Annals of **F**uzzy **M**athematics and **I**nformatics
Volume x, No. x, (Month 201y), pp. 1–xx
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Single valued neutrosophic relations

J. H. Kim, P. K. Lim, J. G. Lee, K. Hur

Received 6 March 2018; Revised 5 April 2018; Accepted 24 June 2018

Abstract.
We introduce the concept of a single valued neutrosophic reflexive, symmetric and transitive relation. And we study single valued neutrosophic analogues of many results concerning relationships between ordinary reflexive, symmetric and transitive relations. Next, we define the concepts of a single valued neutrosophic equivalence class and a single valued neutrosophic partition, and we prove that the set of all single valued neutrosophic equivalence classes is a single valued neutrosophic partition and the single valued neutrosophic equivalence relation is induced by a single valued neutrosophic partition. Finally, we define an α-cut of a single valued neutrosophic relation and investigate some relationships between single valued neutrosophic relations and their α-cuts.

2010 AMS Classification: 04A72

Keywords: Single valued neutrosophic relation, Single valued neutrosophic reflexive [respec., symmetric and transitive]relation, Single valued neutrosophic equivalence relation, Single valued neutrosophic transitive closure, Single valued neutrosophic value.

Corresponding Author: J. H. Kim (`junhikim@wku.ac.kr`)

1. Introduction

In 1965, Zadeh [28] had introduced the concept of a fuzzy set as the generalization of a crisp set. In 1971, he [27] defined the notions of similarity relations and fuzzy orderings as the generalizations of crisp equivalence relations and partial orderings playing basic roles in many fields of pure and applied science. After that time, many researchers [5, 6, 7, 8, 9, 10, 13, 14, 18] studied fuzzy relations. In particular, Chakraborty et al. [5, 6, 7, 8] defined a fuzzy relation over a fuzzy set and obtained many properties. Furthermore, Dib and Youssef [9] defined the fuzzy Cartesian product of two ordinary sets X and Y as the collection of all L-fuzzy sets of $X \times Y$, where $L = I \times I$ and I denotes the unit closed interval. In 2009, Lee [14] obtained many results by using the notion of fuzzy relations introduced by Dib and Youssef.

In 1968, Atanassov [1] defined an intutionistic fuzzy set as a generalization of a fuzzy set. After then, Atanassov and Gargov [2, 3] introduced the concept of an interval-valued intuitionistic fuzzy set an dealt with intuitionistic fuzzy logics. Moreover, Hur et al. [11] studied the category of intuitionistic H-fuzzy relation in the sense of a topological universe. Recently, Liu et al. [15, 16, 17] applied the concepts of an intuitionistic fuzzy set and an interval-valued intuitionistic fuzzy set to multi-attribute group decision making and group decision making, respectively.

In 1998, Smarandache [23] defined the concept of a neutrusophic set as the generalization of an intuitionistic fuzzy set. Also he introduced neutrosophic logics, neutrosophic sets, neutrosophic probabilities, neutrosophic statistics and its applications in [21, 22] . Furthermore, Salama et al. [19, 20] introduced the concept of a neutrusophic relation and studied its some properties. Recently, Bhowmik and Pal [4] introduced the concept of a neutrosophic relation and studied some of its properties. In particular, Wang et al. [24] introduced the notion of a single valued neutrosophic set. Moreover, Yang et al. [25] defined a single valued neutrosophic relation and investigated some of its properties.

In this paper, first, we introduce a single valued neutrosophic relation from a set X to Y and the composition of two single valued neutrosophic relations. Also we introduce some operations between single valued neutrosophic relations and obtain some of their properties. Second, we introduce the concept of a single valued neutrosophic reflexive, symmetric and transitive relation. And we study single valued neutrosophic analogues of many results concerning relationships between ordinary reflexive, symmetric and transitive relations. Third, we define the concepts of a single valued neutrosophic equivalence class and a single valued neutrosophic partition, and we prove that the set of all single valued neutrosophic equivalence classes is a single valued neutrosophic partition and the single valued neutrosophic equivalence relation is induced by a single valued neutrosophic partition. Finally, we define an α-cut of a single valued neutrosophic relation and investigate some relationships between single valued neutrosophic relations and their α-cuts.

2. Preliminaries

In this section, we introduce the concept of single valued neutrosophic set, the complement of a single valued neutrosophic set, the inclusion between two single valued neutrosophic sets, the union and the intersection of two single valued neutrosophic sets.

Definition 2.1 ([22]). Let X be a non-empty set. Then A is called a neutrosophic set (in sort, NS) in X, if A has the form $A = (T_A, I_A, F_A)$, where

$T_A : X \to]^-0, 1^+[$, $I_A : X \to]^-0, 1^+[$, $F_A : X \to]^-0, 1^+[$.

Since there is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, for each $x \in X$,

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+.$$

Moreover, for each $x \in X$, $T_A(x)$ [resp.,$I_A(x)$ and $F_A(x)$] represent the degree of membership [resp.,indeterminacy and non-membership] of x to A.

From Example 2.1.1 in [19], we can see that every IFS (intutionistic fuzzy set) A in a non-empty set X is an NS in X having the form

$$A = (T_A, 1 - (T_A + F_A), F_A),$$

where $(1 - (T_A + F_A))(x) = 1 - (T_A(x) + F_A(x))$.

Definition 2.2 ([22]). Let A and B be two NSs in X. Then we called A is contained in B, denoted by $A \subset B$, if for each $x \in X$, $inf\ T_A(x) \leq inf\ T_B(x)$, $sup\ T_A(x) \leq sup\ T_B(x)$, $inf\ I_A(x) \geq inf\ I_B(x)$, $sup\ I_A(x) \geq sup\ I_B(x)$, $inf\ F_A(x) \geq inf\ F_B(x)$ and $sup\ F_A(x) \geq sup\ F_B(x)$.

Definition 2.3 ([24]). Let X be a space of points (objects) with a generic element in X denoted by x. Then A is called a single valued neutrosophic set (in sort, SVNS) in X, if A has the form $A = (T_A, I_A, F_A)$, where $T_A,\ I_A,\ F_A : X \to [0, 1]$.

In this case, $T_A,\ I_A,\ F_A$ are called truth-membership function, indeterminacy-membership function, falsity-membership function, respectively and we will denote the set of all SVNSs in X as $SVNS(X)$.

Furthermore, we will denote the empty SVNS [resp. the whole SVNS] in X as 0_N [resp. 1_N] and define by $0_N(x) = (0, 1, 1)$ [resp. $1_N = (1, 0, 0)$], for each $x \in X$.

Definition 2.4 ([24]). Let $A \in SVNS(X)$. Then the complement of A, denoted by A^c, is a SVNS in X defined as follows: for each $x \in X$,

$$T_{A^c}(x) = F_A(x),\ I_{A^c}(x) = 1 - I_A(x) \text{ and } F_{A^c}(x) = T_A(x).$$

Definition 2.5 ([26]). Let $A,\ B \in SVNS(X)$. Then

(i) A is said to be contained in B, denoted by $A \subset B$, if for each $x \in X$,

$$T_A(x) \leq T_B(x),\ I_A(x) \geq I_B(x) \text{ and } F_A(x) \geq F_B(x),$$

(ii) A is said to be equal to B, denoted by $A = B$, if $A \subset B$ and $B \subset A$.

Definition 2.6 ([25]). Let $A,\ B \in SVNS(X)$. Then

(i) the intersection of A and B, denoted by $A \cap B$, is a SVNS in X defined as:

$$A \cap B = (T_A \wedge T_B, I_A \vee I_B, F_A \vee F_B),$$

where $(T_A \wedge T_B)(x) = T_A(x) \vee T_B(x)$, $(F_A \vee F_B) = F_A(x) \vee F_B(x)$, for each $x \in X$,

(ii) the union of A and B, denoted by $A \cup B$, is an SVNS in X defined as:

$$A \cup B = (T_A \vee T_B, I_A \wedge I_B, F_A \wedge F_B).$$

Result 2.7 ([25], Proposition 2.1). *Let $A,\ B \in NS(X)$. Then*

(1) $A \subset A \cup B$ *and* $B \subset A \cup B$,

(2) $A \cap B \subset A$ *and* $A \cap B \subset B$,

(3) $(A^c)^c = A$,

(4) $(A \cup B)^c = A^c \cap B^c$, $(A \cap B)^c = A^c \cup B^c$.

3. Single valued neutrosophic relations

In this section, we introduce the concepts of single valued neutrosophic relation, the composition of two single valued neutrosophic relations and the inverse of a single valued neutrosophic relation, and study some properties of each concept.

Let X, Y, Z be ordinary non-empty sets.

Definition 3.1. R is called a single valued neutrosophic relation (in short, SVNR) from X to Y, if it is a SVNS in $X \times Y$ having the form:

$$R = (T_R, I_R, F_R),$$

where $T_R, I_R, F_R : X \times Y \to [0,1]$ denote the truth-membership function, indeterminacy membership function, falsity-membership function, respectively.

For each $(x,y) \in X \times Y$, $T_R(x,y)$ [resp.,$I_R(x,y)$ and $F_R(x,y)$] represent the degree of membership [resp., indeterminacy and non-membership] of (x,y) to R.

In particular, a SVNR from from X to X is called a SVNR in X (See [25]).

The empty SVNR[resp. the whole SVNR] in X is denoted by ϕ_N [resp. X_N] and defined as follows: for each $(x,y) \in X \times X$,

$$\phi_N(x,y) = (0,1,1) \text{ [resp. } X_N(x,y) = (1,0,0)].$$

We will denote the set of all SVNRs in X [resp. from X to Y] as $SVNR(X)$ [resp. $SVNR(X \times Y)$] .

Let $X = \{x_1, x_2, ..., x_m\}$ and let $Y = \{y_1, y_2, ..., y_n\}$. Then $R = (T_R, I_R, F_R) \in SVNR(X \times Y)$ can be expressed by $m \times n$ matrix. This kind of matrix expressing a SVNR will be called a single valued neutrosophic matrix.

