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Single-Valued Neutrosophic Linguistic-Induced Aggregation Distance Measures and Their Application in Investment Multiple Attribute Group Decision Making

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Abstract: This paper studied the single-valued neutrosophic linguistic distance measures based on the induced aggregation method. Firstly, we proposed a single-valued neutrosophic linguistic-induced ordered weighted averaging distance (SVNLIOWAD) measure, which is a new extension of the existing distance measures based on the induced aggregation view. Then, based on the proposed SVNLIOWAD, a novel induced distance for single-valued neutrosophic linguistic sets, namely the single-valued neutrosophic linguistic weighted induced ordered weighted averaging distance (SVNLWIOWAD), was developed to eliminate the defects of the existing methods. The relationship between the two proposed distance measures was also explored. A multiple attribute group decision making (MAGDM) model was further presented based on the proposed SVNLWIOWAD measure. Finally, a numerical example concerning an investment selection problem was provided to demonstrate the usefulness of the proposed method under a single-valued neutrosophic linguistic environment and, then, a comparison analysis was carried out to verify the flexibility and effectiveness of the proposed work.

Keywords: single-valued neutrosophic linguistic set; distance measure; weighted induced aggregation; MAGDM; investment selection

1. Introduction

The growing uncertainties and complexities in multiple attribute decision making (MADM) make it increasingly difficult for people to judge their attributes accurately. Accordingly, how to measure such complex and uncertain information effectively has become a key issue during the process of decision making. Several tools, such as fuzzy set [1], intuitionistic fuzzy set (IFS) [2], picture fuzzy set [3-4], linguistic term [5], and neutrosophic set [6], have been introduced to deal with inaccurate and uncertain information. The single-valued neutrosophic linguistic set (SVNLS), introduced by Ye [7], is an up-to-date tool to measure uncertainty or inaccuracy of information by combining the advantages of single-valued neutrosophic set [8] and linguistic terms [5]. The basic element of the SVNLS is the single-valued neutrosophic linguistic number (SVNLN), which makes it more suitable for solving uncertain and imprecise information than the existing tools. Ye [7] extended the conventional the technique for order preference by similarity to ideal solutions (TOPSIS) [9] approach to SVNLS environment and explored its application in investment selection problems. Wang et al. [10] studied the operational laws for SVNLS and presented the SVNL Maclaurin symmetric mean aggregation operator. Chen et al. [11] studied the ordered weighted distance measure between SVNLSs. Wu et al. [12] studied the application of the SVNLS in a 2-tuple MADM environment. Kazimieras et al. [13] presented a weighted aggregated sum product assessment approach for SVN decision making problems. Garg and Nancy [14] proposed some SVNLS aggregation operators based on the prioritized method to solve the attributes' priority in MADM problems. Cao et al. [15] studied the SVNL decision making approach based on a combination of ordered and weighted distances measures.

Distance measure is one of the most popular tools to express the deviation degree between two sets or variables. Consequently, many types of distance measures have been investigated and proposed in the existing literature, such as the weighted distance (WD) measure [16], ordered weighted averaging distance (OWAD) measure [17], combined weighted distance (CWD) measure [18], and induced OWAD (IOWAD) measure [19]. Among them, the IOWAD measure is a widely used one, recently proposed by Merigó and Casanovas [19]. The key advantage of the IOWAD is that it summarizes the minimun and maximum distance measures and can use induced-ordering variables to depict the intricate attitudinal characteristics. Now, the IOWAD operator has been widely used in MADM problems and extended to accommodate several fuzzy environments, such as fuzzy IOWAD (FIOWAD) [20], fuzzy linguistic IOWAD [21], intuitionistic fuzzy IOWAD (IFIOWAD) [22], and 2-tuple linguistic IOWAD (2LIOWAD) [23].

However, as far as we know, there is no research on the application of the SVNLS with the IOWAD method. In accordance with the previous analysis, the SVNLS is an excellent method to describe fuzzy and uncertain information, while the IOWAD is a new tool that can be well integrated into the complex attitudes of decision makers. In order to develop and enrich the measure theory of SVNLS, this study explored the usefulness of the IOWAD measure in SVNL environments. For this purpose, the rest of the article is set out as follows: in Section 2, we briefly introduce some basic concepts. Section 3 firstly develops the single-valued neutrosophic linguistic induced ordered weighted averaging distance (SVNLIOWAD) operator, which is the extension of the IOWAD operator with SVNL information. Furthermore, the single-valued neutrosophic linguistic weighted induced ordered weighted averaging distance (SVNLWIOWAD) is then introduced to overcome the defects of the SVNLIOWAD operator and other existing induced aggregation distances. In Section 4, a MAGDM model based on the SVNLWIOWAD operator is formulated and a financial decision making problem is also provided to demonstrate the usefulness of the proposed method. Finally, Section 5 gives a conclusion for the paper.

2. Preliminaries

In this section, we mainly recap some basic concepts of the SVNLS and the IOWAD operator.

