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Single–Valued Neutrosophic Filters in EQ–algebras

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Abstract. This paper introduces the concept of single–valued neutrosophic EQ–subalgebras, single–valued neutrosophic EQ–prefilters and single–valued neutrosophic EQ–filters. We study some properties of single–valued neutrosophic EQ–prefilters and show how to construct single–valued neutrosophic EQ–filters. Finally, the relationship between single–valued neutrosophic EQ–filters and EQ–filters are studied.

Keywords: (hyper)Single–valued neutrosophic EQ–algebras, Single–valued neutrosophic EQ–filters.

1. Introduction

EQ-algebra as an alternative to residuated lattices is a special algebra that was presented for the first time by V. Nov$\acute{a}$k [10,11]. Its original motivation comes from fuzzy type theory, in which the main connective is fuzzy equality and stems from the equational style of proof in logic [15]. EQ-algebras are intended to become algebras of truth values for fuzzy type theory (FTT) where the main connective is a fuzzy equality. Every EQ–algebra has three operations meet "$\wedge$", multiplication "$\otimes$", and fuzzy equality "$\sim$" and a unit element, while the implication "$\rightarrow$" is derived from fuzzy equality "$\sim$". This basic structure in fuzzy logic is ordering, represented by $\wedge$–semilattice, with maximal element "1". Further materials regarding EQ–algebras are available in the literature too [6,7,9,12]. Algebras including EQ-algebras have played an important role in recent years and have had its comprehensive applications in many aspects including dynamical systems and genetic code of biology [2]. From the point of view of logic, the main difference between residuated lattices and EQ–algebras lies in the way the implication operation is obtained. While in residuated lattices it is obtained from (strong) conjunction, in EQ–algebras it is obtained from equivalence. Consequently, the two kinds of algebras differ in several essential points despite their many similar or identical properties.

Filter theory plays an important role in studying various logical algebras. From logical point of view, filters correspond to sets of provable formulae. Filters are very important in the proof of the completeness of various logic algebras. Many researchers have studied the filter theory of various logical algebras [3,4,5].

Neutrosophy, as a newlyâĂŞborn science, is a branch of philosophy that studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra. It can be defined as the incidence of the application of a law, an axiom, an idea, a conceptual accredited construction on an unclear, indeterminate phenomenon, contradictory to the purpose of making it intelligible. Neutrosophic set and

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neutrosophic logic are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic) are tools for publications on advanced studies in neutrosophy. In neutrosophic logic, a proposition has a degree of truth (T), indeterminacy (I) and falsity (F), where T, I, F are standard or non–standard subsets of $]^{-}0, 1^{+}[$. In 1995, Smarandache talked for the first time about neutrosophy and in 1999 and 2005 [14] he initiated the theory of neutrosophic set as a new mathematical tool for handling problems involving imprecise, indeterminacy, and inconsistent data. Alkhazaleh et al. generalized the concept of fuzzy soft set to neutrosophic soft set and they gave some applications of this concept in decision making and medical diagnosis [1].

Regarding these points, this paper aims to introduce the notation of single–valued neutrosophic EQ–subalgebras and single–valued neutrosophic EQ–filters. We investigate some properties of single–valued neutrosophic EQ–subalgebras and single–valued neutrosophic EQ–filters and prove them. Indeed show that how to construct single–valued neutrosophic EQ–subalgebras and single–valued neutrosophic EQ–filters. We applied the concept of homomorphisms in EQ–algebras and with this regard, new single–valued neutrosophic EQ–subalgebras and single–valued neutrosophic EQ–filters are generated.

2. Preliminaries

In this section, we recall some definitions and results are indispensable to our research paper.

Definition 2.1. [8] *An algebra $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ of type $(2, 2, 2, 0)$ is called an EQ–algebra, if for all $x, y, z, t \in E$:*

$(E1)$ $(E, \wedge, 1)$ *is a commutative idempotent monoid* (*i.e.* $\wedge$*–semilattice with top element* "1")*;*
$(E2)$ $(E, \otimes, 1)$ *is a monoid and* $\otimes$ *is isotone w.r.t.* "$\leq$" (*where* $x \leq y$ *is defined as* $x \wedge y = x$)*;*
$(E3)$ $x \sim x = 1$*;* (*reflexivity axiom*)
$(E4)$ $((x \wedge y) \sim z) \otimes (t \sim x) \leq z \sim (t \wedge y)$*;* (*substitution axiom*)
$(E5)$ $(x \sim y) \otimes (z \sim t) \leq (x \sim z) \sim (y \sim t)$*;* (*congruence axiom*)
$(E6)$ $(x \wedge y \wedge z) \sim x \leq (x \wedge y) \sim x$*;* (*monotonicity axiom*)
$(E7)$ $x \otimes y \leq x \sim y$*,* (*boundedness axiom*).

The binary operation "$\wedge$" is called meet (*infimum*)*, "$\otimes$" is called multiplication and "$\sim$" is called fuzzy equality.* $(E, \wedge, \otimes, \sim, 1)$ *is called a separated EQ–algebra if* $1 = x \sim y$*, implies that* $x = y$*.*

Proposition 2.2. [8] Let $\mathcal{E}$ be an EQ–algebra, $x \to y := (x \wedge y) \sim x$ and $\tilde{x} = x \sim 1$. Then for all $x, y, z \in E$, the following properties hold:

(i) $x \otimes y \leq x, y, \quad x \otimes y \leq x \wedge y$;
(ii) $x \sim y = y \sim x$;
(iii) $(x \wedge y) \sim x \leq (x \wedge y \wedge z) \sim (x \wedge z)$;
(iv) $x \to x = 1$;
(v) $(x \sim y) \otimes (y \sim z) \leq x \sim z$;
(vi) $(x \to y) \otimes (y \to z) \leq x \to z$;
(vii) $x \leq \tilde{x}, \quad \tilde{1} = 1$.

Proposition 2.3. [8] Let $\mathcal{E}$ be an EQ–algebra. Then for all $x, y, z \in E$, the following properties hold:

(i) $x \otimes (x \sim y) \leq \overline{y}$;
(ii) $(z \to (x \wedge y)) \otimes (x \sim t) \leq z \to (t \wedge y)$;
(iii) $(y \to z) \otimes (x \to y) \leq x \to z$;
(iv) $(x \to y) \otimes (y \to x) \leq x \sim y$;
(v) if $x \leq y \to z$, then $x \otimes y \leq \overline{z}$;
(vi) if $x \leq y \leq z$, then $z \sim x \leq z \sim y$ and $x \sim z \leq x \sim y$;
(vi) $x \to (y \to x) = 1$.

Definition 2.4. [8] Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be a separated EQ–algebra. A subset F of E is called an EQ–filter of E if for all $a, b, c \in E$ it holds that

(i) $1 \in F$,
(ii) if $a, a \to b \in F$, then $b \in F$,
(iii) if $a \to b \in F$, then $a \otimes c \to b \otimes c \in F$ and $c \otimes a \to c \otimes b \in F$.

Theorem 2.5. [8] Let F be a prefilter of separated EQ–algebra $\mathcal{E}$. Then for all $a, b, c \in E$ it holds that

(i) if $a \in F$ and $a \leq b$, then $b \in F$;
(ii) if $a, a \sim b \in F$, then $b \in F$;
(iii) If $a, b \in F$, then $a \wedge b \in F$;
(iv) If $a \sim b \in F$ and $b \sim c \in F$ then $a \sim c \in F$.

Definition 2.6. [17] *Let $\mathcal{E}$ be an EQ–algebras. A fuzzy subset μ of E is called a fuzzy prefilter of $\mathcal{E}$, if for all $x, y, z \in E$:*

$(FH1)$ $\nu(1) \geq \nu(x)$*;*
$(FH2)$ $\nu(y) \geq \nu\big((x \wedge y) {\sim} y\big) \wedge \nu(x)$*.*

A fuzzy EQ–prefilter is called a fuzzy EQ–filter if it satisfies :

$(FH3)$ $\nu\big((x\wedge y)\sim y\big) \leq \nu\Big(\big((x\otimes z)\wedge(y\otimes z)\big)\sim(y\otimes z)\Big)$.

Definition 2.7. [16] *Let X be a set. A single valued neutrosophic set A in X (SVN–S A) is a function $A : X \to [0,1]\times[0,1]\times[0,1]$ with the form $A = \{(x, T_A(x), I_A(x), F_A(x)) \mid x \in X\}$ where the functions T_A, I_A, F_A define respectively the truth–membership function, an indeterminacy–membership function, and a falsity–membership function of the element $x \in X$ to the set A such that $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. Moreover, $Supp(A) = \{x \mid T_A(x) \neq 0, I_A(x) \neq 0, F_A(x) \neq 0\}$ is a crisp set.*

3. Single–Valued Neutrosophic EQ–subalgebras

In this section, we introduce the concept of single–valued neutrosophic EQ–subalgebra and prove some their properties.

Definition 3.1. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra. A map A in E, is called a single–valued neutrosophic EQ–subalgebra of $\mathcal{E}$, if for all $x, y \in E$,*

(i) $T_A(x\wedge y) = T_A(x)\wedge T_A(y), I_A(x\wedge y) = I_A(x)\wedge I_A(y)$ and $F_A(x\wedge y) = F_A(x)\vee F_A(y)$,
(ii) $T_A(x\sim y) \geq T_A(x)\wedge T_A(y), I_A(x\sim y) \geq I_A(x)\wedge I_A(y)$ and $F_A(x\sim y) \leq F_A(x)\vee F_A(y)$.

