Applied Mathematical Modelling 38 (2014) 1170–1175

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Short communication

Single valued neutrosophic cross-entropy for multicriteria decision making problems

Jun Ye *

Faculty of Engineering, Shaoxing University, 508# Huancheng West Road, Shaoxing, Zhejiang Province 312000, PR China

ARTICLE INFO

Article history:
Received 19 October 2011
Received in revised form 8 March 2013
Accepted 4 July 2013
Available online 23 July 2013

Keywords:
Cross entropy
Single valued neutrosophic set
Single valued neutrosophic cross entropy
Decision-making

ABSTRACT

A single valued neutrosophic set (SVNS) is an instance of a neutrosophic set, which give us an additional possibility to represent uncertainty, imprecise, incomplete, and inconsistent information which exist in real world. It would be more suitable to apply indeterminate information and inconsistent information measures. In this paper, the cross entropy of SVNSs, called single valued neutrosophic cross entropy, is proposed as an extension of the cross entropy of fuzzy sets. Then, a multicriteria decision-making method based on the proposed single valued neutrosophic cross entropy is established in which criteria values for alternatives are SVNSs. In decision making process, we utilize the single-valued neutrosophic weighted cross entropy between the ideal alternative and an alternative to rank the alternatives corresponding to the cross entropy values and to select the most desirable one(s). Finally, a practical example of the choosing problem of suppliers is provided to illustrate the application of the developed approach.

1. Introduction

Entropy is very important for measuring uncertain information. The fuzzy entropy was first introduced by Zadeh [1,2]. The starting point for the cross entropy approach is information theory as developed by Shannon [3]. Kullback–Leibler [4] proposed a measure of the "cross entropy distance" between two probability distributions. Later, Lin [5] proposed a modified cross-entropy measure. Shang and Jiang [6] proposed a fuzzy cross entropy measure and a symmetric discrimination information measure between fuzzy sets. As an intuitionistic fuzzy set is a generalization of a fuzzy set, Vlachos and Sergiadis [7] proposed an intuitionistic fuzzy cross-entropy based on an extension of the De Luca-Termini nonprobabilistic entropy [8] and applied it to the pattern recognition, medical diagnosis, and image segmentation. Then, Zhang and Jiang [9] defined a vague cross-entropy between vague sets (VSs) by analogy with the cross entropy of probability distributions and applied it to the pattern recognition and medical diagnosis, and then Ye [10] has investigated the fault diagnosis problem of turbine according to the cross entropy of vague sets. Furthermore, Ye [11] has applied the intuitionistic fuzzy cross entropy to multicriteria fuzzy decision-making problems. Ye [12] proposed an interval-valued intuitionistic fuzzy cross-entropy based on the generalization of the vague cross-entropy [9] and applied it to multicriteria decision-making problems.

Smarandache [13] originally introduced neutrosophy. It is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra. Neutrosophic set is a powerful general formal framework, which generalizes the concept of the classic set, fuzzy se, interval valued fuzzy set, intuitionistic fuzzy set, interval-valued intuitionistic fuzzy set, paraconsistent set, dialetheist set, paradoxist set, and tautological set [13]. In the neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership, and

* Tel.: +86 575 88327323.
E-mail addresses: yejnn@yahoo.com.cn, yehjun@aliyun.com.

0307-904X/$ - see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.apm.2013.07.020

falsity-membership are independent. This assumption is very important in many applications such as information fusion in which the data are combined from different sensors. Recently, neutrosophic sets had mainly been applied to image processing [14,15] in engineering field.

Intuitionistic fuzzy sets and interval valued intuitionistic fuzzy sets can only handle incomplete information but not the indeterminate information and inconsistent information which exist commonly in real situations. For example, when we ask the opinion of an expert about certain statement, he or she may that the possibility that the statement is true is between 0.5 and 0.7, and the statement is false is between 0.2 and 0.4, and the degree that he or she is not sure is between 0.1 and 0.3. For neutrosophic notation, it can be expressed as $x([0.5, 0.7], [0.1, 0.3], [0.2, 0.4])$. Here is another example, suppose there are 10 voters during a voting process. In time t_1, four vote "yes", three vote "no" and three are undecided. For neutrosophic notation, it can be expressed as $x(0.4, 0.3, 0.3)$; in time t_2, two vote "yes", three vote "no", two give up, and three are undecided, then it can be expressed as $x(0.2, 0.3, 0.3)$. The mentioned information is beyond the scope of the intuitionistic fuzzy set. So the notion of neutrosophic set is more general and overcomes the aforementioned issues.

