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Separation Axioms in Neutrosophic Crisp Topological Spaces

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Abstract.The main idea of this research is to define a new neutrosophic crisp points in neutrosophic crisp topological space namely [NCP_N] ,the concept of neutrosophic crisp limit point was defind using [NCP_N],with some of its properties, the separation axioms [N-$\mathcal{T}_i$-space,i= 0,1,2] were constructed in neutrosophic crisp topological space using [NCP_N] and examine the relationship between them in details.
Keywords: Neutrosophic crisp topological spaces, neutrosophic crisp limit point, separation axioms.

Introduction

Smarandache [1,2,3] introduced the notions of neutrosophic theory and introduced the neutrosophic. components (T,I,F) which represent the membership , indeterminacy , and non membership values respectively, where $]-0,1^+[$ is a non standard unit interval. In [4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19 ,20] many scientists presented the concepts of the neutrosophic set theory in their works. Salama et al. [21,22] provided natural foundations to put mathematical treatments for the neutrosophic pervasively phenomena in our real world and for building new branches of neutrosophic mathematics.

Salama et al [23,24] put some basic concepts of the neutrosophic crisp set and their operations, and because of their wide applications and their grate flexibility to solve the problem, we used these concepts to define new types of neutrosophic points, that we called neutrosophic crisp points [NCP_N].

Fainally ,we used these points [NCP_N] to define the concept of neutrosophic crisp limit point, with some of its properties and constructe the separation axioms[N-$\mathcal{T}_i$-space,i=0,1,2] in neutrosophic crisp topological and examine the relationship between them in details.

Throughout this paper,(NCTS) means a neutrosophic crisp topological space. Also, simply we denote neighborhood by (nhd).

1 Basic Concepts

1.1 Definition [25]

Let $\mathcal{X}$ be a non-empty fixed set . A neutrosophic crisp set [NCS for short] B is an object having the form $B = < B_1, B_2, B_3 >$ where B_1, B_2 and B_3 are subsets of $\mathcal{X}$.

1.2 Definition [25]

The object having the form $B=<B_1 , B_2 , B_3>$ is called :

1. A neutrosophic crisp set of Type1 [NCS/Type1] if satisfying
$B_1 \cap B_2 = \emptyset, B_1 \cap B_3 = \emptyset$ and $B_2 \cap B_3 = \emptyset$.
2. A neutrosophic crisp set of Type2 [NCS/Type2] if satisfying
$B_1 \cap B_2 = \emptyset, B_1 \cap B_3 = \emptyset$ and $B_2 \cap B_3 = \emptyset$, $B_1 \cup B_2 \cup B_3 = \mathcal{X}$.
3. A neutrosophic crisp set of Type3 [NCS/Type3] if satisfying
$B_1 \cap B_2 \cap B_3 = \emptyset$, $B_1 \cup B_2 \cup B_3 = \mathcal{X}$

1.3 Definition [25]

Types of NCSs $\emptyset_N$ & $\mathcal{X}_N$ in $\mathcal{X}$ as follows :

1 . $\emptyset_N$ may be defined in many ways as a NCS as follows :
1. Type1 : $\emptyset_N = <\varphi , \varphi , \mathcal{X}>$
2. Type2 : $\emptyset_N = <\varphi , \mathcal{X} , \mathcal{X}>$
3. Type3 : $\emptyset_N = <\varphi , \mathcal{X}, \varphi>$
4. Type4 : $\emptyset_N = <\varphi , \varphi , \varphi>$

2 . $\mathcal{X}_N$ may be defined in many ways as a NCS as follows :
1. Type1: $\mathcal{X}_N = <\mathcal{X}, \varphi , \varphi>$

2. Type2: $\mathcal{X}_N = < \mathcal{X}, \mathcal{X}, \varphi >$
3. Type3: $\mathcal{X}_N = < \mathcal{X}, \varphi, \mathcal{X} >$
4. Type4: $\mathcal{X}_N = < \mathcal{X}, \mathcal{X}, \mathcal{X} >$