2.1. The Single-Valued Neutrosophic Set (SVNS)

Definition 1 [24]. Let u be an element in a finite set U. A single-valued neutrosophic set (SVNS) A in U can be defined as in (1):

$$A = \left\{ \left\langle u, T_A(u), I_A(u), F_A(u) \right\rangle \middle| u \in U \right\}, \tag{1}$$

where $T_A(u)$, $I_A(u)$, and $F_A(u)$ are called the truth-membership function, indeterminacy-membership function, and falsity-membership function, respectively, which satisfy the following conditions:

$$0 \le T_A(u), I_A(u), F_A(u) \le 1, \ \ 0 \le T_A(u) + I_A(u) + F_A(u) \le 3. \tag{2}$$

A single-valued neutrosophic number (SVNN) is expressed as $(T_A(u), I_A(u), F_A(u))$ and is simply termed as $u = (T_u, I_u, F_u)$. The mathematical operational laws between SVNNs $u = (T_u, I_u, F_u)$ and $v = (T_v, I_v, F_v)$ are defined as follows:

(1) $u \oplus v = (T_u + T_v - T_u * T_v, I_u * T_v, F_u * F_v)$;

(2) $\lambda u = (1-(1-T_u)^{\lambda}, (I_u)^{\lambda}, (F_u)^{\lambda})$, $\lambda > 0$;

(3) $u^{\lambda} = ((T_u)^{\lambda}, 1-(1-I_u)^{\lambda}, 1-(1-F_u)^{\lambda})$, $\lambda > 0$.

2.2. The Linguistic Set

Let $S = \{s_{\alpha} | \alpha = 1,\ldots,l\}$ be a finite and totally ordered discrete term set, where s_{α} indicates a possible value for a linguistic variable (LV) and l is an odd value. For instance, given $l = 7$, then a linguistic term set S could be specified $S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$ = *{extremely poor, very poor, poor, fair, good, very good, extremely good}*. Then, for any two LVs, s_i and s_j in S, should satisfy rules (1)–(4) [24]:

(1) $s_i \le s_j \Leftrightarrow i \le j$;

(2) $Neg(s_i) = s_{-i}$;

(3) $\max(s_i, s_j) = s_j$, if $i \le j$;

(4) $\min(s_i, s_j) = s_i$, if $i \le j$.

The discrete term set S is also extended to a continuous set $\overline{S} = \{s_{\alpha} | \alpha \in R\}$ for reducing the loss of information during the operational process. The operational rules for LVs $s_{\alpha}, s_{\beta} \in \overline{S}$ are defined as follows [25]:

(1) $s_{\alpha} \oplus s_{\beta} = s_{\alpha+\beta}$;

(2) $\mu s_{\alpha} = s_{\mu\alpha}$, $\mu \ge 0$.

2.3. The Single-Valued Neutrosophic Linguistic Set (SVNLS)

Definition 2 [7]. Let U be a finite universe set and $\overline{S}$ be a continuous linguistic set, a SVNLS B in U is defined as in (3):

$$B = \left\{\left\langle u, [s_{\theta(u)}, (T_B(u), I_B(u), F_B(u))]\right\rangle \middle| u \in U\right\}, \tag{3}$$

where $s_{\theta(u)} \in \overline{S}$, the truth-membership function $T_B(u)$, the indeterminacy-membership function $I_B(u)$, and the falsity-membership function $F_B(u)$ satisfy condition (4):

$$0 \le T_B(u), I_B(u), F_B(u) \le 1, \quad 0 \le T_B(u) + I_B(u) + F_B(u) \le 3. \tag{4}$$

For an SVNLS B in U, the SVNLN $\langle s_{\theta(u)}, (T_B(u), I_B(u), F_B(u))\rangle$ is simply termed as $u = \langle s_{\theta(u)}, (T_u, I_u, F_u)\rangle$. The operational rules for SVNLNs $u_i = \langle s_{\theta(u_i)}, (T_{u_i}, I_{u_i}, F_{u_i})\rangle (i = 1, 2)$ are defined as follows:

(1) $u_1 \oplus u_2 = \langle s_{\theta(u_1)+\theta(u_2)}, (T_{u_1} + T_{u_2} - T_{u_1} * T_{u_2}, I_{u_1} * T_{u_2}, F_{u_1} * F_{u_2})\rangle$;

(2) $\lambda u_1 = \langle s_{\lambda\theta(u_1)}, (1-(1-T_{u_1})^{\lambda}, (I_{u_1})^{\lambda}, (F_{u_1})^{\lambda})\rangle$, $\lambda > 0$;

(3) $u_1^{\lambda} = \langle s_{\theta^{\lambda}(u_1)}, ((T_{u_1})^{\lambda}, 1-(1-I_{u_1})^{\lambda}, 1-(1-F_{u_1})^{\lambda})\rangle$, $\lambda > 0$.

Definition 3 [7]. Given two SVNLNs $u_i = \langle s_{\theta(u_i)}, (T_{u_i}, I_{u_i}, F_{u_i})\rangle (i = 1, 2)$, their distance measure is defined using the following formula:

$$d(u_1,u_2)=\left[\left|\theta(u_1)T_{u_1}-\theta(u_2)T_{u_2}\right|^l+\left|\theta(u_1)I_{u_1}-\theta(u_2)I_{u_2}\right|^l+\left|\theta(u_1)F_{u_1}-\theta(u_2)F_{u_2}\right|^l\right]^{1/l}, \tag{5}$$

where $l\in(0,+\infty)$. If we consider different weights associated with individual distances of SVNLVs, then we can get the single-valued neutrosophic linguistic weighted distance (SVNLWD) measure [10].