The neutrosophic set generalizes the above mentioned sets from philosophical point of view. From scientific or engineering point of view, the neutrosophic set and set-theoretic operators need to be specified. Otherwise, it will be difficult to apply in the real applications. Therefore, Wang et al. [16] proposed a single valued neutrosophic set (SVNS) and provide the set-theoretic operators and various properties of SVNSs. Recently, Ye [17] proposed Similarity measures between interval neutrosophic sets and applied them to multicriteria decision-making problems under the interval neutrosophic environment.

On one hand, a SVNS is an instance of a neutrosophic set, which give us an additional possibility to represent uncertainty, imprecise, incomplete, and inconsistent information which exist in real world. It would be more suitable to apply indeterminate information and inconsistent information measures in decision-making. However, the connector in the fuzzy set is defined with respect to T, i.e. membership only, hence the information of indeterminacy and nonmembership is lost. The connectors in the intuitionistic fuzzy set are defined with respect to T and F, i.e. membership and nonmembership only, hence the indeterminacy is what is left from 1. While in the SVNS, they can be defined with respect to any of them (no restriction). So the notion of SVNSs is more general and overcomes the aforementioned issues. On the other hand, SVNSs can be used for the scientific and engineering applications because SVNS theory is valuable in modeling uncertain, imprecision and inconsistent information. Due to its ability to easily reflect the ambiguous nature of subjective judgments, the SVNS is suitable for capturing imprecise, uncertain, and inconsistent information in the multicriteria decision-making analysis. However, to the best of our knowledge, there is no work addressing the multicriteria decision-making problems in single valued neutrosophic setting. Therefore, the main purposes of this paper were (1) to present the cross entropy of SVNSs, called single valued neutrosophic cross-entropy, and (2) to establish a multicriteria decision-making method by use of the cross entropy of SVNSs. In the decision-making process, we utilize the single-valued neutrosophic weighted cross entropy between the ideal alternative and an alternative to rank the alternatives corresponding to the cross entropy values, and to select the most desirable one(s).

The rest of paper is organized as follows. In Section 2, we introduce the some concepts of neutrosophic sets and SVNSs, and some operators for SVNSs. In Section 3, a cross-entropy measure between SVNSs (called single valued neutrosophic cross-entropy) is proposed as an extension of the fuzzy cross entropy measure. Section 4 establishes a multicriteria decision-making method based on the proposed cross-entropy of SVNSs. In Section 5, a practical example of the choosing problem of suppliers is given to illustrate the application of the developed approach. Finally, some final remarks and further work are given in Section 6.

2. Some concepts of neutrosophic sets and SVNSs

2.1. Neutrosophic sets

Neutrosophic set is a part of neutrosophy, which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra [13], and is a powerful general formal framework, which generalizes the above mentioned sets from philosophical point of view.

Smarandache [13] gave the following definition of a neutrosophic set.

Definition 1 [13]. Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, a indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$. That is $T_A(x)$: $X \to]0^-, 1^+[$, $I_A(x)$: $X \to]0^-, 1^+[$, and $F_A(x)$: $X \to]0^-, 1^+[$.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leqslant \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leqslant 3^+$.

Definition 2 [13]. The complement of a neutrosophic set A is denoted by A^c and is defined as $T_A{}^c(x) = \{1^+\} \ominus T_A(x)$, $I_A{}^c(x) = \{1^+\} \ominus I_A(x)$, and $F_A{}^c(x) = \{1^+\} \ominus F_A(x)$ for every x in X.

Definition 3 [13]. A neutrosophic set A is contained in the other neutrosophic set B, $A \subseteq B$ if and only if $\inf T_A(x) \leqslant \inf T_B(x)$, $\sup T_A(x) \leqslant \sup T_B(x)$, $\inf I_A(x) \geqslant \inf I_B(x)$, $\sup I_A(x) \geqslant \sup I_B(x)$, $\inf F_A(x) \geqslant \inf F_B(x)$, and $\sup F_A(x) \geqslant \sup F_B(x)$ for every x in X.