1.4 Definition [25]

Let $\mathcal{X}$ be a non-empty set and the NCSs C & D in the form $C = <C_1,C_2,C_3>$, $D = <D_1,D_2,D_3>$ then we may consider two possible definitions for subsets $C \subseteq D$, may be defined in two ways :

1. $C \subseteq D \Leftrightarrow C_1 \subseteq D_1$, $C_2 \subseteq D_2$ and $D_3 \subseteq C_3$
2. $C \subseteq D \Leftrightarrow C_1 \subseteq D_1$, $D_2 \subseteq C_2$ and $D_3 \subseteq C_3$

1.5 Definition [25]

Let $\mathcal{X}$ be a non-empty set and the NCSs C & D in the form $C = <C_1,C_2,C_3>$, $D = <D_1,D_2,D_3>$ then :

1. $C \cap D$ may be defined in two ways as a NCS as follows :
   - $C \cap D = [C_1 \cap D_1], [C_2 \cup D_2], [C_3 \cup D_3]$
   - $C \cap D = [C_1 \cap D_1], [C_2 \cap D_2], [C_3 \cup D_3]$
2. $C \cup D$ may be defined in two ways as a NCS as follows :
   - $C \cup D = [C_1 \cup D_1], [C_2 \cup D_2], [C_3 \cap D_3]$
   - $C \cup D = [C_1 \cup D_1], [C_2 \cap D_2], [C_3 \cap D_3]$

1.6 Definition [25]

A neutrosophic crisp topology (NCT) on a non-empty set $\mathcal{X}$ is a family $\mathcal{T}$ of neutrosophic crisp subsets in $\mathcal{X}$ satisfying the following axioms :

1. $\emptyset_N, \mathcal{X}_N \in \mathcal{T}$
2. $C \cap D \in \mathcal{T}$, for any $C, D \in \mathcal{T}$
3. The union of any number of sets in $\mathcal{T}$ belongs to $\mathcal{T}$

The pair $(\mathcal{X}, \mathcal{T})$ is said to be a neutrosophic crisp topological space (NCTS) in $\mathcal{X}$. Moreover, The elements in $\mathcal{T}$ are said to be neutrosophic crisp open sets (NCOS), a neutrosophic crisp set F is closed (NCCS) iff its complement F^C is an open neutrosophic crisp set.

1.7 Definition [25]

Let $\mathcal{X}$ be a non-empty set and the NCS D in the form $D = <D_1,D_2,D_3>$ .Then D^c may be defined in three ways as a NCS as follows :

$D^c = < D_1^c, D_2^c, D_3^c>$, $D^c = < D_3,D_2,D_1>$ or $D^c = < D_3,D_2^c,D_1>$

1.8 Definition [25]

Let $(\mathcal{X}, \mathcal{T})$ be neutrosophic crisp topological space (NCTS). A be neutrosophic crisp set then: The intersection of any neutrosophic crisp closed sets contained, A is called neutrosophic crisp closure of A (NC-Cl(A) for short) .

2 Neutrosophic crisp limit point :

In this section, we will introduce the neutrosophic crisp limit points with some of its properties. This work contains an adjustment for the above-mentioned definitions 1.4 & 1.5, this was necessary to homogeneous suitable results for the upgrade of this research.

2.1 Definition

Let $\mathcal{X}$ be a non-empty set and the NCSs C & D in the form $C = <C_1,C_2,C_3>$, $D = <D_1,D_2,D_3>$ then the additional new ways for the intersection , union and inclusion between C & D are

$C \cap D = [C_1 \cap D_1], [C_2 \cap D_2], [C_3 \cap D_3]$

$C \cup D = [C_1 \cup D_1], [C_2 \cup D_2], [C_3 \cup D_3]$

$C \subseteq D \Leftrightarrow C_1 \subseteq D_1$, $C_2 \subseteq D_2$ and $C_3 \subseteq D_3$

2.2 Definition

For all x, y, z belonging to a non-empty set $\mathcal{X}$. Then the neutrosophic crisp points related to x, y, z are defined as follows :

- $x_{N_1} = <\{x\},\emptyset,\emptyset>$, is called a neutrosophic crisp point (NCP_{N_1}) in $\mathcal{X}$.
- $y_{N_2} = <\emptyset,\{y\},\emptyset>$, is called a neutrosophic crisp point (NCP_{N_2}) in $\mathcal{X}$.
- $z_{N_3} = <\emptyset,\emptyset,\{z\}>$, is called a neutrosophic crisp point (NCP_{N_3}) in $\mathcal{X}$.