From now on, when we say $(\mathcal{E}, A)$ is a single–valued neutrosophic EQ–subalgebra, means that $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ is an EQ–algebra and A is a single–valued neutrosophic EQ–subalgebra of $\mathcal{E}$.

Theorem 3.2. *Let $(\mathcal{E}, A)$ be a single–valued neutrosophic EQ–subalgebra. Then for all $x, y \in H$,*

(i) if $x \leq y$, then $T_A(x) \leq T_A(y)$,
(ii) if $x \leq y$, then $I_A(x) \leq I_A(y)$,
(iii) if $x \leq y$, then $F_A(x) \geq F_A(y)$,
(iv) $T_A(x) \leq T_A(1), I_A(x) \leq I_A(1)$ and $F_A(x) \geq F_A(1)$,
(v) $T_A(x\otimes y) \leq T_A(x)\wedge T_A(y)$,
(vi) $I_A(x\otimes y) \leq I_A(x)\wedge T_A(y)$,
(vii) $F_A(x\otimes y) \geq F_A(x)\vee F_A(y)$,
$(viii)$ $T_A(x\to y) \geq T_A(x)\wedge T_A(y)$,
(ix) $I_A(x\to y) \geq I_A(x)\wedge I_A(y)$,
(x) $F_A(x\to y) \leq F_A(x)\vee F_A(y)$.

Proof. $(i), (ii), (iii), (iv)$ Let $x, y \in E$. Since $x \leq y$, we get that $x\wedge y = x$ and so $T_A(x)\wedge T_A(y) = T_A(x\wedge y) = T_A(x)$. It follows that $T_A(x) \leq T_A(y)$. In a similar way $I_A(x) \leq I_A(y)$ and $F_A(x) \geq F_A(y)$ are obtained.

$(v), (vi), (vii)$ By the previous items, for all $x, y \in E$, $x\otimes y \leq x\wedge y$ implies that $T_A(x\otimes y) \leq T_A(x)\wedge T_A(y)$, $I_A(x\otimes y) \leq I_A(x)\wedge I_A(y)$ and $F_A(x\otimes y) \geq F_A(x)\vee F_A(y)$.

$(viii), (ix), (x)$ Since $(x\sim y) \leq (x\to y)$, by the previous items we get that $T_A(x\to y) \geq T_A(x)\wedge T_A(y)$, $I_A(x\to y) \geq I_A(x)\wedge T_A(y)$ and $F_A(x\to y) \leq F_A(x)\vee F_A(y)$. □

Example 3.3. *Let $E = \{a_1, a_2, a_3, a_4, a_5, a_6\}$. Define operations "$\otimes, \sim$" and "$\wedge$" on E as follows:*

$\wedge$	a_1	a_2	a_3	a_4	a_5	a_6
a_1	a_1	a_1	a_1	a_1	a_1	a_1
a_2	a_1	a_2	a_2	a_2	a_2	a_2
a_3	a_1	a_2	a_3	a_3	a_3	a_3
a_4	a_1	a_2	a_3	a_4	a_4	a_4
a_5	a_1	a_2	a_3	a_4	a_5	a_5
a_6	a_1	a_2	a_3	a_4	a_5	a_6

,

$\otimes$	a_1	a_2	a_3	a_4	a_5	a_6
a_1	a_1	a_1	a_1	a_1	a_1	a_1
a_2	a_1	a_1	a_1	a_1	a_1	a_2
a_3	a_1	a_1	a_1	a_1	a_2	a_3
a_4	a_1	a_1	a_1	a_2	a_2	a_4
a_5	a_1	a_1	a_2	a_2	a_2	a_5
a_6	a_1	a_2	a_3	a_4	a_5	a_6

and

$\sim$	a_1	a_2	a_3	a_4	a_5	a_6
a_1	a_6	a_4	a_3	a_2	a_1	a_1
a_2	a_4	a_6	a_3	a_2	a_2	a_2
a_3	a_3	a_3	a_6	a_3	a_3	a_3
a_4	a_2	a_2	a_3	a_6	a_4	a_4
a_5	a_1	a_2	a_3	a_4	a_6	a_5
a_6	a_1	a_2	a_3	a_4	a_5	a_6

.

Then $\mathcal{E} = (E, \wedge, \otimes, \sim, a_6)$ is an EQ–algebra. Define a single valued neutrosophic set map A in E as follows:

T_A	a_1	a_2	a_3	a_4	a_5	a_6
	0.22	0.33	0.44	0.55	0.66	0.77

,

I_A	a_1	a_2	a_3	a_4	a_5	a_6
	0.21	0.31	0.41	0.51	0.61	0.71

and

F_A	a_1	a_2	a_3	a_4	a_5	a_6
	0.98	0.88	0.78	0.68	0.58	0.48

Hence $(A, \mathcal{E})$ is a single–valued neutrosophic EQ–subalgebra.

Corollary 3.4. *Let $(\mathcal{E}, A)$ be a single–valued neutrosophic EQ–subalgebra. Then for all $x, y \in H$,*

(i) if $x \leq y$, then $T_A(y\to x) = T_A(x\sim y)$,
(ii) if $x \leq y$, then $I_A(y\to x) = I_A(x\sim y)$,
(iii) if $x \leq y$, then $F_A(y\to x) = F_A(x\sim y)$.

3.1. Single–Valued Neutrosophic EQ–prefilters

In this section, we introduce the concept of single–valued neutrosophic EQ–prefilters and show how to construct of single–valued neutrosophic EQ–prefilters.

Definition 3.5. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra. A map A in E, is called a single–valued neutrosophic EQ–prefilter of $\mathcal{E}$, if for all $x, y \in E$,*

$(SVNF1)$ $T_A(x) \leq T_A(1), I_A(x) \geq I_A(1)$ *and* $F_A(x) \leq F_A(1)$,
$(SVNF2)$ $\wedge\{T_A(x), T_A(x \to y)\} \leq T_A(y)$, $\vee\{I_A(x), I_A(x \to y)\} \geq I_A(y)$ *and* $\wedge\{F_A(x), F_A(x \to y)\} \leq F_A(y)$.

In the following theorem, we will show that how to construct of single–valued neutrosophic EQ–prefilters in EQ–algebras.

Theorem 3.6. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra, A be a single–valued neutrosophic EQ–prefilter of $\mathcal{E}$ and $x, y \in E$.*

(i) *If* $x \leq y$, *then* $\wedge\{T_A(x), T_A(x \to y)\} = T_A(x)$,
(ii) *If* $x \leq y$, *then* $\vee\{I_A(x), I_A(x \to y)\} = I_A(x)$,
(iii) *If* $x \leq y$, *then* $\wedge\{F_A(x), F_A(x \to y)\} = F_A(x)$,
(iv) *If* $x \leq y$, *then* $T_A(x) \leq T_A(y)$ *and* $F_A(x) \leq F_A(y)$,
(v) *If* $x \leq y$, *then* $I_A(y) \leq I_A(x)$.

Proof. $(i), (ii), (iii)$ Since $x \leq y$ we get that $x \to y = 1$, so by definition, $\wedge\{T_A(x), T_A(x \to y)\} = T_A(x)$, $\vee\{I_A(x), I_A(x \to y)\} = I_A(x)$ and $\wedge\{F_A(x), F_A(x \to y)\} = F_A(x)$.

(iv) Since $x \leq y$, by (i) we have $\wedge\{T_A(x), T_A(x \to y)\} = T_A(x)$. So by definition we get $T_A(x) = \wedge\{T_A(x), T_A(x \to y)\} \leq T_A(y)$. In a similar way $x \leq y$ implies that $F_A(x) \leq F_A(x)$.

(v) Since $x \leq y$, by (ii) we have $\vee\{I_A(x), I_A(x \to y)\} = I_A(x)$. Thus by definition we get $I_A(y) \leq \vee\{I_A(x), I_A(x \to y)\} = I_A(x)$ and it follows that $I_A(x) \geq I_A(y)$. □

Corollary 3.7. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra, A be a single–valued neutrosophic EQ–prefilter of $\mathcal{E}$ and $0 \in E$. If for every $y \in E, 0 \wedge y = 0$, then*

(i) $\wedge\{T_A(0), T_A(0 \to y)\} = T_A(0)$, $\vee\{I_A(0), I_A(0 \to y)\} = I_A(0)$,
(ii) $\wedge\{T_A(1), T_A(1 \to y)\} = T_A(\overline{y})$, $\vee\{I_A(1), I_A(1 \to y)\} = I_A(\overline{y})$,
(iii) $\wedge\{T_A(y), T_A(y \to 1)\} = T_A(y)$, $\vee\{I_A(y), I_A(y \to 1)\} = I_A(y)$,
(iv) $\wedge\{T_A(y), T_A(y \to y)\} = T_A(y)$, $\vee\{I_A(y), I_A(y \to y)\} = I_A(y)$,
(v) $T_A(0) \leq T_A(1)$ *and* $I_A(1) \leq I_A(0)$,
(vi) $T_A(x) \leq T_A(y \to x)$ *and* $I_A(x \to y) \geq I_A(y)$,
(vii) $T_A(x \otimes y) \leq T_A(y \sim x)$ *and* $I_A(x \otimes y) \geq I_A(y \sim x)$.