Definition 3.2 (See [25]). Let $R \in SVNR(X \times Y)$. Then

(i) the inverse of R, denoted by R^{-1}, is a SVNR from Y to X defined as follows: for each $(y,x) \in Y \times X$, $R^{-1}(x,y) = R(y,x)$, i.e.,

$$T_R^{-1}(y,x) = T_R(x,y), I_R^{-1}(y,x) = I_R(x,y), F_R^{-1}(y,x) = F_R(x,y).$$

(ii) the complement of R, denoted by R^c, is a SVNR from X to Y defined as follows: for each $(x,y) \in X \times Y$,

$$T_R^c(x,y) = F_R(x,y), I_R^c(x,y) = 1 - I_R(x,y), F_R^c(x,y) = T_R(x,y).$$

Example 3.3. Let $X = \{a,b,c\}$ and let R be a SVNR in X given by the single valued neutrosophic matrix:

$$R = \begin{pmatrix} (0.2,0.4,0.3) & (1,0.2,0) & (0.4,1,0.7) \\ (0,0,0) & (0.6,0.2,0.1) & (0.3,0.2,0.6) \\ (0,0,0) & (0,0,0) & (0.2,0.4,0.1) \end{pmatrix}.$$

Then the inverse and the complement of R are given as below:

$$R^{-1} = \begin{pmatrix} (0.2,0.4,0.3) & (0,0,0) & (0,0,0) \\ (1,0.2,0) & (0.6,0.2,0.1) & (0,0,0) \\ (0.4,1,0.7) & (0.3,0.2,0.6) & (0.2,0.4,0.1) \end{pmatrix},$$

$$R^c = \begin{pmatrix} (0.3,0.6,0.2) & (0,0.8,1) & (0.7,0,0.4) \\ (0,1,0) & (0.1,0.8,0.6) & (0.6,0.8,0.3) \\ (0,1,0) & (0,1,0) & (0.1,0.6,0.2) \end{pmatrix}.$$

Remark 3.4. For each $R \in SVNR(X)$, $R \cap R^c = \phi_N$ and $R \cup R^c = X_N$ do not hold, in general.

Consider the SVNR R in Example 3.3. Then

$$R \cap R^c = \begin{pmatrix} (0.2, 0.6, 0.3) & (0, 0.8, 1) & (0.4, 1, 0.7) \\ (0, 1, 0) & (0.1, 0.8, 0.6) & (0.3, 0.8, 0.6) \\ (0, 1, 0) & (0, 1, 0) & (0.1, 0.6, 0.2) \end{pmatrix} \neq \phi_N,$$

$$R \cup R^c = \begin{pmatrix} (0.3, 0.4, 0.2) & (1, 0.2, 0) & (0.7, 0, 0.4) \\ (0, 0, 0) & (0.6, 0.2, 0.1) & (0.6, 0.2, 0.3) \\ (0, 0, 0) & (0, 0, 0) & (0.2, 0.4, 0.1) \end{pmatrix} \neq X_N.$$

Definition 3.5 (See [25])**.** Let $R,\ S \in SVNR(X \times Y)$. Then
(i) R is said to be contained in S, denoted by $R \subset S$, if
$T_R(x, y) \leq T_S(x, y)$, $I_R(x, y) \geq I_S(x, y)$ and $F_R(x, y) \geq F_S(x, y)$, for each $(x, y) \in X \times Y$,
(ii) R is said to equal to S, denoted by $R = S$, if $R \subset S$ and $S \subset R$,
(iii) the intersection of R and S, denoted by $R \cap S$, is a SVNR from X to Y defined as:

$$A \cap B = (T_A \wedge T_B, I_A \vee I_B, F_A \vee F_B),$$

where $(T_A \wedge T_B)(x, y) = T_A(x, y) \wedge T_B(x, y)$, $(F_A \vee F_B)(x, y) = F_A(x, y) \vee F_B(x, y)$, for each $(x, y) \in X \times Y$,
(iv) the union of R and S, denoted by $R \cup S$, is a SVNR in X to Y defined as:

$$A \cup B = (T_A \vee T_B, I_A \wedge I_B, F_A \wedge F_B).$$

Proposition 3.6 (See [25], Theorem 3.1)**.** *Let $R,\ S,\ P \in SVNR(X \times Y)$. Then*
(1) $(R^c)^{-1} = (R^{-1})^c$,
(2) $(R^{-1})^{-1} = R$, $(R^c)^c = R$,
(3) $R \subset R \cup S$ *and* $S \subset R \cup S$,
(4) $R \cap S \subset R$ *and* $R \cap S \subset S$,
(5) *if* $R \subset S$, *then* $R^{-1} \subset S^{-1}$,
(6) *if* $R \subset P$ *and* $S \subset P$, *then* $R \cup S \subset P$,
(7) *if* $P \subset R$ *and* $P \subset S$, *then* $P \subset R \cap S$,
(8) *if* $R \subset S$, *then* $R \cup S = S$ *and* $R \cap S = R$,
(9) $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$, $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$,
(10) $(R \cup S)^c = R^c \cap S^c$, $(R \cap S)^c = R^c \cup S^c$.

Proof. The proofs are similar to Theorem 3.1 in [25]. □

From Definitions 3.2 and 3.5, we can easily obtain the following results.

Proposition 3.7. *Let $R, S, P \in SVNR(X \times Y)$. Then*
(1) (Idempotent laws): $R \cup R = R$, $R \cap R = R$,
(2) (Commutative laws): $R \cup S = S \cup R$, $R \cap S = S \cap R$,
(3) (Associative laws): $R \cup (S \cup P) = (R \cup S) \cup P$, $R \cap (S \cap P) = (R \cap S) \cap P$,
(4) (Distributive laws): $R \cup (S \cap P) = (R \cup S) \cap (R \cup P)$,
$R \cap (S \cup P) = (R \cap S) \cup (R \cap P)$,
(5) (Absorption laws): $R \cup (R \cap S) = R$, $R \cap (R \cup S) = R$.

Definition 3.8. Let $(R_j)_{j \in J} \subset SVNR(X \times Y)$. Then

(i) the the intersection of $(R_j)_{j\in J}$, denoted by $\bigcap_{j\in J} R_j$ (simply, $\bigcap R_j$), is a SVNR from X to Y defined as:

$$\bigcap R_j = (\bigwedge T_{R_j}, \bigvee I_{R_j}, \bigvee F_{R_j}),$$

(ii) the the union of $(R_j)_{j\in J}$, denoted by $\bigcup_{j\in J} R_j$ (simply, $\bigcup R_j$), is a SVNR from X to Y defined as:

$$\bigcup R_j = (\bigvee T_{R_j}, \bigwedge I_{R_j}, \bigwedge F_{R_j}).$$

The followings are the immediate result of Definitions 3.2, 3.5 and 3.8

Proposition 3.9. *Let $R \in SVNR(X\times Y)$ and let $(R_j)_{j\in J} \subset SVNR(X\times Y)$. Then*
(1) $(\bigcap R_j)^c = \bigcup R_j^c$, $(\bigcup R_j)^c = \bigcap R_j^c$,
(2) $R \cap (\bigcup R_j) = \bigcup (R \cap R_j)$, $R \cup (\bigcap R_j) = \bigcap (R \cup R_j)$.

Definition 3.10. Let $R \in SVNR(X \times Y)$ and let $S \in SVNR(Y \times Z)$. Then the composition of R and S, denoted by $S \circ R$, is a SVNR from X to Z defined as:

$$S \circ R = (T_{S\circ R}, I_{S\circ R}, F_{S\circ R}),$$

where for each $(x, z) \in X \times Z$,

$$T_{S\circ R}(x, z) = \bigvee_{y\in Y} (T_R(x, y) \wedge T_S(y, z)),$$
$$I_{S\circ R}(x, z) = \bigwedge_{y\in Y} (I_R(x, y) \vee I_S(y, z)),$$
$$F_{S\circ R}(x, z) = \bigwedge_{y\in Y} (F_R(x, y) \vee F_S(y, x)).$$

Proposition 3.11. (1) *$P \circ (S \circ R) = (P \circ S) \circ R)$, where $R \in SVNR(X \times Y)$, $S \in SVNR(Y \times Z)$ and $P \in SVNR(Z \times W)$.*
(2) *$P \circ (R \cup S) = (P \circ R) \cup (P \circ S)$, where $R, S \in SVNR(X \times Y)$ and $P \in SVNR(Y \times Z)$.*
(3) *If $R \subset S$, then $P \circ R \subset P \circ S$, where $R, S \in SVNR(X \times Y)$ and $P \in SVNR(Y \times Z)$.*
(4) *$(S \circ R)^{-1} = R^{-1} \circ S^{-1}$, where $R \in SVNR(X \times Y)$ and $S \in SVNR(Y \times Z)$.*

Proof. (1) Let $R \in SVNR(X \times Y)$, $S \in SVNR(Y \times Z)$ and $P \in SVNR(Z \times W)$ and let $(x, w) \in (X \times Z)$. Then

$$\begin{aligned} T_{P\circ(S\circ R)}(x, w) &= \bigvee_{z\in Z} (T_{S\circ R}(x, z) \wedge T_P(z, w)) \\ &= \bigvee_{z\in Z} ([\bigvee_{y\in Y} (T_R(x, y) \wedge T_S(y, z)] \wedge T_P(z, w)) \\ &= \bigvee_{y\in Y} (T_R(x, y) \wedge [\bigvee_{z\in Z} (T_S(y, z) \wedge T_P(z, w))]) \\ &= \bigvee_{y\in Y} (T_R(x, y) \wedge T_{P\circ S}(y, w)) \\ &= T_{(P\circ S)\circ R)}(x, w). \end{aligned}$$

Similarly, we can prove that $I_{P\circ(S\circ R)}(x, w) = I_{(P\circ S)\circ R)}(x, w)$ and $F_{P\circ(S\circ R)}(x, w) = F_{(P\circ S)\circ R)}(x, w)$. Thus the result holds.