Definition 4. Let $u_j,u_j^{'}(j=1,2,...,n)$ be the two collections of SVNLNs, a single-valued neutrosophic linguistic weighted distance measure is defined as following formula:

$$SVNLWD\left((u_1,u_1^{'}),...,(u_n,u_n^{'})\right)=\sum_{j=1}^{n}w_j d(u_j,u_j^{'}), \tag{6}$$

where the associated weighting vector w_j satisfies $w_j\in[0,1]$ and $\sum_{j=1}^{n}w_j=1$.

2.4. The Single-Valued Neutrosophic Linguistic Set (SVNLS)

Motivated by the induced ordered weighted averaging (IOWA) operator [26], Merigó and Casanovas [19] developed the IOWAD operator. For two crisp sets $X=(x_1,..,x_n)$ and $Y=(y_1,...,y_n)$, the IOWAD operator be easily obtained as follows:

Definition 5. An IOWAD operator is defined by a weight vector $W=(w_1,...,w_n)^T$ with $0\le w_j\le 1$ and $\sum_{j=1}^{n}w_j=1$ and an order-inducing vector $T=(t_1,...,t_n)$, such that:

$$IOWAD\left(\langle t_1,x_1,y_1\rangle,...,\langle t_n,x_n,y_n\rangle\right)=\sum_{j=1}^{n}w_j D_j, \tag{7}$$

where $(D_1,...,D_n)$ is recorded $(d_1,...,d_n)$, induced by the decreasing order of $(t_1,...,t_n)$, and $d_i=d(x_i,y_i)=\left|x_i-y_i\right|$ is the distance between x_i and y_i.

3. Single-Valued Neutrosophic Linguistic-Induced Aggregation Distance Measures

3.1. SVNLIOWAD Measure

Previous analysis has shown that the IOWAD is a very practical tool to measure deviation in many fields, such as clustering analysis and decision making. In this section, we explore the application of the IOWAD operator in an SVNL situation and develop the SVNLIOWAD operator.

Definition 6. Let $u_j,u_j^{'}(j=1,2,...,n)$ be two sets of SVNLNs, then the SVNLIOWAD operator is defined by a weight vector $W=(w_1,...,w_n)^T$ with $0\le w_j\le 1$ and $\sum_{j=1}^{n}w_j=1$ and an order-inducing vector $T=(t_1,...,t_n)$, such that:

$$SVNLIOWAD\left(\langle t_1,u_1,u_1^{'}\rangle,...,\langle t_n,u_n,u_n^{'}\rangle\right)=\sum_{j=1}^{n}w_j D_j, \tag{8}$$

where $(D_1,...,D_n)$ is recorded $(d_1,...,d_n)$,induced by the decreasing order of $(t_1,...,t_n)$, $d_i=d(u_i,u'_i)=\left|u_i-u'_i\right|$ is the distance between SVNLNs, defined in Equation (5).

Using a similar analysis with the IOWAD operator [18, 19, 27, 28], it is easy to derive the following useful properties for the SVNLIOWAD operator:

Theorem 1 (Idempotency). If $d_i = d(u_i, u'_i) = |u_i - u'_i| = d$ for all i, then

$$SVNLIOWAD\left(\langle t_1, u_1, u_1' \rangle, \ldots, \langle t_n, u_n, u_n' \rangle\right) = d. \tag{9}$$

Theorem 2 (Boundedness). Let $\min_i(|u_i - u_i'|) = x$ and $\max_i(|u_i - u_i'|) = y$, then

$$x \le SVNLIOWAD\left(\langle t_1, u_1, u_1' \rangle, \ldots, \langle t_n, u_n, u_n' \rangle\right) \le y. \tag{10}$$

Theorem 3 (Monotonicity). If $|u_i - u_i'| \ge |v_i - v_i'|$ for all i, then

$$SVNLIOWAD\left(\langle t_1, u_1, u_1' \rangle, \ldots, \langle t_n, u_n, u_n' \rangle\right) \ge SVNLIOWAD\left(\langle t_1, v_1, v_1' \rangle, \ldots, \langle t_n, v_n, v_n' \rangle\right). \tag{11}$$

Theorem 4 (Commutativity-IOWA operator aggregation). Let $\left(\langle t_1, u_1, u_1' \rangle, \ldots, \langle t_n, u_n, u_n' \rangle\right)$ $(i = 1, 2, \ldots, n)$ be any possible permutation of the argument vector $\left(\langle t_1, v_1, v_1' \rangle, \ldots, \langle t_n, v_n, v_n' \rangle\right)$, then

$$SVNLIOWAD\left(\langle t_1, u_1, u_1' \rangle, \ldots, \langle t_n, u_n, u_n' \rangle\right) = SVNLIOWAD\left(\langle t_1, v_1, v_1' \rangle, \ldots, \langle t_n, v_n, v_n' \rangle\right). \tag{12}$$

We can also illustrate the property of commutativity by considering the distance measure:

$$SVNLIOWAD\left(\langle t_1, u_1, u_1' \rangle, \ldots, \langle t_n, u_n, u_n' \rangle\right) = SVNLIOWAD\left(\langle t_1, u_1', u_1 \rangle, \ldots, \langle t_n, u_n', u_n \rangle\right). \tag{13}$$

By considering different cases of the weighted vector in the SVNLIOWAD operator, we can get several special distance measures. For example:

- If $w_1 = \cdots = w_n = \frac{1}{n}$, we obtain the SVNLWD;
- If the ordering of weight w_j is same as the order-inducing t_j for all j, then the SVNLIOWAD reduces to the SVNLOWAD measure [15];
- If $T = (t, 0, \cdots, 0)$, then

$$SVNLIOWAD\left(\langle t_1, u_1, u_1' \rangle, \ldots, \langle t_n, u_n, u_n' \rangle\right) = D_1. \tag{14}$$

Next, a numerical example is given to show the aggregation process of the SVNLIOWAD operator.

