Article

Single-Valued Neutrosophic Hesitant Fuzzy Choquet Aggregation Operators for Multi-Attribute Decision Making

Xin Li [1] and **Xiaohong Zhang [1,2,]***

[1]   College of Arts and Sciences, Shanghai Maritime University, Shanghai 201306, China;
     Smu201631010013@163.com
[2]   School of Arts and Sciences, Shaanxi University of Science & Technology, Xi'an 710021, China
*   Correspondence: zhangxiaohong@sust.edu.cn or zhangxh@shmtu.edu.cn

Abstract: This paper aims at developing new methods for multi-attribute decision making (MADM) under a single-valued neutrosophic hesitant fuzzy environment, in which each element has sets of possible values designed by truth, indeterminacy, and falsity membership hesitant functions. First, taking advantage of the Choquet integral and that it can reflect more correlations of attributes in MADM, two aggregation operators are defined based on the Choquet integral, specifically, the single-valued neutrosophic hesitant fuzzy Choquet ordered averaging (SVNHFCOA) operator and single-valued neutrosophic hesitant fuzzy Choquet ordered geometric (SVNHFCOG) operator, and their properties are also discussed in detail. Then, novel MADM approaches based on the SVNHFCOA and SVNHFCOG operators are established to process single-valued neutrosophic hesitant fuzzy information. Finally, this work provides a numerical example of investment alternatives to validate the application and effectiveness of the proposed approaches.

Keywords: multi-attribute decision making (MADM); aggregation operator; single-valued neutrosophic hesitant fuzzy set (SVNHFS); the Choquet integral

1. Introduction

Multi-attribute decision making (MADM) is a common approach to help people to select the most desirable alternative(s) from some feasible choices. However, in many cases, decision makers find it is hard to precisely express their preferences under imprecise and uncertain environment [1]. Then, fuzzy set (FS) theory, as proposed by Zadeh, is proven to be useful to deal with fuzzy MADM problems, especially when subjective assessments are involved [2]. On basis of Zadeh's work, rough set (RS), fuzzy multiset (FMS), hesitant fuzzy set (HFS), Interval-valued intuitionistic fuzzy set (IVIFS), and neutrosophic set (NS) have been successively proposed and successfully applied in dealing with various uncertain problems in artificial intelligence, pattern recognition, information fusion, and so on [3–7]. In the practical application of FSs and its extensions, utilizing various techniques to aggregate information in MADM is of vital importance, and the aggregation operators are one of these effective techniques to aggregate data in the information fusion process [8,9].

As a generalization of FSs, the NS was introduced by Smarandache to deal with incomplete, indeterminate, and inconsistent decision information, which include truth, falsity, and indeterminacy memberships, and their correspondingly membership functions $T_A(x)$, $I_A(x)$, $F_A(x)$ are non-standard subsets of $]^-0, 1^+[$ [7]. However, without a specific description, it is difficult to apply the NS in real scientific and other areas. Therefore, some researchers proposed the interval neutrosophic set (INS), single-valued neutrosophic set (SVNS), multi-valued neutrosophic set (MVNS), and rough neutrosophic set (RNS) in [10–14], and studied their related properties in detail, among them,

the membership functions of SVNSs are standard subsets of [0, 1]. The study on aggregation operators is a practical subject, which has been received growing attention. For SVNSs, single-valued neutrosophic averaging/geometrical (SVNA/G) operators, correlated aggregation operators, Hamacher aggregation operators, and Choquet integral operators under single-valued neutrosophic environment are proposed in recent years [15–17]. As another generalization of FSs, the HFS was defined by Torra, which permit its membership function to have a set of possible values [18,19]. Some hesitant fuzzy aggregation operators have also been put forward, such as the hesitant fuzzy weighted averaging/geometric (HFWA/G) operators, the hesitant fuzzy ordered weighted averaging/geometric (HFOWA/G) operators, the hesitant fuzzy Choquet ordered averaging/geometric (HFCOA/G) operators, the generalized hesitant fuzzy Choquet ordered averaging/geometric (GHFCOA/G) operators, and etc, which have been widely used to deal with MADM problems under hesitant fuzzy environment [20–23].

Furthermore, Ye proposed the concept of the single-valued neutrosophic hesitant fuzzy set (SVNHFS) by combining the advantages of the SVNS and HFS, which encompasses FS, HFS, DHFS, SVNHFS as special cases, and permits its membership functions to have sets of possible values, which are denoted by truth, indeterminacy, and falsity membership hesitant functions [24]. On basis of the SVNS and HFS, some properties of SVNHFSs were discussed to solve MADM problems; for instance, Ye introduced the basic operational relations and cosine measure function of SVNHFSs and developed the single-valued neutrosophic hesitant fuzzy weighted averaging/geometric (SVNHFWA/G) operators [24]. Sahin further studied the correlation and correlation coefficient of SVNHFSs [25]. Distance and similarity measures for MADM with single-valued neutrosophic hesitant fuzzy information are defined in [26]. Liu. P defined the Hamming distance measure and neutrosophic hesitant fuzzy Heronian mean aggregation operators of SVNHFSs and then extended the VIKOR method to process the single-valued neutrosophic hesitant fuzzy information [27]. BISWAS put forward some weighted distance measures of SVNHFSs [28]. Liu. C proposed the single-valued neutrosophic hesitant fuzzy ordered weighted averaging/geometric (SVNHFOWA/G) operators and applied them to deal with practical MADM problems [29].

In the existing research on MADM methods with single-valued neutrosophic hesitant fuzzy information, it is generally assumed that the attributes are independent, which are characterized by an independent index. However, for practical MADM problems, due to the dependence among attributes, more correlations are required to be considered. For example, when we use three courses as attributes to measure comprehensive performances of students, i.e., English, probability theory, and mathematical statistics, obviously, students with high performance in probability theory are highly likely to have high scores in mathematical statistics, so the three indexes cannot be regarded as independent from each other, we need to further consider the interrelationships among these attributes. The Choquet integral provides an approach to process the correlation among the attributes in MADM with respect to fuzzy measure, and various kinds of hesitant fuzzy Choquet aggregation operators, intuitionistic fuzzy Choquet aggregation operators, neutrosophic Choquet aggregation operators have been put forward to deal with MADM problems under fuzzy environment [30–34].

Therefore, it is necessary to develop some single-valued neutrosophic hesitant fuzzy operators based on Choquet integral to consider more correlations between attributes in MADM problem. Thus, the purposes of this article are to (i) introduce the Choquet integral with respect to fuzzy measures to into SVNHFSs to consider more correlations among attributes in MADM, (ii) propose a single-valued neutrosophic hesitant fuzzy Choquet ordered averaging (SVNHFCOA) operator and a single-valued neutrosophic hesitant fuzzy Choquet ordered geometric (SVNHFCOG) operator (iii) establish MADM methods based on the SVNHFCOA and SVNHFCOG operators under single-valued neutrosophic hesitant fuzzy environment.

To do so, the rest of this paper is organized as follows: Section 2 recalls some basic concepts related to the Choquet integral, SVNS, HFS, and SVNHFS; In Section 3, the SVNHFCOA and SVNHFCOG operators are put forward and basic properties of them are discussed; In Section 4,

we put forward MADM methods based on the SVNHFCOA and SVNHFCOG operators under single-valued neutrosophic hesitant fuzzy environment; Section 5 utilizes an illustrative example to validate the proposed MADM approaches. Finally, conclusions and future research directions are drawn in Section 6.

2. Preliminaries

Some basic concepts related to the SVNHFS and Choquet integral are briefly reviewed in this section.

2.1. Single-Valued Neutrosophic Hesitant Fuzzy Sets (SVNHFS)

As a generalization of FSs, the SVNHFS is a combination of the SVNS with HFS.

Definition 1. ([10]) *Let X be a non-empty fixed set, a SVNS on X is defined as:*

$$A = \{(x, T_A(x), I_A(x), F_A(x)) | x \in X\}, \tag{1}$$

where $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$, denoting the truth, indeterminacy and falsity membership degree of the element $x \in X$, respectively, and satisfying the limit: $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2. ([18]) *Let X be a non-empty fixed set, a HFS on X is represented by:*

$$E = \{\langle x, h_E(x) \rangle | x \in X\}, \tag{2}$$

where $h_E(x)$ is a set of values in $[0, 1]$, denoting the membership hesitant degrees of $x \in X$.

Definition 3. ([24]) *Let X be a non-empty fixed set, a SVNHFS on X is expressed by:*

$$N = \{(x, \widetilde{t}(x), \widetilde{i}(x), \widetilde{f}(x)) | x \in X\}, \tag{3}$$

where $\widetilde{t}(x) = \{\gamma | \gamma \in \widetilde{t}(x)\}$, $\widetilde{i}(x) = \{\delta | \delta \in \widetilde{i}(x)\}$, $\widetilde{f}(x) = \{\eta | \eta \in \widetilde{f}(x)\}$ are three sets with values in $[0, 1]$, representing truth, indeterminacy and falsity membership hesitant degrees of the element $x \in X$, which satisfy limits: $\gamma \in [0, 1]$, $\delta \in [0, 1]$, $\eta \in [0, 1]$ and $0 \leq \sup \gamma + \sup \delta + \sup \eta \leq 3$.

