic structures. In Sections 4 and 5, we construct two reliable decision making algorithms named a soft max-row decision making algorithm and a multi-soft distributive max–min decision making algorithm. Moreover, we give various applications to analyze their performances and results.

2. Preliminaries

Throughout this work, U is an initial universe set, E is a set of parameters, $P(U)$ is the power set of U and $A \subseteq E$.
Molodtsov [34] defined the concept of soft set as follows:

Definition 1. [34]

A pair (F, A) is called a soft set over U, where F is a mapping given by

$$F : A \to P(U).$$

A soft set (F, A) will be denoted by F_A. According to this definition a soft set F_A can be written as a set of ordered pairs

$$F_A = \{(x, F(x)) | x \in E, F(x) \in P(U)\}$$

where $F(x) = \emptyset$ if $x \notin A$.

In other words, a soft set over U is a parameterized family of subsets of the universe U.

Notation: The set of all soft sets over common universe U will be denoted by $S(U)$.

Definition 2. [14]

Let $U = \{u_1, u_2, \ldots, u_n\}$, $E = \{e_1, e_2, \ldots, e_m\}$, $A \subseteq E$ and let F_A be a soft set over U. If

$$a_{ip} = \begin{cases} 1, & \text{if} \quad u_i \in F(e_p) \\ 0, & \text{if} \quad u_i \notin F(e_p) \end{cases}$$

then the matrix

$$[a_{ip}]_{n \times m} = \begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1m} \\ a_{21} & a_{22} & . & . & . & a_{2m} \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ a_{n1} & a_{n2} & . & . & . & a_{nm} \end{bmatrix}.$$

is called an $n \times m$ soft matrix of the soft set F_A over U. The set of all $n \times m$ soft matrices over U will be denoted by $SM_{n \times m}$. From now on, $[a_{ip}] \in SM_{n \times m}$ means that $[a_{ip}]$ is an $n \times m$ soft matrix.

According to the definition of soft matrix, a soft set F_A is uniquely characterized by the matrix $[a_{ip}]$. It means that a soft set F_A is formally equal to its soft matrix $[a_{ip}]$ [14].

Example 1. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be a universe set and $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ be a set of parameters. If $A = \{e_1, e_3, e_5\}$ and $F : A \to P(U)$ such that $F(e_1) = \{u_1, u_2, u_5\}$, $F(e_3) = \{u_2, u_3, u_4, u_6\}$, $F(e_5) = \{u_6\}$ then we write a soft set

$$F_A = \{(e_1, \{u_1, u_2, u_5\}), (e_3, \{u_2, u_3, u_4, u_6\}), (e_5, \{u_6\})\}.$$

Hence, the soft matrix $[a_{ip}] \in SM_{6 \times 6}$ corresponding to the soft set F_A is

$$[a_{ip}] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}.$$

Definition 3. [14]

Let $[a_{ip}], [b_{ip}] \in SM_{n \times m}$.

1. If $c_{ip} = \max\{a_{ip}, b_{ip}\}$ for all i, p, then the soft matrix $[c_{ip}]$ is said to be union of $[a_{ip}]$ and $[b_{ip}]$ and it is denoted by $[c_{ip}] = [a_{ip}]\tilde{\cup}[b_{ip}]$.
2. If $c_{ip} = \min\{a_{ip}, b_{ip}\}$ for all i, p, then the soft matrix $[c_{ip}]$ is said to be intersection of $[a_{ip}]$ and $[b_{ip}]$ and it is denoted by $[c_{ip}] = [a_{ip}]\tilde{\cap}[b_{ip}]$.
3. If $c_{ip} = 1 - a_{ip}$ for all i, p, then the soft matrix $[c_{ip}]$ is said to be complement of $[a_{ip}]$ and it is denoted by $[c_{ip}] = [a_{ip}]^c$.

Definition 4. [9]

Let $U = \{u_1, u_2, \ldots, u_n\}$, $E = \{e_1, e_2, \ldots, e_m\}$, $A \subseteq E$ and let cardinality of the set A be m_1. Consider the soft set F_A. If

$$a_{ip} = \begin{cases} 1, & \text{if} \quad e_p \in A \quad \text{and} \quad u_i \in F(e_p) \\ 0, & \text{if} \quad e_p \in A \quad \text{and} \quad u_i \notin F(e_p) \end{cases}$$

then the matrix $[a_{ip}]_{n \times m_1}$ is called a *reduced soft matrix* of the soft set F_A over U. Here $1 \le m_1 \le m$.

In other words, let the numbers of elements of U, E and A be n, m and m_1, respectively. Let F_A be a soft set over U. Since $x \notin A$ implies $F(x) = \emptyset$, by eliminating the parameters in $E \backslash A$ from the set E, we can construct the soft matrix corresponding to soft set F_A, and then the type of this soft matrix will be $n \times m_1$.

Example 2. Consider the soft set F_A given in Example 1. Then, the reduced soft matrix $[a_{ip}] \in SM_{6 \times 3}$ of F_A is

$$[a_{ip}] = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

Let $[a_{ip}] \in SM_{n\times m_1}$ and $[b_{ir}] \in SM_{n\times m_2}$ be (reduced) soft matrices of the soft sets F_A and F_B over common universe U, respectively.

Definition 5. [9]

Generalized And-product of $[a_{ip}]$ and $[b_{ir}]$ denoted by $\curlywedge$ is defined by

$$\begin{aligned} \curlywedge : SM_{n\times m_1} \times SM_{n\times m_2} &\longrightarrow SM_{n\times m_1 m_2} \\ [a_{ip}], \quad [b_{ir}] &\longrightarrow [a_{ip}] \curlywedge [b_{ir}] = [c_{is}] \end{aligned}$$

where $c_{is} = \min\{a_{ip}, b_{ir}\}$ such that $p = \alpha$, $s = (\alpha - 1)m_2 + r$ and α is the smallest positive integer which satisfies $s \le \alpha m_2$.

Definition 6. [9]

Generalized Or-product of $[a_{ip}]$ and $[b_{ir}]$ denoted by $\curlyvee$ is defined by

$$\begin{aligned} \curlyvee : SM_{n\times m_1} \times SM_{n\times m_2} &\longrightarrow SM_{n\times m_1 m_2} \\ [a_{ip}], \quad [b_{ir}] &\longrightarrow [a_{ip}] \curlyvee [b_{ir}] = [c_{is}] \end{aligned}$$

where $c_{is} = \max\{a_{ip}, b_{ir}\}$ such that $p = \alpha$, $s = (\alpha - 1)m_2 + r$ and α is the smallest positive integer which satisfies $s \le \alpha m_2$.

Definition 7. [9]

Generalized And-Not-product of $[a_{ip}]$ and $[b_{ir}]$ denoted by $\bar{\curlywedge}$ is defined by

$$\begin{aligned} \bar{\curlywedge} : SM_{n\times m_1} \times SM_{n\times m_2} &\longrightarrow SM_{n\times m_1 m_2} \\ [a_{ip}], \quad [b_{ir}] &\longrightarrow [a_{ip}] \bar{\curlywedge} [b_{ir}] = [c_{is}] \end{aligned}$$

where $c_{is} = \min\{a_{ip}, 1 - b_{ir}\}$ such that $p = \alpha$, $s = (\alpha - 1)m_2 + r$ and α is the smallest positive integer which satisfies $s \le \alpha m_2$.

Definition 8. [9]

Generalized Or-Not-product of $[a_{ip}]$ and $[b_{ir}]$ denoted by $\underline{\curlyvee}$ is defined by

$$\begin{aligned} \underline{\curlyvee} : SM_{n\times m_1} \times SM_{n\times m_2} &\longrightarrow SM_{n\times m_1 m_2} \\ [a_{ip}], \quad [b_{ir}] &\longrightarrow [a_{ip}] \underline{\curlyvee} [b_{ir}] = [c_{is}] \end{aligned}$$

where $c_{is} = \max\{a_{ip}, 1 - b_{ir}\}$ such that $p = \alpha$, $s = (\alpha - 1)m_2 + r$ and α is the smallest positive integer which satisfies $s \le \alpha m_2$.

In Definitions 5–8, we note that the type of soft matrix $[c_{is}]$ is $n \times m_1 m_2$.

Example 3. Suppose that $[a_{ip}] \in SM_{5\times 4}$ and $[b_{ir}] \in SM_{5\times 3}$ are given as follows:

$$[a_{ip}] = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \text{ and } [b_{ir}] = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

Then, we obtain

$$[c_{is}] = [a_{ip}] \curlywedge [b_{ir}] = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Therefore, the type of soft matrix $[c_{is}]$ is 5×12.