(2) Let $R, S \in SVNR(X \times Y)$ and $P \in SVNR(Y \times Z)$ and let $(x, z) \in X \times Z$. Then

$$\begin{aligned} T_{P\circ(R\cup S)}(x, z) &= \bigvee_{y\in Y} (T_{R\cup S}(x, y) \wedge T_P(y, z)) \\ &= \bigvee_{y\in Y} ([T_R(x, y) \vee T_S(x, y)] \wedge T_P(y, z)) \\ &= [\bigvee_{y\in Y} (T_R(x, y) \wedge T_P(y, z)] \vee [\bigvee_{y\in Y} (T_S(x, y) \wedge T_P(y, z)] \\ &= T_{P\circ R}(x, z) \vee T_{P\circ S}(x, z) \end{aligned}$$

$$= T_{(P\circ R)\cup(P\circ S)}(x,z).$$

Similarly, we can see that $I_{P\circ(R\cup S)}(x,z) = I_{(P\circ R)\cup(P\circ S)}(x,z)$ and $F_{P\circ(R\cup S)}(x,z) = F_{(P\circ R)\cup(P\circ S)}(x,z)$. Thus the result holds.

(3) Let $R, S \in SVNR(X\times Y)$ and $P \in SVNR(Y\times Z)$. Suppose $R \subset S$ and let $(x,z) \in X\times Z$. Then

$$\begin{aligned} T_{P\circ R}(x,z) &= \bigvee_{y\in Y}(T_R(x,y)\wedge T_P(y,z)) \\ &\leq \bigvee_{y\in Y}(T_S(x,y)\wedge T_P(y,z)) \\ &\quad [\text{Since } R\subset S,\ T_R(x,y)\leq T_S(x,y)] \\ &= T_{P\circ S}(x,z). \end{aligned}$$

Similarly, we can prove that $I_{P\circ R}(x,z) \geq I_{P\circ S}(x,z)$ and $F_{P\circ R}(x,z) \geq F_{P\circ S}(x,z)$. Thus the result holds.

(4) Let $R \in SVNR(X\times Y)$ and $S \in SVNR(Y\times Z)$ and let $(x,z) \in X\times Z$. Then

$$\begin{aligned} T_{(S\circ R)^{-1}}(z,x) &= T_{(S\circ R)}(x,z) \\ &= \bigvee_{y\in Y}(T_R(x,y)\wedge T_S(y,z)) \\ &= \bigvee_{y\in Y}(T_{S^{-1}}(z,y)\wedge T_{R^{-1}}(y,x)) \\ &= T_{R^{-1}\circ S^{-1}}(z,x). \end{aligned}$$

Similarly we can see that $I_{(S\circ R)^{-1}}(z,x) = I_{R^{-1}\circ S^{-1}}(z,x)$ and $F_{(S\circ R)^{-1}}(z,x) = F_{R^{-1}\circ S^{-1}}(z,x)$. Thus the result holds. □

Remark 3.12. (1) For any SVNRs R and S, $S\circ R \neq R\circ S$, in general.

(2) For any $R, S \in SVNR(X\times Y)$ and $P \in SVNR(Y\times Z)$, $P\circ(R\cap S) \neq (P\circ R)\cap(P\circ S)$, in general.

Example 3.13. Let $X = Y = \{a,b\}$, $Z = \{x,y\}$. Consider two SVNRs R and S in X, and an SVNR P from X to Z given by following single valued neutrosophic matrices:

$$R = \begin{pmatrix} (0.6,0.3,0.4) & (0.7,0.2,0.1) \\ (0.4,0.6,0.3) & (0.6,0.4,0.2) \end{pmatrix},$$

$$S = \begin{pmatrix} (0.7,0.4,0.2) & (0.4,0.6,0.4) \\ (0.5,0.2,0.6) & (0.3,0.6,0.5) \end{pmatrix}$$

and

$$P = \begin{pmatrix} (0.7,0.2,0.3) & (0.4,0.6,0.4) \\ (0.4,0.6,0.2) & (0.8,0.2,0.3) \end{pmatrix}.$$

Then $T_{P\circ(R\cap S)}(a,x) = 0.6 \neq 0.4 = T_{(P\circ R)\cap(P\circ S)}(a,x)$. Thus $P\circ(R\cap S) \neq (P\circ R)\cap(P\circ S)$.

4. Single valued neutrosophic reflexve, smmetric and ansitive relations

In this section, we introduce single valued neutrosophic reflexve, smmetric and ansitive relations and obtain some properties related to them.

Definition 4.1 ([25])**.** The single valued neutrosophic identity relation in X, denoted by I_X (simply, I), is a SVNR in X defined as: for each $(x,y) \in X\times X$,

$$T_{I_X}(x,y) = \begin{cases} 1 & \text{if } x=y \\ 0 & \text{if } x\neq y, \end{cases} \quad I_{I_X}(x,y) = \begin{cases} 0 & \text{if } x=y \\ 1 & \text{if } x\neq y, \end{cases} \quad F_{I_X}(x,y) = \begin{cases} 0 & \text{if } x=y \\ 1 & \text{if } x\neq y. \end{cases}$$

It is clear that $I = I^{-1}$ and $I^c = (I^c)^{-1}$.

Definition 4.2 ([25]). $R \in SVNR(X)$ is said to be:
(i) reflexive, if for each $x \in X$, $T_R(x,x)1, I_R(x,x) = F_R(x,x) = 0$,
(ii) anti-reflexive, if for each $x \in X$, $T_R(x,x) = 0, I_R(x,x) = F_R(x,x) = 1$.

From Definitions 4.1 and 4.2, it is obvious that R is neutrosophic reflexive if and only if $I \subset R$.
The followings are the immediate results of the above definition.

Proposition 4.3 (See [19], Theorem 2.5.2). *Let $R \in SVNR(X)$.*
(1) *R is reflexive if and only if R^{-1} is reflexive.*
(2) *If R is reflexive, then $R \cup S$ is reflexive, for each $S \in SVNR(X)$.*
(3) *If R is reflexive, then $R \cap S$ is reflexive if and only if $S \in SVNR(X)$ is reflexive.*

The followings are the immediate result of Definitions 3.2, 3.5 and 4.2.

Proposition 4.4. *Let $R \in SVNR(X)$.*
(1) *R is anti-reflexive if and only R^{-1} is anti-reflexive.*
(2) *If R is anti-reflexive, then $R \cup S$ is anti-reflexive if and only if $S \in SVNR(X)$ is anti-reflexive.*
(3) *If R is anti-reflexive, then $R \cap S$ is anti-reflexive, for each $S \in SVNR(X)$.*

Proposition 4.5. *Let $R, S \in SVNR(X)$. If R and S are reflexive, then $S \circ R$ is reflexive.*

Proof. Let $x \in X$. Since R and S are reflexive,

$$T_R(x,x) = 1, I_R(x,x) = F_R(x,x) = 0$$

and

$$T_S(x,x) = 1, I_R(x,x) = F_S(x,x) = 0.$$

Thus

$$\begin{aligned} T_{S\circ R} &= \bigvee_{y\in X}(T_R(x,y) \wedge T_S(y,x)) \\ &= [\bigvee_{x\neq y\in X}(T_R(x,y) \wedge T_S(y,x))] \vee (T_R(x,x) \wedge T_S(x,x)) \\ &= [\bigvee_{x\neq y\in X}(T_R(x,y) \wedge T_S(y,x))] \vee (1 \wedge 1) \\ &= 1. \end{aligned}$$

On the other hand,

$$\begin{aligned} I_{S\circ R} &= \bigwedge_{y\in X}(I_R(x,y) \vee I_S(y,x)) \\ &= [\bigwedge_{x\neq y\in X}(I_R(x,y) \vee I_S(y,x))] \wedge (I_R(x,x) \vee I_S(x,x)) \\ &= [\bigwedge_{x\neq y\in X}(I_R(x,y) \vee I_S(y,x))] \wedge (0 \vee 0) \\ &= 0. \end{aligned}$$

Similarly, $F_{S\circ R} = 0$. So $S \circ R$ is reflexive. □

Definition 4.6. Let $R = (T_R, I_R, F_R) \in SVNR(X)$. Then
(i)[19, 25] R is said to be symmetric, if for each $x, y \in X$,

$$T_R(x,y) = T_R(y,x), I_R(x,y) = I_R(y,x), F_R(x,y) = F_R(y,x),$$

(ii)[19] R is said to be anti-symmetric, if for each $(x,y) \in X \times X$ with $x \neq y$,

$$T_R(x,y) \neq T_R(y,x), I_R(x,y) \neq I_R(y,x), F_R(x,y) \neq F_R(y,x),$$

From Definitions 4.2 and 4.6, it is obvious that ϕ_N is a symmetric and anti-reflexive SVNR, X_N and I are symmetric and reflexive SVNRs and I^c is an anti-reflexive SVNR.

The following is the immediate result of Definitions 3.5 and 4.6.

Result 4.7 ([25], Theorem 3.1)**.** *Let $R \in SVNR(X)$. Then R is symmetric iff $R = R^{-1}$.*

Proposition 4.8. *Let $R \in SVNR(X)$. If R is symmetric, then R^{-1} is symmetric.*

Proposition 4.9. *Let $R, S \in SVNR(X)$. If R and S are symmetric, then $R \cup S$ and $R \cap S$ are symmetric.*

Proof. Let $(x, y) \in X \times X$. Since R and S and are symmetric,

$$T_R(x, y) = T_R(y, x), I_R(x, y) = I_R(y, x), F_R(x, y) = F_R(y, x)$$

and

$$T_S(x, y) = T_S(y, x), I_S(x, y) = I_S(y, x), F_S(x, y) = F_S(y, x).$$

Thus $T_{R\cup S}(x, y) = T_R(x, y) \vee S_R(x, y) = T_R(y, x) \vee S_R(y, x) = T_{R\cup S}(y, x)$. Similarly, we can see that $I_{R\cup S}(x, y) = I_{R\cup S}(y, x)$ and $F_{R\cup S}(x, y) = F_{R\cup S}(y, x)$. So $R \cup S$ is symmetric.

Similarly, we can prove that $R \cap S$ is symmetric. □

Remark 4.10. R and S are nsymmetric, but $S \circ R$ is not symmetric, in general.