Example 1. Assuming that:

$$U = (u_1, u_2, u_3, u_4, u_5)$$
$$= \left(\langle s_2, (0.5, 0.3, 0.4) \rangle, \langle s_5, (0.3, 0.3, 0.6) \rangle, \langle s_5, (0.5, 0.2, 0.2) \rangle, \langle s_7, (0.5, 0.8, 0.2) \rangle, \langle s_2, (0.1, 0.4, 0.6) \rangle\right)$$

and

$$V=(v_1,v_2,v_3,v_4,v_5)$$
$$=\left(\langle s_3,(0.7,0.8,0)\rangle,\langle s_5,(0.4,0.4,0.5)\rangle,\langle s_3,(0.5,0.7,0.2)\rangle,\langle s_3,(0.4,0.2,0.6)\rangle,\langle s_4,(0.5,0.7,0.2)\rangle\right),$$

are two SVNLNs defined in linguist term set $S=\{s_1,s_2,s_3,s_4,s_5,s_6,s_7\}$ and suppose $w=(0.20,0.30,0.15,0.10,0.25)^T$ and $T=(5,8,4,2,7)$ are the weight vector and order-inducing variable vector of the SVNLIOWAD operator, respectively. Then, the calculation steps of the SVNLIOWAD are displayed as follows:

(1) Calculate the individual distances $d(u_i,v_i)$ $(i=1,2,\ldots,5)$ (let $\lambda=1$) according to Equation (5):

$$d(u_1,v_1)=|2\times 0.5-3\times 0.7|+|2\times 0.3-3\times 0.8|+|2\times 0.4-3\times 0|=3.7\,.$$

Similarly, we get

$$d(u_2,v_2)=1.5,\ d(u_3,v_3)=2.4,\ d(u_4,v_4)=7.7,\ d(u_5,v_5)=3.2;$$

(2) Sort the $d(u_i,v_i)$ $(i=1,2,\ldots,5)$ according to the decreasing order of the order-inducing variable:

$$D_1=d(u_2,v_2)=1.5,\ D_2=d(u_5,v_5)=3.2,\ D_3=d(u_1,v_1)=3.7,$$

$$D_4=d(u_3,v_3)=2.4,\ d(u_4,v_4)=7.7;$$

(3) Utilize the SVNLIOWAD operator defined in Equation (8) to perform the following aggregation:

$$SVNLIOWAD(U,V)$$
$$=0.20\times 1.5+0.30\times 3.2+0.15\times 3.7+0.10\times 2.4+0.25\times 7.7=3.71\,.$$

From the aggregation process of the SVNLIOWAD operator, as well as the existing other induced aggregation distances, we see that the order-inducing variables are not really infused in the aggregation results, which fail to express the variation caused by the change of order-inducing variables. Thus, we needed to develop a new induced aggregation distance operator for SVNLSs to overcome this defect.

3.2. SVNLWIOWAD Measure

The special feature of the SVNLWIOWAD operator is that its induced ordering-variables play a dual role in the aggregation process. One role is, as the previous SVNLIOWAD operator, to induce the order of the arguments and the other is to adjust the associated weights. Thus it can better reflect the influence of the induced variables on the ensemble results. The SVNLWIOWAD operator can be defined as follows.

Definition 7. Let $u_j,u_j^{'}(j=1,2,\ldots,n)$ be two sets of SVNLNs, the SVNLWIOWAD operator is defined by a weight vector $W=(w_1,\ldots,w_n)^T$ with $0\le w_j\le 1$ and $\sum_{j=1}^{n} w_j=1$; and an order-inducing vector $T=(t_1,\ldots,t_n)$, such that:

$$SVNLWIOWAD\left(\langle t_1,u_1,u_1^{'}\rangle,\ldots,\langle t_n,u_n,u_n^{'}\rangle\right)=\sum_{j=1}^{n}\varpi_j D_j, \tag{15}$$

where $(D_1,\ldots,D_n)$ is recorded $(d_1,\ldots,d_n)$ induced by the decreasing order of $(t_1,\ldots,t_n)$, $d_i=|u_i-u'_i|$ is the distance between SVNLNs, defined in Equation (5). $\varpi_j\,(j=1,2,\ldots,n)$ is a moderated weight that is relatively determined by the weight $w_j \in W$ and order-inducing variable $t_j \in T$:

$$\varpi_j = \frac{w_j t_{\sigma(j)}}{\sum_{j=1}^{n} w_j t_{\sigma(j)}}, \tag{16}$$

where $(\sigma(1),...,\sigma(n))$ is a permutation of $(1,...,n)$ such that $t_{\sigma(j-1)} \geq t_{\sigma(j)}$ for all $j>1$. Example 2 illustrates the performance of the SVNLWIOWAD operator.