3. Row-products of (reduced) soft matrices

The parameter set E is taken as $|E| = m$, that's, it consists of m-parameters. Consider $A \subseteq E$ ($|A| = m' \le m$), $|U_1| = n_1$ and $|U_2| = n_2$. Let $F_A \in S(U_1)$ and $G_A \in S(U_2)$, and let $[a_{ip}]$ and $[b_{jp}]$ be (reduced) soft matrices of F_A and G_A, respectively.

Definition 9. And row-product of $[a_{ip}]$ and $[b_{jp}]$ denoted by $\curlywedge_r$ is defined by

$$\begin{aligned}\curlywedge_r : SM_{n_1\times m'} \times SM_{n_2\times m'} &\longrightarrow SM_{n_1 n_2\times m'}\\ [a_{ip}],\quad [b_{jp}] &\longrightarrow [a_{ip}]\curlywedge_r[b_{jp}] = [c_{vp}]\end{aligned}$$

where $c_{vp} = \min\{a_{ip}, b_{jp}\}$ such that $i = \beta$, $v = (\beta-1)n_2 + j$ and β is the smallest positive integer which satisfies $v \le \beta n_2$.

Definition 10. Or row-product of $[a_{ip}]$ and $[b_{jp}]$ denoted by $\curlyvee_r$ is defined by

$$\begin{aligned}\curlyvee_r : SM_{n_1\times m'} \times SM_{n_2\times m'} &\longrightarrow SM_{n_1 n_2\times m'}\\ [a_{ip}],\quad [b_{jp}] &\longrightarrow [a_{ip}]\curlyvee_r[b_{jp}] = [c_{vp}]\end{aligned}$$

where $c_{vp} = \max\{a_{ip}, b_{jp}\}$ such that $i = \beta$, $v = (\beta-1)n_2 + j$ and β is the smallest positive integer which satisfies $v \le \beta n_2$.

Definition 11. And-Not row-product of $[a_{ip}]$ and $[b_{jp}]$ denoted by $\bar{\curlywedge}_r$ is defined by

$$\begin{aligned}\bar{\curlywedge}_r : SM_{n_1\times m'} \times SM_{n_2\times m'} &\longrightarrow SM_{n_1 n_2\times m'}\\ [a_{ip}],\quad [b_{jp}] &\longrightarrow [a_{ip}]\bar{\curlywedge}_r[b_{jp}] = [c_{vp}]\end{aligned}$$

where $c_{vp} = \min\{a_{ip}, 1-b_{jp}\}$ such that $i = \beta$, $v = (\beta-1)n_2 + j$ and β is the smallest positive integer which satisfies $v \le \beta n_2$.

Definition 12. Or-Not row-product of $[a_{ip}]$ and $[b_{jp}]$ denoted by $\underline{\curlyvee_r}$ is defined by

$$\begin{aligned}\underline{\curlyvee_r} : SM_{n_1\times m'} \times SM_{n_2\times m'} &\longrightarrow SM_{n_1 n_2\times m'}\\ [a_{ip}],\quad [b_{jp}] &\longrightarrow [a_{ip}]\underline{\curlyvee_r}[b_{jp}] = [c_{vp}]\end{aligned}$$

where $c_{vp} = \max\{a_{ip}, 1-b_{jp}\}$ such that $i = \beta$, $v = (\beta-1)n_2 + j$ and β is the smallest positive integer which satisfies $v \le \beta n_2$.

Example 4. Let $U_1 = \{x_1, x_2, x_3, x_4\}$ and $U_2 = \{y_1, y_2, y_3\}$ be two universe sets and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters. If the soft sets are created as $F_A = \{(e_2, \{x_3, x_4\}), (e_3, \{x_1, x_3, x_4\}), (e_4, \{x_1\}), (e_5, \emptyset)\}$ and $G_A = \{(e_2, \emptyset), (e_3, \{y_1, y_3\}), (e_4, U), (e_5, \{y_1\})\}$ for the set $A = \{e_2, e_3, e_4, e_5\} \subset E$, then the soft matrices $[a_{ip}] \in SM_{4\times 4}$ and $[b_{jp}] \in SM_{3\times 4}$ corresponding to the soft sets F_A and G_A are obtained as follows:

$$[a_{ip}] = \begin{bmatrix} 0 & 1 & 1 & 0\\ 0 & 0 & 0 & 0\\ 1 & 1 & 0 & 0\\ 1 & 1 & 0 & 0\end{bmatrix} \text{ and } [b_{jp}] = \begin{bmatrix} 0 & 1 & 1 & 1\\ 0 & 0 & 1 & 0\\ 0 & 1 & 1 & 0\end{bmatrix}.$$

Then, the row product of the soft matrices is

$$[c_{vp}] = [a_{ip}]\curlywedge_r[b_{jp}] = \begin{bmatrix} 0 & 1 & 1 & 0\\ 0 & 0 & 1 & 0\\ 0 & 1 & 1 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}.$$

The type of soft matrix $[c_{vp}]$ is 12×5. Also, the soft set corresponding to the soft matrix $[c_{vp}]$ is as follows:

$$H_A = \{(e_1, \emptyset), (e_2, \emptyset), (e_3, \{(x_1, y_1), (x_1, y_3), (x_3, y_1), (x_3, y_3), (x_4, y_1), (x_4, y_3)\}), (e_4, \{(x_1, y_1), (x_1, y_2), (x_1, y_3)\}), (e_5, \emptyset)\} \in S(U_1 \times U_2).$$

Remark 1. In general, the row-products of soft matrices are not commutative.

Example 5. Consider the soft matrices $[a_{ip}]$, $[b_{jp}]$ and $[c_{vp}]$ given in Example 4. Then, it is obtained the soft matrix

$$[d_{wp}] = [b_{jp}] \curlywedge_r [a_{ip}] = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

Proposition 1. *Let $[a_{ip}] \in SM_{n_1 \times m'}$, $[b_{jp}] \in SM_{n_2 \times m'}$ be two soft matrices. Then, the following De Morgan's Laws are valid for the row-products:*

(i) $([a_{ip}] \curlyvee_r [b_{jp}])^c = ([a_{ip}])^c \curlywedge_r ([b_{jp}])^c$
(ii) $([a_{ip}] \curlywedge_r [b_{jp}])^c = ([a_{ip}])^c \curlyvee_r ([b_{jp}])^c$
(iii) $([a_{ip}] \underline{\curlyvee_r} [b_{jp}])^c = ([a_{ip}])^c \bar{\curlywedge}_r ([b_{jp}])^c$
(iv) $([a_{ip}] \bar{\curlywedge}_r [b_{jp}])^c = ([a_{ip}])^c \underline{\curlyvee_r} ([b_{jp}])^c$

Proof. (i) Let $[a_{ip}] \in SM_{n_1 \times m'}$, $[b_{jp}] \in SM_{n_2 \times m'}$ be two soft matrices and let $[c_{kp}] = ([a_{ip}] \curlyvee_r [b_{jp}])^c$, $[d_{lp}] = ([a_{ip}])^c \curlywedge_r ([b_{jp}])^c$. Since the types of $[c_{kp}]$ and $[d_{lp}]$ are $n_1 n_2 \times m'$, and $c_{kp} = 1 - \max\{a_{ip}, b_{jp}\} = \min\{1 - a_{ip}, 1 - b_{jp}\} = d_{lp}$ for all i, j, p by Definitions 9 and 10, we have $[c_{kp}] = [d_{lp}]$.