For convenience, we call $\widetilde{n} = (\widetilde{t}(x), \widetilde{i}(x), \widetilde{f}(x))$ a neutrosophic hesitant fuzzy element (SVNHFE), some basic operations of SVNHFEs are defined by Ye [24], as follows:

Definition 4. ([24]) *Let $\widetilde{n_1} = (\widetilde{t_1}, \widetilde{i_1}, \widetilde{f_1})$ and $\widetilde{n_2} = (\widetilde{t_2}, \widetilde{i_2}, \widetilde{f_2})$ be two SNVHFEs, then:*

1. $\widetilde{n_1} \cup \widetilde{n_2} = \{\widetilde{t_1} \cup \widetilde{t_2}, \widetilde{i_1} \cap \widetilde{i_2}, \widetilde{f_1} \cap \widetilde{f_2}\};$
2. $\widetilde{n_1} \cap \widetilde{n_2} = \{\widetilde{t_1} \cap \widetilde{t_2}, \widetilde{i_1} \cup \widetilde{i_2}, \widetilde{f_1} \cup \widetilde{f_2}\};$
3. $\widetilde{n_1} \oplus \widetilde{n_2} = \cup_{\gamma_1 \in \widetilde{t_1}, \sigma_1 \in \widetilde{i_1}, \eta_1 \in \widetilde{f_1}} \gamma_2 \in \widetilde{t_2}, \sigma_2 \in \widetilde{i_2}, \eta_2 \in \widetilde{f_2}} \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2, \sigma_1\sigma_2, \eta_1\eta_2\};$
4. $\widetilde{n_1} \otimes \widetilde{n_2} = \cup_{\gamma_1 \in \widetilde{t_1}, \sigma_1 \in \widetilde{i_1}, \eta_1 \in \widetilde{f_1}} \gamma_2 \in \widetilde{t_2}, \sigma_2 \in \widetilde{i_2}, \eta_2 \in \widetilde{f_2}} \{\gamma_1\gamma_2, \sigma_1 + \sigma_2 - \sigma_1\sigma_2, \eta_1 + \eta_2 - \eta_1\eta_2\};$
5. $k\widetilde{n_1} = \cup_{\gamma_1 \in \widetilde{t_1}, \sigma_1 \in \widetilde{i_1}, \eta_1 \in \widetilde{f_1}} \{1 - (1 - \gamma_1)^k, \sigma_1^k, \eta_1^k\};$
6. $\widetilde{n_1}^k = \cup_{\gamma_1 \in \widetilde{t_1}, \sigma_1 \in \widetilde{i_1}, \eta_1 \in \widetilde{f_1}} \{\gamma_1^k, 1 - (1 - \sigma_1)^k, 1 - (1 - \eta_1)^k\}.$

The score function and accuracy function are effective tools to rank SVNHFEs, and here we give definitions of these functions:

Definition 5. ([29]) *For $\widetilde{n}$, the score function $s(\widetilde{n})$ and accuracy function $a(\widetilde{n})$ are defined as:*

$$s(\widetilde{n}) = \left(\frac{1}{l}\sum_{i=1}^{l}\gamma_i + \frac{1}{p}\sum_{i=1}^{p}(1 - \sigma_i) + \frac{1}{q}\sum_{i=1}^{q}(1 - \eta_i)\right)/3; \tag{4}$$

$$a(\tilde{n}) = \frac{1}{l} \sum_{i=1}^{l} \gamma_i - \frac{1}{q} \sum_{i=1}^{q} (1 - \eta_i); \tag{5}$$

where l, p, q are the number of the values in $\tilde{t}, \tilde{i}, \tilde{f}$. Obviously, $s(\tilde{n}), a(\tilde{n}) \in [0, 1]$. If $s(\tilde{n}_1) > s(\tilde{n}_2)$, then $\tilde{n}_1 > \tilde{n}_2$; if $s(\tilde{n}_1) = s(\tilde{n}_2)$ and $a(\tilde{n}_1) > a(\tilde{n}_2)$, then $\tilde{n}_1 > \tilde{n}_2$.

2.2. The Fuzzy Measure and Choquet Integral

The Choquet integral is a powerful operator to aggregate kinds of fuzzy information in MADM with respect to fuzzy measure.

Definition 6. ([31]) *Let $(X, \mathcal{A}, \mu)$ be a measurable space and $\mu : \mathcal{A} \to [0, 1]$, if it satisfies the conditions:*

1. $\mu(\varnothing) = 0$;
2. $\mu(A) \le \mu(B)$ *whenever* $A \subset B, A, B \in \mathcal{A}$;
3. *If* $A_1 \subset A_2 \subset \ldots \subset A_n \subset \ldots, A_n \in \mathcal{A}$, *then* $\mu\left(\cup_{n=1}^{\infty} A_n\right) = \lim_{n \to \infty} \mu(A_n)$;
4. *If* $A_1 \supset A_2 \supset \ldots \supset A_n \supset \ldots, A_n \in \mathcal{A}$, *then* $\mu\left(\cap_{n=1}^{\infty} A_n\right) = \lim_{n \to \infty} \mu(A_n)$;

then we call μ be a fuzzy measure defined by Sugeno M.

In addition, to avoid the problems with computational complexity, g_λ fuzzy measure, a special kind of fuzzy measure, was proposed by Sugeno M [31], which satisfies the additional properties: $\mu(A \cup B) = \mu(A) + \mu(B) + \lambda \mu(A) \mu(B), \lambda \in (-1, \infty)$ for all $A, B \in \mathcal{A}$ and $A \cap B = \varnothing$. Specially, the expression of g_λ fuzzy measure on a finite set can be simplified as follows:

Theorem 1. ([31]) *When X is a finite set ($X = \{x_1, x_2, \ldots, x_m\}$), g_λ fuzzy measure can be expressed as:*

$$\mu(X) = \begin{cases} \frac{1}{\lambda}\left(\prod_{i \in X} (1 + \lambda \mu(x_i)) - 1\right), if\ \lambda \ne 0, \\ \sum_{i \in X} \mu(x_i), if\ \lambda = 0, \end{cases} \tag{6}$$

where $x_i \cap x_j = \varnothing$ for all $i, j = 1, 2, \cdots, m$ and $i \ne j$.

Definition 7. ([32]) *Let μ be a fuzzy measure, $X = \{x_1, x_2, \ldots, x_m\}$ be a finite set. The Choquet integral of a function $f : X \to [0, 1]$ with respect to fuzzy measure μ is expressed as follows:*

$$\int f d\mu = \sum_{i=1}^{m} \left(\mu(F_{\phi(i)}) - \mu(F_{\phi(i-1)})\right) \cdot f(x_{\phi(i)}), \tag{7}$$

where $(\phi(1), \phi(2), \ldots, \phi(m))$ is a permutation of $(1, 2, \ldots, n)$ such that $f(x_{\phi(1)}) \ge f(x_{\phi(2)}), \ldots, F_{\phi(i)} = \{x_{\phi(1)}, x_{\phi(2)}, \ldots, x_{\phi(i)}\}$ and $F_{\phi(0)} = \varnothing$.

3. New Single-Valued Neutrosophic Hesitant Fuzzy Choquet Aggregation Operators

Based on the operational laws of SVNHFEs and Choquet integral, new aggregation operators SVNHFCOA and SVNHFCOG are proposed in this section.

3.1. Single-Valued Neutrosophic Hesitant Fuzzy Choquet Ordered Averaging (SVNHFCOA) Operator

Definition 8. *Let $\tilde{n}_j (j = 1, 2, \ldots, m)$ be a collection of SVNHFEs, X be the set of attributes and μ be fuzzy measures on X, then the SVNHFCOA operator is defined as follows:*

$$SVNHFCOA_\mu\{\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_m\} = \oplus_{j=1}^{m}\left((\mu(F_{\phi(j)}) - \mu(F_{\phi(j-1)}))\tilde{n}_{\phi(j)}\right) \tag{8}$$

where $\mu_{\phi(j)} = \mu(F_{\phi(j)}) - \mu(F_{\phi(j-1)})$ and $(\phi(1), \phi(2), \ldots, \phi(m))$ is a permutation of $(1, 2, \ldots, m)$ such that $\widetilde{n}_{\phi(1)} \geq \widetilde{n}_{\phi(2)}, \ldots, \widetilde{n}_{\phi(m)}$, $F_{\phi(i)} = \{x_{\phi(1)}, x_{\phi(2)}, \ldots, x_{\phi(i)}\}$ and $F_{\phi(0)} = \varnothing$.

Theorem 2. *Let $\widetilde{n}_j (j = 1, 2, \ldots, m)$ be a collection of SVNHFEs, then the aggregated value obtained by the SVNHFCOA operator is also a SVNHFE, and*

$$SVNHFCOA_\mu\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_m\} = \oplus_{j=1}^m ((\mu(F_{\phi(j)}) - \mu(F_{\phi(j-1)}))\widetilde{n}_{\phi(j)})$$

$$= \bigcup_{\gamma_{\phi(j)} \in \widetilde{t_{\phi(j)}},\, \sigma_{\phi(j)} \in \widetilde{i_{\phi(j)}}, \eta_{\phi(j)} \in \widetilde{f_{\phi(j)}}} \{\{1 - \prod_{j=1}^m (1 - \gamma_{\phi(j)})^{\mu_{\phi(j)}}\}, \{\prod_{j=1}^m \sigma_{\phi(j)}{}^{\mu_{\phi(j)}}\}, \{\prod_{j=1}^m \eta_{\phi(j)}{}^{\mu_{\phi(j)}}\}\} \tag{9}$$

Proof. By means of mathematical induction, the proof of Theorem 2 can be done, i.e., the aggregated value with SVNHFCOA operator is also a SVNHFE.

(a) For $m = 1$, since

$$SVNHFCOA_\mu\{\widetilde{n}_1\} = (\mu(F_{\phi(1)}) - \mu(F_{\phi(0)}))\widetilde{n}_{\phi(1)} = \widetilde{n}_{\phi(1)},$$

Obviously, Equation (9) holds for $m = 1$.