Example 4.11. Let $X = \{a, b, c\}$ and consider two NRs R and S in X given by the following single valued neutrosophic matrices:

$$R = \begin{pmatrix} (0.2, 0.4, 0.3) & (1, 0.2, 0) & (0.4, 1, 0.7) \\ (1, 0.2, 0) & (0.6, 0.2, 0.1) & (0.3, 0.2, 0.6) \\ (0.4, 1, 0.7) & (0.3, 0.2, 0.6) & (0.2, 0.4, 0.1) \end{pmatrix}$$

and

$$S = \begin{pmatrix} (0.2, 0.4, 0.3) & (0, 0.2, 0.6) & (0.2, 0.6, 0.3) \\ (0, 0.2, 0.6) & (0.6, 0.2, 0.1) & (0.3, 0.2, 0.6) \\ (0.2, 0.6, 0.3) & (0.3, 0.2, 0.6) & (0.2, 0.4, 0.1) \end{pmatrix}.$$

Then clearly, R and S are symmetric. But

$$T_{S\circ R}(a, b) = 0.6 \neq 0.2 = T_{S\circ R}(b, a).$$

Thus $S \circ R$ is not nsymmetric.

The following gives the condition for its being symmetric.

Proposition 4.12. *Let $R, S \in SVNR(X)$. Let R and S be symmetric. Then $S \circ R$ is symmetric if and only if $S \circ R = R \circ S$.*

Proof. Suppose $S \circ R$ is symmetric. Since R and S and are symmetric, by Result 4.7, $R = R^{-1}$ and $S = S^{-1}$. Thus

$S \circ R = (S \circ R)^{-1}$ [By the hypothesis and Result 4.7]
$= R^{-1} \circ S^{-1}$ [By Proposition 3.11]
$= R \circ S$.

Conversely, suppose $S \circ R = R \circ S$. Then
$(S \circ R)^{-1} = R^{-1} \circ S^{-1}$ [By Proposition 3.11]

$= R \circ S$
[Since R and S and are symmetric, $R = R^{-1}$ and $S = S^{-1}$]
$= S \circ R$. [By the hypothesis]
This completes the proof. □

The following is the immediate result of Proposition 4.12.

Corollary 4.13. *If R is symmetric, then R^n is symmetric, for all positive integer n, where $R^n = R \circ R \circ ...$ n times.*

Definition 4.14. (See [25]) $R \in SVNR(X)$ is said to be transitive, if $R \circ R \subset R$, i.e., $R^2 \subset R$.

Proposition 4.15. *Let $R \in SVNR(X)$. If R is transitive, then R^{-1} is so.*

Proof. Let $(x, y) \in X \times X$. Then

$$\begin{aligned} T_{R^{-1}}(x,y) = T_R(y,x) &\geq T_{R \circ R}(y,x) \\ &= \bigvee_{z \in X}(T_R(y,z) \wedge T_R(z,x)) \\ &= \bigvee_{z \in X}(T_{R^{-1}}(z,y) \wedge T_{R^{-1}}(x,z)) \\ &= \bigvee_{z \in X}(T_{R^{-1}}(x,z) \wedge T_{R^{-1}}(z,y)) \\ &= T_{R^{-1} \circ R^{-1}}(x,y). \end{aligned}$$

Similarly, we can prove that

$$I_{R^{-1}}(x,y) \leq I_{R^{-1} \circ R^{-1}}(x,y) \text{ and } F_{R^{-1}}(x,y) \leq F_{R^{-1} \circ R^{-1}}(x,y).$$

Thus the result holds. □

Proposition 4.16. *Let $R \in SVNR(X)$. If R is transitive, then so is R^2.*

Proof. Let $(x, y) \in X \times X$. Then

$$\begin{aligned} T_{R^2}(x,y) &= \bigvee_{z \in X}(T_R(x,z) \wedge T_R(z,y)) \\ &\geq \bigvee_{z \in X}(T_{R^2}(x,z) \wedge T_{R^2}(z,y)) \\ &= T_{R^2 \circ R^2}(x,y). \end{aligned}$$

Similarly, we can see that $I_{R^2}(x,y) \leq I_{R^2 \circ R^2}(x,y)$ and $F_{R^2}(x,y) \leq F_{R^2 \circ R^2}(x,y)$.
Thus the result holds. □

Proposition 4.17. *Let $R, S \in SVNR(X)$. If R and S are transitive, then $R \cap S$ is transitive.*

Proof. Let $(x, y) \in X \times X$. Then

$$\begin{aligned} T_{(R \cap S) \circ (R \cap S)}(x,y) &= \bigvee_{z \in X}(T_{R \cap S}(y,z) \wedge T_{R \cap S}(z,x)) \\ &= \bigvee_{z \in X}([T_R(x,z) \wedge T_S(x,z)] \wedge [T_R(z,y) \wedge T_S(z,y)]) \\ &= \bigvee_{z \in X}([T_R(x,z) \wedge T_R(z,y)] \wedge [T_S(x,z) \wedge T_S(z,y)]) \\ &= (\bigvee_{z \in X}[T_R(x,z) \wedge T_R(z,y)]) \wedge (\bigvee_{z \in X}[T_S(x,z) \wedge T_S(z,y)]) \\ &= T_{R \circ R}(x,y) \wedge T_{S \circ S}(x,y) \\ &\leq T_R(x,y) \wedge T_S(x,y) \text{ [Since } R \text{ and } S \text{ are transitive]} \\ &= T_{R \cap S}(x,y). \end{aligned}$$

Similarly, we can prove that

$$I_{(R \cap S) \circ (R \cap S)}(x,y) \geq I_{R \cap S}(x,y) \text{ and } F_{(R \cap S) \circ (R \cap S)}(x,y) \geq F_{R \cap S}(x,y).$$

Thus the result holds. □

Remark 4.18. For two single valued neutrosophic transitive relation R and S in X, $R \cup S$ is not transitive, in general.

Example 4.19. Let $X = \{a, b\}$ and consider two SVNRs R and S in X given by following single valed neutrosophic matrices:

$$R = \begin{pmatrix} (0.8, 0.5, 0.4) & (0.6, 0.4, 0.5) \\ (0.7, 0.6, 0.2) & (0.7, 0.6, 0.3) \end{pmatrix}$$

and

$$S = \begin{pmatrix} (0.7, 0.4, 0.2) & (0.4, 0.6, 0.4) \\ (0.5, 0.4, 0.3) & (0.5, 0.4, 0.4) \end{pmatrix}.$$

Then we can easily see that R and S are transitive. On the other hand,

$$R \cup S = \begin{pmatrix} (0.8, 0.4, 0.2) & (0.6, 0.4, 0.4) \\ (0.7, 0.4, 0.2) & (0.7, 0.4, 0.3) \end{pmatrix}.$$

Then $T_{(R\cup S)\circ(R\cup S)}(a, b) = 0.7 \geq 0.6 = T_{R\cup S}(a, b)$. Thus $R \cup S$ is not transitive.

5. Single valurd neutrosophic transitive closure

In this section, we define the concept of the single valued neutrosophic transitive closure of an SVNR and study some of its properties.

Definition 5.1. Let $R \in SVNR(X)$. Then the single valued neutrosophic transitive closure of R, denoted by $\hat{R}$, is defined as:

$$\hat{R} = R \cup R^2 \cup ....$$

The following is the immediate result of Definition 5.1.

Proposition 5.2. *Let $R \in SVNR(X)$. Then*
(1) *$\hat{R}$ is transitive.*
(2) *R is transitive iff $R = \hat{R}$.*

Proposition 5.3. *Let $R,\ S \in SVNR(X)$. If $R \subset S$, then $\hat{R} \subset \hat{S}$.*

Proof. By Definition 5.1, $\hat{R} = R \cup R^2 \cup ...$ and $\hat{S} = S \cup S^2 \cup ....$ Since $R \subset S$, by Proposition 3.11, $R \circ R \subset S \circ R \subset S \circ S$. Then $R^2 \subset S^2$. Thus $R^3 \subset S^3$ and so on. So $\hat{R} \subset \hat{S}$. □

Proposition 5.4. *Let $R, S \in SVNR(X)$. If R is symmetric, then $\hat{R}$ is symmetric.*

Proof. By Corollary 4.13, R^2, R^3, ..., are symmetric. Then by Proposition 4.9, $\hat{R}$ is symmetric. □

Proposition 5.5. *Let $R \in SVNR(X)$. Then $(\hat{R})^{-1} = \hat{R^{-1}}$.*

Proof. $(R^n)^{-1} = (R \circ R \circ ... \circ R)^{-1}$ $n$times
$= R^{-1} \circ R^{-1} \circ ... \circ R^{-1} = (R^{-1})^n = (R^{-1})^n.$

Then

$$\begin{aligned} (\hat{R})^{-1} &= (R \cup R^2 \cup ...)^{-1} \\ &= R^{-1} \cup (R^2)^{-1} \cup ... \\ &= R^{-1} \cup (R^{-1})^2 \cup ... \\ &= \hat{R^{-1}}. \end{aligned}$$

□

Proposition 5.6. *For any $R \in SVNR(X)$, Then $\hat{R}$ is the intersection of all single valued neutrosophic transitive relations containing R.*

Proof. Let $R \in SVNR(X)$ and let

$R^* = \bigcap\{R^T : R^T \text{is a transitive relation containing } R\}$.

Then clearly, R^* is the smallest transitive relation containing R. Since $\hat{R}$ is a transitive relation containing R, $R^* \subset \hat{R}$.

Conversely, let R^T be any transitive relation containing R. Then by Proposition 5.3, $\hat{R} \subset \hat{R^T}$. Since R^T is transitive, by Proposition 5.2, $\hat{R^T} = R^T$. Thus $\hat{R} \subset \hat{R^T}$, for each R^T. So $\hat{R} \subset R^*$. This completes the proof. □

6. Single valued neutrosophic equivalence relation

In this section, we define the concept of a single valued neutrosophic equivalence class and a single valued neutrosophic partition, and we prove that the set of all single valued neutrosophic equivalence classes is a neutrosophic partition and induce the single valued neutrosophic equivalence relation from a single valued neutrosophic partition.