2.2. Single valued neutrosophic sets

A SVNS is an instance of a neutrosophic set, which can be used in real scientific and engineering applications. In the following, we introduce the definition of a SVNS [16].

Definition 4 [16]. Let X be a space of points (objects) with generic elements in X denoted by x. A SVNS A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$, and falsity-membership function $F_A(x)$. For each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0,1]$.

Therefore, a SVNS A can be written as

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}.$$

The following expressions are defined in [16] for SVNSs A, B:

(1) $A \subseteq B$ if and only if $T_A(x) \leqslant T_B(x)$, $I_A(x) \geqslant I_B(x)$, $F_A(x) \geqslant F_B(x)$ for any x in X,
(2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(3) $A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle | x \in X\}$.

For convenience, a SVNS A is denoted by the simplified symbol $A = \langle T_A(x), I_A(x), F_A(x)\rangle$ for any x in X. For two SVNSs A and B, the operational relations are defined by [16]

(1) $A \cup B = \langle \max(T_A(x),T_B(x)), \min(I_A(x),I_B(x)), \min(F_A(x),F_B(x)) \rangle$ for any x in X,
(2) $A \cap B = \langle \min(T_A(x),T_B(x)), \max(I_A(x),I_B(x)), \max(F_A(x),F_B(x))\rangle$ for any x in X,
(3) $A \times B = \langle T_A(x) + T_B(x) - T_A(x)T_B(x), I_A(x)I_B(x), F_A(x)F_B(x)\rangle$ for any x in X.

Wang et al. [16] defined the two operators of truth-favorite (Δ) and falsity-favorite (∇) to remove the indeterminacy in the single valued neutrosophic sets and transform them into intuitionistic fuzzy sets or paraconsistent sets. These two operators are unique on single valued neutrosophic sets, which are given as [16]

(1) $\Delta A = \langle \min(T_A(x) + I_A(x), 1), 0, F_A(x)\rangle$ for any x in X,
(2) $\nabla A = \langle T_A(x), 0, \min(F_A(x) + I_A(x), 1)\rangle$ for any x in X.

3. Cross-entropy between SVNSs

This section proposes cross-entropy and discrimination information measures between two SVNSs based on the extension of the concept of cross-entropy between two fuzzy sets.

To do this, we firstly introduce the concepts of cross-entropy and symmetric discrimination information measures between two fuzzy sets which were proposed by Shang and Jiang [6].

Definition 5 [6]. Assume that $A = (A(x_1), A(x_2), \ldots, A(x_n))$ and $B = (B(x_1), B(x_2), \ldots, B(x_n))$ are two fuzzy sets in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$. The fuzzy cross entropy of A from B is defined as follows:

$$H(A,B) = \sum_{i=1}^{n}\left(A(x_i)\log_2\frac{A(x_i)}{\frac{1}{2}(A(x_i)+B(x_i))} + (1 - A(x_i))\log_2\frac{1 - A(x_i)}{1 - \frac{1}{2}(A(x_i)+B(x_i))}\right), \tag{1}$$

which indicates the degree of discrimination of A from B.

However, $H(A,B)$ is not symmetric with respect to its arguments. Shang and Jiang [6] proposed a symmetric discrimination information measure:

$$I(A,\ B) = H(A,\ B) + H(B,\ A). \tag{2}$$

Moreover, there are $I(A,B) \geqslant 0$ and $I(A,B) = 0$ if and only if $A = B$.