(b) When $m = 2$, since

$$\mu_{\phi(1)}\widetilde{n}_{\phi(1)} = \bigcup_{\gamma_{\phi(1)} \in \widetilde{t_{\phi(1)}},\, \sigma_{\phi(1)} \in \widetilde{i_{\phi(1)}}, \eta_{\phi(1)} \in \widetilde{f_{\phi(1)}}} \{1 - (1 - \gamma_{\phi(1)})^{\mu_{\phi(1)}}, \sigma_{\phi(1)}{}^{\mu_{\phi(1)}}, \eta_{\phi(1)}{}^{\mu_{\phi(1)}}\},$$

$$\mu_{\phi(2)}\widetilde{n}_{\phi(2)} = \bigcup_{\gamma_{\phi(2)} \in \widetilde{t_{\phi(2)}},\, \sigma_{\phi(2)} \in \widetilde{i_{\phi(2)}}, \eta_{\phi(2)} \in \widetilde{f_{\phi(2)}}} \{1 - (1 - \gamma_{\phi(2)})^{\mu_{\phi(2)}}, \sigma_{\phi(2)}{}^{\mu_{\phi(2)}}, \eta_{\phi(2)}{}^{\mu_{\phi(2)}}\}$$

then, we have

$$SVNHFCOA_\mu\{\widetilde{n}_1, \widetilde{n}_2\} = \mu_{\phi(1)}\widetilde{n}_{\phi(1)} \oplus \mu_{\phi(2)}\widetilde{n}_{\phi(2)}$$

$$= \bigcup_{\gamma_{\phi(j)} \in \widetilde{t_{\phi(j)}},\, \sigma_{\phi(j)} \in \widetilde{i_{\phi(j)}}, \eta_{\phi(j)} \in \widetilde{f_{\phi(j)}}} \{\{1 - (1 - \gamma_{\phi(1)})^{\mu_{\phi(1)}}(1 - \gamma_{\phi(2)})^{\mu_{\phi(2)}}\}, \{\sigma_{\phi(1)}{}^{\mu_{\phi(1)}}\sigma_{\phi(2)}{}^{\mu_{\phi(2)}}\}, \{\eta_{\phi(1)}{}^{\mu_{\phi(1)}}\eta_{\phi(2)}{}^{\mu_{\phi(2)}}\}\}.$$

Thus, Equation (9) holds for $m = 2$.

(c) If Equation (9) holds for $m = k$, then

$$SVNHFCOA_\mu\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_k\} = \oplus_{j=1}^k \mu_{\phi(j)}\widetilde{n}_{\phi(j)}$$

$$= \bigcup_{\gamma_{\phi(j)} \in \widetilde{t_{\phi(j)}},\, \sigma_{\phi(j)} \in \widetilde{i_{\phi(j)}}, \eta_{\phi(j)} \in \widetilde{f_{\phi(j)}}} \{\{1 - \prod_{j=1}^k (1 - \gamma_{\phi(j)})^{\mu_{\phi(j)}}\}, \{\prod_{j=1}^k \sigma_{\phi(j)}{}^{\mu_{\phi(j)}}\}, \{\prod_{j=1}^k \eta_{\phi(j)}{}^{\mu_{\phi(j)}}\}\}$$

When $m = k + 1$, for $\forall \gamma_{\phi(j)} \in \widetilde{t_{\phi(j)}}$, $\sigma_{\phi(j)} \in \widetilde{i_{\phi(j)}}, \eta_{\phi(j)} \in \widetilde{f_{\phi(j)}}$, then,

$$SVNHFCOA_\mu\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_{k+1}\} = \oplus_{j=1}^{k+1} \mu_{\phi(j)}\widetilde{n}_{\phi(j)}$$

$$= \bigcup_{\gamma_{\phi(j)} \in \widetilde{t_{\phi(j)}},\, \sigma_{\phi(j)} \in \widetilde{i_{\phi(j)}}, \eta_{\phi(j)} \in \widetilde{f_{\phi(j)}}} \{\{1 - \prod_{j=1}^k (1 - \gamma_{\phi(j)})^{\mu_{\phi(j)}}\}, \{\prod_{j=1}^k \sigma_{\phi(j)}{}^{\mu_{\phi(j)}}\}, \{\prod_{j=1}^k \eta_{\phi(j)}{}^{\mu_{\phi(j)}}\}\} \oplus$$

$$\bigcup_{\gamma_{\phi(k+1)} \in \widetilde{t_{\phi(k+1)}},\, \sigma_{\phi(k+1)} \in \widetilde{i_{\phi(k+1)}}, \eta_{\phi(k+1)} \in \widetilde{f_{\phi(k+1)}}} \{1 - (1 - \gamma_{\phi(k+1)})^{\mu_{\phi(k+1)}}, \sigma_{\phi(k+1)}{}^{\mu_{\phi(k+1)}}, \eta_{\phi(k+1)}{}^{\mu_{\phi(k+1)}}\}$$

which can be obtained by $(\oplus)$ calculation,

$$SVNHFCOA_\mu\{\widetilde{n}_1,\widetilde{n}_2,\ldots,\widetilde{n}_{k+1}\} = \oplus_{j=1}^{k+1}\mu_{\phi(j)}\widetilde{n}_{\phi(j)}$$

$$= \bigcup_{\gamma_{\phi(j)}\in\widetilde{t}_{\phi(j)},\ \sigma_{\phi(j)}\in\widetilde{i}_{\phi(j)},\eta_{\phi(j)}\in\widetilde{f}_{\phi(j)}} \left\{\left\{1-\prod_{j=1}^{k+1}(1-\gamma_{\phi(j)})^{\mu_{\phi(j)}}\right\},\left\{\prod_{j=1}^{k+1}\sigma_{\phi(j)}{}^{\mu_{\phi(j)}}\right\},\left\{\prod_{j=1}^{k+1}\eta_{\phi(j)}{}^{\mu_{\phi(j)}}\right\}\right\}.$$

i.e., Equation (9) holds for m = k + 1, thus we confirm Equation (9) holds for all m. □

Some special cases of the SVNHFCOA operator are given as follows:

1. If $\mu(F) \equiv 1$, then $SVNHFCOA_\mu\{\widetilde{n}_1,\widetilde{n}_2,\ldots,\widetilde{n}_m\} = max\{\widetilde{n}_1,\widetilde{n}_2,\ldots,\widetilde{n}_m\}$;
2. If $\mu(F) \equiv 0$, then $SVNHFCOA_\mu\{\widetilde{n}_1,\widetilde{n}_2,\ldots,\widetilde{n}_m\} = min\{\widetilde{n}_1,\widetilde{n}_2,\ldots,\widetilde{n}_m\}$;
3. The SVNHFCOA operator reduces to the single-valued neutrosophic hesitant fuzzy weighted averaging (SVNHFWA) operator, if the independent condition $\mu(x_{\phi(j)}) = \mu(F_{\phi(j)}) - \mu(F_{\phi(j-1)})$ holds.

$$SVNHFWA\{\widetilde{n}_1,\widetilde{n}_2,\ldots,\widetilde{n}_m\} = \oplus_{j=1}^{m}(\mu(x_j)\cdot\widetilde{n}_j)$$

$$= \bigcup_{\gamma_j\in\widetilde{t}_j,\sigma_j\in\widetilde{iz}_j,\eta_j\in\widetilde{f}_j}\left\{\left\{1-\prod_{j=1}^{m}(1-\gamma_j)^{\mu(x_j)}\right\},\left\{\prod_{j=1}^{m}\sigma_j{}^{\mu(x_j)}\right\},\left\{\prod_{j=1}^{m}\eta_j{}^{\mu(x_j)}\right\}\right\}.$$

4. If $\mu(x_j) = 1/m$, for $j = 1,2,\ldots,m$, then both the SVNHFCOA and SVNHFWA operators reduce to the single-valued neutrosophic hesitant fuzzy averaging (SVNHFA) operator, which is shown as follows:

$$SVNHFWA\{\widetilde{n}_1,\widetilde{n}_2,\ldots,\widetilde{n}_m\} = \bigcup_{\gamma_j\in\widetilde{t}_j,\ \sigma_j\in\widetilde{i}_j,\eta_j\in\widetilde{f}_j}\left\{\left\{1-\prod_{j=1}^{m}(1-\gamma_j)^{\frac{1}{m}}\right\},\left\{\prod_{j=1}^{m}(\sigma_j)^{\frac{1}{m}}\right\},\left\{\prod_{j=1}^{m}(\eta_j)^{\frac{1}{m}}\right\}\right\}.$$

5. If $\mu(F) = \sum_{j=1}^{|F|}\omega_j$ for all $F\subseteq X$, where $|F|$ is the number of elements in F, then $\omega_j = \mu(F_{\phi(j)}) - \mu(F_{\phi(j-1)})$, $j = 1,2,\ldots,m$, where $\omega = (\omega_1,\omega_2,\ldots,\omega_m)^T$ such that $\omega_j \geq 0$ and $\sum_{j=1}^{m}\omega_j = 1$. In this case, the SVNHFCOA operator reduces to the single-valued neutrosophic hesitant fuzzy ordered weighted averaging (SVNHFOWA) operator as:

$$SVNHFOWA\{\widetilde{n}_1,\widetilde{n}_2,\ldots,\widetilde{n}_m\} = \bigcup_{\gamma_j\in\widetilde{t}_j,\ \sigma_j\in\widetilde{iz}_j,\eta_j\in\widetilde{f}_j}\left\{\left\{1-\prod_{j=1}^{m}(1-\gamma_{\omega_j})^{\omega_j}\right\},\left\{\prod_{j=1}^{m}\sigma_j{}^{\omega_j}\right\},\left\{\prod_{j=1}^{m}\eta_j{}^{\omega_j}\right\}\right\};$$

Particularly, if $\mu(F) = |F|/m$, for all $F\subseteq X$, then both the SVNHFCOA and SVNHFOWA operators reduce to the SVNHFA operator.