Example 3.8. *Let $E = \{a, b, c, d, 1\}$. Define operations "$\otimes, \sim$" and an operation "$\wedge$" on E as follows:*

$\wedge$	a	b	c	d	1
a	a	a	a	a	a
b	a	b	b	b	b
c	a	b	c	c	c
d	a	b	c	d	d
1	a	b	c	d	1

,

$\otimes$	a	b	c	d	1
a	a	a	a	a	a
b	a	a	a	a	b
c	a	a	a	c	c
d	a	a	a	d	d
1	a	b	c	d	1

and

$\sim$	a	b	c	d	1
a	1	b	a	a	a
b	b	1	b	b	b
c	a	b	1	c	c
d	a	b	c	1	d
1	a	b	c	d	1

.

Then $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ is an EQ–algebra and obtain the operation "$\to$" as follows: Define a single valued neutrosophic set map A in E as follows:

$\to$	a	b	c	d	1
a	1	1	1	1	1
b	b	1	1	1	1
c	a	b	1	1	1
d	a	b	c	1	1
1	a	b	c	d	1

.

T_A	a	b	c	d	1
	0.1	0.2	0.3	0.4	0.5

,

F_A	a	b	c	d	1
	0.55	0.45	0.35	0.25	0.15

and

I_A	a	b	c	d	1
	0.17	0.27	0.37	0.47	0.57

Hence A is a single–valued neutrosophic EQ–prefilter of $\mathcal{E}$.

Theorem 3.9. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra, A be a single–valued neutrosophic EQ–prefilter of $\mathcal{E}$ and $x, y \in E$. Then*

(i) $\wedge\{T_A(x), T_A(x \sim y)\} \leq T_A(y)$ *and* $(I_A(x) \vee I_A(x \sim y)) \geq I_A(y)$,
(ii) $\wedge\{T_A(x), T_A(x \otimes y)\} \leq T_A(y)$ *and* $(I_A(x) \vee I_A(x \otimes y)) \geq I_A(y)$,

(iii) $\wedge\{T_A(x), T_A(x \wedge y)\} \leq T_A(y)$ *and* $(I_A(x) \vee I_A(x \wedge y)) \geq I_A(y)$,
(iv) $T_A(x) \wedge T_A(y) \leq T_A(x) \wedge T_A(x \to y)$,
(v) $I_A(x) \vee I_A(x \to y) \leq I_A(x) \vee I_A(y)$,
(vi) $T_A(x \otimes y) \leq T_A(x) \wedge T_A(x)$,
(vii) $I_A(x \otimes y) \geq I_A(x) \vee I_A(x)$.

Proof. $(i),(ii),(iii)$ Let $x, y \in E$. Since $x \sim y \leq x \to y$ and T_A ia a monotone map, we get that $T_A(x \sim y) \leq T_A(x \to y)$. Hence

$$\wedge\{T_A(x), T_A(x \sim y)\} \leq \wedge\{T_A(x), T_A(x \to y)\} \leq T_A(y).$$

In addition, since I_A is an antimonotone map, $x \sim y \leq x \to y$ concludes that $I_A(x \sim y) \geq I_A(x \to y)$. Hence $\vee\{I_A(x), I_A(x \sim y)\} \geq \vee\{I_A(x), I_A(x \to y)\} \geq I_A(y)$. In a similar way $x \wedge y \leq y$ and $x \otimes y \leq x \to y$, imply that $\wedge\{T_A(x), T_A(x \otimes y)\} \leq T_A(y)$, $\wedge\{T_A(x), T_A(x \wedge y)\} \leq T_A(y)$, $(I_A(x) \vee I_A(x \otimes y)) \geq I_A(y)$ and $(I_A(x) \vee I_A(x \wedge y)) \geq I_A(y)$.

$(iv),(v)$ Let $x, y \in E$. Since $y \leq (x \to y)$, we get that

$$(T_A(x) \wedge T_A(y)) \leq (T_A(x) \wedge T_A(x \to y)) \leq T_A(y).$$

In a similar way we conclude that $I_A(y) \leq (I_A(x) \vee I_A(x \to y)) \leq I_A(x) \vee I_A(y)$.

$(vi),(vii)$ Since $x \otimes y \leq (x \wedge y)$ and T_A is a monotone map, then we get that $T_A(x \otimes y) \leq T_A(x \wedge y) \leq T_A(x) \wedge T_A(y)$. In a similar way since I_A is an antimonotone map, then we get that $I_A(x \otimes y) \geq T_A(x \wedge y) \geq I_A(x) \vee I_A(y)$. □

Corollary 3.10. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra, A be a single–valued neutrosophic EQ–prefilter of $\mathcal{E}$ and $x, y \in E$. Then*

(i) $\wedge\{F_A(x), F_A(x \sim y)\} \leq F_A(y)$,
(ii) $\wedge\{F_A(x), F_A(x \otimes y)\} \leq F_A(y)$,
(iii) $\wedge\{F_A(x), F_A(x \wedge y)\} \leq F_A(y)$,
(iv) $F_A(x) \wedge F_A(y) \leq F_A(x) \wedge F_A(x \to y)$,
(v) $F_A(x \otimes y) \leq F_A(x) \wedge F_A(x)$.

Theorem 3.11. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra, A be a single–valued neutrosophic EQ–prefilter of $\mathcal{E}$ and $x, y, z \in E$.*

(i) *If $x \leq y$, then $T_A(x) \wedge T_A(x \sim y) = T_A(x) \wedge T_A(y \to x)$,*
(ii) *If $x \leq y$, then $T_A(z) \wedge T_A(z \to x) \leq T_A(y)$,*
(iii) *If $x \leq y$, then $T_A(x) \wedge T_A(y \to z) = T_A(x) \wedge T_A(z)$,*
(iv) *If $x \leq y$, then $I_A(x) \vee I_A(x \sim y) = I_A(x) \vee I_A(y \to x)$,*
(v) *If $x \leq y$, then $I_A(z) \vee I_A(z \to x) = I_A(x) \vee I_A(z)$,*
(vi) *If $x \leq y$, then $I_A(x) \vee I_A(y \to z) = I_A(x) \vee I_A(z)$.*

Proof. (i) Let $x, y \in E$. Then $x \leq y$ follows that $x \sim y = y \to x$ and so $T_A(x) \wedge T_A(x \sim y) = T_A(x) \wedge T_A(y \to x)$.

(ii) Let $x, y, z \in E$. Since $z \to x \leq z \to y$, we get that $T_A(z \to x) \leq T_A(z \to y)$ and so $T_A(z) \wedge T_A(z \to x) \leq T_A(z) \wedge T_A(z \to y) \leq T_A(y)$.

(iii) Let $x, y, z \in E$. Since $y \to z \leq x \to z$, we get that $T_A(y \to z) \leq T_A(x \to z)$ and so $T_A(x) \wedge T_A(y \to z) \leq T_A(x) \wedge T_A(x \to z) \leq T_A(z)$. Moreover, $z \leq y \to z$ implies that $T_A(z) \leq T_A(y \to z)$, hence $T_A(z) \wedge T_A(x) \leq T_A(x) \wedge T_A(y \to z) \leq T_A(z) \wedge T_A(x)$ and so $T_A(x) \wedge T_A(y \to z) = T_A(z) \wedge T_A(x)$.

(v) Let $x, y, z \in E$. Since $z \to x \leq z \to y$, we get that $I_A(z \to y) \leq I_A(z \to x)$ and so $I_A(z) \vee I_A(z \to y) \leq I_A(z) \vee I_A(z \to x)$. Moreover, $x \leq y$ implies that $I_A(x) \vee I_A(y) = I_A(x)$, hence by Theorem 3.9, $I_A(z) \vee I_A(x) \vee I_A(y) \leq I_A(z) \vee I_A(z \to x) \leq T_A(x) \vee I_A(z)$ and so $T_A(z) \wedge I_A(z \to x) = I_A(z) \vee I_A(x)$.

(iv) and (vi) in a similar way are obtained. □

Corollary 3.12. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra, A be a single–valued neutrosophic EQ–prefilter of $\mathcal{E}$ and $x, y, z \in E$.*

(i) *If $x \leq y$, then $F_A(x) \wedge F_A(x \sim y) = F_A(x) \wedge F_A(y \to x)$,*
(ii) *If $x \leq y$, then $F_A(z) \wedge F_A(z \to x) = F_A(x) \wedge F_A(z)$,*
(iii) *If $x \leq y$, then $F_A(x) \wedge F_A(y \to z) = F_A(x) \wedge F_A(z)$.*

Theorem 3.13. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra, A be a single–valued neutrosophic EQ–prefilter of $\mathcal{E}$ and $x, y, z \in E$. Then*

(i) $T_A(x \wedge y) = T_A(x) \wedge T_A(y)$,
(ii) $T_A(x) \wedge T_A(x \sim y) \leq T_A(x) \wedge T_A(y)$,

Proof. (i) Since T_A is a monotone map, $x \wedge y \leq x$ and $x \wedge y \leq y$, we obtain $T_A(x \wedge y) \leq T_A(x) \wedge T_A(y)$. In addition from $y \leq x \to (x \wedge y)$ and Theorem 3.9, we conclude that $T_A(x) \wedge T_A(y) \leq (T_A(x) \wedge T_A(x \to (x \wedge y))) \leq T_A(x \wedge y)$. Hence $T_A(x \wedge y) = T_A(x) \wedge T_A(y)$.