Then, the cross-entropy and symmetric discrimination information measures between two fuzzy sets are extended to these measures between SVNSs. In order to do so, let us consider two SVNSs A and B in a universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$, which are denoted by $A = \{\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle | x_i \in X\}$ and $B = \{\langle x_i, T_B(x_i), I_B(x_i), F_B(x_i)\rangle | x_i \in X\}$, where $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i)$, $T_B(x_i)$, $I_B(x_i)$, $F_B(x_i) \in [0,1]$ for every $x_i \in X$. The information carried by the truth-membership, the indeterminacy-membership, and the falsity-membership in SVNSs A and B can be considered as fuzzy spaces with the three elements. Thus based on Eq. (1), the amount of information for discrimination of $T_A(x_i)$ from $T_B(x_i)$ $(i = 1,2,\ldots,n)$ can be given by

$$E^T(A,B;x_i) = T_A(x_i)\log_2\frac{T_A(x_i)}{T_B(x_i)} + (1 - T_A(x_i))\log_2\frac{1 - T_A(x_i)}{1 - \frac{1}{2}(T_A(x_i) + T_B(x_i))}, \tag{3}$$

Therefore, the expected information based on the single membership for discrimination of A against B is expressed by

$$E^T(A,B)=\sum_{i=1}^{n}\left[T_A(x_i)\log_2\frac{T_A(x_i)}{T_B(x_i)}+(1-T_A(x_i))\log_2\frac{1-T_A(x_i)}{1-\frac{1}{2}(T_A(x_i)+T_B(x_i))}\right]. \tag{4}$$

Similarly, considering the indeterminacy-membership function and the falsity-membership function, we have the following amounts of information:

$$E^I(A,B)=\sum_{i=1}^{n}\left[I_A(x_i)\log_2\frac{I_A(x_i)}{I_B(x_i)}+(1-I_A(x_i))\log_2\frac{1-I_A(x_i)}{1-\frac{1}{2}(I_A(x_i)+I_B(x_i))}\right], \tag{5}$$

$$E^F(A,B)=\sum_{i=1}^{n}\left[F_A(x_i)\log_2\frac{F_A(x_i)}{F_B(x_i)}+(1-F_A(x_i))\log_2\frac{1-F_A(x_i)}{1-\frac{1}{2}(F_A(x_i)+F_B(x_i))}\right]. \tag{6}$$

Hence, a novel single valued neutrosophic cross-entropy measure between A and B is obtained as the sum of the three amounts:

$$\begin{aligned}E(A,B)=&\sum_{i=1}^{n}\left[T_A(x_i)\log_2\frac{T_A(x_i)}{\frac{1}{2}(T_A(x_i)+T_B(x_i))}+(1-T_A(x_i))\log_2\frac{1-T_A(x_i)}{1-\frac{1}{2}(T_A(x_i)+T_B(x_i))}\right]\\&+\sum_{i=1}^{n}\left[I_A(x_i)\log_2\frac{I_A(x_i)}{\frac{1}{2}(I_A(x_i)+I_B(x_i))}+(1-I_A(x_i))\log_2\frac{1-I_A(x_i)}{1-\frac{1}{2}(I_A(x_i)+I_B(x_i))}\right]\\&+\sum_{i=1}^{n}\left[F_A(x_i)\log_2\frac{F_A(x_i)}{\frac{1}{2}(F_A(x_i)+F_B(x_i))}+(1-F_A(x_i))\log_2\frac{1-F_A(x_i)}{1-\frac{1}{2}(F_A(x_i)+F_B(x_i))}\right],\end{aligned} \tag{7}$$

which also indicates discrimination degree of A from B. According to Shannon's inequality [5], one can easily prove that $E(A,B)\geqslant 0$, and $E(A,B)=0$ if and only if $T_A(x_i)=T_B(x_i)$, $I_A(x_i)=I_B(x_i)$, and $F_A(x_i)=F_B(x_i)$ for any $x_i\in X$. Moreover, we can easily see that $E(A^c,B^c)=E(A,B)$, where A^c and B^c are the complement of SVNSs A and B, respectively.

Then, $E(A,B)$ is not symmetric. So it should be modified to a symmetric discrimination information measure for SVNSs as

$$D(A,\ B)=E(A,\ B)+E(B,\ A). \tag{8}$$

The larger the difference between A and B is, the larger $D(A,B)$ is.

4. Multicriteria decision-making method based on the cross-entropy of SVNSs

A multi-criteria decision making problem is the process of finding the best alternative from all of the feasible alternatives where all the alternatives can be evaluated according to a number of criteria or attributes. In general, the multi-criteria decision making problem includes uncertainty, imprecise, incomplete, and inconsistent information which exist in real world. Then SVNS can represent this information. In this section, we present a handling method for the multicriteria decision-making problem under single valued neutrosophic environment (or called a single valued neutrosophic multicriteria decision-making method) by means of the proposed cross entropy measure of SVNSs.