Definition 6.1. $R \in SVNR(X \times X)$ is called a:

(i) tolerance relation on X, if it is reflexive and symmetric,

(ii) similarity (or equivalence) relation on X, if it is reflexive, symmetric and transitive.

(iii) order relation on X, if it is reflexive, anti-symmetric and transitive.

We will denote the set of all tolerance [resp., equivalence and order] relations on X as $SVNT(X)$ [resp., $SVNE(X)$ and $SVNO(X)$].

The following is the immediate result of Propositions 4.3, 4.9 and 4.17.

Proposition 6.2. *Let $(R_j)_{j\in J} \subset SVNT(X)$ [resp., $SVNE(X)$ and $SVNO(X)$]. Then $\bigcap R_j \in SVNT(X)$ [resp., $SVNE(X)$ and $SVNO(X)$].*

Proposition 6.3. *Let $R \in SVNE(X)$. Then $R = R \circ R$.*

Proof. From Definition 4.14, it is clear that $R \circ R \subset R$.

Let $(x, y) \in X \times X$. Then

$$\begin{aligned} T_{R\circ R}(x,y) &= \textstyle\bigvee_{z\in X}(T_R(x,z) \wedge T_R(z,y)) \\ &\geq T_R(x,x) \wedge T_R(x,y) \\ &= 1 \wedge T_R(x,y) \text{ [Since } R \text{ is reflexive]} \\ &= T_R(x,y) \end{aligned}$$

and

$$\begin{aligned} I_{R\circ R}(x,y) &= \textstyle\bigwedge_{z\in X}(I_R(x,z) \vee I_R(z,y)) \\ &\leq I_R(x,x) \vee I_R(x,y) \\ &= 0 \vee I_R(x,y) \text{ [Since } R \text{ is reflexive]} \\ &= I_R(x,y). \end{aligned}$$

Similarly, $F_{R\circ R}(x,y) \leq F_R(x,y)$. Thus $R \circ R \supset R$. So $R \circ R = R$. □

Definition 6.4. Let $A \in SVNS(X)$. Then A is said to be normal, if $\bigvee_{x\in X} T_A(x) = 1$, $\bigwedge_{x\in X} I_A(x) = \bigwedge_{x\in X} F_A(x) = 0$.

Definition 6.5. Let $R \in SVNE(X)$ and let $x \in X$. Then the single valued neutrosophic equivalence class of x by R, denoted by R_x, is a SVNS in X defined as:

$$R_x = (T_{R_x}, I_{R_x}, F_{R_x}),$$

where $T_{R_x},\ I_{R_x},\ F_{R_x} : X \to [0,1]$ are mappings
and

$$T_{R_x}(y) = T_R(x,y), I_{R_x}(y) = I_R(x,y), F_{R_x}(y) = F_R(x,y), \text{ for each } y \in X.$$

We will denote the set of all single valued neutrosophic equivalence class by R as X/R and it will be called the single valued neutrosophic quotient set of X by R.

Proposition 6.6. *Let $R \in SVNE(X)$ and let $x, y \in X$. Then*
(1) *R_x is normal, in fact, $R_x \neq 0_N$,*
(2) *$R_x \cap R_y = 0_N$ iff $R(x,y) = (0,1,1)$,*
(3) *$R_x = R_y$ iff $R(x,y) = (1,0,0)$.*

Proof. (1) Since R is reflexive,

$$T_R(x,x) = T_{R_x}(x) = 1,\ I_R(x,x) = I_{R_x}(x) = 0 \text{ and } F_R(x,x) = F_{R_x}(x) = 0.$$

Thus $\bigvee_{y \in X} T_{R_x}(y) = 1, \bigwedge_{y \in X} I_{R_x}(y) = 0$ and $\bigwedge_{y \in X} F_{R_x}(y) = 0$. So R_x is normal. Moreover, $R_x = (1,0,0) \neq (0,1,1) = 0_N(x)$. Hence $R_x \neq 0_N$.

(2) Suppose $R_x \cap R_y = 0_N$ and let $z \in X$. Then

$$\begin{aligned} 0 &= T_{R_x \cap R_y}(z) \\ &= T_{R_x}(z) \wedge T_{R_y}(z) \\ &= T_R(x,z) \wedge T_R(y,z) \text{ [By Definition 6.5]} \\ &= T_R(x,z) \wedge T_R(z,y) \text{ [Since } R \text{ is symmetric]} \end{aligned}$$

and

$$\begin{aligned} 1 &= I_{R_x \cup R_y}(z) \\ &= I_{R_x}(z) \vee I_{R_y}(z) \\ &= I_R(x,z) \vee I_R(y,z) \text{ [By Definition 6.5]} \\ &= I_R(x,z) \vee FIR(z,y). \text{ [Since } R \text{ is symmetric]} \end{aligned}$$

Thus

$$\begin{aligned} 0 &= \textstyle\bigvee_{z \in X} (T_R(x,z) \wedge T_R(z,y)) \\ &= T_{R \circ R}(x,y) \\ &= T_R(x,y) \text{ [By Proposition 6.3]} \end{aligned}$$

and

$$\begin{aligned} 1 &= \textstyle\bigwedge_{z \in X} (I_R(x,z) \vee I_R(z,y)) \\ &= I_{R \circ R}(x,y) \\ &= I_R(x,y) \text{ [By Proposition 6.3]}. \end{aligned}$$

Similarly, $F_R(x,y) = 1$. So $R(x,y) = (0,1,1)$.

The sufficient condition is easily proved.

(3) Suppose $R_x = R_y$ and let $z \in X$. Then $R(x,z) = R(y,z)$. In particular, $R(x,y) = R(y,y)$. Since R is reflexive, $R(x,y) = (1,0,0)$.

Conversely, suppose $R(x,y) = (1,0,0)$ and let $z \in X$. Since R is transitive, $R \circ R \subset R$. Then

$$T_R(x,y) \wedge T_R(y,z) \leq T_R(x,z),$$
$$I_R(x,y) \vee I_R(y,z) \geq I_R(x,z),$$
$$F_R(x,y) \vee F_R(y,z) \geq F_R(x,z).$$

Since $R(x,y) = (1,0,0)$, $T_R(x,y) = 1$ and $I_R(x,y) = F_R(x,y) = 0$. Thus

$$T_R(y,z) \leq T_R(x,z),\ I_R(y,z) \geq I_R(x,z),\ F_R(y,z) \geq F_R(x,z).$$

So $T_{R_y}(z) \leq T_{R_x}(z),\ I_{R_y}(z) \geq I_{R_x}(z), F_{R_y}(z) \geq F_{R_x}(z)$. Hence $R_y \subset R_x$.

Similarly, we can see that $R_x \subset R_y$. Therefore $R_x = R_y$. □

Definition 6.7. Let $\Sigma = (A_j)_{j\in J} \subset SVNS(X)$. Then Σ is called a single valued neutrosophic partition of X, if it satisfies the followings:
(i) A_j is normal, for each $j \in J$,
(ii) either $A_j = A_k$ or $A_j \neq A_k$, for any $j, k \in J$,
(iii) $\bigcup_{j\in J} A_j = 1_N$.

The following is the immediate result of Proposition 6.6 and Definition 6.7.

Corollary 6.8. *Let $R \in SVNE(X)$. Then X/R is a single valued neutrosophic partition of X.*

Proposition 6.9. *Let Σ be a single valued neutrosophic partition of X. We define $R(\Sigma) = (T_{R(\Sigma)}, I_{R(\Sigma)}, F_{R(\Sigma)})$ as: for each $(x, y) \in X \times X$,*

$$T_{R(\Sigma)}(x,y) = \bigvee_{A\in\Sigma} [T_A(x) \wedge T_A(y)],$$

$$I_{R(\Sigma)}(x,y) = \bigwedge_{A\in\Sigma} [I_A(x) \vee I_A(y)],$$

$$F_{R(\Sigma)}(x,y) = \bigwedge_{A\in\Sigma} [F_A(x) \vee F_A(y)],$$

where $T_{R(\Sigma)}, I_{R(\Sigma)}, F_{R(\Sigma)} : X \times X \to [0,1]$ are mappings.
Then $R(\Sigma) \in SVNE(X)$.

Proof. Let $x \in X$. Then by Definition 6.7 (iii),

$$T_{R(\Sigma)}(x,x) = \bigvee_{A\in\Sigma} (T_A(x) \wedge T_A(x) = \bigvee_{A\in\Sigma} (T_A(x) = 1$$

and

$$I_{R(\Sigma)}(x,y) = \bigwedge_{A\in\Sigma} (I_A(x) \vee I_A(y) = \bigwedge_{A\in\Sigma} (I_A(x) = 0 = F_{R(\Sigma)}(x,y).$$

Thus $R(\Sigma)$ is reflexive.

From the definition of $R(\Sigma)$, it is clear that $R(\Sigma)$ is symmetric.

Let $(x, y) \in X \times X$. Then

$T_{R(\Sigma)\circ R(\Sigma)}(x,y)$
$= \bigvee_{z\in X}[T_{R(\Sigma)}(x,z) \wedge T_{R(\Sigma)}(z,y)]$
$= \bigvee_{z\in X}[\bigvee_{A\in\Sigma}(T_A(x) \wedge T_A(z)) \wedge \bigvee_{B\in\Sigma}(T_B(z) \wedge T_B(y))]$
$= \bigvee_{z\in X}[(\bigvee_{A\in\Sigma} T_A(z) \wedge \bigvee_{B\in\Sigma} T_B(z)) \wedge (T_A(x) \wedge T_B(y))]$
$= \bigvee_{z\in X}[(1 \wedge 1) \wedge (T_A(x) \wedge T_B(y))]$ [Since A and B are normal]
$= \bigvee_{z\in X}[T_A(x) \wedge T_B(y)]$
$= T_{R(\Sigma)}(x,y)$.