Theorem 3. *The SVNHFCOA operator has the following desirable properties:*

1. *(Idempotency) Let $\widetilde{n}_j = \widetilde{n}$ for all $j = 1,2,\ldots,m$, and $\widetilde{n} = \{\{\gamma\},\{\sigma\},\{\eta\}\}$, then:*

$$SVNHFCOA_\mu\{\widetilde{n}_1,\widetilde{n}_2,\ldots,\widetilde{n}_m\} = \{\{\gamma\},\{\sigma\},\{\eta\}\}.$$

2. *(Boundedness) Let $\widetilde{n}^- = \{min\{\gamma_j\},\ max\{\sigma_j\},\ max\{\eta_j\}\}$, $\widetilde{n}^+ = \{max\{\gamma_j\},min\{\sigma_j\},min\{\eta_j\}\}$, so:*

$$\widetilde{n}^- \leq SVNHFCOA_\mu\{\widetilde{n}_1,\widetilde{n}_2,\ldots,\widetilde{n}_m\} \leq \widetilde{n}^+.$$

3. *(Commutativity) If $\{\widetilde{n}'_1,\widetilde{n}'_2,\ldots,\widetilde{n}'_m\}$ is a permutation of $\{\widetilde{n}_1,\widetilde{n}_2,\ldots,\widetilde{n}_m\}$, then,*

$$SVNHFCOA_\mu\{\widetilde{n}_1,\widetilde{n}_2,\ldots,\widetilde{n}_m\} = SVNHFCOA_\mu\{\widetilde{n}'_1,\widetilde{n}'_2,\ldots,\widetilde{n}'_m\}.$$

4. *(Monotonity)* If $\tilde{n}_j \le \tilde{n}'_j$ for $\forall j \in \{1, 2, \dots, n\}$, then,

$$SVNHFCOA_\mu\{\tilde{n}_1, \tilde{n}_2, \dots, \tilde{n}_m\} \le SVNHFCOA_\mu\{\tilde{n}'_1, \tilde{n}'_2, \dots, \tilde{n}'_m\}.$$

Proof. Suppose $(1, 2, \dots, m)$ is a permutation such that $\tilde{n}_1 \ge \tilde{n}_2, \dots, \ge \tilde{n}_m$.

1. For $\tilde{n} = \{\{\gamma\}, \{\sigma\}, \{\eta\}\}$, according to Theorem 1, it follows that

$$SVNHFCOA_\mu\{\tilde{n}_1, \tilde{n}_2, \dots, \tilde{n}_m\} =$$
$$\{\{1 - (1 - \gamma)^{\sum_{j=1}^m (\mu(F_j) - \mu(F_{j-1}))}\}, \{\sigma^{\sum_{j=1}^m (\mu(F_j) - \mu(F_{j-1}))}\}, \{\eta^{\sum_{j=1}^m (\mu(F_j) - \mu(F_{j-1}))}\}\} = \{\{\gamma\}, \{\sigma\}, \{\eta\}\}.$$

2. Since $y = x^a (0 < a < 1)$ is a monotone increasing function when $x > 0$, therefore, it holds

$$1 - \prod_{j=1}^m (1 - \min\{\gamma_j\})^{(\mu(F_j) - \mu(F_{j-1}))} \le 1 - \prod_{j=1}^m (1 - \gamma_j)^{(\mu(F_j) - \mu(F_{j-1}))}$$
$$\le 1 - \prod_{j=1}^m (1 - \max\{\gamma_j\})^{(\mu(F_j) - \mu(F_{j-1}))},$$

which is equivalent to

$$1 - (1 - \min\{\gamma_j\})^{\sum_{j=1}^m (\mu(F_j) - \mu(F_{j-1}))} \le 1 - \prod_{j=1}^m (1 - \gamma_j)^{(\mu(F_j) - \mu(F_{j-1}))}$$
$$\le 1 - (1 - \max\{\gamma_j\})^{\sum_{j=1}^m (\mu(F_j) - \mu(F_{j-1}))}$$

i.e.,

$$\min\{\gamma_j\} \le 1 - \prod_{j=1}^m (1 - \gamma_j)^{(\mu(F_j) - \mu(F_{j-1}))} = \gamma \le \max\{\gamma_j\}$$

Analogously, we have

$$\min\{\sigma_j\} \le \prod_{j=1}^m \sigma_j^{(\mu(F_j) - \mu(F_{j-1}))} \le \max\{\sigma_j\} \text{ and } \min\{\eta_j\} \le \prod_{j=1}^m \eta_j^{(\mu(F_j) - \mu(F_{j-1}))} \le \max\{\eta_j\}.$$

Since $s(\tilde{n}^-) \le S(\tilde{n}) \le s(\tilde{n}^+)$, namely, $\tilde{n}^- \le SVNHFCOA_\mu\{\tilde{n}_1, \tilde{n}_2, \dots, \tilde{n}_m\} \le \tilde{n}^+$.

3. Suppose $(\phi(1), \phi(2), \dots, \phi(m))$ is a permutation of both $\{\tilde{n}'_1, \tilde{n}'_2, \dots, \tilde{n}'_m\}$ and $\{\tilde{n}_1, \tilde{n}_2, \dots, \tilde{n}_m\}$, such that $\tilde{n}_{\phi(1)} \ge \tilde{n}_{\phi(2)}, \dots, \ge \tilde{n}_{\phi(m)}, F_{\phi(i)} = \{x_{\phi(1)}, x_{\phi(2)}, \dots, x_{\phi(i)}\}$, then,

$$SVNHFCOA_\mu\{\tilde{n}_1, \tilde{n}_2, \dots, \tilde{n}_m\} = SVNHFCOA_\mu\{\tilde{n}'_1, \tilde{n}'_2, \dots, \tilde{n}'_m\} = \oplus_{j=1}^m ((\mu(F_{\phi(j)}) - \mu(F_{\phi(j-1)}))\tilde{n}_{\phi(j)}).$$

4. Considering $\tilde{n}_j \le \tilde{n}'_j$ for $\forall j \in \{1, 2, \dots, n\}$, we have

$$\gamma = 1 - \prod_{j=1}^m (1 - \gamma_j)^{(\mu(F_j) - \mu(F_{j-1}))} \le 1 - \prod_{j=1}^m (1 - \gamma'_j)^{(\mu(F_j) - \mu(F_{j-1}))} = \gamma',$$

$$\sigma = \prod_{j=1}^m \sigma_j^{\mu(F_j) - \mu(F_{j-1})} \ge \prod_{j=1}^m (\sigma'_j)^{\mu(F_j) - \mu(F_{j-1})} = \sigma',$$

$$\eta = \prod_{j=1}^m \eta_j^{\mu(F_j) - \mu(F_{j-1})} \ge \prod_{j=1}^m (\eta'_j)^{\mu(F_j) - \mu(F_{j-1})} = \eta'.$$

Since $s(\tilde{n}_j) \le s(\tilde{n}'_j)$, namely, $SVNHFCOA_\mu\{\tilde{n}_1, \tilde{n}_2, \dots, \tilde{n}_m\} \le SVNHFCOA_\mu\{\tilde{n}'_1, \tilde{n}'_2, \dots, \tilde{n}'_m\}$. $\square$

3.2. Single-Valued Neutrosophic Hesitant Fuzzy Choquet Ordered Geometric (SVNHFCOG) Operator

Similarly, we can develop the SVNHFCOG operator for SVNHFSs:

Definition 9. *Let $\widetilde{n}_j(j = 1, 2, \ldots, m)$ be a collection of SVNHFEs, X be the set of attributes and μ be fuzzy measures on X, then the SVNHFCOG operator is defined as follows:*

$$SVNHFCOG_\mu\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_m\} = \otimes_{j=1}^m (\widetilde{n}_{\phi(j)}{}^{(\mu(F_{\phi(j)}) - \mu(F_{\phi(j-1)}))}), \tag{10}$$

where $\mu_{\phi(j)} = \mu(F_{\phi(j)}) - \mu(F_{\phi(j-1)})$ and $(\phi(1), \phi(2), \ldots, \phi(m))$ is a permutation of $(1, 2, \ldots, m)$ such that $\widetilde{n}_{\phi(1)} \geq \widetilde{n}_{\phi(2)}, \ldots, \widetilde{n}_{\phi(m)}$, $F_{\phi(i)} = \{x_{\phi(1)}, x_{\phi(2)}, \ldots, x_{\phi(i)}\}$ and $F_{\phi(0)} = \varnothing$.