The set of all neutrosophic crisp points ($NCP_{N_1}, NCP_{N_2}, NCP_{N_3}$) is denoted by NCP_N.

2.3 Definition

Let $\mathcal{X}$ be to a non-empty set and $x, y, z \in \mathcal{X}$. Then the neutrosophic crisp point:

- x_{N_1} is belonging to the neutrosophic crisp set $B=<B_1,B_2,B_3>$, denoted by $x_{N_1} \in B$, if $x \in B_1$,wherein x_{N_1} does not belong to the neutrosophic crisp set B denoted by $x_{N_1} \notin B$, if $x \notin B_1$.
- y_{N_2} is belonging to the neutrosophic crisp set $B=<B_1,B_2,B_3>$, denoted by $y_{N_2} \in B$, if $y \in B_2$. In contrast y_{N_2} does not belong to the neutrosophic crisp set B, denoted by $y_{N_2} \notin B$, if $y \notin B_2$.
- z_{N_3} is belonging to the neutrosophic crisp set $B=<B_1,B_2,B_3>$, denoted by $z_{N_3} \in B$, if $z \in B_3$. In contrast z_{N_3} does not belong to the neutrosophic crisp set B ,denoted by $z_{N_3} \notin B$, if $z \notin B_3$.

2.4 Remark

If $B = <B_1,B_2,B_3>$ is a NCS in a non-empty set $\mathcal{X}$ then :

$B\backslash x_{N_1} = < B_1\backslash\{x\}, B_2, B_3 >$. $B\backslash x_{N_1}$ means that the component B doesn't contain x_{N_1}.

$B\backslash y_{N_2} = < B_1, B_2\backslash\{y\}, B_3 >$. $B\backslash y_{N_2}$ means that the component B doesn't contain y_{N_2}.

$B\backslash z_{N_3} = < B_1, B_2, B_3\backslash\{z\} >$. $B\backslash z_{N_3}$ means that the component B doesn't contain z_{N_3}.

2.5 Example

If $B = <\{a, b\},\{c, b\},\{c, a\}>$ is an NCS in $\mathcal{X} = \{a, b, c\}$, then:

$B\backslash a_{N_1} = <\{ \ b\}, \{c, b\}, \{c, a\}>$

$B\backslash b_{N_2} = <\{a, b\} , \{c\}, \{c, a\}>$

$B\backslash c_{N_3} = <\{a, b\} , \{c, b\}, \{ \ b\}>$

2.6 Remark

If $B = <B_1,B_2,B_3>$ is a NCS in a non-empty set $\mathcal{X}$ then :

$B = (\cup\{ x_{N_1} : x_{N_1} \in B \}) \cup (\cup\{ y_{N_2} : y_{N_2} \in B \}) \cup (\cap\{ z_{N_3} : z_{N_3} \in B \})$

$= (\cup\{<\{x\}, \emptyset,\emptyset> : x \in \mathcal{X}\}) \cup (\cup\{<\emptyset, \{y\}, \emptyset> : y \in \mathcal{X}\}) \cup (\cap\{<\emptyset, \emptyset, \{z\}> : z \in \mathcal{X}\})$

or $B = (\cup\{ x_{N_1} : x_{N_1} \in B \}) \cup (\cup\{ y_{N_2} : y_{N_2} \in B \}) \cup (\cup\{ z_{N_3} : z_{N_3} \in B \})$

$= (\cup\{<\{x\}, \emptyset,\emptyset> : x \in \mathcal{X}\}) \cup (\cup\{<\emptyset, \{y\}, \emptyset> : y \in \mathcal{X}\}) \cup (\cup\{<\emptyset, \emptyset,\{z\}> : z \in \mathcal{X}\})$.