(ii) Let $x, y \in E$. Then by Theorem 3.9, $T_A(x) \wedge T_A(x \sim y) \leq T_A(y)$. Since $x \sim y = y \sim x$, we obtain $T_A(x) \wedge T_A(x \sim y) = T_A(x) \wedge T_A(y \sim x) \leq T_A(x)$. So $T_A(x) \wedge T_A(x \sim y) \leq T_A(x) \wedge T_A(y)$.

□

Corollary 3.14. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra, A be a single–valued neutrosophic EQ–prefilter of $\mathcal{E}$ and $x, y, z \in E$. Then*

(i) $F_A(x \wedge y) = F_A(x) \wedge F_A(y)$,
(ii) $F_A(x) \wedge F_A(x \sim y) \leq F_A(x) \wedge F_A(y)$,

Theorem 3.15. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra, A be a single–valued neutrosophic EQ–prefilter of $\mathcal{E}$ and $x, y \in E$. Then*

(i) $I_A(x \wedge y) = I_A(x) \vee I_A(y)$,
(ii) $I_A(x) \vee I_A(x \sim y) \geq I_A(x \wedge y)$,

Proof. (i) Since I_A is an antimonotone map, $x \wedge y \leq x$ and $x \wedge y \leq y$, we obtain $I_A(x \wedge y) \geq I_A(x) \vee I_A(y)$. In addition from $y \leq x \to (x \wedge y)$, we conclude that

$$I_A(x) \vee I_A(y) \geq (I_A(x) \vee I_A(x \to (x \wedge y))) \geq I_A(x \wedge y).$$

Hence $I_A(x \wedge y) = I_A(x) \vee I_A(y)$.

(ii) Let $x, y \in E$. Then, $I_A(x) \vee I_A(x \sim y) \geq I_A(y)$. Since $x \sim y = y \sim x$, we obtain $I_A(x) \vee I_A(x \sim y) = I_A(x) \vee I_A(y \sim x) \geq I_A(x)$. So $I_A(x) \vee I_A(x \sim y) \geq I_A(x) \vee I_A(y)$.

□

Corollary 3.16. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra, A be a single–valued neutrosophic EQ–prefilter of $\mathcal{E}$ and $x, y \in E$. Then $x = y$, implies that $I_A(x) \vee I_A(x \sim y) = I_A(x \wedge y)$.*

In Example 3.8, for $x = a$ and $y = d$, we have $I_A(x) \vee I_A(x \sim y) = I_A(x \wedge y)$, while $x \neq y$.

4. Single–Valued Neutrosophic EQ–filters

In this section, we introduce the concept of single–valued neutrosophic EQ–filters as generalization of single–valued neutrosophic EQ–prefilters and prove some their properties.

Definition 4.1. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra. A map A in E, is called a single–valued neutrosophic EQ–filter of $\mathcal{E}$, if for all $x, y, z \in E$,*

$(SVNF1)$ $T_A(x) \leq T_A(1), I_A(x) \geq I_A(1)$ *and* $F_A(x) \leq F_A(1)$,
$(SVNF2)$ $\wedge\{T_A(x), T_A(x \to y)\} \leq T_A(y)$, $\vee\{I_A(x), I_A(x \to y)\} \geq I_A(y)$ *and* $\wedge\{F_A(x), F_A(x \to y)\} \leq F_A(y)$,
$(SVNF3)$ $T_A(x \to y) \leq T_A((x \otimes z) \to (y \otimes z)), I_A(x \to y) \geq I_A((x \otimes z) \to (y \otimes z))$, *and* $F_A(x \to y) \leq F_A((x \otimes z) \to (y \otimes z))$.

In the following theorem, we will show that how to construct of single–valued neutrosophic EQ–prefilters in EQ–algebras.

Theorem 4.2. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra, A be a single–valued neutrosophic EQ–filter of $\mathcal{E}$ and $x, y \in E$.*

(i) *If* $T_A(x \to y) = T_A(1)$, *then for every* $z \in E$, $T_A((x \otimes z) \to (y \otimes z)) = T_A(x \to y)$.
(ii) *If* $x \leq y$, *then for every* $z \in E$, $T_A((x \otimes z) \to (y \otimes z)) = T_A(x \to y)$.
(iii) *If* $T_A(x \to y) = T_A(0)$, *then for every* $z \in E$, $T_A((x \otimes z) \to (y \otimes z)) \geq T_A(x \to y)$.
(iv) *If* $I_A(x \to y) = I_A(1)$, *then for every* $z \in E$, $I_A((x \otimes z) \to (y \otimes z)) = I_A(x \to y)$.
(v) *If* $x \leq y$, *then for every* $z \in E$, $I_A((x \otimes z) \to (y \otimes z)) = I_A(x \to y)$.
(vi) *If* $I_A(x \to y) = I_A(0)$, *then for every* $z \in E$, $I_A((x \otimes z) \to (y \otimes z)) \leq I_A(x \to y)$.

Proof. $(i), (iii), (iv)$ and (vi) by definition are obtained.

(ii) Since $x \leq y$ we get that $x \to y = 1$ and by definition $x \otimes z \leq y \otimes z$. Hence by item (i), we have $T_A((x \otimes z) \to (y \otimes z)) = T_A(x \to y)$.

(v) It is similar to the item (ii).

□

Corollary 4.3. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra, A be a single–valued neutrosophic EQ–prefilter of $\mathcal{E}$ and $0, x, y, z \in E$. If for every $y \in E, 0 \wedge y = 0$, Then*

(i) $T_A(0 \to y) = T_A((x \otimes z) \to (y \otimes z))$,
(ii) $T_A(x \to x) = T_A((x \otimes z) \to (y \otimes z))$,
(iii) $T_A(x \to 1) = T_A((x \otimes z) \to (y \otimes z))$,
(iv) $I_A(0 \to y) = I_A((x \otimes z) \to (y \otimes z))$,
(v) $I_A(x \to x) = I_A((x \otimes z) \to (y \otimes z))$,
(vi) $I_A(x \to 1) = I_A((x \otimes z) \to (y \otimes z))$.

Corollary 4.4. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra, A be a single–valued neutrosophic EQ–filter of $\mathcal{E}$ and $x, y \in E$.*

(i) *If* $F_A(x \to y) = F_A(1)$*, then for every* $z \in E$*,* $F_A((x \otimes z) \to (y \otimes z)) = F_A(x \to y)$*,*
(ii) *If* $x \leq y$*, then for every* $z \in E$*,* $F_A((x \otimes z) \to (y \otimes z)) = F_A(x \to y)$*.*
(iii) *If* $F_A(x \to y) = F_A(0)$*, then for every* $z \in E$*,* $F_A((x \otimes z) \to (y \otimes z)) \geq F_A(x \to y)$*.*

Example 4.5. *Let* $E = \{0, a, b, c, 1\}$*. Define operations "*$\otimes, \sim$*" and an operation "*$\wedge$*" on* E *as follows:*

$\wedge$	0	a	b	c	1
0	0	0	0	0	0
a	0	a	a	a	a
b	0	a	b	$-$	b
c	0	a	$-$	c	c
1	0	a	b	c	1

,

$\otimes$	0	a	b	c	1
0	0	0	0	0	0
a	0	0	0	a	a
b	0	a	b	a	b
c	0	0	0	c	c
1	0	a	b	c	1

and

$\sim$	0	a	b	c	1
0	1	0	0	0	0
a	0	1	a	a	a
b	0	a	1	a	b
c	0	a	a	1	c
1	0	a	b	c	1

.

Then $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ *is an EQ–algebra, where b and c are non-comparable. Now, obtain the operation "*$\to$*" as follows:*

$\to$	0	a	b	c	1
0	1	1	1	1	1
a	0	1	1	1	1
b	0	a	1	c	1
c	0	a	b	1	1
1	0	a	b	c	1

.

Define a single valued neutrosophic set map A *in* E *as follows:*

T_A	0	a	b	c	1
	0.4	0.4	0.4	0.4	0.5

,

I_A	0	a	b	c	1
	0.62	0.62	0.62	0.62	0.11

and

F_A	0	a	b	c	1
	0.2	0.2	0.2	0.2	0.6

Hence A *is a single–valued neutrosophic EQ–filter of* $\mathcal{E}$*.*

Theorem 4.6. *Let* $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ *be an EQ–algebra,* A *be a single–valued neutrosophic EQ–filter of* $\mathcal{E}$ *and* $x, y \in E$*. Then*

(i) $T_A(x \otimes y) = T_A(x) \wedge T_A(y)$*,*
(ii) $I_A(x \otimes y) = I_A(x) \vee I_A(y)$*,*
(iii) $T_A(x \sim y) \leq T_A(y \to x)$*,*
(iv) $T_A(z) \wedge T_A(y) \leq T_A(x \to z)$*,*
(v) $T_A(x \sim y) \wedge T_A(y \sim z) \leq T_A(x \sim z)$*,*
(vi) $I_A(x \sim y) \geq I_A(y \to x)$*,*
(vii) $I_A(z) \vee I_A(y) \geq I_A(x \to z)$*,*
$(viii)$ $I_A(x \sim y) \vee I_A(y \sim z) \geq I_A(x \sim z)$*.*

Proof. (i) Let $x, y \in E$. Since A is a single–valued neutrosophic EQ–filter of $\mathcal{E}$, we get that

$$T_A(1 \to y) \leq T_A((1 \otimes x) \to (y \otimes x)) = T_A(x \to (y \otimes x)).$$

In addition by the item $(SVNF2)$, we have

$$T_A(x) \wedge T_A(x \to (y \otimes x)) \leq T_A(y \otimes x).$$

Hence

$$T_A(x) \wedge T_A(y) \leq T_A(x) \wedge T_A(1 \to y) \leq T_A(y \otimes x).$$

We apply Theorem 3.9 and obtain $T_A(x) \wedge T_A(y) = T_A(y \otimes x)$.