Example 2 (Example 1 continuation). To utilize the SVNLWIOWAD operator, we calculated the moderated weight ϖ_j defined in Equation (16):

$$\varpi_1 = \frac{w_1 t_{\sigma(1)}}{\sum_{j=1}^{5} w_j t_{\sigma(j)}} = \frac{0.20\times 8}{0.20\times 8 + 0.30\times 7 + 0.15\times 5 + 0.10\times 4 + 0.25\times 4} = 0.274.$$

Similarly,

$$\varpi_2 = 0.359, \quad \varpi_3 = 0.128, \quad \varpi_4 = 0.068, \quad \varpi_5 = 0.171.$$

Thus, based on the results of Example 1, we can get the aggregation result of the SVNLWIOWAD operator:

$$SVNLWIOWAD(U,V)$$

$$= 0.274\times 1.5 + 0.359\times 3.2 + 0.128\times 3.7 + 0.068\times 2.4 + 0.171\times 7.7 = 3.462$$

Obviously, we got a different result compared with the SVNLIOWAD operator in Example 1. The main reason for the difference is that the order-inducing variables in the SVNLIOWAD operator (including the existing IOWAD and its numerous extensions) only act as inducers for the arguments, and do not participate in the actual calculation process. However, the SVNLWIOWAD's order-inducing variables can not only act as the inducer, but also participate in the actual calculation progress by adjusting the associated weights. Therefore, it can measure the effect of order-inducing variables on the aggregation results. Consequently, the SVNLWIOWAD can achieve a more reasonable and scientific measurement over the SVNLIOWAD operator.

The following theorems show some useful properties of the SVNLWIOWAD operator:

Theorem 5 (Idempotency). Let Q be the SVNLWIOWAD operator, if all $d_i = \left|u_i - u_i^{'}\right| = d$ for all i, then:

$$Q\left(\left\langle t_1, u_1, u_1^{'}\right\rangle, ..., \left\langle t_n, u_n, u_n^{'}\right\rangle\right) = d. \tag{17}$$

Proof. Because $d_i = \left|u_i - u_i^{'}\right| = d$, then $D_j = d$ for $j = 1,2,...,n$, and we have:

$$Q\left(\left\langle t_1, u_1, u_1^{'}\right\rangle, ..., \left\langle t_n, u_n, u_n^{'}\right\rangle\right) = \sum_{j=1}^{n} \varpi_j D_j = d\sum_{j=1}^{n} \varpi_j .$$

Note that $\sum_{j=1}^{n} \varpi_j = 1,$ thus we obtain $Q\left(\left\langle t_1, u_1, u_1^{'}\right\rangle, ..., \left\langle t_n, u_n, u_n^{'}\right\rangle\right) = d\sum_{j=1}^{n} \varpi_j = d.$ □

Theorem 6 (Boundedness). Let $\min_i(|u_i - u_i^{'}|) = x$ and $\max_i(|u_i - u_i^{'}|) = y$, then:

$$x \le Q\left(\left\langle t_1, u_1, u_1^{'}\right\rangle, \ldots, \left\langle t_n, u_n, u_n^{'}\right\rangle\right) \le y. \tag{18}$$

Proof. Because $\varpi_j \in [0,1]$ and $\sum_{j=1}^{n} \varpi_j = 1$, then:

$$Q\left(\left\langle t_1, u_1, u_1^{'}\right\rangle, \ldots, \left\langle t_n, u_n, u_n^{'}\right\rangle\right) = \sum_{j=1}^{n} \varpi_j D_j \le \sum_{j=1}^{n} \varpi_j y = y \sum_{j=1}^{n} \varpi_j = y.$$

Similarly,

$$Q\left(\left\langle t_1, u_1, u_1^{'}\right\rangle, \ldots, \left\langle t_n, u_n, u_n^{'}\right\rangle\right) = \sum_{j=1}^{n} \varpi_j D_j \ge \sum_{j=1}^{n} \varpi_j x = x \sum_{j=1}^{n} \varpi_j = x.$$

Thus, we get

$$x \le Q\left(\left\langle t_1, u_1, u_1^{'}\right\rangle, \ldots, \left\langle t_n, u_n, u_n^{'}\right\rangle\right) \le y.$$

□

Theorem 7 (Monotonicity). If $|u_i - u_i^{'}| \ge |v_i - v_i^{'}|$ for all i, then:

$$Q\left(\left\langle t_1, u_1, u_1^{'}\right\rangle, \ldots, \left\langle t_n, u_n, u_n^{'}\right\rangle\right) \ge Q\left(\left\langle t_1, v_1, v_1^{'}\right\rangle, \ldots, \left\langle t_n, v_n, v_n^{'}\right\rangle\right). \tag{19}$$

Proof. Let

$$Q\left(\left\langle t_1, u_1, u_1^{'}\right\rangle, \ldots, \left\langle t_n, u_n, u_n^{'}\right\rangle\right) = \sum_{j=1}^{n} \varpi_j D_j,$$

$$Q\left(\left\langle t_1, v_1, v_1^{'}\right\rangle, \ldots, \left\langle t_n, v_n, v_n^{'}\right\rangle\right) = \sum_{j=1}^{n} \varpi_j D_j'.$$

As $|u_i - u_i^{'}| \ge |v_i - v_i^{'}|$ for all i, it follows $D_j \ge D_j'$ for all j, therefore

$$Q\left(\left\langle t_1, u_1, u_1^{'}\right\rangle, \ldots, \left\langle t_n, u_n, u_n^{'}\right\rangle\right) = \sum_{j=1}^{n} \varpi_j D_j \ge \sum_{j=1}^{n} \varpi_j D_j' = Q\left(\left\langle t_1, v_1, v_1^{'}\right\rangle, \ldots, \left\langle t_n, v_n, v_n^{'}\right\rangle\right).$$