Let $A=\{A_1,A_2,\ldots,A_m\}$ be a set of alternatives and $C=\{C_1,C_2,\ldots,C_n\}$ be a set of criteria. Assume that the weight of the criterion C_j ($j=1,2,\ldots,n$), entered by the decision-maker, is w_j, $w_j\in[0,1]$ and $\sum_{j=1}^{n}x_j=1$. In this case, the characteristic of the alternative A_i ($i=1,2,\ldots,m$) is represented by the following SVNS:

$$A_i=\{\langle C_j,T_{A_i}(C_j),I_{A_i}(C_j),F_{A_i}(C_j)\rangle|C_j\in C\},$$

where $T_{A_i}(C_j)$, $I_{A_i}(C_j)$, $F_{A_i}(C_j)\in[0,1]$, $j=1,2,\ldots,n$, and $i=1,2,\ldots,m$. Here, $T_{A_i}(C_j)$ indicates the degree to which the alternative A_i satisfies the criterion C_j, $I_{A_i}(C_j)$ indicates the indeterminacy degree to which the alternative A_i satisfies or does not satisfy the criterion C_j, $F_{A_i}(C_j)$ indicates the degree to which the alternative A_i does not satisfy the criterion C_j. For the sake of simplicity, a criterion value $\langle C_j,T_{A_i}(C_j),I_{A_i}(C_j),F_{A_i}(C_j)\rangle$ in A_i is denoted by the symbol $a_{ij}=\langle T_{ij},I_{ij},F_{ij}\rangle$ ($j=1,2,\ldots,n$, and $i=1,2,\ldots,m$), which is usually derived from the evaluation of an alternative A_i with respect to a criterion C_j by means of a score law and data processing in practice [12,17]. Therefore, we can elicit a single valued neutrosophic decision matrix $A=(a_{ij})_{m\times n}$:

$$A=\begin{pmatrix}a_{11}&a_{12}&\cdots&a_{1n}\\a_{21}&a_{22}&\cdots&a_{2n}\\\vdots&\vdots&\vdots&\vdots\\a_{m1}&a_{m2}&\cdots&a_{mn}\end{pmatrix}=\begin{pmatrix}\langle T_{11},I_{11},F_{11}\rangle&\langle T_{12},I_{12},F_{12}\rangle&\cdots&\langle T_{1n},I_{1n},F_{1n}\rangle\\\langle T_{21},I_{21},F_{21}\rangle&\langle T_{22},I_{22},F_{22}\rangle&\cdots&\langle T_{2n},I_{2n},F_{2n}\rangle\\\vdots&\vdots&\vdots&\vdots\\\langle T_{m1},I_{m1},F_{m1}\rangle&\langle T_{m2},I_{m2},F_{m2}\rangle&\cdots&\langle T_{mn},I_{mn},F_{mn}\rangle\end{pmatrix}.$$

In multicriteria decision-making environments, the concept of ideal point has been used to help identify the best alternative in the decision set. Although the ideal alternative does not exist in real world, it does provide a useful theoretical construct against which to evaluate alternatives [12]. Hence, we can define an ideal criterion value $a_j^*=\left\langle T_j^*,I_j^*,F_j^*\right\rangle=\langle 1,0,0\rangle$ ($j=1,2,\ldots,n$) in the ideal alternative A^*.

Thus, by applying Eqs. (7) and (8) the weighted cross entropy between an alternative A_i and the ideal alternative A^* can be expressed by

$$\begin{aligned} D_i(A^*,A_i) &= \sum_{j=1}^{n} w_j \left[\log_2 \frac{1}{\frac{1}{2}(1+T_{ij})} + \log_2 \frac{1}{1-\frac{1}{2}(I_{ij})} + \log_2 \frac{1}{1-\frac{1}{2}(F_{ij})}\right] \\ &\quad + \sum_{j=1}^{n} w_j \left[T_{ij}\log_2 \frac{T_{ij}}{\frac{1}{2}(1+T_{ij})} + (1-T_{ij})\log_2 \frac{1-T_{ij}}{1-\frac{1}{2}(1+T_{ij})}\right] + \sum_{j=1}^{n} w_j \left[I_{ij} + (1-I_{ij})\log_2 \frac{1-I_{ij}}{1-\frac{1}{2}(I_{ij})}\right] \\ &\quad + \sum_{j=1}^{n} w_j \left[F_{ij} + (1-F_{ij})\log_2 \frac{1-F_{ij}}{1-\frac{1}{2}(F_{ij})}\right]. \end{aligned} \tag{9}$$