Theorem 4. *Let $\widetilde{n}_j(j = 1, 2, \ldots, m)$ be a collection of SVNHFEs, then the aggregated value obtained by the SVNHFCOA operator is also a SVNHFE, and*

$$
\begin{aligned}
SVNHFCOG_\mu\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_m\} &= \otimes_{j=1}^m (\widetilde{n}_{\phi(j)}{}^{(\mu(F_{\phi(j)}) - \mu(F_{\phi(j-1)}))}) \\
&= \bigcup_{\substack{\gamma_{\phi(j)} \in \widetilde{t}_{\phi(j)}, \sigma_{\phi(j)} \in \widetilde{i}_{\phi(j)}, \\ \eta_{\phi(j)} \in \widetilde{f}_{\phi(j)}}} \left\{\left\{\prod_{j=1}^m (\gamma_{\phi(j)})^{\mu_{\phi(j)}}\right\}, \left\{1 - \prod_{j=1}^m (1 - \sigma_{\phi(j)})^{\mu_{\phi(j)}}\right\}, \left\{1 - \prod_{j=1}^m (1 - \eta_{\phi(j)})^{\mu_{\phi(j)}}\right\}\right\}
\end{aligned} \tag{11}
$$

Some special cases of the SVNHFCOA operator are given as follows:

1.　If $\mu(F) \equiv 1$, then $SVNHFCOG_\mu\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_m\} = max\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_m\}$;

2.　If $\mu(F) \equiv 0$, then $SVNHFCOG_\mu\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_m\} = min\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_m\}$;

3.　The SVNHFCOG operator reduces to the single-valued neutrosophic hesitant fuzzy weighted geometric (SVNHFWG) operator, if the independent condition $\mu(x_{\phi(j)}) = \mu(F_{\phi(j)}) - \mu(F_{\phi(j-1)})$ holds.

$$
\begin{aligned}
SVNHFWG\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_m\} &= \otimes_{j=1}^m (\mu(x_j) \otimes \widetilde{n}_j) \\
&= \bigcup_{\gamma_j \in \widetilde{t}_j, \sigma_j \in \widetilde{iz}_j, \eta_j \in \widetilde{f}_j} \left\{\left\{\prod_{j=1}^m \gamma_j{}^{\mu(x_j)}\right\}, \left\{1 - \prod_{j=1}^m (1 - \sigma_j)^{\mu(x_j)}\right\}, \left\{1 - \prod_{j=1}^m (1 - \eta_j)^{\mu(x_j)}\right\}\right\}.
\end{aligned}
$$

4.　If $\mu(x_j) = 1/m$, for $j = 1, 2, \ldots, m$, then both the SVNHFCOG and SVNHFWG operators reduce to the single-valued neutrosophic hesitant fuzzy geometric (SVNHFG) operator, which is shown as follows:

$$
\begin{aligned}
&SVNHFWG\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_m\} \\
&= \bigcup_{\gamma_j \in \widetilde{t}_j, \sigma_j \in \widetilde{i}_j, \eta_j \in \widetilde{f}_j} \left\{\left\{\prod_{j=1}^m (\gamma_j)^{\frac{1}{m}}\right\}, \left\{1 - \prod_{j=1}^m (1 - \sigma_j)^{\frac{1}{m}}\right\}, \left\{1 - \prod_{j=1}^m (1 - \eta_j)^{\frac{1}{m}}\right\}\right\}.
\end{aligned}
$$

5.　If $\mu(F) = \sum_{j=1}^{|F|} \omega_j$ for all $F \subseteq X$, where $|F|$ is the number of elements in F, then $\omega_j = \mu(F_{\phi(j)}) - \mu(F_{\phi(j-1)})$, $j = 1, 2, \ldots, m$, where $\omega = (\omega_1, \omega_2, \ldots, \omega_m)^T$ such that $\omega_j \geq 0$ and $\sum_{j=1}^m \omega_j = 1$. Then, the SVNHFCOG operator reduces to the single-valued neutrosophic hesitant fuzzy ordered weighted geometric (SVNHFOWG) operator as follows:

$$
\begin{aligned}
&SVNHFOWG\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_m\} \\
&= \bigcup_{\gamma_j \in \widetilde{t}_j, \sigma_j \in \widetilde{iz}_j, \eta_j \in \widetilde{f}_j} \left\{\left\{\prod_{j=1}^m (\gamma_j)^{\omega_j}\right\}, \left\{1 - \prod_{j=1}^m (1 - \sigma_j)^{\omega_j}\right\}, \left\{1 - \prod_{j=1}^m (1 - \eta_j)^{\omega_j}\right\}\right\}.
\end{aligned}
$$

Particularly, if $\mu(F) = |F|/m$, for all $F \subseteq X$, then the SVNHFCOG and SVNHFOWG operators reduce to the SVNHFG operator.

Theorem 5. *The SVNHFCOG operator has the following desirable properties:*

1. *(Idempotency) Let $\widetilde{n}_j = \widetilde{n}$ for all $j = 1, 2, \ldots, m$, and $\widetilde{n} = \{\{\gamma\}, \{\sigma\}, \{\eta\}\}$, then:*

$$SVNHFCOG_\mu\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_m\} = \{\{\gamma\}, \{\sigma\}, \{\eta\}\}.$$

2. *(Boundedness) Let $\widetilde{n}^- = \{min\{\gamma_j\}, \ max\{\sigma_j\}, \ max\{\eta_j\}\}, \widetilde{n}^+ = \{ \ max\{ \ \gamma_j\}, min\{\sigma_j\}, min\{\eta_j\}\}$, so:*

$$\widetilde{n}^- \leq \ SVNHFCOG_\mu\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_m\} \ \leq \ \widetilde{n}^+$$

3. *(Commutativity) If $\{\widetilde{n}'_1, \widetilde{n}'_2, \ldots, \widetilde{n}'_m\}$ is a permutation of $\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_m\}$, then,*

$$SVNHFCOG_\mu\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_m\} = SVNHFCOG_\mu\{\widetilde{n}'_1, \widetilde{n}'_2, \ldots, \widetilde{n}'_m\}$$

4. *(Monotonity) If $\widetilde{n}_j \leq \widetilde{n}'_j$ for$\forall j \in \{1, \ 2, \ldots, n\}$, then,*

$$SVNHFCOG_\mu\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_m\} \leq SVNHFCOG_\mu\{\widetilde{n}'_1, \widetilde{n}'_2, \ldots, \widetilde{n}'_m\}.$$

Theorem 6. *Let $\widetilde{n}_j (j = 1, 2, \ldots, m)$ be a collection of SVNHFEs, X be the set of attributes and m be a fuzzy measure on X, then we have*

$$SVNHFCOG_\mu\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_m\} \leq SVNHFCOA_\mu\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_m\}$$

Proof. Based on Lemma 1, $\forall \ \widetilde{n}_j = \{\{\gamma_j\}, \{\sigma_j\}, \{\eta_j\}\}, \ (j = 1, 2, \ldots, m)$, it follows that

$$\prod_{j=1}^{m} \gamma_j{}^{\mu_{\phi(j)}} \leq \sum_{j=1}^{m} \mu_{\phi(j)} \gamma_j = 1 - \sum_{j=1}^{m} (\mu_{\phi(j)})(1 - \gamma_j) \leq 1 - \prod_{j=1}^{m} (1 - \gamma_j)^{(\mu_{\phi(j)})}$$

Analogously, we have

$$\prod_{j=1}^{m} \sigma_j{}^{\mu_{\phi(j)}} \leq 1 - \prod_{j=1}^{m} (1 - \sigma_j)^{\mu_{\phi(j)}}; \prod_{j=1}^{m} \eta_j{}^{\mu_{\phi(j)}} \leq 1 - \prod_{j=1}^{m} (1 - \eta_j)^{\mu_{\phi(j)}}.$$

Next, by calculation of their score functions, we can know that

$$SVNHFCOG_\mu\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_m\} \leq SVNHFCOA_\mu\{\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_m\}$$

□

4. Approaches for MADM with Single-Valued Neutrosophic Hesitant Fuzzy Information

For a MADM problem with single-valued neutrosophic hesitant fuzzy information, assume that there are n alternatives $A = \{a_1, a_2, \ldots, a_n\}$ and m interrelated attributes $X = \{x_1, x_2, \ldots, x_m\}$. Thus, let $N = (\widetilde{n}_{ij} = \{\widetilde{t}_{ij}, \widetilde{i}_{ij}, \widetilde{f}_{ij}\})_{n \times m}$ denotes values assigned to n alternatives with respect to m attributes, in detail, $\widetilde{t}_{ij}, \widetilde{i}_{ij}, \widetilde{f}_{ij}$ indicate the truth, indeterminacy and falsity membership hesitant functions of a_i satisfying x_j given by decision-makers, respectively. Then, to determine the most desirable alternative(s), the SVNHFCOA and SVNHFCOG operators are utilized to establish MADM methods with single-valued neutrosophic hesitant fuzzy information, which involves the following steps:

Step 1. Reorder the Decision Matrix

Reorder m SVNHFEs $\widetilde{n}_{ij}$ from smallest to largest of a_i $(i = 1, 2, \ldots, n)$ in $N = (\widetilde{n}_{ij})_{n \times m}$: (1) if their score values $s(\widetilde{n}_{ij})$ are different, we can rank them by Equation (12); (2) if some of their score values are same, we continue to calculate accuracy values $a(\widetilde{n}_{ij})$ by Equation (13). Then, the reorder sequence is expressed as $\widetilde{n}_{i\phi(1)}, \widetilde{n}_{i\phi(2)}, \ldots, \widetilde{n}_{i\phi(m)}$, where $(i\phi(1), i\phi(2), \ldots, i\phi(m))$ is a permutation of $(i1, i2, \ldots, im)$, such that $\widetilde{n}_{i\phi(1)} \geq \widetilde{n}_{i\phi(2)}, \ldots, \geq \widetilde{n}_{i\phi(m)}$.

$$s(\widetilde{n}_{ij}) = \left[\frac{1}{\#\gamma} \sum_{l=1}^{\#\gamma} \gamma_{ij} + \frac{1}{\#\sigma} \sum_{p=1}^{\#\sigma} (1 - \sigma_{ij}) + \frac{1}{\#\eta} \sum_{q=1}^{\#\eta} (1 - \eta_{ij}) \right] / 3, \tag{12}$$

$$a(\widetilde{n}_{ij}) = \frac{1}{\#\gamma} \sum_{l=1}^{\#\gamma} \gamma_{ij} - \frac{1}{\#\eta} \sum_{q=1}^{\#\eta} (1 - \eta_{ij}). \tag{13}$$