2.7 Definition

Let $(\mathcal{X},\mathcal{T})$ be NCTS , $P \in NCP_N$ in $\mathcal{X}$, a neutrosophic crisp set $B = <B_1,B_2,B_3> \in \mathcal{T}$ is called neutrosophic crisp open nhd of P in $(\mathcal{X},\mathcal{T})$ if $P \in B$.

2.8 Definition

Let $(\mathcal{X},\mathcal{T})$ be NCTS , $P \in NCP_N$ in $\mathcal{X}$, a neutrosophic crisp set $B = < B_1 , B_2 , B_3 > \in \mathcal{T}$ is called neutrosophic crisp nhd of P in $(\mathcal{X},\mathcal{T})$, if there is neutrosophic crisp open set $A =< A_1, A_2, A_3 >$ containing P such that $A \subseteq B$.

2.9 Note

Every neutrosophic crisp open nhd of any point $P \in NCP_N$ in $\mathcal{X}$ is neutrosophic crisp nhd of P, but in general the inverse is not true , the following example illustrates this fact.

2.10 Example

If $\mathcal{X} = \{x, y, z\}$, $\mathcal{T} = \{\mathcal{X}_N , \emptyset_N , A, B, C\}$,

$A = <\{x\}, \emptyset, \emptyset>$, $B = <\{y\}, \emptyset, \emptyset>$, $G = <\{x, y\}, \emptyset, \emptyset>$

If we take $U = <\{x, y\},\{z\}, \emptyset>$.

Then $G = <\{x, y\}, \emptyset, \emptyset>$ is an open set containing $P = x_{N_1} =< \{x\}, \emptyset, \emptyset>$ and $G \subseteq U$. That is U is a neutrosophic crisp nhd of P in $(\mathcal{X},\mathcal{T})$, while it is not a neutrosophic crisp open nhd of P .

2.11 Definition

Let $(\mathcal{X},\mathcal{T})$ be NCTS and $B = < B_1, B_2, B_3 >$ be NCS of $\mathcal{X}$. A neutrosophic crisp point $P \in NCP_N$ in $\mathcal{X}$ is called a neutrosophic crisp limit point of $B = < B_1, B_2, B_3 >$ iff every neutrosophic crisp open set containing P must contains at least one neutrosophic crisp point of B different from P. It is easy to say that the point P is not neutrosophic crisp limit point of B if there is a neutrosophic crisp open set G of P and $B \cap (G \setminus P) = \emptyset_N$.

2.12 Definition

The set of all neutrosophic crisp limit points of a neutrosophic crisp set B is called neutrosophic crisp derived set of B, denoted by NCD(B).

2.13 Example

If $\mathcal{X} = \{x,y,z\}$, $\mathcal{T} = \{\mathcal{X}_N, \emptyset_N, A, B, C\}$, $A = < \{x\}, \emptyset, \emptyset >$, $B = < \{y\}, \emptyset, \emptyset >$, $G = < \{x, y\}, \emptyset, \emptyset >$. If we take $D = < \{x, y\}, \emptyset, \emptyset >$, Then $P = Z_{N_1} = < \{Z\}, \emptyset, \emptyset >$ is the only neutrosophic crisp limit point of D. i.e. $NCD(D) = \{Z_{N_1}\}$

2.14 Remarks

- Let B be any neutrosophic crisp set of $\mathcal{X}$, If $P = < \{x\}, \emptyset, \emptyset > \in \mathcal{T}$ in any NCT space $(\mathcal{X},\mathcal{T})$, then $P \in NCD(B)$.
- Let B be any neutrosophic crisp set of $\mathcal{X}$, the following facts is true:
  $NCD(B) \not\subset B$, $B \not\subset NCD(B)$, and sometimes $NCD(B) \cap B = \emptyset_N$ or $NCD(B) \cap B \neq \emptyset_N$.
- In any NCT space $(\mathcal{X},\mathcal{T})$, we have $NCD(\emptyset) = \emptyset_N$.