(ii) Let $x, y \in E$. By item $(SVNF2)$, we have

$$I_A(1 \to y) \geq I_A(1 \otimes x) \to (y \otimes x).$$

Then $I_A(x) \vee I_A(1 \to y) \geq I_A(x) \vee I_A(x \to (y \otimes x)) \geq I_A(y \otimes x)$. It follows that $I_A(x) \vee I_A(y) \geq I_A(x) \vee I_A(1 \to y) \geq I_A(y \otimes x)$. Therefore, Theorem 3.9 implies that $I_A(x) \vee I_A(y) = I_A(y \otimes x)$.

(iii) Let $x, y \in E$. Then $x \sim y \leq (x \to y) \wedge (y \to x)$ implies that $T_A(x \sim y) \leq T_A(y \to x)$.

(iv) Let $x, y, z \in E$. Since $(x \to y) \otimes (y \to z) \leq (x \to z)$, by item (i), we get that

$$\begin{aligned} T_A(y) \wedge T_A(z) &\leq T_A(x \to y) \wedge T_A(y \to z) \\ &= T_A((x \to y) \otimes (y \to z)) \\ &\leq T_A(x \to z). \end{aligned}$$

(v) Let $x, y, z \in E$. Since $(x \sim y) \otimes (y \sim z) \leq x \sim z$, we get that $T_A((x \sim y) \otimes (y \sim z)) \leq T_A(x \sim z)$. Now by item (i), we get that $T_A(x \sim y) \wedge T_A(y \sim z) = T_A((x \sim y) \otimes (y \sim z)) \leq T_A(x \sim z)$. (vi), (vii) and $(viii)$ in a similar way are obtained. $\square$

Example 4.7. *Consider the EQ–algebra and the single–valued neutrosophic EQ–prefilter* A *of* $\mathcal{E}$ *which are defined in Example 3.8. Since* $0.1 = T_A(a) = T_A(d \otimes c) \neq 0.3 = 0.4 \wedge 0.3 = T_A(d) \wedge T_A(c)$*, we conclude that* A *is not a single–valued neutrosophic EQ–filter* A *of* $\mathcal{E}$*.*

Corollary 4.8. *Let* $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ *be an EQ–algebra,* A *be a single–valued neutrosophic EQ–filter of* $\mathcal{E}$ *and* $x, y, z \in E$*. Then*

(i) $F(x \otimes y) = F_A(x) \wedge F_A(y)$,
(ii) $F_A(x \sim y) \leq F_A(y \to x)$,
(iii) $F_A(z) \wedge F_A(y) \leq F_A(x \to z)$,
(iv) $F_A(x \sim y) \wedge F_A(y \sim z) \leq F_A(x \sim z)$.

4.1. Special single–valued neutrosophic EQ–filters

In this section, we apply the concept of homomorphisms and (α, β, γ)–level sets to construct of single–valued neutrosophic EQ–filters.

Theorem 4.9. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra and $\{A_i = (T_{A_i}, F_{A_i}, I_{A_i})\}_{i \in I}$ be a family of single–valued neutrosophic EQ–filters of $\mathcal{E}$. Then $\bigcap\limits_{i \in I} A_i$ is a single–valued neutrosophic EQ–filter of $\mathcal{E}$.*

Proof. Let $x \in E$, then for any $i \in I, T_{A_i}(x) \leq T_{A_i}(1), F_{A_i}(x) \leq F_{A_i}(1), I_{A_i}(x) \geq I_{A_i}(1)$ and so $(\bigcap\limits_{i \in I} T_{A_i})(x) = \bigwedge\limits_{i \in I} T_{A_i}(x) \leq T_{A_i}(1), (\bigcap\limits_{i \in I} F_{A_i})(x) = \bigwedge\limits_{i \in I} F_{A_i}(x) \leq F_{A_i}(1)$ and $(\bigcap\limits_{i \in I} I_{A_i})(x) = \bigwedge\limits_{i \in I} I_{A_i}(x) \geq I_{A_i}(1)$. Let $x, y \in E$. Then

$$(\bigcap_{i \in I} T_{A_i})(x) \wedge (\bigcap_{i \in I} T_{A_i})(x \to y)$$
$$= \bigwedge_{i \in I} T_{A_i}(x) \wedge \bigwedge_{i \in I} T_{A_i}(x \to y) \leq \bigwedge_{i \in I} T_{A_i}(y)$$
$$= \bigcap_{i \in I} T_{A_i}(y),$$
$$(\bigcap_{i \in I} F_{A_i})(x) \wedge (\bigcap_{i \in I} F_{A_i})(x \to y)$$
$$= \bigwedge_{i \in I} F_{A_i}(x) \wedge \bigwedge_{i \in I} F_{A_i}(x \to y) \leq \bigwedge_{i \in I} F_{A_i}(y)$$
$$= \bigcap_{i \in I} F_{A_i}(y) \text{ and}$$
$$(\bigcap_{i \in I} I_{A_i})(x) \vee (\bigcap_{i \in I} I_{A_i})(x \to y)$$
$$= \bigwedge_{i \in I} I_{A_i}(x) \vee \bigwedge_{i \in I} I_{A_i}(x \to y) \geq \bigwedge_{i \in I} I_{A_i}(y)$$
$$= \bigcap_{i \in I} I_{A_i}(y).$$

Let $x, y, z \in E$. Then

$$(\bigcap_{i \in I} T_{A_i})(x \to y) = \bigwedge_{i \in I} T_{A_i}(x \to y)$$
$$\leq \bigwedge_{i \in I} T_{A_i}(x \otimes z \to y \otimes z)$$
$$= \bigcap_{i \in I} T_{A_i}(x \otimes z \to y \otimes z),$$
$$(\bigcap_{i \in I} F_{A_i})(x \to y) = \bigwedge_{i \in I} F_{A_i}(x \to y)$$
$$\leq \bigwedge_{i \in I} F_{A_i}(x \otimes z \to y \otimes z)$$
$$= \bigcap_{i \in I} I_{A_i}(x \otimes z \to y \otimes z) \text{ and}$$
$$(\bigcap_{i \in I} I_{A_i})(x \to y) = \bigwedge_{i \in I} I_{A_i}(x \to y)$$
$$\leq \bigwedge_{i \in I} I_{A_i}(x \otimes z \to y \otimes z)$$
$$= \bigcap_{i \in I} I_{A_i}(x \otimes z \to y \otimes z).$$

Thus $\bigcap\limits_{i \in I} A_i$ is a single–valued neutrosophic EQ–filter of $\mathcal{E}$. □

Definition 4.10. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra, A be a single–valued neutrosophic EQ–filter of $\mathcal{E}$ and $\alpha, \beta, \gamma \in [0,1]$. Consider $T_A^{\alpha} = \{x \in E \mid T_A(x) \geq \alpha\}, F_A^{\beta} = \{x \in E \mid F_A(x) \geq \beta\}, I_A^{\gamma} = \{x \in E \mid T_A(x) \leq \gamma\}$ and define $A^{(\alpha,\beta,\gamma)} = \{x \in E \mid T_A(x) \geq \alpha, F_A(x) \geq \beta, I_A(x) \leq \gamma\}$. For any $\alpha, \beta, \gamma \in [0,1]$ the set $A^{(\alpha,\beta,\gamma)}$ is called an (α, β, γ)-level set.*

Example 4.11. *Consider the EQ–algebra $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$, single–valued neutrosophic EQ–filter A of $\mathcal{E}$ which are defind in Example 4.5. If $\alpha = 0.3, \beta = 0.4$ and $\gamma = 0.5$, then $T_A^{\alpha} = E, F_A^{\beta} = \{1\}, I_A^{\gamma} = \{1\}$ and $A^{(\alpha,\beta,\gamma)} = \{1\}$.*

Theorem 4.12. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra, A be a single–valued neutrosophic EQ–filter of $\mathcal{E}$ and $\alpha, \beta, \gamma \in [0,1]$. Then*

(i) $A^{(\alpha,\beta,\gamma)} = T_A^{\alpha} \cap I_A^{\beta} \cap F_A^{\gamma}$,
(ii) if $\emptyset \neq A^{(\alpha,\beta,\gamma)}$, then $A^{(\alpha,\beta,\gamma)}$ is an EQ–filter of $\mathcal{E}$,
(ii) if $A^{(\alpha,\beta,\gamma)}$ is an EQ–filter of $\mathcal{E}$, then A is a single–valued neutrosophic EQ–filter in $\mathcal{E}$.

Proof. (i) It is obtained by definition.

(ii) $\emptyset \neq A^{(\alpha,\beta,\gamma)}$, implies that there exists $x \in A^{(\alpha,\beta,\gamma)}$. By Theorem 3.6, we conclude that $\alpha \leq T_A(x) \leq T_A(1), \beta \leq F_A(x) \leq F_A(1)$ and $\gamma \geq I_A(x) \geq I_A(1)$. Therefore, $1 \in A^{(\alpha,\beta,\gamma)}$.