Step 2. Confirm Fuzzy Measures of m Attributes

Since the existing MADM methods using single-valued neutrosophic hesitant fuzzy information ignore the interrelationships of attributes, thus, by combining the Choquet integral with respect to fuzzy measures, more correlations can be considered in the MADM process. So, we use g_λ fuzzy measure to determine fuzzy measures μ of X.

$$\mu_{i\phi(j)} = \mu(F_{i\phi(j)}) - \mu(F_{i\phi(j-1)}), (i = 1, 2, \ldots n, j = 1, 2, \ldots, m) \tag{14}$$

Step 3. Aggregate All Decision Information by SVNHFCOA or SVNHFCOG Operator

Aggregate m SVNHFEs $\widetilde{n}_{i\phi(j)}$ of a_i based on the SVNHFCOA or SVNHFCOG operator in Equation (15) or (16).

$$\widetilde{n}_i = \text{SVNHFCOA}_\mu \{ \widetilde{n}_{i1}, \widetilde{n}_{i2}, \ldots, \widetilde{n}_{im} \} = \oplus_{j=1}^{m} ((\mu(F_{i\phi(j)}) - \mu(F_{i\phi(j-1)}))\widetilde{n}_{i\phi(j)}), \tag{15}$$

$$\widetilde{n}_i = \text{SVNHFCOG}_\mu \{ \widetilde{n}_{i1}, \widetilde{n}_{i2}, \ldots, \widetilde{n}_{im} \} = \otimes_{j=1}^{m} (\widetilde{n}_{i\phi(j)}^{(\mu(F_{i\phi(j)}) - \mu(F_{i\phi(j-1)}))}). \tag{16}$$

Step 4. Rank the Alternatives

Calculate and rank n alternatives to select the most desirable one: (1) if their score values $s(\widetilde{n}_j)$ are different, we can rank n alternatives by Equation (17); (2) if some of their score values $s(\widetilde{n}_j)$ are same, we continue to calculate accuracy values $a(\widetilde{n}_j)$ by Equation (18) to get the best choice.

$$s(\widetilde{n}_j) = \left[\frac{1}{\#\gamma} \sum_{l=1}^{\#\gamma} \gamma_j + \frac{1}{\#\sigma} \sum_{p=1}^{\#\sigma} (1 - \sigma_j) + \frac{1}{\#\eta} \sum_{q=1}^{\#\eta} (1 - \eta_j) \right] / 3, \tag{17}$$

$$a(\widetilde{n}_j) = \frac{1}{\#\gamma} \sum_{l=1}^{\#\gamma} \gamma_j - \frac{1}{\#\eta} \sum_{q=1}^{\#\eta} (1 - \eta_j). \tag{18}$$

5. Numerical Example and Analysis

5.1. Numerical Example

An illustrative example about investment alternatives for a MADM problem adapted from Ye [24] is utilized to illustrate the applications of the proposed approaches in this paper, and to demonstrate their feasibility and effectiveness.

Suppose that there is a problem to deal with potential evaluation of emerging technology commercialization, which is a typical MADM problem. There is a panel with four possible emerging

technology enterprises denoted by a_1, a_2, a_3, a_4. During the decision making process, three attributes need to be considered: (1) x_1 is the potential market and market risk; (2) x_2 is the industrialization infrastructure, human resources, and financial conditions; (3) x_3 is the employment creation and development of science and technology.

To avoid influence others, decision makers are required to evaluate the four possible emerging technology enterprises a_i ($i = 1, 2, 3, 4$) under the above three attributes in anonymity, and single-valued neutrosophic hesitant fuzzy decision matrix $N = (\tilde{n}_{ij})_{4 \times 3}$ is constructed, shown in Table 1.

Table 1. Single-valued Neutrosophic Hesitant Fuzzy Decision Matrix N.

	x_1	x_2	x_3
a_1	$\{\{0.3, 0.4, 0.5\}, \{0.1\}, \{0.3, 0.4\}\}$	$\{\{0.5, 0.6\}, \{0.2, 0.3\}, \{0.3, 0.4\}\}$	$\{\{0.2, 0.3\}, \{0.1, 0.2\}, \{0.5, 0.6\}\}$
a_2	$\{\{0.6, 0.7\}, \{0.1, 0.2\}, \{0.2, 0.3\}\}$	$\{\{0.6, 0.7\}, \{0.1\}, \{0.3\}\}$	$\{\{0.6, 0.7\}, \{0.1, 0.2\}, \{0.1, 0.2\}\}$
a_3	$\{\{0.5, 0.6\}, \{0.4\}, \{0.2, 0.3\}\}$	$\{\{0.6\}, \{0.3\}, \{0.4\}\}$	$\{\{0.5, 0.6\}, \{0.1\}, \{0.3\}\}$
a_4	$\{\{0.7, 0.8\}, \{0.1\}, \{0.1, 0.2\}\}$	$\{\{0.6, 0.7\}, \{0.1\}, \{0.2\}\}$	$\{\{0.3, 0.5\}, \{0.2\}, \{0.1, 0.2, 0.3\}\}$

Step 1. Get the score matrix of $\tilde{n}_{ij}$ calculated by Equation (11), shown in Table 2, and the reordered decision matrix shown in Table 3.

Table 2. Score values of $\tilde{n}_{ij}$.

	x_1	x_2	x_3
a_1	0.65	0.65	0.52
a_2	0.75	0.75	0.78
a_3	0.63	0.57	0.72
a_4	0.83	0.78	0.67

Table 3. Reordered decision matrix N'.

	$x_{\sigma(1)}$	$x_{\sigma(2)}$	$x_{\sigma(3)}$
a_1	$\{\{0.2, 0.3\}, \{0.1, 0.2\}, \{0.5, 0.6\}\}$	$\{\{0.3, 0.4, 0.5\}, \{0.1\}, \{0.3, 0.4\}\}$	$\{\{0.5, 0.6\}, \{0.2, 0.3\}, \{0.3, 0.4\}\}$
a_2	$\{\{0.6, 0.7\}, \{0.1, 0.2\}, \{0.2, 0.3\}\}$	$\{\{0.6, 0.7\}, \{0.1\}, \{0.3\}\}$	$\{\{0.6, 0.7\}, \{0.1, 0.2\}, \{0.1, 0.2\}\}$
a_3	$\{\{0.6\}, \{0.3\}, \{0.4\}\}$	$\{\{0.5, 0.6\}, \{0.4\}, \{0.2, 0.3\}\}$	$\{\{0.5, 0.6\}, \{0.1\}, \{0.3\}\}$
a_4	$\{\{0.3, 0.5\}, \{0.2\}, \{0.1, 0.2, 0.3\}\}$	$\{\{0.6, 0.7\}, \{0.1\}, \{0.2\}\}$	$\{\{0.7, 0.8\}, \{0.1\}, \{0.1, 0.2\}\}$

Since $s(\tilde{n}_{11}) = s(\tilde{n}_{12}) = 0.65$, $s(\tilde{n}_{21}) = s(\tilde{n}_{22}) = 0.75$, we calculate accuracy values and get $a(\tilde{n}_{11}) = -0.25$, $a(\tilde{n}_{12}) = -0.1$; $a(\tilde{n}_{21}) = -0.1$, $a(\tilde{n}_{22}) = -0.05$. Thus, reorder sequences for a_1, a_2, a_3, a_4 are as follows: $\tilde{n}_{1_{\sigma(1)}} = \tilde{n}_{13}$, $\tilde{n}_{1_{\sigma(2)}} = \tilde{n}_{11}, \tilde{n}_{1_{\sigma(3)}} = \tilde{n}_{12}; \tilde{n}_{2_{\sigma(1)}} = \tilde{n}_{21}, \tilde{n}_{2_{\sigma(2)}} = \tilde{n}_{22}$, $\tilde{n}_{2_{\sigma(3)}} = \tilde{n}_{23}; \tilde{n}_{3_{\sigma(1)}} = \tilde{n}_{32}, \tilde{n}_{3_{\sigma(2)}} = \tilde{n}_{31}, \tilde{n}_{3_{\sigma(3)}} = \tilde{n}_{33}; \tilde{n}_{4_{\sigma(1)}} = \tilde{n}_{43}, \tilde{n}_{4_{\sigma(2)}} = \tilde{n}_{42}, \tilde{n}_{4_{\sigma(3)}} = \tilde{n}_{41}$.