□

Theorem 8 (Commutativity-IOWA operator aggregation). Let $\left(\left\langle t_1, u_1, u_1^{'}\right\rangle, \ldots, \left\langle t_n, u_n, u_n^{'}\right\rangle\right)$ $(i = 1, 2, \ldots, n)$ be any possible permutation of the argument vector $\left(\left\langle t_1, v_1, v_1^{'}\right\rangle, \ldots, \left\langle t_n, v_n, v_n^{'}\right\rangle\right)$, then:

$$Q\left(\left\langle t_1, u_1, u_1^{'}\right\rangle, \ldots, \left\langle t_n, u_n, u_n^{'}\right\rangle\right) = Q\left(\left\langle t_1, v_1, v_1^{'}\right\rangle, \ldots, \left\langle t_n, v_n, v_n^{'}\right\rangle\right). \tag{20}$$

Proof. The permutation between $\left(\left\langle t_1, u_1, u_1^{'}\right\rangle, \ldots, \left\langle t_n, u_n, u_n^{'}\right\rangle\right)$ and $\left(\left\langle t_1, v_1, v_1^{'}\right\rangle, \ldots, \left\langle t_n, v_n, v_n^{'}\right\rangle\right)$ $(i = 1, 2, \ldots, n)$ follows that the corresponding rearranged arguments $D_j = D_j'$ for all j, therefore

$$Q\left(\left\langle t_1, u_1, u_1^{'}\right\rangle, \ldots, \left\langle t_n, u_n, u_n^{'}\right\rangle\right) = \sum_{j=1}^{n} \varpi_j D_j = \sum_{j=1}^{n} \varpi_j D_j' = Q\left(\left\langle t_1, v_1, v_1^{'}\right\rangle, \ldots, \left\langle t_n, v_n, v_n^{'}\right\rangle\right)$$

We can also illustrate the property of commutativity by considering the distance measure:

$$Q\left(\left\langle t_1, u_1, u_1^{'}\right\rangle, \ldots, \left\langle t_n, u_n, u_n^{'}\right\rangle\right) = Q\left(\left\langle t_1, u_1^{'}, u_1\right\rangle, \ldots, \left\langle t_n, u_n^{'}, u_n\right\rangle\right). \tag{21}$$

Note that $\left|u_i - u_i^{'}\right| = \left|u_i^{'} - u_i\right|$ for all i, thus the Equation (20) is easy to prove. □

In light of the similar analysis methods in [29-34], some particular cases of the SVNLWIOWAD operator can be achieved by exploring the weight vector and order-inducing values.

4. A New MAGDM Approach Based on the SVNLWIOWAD Operator

4.1. Steps of the MAGDM Method Based on the SVNWIOWAD Operator

On the basis of the analysis reviewed in the Introduction, it is customary for decision makers to express their opinions on alternatives over attributes by SVNLNs because of their cognition with uncertainty and vagueness. Therefore, it is well worth investigating the application of the proposed SVNLWIOWAD under the SVNL framework. For an MAGDM problem with n alternatives $A = \{A_1, A_2, \ldots, A_n\}$ assessed by decision makers with respect to m schemes (attributes) $C = \{C_1, C_2, \ldots, C_m\}$, the decision steps based on the SVNLWIOWAD are listed as follows:

Step 1: Each expert $d_k (k = 1, 2, \ldots, l)$ (whose weight is ε_k, meeting $\varepsilon_k \geq 0$ and $\sum_{k=1}^{l} \varepsilon_k = 1$) provides his or her performance of attributes by the SVNLNs. Afterwards, the individual decision matrix $U^k = \left(u_{ij}^{(k)}\right)_{m \times n}$ is obtained, where $u_{ij}^{(k)}$ is the *k*-th expert's evaluation of the alternative A_j with respect to the attribute C_i;

Step 2: Aggregate all performances of the individual experts into a collective one and then form the group decision matrix:

$$U = \left(u_{ij}\right)_{m \times n} = \begin{pmatrix} u_{11} & \cdots & u_{1n} \\ \vdots & \ddots & \vdots \\ u_{m1} & \cdots & u_{mn} \end{pmatrix}, \tag{22}$$

where $u_{ij} = \sum_{k=1}^{l} \varepsilon_k u_{ij}^{(k)}$;

Step 3: Find the ideal levels for each attribute to construct the ideal scheme, listed in the table 1;

Table 1. Ideal scheme.

	C_1	C_2	…	C_n
I	I_1	I_2	…	I_n

Step 4: Utilize Equation (15) to calculate the distance $SVNLWIOWAD(A_i, I)$ between different alternatives $A_i (i = 1, 2, ..., m)$ and the ideal scheme I;

Step 5: Rank the alternatives and identify the best one(s) according to $SVNLWIOWAD(A_i, I)$, where the smaller the value of $SVNLWIOWAD(A_i, I)$, the better the alternative $A_i (i = 1, 2, ..., m)$.