2.15 Theorem

Let $(\mathcal{X},\mathcal{T})$ be NCTS and $B = < B_1, B_2, B_3 >$ be a neutrosophic crisp set of $\mathcal{X}$, then B is neutrosophic crisp closed set (NCCS for short) iff $NCD(B) \subseteq B$

Proof

Let B be NCCS, then $(\mathcal{X}\setminus B)$ is neutrosophic crisp open set (NCOS for short) this implies that for each neutrosophic crisp point $P \in NCP_N$ in $(\mathcal{X}\setminus B)$, $P \notin B$, there is a neutrosophic crisp open set G of P and $G \subseteq (\mathcal{X}\setminus B)$.
Since $B \cap (\mathcal{X}\setminus B) = \emptyset_N$, then P is not neutrosophic crisp limit point of B, thus $G \cap B = \emptyset_N$, which implies that $P \notin NCD(B)$. Hence $NCD(B) \subseteq B$
Conversely, assume that $P \notin NCD(B)$, implis that P is not neutrosophic crisp limit point of B, hence, there is a neutrosophic crisp open set G of P and $G \cap B = \emptyset_N$ which means that $G \subseteq (\mathcal{X}\setminus B)$ and since $(\mathcal{X}\setminus B)$ is a neutrosophic crisp open set. Hence B is neutrosophic crisp closed set. .

2.16 Theorem

Let $(\mathcal{X},\mathcal{T})$ be NCTS, B, G be a neutrosophic crisp sets of $\mathcal{X}$, then the following properties hold:
(1) $NCD(\emptyset_N) = \emptyset_N$
(2) If $B \subseteq G$, then $NCD(B) \subseteq NCD(G)$
(3) $NCD(B \cap G) \subseteq NCD(B) \cap NCD(G)$
(4) $NCD(B \cup G) = NCD(B) \cup NCD(G)$

Proof (1) the proof is, directly.

Proof (2)

Assume that NCD(B) be a neutrosophic crisp set containing a neutrosophic crisp point $P \in NCP_N$, then by definition 2.11, for each neutrosophic crisp open set V of P, we have $B \cap V\setminus P \neq \emptyset_N$, but $B \subseteq G$, hence $G \cap V\setminus P \neq \emptyset_N$, this means that $P \in NCD(G)$. Hence, $NCD(B) \subseteq NCD(G)$

Proof (3)

Since $B \cap G \subseteq B$, then by (2) $NCD(B \cap G) \subseteq NCD(B)$ (1)
$B \cap G \subseteq G$, implies $NCD(B \cap G) \subseteq NCD(G)$ (2)
From (1) & (2) $NCD(B \cap G) \subseteq NCD(B) \cap NCD(G)$

Proof (4)

Let $P \in NCP_N$ such that $P \notin NCD(B) \cup NCD(G)$, then either $P \notin NCD(B)$ and $P \notin NCD(G)$, then there is a neutrosophic crisp open set K of P and $B \cap K\setminus P = \emptyset_N$ and $G \cap K\setminus P = \emptyset_N$, this implies that $(B \cup G) \cap K\setminus P = \emptyset_N$, i.e $P \notin NCD(B \cup G)$, hence $NCD(B \cup G) \subseteq NCD(B) \cup NCD(G)$ (3)

Conversely, since $B \subseteq B \cup G$,$G \subseteq B \cup G$,then by property (2) $NCD(B) \subseteq NCD(B \cup G)$ and $NCD(G) \subseteq NCD(B \cup G)$, thus $NCD(B \cup G) \supseteq NCD(B) \cup NCD(G)$ (4)
from (3) and (4) we have $NCD(B \cup G) = NCD(B) \cup NCD(G)$.

2.17 Remark

In general, the inverse of property 2 & 3 in Th.(2.16) is not true. The following examples act as an evidence to this claim.