Therefore, the smaller the value of $D_i(A^*,A_i)$ is, the better the alternative A_i is. In this case, the alternative A_i is close to the ideal alternative A^*. Through the weighted cross entropy $D_i(A^*,A_i)$ $(i = 1,2,\ldots,m)$ between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best one can be easily identified as well.

5. Practical example

In order to demonstrate the application of the proposed approach, a multi-criteria decision making problem adapted from Tan and Chen [18] is concerned with a manufacturing company which wants to select the best global supplier according to the core competencies of suppliers. Now suppose that there are a set of four suppliers $A = \{A_1,A_2,A_3,A_4\}$ whose core competencies are evaluated by means of the following four criteria (C_1,C_2,C_3,C_4):

(1) the level of technology innovation (C_1),
(2) the control ability of flow (C_2),
(3) the ability of management (C_3),
(4) the level of service (C_4).

Then, the weight vector for the four criteria is $w = (0.3,0.25,0.25,0.2)$.

The proposed decision making method is applied to solve this problem for selecting suppliers.

For the evaluation of an alternative A_i $(i = 1,2,3,4)$ with respect to a criterion C_j $(j = 1,2,3,4)$, it is obtained from the questionnaire of a domain expert. For example, when we ask the opinion of an expert about an alternative A_1 with respect to a criterion C_1, he or she may say that the possibility in which the statement is good is 0.5 and the statement is poor is 0.3 and the degree in which he or she is not sure is 0.1. For the neutrosophic notation, it can be expressed as $a_{11} = \langle 0.5,0.1,0.3\rangle$. Thus, when the four possible alternatives with respect to the above four criteria are evaluated by the similar method from the expert, we can obtain the following single valued neutrosophic decision matrix A:

$$A = \begin{pmatrix} \langle 0.5,0.1,0.3\rangle & \langle 0.5,0.1,0.4\rangle & \langle 0.7,0.1,0.2\rangle & \langle 0.3,0.2,0.1\rangle \\ \langle 0.4,0.2,0.3\rangle & \langle 0.3,0.2,0.4\rangle & \langle 0.9,0.0,0.1\rangle & \langle 0.5,0.3,0.2\rangle \\ \langle 0.4,0.3,0.1\rangle & \langle 0.5,0.1,0.3\rangle & \langle 0.5,0.0,0.4\rangle & \langle 0.6,0.2,0.2\rangle \\ \langle 0.6,0.1,0.2\rangle & \langle 0.2,0.2,0.5\rangle & \langle 0.4,0.3,0.2\rangle & \langle 0.7,0.2,0.1\rangle \end{pmatrix}.$$

By applying Eq. (9), the cross entropy values between an alternative A_i $(i = 1,2,3,4)$ and the ideal alternative A^* are as follows:

$$D_1(A^*,\ A_1) = 1.1101, \quad D_2(A^*,\ A_2) = 1.1801, \quad D_3(A^*,\ A_3) = 0.9962, \text{ and } D_4(A^*,\ A_4) = 1.2406.$$

According to the cross entropy values, thus the ranking order of the four suppliers is A_3, A_1, A_2, and A_4. Hence, the best supplier is A_3.

From the example, we can see that the proposed single valued neutrosophic multicriteria decision-making method is more suitable for real scientific and engineering applications because it can handle not only incomplete information but also the indeterminate information and inconsistent information which exist commonly in real situations. The technique proposed in this paper extends existing fuzzy decision-making methods and provides a new way for decision-makers.