The parts (ii), (iii) and (iv) can be proved similarly, hence omitted. □

Theorem 1. *The operation And row-product is associative. That's, let $[a_{ip}] \in SM_{n_1 \times m'}$, $[b_{jp}] \in SM_{n_2 \times m'}$ and $[c_{kp}] \in SM_{n_3 \times m'}$ then*

$$([a_{ip}] \curlywedge_r [b_{jp}]) \curlywedge_r [c_{kp}] = [a_{ip}] \curlywedge_r ([b_{jp}] \curlywedge_r [c_{kp}]).$$

Proof. Let $[a_{ip}] \in SM_{n_1 \times m'}$, $[b_{jp}] \in SM_{n_2 \times m'}$ and $[c_{kp}] \in SM_{n_3 \times m'}$. By Definition 9, we can write $[a_{ip}] \curlywedge_r [b_{jp}] = [d_{vp}] \in SM_{n_1 n_2 \times m'}$ where $d_{vp} = \min\{a_{ip}, b_{jp}\}$ such that $i = \beta_1$, $v = (\beta_1 - 1)n_2 + j$ and β_1 is the smallest positive integer which satisfies $v \leq \beta_1 n_2$. In the same way, we can write $[d_{vp}] \curlywedge_r [c_{kp}] = [e_{wp}] \in SM_{n_1 n_2 n_3 \times m'}$ where $e_{wp} = \min\{d_{vp}, c_{kp}\}$ such that $v = \beta_2$, $w = (\beta_2 - 1)n_3 + k$ and β_2 is the smallest positive integer which satisfies $w \leq \beta_2 n_3$. We then obtain

$$\begin{aligned} w &= (\beta_2 - 1)n_3 + k \\ &= (v - 1)n_3 + k \\ &= ((\beta_1 - 1)n_2 + j - 1)n_3 + k \\ &= (\beta_1 - 1)n_2 n_3 + (j - 1)n_3 + k \\ &= (i - 1)n_2 n_3 + (j - 1)n_3 + k \end{aligned} \tag{1}$$

Similarly, we can write $[b_{jp}] \curlywedge_r [c_{kp}] = [f_{up}] \in SM_{n_2 n_3 \times m'}$ where $f_{up} = \min\{b_{jp}, c_{kp}\}$ such that $j = \beta_3$, $u = (\beta_3 - 1)n_3 + k$ and β_3 is the smallest positive integer which satisfies $u \leq \beta_3 n_3$. In the same way, we can write $[a_{ip}] \curlywedge_r [f_{up}] = [g_{zp}] \in SM_{n_1 n_2 n_3 \times m'}$ where $g_{zp} = \min\{a_{ip}, f_{up}\}$ such that $i = \beta_4$, $z = (\beta_4 - 1)n_2 n_3 + u$ and β_4 is the smallest positive integer which satisfies $z \leq \beta_4 n_2 n_3$. We then obtain

$$\begin{aligned} z &= (\beta_4 - 1)n_2 n_3 + u \\ &= (\beta_4 - 1)n_2 n_3 + (\beta_3 - 1)n_3 + k \\ &= (i - 1)n_2 n_3 + (j - 1)n_3 + k \end{aligned} \tag{2}$$

Therefore, we obtain $[e_{wp}] = [g_{zp}]$ by (1), (2) and

$$\min\{\min\{a_{ip}, b_{jp}\}, c_{kp}\} = \min\{a_{ip}, \min\{b_{jp}, c_{kp}\}\}.$$

Theorem 2. *The operation Or row-product is associative. That's, let $[a_{ip}] \in SM_{n_1 \times m'}$, $[b_{jp}] \in SM_{n_2 \times m'}$ and $[c_{kp}] \in SM_{n_3 \times m'}$ then*

$$([a_{ip}] \curlyvee_r [b_{jp}]) \curlyvee_r [c_{kp}] = [a_{ip}] \curlyvee_r ([b_{jp}] \curlyvee_r [c_{kp}]).$$

Proof. It can be proved similarly to the proof of Theorem 1. □

The operation And-Not row-product and Or-Not row-product are not associative. To verify this, we present the following example:

Example 6. Assume that $[a_{ip}] \in SM_{2\times 4}$, $[b_{jp}] \in SM_{3\times 4}$ and $[c_{kp}] \in SM_{1\times 4}$ are given as below:

$$[a_{ip}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \ [b_{jp}] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \text{ and } [c_{kp}] = \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix}.$$

$$([a_{ip}]\bar{\curlywedge}_r[b_{jp}])\bar{\curlywedge}_r[c_{kp}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ [a_{ip}]\bar{\curlywedge}_r([b_{jp}]\bar{\curlywedge}_r[c_{kp}]) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

Likewise, it is obtained that $([a_{ip}]\underline{\curlyvee}_r[b_{jp}])\underline{\curlyvee}_r[c_{kp}] \neq [a_{ip}]\underline{\curlyvee}_r([b_{jp}]\underline{\curlyvee}_r[c_{kp}])$ since

$$([a_{ip}]\underline{\curlyvee}_r[b_{jp}])\underline{\curlyvee}_r[c_{kp}] = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}, \ [a_{ip}]\underline{\curlyvee}_r([b_{jp}]\underline{\curlyvee}_r[c_{kp}]) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

Proposition 2.

(i) *According to operation And row-product,* $[1]_{1\times k}$ *is a unit element of* $SM_{n\times k}$ *for each* $k \leq m$.
(ii) *According to operation Or row-product,* $[0]_{1\times k}$ *is a unit element of* $SM_{n\times k}$ *for each* $k \leq m$.

Proof.

(i) Let $[a_{ip}] \in SM_{n\times k}$ be a soft matrix and let $[c_{vp}] = [a_{ip}]\bar{\curlywedge}_r[1]_{1\times k}$. Since the type of soft matrix $[c_{vp}]$ is $n \times k$ and $c_{vp} = \min\{a_{ip}, 1\} = a_{ip}$ by Definition 9, it is obtained that $[c_{vp}] = [a_{ip}]$.

(ii) It can be proved similarly. □

By Theorem 1 and 2 and Proposition 2, we have the following theorem.

Theorem 3.

(i) *According to operation And row-product,* $SM_{n\times k}$ *is a monoid.*
(ii) *According to operation Or row-product,* $SM_{n\times k}$ *is a monoid.*

4. Soft max-row decision making

In this section, we firstly define soft max-row function. Later on, we construct a soft max-row decision making (*SMrDM*) algorithm. By employing the algorithm, it can be selected the compatible optimal objects from the multi-disjoint universe sets under the choice parameters. Lastly, we give an example for better understanding of the steps of the algorithm.

Definition 13. Let $[a_{ip}] \in SM_{n\times m'}$ be a soft matrix. A soft max-row function, denoted by M_r, is defined as follows:

$$M_r : SM_{n\times m'} \longrightarrow SM_{n\times 1}, M_r([a_{ip}]) = [c_{i1}]$$

where $c_{i1} = \max_{p\in\{1,2,\ldots,m'\}}\{a_{ip}\}$.

The single column soft matrix $M_r([a_{ip}]) = [c_{i1}]$ is called a max-row soft matrix.

Example 7. Suppose that $[a_{ip}] \in SM_{4\times 3}$ is given as below:

$$[a_{ip}] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

Then, it is obtained the max-row soft matrix

$$M_r([a_{ip}]) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}.$$

Here, the type of soft matrix $M_r([a_{ip}])$ is 4×1.

Definition 14.

(i) Let $[f_{vt}] \in SM_{n_1n_2 \times m_1m_2}$ (or $[f_{vt}] \in SM_{n_1n_2n_3 \times m_1m_2m_3}$) be a soft matrix. The value $\kappa_v = \sum_{t=1}^{w} f_{vt}$ is said to be a vth sum-row of the matrix $[f_{vt}]$ where $w = m_1m_2$ (or $w = m_1m_2m_3$).
(ii) Let $[g_{v1}]$ be a max-row soft matrix of $[f_{vt}]$. The value $\tau_v = \frac{\kappa_v}{w}$ for each $g_{v1} = 1$ is said to be a vth max-row decision value of $[g_{v1}]$. Here, $0 \leq \tau_v \leq 1$.

Definition 15.

(a) Let $U_1 = \{x_1, x_2, \ldots, x_{n_1}\}$ and $U_2 = \{y_1, y_2, \ldots, y_{n_2}\}$ be two disjoint universe sets. Assume that $[f_{vt}] \in SM_{n_1n_2 \times m_1m_2}$ and $M_r([f_{vt}]) = [g_{v1}]$, then the subset of $U_1 \times U_2$ given by

$$opt_{[h_{v1}]}(U_1 \times U_2) = \{u_v : u_v = (x_i, y_j) \in U_1 \times U_2, \quad \max\{\tau_v\}\}$$

is called an optimum set of $U_1 \times U_2$.

Here, $i = \beta$, $v = (\beta - 1)n_2 + j$ and β is the smallest positive integer which satisfies $v \leq \beta n_2$.

(b) Let $U_1 = \{x_1, x_2, \ldots, x_{n_1}\}$, $U_2 = \{y_1, y_2, \ldots, y_{n_2}\}$ and $U_3 = \{z_1, z_2, \ldots, z_{n_3}\}$ be three disjoint universe sets. Assume that $[f_{vt}] \in SM_{n_1n_2n_3 \times m_1m_2m_3}$ and $M_r([f_{vt}]) = [g_{v1}]$, then the following subset of $U_1 \times U_2 \times U_3$ given by

$$opt_{[h_{v1}]}(U_1 \times U_2 \times U_3) = \{u_v : u_v = (x_i, y_j, z_k) \in U_1 \times U_2 \times U_3, \quad \max\{\tau_v\}\}$$

is called an optimum set of $U_1 \times U_2 \times U_3$.