2.18 Example

If $\mathcal{X} = \{x, y, z\}$, $\mathcal{T} = \{\mathcal{X}_N , \emptyset_N, B\}$, $B = <\emptyset,\{x\},\emptyset>$. If we take $A = <\emptyset,\{x\},\emptyset>$, $G = <\emptyset,\{y\},\emptyset>$ Notes that; $NCD(A) = <\emptyset,\{y, z\},\emptyset>$, $NCD(G) = <\emptyset,\{y, z\},\emptyset>$ and $NCD(A) \subseteq NCD(G)$, but $A \not\subset G$.

2.19 Example

If $\mathcal{X} = \{x, y, z\}$, $\mathcal{T} = \{\mathcal{X}_N , \emptyset_N, B\}$, $B = <\emptyset,\{x\},\emptyset>$. If we take $A = <\emptyset,\{x\},\emptyset>$, $G = <\emptyset,\{y\},\emptyset>$.
Notes that; $NCD(B \cap G) \not\supset NCD(B) \cap NCD(G)$.

2.20 Theorem

For any neutrosophic crisp set B over the universe $\mathcal{X}$, then NC-Cl(B) = $B \cup NCD(B)$

Proof

Let us first prove that $B \cup NCD(B)$ is a neutrosophic crisp closed set that is $\mathcal{X}_N \backslash (B \cup NCD(B)) = (\mathcal{X}_N \backslash B) \cap (\mathcal{X}_N \backslash NCD(B))$ is a neutrosophic crisp open set .
Now for a neutrosophic crisp point $P \in (\mathcal{X}_N \backslash (B)) \cap (\mathcal{X}_N \backslash NCD(B))$, then $P \in (\mathcal{X}_N \backslash (B))$ and $P \in \mathcal{X}_N \backslash NCD(B)$, thus $P \notin B$ and $P \notin NCD(B)$. So by definition 2.12, there is a neutrosophic crisp set R of P S.t $R \cap B = \emptyset_N$, hence $R \subseteq \mathcal{X}_N \backslash B$.
Now for each $P_1 \in R$, then $P_1 \notin NCD(B)$, then $R \cap NCD(B) = \emptyset_N$, this implies that $R \subseteq \mathcal{X}_N \backslash NCD(B)$ [i.e $R \subseteq (\mathcal{X}_N \backslash B) \cap (\mathcal{X}_N \backslash NCD(B))$] .Thus $(\mathcal{X}_N \backslash B) \cap (\mathcal{X}_N \backslash NCD(B))$ is a neutrosophic crisp nhd of all its elements and hence $(\mathcal{X}_N \backslash B) \cap (\mathcal{X}_N \backslash NCD(B))$ is a neutrosophic crisp open set and thus $B \cup NCD(B)$ is a neutrosophic crisp closed set containing B, therefore NC-Cl(B) $\subseteq B \cup NCD(B)$. S ince NC-Cl(B) is a neutrosophic crisp closed set (see definition 2.12) and NC-Cl(B) contains all its neutrosophic crisp limits points .Thus $NCD(B) \subseteq$ NC-Cl(B) and $B \subseteq$ NC-Cl(B), hence NC-Cl(B) = $B \cup NCD(B)$.

3 Separation Axioms In a neutrosophic Crisp Topological Space

3.1 Definition

A neutrosophic crisp topological space $(\mathcal{X}, \mathcal{T})$ is called:

- N_1-$\mathcal{T}_o$-space if $\forall x_{N_1} \neq y_{N_1} \in \mathcal{X}$ $\exists$ a neutrosophic crisp open set G in $\mathcal{X}$ containing one of them but not the other.
- N_2-$\mathcal{T}_o$-space if $\forall x_{N_2} \neq y_{N_2} \in \mathcal{X}$ $\exists$ a neutrosophic crisp open set G in $\mathcal{X}$ containing one of them but not the other .
- N_3-$\mathcal{T}_o$-space if $\forall x_{N_3} \neq y_{N_3} \in \mathcal{X}$ $\exists$ a neutrosophic crisp open set G in $\mathcal{X}$ containing one of them but not the other .
- N_1-$\mathcal{T}_1$-space if $\forall x_{N_1} \neq y_{N_1} \in \mathcal{X}$ $\exists$ a neutrosophic crisp open sets G_1 , G_2 in $\mathcal{X}$ such that $x_{N_1} \in G_1$, $y_{N_1} \notin G_1$ and $x_{N_1} \notin G_2$, $y_{N_1} \in G_2$
- N_2-$\mathcal{T}_1$-space if $\forall x_{N_2} \neq y_{N_2} \in \mathcal{X}$ $\exists$ a neutrosophic crisp open sets G_1 , G_2 in $\mathcal{X}$ such that $x_{N_2} \in G_1$, $y_{N_2} \notin G_1$ and $x_{N_2} \notin G_2$, $y_{N_2} \in G_2$
- N_3-$\mathcal{T}_1$-space if $\forall x_{N_3} \neq y_{N_3} \in \mathcal{X}$ $\exists$ a neutrosophic crisp open sets G_1 , G_2 in $\mathcal{X}$ such that $x_{N_3} \in G_1$, $y_{N_3} \notin G_1$ and $x_{N_3} \notin G_2$, $y_{N_3} \in G_2$
- N_1-$\mathcal{T}_2$-space if $\forall x_{N_1} \neq y_{N_1} \in \mathcal{X}$ $\exists$ a neutrosophic crisp open sets G_1 , G_2 in $\mathcal{X}$ such that $x_{N_1} \in G_1$, $y_{N_1} \notin G_1$ and $x_{N_1} \notin G_2$, $y_{N_1} \in G_2$ with $G_1 \cap G_2 = \emptyset$.
- N_2-$\mathcal{T}_2$-space if $\forall x_{N_2} \neq y_{N_2} \in \mathcal{X}$ $\exists$ a neutrosophic crisp open sets G_1 , G_2 in $\mathcal{X}$ such that $x_{N_2} \in G_1$, $y_{N_2} \notin G_1$ and $x_{N_2} \notin G_2$, $y_{N_2} \in G_2$ with $G_1 \cap G_2 = \emptyset$.

- N_3-$\mathcal{T}_2$-space if $\forall\, x_{N_3} \neq y_{N_3} \in \mathcal{X}$ $\exists$ a neutrosophic crisp open sets G_1 , G_2 in $\mathcal{X}$ such that $x_{N_3} \in G_1$, $y_{N_3} \notin G_1$ and $x_{N_3} \notin G_2$, $y_{N_3} \in G_2$ with $G_1 \cap G_2 = \emptyset$.

3.2 Definition

A neutrosophic crisp topological space ($\mathcal{X},\mathcal{T}$) is called:

- N-$\mathcal{T}_o$-space if $(\mathcal{X},\mathcal{T})$ is N_1-$\mathcal{T}_o$-space , N_2-$\mathcal{T}_o$-space and N_3-$\mathcal{T}_o$-space
- N-$\mathcal{T}_1$-space if $(\mathcal{X},\mathcal{T})$ is N_1-$\mathcal{T}_1$-space , N_2-$\mathcal{T}_1$-space and N_3-$\mathcal{T}_1$-space
- N-$\mathcal{T}_2$-space if $(\mathcal{X},\mathcal{T})$ is N_1-$\mathcal{T}_2$-space , N_2-$\mathcal{T}_2$-space and N_3-$\mathcal{T}_2$-space

3.3 Remark

For a neutrosophic crisp topological space $(\mathcal{X},\mathcal{T})$

- Every N-$\mathcal{T}_o$-space is N_1-$\mathcal{T}_o$-space
- Every N-$\mathcal{T}_0$- space is N_2-$\mathcal{T}_0$-space
- Every N-$\mathcal{T}_0$- space is N_3-$\mathcal{T}_0$-space

Proof the proof is directly from definition 3.2 .

The inverse of remark 3.3 is not true, the following example explain this state.