Let $x \in A^{(\alpha,\beta,\gamma)}$ and $x \leq y$. Since T_A and F_A are monotone maps and I_A is an antimonotone map, we get that $\alpha \leq T_A(x) \leq T_A(y), \beta \leq F_A(x) \leq F_A(y)$ and $\gamma \geq I_A(x) \geq I_A(y)$. Hence $y \in A^{(\alpha,\beta,\gamma)}$.

Let $x \in A^{(\alpha,\beta,\gamma)}$ and $x \to y \in A^{(\alpha,\beta,\gamma)}$. Since A is a single–valued neutrosophic EQ–filter of $\mathcal{E}$, by definition we get that $\alpha \leq T_A(x) \wedge T_A(x \to y) \leq T_A(y), \beta \leq F_A(x) \wedge F_A(x \to y) \leq F_A(y)$ and $\gamma \geq I_A(x) \vee I_A(x \to y) \geq I_A(y)$. So $y \in A^{(\alpha,\beta,\gamma)}$.

Let $x \to y \in A^{(\alpha,\beta,\gamma)}$ and $z \in E$. Since A is a single–valued neutrosophic EQ–filter of $\mathcal{E}$, by definition we get that $\alpha \leq T_A(x \to y) \leq T_A((x \otimes z) \to (y \otimes z)), \gamma \geq I_A(x \to y) \geq I_A((x \otimes z) \to (y \otimes z))$ and $\beta \leq F_A(x \to y) \leq F_A((x \otimes z) \to (y \otimes z))$. It follows that $(x \otimes z) \to (y \otimes z) \in A^{(\alpha,\beta,\gamma)}$ and so $A^{(\alpha,\beta,\gamma)}$ is an EQ–filter of $\mathcal{E}$.

(iii) Let $x, y, z \in E$. Consider $\alpha_x = T_A(x), \beta_x = F_A(x)$ and $\gamma_x = I_A(x)$. Since $A^{(\alpha,\beta,\gamma)}$ is an EQ–filter of $\mathcal{E}$, then $1 \in A^{(\alpha,\beta,\gamma)}$ implies that

$$T_A(1) \geq \alpha_x = T_A(x), F_A(1) \geq \beta_x = F_A(x),$$

$I_A(1) \leq \gamma_x = I_A(x)$.

Let $\alpha_{x\to y} = T_A(x \to y), \beta_{x\to y} = F_A(x \to y), \gamma_{x\to y} = I_A(x \to y), \alpha = \alpha_x \wedge \alpha_{x\to y}, \beta = \beta_x \wedge \beta_{x\to y}$ and $\gamma = \gamma_x \vee \gamma_{x\to y}$. We have $T_A(x) = \alpha_x \geq \alpha, T_A(x \to y) = \alpha_{x\to y} \geq \alpha, F_A(x) = \beta_x \geq \beta, F_A(x \to y) = \beta_{x\to y} \geq \beta$ and $I_A(x) = \gamma_x \leq \gamma, I_A(x \to y) = \gamma_{x\to y} \leq \gamma$, so $x, x \to y \in A^{(\alpha,\beta,\gamma)}$. Since $A^{(\alpha,\beta,\gamma)}$ is an EQ–filter of $\mathcal{E}$ we get $y \in A^{(\alpha,\beta,\gamma)}$. Thus we conclude that

$$T_A(y) \geq \alpha = \alpha_x \wedge \alpha_{x\to y} = T_A(x) \wedge T_A(x \to y),$$

$$F_A(y) \geq \beta = \beta_x \wedge \beta_{x\to y} = F_A(x) \wedge F_A(x \to y)$$

and $I_A(y) \leq \gamma = \gamma_x \vee \gamma_{x\to y} = I_A(x) \vee I_A(x \to y)$. We have $T_A(x \to y) = \alpha_{x\to y} \geq \alpha_{x\to y}, F_A(x \to y) = \beta_{x\to y} \geq \beta_{x\to y}$ and $I_A(x \to y) = \gamma_{x\to y} \leq \gamma_{x\to y}$, so $x \to y \in A^{(\alpha_{x\to y},\beta_{x\to y},\gamma_{x\to y})}$. Since $A^{(\alpha_{x\to y},\beta_{x\to y},\gamma_{x\to y})}$ is an EQ–filter of $\mathcal{E}$ we get $x \otimes z \to y \otimes z \in A^{(\alpha_{x\to y},\beta_{x\to y},\gamma_{x\to y})}$. Thus we conclude that

$$T_A((x \otimes z) \to (y \otimes z)) \geq \alpha_{x\to y} = T_A(x \to y),$$

$$F_A((x \otimes z) \to (y \otimes z)) \geq \beta_{x\to y} = F_A(x \to y)$$

and $I_A((x \otimes z) \to (y \otimes z)) \geq \gamma_{x\to y} = I_A(x \to y)$. It follows that A is a single–valued neutrosophic EQ–filter $\mathcal{E}$. □

Corollary 4.13. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra, A be a single–valued neutrosophic EQ–filter of $\mathcal{E}$, $\alpha, \beta, \gamma \in [0,1]$ and $\emptyset \neq A^{(\alpha,\beta,\gamma)}$.*

(i) $A^{(\alpha,\beta,\gamma)}$ is an EQ–filter of $\mathcal{E}$ if and only if A is a single–valued neutrosophic EQ–filter in $\mathcal{E}$.

(ii) If $G_A = \{x \in E \mid T_A(1) = F_A(1) = 1, I_A(0) = 1\}$, then G_A is an EQ–filter in $\mathcal{E}$

Let $A = (T_A, F_A, I_A)$ be a single–valued neutrosophic EQ–filter in $\mathcal{E}, \alpha, \alpha', \beta, \beta', \gamma, \gamma' \in [0,1]$ and $\emptyset \neq H \subseteq \mathcal{E}$. Consider $T_{A,H}^{[\alpha,\alpha']} = \begin{cases} \alpha & \text{if } x \in H, \\ \alpha' & \text{otherwise,} \end{cases}$ $F_{A,H}^{[\alpha,\alpha']} = \begin{cases} \beta & \text{if} x \in H \\ \beta' & \text{o.w,} \end{cases}$

and $I_{A,H}^{[\alpha,\alpha']} = \begin{cases} \gamma & \text{if } x \in H, \\ \gamma' & \text{otherwise.} \end{cases}$ Then we have the following corollary.

Corollary 4.14. *Let $A = (T_A, F_A, I_A)$ be a single–valued neutrosophic EQ–filter in $\mathcal{E}$. Then*

(i) $T_{A,H}^{[\alpha,\alpha']}, F_{A,G}^{[\alpha,\alpha']}$ and $I_{A,G}^{[\alpha,\alpha']}$ are fuzzy subsets,

(ii) $T_{A,H}^{[\alpha,\alpha']}$ is a fuzzy filter in E if and only if G is an EQ–filter of $\mathcal{E}$,

(iii) $F_{A,H}^{[\alpha,\alpha']}$ is a fuzzy filter in E if and only if G is an EQ–filter of $\mathcal{E}$,

(iv) $I_{A,H}^{[\alpha,\alpha']}$ is a fuzzy filter in E if and only if G is an EQ–filter of $\mathcal{E}$.

Definition 4.15. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra, A be a single–valued neutrosophic EQ–filter of $\mathcal{E}$. Then A is said to be a normal single–valued neutrosophic EQ–filter of $\mathcal{E}$ if there exists $x, y, z \in E$ such that $T_A(x) = 1, I_A(y) = 1$ and $F_A(z) = 1$.*

Example 4.16. *Consider the EQ–algebra $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$, which is defind in Example 4.5. If Define a single valued neutrosophic set map A in E as follows:*

T_A	0	a	b	c	1
	0.4	0.4	0.4	0.4	1

,

I_A	0	a	b	c	1
	1	1	1	1	0.11

and

F_A	0	a	b	c	1
	0.2	0.2	0.2	0.2	1

Hence A is a normal single–valued neutrosophic EQ–filter of $\mathcal{E}$.

Theorem 4.17. *Let $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ be an EQ–algebra and A be a single–valued neutrosophic EQ–filter of $\mathcal{E}$. Then A is a normal single–valued neutrosophic EQ–filter of $\mathcal{E}$ if and only if $T_A(1) = 1, F_A(1) = 1$ and $I_A(0) = 1$.*

Proof. By Corollary 3.7, it is straightforward. □

Corollary 4.18. *Let $A = (T_A, I_A, F_A)$ be a single–valued neutrosophic EQ–filter of $\mathcal{E}$. Then*

- (i) *A is a normal single–valued neutrosophic EQ–filter of $\mathcal{E}$ if and only if T_A, F_A and I_A are normal fuzzy subset.*
- (ii) *If there exists a sequence $\{(x_n, y_n, z_n)\}_{n=1}^{\infty}$ of elements E in such a way that*

$$\{(T_A(x_n), I_A(y_n), F_A(z_n))\} \to (1, 1, 1),$$

then $A(1, 0, 1) = (1, 1, 1)$.