Here, $i = \beta$ such that β is the smallest positive integer which satisfies $v \leq \beta n_2 n_3$. $j = \gamma - (\alpha - 1)n_2$ such that γ and α are the smallest positive integers which satisfy $v \leq \gamma n_3$ and $\gamma \leq \alpha n_2$, respectively. $k = w - (\delta - 1)n_3$ such that $w = v - (\beta - 1)n_2 n_3$ and δ is the smallest positive integer which satisfies $w \leq \delta n_3$.

By using the Definitions 13 and 15, we construct an ($SMrDM$) algorithm and its Scilab algorithm, and present the steps of them in the following table:

Algorithm	**Scilab algorithm of $(SMrDM)$ for the following example**

Step 1: Construct the soft matrices of the experts (decision makers) for each of the disjoint universe sets.	**Step 1:**
	`a1,a2,a3=input('enter soft matrix1,2,3')`
	`b1,b2,b3=input('enter soft matrix4,5,6')`
	`c1,c2,c3=input('enter soft matrix7,8,9')`

Step 2: Using one of the convenient row-products $\curlywedge_r$, $\curlyvee_r$, $\bar{\curlywedge}_r$ and $\underline{\curlyvee}_r$ for each of the experts (decision makers), find the row-products of soft matrices for each of the experts (decision makers).	**Step 2:** For $\curlywedge_r$-product:
	`function d=rowandproduct(a1,a2)`
	`[n1,m1]=size(a1);`
	`[n2,m1]=size(a2);`
	`a=zeros(n1*n2,m1);`
	`for p=1:m1`
	`for i=1:n1`
	`for j=1:n2`
	`t=(i-1)*n2+j;`
	`d(t,p)=a1(i,p)*a2(j,p);`
	`end`
	`end`
	`end`
	`endfunction`
	
	`function a=rowandmulti(varargin)`
	`r=argn(2);`
	`X=varargin(1);`
	`for i=2:r`
	`X=rowandproduct(X,varargin(i));`
	`end`
	`a=X`
	`endfunction`
	
	For the following example, it is taken a=rowandmulti(a1,a2,a3) in this code. Similarly, it is obtained b and c.

<table>
<tr><td>Step 3: Using one of the convenient generalized products $\curlywedge$, $\bar{\curlywedge}$, $\curlyvee$ and $\underline{\curlyvee}$ according to the decision problem, find the soft matrix $[f_{vt}]$.</td><td>Step 3: For $\curlywedge$-product:<pre>
function e=andproduct(a,b)
 [n1*n2*n3,m1]=size(a);
 [n1*n2*n3,m2]=size(b);
 e=zeros(n1*n2*n3,m1*m2);
   for v=1:n1*n2*n3
    for p=1:m1
      for r=1:m2
          s=(p-1)*m2+r;
          e(v,s)=a(v,p)*b(v,r);
      end
    end
  end
endfunction

function f=andmulti(varargin)
r=argn(2);
 X=varargin(1);
 for i=2:r
  X=andproduct(X,varargin(i));
 end
  f=X
endfunction
</pre>For the following example, it is taken f=andmulti(a,b,c) in this code.</td></tr>
<tr><td>Step 4: Find the max-row soft matrix $[g_{v1}] = M_r([f_{vt}])$ and calculate the max-row decision value τ_v for each $g_{v1} = 1$.</td><td>Step 4: For $(SMrDM)$:<pre>
function t=maxrow(f)
    g=max(f,'c');
    s=sum(f,'c');
    w=size(f,2);
    t=g.*s/w
endfunction
</pre></td></tr>
<tr><td>Step 5: Find an optimum set according to the number of disjoint universe sets.</td><td>Step 5: Find an optimum set according to the number of disjoint universe sets.</td></tr>
</table>

Now using the And row-product and generalized And product, we present an application of the novel method in the multiple-disjoint decision making.

Example 8.

Adapted from [6,16]

Suppose that there are three universes U_1, U_2, and U_3. Suppose that Mr. X has a budget to buy a house, a car and rent a venue to hold a wedding celebration. Let the sets of "houses," "cars," and "hotels" that Mr. X is considering for accommodation purchase, transportation purchase, and a venue to hold a wedding celebration are $U_1 = \{x_1, x_2, x_3\}$, $U_2 = \{y_1, y_2\}$ and $U_3 = \{z_1, z_2, z_3\}$, respectively. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$$\begin{aligned}
E_{U_1} &= \{e_1^{U_1} = expensive, \quad e_2^{U_1} = cheap, \quad e_3^{U_1} = wooden, \quad e_4^{U_1} = in \quad green \quad surroundings\}\\
E_{U_2} &= \{e_1^{U_2} = expensive, \quad e_2^{U_2} = cheap, \quad e_3^{U_2} = sporty\},\\
E_{U_3} &= \{e_1^{U_3} = expensive, \quad e_2^{U_3} = cheap, \quad e_3^{U_3} = in \quad Kuala \quad Lumpur, \quad e_4^{U_3} = majestic\}.
\end{aligned}$$

Let the sets of choice parameters be as below:

$$\begin{aligned}
A &= \{a_1 = (a_{11} = e_2^{U_1}, a_{12} = e_2^{U_2}, a_{13} = e_2^{U_3}), \quad a_2 = (a_{21} = e_4^{U_1}, a_{22} = e_3^{U_2}, a_{23} = e_1^{U_3})\},\\
B &= \{b_1 = (b_{11} = e_1^{U_1}, b_{12} = e_2^{U_2}, b_{13} = e_2^{U_3}), \quad b_2 = (b_{21} = e_2^{U_1}, b_{22} = e_3^{U_2}, b_{23} = e_3^{U_3}), b_3 = (b_{31} = e_4^{U_1}, b_{32} = e_1^{U_2}, b_{33} = e_4^{U_3})\},\\
C &= \{c_1 = (c_{11} = e_2^{U_1}, c_{12} = e_2^{U_2}, c_{13} = e_1^{U_3}), \quad c_2 = (c_{21} = e_3^{U_1}, c_{22} = e_3^{U_2}, c_{23} = e_4^{U_3})\}.
\end{aligned}$$

Suppose Mr. X wants to select the optimal objects by using the following observations of three experts analyzing the objects of the three universe sets with respect to the sets of choice parameters.

The observations of first expert under the parameter set A are

$(F, E_{U_1})_A$	a_{11}	a_{21}
x_1	0	0
x_2	1	1
x_3	1	0

$(G, E_{U_2})_A$	a_{12}	a_{22}
y_1	1	0
y_2	1	1

$(H, E_{U_3})_A$	a_{13}	a_{23}
z_1	0	1
z_2	1	0
z_3	1	0

The observations of second expert under the parameter set B are

$(F, E_{U_1})_B$	b_{11}	b_{21}	b_{31}
x_1	1	0	0
x_2	1	0	1
x_3	0	1	1

$(G, E_{U_2})_B$	b_{12}	b_{22}	b_{32}
y_1	1	1	0
y_2	0	0	1

$(H, E_{U_3})_B$	b_{13}	b_{23}	b_{33}
z_1	1	1	0
z_2	0	0	1
z_3	1	0	0

The observations of third expert under the parameter set C are

$(F, E_{U_1})_C$	c_{11}	c_{21}
x_1	0	1
x_2	0	1
x_3	1	0

$(G, E_{U_2})_C$	c_{12}	c_{22}
y_1	1	1
y_2	0	1

$(H, E_{U_3})_C$	c_{13}	c_{31}
z_1	1	0
z_2	0	1
z_3	1	1

Then, Mr. X can apply the soft max-row decision algorithm to select the optimum object as follows:

Step 1: He constructs the following soft matrices corresponding to the soft sets of each of the experts. The soft matrices corresponding to the observation tables of first expert are

$$[a^1_{ip}] = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}, \ [a^2_{jp}] = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \text{ and } [a^3_{kp}] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}.$$

The soft matrices corresponding to the observation tables of second expert are

$$[b^1_{ir}] = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \ [b^2_{jr}] = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } [b^3_{kr}] = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

The soft matrices corresponding to the observation tables of third expert are

$$[c^1_{is}] = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \ [c^2_{js}] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \text{ and } [c^3_{ks}] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}.$$

Step 2: He obtains row-products of the soft matrices for each of the experts by using And row-product as follows:

For the first, second and third experts, it is obtained the soft matrices $[a_{vp}] = [a^1_{ip}] \curlywedge_r [a^2_{jp}] \curlywedge_r [a^3_{kp}]$, $[b_{vr}] = [b^1_{ir}] \curlywedge_r [b^2_{jr}] \curlywedge_r [b^3_{kr}]$ and $[c_{vs}] = [c^1_{is}] \curlywedge_r [c^2_{js}] \curlywedge_r [c^3_{ks}]$, respectively as follows:

$$[a_{vp}] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad [b_{vr}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad [c_{vs}] = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Note: If any of soft matrices $[a_{vp}]$, $[b_{vr}]$, $[c_{vs}]$ is equal the zero soft matrix $[0]$, then it can be taken $\curlyvee_r$ instead of $\curlywedge_r$.