4.2. An Illustrative Example: Investment Selection

We explored the application of the proposed approach in an investment selection problem where three decision makers were invited to assess a suitable strategy. There were four companies (alternatives) considered as potential investment options, chemical company (A_1), food company (A_2), car company (A_3) and furniture company (A_4), according to following possible situations (attributes) for the next year: C_1 was the risk, C_2 was the growth, C_3 was the environmental impact, and C_4 was other impacts. The evaluation presented by the decision makers with respect to the four attributes formed individual SVNL decision matrices under the linguistic term set $S = \{s_1$ = extremely poor, s_2 = very poor, s_3 = poor, s_4 = fair, s_5 = good, s_6 = very good, and s_7 = extremely good$\}$, as shown in Tables 2–4.

Table 2. Single-valued neutrosophic linguistic (SVNL) decision matrix U^1.

	C_1	C_2	C_3	C_4
A_1	$\langle s_4^{(1)}, (0.3, 0.2, 0.3) \rangle$	$\langle s_3^{(1)}, (0.5, 0.3, 0.1) \rangle$	$\langle s_4^{(1)}, (0.5, 0.2, 0.3) \rangle$	$\langle s_5^{(1)}, (0.3, 0.5, 0.2) \rangle$
A_2	$\langle s_6^{(1)}, (0.6, 0.1, 0.2) \rangle$	$\langle s_4^{(1)}, (0.5, 0.2, 0.2) \rangle$	$\langle s_5^{(1)}, (0.6, 0.1, 0.2) \rangle$	$\langle s_3^{(1)}, (0.6, 0.2, 0.4) \rangle$
A_3	$\langle s_5^{(1)}, (0.7, 0.0, 0.1) \rangle$	$\langle s_3^{(1)}, (0.3, 0.1, 0.2) \rangle$	$\langle s_4^{(1)}, (0.6, 0.1, 0.2) \rangle$	$\langle s_6^{(1)}, (0.6, 0.1, 0.2) \rangle$
A_4	$\langle s_5^{(1)}, (0.4, 0.2, 0.3) \rangle$	$\langle s_3^{(1)}, (0.3, 0.2, 0.5) \rangle$	$\langle s_5^{(1)}, (0.4, 0.2, 0.3) \rangle$	$\langle s_4^{(1)}, (0.5, 0.3, 0.3) \rangle$

Table 3. SVNL decision matrix U^2.

	C_1	C_2	C_3	C_4
A_1	$\langle s_6^{(2)}, (0.4, 0.2, 0.4) \rangle$	$\langle s_4^{(2)}, (0.6, 0.1, 0.3) \rangle$	$\langle s_6^{(2)}, (0.6, 0.3, 0.4) \rangle$	$\langle s_5^{(2)}, (0.4, 0.4, 0.1) \rangle$
A_2	$\langle s_6^{(2)}, (0.7, 0.2, 0.3) \rangle$	$\langle s_5^{(2)}, (0.6, 0.2, 0.2) \rangle$	$\langle s_6^{(2)}, (0.7, 0.2, 0.3) \rangle$	$\langle s_4^{(2)}, (0.5, 0.4, 0.2) \rangle$
A_3	$\langle s_4^{(2)}, (0.8, 0.1, 0.2) \rangle$	$\langle s_4^{(2)}, (0.4, 0.2, 0.2) \rangle$	$\langle s_5^{(2)}, (0.7, 0.2, 0.3) \rangle$	$\langle s_6^{(2)}, (0.6, 0.3, 0.3) \rangle$
A_4	$\langle s_5^{(2)}, (0.4, 0.3, 0.4) \rangle$	$\langle s_5^{(2)}, (0.3, 0.1, 0.6) \rangle$	$\langle s_6^{(2)}, (0.5, 0.1, 0.2) \rangle$	$\langle s_3^{(2)}, (0.7, 0.1, 0.1) \rangle$

Table 4. SVNL decision matrix U^3.

	C_1	C_2	C_3	C_4
A_1	$\langle s_6^{(3)}, (0.5, 0.1, 0.3) \rangle$	$\langle s_4^{(3)}, (0.6, 0.2, 0.1) \rangle$	$\langle s_5^{(3)}, (0.6, 0.1, 0.3) \rangle$	$\langle s_4^{(3)}, (0.3, 0.6, 0.2) \rangle$
A_2	$\langle s_5^{(3)}, (0.5, 0.2, 0.3) \rangle$	$\langle s_5^{(3)}, (0.7, 0.2, 0.1) \rangle$	$\langle s_4^{(3)}, (0.7, 0.2, 0.2) \rangle$	$\langle s_6^{(3)}, (0.4, 0.6, 0.2) \rangle$
A_3	$\langle s_4^{(3)}, (0.6, 0.1, 0.2) \rangle$	$\langle s_3^{(3)}, (0.4, 0.1, 0.1) \rangle$	$\langle s_4^{(3)}, (0.5, 0.2, 0.2) \rangle$	$\langle s_5^{(3)}, (0.7, 0.2, 0.1) \rangle$
A_4	$\langle s_6^{(3)}, (0.5, 0.2, 0.3) \rangle$	$\langle s_5^{(3)}, (0.2, 0.1, 0.6) \rangle$	$\langle s_6^{(3)}, (0.6, 0.2, 0.4) \rangle$	$\langle s_4^{(3)}, (0.5, 0.2, 0.3) \rangle$

Assuming that the weights of the experts were $\varepsilon_1 = 0.30$, $\varepsilon_2 = 0.37$, and $\varepsilon_3 = 0.33$, respectively, then the group SVNL decision matrix could be obtained through aggregating the three individual decision matrices. The results are listed in the table 5.

Table 5. Group SVNL decision matrix U .