Step 2. Suppose that the fuzzy measures of attributes of X are given as follows: $\mu(x_1) = 0.362, \mu(x_2) = 0.2, \mu(x_3) = 0.438$. Firstly, according to Equation (7), the value of λ is obtained: $\lambda = 0.856$. Thus, $\mu(x_1, x_2) = 0.626$, $\mu(x_2, x_3) = 0.713$, $\mu(x_1, x_3) = 0.936$, $\mu(X) = 1$, thus,

$$\mu(x_{\phi(1)}) = 0.362; \mu(x_{\phi(2)}) = 0.264; \mu(x_{\phi(3)}) = \mu(X) - \mu(x_1, x_2) = 0.374.$$

Step 3. Aggregate $\widetilde{n}_{ij}(i = 1, 2, 3, 4; j = 1, 2, 3)$ by using the SVNHFCOA operator to derive the comprehensive score value $\widetilde{n}_i$ for a_i $(i = 1, 2, 3, 4)$. Take a_1 for an example, the comprehensive score value $\widetilde{n}_1$ of a_1 is calculated as follows:

$$
\begin{aligned}
\widetilde{n}_1 = \quad &\{\{1 - (1 - 0.2)^{0.362}(1 - 0.3)^{0.264}(1 - 0.5)^{0.374}, 1 - (1 - 0.2)^{0.362}(1 - 0.3)^{0.264}(1 - 0.6)^{0.374}, \\
&1 - (1 - 0.2)^{0.362}(1 - 0.4)^{0.264}(1 - 0.5)^{0.374}, 1 - (1 - 0.2)^{0.362}(1 - 0.4)^{0.264}(1 - 0.6)^{0.374}, \\
&1 - (1 - 0.2)^{0.362}(1 - 0.5)^{0.264}(1 - 0.5)^{0.374}, 1 - (1 - 0.2)^{0.362}(1 - 0.5)^{0.264}(1 - 0.6)^{0.374}, \\
&1 - (1 - 0.3)^{0.362}(1 - 0.3)^{0.264}(1 - 0.5)^{0.374}, 1 - (1 - 0.3)^{0.362}(1 - 0.3)^{0.264}(1 - 0.6)^{0.374}, \\
&1 - (1 - 0.3)^{0.362}(1 - 0.4)^{0.264}(1 - 0.5)^{0.374}, 1 - (1 - 0.3)^{0.362}(1 - 0.4)^{0.264}(1 - 0.6)^{0.374}, \\
&1 - (1 - 0.3)^{0.362}(1 - 0.5)^{0.264}(1 - 0.5)^{0.374}, 1 - (1 - 0.3)^{0.362}(1 - 0.5)^{0.264}(1 - 0.6)^{0.374} \\
&\{0.1^{0.362}0.1^{0.264}0.2^{0.374}, 0.1^{0.362}0.1^{0.264}0.3^{0.374}, 0.2^{0.362}0.1^{0.264}0.2^{0.374}, 0.2^{0.362}0.1^{0.264}0.3^{0.374}\}, \\
&\{0.5^{0.362}0.3^{0.264}0.3^{0.374}, 0.5^{0.362}0.3^{0.264}0.4^{0.374}, 0.5^{0.362}0.4^{0.264}0.3^{0.374}, 0.5^{0.362}0.4^{0.264}0.4^{0.374}, \\
&0.6^{0.362}0.3^{0.264}0.3^{0.374}, 0.6^{0.362}0.3^{0.264}0.4^{0.374}, 0.6^{0.362}0.4^{0.264}0.3^{0.374}, 0.6^{0.362}0.4^{0.264}0.4^{0.374}\}\},
\end{aligned}
$$

and obtain the following collective SVNHFEs:

$\widetilde{n}_1 = \{\{0.352, 0.401, 0.386, 0.432, 0.424, 0.468, 0.378, 0.425, 0.41, 0.455, 0.447, 0.488\}, \{0.127, 0.147, 0.157, 0.181\}, \{0.35, 0.387, 0.387, 0.428, 0.369, 0.408, 0.408, 0.452\}\}.$

$\widetilde{n}_2 = \{\{0.6, 0.629, 0.64, 0.641, 0.666, 0.667, 0.676, 0.7\}, \{0.1, 0.129, 0.13, 0.167\}, \{0.172, 0.199, 0.223, 0.258\}\};$

$\widetilde{n}_3 = \{\{0.538, 0.568, 0.572, 0.6\}, \{0.223\}, \{0.294, 0.332\}\};$

$\widetilde{n}_4 = \{\{0.56, 0.566, 0.592, 0.598, 0.611, 0.622, 0.639, 0.65\}, \{0.129\}, \{0.12, 0.154, 0.156, 0.179, 0.2, 0.232\}\}.$

Step 4. Based on the score function of SVNHFEs, we get:

$$s(\widetilde{n}_1) = 0.599, \ s(\widetilde{n}_2) = 0.77, \ s(\widetilde{n}_3) = 0.678, \ s(\widetilde{n}_4) = 0.767.$$

Therefore, we can see that $a_2 > a_4 > a_3 > a_1$ and a_2 is the best choice.

If we utilize the SVNHFCOG operator for the MADM problem, the decision-making procedure can be described as follows:

Step 3′. Aggregate $\widetilde{n}_{ij}$ $(i = 1, 2, 3, 4; j = 1, 2, 3)$ by using the SVNHFCOG operator to derive the comprehensive score value $\widetilde{n}'_i$ for a_i $(i = 1, 2, 3, 4)$.

$$
\begin{aligned}
\widetilde{n}'_1 = \quad &\{\{0.2^{0.362}0.3^{0.264}0.5^{0.374}, 0.2^{0.36}0.3^{0.264}0.6^{0.374}, 0.2^{0.36}0.4^{0.264}0.5^{0.374}, 0.2^{0.36}0.4^{0.264}0.6^{0.374}, \\
&0.2^{0.362}0.5^{0.264}0.5^{0.374}, 0.2^{0.36}0.5^{0.264}0.6^{0.374}, 0.3^{0.36}0.3^{0.264}0.5^{0.374}, 0.3^{0.36}0.3^{0.264}0.6^{0.374}, \\
&0.3^{0.362}0.4^{0.264}0.5^{0.374}, 0.3^{0.36}0.4^{0.264}0.6^{0.374}, 0.3^{0.36}0.5^{0.264}0.5^{0.374}, 0.3^{0.36}0.5^{0.264}0.6^{0.374} \\
&1 - (1 - 0.1)^{0.362}(1 - 0.1)^{0.264}(1 - 0.2)^{0.374}, 1 - (1 - 0.1)^{0.362}(1 - 0.1)^{0.264}(1 - 0.3)^{0.374}, \\
&1 - (1 - 0.2)^{0.362}(1 - 0.1)^{0.264}(1 - 0.2)^{0.374}, 1 - (1 - 0.2)^{0.362}(1 - 0.1)^{0.264}(1 - 0.3)^{0.374}\}, \\
&\{1 - (1 - 0.6)^{0.362}(1 - 0.3)^{0.264}(1 - 0.3)^{0.374}, 1 - (1 - 0.6)^{0.362}(1 - 0.3)^{0.264}(1 - 0.4)^{0.374}, \\
&1 - (1 - 0.6)^{0.362}(1 - 0.4)^{0.264}(1 - 0.3)^{0.374}, 1 - (1 - 0.6)^{0.362}(1 - 0.4)^{0.264}(1 - 0.4)^{0.374}, \\
&1 - (1 - 0.5)^{0.362}(1 - 0.3)^{0.264}(1 - 0.3)^{0.374}, 1 - (1 - 0.5)^{0.362}(1 - 0.3)^{0.264}(1 - 0.4)^{0.374}, \\
&1 - (1 - 0.5)^{0.362}(1 - 0.4)^{0.264}(1 - 0.3)^{0.374}, 1 - (1 - 0.5)^{0.362}(1 - 0.4)^{0.264}(1 - 0.4)^{0.374}\}\},
\end{aligned}
$$

and obtain the following collective SVNHF $\widetilde{n}_1'$:

$\widetilde{n}_1' = \{\{0.318, 0.339, 0.351, 0.374, 0.38, 0.405, 0.359, 0.382, 0.397, 0.423, 0.429, 0.457\}, \{0.136, 0.176, 0.166, 0.204\},$
$\{0.408, 0.439, 0.439, 0.469, 0.367, 0.4, 0.4, 0.432\}\}.$
$\widetilde{n}_2' = \{\{0.6, 0.625, 0.634, 0.636, 0.661, 0.662, 0.672, 0.7\}, \{0.1, 0.138, 0.139, 0.175\}, \{0.193, 0.228, 0.231, 0.264\}\};$
$\widetilde{n}_3' = \{\{0.533, 0.568, 0.563, 0.6\}, \{0.27\}, \{0.31, 0.337\}\};$
$\widetilde{n}_4' = \{\{0.495, 0.515, 0.52, 0.541, 0.595, 0.62, 0.625, 0.651\}, \{0.138\}, \{0.128, 0.164, 0.165, 0.2, 0.203, 0.238\}\}.$

Step 4′. Based on the score function of SVNHFEs, we get:

$$\text{s}(\widetilde{n}_1) = 0.599, \ \text{s}(\widetilde{n}_2) = 0.761, \ \text{s}(\widetilde{n}_3) = 0.658, \ \text{s}(\widetilde{n}_4) = 0.75.$$

Rank a_i according to the score values $a_2 > a_4 > a_3 > a_1$. Therefore, we can see that a_2 is the best choice.

Obviously, the above two kinds of ranking orders are the same as the ones in Ref [24]. Thus, the two kinds of ranking orders coincide with the results that are obtained in other references; therefore, the above example clearly indicates that the proposed decision-making methods are applicable and effective under a single-valued neutrosophic hesitant fuzzy environment.

5.2. Comparison Analysis and Discussion

To further validate the feasibility of above MADM methods, a comparison analysis was conducted with other methods, it should be noted that all these approaches are not clarify how to solve a situation where the attributes are inter-related. To be specific, the comparative study was based on the same illustrative example in which the weight of attributes is $\omega = (0.35, 0.25, 0.4)$. Then, the results by utilizing different approaches with complete weight information are shown in Table 4.

Table 4. Results obtained by utilizing the different methods based on the same illustrative example.

Methods	Final Ranking	Best Alternative	Worst Alternative
SVNHFWA operator [24]	$a_4 > a_2 > a_3 > a_1$	a_4	a_1
SVNHFWG operator [24]	$a_2 > a_4 > a_3 > a_1$	a_2	a_1
Correlation coefficient [25]	$a_2 > a_4 > a_3 > a_1$	a_2	a_1
Hamming distance [26]	$a_2 > a_4 > a_3 > a_1$	a_2	a_1
SVNHFCOA operator	$a_2 > a_4 > a_3 > a_1$	a_2	a_1
SVNHFCOG operator	$a_2 > a_4 > a_3 > a_1$	a_2	a_1

For the compared methods in [24], Ye proposes two kinds of aggregation operators, the SVNHFWA and SVNHFWG operators, which are applied to MADM problems with single-valued neutrosophic hesitant fuzzy information in [25].