Corollary 4.19. *Let $\{A_i = (T_{A_i}, F_{A_i}, I_{A_i})\}_{i\in I}$ be a family of normal single–valued neutrosophic EQ–filters of $\mathcal{E}$. Then $\bigcap\limits_{i\in I} A_i$ is a normal single–valued neutrosophic EQ–filter of $\mathcal{E}$.*

Let $A = (T_A, I_A, F_A)$ be a single–valued neutrosophic EQ–filter of $\mathcal{E}, x \in E$ and $p \in [1, +\infty)$. Consider $T_A^{+p}(x) = \dfrac{1}{p}(p + T_A(x) - T_A(1))$, $F_A^{+p}(x) = \dfrac{1}{p}(p + F_A(x) - F_A(1))$ and $I_A^{+p}(x) = \dfrac{1}{p}(p + I_A(x) - I_A(0))$.

Theorem 4.20. *Let $A = (T_A, I_A, F_A)$ be a single–valued neutrosophic EQ–filter of $\mathcal{E}$. Then*

- (i) *T_A^{+p} is a normal EQ–filter of $\mathcal{E}$,*
- (ii) *I_A^{+p} is a normal EQ–filter of $\mathcal{E}$,*
- (iii) *$(T_A^{+p})^{+p} = T_A^{+p}$ if and only if $p = 1$,*
- (iv) *$(I_A^{+p})^{+p} = I_A^{+p}$ if and only if $p = 1$,*
- (v) *$(T_A^{+p})^{+p} = T_A$ if and only if T_A is normal EQ–filter,*
- (vi) *$(I_A^{+p})^{+p} = I_A$ if and only if $I_A(0) = 1$.*

Proof. (i) Let $x \in E$. Because $T_A(x) \le T_A(1)$, then we have $T_A^{+p}(x) = \dfrac{1}{p}(p + T_A(x) - T_A(1)) \le 1$. Assume that $x, y \in E$. Using $(SVNF2)$, we get that

$$\begin{aligned}
T_A^{+p}(x) \wedge T_A^{+p}(x \to y) &= \frac{1}{p}(p + T_A(x) - T_A(1)) \\
&\wedge \frac{1}{p}(p + T_A(x \to y) - T_A(1)) \\
&= \frac{1}{p}\big[(p + T_A(x) - T_A(1)) \\
&\wedge (p + T_A(x \to y) - T_A(1))\big] \\
&= \frac{1}{p}\big[((p \wedge p) + (T_A(x) \wedge T_A(x \to y)) \\
&- (T_A(1) \wedge T_A(1))\big] \\
&\le \frac{1}{p}(p + T_A(y) - T_A(1)) = T_A^{+p}(y).
\end{aligned}$$

Suppose that $x, y, z \in E$. Using $(SVNF3)$, we get that

$$\begin{aligned}
T_A^{+p}(x \to y) &= \frac{1}{p}(p + T_A(x \to y) - T_A(1)) \\
&\le \frac{1}{p}(p + T_A(x \otimes z \to y \otimes z) \\
&- T_A(1)) = T_A^{+p}(x \otimes z \to y \otimes z).
\end{aligned}$$

Thus T_A^{+p} is an EQ–filter of $\mathcal{E}$. In addition the equality $T_A^{+p}(1) = \dfrac{1}{p}(p + T_A(1) - T_A(1)) = 1$, implies that T_A^{+p} is a normal EQ–filter of $\mathcal{E}$.

(ii) Let $x \in E$. Since $I_A(x) \le I_A(0)$ we get that $I_A^{+p}(x) = \dfrac{1}{p}(p + I_A(x) - I_A(0)) \le 1$. Items $(SVNF2)$ and $(SVNF3)$ are obtained similar to the item (i).

(iii) Assume that $x \in E$. Then

$$\begin{aligned}
(T_A^{+p})^{+p}(x) &= [\frac{1}{p}(p + T_A(x) - T_A(1))]^{+p} \\
&= \frac{1}{p}[p + \frac{1}{p}(p + T_A(x) - T_A(1)) \\
&- \frac{1}{p}(p + T_A(1) - T_A(1))] \\
&= \frac{1}{p}(p + \frac{1}{p}(T_A(x) - T_A(1))).
\end{aligned}$$

So

$$
\begin{aligned}
(T_A^{+p})^{+p}(x) &= T_A^{+p}(x) \\
&\Longleftrightarrow \frac{1}{p}(p + \frac{1}{p}(T_A(x) - T_A(1))) \\
&= \frac{1}{p}(p + T_A(x) - T_A(1)) \\
&\Longleftrightarrow p = 1.
\end{aligned}
$$

(iv) It is similar to (iii).
(v) Let $x \in E$. $(T_A^{+p})^{+p} = T_A$ if and only if

$$\frac{1}{p}(p + \frac{1}{p}(T_A(x) - T_A(1))) = T_A(x)$$

$$\Longleftrightarrow T_A(1) = (1 - p^2)T_A(x) + p^2$$

$$\Longleftrightarrow p = 1 \Longleftrightarrow T_A(1) = 1.$$

(vi) It is similar to (v). □

Example 4.21. *Let $E = \{0, a, b, c, 1\}$. Define operations "$\otimes, \sim$" and an operation "$\wedge$" on E as follows:*

$\wedge$	0	a	b	c	1
0	0	0	0	0	0
a	0	a	a	a	a
b	0	a	b	b	b
c	0	a	b	c	c
1	0	a	b	c	1

,

$\otimes$	0	a	b	c	1
0	0	0	0	0	0
a	0	0	0	0	a
b	0	0	0	a	b
c	0	0	0	a	c
1	0	a	b	c	1

and

$\sim$	0	a	b	c	1
0	1	c	b	a	0
a	c	1	c	b	a
b	b	c	1	c	b
c	a	b	c	1	c
1	0	a	b	c	1

.

Then $\mathcal{E} = (E, \wedge, \otimes, \sim, 1)$ is an EQ–algebra, where b and c are non-comparable. Now, obtain the operation "$\rightarrow$" as follows:

$\rightarrow$	0	a	b	c	1
0	1	1	1	1	1
a	c	1	1	1	1
b	b	c	1	1	1
c	a	b	c	1	1
1	0	a	b	c	1

.

Define a single valued neutrosophic set map A in E as follows:

T_A	0	a	b	c	1
	0.41	0.42	0.43	0.44	0.5

,

I_A	0	a	b	c	1
	0.69	0.68	0.67	0.66	0.65

and

F_A	0	a	b	c	1
	0.21	0.22	0.23	0.24	0.25

Hence A is a single–valued neutrosophic EQ–prefilter of $\mathcal{E}$. Consider $p = 3$, then we obtain a single–valued neutrosophic EQ–prefilter A^{+3} in E as follows:

T_A^{+3}	0	a	b	c	1
	0.97	0.973	0.977	0.98	1

,

I_A^{+3}	0	a	b	c	1
	1	0.977	0.973	0.99	0.987

and

F_A^{+3}	0	a	b	c	1
	0.987	0.99	0.993	0.997	1

.

Corollary 4.22. *Let $A = (T_A, I_A, F_A)$ be a single–valued neutrosophic EQ–filter of $\mathcal{E}$. Then*

(i) F_A^{+p} is a normal EQ–filter of $\mathcal{E}$,
(ii) $(F_A^{+p})^{+p} = F_A^{+p}$ if and only if $p = 1$,
(iii) $(F_A^{+p})^{+p} = F_A$ if and only if F_A is normal EQ–filter.

Corollary 4.23. *Let $A = (T_A, I_A, F_A)$ be a single–valued neutrosophic EQ–filter of $\mathcal{E}$. Then*

(i) $A^{+p} = (T_A^{+p}, I_A^{+p}, F_A^{+p})$ is a normal single–valued neutrosophic EQ–filter of $\mathcal{E}$,
(ii) $(A^{+p})^{+p} = A^{+p}$ if and only if $p = 1$,
(ii) $(A^{+p})^{+p} = A$ if and only if A is a normal single–valued neutrosophic EQ–filter.

Proof. It is trivial by Theorem 4.20 and Corollary 4.22. □

Let $A = (T_A, I_A, F_A)$ be a single–valued neutrosophic EQ–filter of $\mathcal{E}$ and g be an endomorphism on $\mathcal{E}$. Now we define $A^g = (T_A^g, I_A^g, F_A^g)$ by $T_A^g(x) = T_A(g(x)), F_A^g(x) = F_A(g(x))$ and $I_A^g(x) = I_A(g(x))$.

Theorem 4.24. *Let $A = (T_A, I_A, F_A)$ be a single–valued neutrosophic EQ–filter of $\mathcal{E}$ and $x, y \in E$. Then*

(i) if $x \leq y$, then $T_A^g(x) \leq T_A^g(y), F_A^g(x) \leq F_A^g(y)$ and $I_A^g(x) \geq I_A^g(y)$,
(ii) A^g is a single–valued neutrosophic EQ–filter of $\mathcal{E}$,
(iii) $T'_A(x) = \frac{1}{2}(T_A^g(x) + T_A(x))$ is a fuzzy filter in E,
(iv) $F'_A(x) = \frac{1}{2}(F_A^g(x) + F_A(x))$ is a fuzzy filter in E,

(v) $A'^g = (T'_A, I'_A, F'_A)$ *is a single–valued neutrosophic EQ–filter of* $\mathcal{E}$.

Proof. (i) Let $x, y \in E$. If $x \leq y$, then $g(x) \leq g(y)$. It follows that $T^g_A(x) = T_A(g(x)) \leq T_A(g(y)), F^g_A(x) = F_A(g(x)) \leq F_A(g(y))$ and $I^g_A(x) = I_A(g(x)) \geq I_A(g(y))$.

