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SINGLE VALUED NEUTROSOPHIC TREES

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Abstract. The edge connectivity plays important role in computer network problems and path problems. In this paper, we introduce special types of single valued neutrosophic (SVN) bridges, single valued neutrosophic cut-vertices, single valued neutrosophic cycles and single valued neutrosophic trees in single valued neutrosophic graphs, and introduced some of their properties.

Keywords: SVN-cycles, SVN-trees, SVN-bridges, SVN-cut-vertices and SVN-levels.

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1. Introduction

Neutrosopic sets are introduced by Smarandache [4] which are the generalization of fuzzy sets and intuitionistic fuzzy sets. The Neutrosophic sets has many applications in medical, management sciences, life sciences and engineering, graph theory, robotics, automata theory and computer science. The measure of SVNSs introduced by Sahin and kucuk [5]. Single valued neutrosophic graphs was introduced by Brumi, Talea, Bakali and Smarandache [1, 2, 3]. The degree, order and size of fuzzy graph discussed Gani [10]. Cycles and co-cycles was introduced by Mordeson and Nair [9]. Sunitha and Vijayakumar [6] gives the definition of complement of a fuzzy graph for understand and utilize in general concept of fuzzy graphs with respect to complement properties. Sunitha and Vijay kumar [7, 8] also introduced properties of fuzzy cut vertex, fuzzy tree and fuzzy bridges and obtained many results on metric spaces.
Single valued neutrosophic graphs have many applications in computer science such as image segmentation, clustering and network problems. In this paper we discuss the concepts of single valued neutrosophic bridges, single valued neutrosophic cycles, single valued neutrosophic trees, single valued neutrosophic firm and single valued neutrosophic blocks on the basis of weight of edge connectivity.

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2. Preliminaries

In this section, we recall definitions to understand the concepts of SVN trees.

Definition 2.1. [4] *Let Z be a crisp set, A single valued neutrosophic set (SVNS) C is characterized by truth membership function $T_C(p)$, an indeterminacy membership function $I_C(p)$ and a falsity membership function $F_C(p)$. For every point $p \in Z$; $T_C(p)$, $I_C(p)$, $F_C(p)$ $\in$ [0, 1].*

Definition 2.2. [1, 2, 3] *A single valued neutrosophic graph (SVNG) is a pair $G = (C, D)$ of a crisp graph $G^* = (V, E)$, where C is SVNS on V and D is SVNS on E such that*

$$T_D(p, q) \leq \min(T_C(p), T_C(q))$$

$$I_D(p, q) \geq \max(I_C(p), I_C(q))$$

$$F_D(p, q) \geq \max(F_C(p), F_C(q))$$

where

$$0 \leq T_D(p, q) + I_D(p, q) + F_D(p, q) \leq 3$$

$\forall x, y \in V$.

Definition 2.3. [1, 2, 3] *Let $G = (C, D)$ be a SVNG of a crisp graph $G^* = (V, E)$, G is said to be complete SVNG, if*

$$T_D(p, q) = \min(T_C(p), T_C(q))$$

$$I_D(p, q) = \max(I_C(p), I_C(q))$$

$$F_D(p, q) = \max(F_C(p), F_C(q))$$

$\forall p, q \in V$.

Definition 2.4. [1, 2, 3] *A path P in a SVNG $G = (A, B)$ is a sequence of distinct vertices $p_1, p_2, p_3, \ldots, p_m$ such that $T_B(p_jp_{j+1}) > 0$, $I_B(p_jp_{j+1}) > 0$, $F_B(p_jp_{j+1}) > 0$ for $1 \leq j \leq m$.*

Definition 2.5. [1, 2, 3] *If there is at least one path between every pair of vertices in SVNG $G = (A, B)$ Then G is said to be connected, else G is disconnected.*

Definition 2.6. [1, 2, 3] *The partial SVN subgraph of SVNG $G = (C, D)$ on a crisp graph $G^* = (V, E)$ is a SVNG $H = (C', D')$, such that*
(1) *$C' \subseteq C$, that is $\forall x \in V$*

$$T_{C'}(p) \leq T_C(p),\ I_{C'}(p) \geq I_C(p),\ F_{C'}(p) \geq F_C(p).$$

(2) *$D' \subseteq D$, that is $\forall pq \in E$*

$$T_{D'}(pq) \leq T_D(pq),\ I_{D'}(pq) \geq I_D(pq),\ F_{D'}(pq) \geq F_D(pq).$$

Definition 2.7. [1, 2, 3] *The SVN subgraph of SVNG $G = (C, D)$ of crisp graph $G^* = (V, E)$ is a SVNG $H = (C', D')$ on a $H^* = (V', E')$, such that*
(1) *$C' = C$, that is $\forall p \in V'$*

$$T_{C'}(p) = T_C(p),\ I_{C'}(p) = I_C(p),\ F_{C'}(p) = F_C(p).$$

(2) *$D' = D$, that is $\forall pq \in E$ in the edge set E'*

$$T_{D'}(pq) = T_D(pq),\ I_{D'}(pq) = I_D(pq),\ F_{D'}(pq) = F_D(pq).$$

3. Single valued neutrosophic trees

The concept of connectivity plays an important role in path and network problems, we introduce here the basic concept of bridge, cycle, tree, cut vertex and levels of SVNG.