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Step 3: He finds the products of soft matrices $[a_{vp}]$ and $[b_{vr}]$, $[c_{vs}]$ by using generalized And-product as follows:

$$[f_{vt}] = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}.$$

Note: If $[f_{vt}] = [0]$, then it can be taken $\curlyvee$ instead of $\curlywedge$.
Step 4: He obtains a max-row soft matrix of $[f_{vt}]$ as

$$[g_{v1}] = M_r([f_{vt}]) = \begin{bmatrix} 0\\0\\0\\0\\0\\0\\0\\0\\1\\0\\1\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix}.$$

Then it is obtained $\tau_9 = \tau_{11} = \frac{1}{12}$ for $g_{(9)1} = g_{(11)1} = 1$, hence $\max\{\tau_v\} = \{\tau_9, \tau_{11}\}$.
Step 5: Finally, Mr. X finds an optimum set of $U_1 \times U_2 \times U_3$ according to $[g_{v1}] = M_r([f_{vt}])$ as

$$opt_{[g_{v1}]}(U_1 \times U_2 \times U_3) = \{u_9 = \{(x_2, y_1, z_3), u_{11} = \{(x_2, y_2, z_2)\}\}.$$

Then, Mr. X obtains the optimal house-car-hotel as (x_2, y_1, z_3) or (x_2, y_2, z_2) in the direction of experts' observations. From here, it can be understood that if Mr. X wants to choose the car y_1 then he should choose the house x_2 and the hotel z_3, but he can't choose the hotel z_2.

5. Multi-soft distributive max–min decision making

In this section, we construct a multi-soft distributive max–min decision making (*Multi-SDMmDM*) method using soft distributive max–min decision function defined in the following. By employing this method, it can be selected the compatible optimal objects from the multi-disjoint universe sets. To illustrate the performance of the method, we present two examples containing two disjoint universe sets and three disjoint universe sets.

Definition 16.

(i) Let $[c_{vs}] \in SM_{n \times m_1 m_2}$ and let

$$I_k = \{s : \exists v, c_{vs} \neq 0, (k-1)m_2 < s \leq km_2\}$$

for all $k \in I = \{1, 2, \ldots, m_1\}$. Then *soft left-distributive max–min decision function*, denoted by $D_l f$, is defined as follows:

$$D_l f : SM_{n\times m_1 m_2} \longrightarrow SM_{n\times 1}, D_l f([c_{vs}]) = [\max_{k\in I}\{l_k\}]$$

where

$$l_k = \begin{cases} \min\limits_{s\in I_k}\{c_{vs}\}, & \text{if} \quad I_k \neq \emptyset, \\ 0, & \text{if} \quad I_k = \emptyset. \end{cases}$$

The soft matrix $[e_{v1}] = D_l f([c_{vs}])$ is called *a left-distributive max–min decision soft matrix*.

(ii) Let $[d_{vt}] \in SM_{n\times m_2 m_1}$ and let

$$J_\ell = \{t : \exists v, f_{vt} \neq 0, (\ell-1)m_1 < t \leq \ell m_1\}$$

for all $\ell \in J = \{1, 2, \ldots, m_2\}$. Then *soft right-distributive max–min decision function*, denoted by $D_r f$, is defined as follows:

$$D_r f : SM_{n\times m_2 m_1} \longrightarrow SM_{n\times 1}, D_r f([d_{vt}]) = [\max_{\ell\in J}\{r_\ell\}]$$

where

$$r_\ell = \begin{cases} \min\limits_{s\in J_\ell}\{d_{vt}\}, & \text{if} \quad J_\ell \neq \emptyset, \\ 0, & \text{if} \quad J_\ell = \emptyset. \end{cases}$$

The soft matrix $[f_{v1}] = D_r f([d_{vt}])$ is called *a right-distributive max–min decision soft matrix*.

(iii) *Soft distributive max–min decision function*, denoted by Df, is defined as follows:

$$Df : SM_{n\times 1} \times SM_{n\times 1} \longrightarrow SM_{n\times 1}, Df([e_{v1}], [f_{v1}]) = [e_{v1}]\tilde{\cup}[f_{v1}]$$

where $[e_{v1}] = D_l f([c_{vs}])$ and $[f_{v1}] = D_r f([d_{vt}])$. The soft matrix $[g_{v1}] = D_l f([c_{vs}])\tilde{\cup} D_r f([d_{vt}])$ is called *a distributive max–min decision soft matrix*.

Since the operations $\curlywedge$, $\curlyvee$, $\bar{\curlywedge}$ and $\underline{\curlyvee}$ are not commutative, for the sake of clarity, in the above equations, whenever the first factor is $[a_{vp}]$ and the second factor is $[b_{vr}]$ we denote product by $[c_{vs}]$, otherwise if the first factor is $[b_{vr}]$ and the second factor is $[a_{vp}]$, we denote product by $[d_{vt}]$, i.e.

If $[c_{vs}] = [a_{vp}] \curlywedge [b_{vr}]$, then $[d_{vt}] = [b_{vr}] \curlywedge [a_{vp}]$.
If $[c_{vs}] = [a_{vp}] \curlyvee [b_{vr}]$, then $[d_{vt}] = [b_{vr}] \curlyvee [a_{vp}]$.
If $[c_{vs}] = [a_{vp}]\bar{\curlywedge}[b_{vr}]$, then $[d_{vt}] = [b_{vr}]^c\bar{\curlywedge}[a_{vp}]^c$.
If $[c_{vs}] = [a_{vp}]\underline{\curlyvee}[b_{vr}]$, then $[d_{vt}] = [b_{vr}]^c\underline{\curlyvee}[a_{vp}]^c$.

Definition 17.

(a) Let $U_1 = \{x_1, x_2, \ldots, x_{n_1}\}$ and $U_2 = \{y_1, y_2, \ldots, y_{n_2}\}$ be two disjoint universe sets. Then, using distributive max–min decision soft matrix $[g_{v1}]$, an optimum set of $U_1 \times U_2$ is defined as

$$opt_{[g_{v1}]}(U_1 \times U_2) = \{u_v = (x_i, y_j) | (x_i, y_j) \in U_1 \times U_2, g_{v1} = 1\}.$$

Here $i = \beta$, $v = (\beta-1)n_2 + j$ and β is the smallest positive integer which satisfies $v \leq \beta n_2$.

(b) Let $U_1 = \{x_1, x_2, \ldots, x_{n_1}\}$, $U_2 = \{y_1, y_2, \ldots, y_{n_2}\}$ and $U_3 = \{z_1, z_2, \ldots, z_{n_3}\}$ be three disjoint universe sets. Then, using distributive max–min decision soft matrix $[g_{v1}]$, an optimum set of $U_1 \times U_2 \times U_3$ is defined as

$$opt_{[g_{v1}]}(U_1 \times U_2 \times U_3) = \{u_v = (x_i, y_j, z_k) | (x_i, y_j, z_k) \in U_1 \times U_2 \times U_3, g_{v1} = 1\}.$$

Here $i = \beta$ such that β is the smallest positive integer which satisfies $v \leq \beta n_2 n_3$. $j = \gamma - (\alpha-1)n_2$ such that γ and α are the smallest positive integers which satisfy $v \leq \gamma n_3$ and $\gamma \leq \alpha n_2$, respectively. $k = w - (\delta-1)n_3$ such that $w = v - (\beta-1)n_2 n_3$ and δ is the smallest positive integer which satisfies $w \leq \delta n_3$.

By using the Definitions 16 and 17, we construct the following (*Multi-SDMmDM*) algorithm and its Scilab algorithm.