(ii) Since $g(x \to y) = g(x) \to g(y)$, we get that

$$\begin{aligned} &T^g_A(x) \wedge T^g_A(x \to y) \\ &\quad= T_A(g(x)) \wedge T_A(g(x) \to g(y)) \\ &\quad\leq T_A(g(y)) = T^g_A(y), F^g_A(x) \wedge F^g_A(x \to y) \\ &\quad= F_A(g(x)) \wedge F_A(g(x) \to g(y)) \\ &\quad\leq F_A(g(y)) = F^g_A(y) \end{aligned}$$

and $I^g_A(x) \vee I^g_A(x \to y) = I_A(g(x)) \vee I_A(g(x) \to g(y)) \leq I_A(g(y)) = I^g_A(y)$.

Let $z \in E$. Since $g(x \otimes z \to y \otimes z) = g(x \otimes z) \to g(y \otimes z)$, we get that

$$\begin{aligned} T^g_A(x \to y) &= T_A(g(x) \to g(y)) \\ &\leq T_A(g(x \otimes z \to y \otimes z)) \\ &= T_A(g(x \otimes z) \to (y \otimes z)) \\ &= T^g_A(x \otimes z \to y \otimes z), \\ F^g_A(x \to y) &= F_A(g(x) \to g(y)) \\ &\leq F_A(g(x \otimes z \to y \otimes z)) \\ &= F_A(g(x \otimes z) \to (y \otimes z)) \\ &= F^g_A(x \otimes z \to y \otimes z), \\ I^g_A(x \to y) &= I_A(g(x) \to g(y)) \\ &\geq I_A(g(x \otimes z \to y \otimes z)) \\ &= I_A(g(x \otimes z) \to (y \otimes z)) \\ &= I^g_A(x \otimes z \to y \otimes z). \end{aligned}$$

So by the item (i), A^g is a single–valued neutrosophic EQ–filter of $\mathcal{E}$.

(iii), (iv) Let $x \in E$. Since $g(1) = 1$, so $T_A(x) + T_A(g(x)) \leq 2$ implies that $T'_A(x) = \frac{1}{2}(T^g_A(x) + T_A(x)) \leq T'_A(1)$. In a similar way $F'_A(x) \leq F'_A(1)$ and $I'_A(x) \geq I'_A(1)$ are obtained. Suppose that $x, y \in E$. Then we have

$$\begin{aligned} &T'_A(x) \wedge T'_A(x \to y) \\ &= \frac{1}{2}(T^g_A(x) + T_A(x)) \\ &\wedge \frac{1}{2}(T^g_A(x \to y) + T_A(x \to y)) \\ &= \frac{1}{2}(T^g_A(x) \wedge T^g_A(x \to y)) \\ &+ \frac{1}{2}(T_A(x) + T_A(x \to y)) \\ &\leq \frac{1}{2}(T^g_A(y) + T_A(y)) = T'_A(y). \end{aligned}$$

We can show that $F'_A(x) \wedge F'_A(x \to y) \leq F'_A(y)$ and $I'_A(x) \vee I'_A(x \to y) \geq I'_A(y)$. Let $x, y, z \in E$. Then

$$\begin{aligned} &T'_A(x \to y) = \frac{1}{2}(T^g_A(x \to y) + T_A(x \to y)) \\ &= \frac{1}{2}(T_A(g(x \to y)) + T_A(x \to y)) \\ &\leq \frac{1}{2}(T_A(g(x \otimes z \to y \otimes z)) + T_A(x \otimes z \to y \otimes z)) \\ &= \frac{1}{2}(T^g_A((x \otimes z \to y \otimes z)) + T_A(x \otimes z \to y \otimes z)) \\ &= T'_A(x \otimes z \to y \otimes z). \end{aligned}$$

In a similar way can see that $F'_A(x \to y) \leq F'_A(x \otimes z \to y \otimes z)$ and $I'_A(x \to y) \geq I'_A(x \otimes z \to y \otimes z)$.

(v) It is obtained from previous items. □

Example 4.25. *Let* $E = \{a_1, a_2, a_3, a_4, a_5, a_6\}$. *Define operations "*$\otimes, \sim$*" and "*$\wedge$*" on* E *as follows:*

$\wedge$	a_1	a_2	a_3	a_4	a_5	a_6
a_1	a_1	a_1	a_1	a_1	a_1	a_1
a_2	a_1	a_2	a_2	a_2	a_2	a_2
a_3	a_1	a_2	a_3	a_3	a_3	a_3
a_4	a_1	a_2	a_3	a_4	a_4	a_4
a_5	a_1	a_2	a_3	a_4	a_5	a_5
a_6	a_1	a_2	a_3	a_4	a_5	a_6

,

$\otimes$	a_1	a_2	a_3	a_4	a_5	a_6
a_1	a_1	a_1	a_1	a_1	a_1	a_1
a_2	a_1	a_1	a_1	a_1	a_1	a_1
a_3	a_1	a_1	a_1	a_1	a_1	a_1
a_4	a_1	a_1	a_1	a_1	a_1	a_1
a_5	a_1	a_1	a_1	a_1	a_5	a_5
a_6	a_1	a_2	a_3	a_4	a_5	a_6

and

$\sim$	a_1	a_2	a_3	a_4	a_5	a_6
a_1	a_6	a_6	a_1	a_1	a_1	a_1
a_2	a_6	a_6	a_1	a_1	a_1	a_1
a_3	a_1	a_1	a_6	a_4	a_4	a_4
a_4	a_1	a_1	a_4	a_6	a_4	a_4
a_5	a_1	a_1	a_4	a_4	a_6	a_5
a_6	a_1	a_1	a_4	a_4	a_5	a_6

.

Now, we obtain the operation "$\rightarrow$" as follows:

$\rightarrow$	a_1	a_2	a_3	a_4	a_5	a_6
a_1	a_6	a_6	a_6	a_6	a_6	a_6
a_2	a_6	a_6	a_6	a_6	a_6	a_6
a_3	a_1	a_1	a_6	a_6	a_6	a_6
a_4	a_1	a_1	a_4	a_6	a_6	a_6
a_5	a_1	a_1	a_4	a_4	a_6	a_6
a_6	a_1	a_1	a_4	a_4	a_5	a_6

Then $\mathcal{E} = (E, \wedge, \otimes, \sim, a_6)$ is an EQ–algebra. Let $g \in End(E)$. Clearly $g(a_6) = a_6$. Since for any $1 \leq i \leq 4, 1 \leq j \leq 6, g(a_1) = g(a_i \otimes a_j) = g(a_i) \otimes g(a_j)$. So $a_1 = g(a_1) = g(a_5 \sim a_2) = g(a_5) \sim g(a_2) = g(a_5) \sim a_1 = a_1$ implies that $g(a_5) = a_1$. Hence define a single valued neutrosophic set map A in E and a map g on E as follows:

T_A	a_1	a_2	a_3	a_4	a_5	a_6
	0.01	0.02	0.03	0.04	0.05	0.06

,

F_A	a_1	a_2	a_3	a_4	a_5	a_6
	0.11	0.12	0.13	0.14	0.15	0.16

,

I_A	a_1	a_2	a_3	a_4	a_5	a_6
	0.61	0.52	0.43	0.34	0.25	0.16

and

g	a_1	a_2	a_3	a_4	a_5	a_6
	a_1	a_1	a_1	a_1	a_5	a_6

.

Hence $(A, \mathcal{E})$ is a single–valued neutrosophic EQ–prefilter. Now, we obtain a single valued neutrosophic EQ–prefilter A^g in E follows:

T_A^g	a_1	a_2	a_3	a_4	a_5	a_6
	0.01	0.01	0.01	0.01	0.05	0.06

,

F_A^g	a_1	a_2	a_3	a_4	a_5	a_6
	0.11	0.11	0.11	0.11	0.15	0.16

and

I_A^g	a_1	a_2	a_3	a_4	a_5	a_6
	0.61	0.61	0.61	0.61	0.25	0.16

.

and obtain a single valued neutrosophic EQ–prefilter A'^g in E follows:

$T_A'^g$	a_1	a_2	a_3	a_4	a_5	a_6
	0.01	0.015	0.02	0.025	0.05	0.06

,

$F_A'^g$	a_1	a_2	a_3	a_4	a_5	a_6
	0.11	0.115	0.12	0.125	0.15	0.16

and

$I_A'^g$	a_1	a_2	a_3	a_4	a_5	a_6
	0.61	0.565	0.52	0.475	0.25	0.16

.

5. Conclusion

The current paper considered the concept of single–valued neutrosophic EQ–algebras and introduce the concepts single–valued neutrosophic EQ–subalgebras, single–valued neutrosophic EQ–prefilters and single–valued neutrosophic EQ–filters.

(i) It is showed that single–valued neutrosophic EQ–subalgebras preserve some binary relation on EQ–algebras under some conditions.

(ii) Using the some properties of single–valued neutrosophic EQ–prefilters, we construct new single–valued neutrosophic EQ–prefilters.

(iii) We considered that single–valued neutrosophic EQ–filters as generalisation of single–valued neutrosophic EQ–prefilters and constructed them.

(iv) We connected the concept of EQ–prefilters to single–valued neutrosophic EQ–prefilters and the concept of EQ–filters to single–valued neutrosophic EQ–filters, so we obtained such structures from this connection.

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