Similarly, we can prove that $I_{R(\Sigma)\circ R(\Sigma)}(x,y) = I_{R(\Sigma)}(x,y)$ and $F_{R(\Sigma)\circ R(\Sigma)}(x,y) = F_{R(\Sigma)}(x,y)$. Thus $R(\Sigma)$ is transitive. So $R(\Sigma) \in SVNE(X)$. □

Proposition 6.10. *Let $R, S \in SVNE(X)$. Then $R \subset S$ iff $R_x \subset S_x$, for each $x \in X$.*

Proof. Suppose $R \subset S$ and let $x \in X$. Let $y \in X$. Then by the hypothesis,

$$T_{R_x}(y) = T_R(x,y) \leq T_S(x,y) = T_{S_x}(y),$$

$$I_{R_x}(y) = I_R(x,y) \geq I_S(x,y) = I_{S_x}(y),$$

$$F_{R_x}(y) = F_R(x,y) \geq F_S(x,y) = F_{S_x}(y).$$

Thus $R_x \subset S_x$.

The converse can be easily proved. □

Proposition 6.11. *Let $R, S \in SVNE(X)$. Then $S \circ R \in NE(X)$ iff $S \circ R = R \circ S$.*

Proof. Suppose $S \circ R = R \circ S$. Since R and S are reflexive, by Proposition 4.5, $S \circ R$ is reflexive. Since R and S are symmetric, by the hypothesis and Proposition 4.12, $S \circ R$ is symmetric. Then it is sufficient to show that $S \circ R$ is transitive.

$$\begin{aligned}(S \circ R) \circ (S \circ R) &= S \circ (R \circ S) \circ R \text{ [By Proposition 3.11 (2)]}\\ &= S \circ (S \circ R) \circ)\\ &= (S \circ S) \circ (R \circ R)\\ &\subset S \circ R.\end{aligned}$$

Thus $S \circ R$ is transitive. So $S \circ R \in SVNE(X)$.

The converse is immediate. □

Proposition 6.12. *Let $R, S \in SVNE(X)$. If $R \cup S = S \circ R$, then $R \cup S \in SVNE(X)$.*

Proof. Suppose $R \cup S = S \circ R$. Since R and S are reflexive, by Result 4.3 (2), $R \cup S$ is neutrosophic reflexive. Since R and S are symmetric, by the hypothesis and Proposition 4.8, $R \cup S$ is symmetric. Then by the hypothesis, $S \circ R$ is symmetric. Thus by Proposition 4.12, $S \circ R = R \circ S$. So by Proposition 6.11, $S \circ R \in SVNE(X)$. Hence $R \cup S \in SVNE(X)$. □

7. Relationships between a neutrosophic relation and its α-cut

For $T_\alpha, I_\alpha, F_\alpha \in [0,1]$, $\alpha = (T_\alpha, I_\alpha, F_\alpha)$ will be called a single valued neutrosophic value. For two single valued neutrosophic values α and β,

(i) $\alpha \leq \beta$ iff $T_\alpha \leq T_\beta$, $I_\alpha \geq I_\beta$ and $F_\alpha \geq F_\beta$.

(ii) $\alpha < \beta$ iff $T_\alpha < T_\beta$, $I_\alpha > I_\beta$ and $F_\alpha > F_\beta$.

In particular, the form $\alpha = (\alpha, 1-\alpha, 1-\alpha)$ is called a single valued neutrosophic constant and denoted by α^*.

We will denote that set of all single valued neutrosophic values [resp. constant] as **SVNV** [resp. **SVNC**].

Definition 7.1. Let $R \in SVNR(X \times Y)$ and let $\alpha \in$ **SVNV**.

(i) The strong α-level subset or strong α-cut of R, denoted by $[R]_{\bar{\alpha}}$, is an ordinary relation from X to Y defined as:

$$[R]_{\bar{\alpha}} = \{(x,y) \in X \times Y : T_R(x,y) > T_\alpha, I_R(x,y) < I_\alpha, F_R(x,y) < F_\alpha\}.$$

(ii) The α-level subset or α-cut of R, denoted by $[R]_\alpha$, is an ordinary relation from X to Y defined as:

$$[R]_\alpha = \{(x,y) \in X \times Y : T_R(x,y) \geq T_\alpha, I_R(x,y) \leq I_\alpha, F_R(x,y) \leq F_\alpha\}.$$

Definition 7.2. Let $R \in SVNR(X \times Y)$ and let $\alpha^* \in$ **SVNC**.

(i) The strong α^*-level subset or strong α^*-cut of R, denoted by $[R]_{\bar{\alpha^*}}$, is an ordinary relation from X to Y defined as:

$$[R]_{\bar{\alpha^*}} = \{(x,y) \in X \times Y : T_R(x,y) > \alpha, I_R(x,y) < 1-\alpha, F_R(x,y) < 1-\alpha\}.$$

(ii) The α^*-level subset or α^*-cut of R, denoted by $[R]_{\alpha^*}$, is an ordinary relation from X to Y defined as:

$$[R]_{\alpha^*} = \{(x,y) \in X \times Y : T_R(x,y) \geq \alpha, I_R(x,y) \leq 1-\alpha, F_R(x,y) \leq 1-\alpha\}.$$

Example 7.3. In Example 3.3,

$[R]_{(0.2,0\bar{.3},0.1)}$
$= \{(x,y) \in X \times X : T_R(x,y) \geq 0.2, I_R(x,y) \geq 0.3, F_R(x,y) \leq 0.1\} = \phi$,
$[R]_{(0.2,0.3,0.1)} = \{(c,c)\}$, and $[R]_{(0.2,0\bar{.3},0.1)} = \phi$,
$[R]_{(0.2,0\bar{.3},0.8)} = \{(c,c)\}$,
$[R]_{(0.2,0.3,0.8)} = \{(a,a),(a,c),(c,c)\}$.
$[R]_{0.2^*} = [R]_{(0.2,0.2,0.9)} = \{(a,a),(c,c)\} = [R]_{0.\bar{2}^*}$.

Proposition 7.4. *Let $R, S \in SVNR(X \times Y)$ and let $\alpha, \beta \in$ **SVNV**.*
(1) *If $R \subset S$, then $[R]_\alpha \subset [S]_\alpha$ and $[R]_{\bar{\alpha}} \subset [S]_{\bar{\alpha}}$.*
(2) *If $\alpha \leq \beta$, then $[R]_\beta \subset [R]_\alpha$ and $[R]_{\bar{\beta}} \subset [R]_{\bar{\alpha}}$.*

Proof. (1) Let $(x,y) \in [R]_\alpha$. Then $T_R(x,y) \geq T_\alpha$, $I_R(x,y) \leq I_\alpha$ and $F_R(x,y) \leq F_\alpha$. Since $R \subset S$, $T_R(x,y) \leq T_S(x,y)$, $I_R(x,y) \geq I_S(x,y)$ and $F_R(x,y) \geq F_S(x,y)$. Thus $S_R(x,y) \geq T_\alpha$, $I_S(x,y) \leq I_\alpha$ and $F_S(x,y) \leq F_\alpha$. Hence $[R]_\alpha \subset [S]_\alpha$.
The proof of the second part is similar.
(2) Let $(x,y) \in [R]_\beta$. Then $T_R(x,y) \geq T_\beta$, $I_R(x,y) \leq I_\beta$ and $F_R(x,y) \leq F_\beta$. Since $\alpha \leq \beta$, $T_\alpha \leq T_\beta$, $I_\alpha \geq I_\beta$ and $F_\alpha \geq F_\beta$. Thus $T_R(x,y) \geq T_\alpha$, $I_R(x,y) \leq I_\alpha$ and $F_R(x,y) \leq F_\alpha$. So $(x,y) \in [R]_\alpha$. Hence $[R]_\beta \subset [R]_\alpha$.
The proof of the second part is similar. □

The following is the particular case of the above Proposition.

Corollary 7.5. *Let $R, S \in SVNR(X \times Y)$ and let $\alpha^*, \beta^* \in$ **SVNC**.*
(1) *If $R \subset S$, then $[R]_{\alpha^*} \subset [S]_{\alpha^*}$ and $[R]_{\bar{\alpha^*}} \subset [S]_{\bar{\alpha^*}}$.*
(2) *If $\alpha^* \leq \beta^*$, then $[R]_{\beta^*} \subset [R]_{\alpha^*}$ and $[R]_{\bar{\beta^*}} \subset [R]_{\bar{\alpha^*}}$.*

Proposition 7.6. *Let $R \in SVNR(X \times Y)$.*
(1) *$[R]_r$ is an ordinary relation from X to Y, for each $r \in$ **SVNV**.*
(2) *$[R]_{\bar{r}}$ is an ordinary relation from X to Y, for each $r \in$ **SVNV**, where $T_r \in [0.1)$ and $I_r,\ F_r \in (0,1]$.*
(3) *$[R]_r = \bigcap_{s<r}[R]_s$, for each $r \in$ **SVNV**, where $T_r \in (0,1]$ and $I_r,\ F_r \in [0,1)$.*
(4) *$[R]_{\bar{r}} = \bigcup_{s>r}[R]_{\bar{s}}$, for each $r \in$ **SVNV**, where $T_r \in [0,1)$ and $I_r,\ F_r \in (0,1]$.*

Proof. The proofs of (1) and (2) are clear from Definition 7.1.
(3) From Proposition 7.4, it is obvious that $\{[R]_r : r \in$ **SVNV**$\}$ is a descending family of ordinary relations from X to Y. Let $r \in$ **SVNV** such that $T_r \in (0,1]$ and $I_r,\ F_r \in [0,1)$. Then clearly, $[R]_r \subset \bigcap_{s<r}[R]_s$. Assume that $(x,y) \notin [R]_r$. Then $T_R(x,y) < T_r$ or $I_R(x,y) > I_r$ or $F_R(x,y) > F_r$.
Suppose $T_R(x,y) < T_r$. Then there exists $T_s \in (0,1]$ such that $T_R(x,y) < T_s < T_r$. Thus $(x,y) \notin [R]_s$, i.e. , $(x,y) \notin \bigcap_{s<r}[R]_s$. So $\bigcap_{s<r}[R]_s \subset [R]_r$. Hence $[R]_r = \bigcap_{s<r}[R]_s$.