For the compared methods in [25], Sahin and Liu motivated by the idea of correlation coefficients derived for HFSs, IFSs, SVNSs, they put forward correlation coefficients of SVNHFSs. Then, an effective MADM example is used to demonstrate its validity.

For the compared methods in [26], Şahin R utilize the distance measure between each alternative and ideal alternative to establish a multiple attribute decision making method under single-valued neutrosophic hesitant fuzzy environment.

Thus, according to the results presented in Table 4, if the SVNHFCOG operator [24] is used, the desirable alternative is a_4, and if other five kinds of methods are utilized, the best choice is a_2, and for all the compared methods, the worst alternative is always a_1. Therefore, for the same single-valued neutrosophic hesitant fuzzy information, the results obtained by the proposed methods in this paper are consistent with those obtained using the compared methods in [24–26], which further demonstrates the effectiveness and feasibility of the SVNHFCOA and SVNHFCOG operators.

6. Conclusions

In this paper, we put forward two novel MADM methods based on the SVNHFCOA and SVNHFCOG operators, their advantages can be summarized below.

First, the SVNHFCOA and SVNHFCOG operators have desirable properties, like: idempotency, boundedness, commutativity, and monotonity, and they can reduce to the existing aggregation operators of SVNHFSs, which illustrate their validity in theory.

Second, when comparing with existing methods for MADM problems under neutrosophic hesitant fuzzy environment, results obtained by the SVNHFCOA and SVNHFCOG operators are consistent and accurate, which illustrates their practicability in application.

Third, the existing approaches cannot consider the interrelationships of attributes in practical application; the proposed methods for MADM in this paper can further consider more correlations between attributes, which means that they have higher accuracy and greater reference value.

Finally, like the Choquet aggregation operators applied and studied under other fuzzy environments, like hesitant fuzzy environments, intuitionistic fuzzy environment, linguistic fuzzy environment, and others, the research of this paper can lay the foundation for the following research, which is of profound significance.

Author Contributions: All authors have contributed equally to this paper. The idea of the whole thesis is put forward by Xiaohong Zhang, he also completed the preparatory work of the paper. Xin Li analyzed the existing work of single-valued neutrosophic hesitant fuzzy sets and wrote the paper. The revision and submission of the paper were completed by Xiaohong Zhang and Xin Li.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Rao, R.V. *Decision Making in the Manufacturing Environment Using Graph Theory and Fuzzy MADM Methods*; Springer: London, UK, 2007; Available online: https://max.book118.com/html/2017/0729/124983015.shtm (accessed on 29 July 2017).
2. Zadeh, L.A. Fuzzy sets. *J. Inform. Control* **1965**, *8*, 338–353. [CrossRef]
3. Zhang, X.H. Fuzzy anti-grouped filters and fuzzy normal filters in pseudo-BCI algebras. *J. Intell. Fuzzy Syst.* **2017**, *33*, 1767–1774. [CrossRef]
4. Miyamoto, S. Remarks on basics of fuzzy sets and fuzzy multisets. *Fuzzy Sets Syst.* **2005**, *156*, 427–431. [CrossRef]
5. Atanassov, K.; Gargov, G. Interval-valued intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1989**, *31*, 343–349. [CrossRef]
6. Dubois, D.; Prade, H. *Fuzzy Sets and Systems: Theory and Applications*; Academic Press: New York, NY, USA, 1980; pp. 32–154.
7. Smarandache, F. A unifying field in logics: Neutrosophic logic. *Multi. Valued Logic* **1999**, *8*, 489–503.
8. Ju, Y.B.; Yang, S.H.; Liu, X.Y. Some new dual hesitant fuzzy aggregation operators based on Choquet integral and their applications to multiple attribute decision making. *J. Intell. Fuzzy Syst.* **2014**, *27*, 2857–2868. [CrossRef]
9. Ju, Y.B. A new method for multiple criteria group decision making with incomplete weight information under linguistic environment. *Appl. Math. Model* **2014**, *38*, 5256–5268. [CrossRef]
10. Wang, H.; Smarandache, F.; Sunderraman, R. Single-valued neutrosophic sets. *Rev. Air Force Acad.* **2013**, *17*, 10–13.
11. Wang, H.; Madiraju, P. Interval-neutrosophic Sets. *J. Mech.* **2004**, *1*, 274–277. [CrossRef]
12. Peng, J.; Wang, J.; Wu, X. Multi-valued neutrosophic sets and power aggregation operators with their applications in multi-criteria group decision-making problems. *Int. J. Comput. Int. Sys.* **2015**, *8*, 345–363. [CrossRef]
13. Broumi, S.; Smarandache, F.; Dhar, M. Rough neutrosophic sets. *Ital. J. Pure. Appl. Math.* **2014**, *32*. [CrossRef]

14.	Zhang, X.H.; Bo, C.X.; Smarandache, F.; Dai, J.H. New inclusion relation of neutrosophic sets with applications and related lattice structrue. *Int. J. Mach. Learn. Cybs.* **2017**. accepted.

15.	Ye, J. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. *J. Intell. Fuzzy Syst.* **2014**, *26*, 2459–2466. [CrossRef]

16.	Ye, J. Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. *Int. J. Gen. Syst.* **2013**, *42*, 386–394. [CrossRef]

17.	Huang, L.H. New distance measure of single-valued neutrosophic sets and its application. *J. Intell. Syst.* **2016**, *31*, 1021–1032. [CrossRef]

18.	Torra, V.; Narukawa, Y. On hesitant fuzzy sets and decision. In Proceedings of the 18th IEEE International Conference on Fuzzy Systems, Jeju Island, Korea, 20–24 August 2009; pp. 1378–1382. [CrossRef]

19.	Torra, V. Hesitant fuzzy sets. *Int. J. Intell. Syst.* **2010**, *25*, 529–539. [CrossRef]

20.	Qu, G.; Li, Y.; Qu, W. Some new Shapley dual hesitant fuzzy Choquet aggregation operators and their applications to multiple attribute group decision making-based TOPSIS. *J. Intell. Fuzzy Syst.* **2017**, *33*, 1–21. [CrossRef]

21.	Xu, Z.S.; Zhang, X.L. Hesitant fuzzy multi-attribute decision making based on TOPSIS with incomplete weight information. *IFIP. Trans. B.* **2013**, *52*, 53–64. [CrossRef]

22.	Xia, M.M.; Xu, Z.S. Hesitant fuzzy information aggregation in decision making. *Int. J. Approx. Reason.* **2011**, *52*, 395–407. [CrossRef]

23.	Zhang, N.; Wei, G.W. Extension of VIKOR method for decision making problem based on hesitant fuzzy set. *Appl. Math. Model.* **2013**, *37*, 4938–4947. [CrossRef]

24.	Ye, J. Multiple-attribute decision-making method under a single-valued neutrosophic hesitant fuzzy environment. *J. Intell. Syst.* **2014**, *24*, 23–36. [CrossRef]

25.	Şahin, R.; Liu, P.D. Correlation coefficient of single-valued neutrosophic hesitant fuzzy sets and its applications in decision making. *Neural. Comput. Appl.* **2017**, *28*, 1387–1395. [CrossRef]

26.	Şahin, R. Distance and similarity measures for *MADM* with single-valued neutrosophic hesitant fuzzy information. In *New trends in Neutrosophic Theory and Applications*; Florentin, S., Surapati, P., Eds.; Pons Editions Brussels: Brussels, Belgium, 2016; ISBN 978-1-59973-498-9. Available online: https://www.researchgate.net/publication/310646060 (accessed on 20 November 2016).

27.	Liu, P.D.; Zhang, L.L. An extended multi-criteria decision making method based on neutrosophic hesitant fuzzy information. *J. Intell. Fuzzy Syst.* **2017**, *32*, 4403–4413. [CrossRef]

28.	Biswas, P.; Pramanik, S.; Giri, B.C. Some distance measures of single-valued neutrosophic hesitant fuzzy sets and their applications to multiple attribute decision making. In *New trends in Neutrosophic Theory and Applications*; Florentin, S., Surapati, P., Eds.; Pons Editions Brussels: Brussels, Belgium, 2016; ISBN 978-1-59973-498-9. Available online: https://www.researchgate.net/publication/310646060 (accessed on 20 November 2016).

29.	Liu, C.F.; Luo, Y.S. New aggregation operators of single-valued neutrosophic hesitant fuzzy set and their application in *MADM*. *Pattern Anal. Appls.* **2017**, *1*, 1–11. [CrossRef]

30.	Choquet, G. Theory of capacities. *Annales del. Institut Fourier.* **1953**, *5*, 131–295. [CrossRef]

31.	Sugeno, M. Theory of fuzzy integral and its application. Ph.D. Thesis, Tokyo Institute of Technology, Tokyo, Japan, 22 January 1975.

32.	Grabisch, M.; Nguyen, H.T.; Walker, E.A. Fuzzy measures and integrals. In fundamentals of uncertainty calculi with applications to fuzzy inference. *Springer Neth.* **1995**, *78*, 563–604. [CrossRef]

33.	Zhang, X.H.; She, Y.H. *Fuzzy Quantifies with Integral Semantics*; Science press: Beijing, China, 2017; ISBN 978-7-03053-480-4. Available online: http://product.dangdang.com/25113577.html (accessed on 1 July 2017).

34.	Qin, J.; Liu, X.; Pedrycz, W. Multi-attribute group decision making based on Choquet integral under interval-valued intuitionistic fuzzy environment. *Int. J. Comput. Int. Sys.* **2016**, *9*, 133–152. [CrossRef]