6. Conclusion

SVNS is an instance of a neutrosophic set, which give us an additional possibility to represent uncertainty, imprecise, incomplete, and inconsistent information which exist in real world. It would be more suitable to apply indeterminate information and inconsistent information measures. Hence, we have proposed a single valued neutrosophic cross-entropy measure and established its multi-criteria decision making method based on the proposed cross entropy under single valued neutrosophic environment, where the characteristics of alternatives on criteria are represented by SVNSs. Finally, a practical example was given to illustrate the application of the proposed multicriteria decision making method.

The proposed method differs from previous approaches for fuzzy multi-criteria decision making not only due to the fact that the proposed method use the SVNS theory, but also due to the consideration of the indeterminacy information besides truth and falsity information in the evaluation of the alternative with respect to criteria, which makes it have more feasible and practical than other traditional decision making methods in real decision making problems. Therefore, its advantage is easily reflecting the ambiguous nature of subjective judgments because SVNSs are suitable for capturing imprecise, uncertain, and inconsistent information in the multicriteria decision-making analysis. In the future, we shall investigate single-valued neutrosophic multicriteria group decision-making problems and apply the single valued neutrosophic cross entropy to solve practical applications in other areas such as expert system, information fusion system, and medical diagnoses.

Acknowledgment

The author is very grateful to the anonymous reviewer and the Editor in Chief, Professor Johann (Hans) Sienz, for their insightful and constructive comments and suggestions, which have been very helpful in improving the paper.

References

[1] L.A. Zadeh, Fuzzy sets, Inf. Control 8 (1965) 338–353.
[2] L.A. Zadeh, Probability measures of fuzzy events, J. Math. Anal. Appl. 23 (1968) 421–427.
[3] C.E. Shannon, A mathematical theory of communication, Bell Syst. Tech. J. 27 (1948) 379–423.
[4] S. Kullback, R.A. Leibler, On information and sufficiency, Ann. Math. Stat. 4 (1951) 99–111.
[5] J. Lin, Divergence measures based on Shannon entropy, IEEE Trans. Inf. Theory 37 (1991) 145–151.
[6] X.G. Shang, W.S. Jiang, A note on fuzzy information measures, Pattern Recognit. Lett. 18 (1997) 425–432.
[7] I.K. Vlachos, G.D. Sergiadis, Intuitionistic fuzzy information – applications to pattern recognition, Pattern Recognit. Lett. 28 (2007) 197–206.
[8] A.S. De Luca, S. Termini, A definition of nonprobabilistic entropy in the setting of fuzzy sets theory, Inf. Control 20 (1972) 301–312.
[9] Q.S. Zhang, S.Y. Jiang, A note on information entropy measures for vague sets, Inf. Sci. 178 (2008) 4184–4191.
[10] J. Ye, Fault diagnosis of turbine based on fuzzy cross entropy of vague sets, Expert Syst. Appl. 36 (2009) 8103–8106.
[11] J. Ye, Multicriteria fuzzy decision-making method based on the intuitionistic fuzzy cross-entropy, in: Y.C. Tang, J. Lawry, V.N. Huynh (Eds.), Proceedings in International Conference on Intelligent Human-Machine Systems and Cybernetics, 1, IEEE Computer Society, 2009, pp. 59–61.
[12] J. Ye, Fuzzy cross entropy of interval-valued intuitionistic fuzzy sets and its optimal decision-making method based on the weights of alternatives, Expert Syst. Appl. 38 (2011) 6179–6183.
[13] F. Smarandache, A unifying field in logics. neutrosophy: neutrosophic probability, set and logic, American Research Press, Rehoboth, 1999.
[14] H.D. Cheng, Y. Guo, A new neutrosophic approach to image thresholding, New Math. Natural Comput. 4 (3) (2008) 291–308.
[15] Y. Guo, H.D. Cheng, New neutrosophic approach to image segmentation, Pattern Recognit. 42 (2009) 587–595.
[16] H. Wang, F. Smarandache, Y.Q. Zhang, et al, Single valued neutrosophic sets, Multispace Multistruct. 4 (2010) 410–413.
[17] J. Ye, Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making, J. Intell. Fuzzy Syst. (2013), http://dx.doi.org/10.3233/IFS-120724.
[18] C. Tan, X. Chen, Intuitionistic fuzzy Choquet integral operator for multi-criteria decision making, Expert Syst. Appl. 37 (2010) 149–157.