Suppose $I_R(x,y) > I_r$ or $F_R(x,y) > F_r$. Then each case can be similarly proved.

(4) Also from Proposition 7.4, it is obvious that $\{[R]_{\bar{r}} : r \in \mathbf{SVNV}]\}$ is a descending family of ordinary relations from X to Y. Let $r \in \mathbf{SVNV}$ such that $T_r, I_r \in [0,1)$ and $F_r \in (0,1]$. Then clearly, $[R]_{\bar{r}} \supset \bigcup_{s>r} R_{\bar{s}}$. Assume that $(x,y) \notin [R]_{\bar{r}}$. Then $T_R(x,y) \leq T_r$ or $I_R(x,y) \leq I_r$ or $F_R(x,y) \geq F_r$.

Suppose $T_R(x,y) \leq T_r$. Then there exists $T_s \in [0,1)$ such that $T_R(x,y) \leq T_r < T_s$. Thus $(x,y) \notin [R]_{\bar{s}}$, i.e., $(x,y) \notin \bigcup_{s>r}[R]_{\bar{s}}$. So $\bigcup_{s>r}[R]_{\bar{s}} \subset [R]_{\bar{r}}$. Hence $[R]_{\bar{r}} = \bigcup_{s>r}[R]_{\bar{s}}$.

Suppose $I_R(x,y) \leq I_r$ or $F_R(x,y) \geq F_r$. Then each case can be similarly proved. □

The following is the particular case of the above Proposition.

Corollary 7.7. *Let $R \in SVNR(X \times Y)$.*

(1) *$[R]_{r^*}$ is an ordinary relation from X to Y, for each $r^* \in$ **SVNC**.*

(2) *$[R]_{\bar{r^*}}$ is an ordinary relation from X to Y, for each $r^* \in$ **SVNC**, where $r \in [0,1)$.*

(3) *$[R]_{r^*} = \bigcap_{s^*<r^*}[R]_{s^*}$, for each $r^* \in$ **SVNV**, where $r \in (0,1]$.*

(4) *$[R]_{\bar{r^*}} = \bigcup_{s^*>r^*}[R]_{\bar{s^*}}$, for each $r^* \in$ **SVNC**, where $r \in [0,1)$.*

Proposition 7.8. *Let X, Y be non-empty sets and let $\{R^r : r \in [0,1]\}$ be a non-empty descending family of ordinary relations from X to Y such that $R^0 = X \times Y$.*

(1) *We define $T_R, I_R, F_R : X \times Y \to [0,1]$ as follows: for each $(x,y) \in X \times Y$,*

$$\begin{aligned} T_R(x,y) &= \bigvee\{r \in [0,1] : (x,y) \in R^r\}, \\ I_R(x,y) &= F_R(x,y) \\ &= \bigwedge\{r \in [0,1] : (x,y) \notin R^r\} \\ &= \bigwedge\{(1-r) \in [0,1] : (x,y) \in R^r\} \\ &= 1 - \bigvee\{r \in [0,1] : (x,y) \in R^r\}. \end{aligned}$$

Then $R \in SVNR(X \times Y)$.

(2) *For each $r \in (0,1]$, if $R^r = \bigcap_{s<r} R^s$, then $[R]_{r^*} = R^r$.*

(3) *For each $r \in [0,1)$, if $R^r = \bigcup_{s>r} R^s$, then $[R]_{\bar{r^*}} = R^r$.*

In the above proposition, R is called the single valued neutrosophic relation from X to Y induced by $\{R^r : r \in [0,1]\}$.

Proof. (1) It is obvious from the definition of R.

(2) Suppose $R^r = \bigcap_{s<r} R^s$, for each $r \in (0,1]$ and let $(x,y) \in R^r$. Then

$$T_R(x,y) = \bigvee\{r \in [0,1] : (x,y) \in R^r\} \geq r$$

and

$$I_R(x,y) = F_R(x,y) = 1 - \bigvee\{r \in [0,1] : (x,y) \notin R^r\} \leq 1 - r.$$

Thus $(x,y) \in R_r$. So $R^r \subset [R]_{r^*}$, for each $r \in (0,1]$.

Now let $(x,y) \in [R]_{r^*}$. Then $T_R(x,y) \geq r, I_R(x,y) \leq 1-r, F_R(x,y) \leq 1-r$, say $T_R(x,y) \geq r$. Thus by the definition of R,

$$T_R(x,y) = \bigvee\{k \in [0,1] : (x,y) \in R^k\} = s \geq r.$$

Let $\epsilon > 0$. Then there exists $k \in (0,1]$ such that $s - \epsilon < k$ and $(x,y) \in R^k$. Thus $r - \epsilon < s - \epsilon < k$ and $(x,y) \in R^k$. So $(x,y) \in R^{r-\epsilon}$. Since $\epsilon > 0$ is arbitrary, by the hypothesis, $(x,y) \in R^r$. Hence $[R]_{r^*} \subset R^r$. Therefore $[R]_{r^*} = R^r$, for each $r \in (0,1]$.

(3) By the similar argument of the proof of (2), it is proved. □

The following is the immediate result of Corollary 7.7 and Proposition 7.8

Corollary 7.9. *Let X, Y be non-empty sets, let $R \in SVNR(X \times Y)$ and let $\{[R]_{r^*} : r \in [0,1]\}$ be a family of all ordinary relations from X to Y. We define mappings $T_S, I_S, F_S : X \times Y \to]^-0, 1^+[$ as follows: for each $(x,y) \in X \times Y$,*

$$T_S(x,y) = \bigvee\{r \in [0,1] : (x,y) \in [R]_{r^*}\},$$
$$I_S(x,y) = F_S(x,y) = 1 - \bigvee\{r \in [0,1] : (x,y) \in [R]_{r^*}\}.$$

Then $S \in SVNR(X \times Y)$ and $R = S$.

From the above corollary, we have the following.

Corollary 7.10. *Let X, Y be non-empty sets and let , $R, S \in SVNR(X \times Y)$. Then $R = S$ iff $[R]_{r^*} = [S]_{r^*}$, for each $r \in [0,1]$, or alternatively, iff $[R]_{\bar{r^*}} = [S]_{\bar{r^*}}$, for each $r \in [0,1]$.*

Definition 7.11. Let X, Y be non-empty sets, let R be an ordinary relation from X to Y and let $R_N \in SVNR(X \times Y)$. Then R_N is said to be compatible with R, if $R = S(R_N)$, where $S(R_N) = \{(x,y) : T_{R_N}(x,y) > 0, I_{R_N}(x,y) < 1, F_{R_N}(x,y) < 1\}$.

Example 7.12. (1) Let X, Y be non-empty sets, let $\phi_{X \times Y}$ be the ordinary empty relation from X to Y and let $0_{N,X \times Y}$ be the single valued neutrosophic empty relation from X to Y defined by $0_{N,X \times Y} = (0,1,1)$, for each $x \in X$. Then clearly, $S(0_{N,X \times Y}) = \phi_{X \times Y}$. Thus $0_{N,X \times Y}$ is compatible with $\phi_{X \times Y}$.

(2) Let X, Y be non-empty sets, let $X \times Y$ be the whole ordinary relation from X to Y and let $1_{N,X \times Y}$ be the single valued neutrosophic whole relation from X to Y defined by $0_{N,X \times Y} = (1,0,0)$, for each $x \in X$. Then clearly, $S(1_{N,X \times Y}) = X \times Y$. Thus $1_{N,X \times Y}$ is compatible with $X \times Y$.

(3) Let X, Y be non-empty sets, let $r \in (0,1)$ be fixed. We define the mappings $T_R, I_R, F_R : X \times Y \to [0,1]$ as follows: for each $(x,y) \in X \times Y$,

$$T_R(x,y) = r, \ I_R(x,y) = F_R(x,y) = 1 - r.$$

Then clearly, $R \in SVNR(X \times Y)$ and $S(R) = \bigcap_{r^* \in SVNC}[R]_{r^*}$. Thus R is compatible with $\bigcap_{r^* \in SVNC}[R]_{r^*}$.

From the following result, every ordinary relation can be consider as a single valued neutrosophic relation.

Proposition 7.13. *Let X, Y be non-empty sets, let R be an ordinary relation from X to Y and let $r \in (0,1]$. Then there exists $R_{r^*} \in SVNR(X \times Y)$ such that R_{r^*} is compatible with R and $[R_{r^*}]_{r^*} = R$.*

In this case, R_{r^*} will be called an r^*-th single valued neutrosophic relation from X to Y.

Proof. We define the mappings $T_R, I_R, F_R : X \times Y \to [0,1]$ as follows: for each $(x,y) \in X \times Y$,

$$T_{R_{r^*}}(x,y) = \begin{cases} r & \text{if} \quad (x,y) \in R \\ 0 & \text{if} \quad (x,y) \notin R, \end{cases}$$

$$I_{R_{r^*}}(x,y) = F_{R_{r^*}}(x,y) = \begin{cases} 1 - r & \text{if} \quad (x,y) \in R \\ 1 & \text{if} \quad (x,y) \notin R. \end{cases}$$

Then clearly, $R_{r^*} \in SVNR(X \times Y)$ and $[R_{r^*}]_{r^*} = R$. Moreover, by the definition of R_{r^*}, $S(R_{r^*}) = R$. Thus R_{r^*} is compatible with R. □

The following is the immediate result of Definitions 3.5 and 7.1.

Proposition 7.14. *Let $R, S \in SVNR(X \times Y)$ and let $\alpha \in$ **SVNV**. Then*
(1) $[R \cup S]_\alpha = [R]_\alpha \cup [S]_\alpha$, $[R \cup S]_{\bar{\alpha}} = [R]_{\bar{\alpha}} \cup [S]_{\bar{\alpha}}$,
(2) $[R \cap S]_\alpha = [R]_\alpha \cap [S]_\alpha$, $[R \cap S]_{\bar{\alpha}} = [R]_{\bar{\alpha}} \cap [S]_{\bar{\alpha}}$.

The following is the immediate result of Definition 7.2 and Proposition 7.14.