	C_1	C_2	C_3	C_4
A_1	$\langle s_{5.26},(0.399,0.163,0.330)\rangle$	$\langle s_{3.37},(0.566,0.185,0.144)\rangle$	$\langle s_{4.96},(0.566,0.186,0.330)\rangle$	$\langle s_{4.70},(0.335,0.491,0.159)\rangle$
A_2	$\langle s_{5.70},(0.611,0.155,0.258)\rangle$	$\langle s_{2.37},(0.602,0.200,0.162)\rangle$	$\langle s_{4.70},(0.666,0.155,0.229)\rangle$	$\langle s_{4.23},(0.514,0.350,0.258)\rangle$
A_3	$\langle s_{4.37},(0.714,0.000,0.155)\rangle$	$\langle s_{3.67},(0.365,0.128,0.163)\rangle$	$\langle s_{4.33},(0.611,0.155,0.229)\rangle$	$\langle s_{5.70},(0.633,0.180,0.186)\rangle$
A_4	$\langle s_{5.30},(0.432,0.229,0.330)\rangle$	$\langle s_{2.37},(0.271,0.129,0.561)\rangle$	$\langle s_{5.63},(0.450,0.159,0.286)\rangle$	$\langle s_{3.67},(0.578,0.185,0.209)\rangle$

The ideal scheme (Table 6) determined by experts represents the optimal results that a supplier should satisfy, which further serves as a reference point in the aggregation process.

Table 6. Ideal scheme.

	C_1	C_2	C_3	C_4
I	$\langle s_7,(0.9,0,0)\rangle$	$\langle s_7,(0.9,0,0.1)\rangle$	$\langle s_7,(1,0,0.1)\rangle$	$\langle s_6,(0.9,0.1,0)\rangle$

We assumed that the weight and the order-inducing vectors of the SVNLWIOWAD were $w=(0.2,0.15,0.3,0.35)^T$ and $T=(5,9,7,4)$, respectively. Based on the available information, we utilized the SVNLWIOWAD to calculate the distances between the alternative A_i and the ideal scheme I:

$$SVNLWIOWAD(A_1,I)=6.440,\quad SVNLWIOWAD(A_2,I)=5.713,$$

$$SVNLWIOWAD(A_3,I)=5.323,\ SVNLWIOWAD(A_4,I)=6.810.$$

Therefore, the ordering of the alternatives through the values of $SVNLWIOWAD(A_i,I)(i=1,2,3,4)$ was $A_3 \succ A_2 \succ A_1 \succ A_4$, which implies that the optimal company A_3 is the best choice for investment.

To conduct a comparative analysis with the existing methods, in this example we utilized the SVNLWD, SVNLOWAD, and SVNLIOWAD to measure the relative performance of all alternatives to the ideal scheme, and the aggregation results are listed in the Table 7.

Table 7. Aggregation results.

	A_1	A_2	A_3	A_4	**Ranking**
$SVNLWD(A_i,I)$	6.828	5.836	5.048	6.444	$A_3 \succ A_2 \succ A_4 \succ A_1$
$SVNLOWAD(A_i,I)$	6.466	5.652	4.802	6.460	$A_3 \succ A_2 \succ A_4 \succ A_1$
$SVNLIOWAD(A_i,I)$	6.770	5.788	4.833	6.460	$A_3 \succ A_2 \succ A_1 \succ A_4$

From the Table 7, it is easy to see that the most desirable alternative was A_3 for the different distance measures used, which was the same as the result obtained from the SVNLWIOWAD operator. We also found that the ranking of alternatives may change for the different distance measures used because the different operators include different information. The SVNLWD uses the importance of attributes and the SVNLOWD focuses on the ordered location of the arguments. The SVNLIOWAD considers the attitudinal character of the decision-makers, while the SVNLIOWAD operator includes more information than the SVNLIOWAD as its design function of the order-induced variables. It is worth pointing out that the SVNLWIOWAD operator not only combines the advantages of the existing methods, but also overcomes some of their shortcomings, so that it can achieve a more scientific and reasonable result.

5. Conclusions

With the help of SVNLNs, decision makers may easily evaluate alternatives by linguistic terms as well as uncertainty degrees, which is very close to human cognition. In order to highlight the theory and application of SVNLS, in this paper, we explored some distance measures for SVNLSs from an induced aggregation point of view. Firstly, we put forward the SVNLIOWAD operator, which is a useful extension of the existing IOWAD operator. Then, a novel induced aggregation distance, namely the single valued neutrosophic linguistic weighted IOWAD (SVNLWIOWAD) operator, was developed to overcome the defects of the existing methods. The key feature of the SVNLWIOWAD is that it extends the functions of the order-inducing variables, which not only induce the order of arguments, but also moderate the associated weights. Compared with the existing methods, wherein the order-inducing variables just play the induced function, this dual role enables the SVNLWIOWAD operator to effectively measure the intrinsic variation of the induced variables on the integration results. Therefore, it can consider the complex attitudinal characteristics as well as reflect the influence of the induced variables on the aggregation results by moderating the associated weights. An MAGDM method, based on the SVNLWIOWAD operator, was further presented, which turned out to be a very powerful approach to handle decision making problems under SVNL situation. Finally, a numerical example on investment selection and comparative analysis were utilized to demonstrate the feasibility and effectiveness of the proposed method.

For future research, we will consider some methodological extensions and application of the proposed method with other decision making approaches, such as moving averaging and probability information.

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