3.4 Example

If $\mathcal{X} = \{x,y\}$, $\mathcal{T}_1 = \{\mathcal{X}_N, \emptyset_N, A\}$,$\mathcal{T}_2 = \{\mathcal{X}_N, \emptyset_N, B\}$,$\mathcal{T}_3 = \{\mathcal{X}_N, \emptyset_N, G\}$, A =<{x},∅,∅>,B =<∅,{y},∅> , G =<∅,∅,{x}>, Then $(\mathcal{X},\mathcal{T}_1)$ is N_1-$\mathcal{T}_o$-space but it is not N-$\mathcal{T}_o$-space, $(\mathcal{X},\mathcal{T}_2)$ is N_2-$\mathcal{T}_o$-space but it is not N-$\mathcal{T}_o$-space, $(\mathcal{X},\mathcal{T}_3)$ is N_3-$\mathcal{T}_o$-space but it is not N-$\mathcal{T}_o$-space.

3.5 Remark

For a neutrosophic crisp topological space $(\mathcal{X},\mathcal{T})$

- Every N-$\mathcal{T}_1$-space is N_1-$\mathcal{T}_1$-space
- Every N-$\mathcal{T}_1$- space is N_2-$\mathcal{T}_1$-space
- Every N-$\mathcal{T}_1$- space is N_3-$\mathcal{T}_1$-space

Proof the proof is directly from definition 3.2 .

The inverse of remark (3.5) is not true as it is shown in the following example,

3.6 Example

If $\mathcal{X} = \{x,y\}$,$\mathcal{T}_1 = \{\mathcal{X}_N , \emptyset_N , A, B\}$, $\mathcal{T}_2 = \{\mathcal{X}_N, \emptyset_N, G , F\}$, A =<{x},{y},∅>, B =<{y},{x},∅>, G =<∅,∅,{x}>, F =<∅,∅,{y}>, Then $(\mathcal{X},\mathcal{T}_1)$ is N_1-$\mathcal{T}_1$-space but it is not N-$\mathcal{T}_1$-space. $(\mathcal{X},\mathcal{T}_1)$ is N_2-$\mathcal{T}_1$-space but it is not N-$\mathcal{T}_1$-space. $(\mathcal{X},\mathcal{T}_2)$ is N_3-$\mathcal{T}_1$-space but it is not N-$\mathcal{T}_1$-space

3.7 Remark

For a neutrosophic crisp topological space $(\mathcal{X},\mathcal{T})$

- Every N-$\mathcal{T}_2$-space is N_1-$\mathcal{T}_2$-space
- Every N-$\mathcal{T}_2$-space is N_2-$\mathcal{T}_2$-space
- Every N-$\mathcal{T}_2$-space is N_3-$\mathcal{T}_2$-space

Proof the proof is directly from definition 3.2 .

The inverse of remark (3.7) is not true as it is shown in the example (3.6).

3.8 Remark

For a neutrosophic crisp topological space $(\mathcal{X},\mathcal{T})$

- Every N-$\mathcal{T}_1$-space is N-$\mathcal{T}_0$-space
- Every N-$\mathcal{T}_2$-space is N-$\mathcal{T}_1$-space

Proof the proof is directly.

The inverse of remark (3.8) is not true as it is shown in the following example :

3.9 Example

If $= \{ x , y \}$, $\mathcal{T} = \{ \mathcal{X}_N , \emptyset_N , A , B , G \}$

A = < { x }, ∅ , ∅ > , B = < ∅, { y }, ∅ > , G = < ∅ , ∅ ,{ x } > ,

Then ($\mathcal{X},\mathcal{T}$) is N-$\mathcal{T}_o$-space but not N-$\mathcal{T}_1$-space

Conclusion

- We defined a new neutrosophic crisp points in neutrosophic crisp topological space
- We introduced the concept of neutrosophic crisp limit point, with some of its properties
- We constructed the separation axioms [N-$\mathcal{T}_i$-space , i= 0,1,2] in neutrosophic crisp topological and examine the relationship between them in details.

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