Algorithm	**Scilab algorithm of** $(Multi{-}SDMmDM)$ **for the following example**
Step 1: Decision makers determine their own parameter sets and create the soft sets for each of disjoint universe sets.	**Step 1:** Decision makers determine their own parameter sets and create the soft sets for each of disjoint universe sets.

Step 2: Construct the soft matrices corresponding to the soft sets.	**Step 2:**
	`a1=input('enter soft matrix1')`
	`a2=input('enter soft matrix2')`
	`b1=input('enter soft matrix3')`
	`b2=input('enter soft matrix4')`

Step 3: Using one of the convenient row-products $\curlywedge_r$, $\overline{\curlywedge}_r$, $\curlyvee_r$ and $\underline{\curlyvee}_r$ for each of the decision makers, find the row- products of soft matrices for each of the decision makers.	**Step 3:** For $\curlywedge_r$-product:
	`function a=rowandproduct(a1,a2)`
	`[n1,m1]=size(a1);`
	`[n2,m1]=size(a2);`
	`a=zeros(n1*n2,m1);`
	`for p=1:m1`
	`for i=1:n1`
	`for j=1:n2`
	`v=(i-1)*n2+j;`
	`a(v,p)=a1(i,p)*a2(j,p);`
	`end`
	`end`
	`end`
	`endfunction`
	
	`function b=rowandproduct(b1,b2)`
	`[n1,m2]=size(b1);`
	`[n2,m2]=size(b2);`
	`b=zeros(n1*n2,m2);`
	`for r=1:m2`
	`for i=1:n1`
	`for j=1:n2`
	`v=(i-1)*n2+j;`
	25 `b(v,r)=b1(i,r)*b2(j,r);`
	`end`
	`end`
	`end`
	`endfunction`

<table>
<tr>
<td>Step 4: Using one of the convenient generalized products $\curlywedge$, $\bar{\curlywedge}$, $\curlyvee$ and $\underline{\curlyvee}$ for given decision making problem, find the soft matrices $[c_{vs}]$ and $[d_{vt}]$.</td>
<td>Step 4: For $\curlywedge$-product:
<pre>
function c=andproduct(a,b)
 [n1*n2,m1]=size(a);
 [n1*n2,m2]=size(b);
 c=zeros(n1*n2,m1*m2);
   for v=1:n1*n2
    for p=1:m1
      for r=1:m2
          s=(p-1)*m2+r;
          c(v,s)=a(v,p)*b(v,r);
      end
    end
  end
endfunction

function d=andproduct(b,a)
 [n1*n2,m1]=size(a);
 [n1*n2,m2]=size(b);
 d=zeros(n1*n2,m1*m2);
   for v=1:n1*n2
    for r=1:m2
      for p=1:m1
          t=(r-1)*m1+p;
          d(v,t)=b(v,r)*a(v,p);
      end
    end
  end
endfunction
</pre></td>
</tr>
</table>

<table>
<tr><td>Step 5: Find a distributive max-min decision soft matrix.</td><td>Step 5: For $(SDMmDM)$:<pre>
function M2=delzero(M)
    Ms=sum(M,1);
    ind=find(Ms==0);
    M(:,ind)=[];
    M2=M;
endfunction

function Dlf=leftdist(c)
[n1*n2,m1]=size(c);
[n1*n2,m2]=size(d);
e=zeros(n1*n2,m1);
 for p=1:m1
  M=delzero(c(:,(p-1)*m2+1:p*m2));
  e(:,p)=min(M,'c');
 end
  Dlf=max(e,'c');
endfunction

function Drf=rightdist(d)
[n1*n2,m1]=size(a);
[n1*n2,m2]=size(b);
f=zeros(n1*n2,m2);
 for p=1:m2
  M=delzero(d(:,(p-1)*m1+1:p*m1));
  f(:,p)=min(M,'c');
 end
  Drf=max(f,'c');
endfunction

function Df=dist(Dlf,Drf)
         Df=max([Dlf Drf],'c');
endfunction
</pre></td></tr>
<tr><td>Step 6: Find an optimum set according to the number of disjoint universe sets.</td><td>Step 6: Find an optimum set according to the number of disjoint universe sets.</td></tr>
</table>

Note that, in Definition 16, if we permute the phrases "min" and "max" then we obtain multi-soft distributive min–max decision making method which is denoted by (*Multi-SDMmDM*). If we take "min" instead of "max" then we obtain multi-soft distributive min-min decision making method which is denoted by (*Multi-SDMmDM*). If we take "max" instead of "min" then we obtain multi-soft distributive max–max decision making method which is denoted by (*Multi-SDMmDM*). One of them may be more useful than other to solve some decision problems.

Now, we present two practical examples applying the emerging method in the multiple-disjoint decision making.

Example 9. Assume that a marketing company wants to select a man and a woman staff to employ in the department of public relations. Four men and five women candidates apply to the company for this job. The sets of men and women candidates are $U_1 = \{x_1, x_2, x_3, x_4\}$ and $U_2 = \{y_1, y_2, y_3, y_4, y_5\}$, respectively. These are characterized by a set of parameters $E = \{e_1, e_2, e_3, e_4, e_5\}$. For $p = 1, 2, 3, 4, 5$, the parameters e_p stand for "experience", "computer knowledge", "graduate degree" "knowing foreign languages" and "proper diction", respectively. The company manager assigns two assistants to achieve this purpose.

When the manager assistants choose their own parameters from the set E, they follow the steps of multi-soft distributive max–min decision algorithm to fill the empty positions as below:

Step 1: The manager assistants determine their own parameters as $A = \{e_1, e_2, e_3, e_5\}$, $B = \{e_1, e_2, e_3, e_4, e_5\}$, respectively.

The soft sets of first manager assistant are

$$(F_1, A) = \{(e_1, \{x_1, x_3, x_4\}), (e_2, U_1), (e_3, \{x_1, x_2, x_4\}), (e_5, \{x_2, x_4\})\},$$
$$(G_1, A) = \{(e_1, \{y_2, y_3\}), (e_2, \{y_2, y_3, y_5\}), (e_3, \{y_2, y_3, y_4, y_5\}), (e_5, \{y_1, y_2\})\}.$$

The soft sets of second manager assistant are

$$(F_2, B) = \{(e_1, \{x_1, x_3, x_4\}), (e_2, U_1), (e_3, \{x_1, x_2, x_4\}), (e_4, \{x_1, x_3\}), (e_5, \{x_1, x_2, x_4\})\},$$
$$(G_2, B) = \{(e_1, \{y_2, y_3, y_4\}), (e_2, \{y_1, y_2, y_3, y_5\}), (e_3, \{y_2, y_3, y_4, y_5\}), (e_4, \{y_3, y_4\}), (e_5, \{y_1, y_5\})\}.$$

Step 2: The soft matrices corresponding to the soft sets are as follows:
The soft matrices of first manager assistant are

$$[F_1, A] = [a^1_{ip}] = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \ [G_1, A] = [a^2_{jp}] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

The soft matrices of second manager assistant are

$$[F_2, B] = [b^1_{ir}] = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}, \ [G_2, B] = [b^2_{jr}] = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

Step 3: They find the following soft matrices $[a_{vp}] = [a^1_{ip}] \curlywedge_r [a^2_{jp}]$ and $[b_{vr}] = [b^1_{ir}] \curlywedge_r [b^2_{jr}]$ by using And row-product.

$$[a_{vp}] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \ [b_{vr}] = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

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Step 4: They find the products of soft matrices $[a_{vp}]$ and $[b_{vr}]$ by using generalized And-product as follows:

$$[c_{vs}]=[a_{vp}]\curlywedge[b_{vr}]=\begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&1&1&0&0&1&1&1&0&0&1&1&1&0&0&0&0&0&0&0\\
1&1&1&1&0&1&1&1&1&0&1&1&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&1&1&0&1&0&1&1&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1\\
0&0&0&0&0&0&1&1&0&0&0&1&1&0&0&0&1&1&0&0\\
0&0&0&0&0&0&1&1&0&0&0&1&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&1&0&1&0&1&1&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&1&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&1&0&1&0&1&1&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1\\
1&1&1&0&0&1&1&1&0&0&1&1&1&0&0&1&1&1&0&0\\
1&1&1&0&0&1&1&1&0&0&1&1&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&1&0&1&0&1&1&0&1&0&0&0&0&0
\end{bmatrix},$$

$$[d_{vt}]=[b_{vr}]\curlywedge[a_{vp}]=\begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&1&1&0&1&1&1&0&1&1&1&0&0&0&0&0&0&0&0&0\\
1&1&1&0&1&1&1&0&1&1&1&0&1&1&1&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&1&1&0&0&1&1&0&0&0&0&0&0&1&1&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&1&1&1&0&1&1&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&1&0&0&1&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&1&0&0&1&1&0&0&0&0&0&0&1&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&1&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&1&0&0&1&1&0&0&0&0&0&0&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&1\\
1&1&1&1&1&1&1&1&1&1&1&1&0&0&0&0&0&0&0&0\\
1&1&1&0&1&1&1&0&1&1&1&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&1&0&0&1&1&0&0&0&0&0&0&1&1&0
\end{bmatrix}.$$