Corollary 7.15. *Let $R, S \in SVNR(X \times Y)$ and let $\alpha^* \in$ **SVNC**. Then*
(1) $[R \cup S]_{\alpha^*} = [R]_{\alpha^*} \cup [S]_{\alpha^*}$, $[R \cup S]_{\bar{\alpha^*}} = [R]_{\bar{\alpha^*}} \cup [S]_{\bar{\alpha^*}}$,
(2) $[R \cap S]_{\alpha^*} = [R]_{\alpha^*} \cap [S]_{\alpha^*}$, $[R \cap S]_{\bar{\alpha^*}} = [R]_{\bar{\alpha^*}} \cap [S]_{\bar{\alpha^*}}$.

From Definitions 4.2, 4.6 and 7.1 it is clear that $R \in SVNR(X)$ is reflexive [resp. symmetric], then $[R]_\alpha$ and $[R]_{\bar{\alpha}}$ are ordinary reflexive [resp. symmetric] on X, for each $\alpha \in$ **SVNV**.

Proposition 7.16. *Let $R \in SVNR(X \times Y)$ and let $\alpha \in$ **SVNV**. If R is transitive, then $[R]_\alpha$ and $[R]_{\bar{\alpha}}$ are ordinary transitive on X.*

Proof. Suppose R is transitive. Then $R \circ R \subset R$, i.e., $T_{R \circ R} \subset T_R$, $I_{R \circ R} \supset I_R$ and $F_{R \circ R} \supset F_R$. Let $(x, z) \in [R]_\alpha \circ [R]_\alpha$. Then there exists $y \in X$ such that $(x, z), (z, y) \in [R]_\alpha$. Thus

$$T_R(x, z) \geq T_\alpha,\ I_R(x, z) \leq I_\alpha,\ F_R(x, z) \leq F_\alpha$$

and

$$T_R(z, y) \geq T_\alpha,\ I_R(z, y) \leq I_\alpha,\ F_R(z, y) \leq F_\alpha.$$

So $T_R(x, z) \wedge T_R(z, y) \geq T_\alpha, I_R(x, z) \vee I_R(z, y) \leq I_\alpha, F_R(x, z) \vee F_R(z, y) \leq F_\alpha$.
Since $R \circ R \subset R$, $T_R(x, y) \geq T_R(x, z) \wedge T_R(z, y), I_R(x, y) \leq I_R(x, z) \vee I_R(z, y)$, $F_R(x, y) \leq F_R(x, z) \wedge F_R(z, y)$. Hence $T_R(x, y) \geq T_\alpha, I_R(x, y) \leq I_\alpha, F_R(x, y) \leq F_\alpha$, i.e., $(x, y) \in [R]_\alpha$. Therefore $[R]_\alpha$ is ordinary transitive.
The prof of the second part is similar. □

From Definitions 4.2, 4.6 and 7.2 it is clear that $R \in NR(X)$ is reflexive [resp. symmetric], then $[R]_{\alpha^*}$ and $[R]_{\bar{\alpha^*}}$ are ordinary reflexive [resp. symmetric] on X, for each $\alpha^* \in$ **NCV**. Moreover, we obtain the following from Proposition 7.16.

Corollary 7.17. *Let $R \in NR(X \times Y)$ and let $\alpha^* \in$ **NCV**. If R is transitive, then $[R]_{\alpha^*}$ and $[R]_{\bar{\alpha^*}}$ are ordinary transitive on X.*

The followings are the immediate results of 4.2, 4.6, Proposition 7.16 and Corollary 7.17.

Corollary 7.18. *Let $R \in SVNE(X)$ and let $\alpha \in$ **SVNV**. Then $[R]_\alpha$ and $[R]_{\bar{\alpha}}$ are ordinary equivalence relation on X*

Corollary 7.19. *Let $R \in SVNE(X)$ and let $\alpha^* \in$ **SVNC**. Then $[R]_{\alpha^*}$ and $[R]_{\bar{\alpha^*}}$ are ordinary equivalence relation on X*

8. Conclusions

From now on, we dealt with properties of single valued neutrosophic reflexive, symmetric, transitive relations and single valued neutrosophic equivalence relations. In particular, we defined a single valued neutrosophic equivalence class of a point in a set X modulo a single valued neutrosophic equivalence relation R and a single valued neutrosophic partition of a set X. And we proved that the set of all single valued neutrosophic equivalence classes is a single valued neutrosophic partition and induced the single valued neutrosophic equivalence relation by a single valued neutrosophic partition. However, we did not deal with the quotient of S by R, for any SVNRs R and S such that $R \subset S$ and decomposition of a mapping $f : X \to Y$ by Vneutrosophic relations. Furthermore, we defined α-cut of a SVNR and investigated some relationships between SVNRs and their α-cuts.

In the future, we will solve by the above two problems and deal with single valued neutrosophic relations in a fixed SVNS A.

References

[1] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87–96.

[2] K. T. Atanassov and G. Gargov, Interval-valued intuitionistic fuzzy sets, Fuzzy Sets and Systems 31 (1989) 343–349.

[3] K. T. Atanassov and G. Gargov, Elements of intuitionistic fuzzy logic. Part I, Fuzzy Sets and Systems 95 (1998) 39–52.

[4] M. Bhowmik and M. Pal, Intuiyionistic neutrosophic set relations and some of its properties, Journal of Information and Computing Science 5 (3) (2010) 183–192.

[5] M. K. Chakraborty and Mili Das, Studies in fuzzy relations over fuzzy subsets, Fuzzy sets and Systems 9 (1983) 79–89.

[6] M. K. Chakraborty and Mili Das, On fuzzy equivalence I, Fuzzy sets and Systems 11 (1983) 185–193.

[7] M. K. Chakraborty and Mili Das, On fuzzy equivalence II, Fuzzy sets and Systems 11 (1983) 299–307.

[8] M. K. Chakraborty and Subhra Sarkar, Fuzzy antisymmetry and order, Fuzzy sets and Systems 21 (1987) 169–182.

[9] K. A. Dib and Nabil L. Youssef, Fuzzy Cartesian product, fuzzy relations and fuzzy functions, Fuzzy sets and Systems 41 (1991) 299–315.

[10] K. C. Gupta and R. K. Gupta, Fuzzy equivalence relation redefined, Fuzzy sets and Systems 79 (1996) 227–233.

[11] K. Hur, S. Y. Jang and H. W. Kang, Intuitionistic H-fuzzy relations, International Journal of Mathematics and Mathematical Sciences, 2005 (17) (2005) 2723–2734.

[12] K. Hur, P. K. Lim, J. G. Lee, J. H. Kim, The category of neutrosophic sets, To be submitted in AFMI.

[13] A. Kaufmann, Introduction to the Theory of Fuzzy Subsets, Vol 1, Academic Press, New York 1975.

[14] K. C. Lee, Fuzzy equivalence relations and fuzzy functions, International Journal of Fuzzy Logic and Intelligent Systems 9 (1) (2009) 20–29.

[15] Peide Liu, Junlin Liu and Jose M. Merigo, Partitioned Heronian means based on linguistic intuitionistic fuzzy numbers for dealing with multi-attribute group decision making, Applied Soft Computing 62 (2018) 395–422.

[16] Peide Liu and Honggang Li, Some intuitionistic fuzzy interaction partitioned Bonferroni mean operators and their application to multi-attribute group decision making, Inform. Sci. 411 (2017) 98ÔÇô-121.

[17] Peide Liu and Honggang Li, Interval-valued intuitionistic fuzzy power Bonferroni aggregation operators and their application to group decision making, Cognitive Computation 9 (4) (2017) 494–512.
[18] V. Murali, Fuzzy equivalence relations, Fuzzy sets and Systems 30 (1989) 155–163.
[19] A. A. Salama and F. Smarandache, Neutrosophic Crisp Set Theory, The Educational Publisher Columbus, Ohio 2015.
[20] A. A. Salama, Said Broumi and F. Smarandache, Some Types of Neutrosophic Crisp Sets and Neutrosophic Crisp relations, I. J. Information Engineering and Electronic Business 2014 (http://www.mecs-press.org/).
[21] F. Smarandache, n-valued refined neutrosophic logic and its applications to Physics , Progress in Physics 4 (2013) 143–146.
[22] F. Smarandache, A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics (sixth edition), InfoLearnQuest, USA 2007. http://fs.gallup.unm.edu/eBook-neutrosophics6.pdf (edition online)
[23] F. Smarandache, Neutrosophy. Neutrisophic Property, Sets, and Logic, American Research Press, Rehoboth, USA 1998.
[24] H. Wang, F. Smarandache, Y. Q. Zhang and R. Sunderraman, Single valued neutrosophic sets, Multispace Multistruct 4 (2010) 410–413.
[25] Hai-Long Yang, Zhi-Lian Guo and Xiuwu Liao, On single valued neutrosophic relations, Journal of Intelligent and Fuzzy Systems 30 (2) (2016) 1045–1056.
[26] J. Ye, A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets, Journal of Intelligent and Fuzzy Systems 26 (5) (2014) 2450–2466.
[27] L. A. Zadeh, Similarity relations and fuzzy orderings, Inform. Sci. 3 (1971) 177–200.
[28] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.

J. Kim (junhikim@wku.ac.kr)
Department of Mathematics Education, Wonkwang University, 460, Iksan-daero, Iksan-Si, Jeonbuk 54538, Korea

P. K. Lim (pklim@wku.ac.kr)
Division of Mathematics and Informational Statistics, Institute of Basic Natural Science, Wonkwang University, 460, Iksan-daero, Iksan-Si, Jeonbuk 54538, Korea

J. G. Lee (jukolee@wku.ac.kr)
Division of Mathematics and Informational Statistics, Institute of Basic Natural Science, Wonkwang University, 460, Iksan-daero, Iksan-Si, Jeonbuk 54538, Korea

K. Hur (kulhur@wku.ac.kr)
Division of Mathematics and Informational Statistics, Institute of Basic Natural Science, Wonkwang University, 460, Iksan-daero, Iksan-Si, Jeonbuk 54538, Korea