Step 5: They obtain a left-distributive max–min decision soft matrix and a right-distributive max–min decision soft matrix as:

$$D_l f([c_{vs}]) = \begin{bmatrix} 0\\0\\1\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix}, \quad D_r f([d_{vt}]) = \begin{bmatrix} 0\\0\\1\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\1\\0\\0\\0 \end{bmatrix}.$$

Later on, they obtain a distributive max–min decision soft matrix as

$$Df(D_l f([c_{vs}]), D_r f([d_{vt}])) = \begin{bmatrix} 0\\0\\1\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\1\\0\\0\\0 \end{bmatrix}$$

Step 6: Finally, according to $Df(D_l f([c_{vs}]), D_r f([d_{vt}]))$, they find an optimum set of $U_1 \times U_2$ as

$$opt_{Df(D_l f([c_{vs}]), D_r f([d_{vt}]))}(U_1 \times U_2) = \{u_3 = (x_1, y_3), u_{17} = (x_4, y_2)\}$$

where (x_1, y_3) and (x_4, y_2) are optimum men-women staff to employ in the department of public relations of the marketing company.

Note: When the set $U_1 \times U_2$ is disjointed as U_1 and U_2, this decision problem can be resolved by using the soft distributive max–min decision making method proposed in [9]. Then, we find the separate solutions that are $\{x_1, x_4\}$ for the universe set U_1 and $\{y_2, y_3\}$ for the universe set U_2. Thus, it can be taken the pairs (x_1, y_2) as an optimum man-woman staff to employ in the department of public relations of the marketing company. However, we argue that the choosing of (x_1, y_3) is more convenient than (x_1, y_2) since x_1 and y_3 have almost the same attributes.

Example 10. Adapted from [2,26,44]

Suppose that there is an investment company which wants to invest a sum of money in the best options from each of sets of the companies in different types. There are three disjoint universe sets which each of them involves possible options to invest the money: $A_1 = \{x_1, x_2, x_3\}$ is a set of car companies; $A_2 = \{y_1, y_2\}$ is a set of food companies and $A_3 = \{z_1, z_2\}$ is a set of computer companies. Two partners of the investment company should take a decision according to the following four parameters: $e_1 = low \quad risk$, $e_2 = high \quad growth \quad rate$, $e_3 = low \quad social - political \quad impact$ and $e_4 = low \quad enviromental \quad impact$.

When two partners evaluate the possible objects of three disjoint universe sets under the above four parameters, we are ready to apply the multi-soft distributive max–min decision making method as follows:

Step 1: The soft sets for each of the universe sets are created by the partners as below:

The soft sets of first partner are

$$(F_1, E) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_3\}), (e_3, \emptyset), (e_4, \{x_1, x_2\})\},$$
$$(G_1, E) = \{(e_1, A_2), (e_2, \{y_1\}), (e_3, \{y_1\}), (e_4, A_2)\},$$
$$(H_1, E) = \{(e_1, \{z_1\}), (e_2, A_3), (e_3, \emptyset), (e_4, A_3)\}.$$

The soft sets of second partner are

$$(F_2, E) = \{(e_1, \{x_1, x_3\}), (e_2, \{x_2, x_3\}), (e_3, \{x_1\}), (e_4, \{x_1, x_2\})\},$$
$$(G_2, E) = \{(e_1, A_2), (e_2, \{y_1\}), (e_3, \emptyset\}), (e_4, A_2)\},$$
$$(H_2, E) = \{(e_1, A_3), (e_2, \{z_1\}), (e_3, \emptyset), (e_4, A_3)\}.$$

Step 2: We can write the following soft matrices corresponding to the soft sets.

The soft matrices of first partner are

$$[a^1_{ip}] = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \ [a^2_{jp}] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \ [a^3_{kp}] = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

The soft matrices of second partner are

$$[b^1_{ir}] = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}, \ [b^2_{jr}] = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \ [b^3_{kr}] = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}.$$

Step 3: The soft matrices $[a_{vp}] = [a^1_{ip}] \curlywedge_r [a^2_{jp}] \curlywedge_r [a^3_{kp}]$ and $[b_{vr}] = [b^1_{ir}] \curlywedge_r [b^2_{jr}] \curlywedge_r [b^3_{kr}]$ by employing And row-product are found as below:

$$[a_{vp}] = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } [b_{vr}] = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

Step 4: The products of soft matrices $[a_{vp}]$ and $[b_{vr}]$ by employing generalized And-product are obtained as below:

$$[c_{vs}] = [a_{vp}] \curlywedge [b_{vr}] = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

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$$[d_{vt}]=[b_{vr}]\curlywedge[a_{vp}]=\begin{bmatrix}1&1&0&1&0&0&0&0&0&0&0&0&1&1&0&1\\0&1&0&1&0&0&0&0&0&0&0&0&0&1&0&1\\1&0&0&1&0&0&0&0&0&0&0&0&1&0&0&1\\0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&1\\0&0&0&0&1&0&0&1&0&0&0&0&1&0&0&1\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\0&1&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\0&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\end{bmatrix}.$$

Step 5: We obtain a left-distributive max–min decision soft matrix and a right-distributive max–min decision soft matrix as

$$D_lf([c_{vs}])=\begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix},\quad D_rf([d_{vt}])=\begin{bmatrix}1\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix}.$$

We then obtain a distributive max–min decision soft matrix as

$$Df(D_lf([c_{vs}]),D_rf([d_{vt}]))=\begin{bmatrix}1\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix}$$

Step 6: Finally, according to $Df(D_lf([c_{vs}]), D_rf([d_{vt}]))$, we can find an optimum set of $A_1\times A_2\times A_3$ as

$$opt_{Df(D_lf([c_{vs}]),D_rf([d_{vt}]))}(A_1\times A_2\times A_3)=\{(x_1,y_1,z_1)\}$$

where (x_1, y_1, z_1) are optimum car-food-computer companies to invest the money of the investment company.

Additionally, we emphasize that the investment company may also invest money to anyone or any two of the companies x_1, y_1 and z_1.

Similarly, we may also use Or row-product, And-Not row-product, Or-Not row-product in Step 3 and generalized Or-product, And-Not-product, Or-Not-product in Step 4 for the other convenient decision problems.

Note: In [2,26,44], the decision making methods based on the fuzzy-valued linguistic soft set, the aggregation operator of linguistic information and the interval-valued intuitionistic fuzzy information are proposed to solve decision making problems involving the sets $A_1=\{x_1\},\quad A_2=\{y_1\}$ and $A_3=\{z_1\}$.

Comment: The decision problems in Examples 9 and 10 can also be solved using the soft distributive max–min decision making method constructed in [9]. When this method is used, we need to obtain a separate solutions for each of the universe sets. That are $\{x_1, x_4\}$ for the universe set U_1, $\{y_2, y_3\}$ for the universe set U_2 in the problem of Example 9, and x_1 for the universe set A_1, y_1 for the universe set A_2, z_1 for the universe set A_3 in the problem of Example 10. This situation is both time-consuming and a lot of work for us. Therefore, using the multi-soft distributive max–min decision making method in Examples 9 and 10, we choose the optimum objects from each of the disjoint universe sets simultaneously. Thus, we demonstrate the concise and effective of our method solving decision making problems involving the multi-disjoint universe sets. As a result, we indicate that our method can be viewed as a generalization of the soft distributive max–min method presented in [9].

Remark: Our methods are insufficient to solve the decision making problems such as the following problem (adapted from Example 2.4.1 in [11]). Because the multi-universe sets in this problem are not disjoint.

The problem: Let S_i, $i \in N$ be a collection of states in a country and U_i, $i \in N$ be a collection of states with availability of land, labor and raw materials. Suppose $U_1 = \{S_1, S_2, S_3\}$ be a set of states with availability of land, $U_2 = \{S_2, S_4, S_6\}$ be a set of states with availability of labor $U_3 = \{S_2, S_4, S_7\}$ be a set of state with availability of raw materials. Let E be a set of decision parameters related to the above universe sets, where $E = \{e_1 = peaceful,\quad e_2 = kidnapping,\quad e_3 = army\quad robbery,\quad e_4 = accessibility, e_5 = market\}$. Assume that the decision makers wants to obtain optimum choice under the parameters from the multi universe sets. They should choose which one?

6. Conclusion

In this paper, we introduced some operations of the soft matrix such as And row-product, And-Not row-product, Or row-product and Or-Not row-product, and then presented two new decision making methods using these row-products. Also, we demonstrated the efficiency of these methods by solving the decision making problems in various fields. As the dimensions of the matrices used in our decision models increase, the computations become more difficult. So, we gave Scilab codes for convenience.

We think that these methods will present a new perspective to handle the decision making problems involving multi-disjoint universe sets and also will greatly impact the decision making based on soft matrix theory in the coming years.

Appendix A. Scilab Algorithms

Since one of the row-products $\curlyvee_r$ and $\underline{\curlyvee_r}$ can be used in the multiple-disjoint decision making, we present the following Scilab codes of these products. As a matter of convenience, we add Scilab codes of multi-row-products.

for $\curlyvee_r$-product

```
function d=roworproduct(a,b)
 [n1,m]=size(a);
 [n2,m]=size(b);
 d=zeros(n1*n2,m);
   for p=1:m
    for i=1:n1
      for j=1:n2
       t=(i-1)*n2+j;
       d(t,p)=a(i,p)+b(j,p);
        for t=1:n1*n2
         if d(t,p)>=2
            d(t,p)=1;
         end
        end
      end
    end
  end
endfunction
```

for $\underline{\curlyvee}_r$-product

```
function d=rowornotproduct(a,b)
 [n1,m]=size(a);
 [n2,m]=size(b);
 d=zeros(n1*n2,m);
   for p=1:m
    for i=1:n1
      for j=1:n2
       t=(i-1)*n2+j;
       d(t,p)=a(i,p)+(1-b(j,p));
        for t=1:n1*n2
         if d(t,p)>=2
            d(t,p)=1;
         end
        end
      end
    end
  end
endfunction
```

for $Multi-row$-products

```
function f=rowandmulti(varargin)
r=argn(2);
 X=varargin(1);
 for i=2:r
  X=rowandproduct(X,varargin(i));
 end
  f=X
endfunction

function f=rowormulti(varargin)
r=argn(2);
 Y=varargin(1);
 for i=2:r
  Y=roworproduct(Y,varargin(i));
 end
  f=Y
endfunction

function f=rowandnotmulti(varargin)
r=argn(2);
 Z=varargin(1);
 for i=2:r
  Z=rowandnotproduct(Z,varargin(i));
 end
  f=Z
endfunction

function f=rowornotmulti(varargin)
r=argn(2);
 W=varargin(1);
 for i=2:r
  W=rowornotproduct(W,varargin(i));
 end
  f=W
endfunction
```

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Since one of the generalized products $\curlyvee$, $\bar{\curlywedge}$ and $\underline{\curlyvee}$ can be used in the multiple-disjoint decision making, we give the Scilab codes of them.

for $\curlyvee$-product

```
function c=orproduct(a,b)
[n,m1]=size(a);
[n,m2]=size(b);
c=zeros(n,m1*m2);
 for v=1:n
  for p=1:m1
   for r=1:m2
    s=(p-1)*m2+r;
    c(v,s)=a(v,p)+b(v,r);
     for s=1:m1*m2
      if c(v,s)>=2
         c(v,s)=1;
      end
     end
   end
  end
 end
endfunction

function d=orproduct(b,a)
[n,m1]=size(a);
[n,m2]=size(b);
d=zeros(n,m1*m2);
 for v=1:n
  for r=1:m2
   for p=1:m1
    t=(r-1)*m1+p;
    d(v,t)=b(v,r)+a(v,p);
     for t=1:m1*m2
      if d(v,t)>=2
         d(v,t)=1;
      end
     end
   end
  end
 end
endfunction
```

for $\bar{\curlywedge}$-product

```
function c=andnotproduct(a,b)
[n,m1]=size(a);
[n,m2]=size(b);
c=zeros(n,m1*m2);
 for v=1:n
  for p=1:m1
   for r=1:m2
   s=(p-1)*m2+r;
   c(v,s)=a(v,p)*(1-b(v,r));
   end
  end
 end
endfunction

function d=andnotproduct(b,a)
[n,m1]=size(a);
[n,m2]=size(b);
d=zeros(n,m1*m2);
 for v=1:n
  for r=1:m2
   for p=1:m1
   t=(r-1)*m1+p;
   d(v,t)=(1-b(v,r))*a(v,p);
   end
  end
 end
endfunction
```

for $\underline{\curlyvee}$-product

```
function c=ornotproduct(a,b)
[n,m1]=size(a);
[n,m2]=size(b);
c=zeros(n,m1*m2);
 for v=1:n
  for p=1:m1
   for r=1:m2
   s=(p-1)*m2+r;
   c(v,s)=a(v,p)+(1-b(v,r));
    for s=1:m1*m2
     if c(v,s)>=2
        c(v,s)=1;
     end
    end
   end
  end
 end
endfunction

function d=ornotproduct(b,a)
[n,m1]=size(a);
[n,m2]=size(b);
d=zeros(n,m1*m2);
 for v=1:n
  for r=1:m2
   for p=1:m1
   t=(r-1)*m1+p;
   d(v,t)=(1-b(v,r))+a(v,p);
    for t=1:m1*m2
     if d(v,t)>=2
        d(v,t)=1;
     end
    end
   end
  end
 end
endfunction
```

Since one of the decision making methods (*Multi-SDMmDM*), (*Multi-SDMmDM*) and (*Multi-SDMmDM*) can be used in the multiple-disjoint decision making, we also provide the Scilab codes of these methods.

for $(SDmMDM)$

```
function M2=delzero(M)
    Ms=sum(M,1);
    ind=find(Ms==0);
    M(:,ind)=[];
    M2=M;
endfunction

function Dlf=leftdist(c)
[n,m1]=size(a);
[n,m2]=size(b);
e=zeros(n,m1);
for p=1:m1
M=delzero(c(:,(p-1)*m2+1:p*m2));
e(:,p)=max(M,'c');
end
Dlf=min(e,'c');
endfunction}

function Drf=rightdist(d)
[n,m1]=size(a);
[n,m2]=size(b);
f=zeros(n,m2);
for r=1:m2
M=delzero(d(:,(r-1)*m1+1:r*m1));
f(:,r)=max(M,'c');
end
Drf=min(f,'c');
endfunction

function Df=dist(Dlf,Drf)
        Df=max([Dlf Drf],'c');
endfunction
```

for $(SDmmDM)$

```
function M2=delzero(M)
    Ms=sum(M,1);
    ind=find(Ms==0);
    M(:,ind)=[];
    M2=M;
endfunction

function Dlf=leftdist(c)
[n,m1]=size(a);
[n,m2]=size(b);
e=zeros(n,m1);
for p=1:m1
M=delzero(c(:,(p-1)*m2+1:p*m2));
e(:,p)=min(M,'c');
end
Dlf=min(e,'c');
endfunction

function Drf=rightdist(d)
[n,m1]=size(a);
[n,m2]=size(b);
f=zeros(n,m2);
for r=1:m2
M=delzero(d(:,(r-1)*m1+1:r*m1));
f(:,r)=min(M,'c');
end
Drf=min(f,'c');
endfunction

function Df=dist(Dlf,Drf)
        Df=max([Dlf Drf],'c');
endfunction
```

for $(SDMMDM)$

```
function M2=delzero(M)
    Ms=sum(M,1);
    ind=find(Ms==0);
    M(:,ind)=[];
    M2=M;
endfunction

function Dlf=leftdist(c)
[n,m1]=size(a);
[n,m2]=size(b);
e=zeros(n,m1);
for p=1:m1
M=delzero(c(:,(p-1)*m2+1:p*m2));
e(:,p)=max(M,'c');
end
Dlf=max(e,'c');
endfunction

function Drf=rightdist(d)
[n,m1]=size(a);
[n,m2]=size(b);
f=zeros(n,m2);
for r=1:m2
M=delzero(d(:,(r-1)*m1+1:r*m1));
f(:,r)=max(M,'c');
end
Drf=max(f,'c');
endfunction

function Df=dist(Dlf,Drf)
        Df=max([Dlf Drf],'c');
endfunction
```

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