Definition 3.1. *Let $C = (T_C, I_C, F_C)$ be a SVNS on X, the support of C is denoted and defined by $supp(C) = supp(T_C) \cup supp(I_C) \cup supp(F_C)$, where*

$$supp(T_C) = \{x : x \in X,\ T_C(x) > 0\},$$

$$supp(I_C) = \{x : x \in X,\ I_C(x) > 0\},$$

$$supp(F_C) = \{x : x \in X,\ F_C(x) > 0\}.$$

We call $supp(T_C)$, $supp(I_C)$ and $supp(F_C)$ truth support, indeterminacy support and falsity support respectively.

Definition 3.2. *Let $C = (T_C, I_C, F_C)$ be a SVNS on X, the (ξ, η, ζ)-level subset of C is denoted and defined by $A^{(\xi,\eta,\zeta)} = C^\xi \cup C^\eta \cup C^\zeta$, where*

$$C^\xi = \{x : x \in X,\ T_C(x) \geq \xi\},$$

$$C^\eta = \{x : x \in X,\ I_C(x) \leq \eta\},$$

$$C^\zeta = \{x : x \in X,\ F_C(x) \leq \zeta\}.$$

Definition 3.3. *Let $C = (T_C, I_C, F_C)$ be a SVNS on X, the height of C is denote and defined by $h(C) = (h_T(C), h_I(C), h_F(C))$, where*

$$h_T(C) = \sup\{T_C(x) : x \in X\},$$

$$h_T(C) = \inf\{I_C(x) : x \in X\},$$

$$h_T(C) = \inf\{F_C(x) : x \in X\}.$$

The SVNS C is normal if there is $p \in X$ such that $T_C(p) = 1$, $I_C(p) = 0$ and $F_C(p) = 0$.

Definition 3.4. *Let $C = (T_C, I_C, F_C)$ be a SVNS on X, the depth of A is denote and defined by $d(C) = (d_T(C), d_I(C), d_F(C))$, where*

$$d_T(C) = \inf\{T_C(x) : x \in X\},$$

$$d_T(C) = \sup\{I_C(x) : x \in X\},$$

$$d_T(C) = \sup\{F_C(x) : x \in X\}.$$

Definition 3.5. *The crisp graph of a SVNG $G = (A, B)$ is $G^* = (A^*, B^*)$, where $A^* = supp(A)$ and $B^* = supp(B)$. Let $G^{(\xi,\eta,\zeta)} = (A^{(\xi,\eta,\zeta)}, B^{(\xi,\eta,\zeta)})$ where $\xi, \eta, \zeta \in [0, 1]$,*

$$A^{(\xi,\eta,\zeta)} = \{x : x \in V,\ T_A(x) \geq \xi,\ I_A(x) \leq \eta,\ F_A(x) \leq \zeta\}$$

is the (ξ, η, ζ)-level subset of A and

$$B^{(\xi,\eta,\zeta)} = \{xy : xy \in E,\ T_B(xy) \geq \xi,\ I_B(xy) \leq \eta,\ F_B(xy) \leq \zeta\}$$

is the (ξ, η, ζ)-level subset of B. Note that $G^{(\xi,\eta,\zeta)}$ is a crisp graph.

Definition 3.6. *A bridge in SVNG $G = (A, B)$ is said to be T-bridge, if removing the edge xy decreases the T-strength of connectivity of some two vertices. A bridge in G is said to be I-bridge, if removing the edge xy increases the I-strength of connectedness of two vertices . A bridge in G is said to be F-bridge, if by removing the edge xy increases the F-strength of connectedness of some two vertices. A bridge in SVNG G is said to be SVN-bridge xy if it is T-bridge, I-bridge and F-bridge.*

Definition 3.7. *Let $G = (A, B)$ be a SVNG on the crisp graph $G^* = (V, E)$, the T-strength of connectedness between x and y in V is*

$$T_B^\infty(xy) = \sup\{T_B^k(xy) : k = 1, 2, \ldots, n\},$$

$$T_B^\infty(xy) = \sup\{T_B(xv_1)\wedge T_B(v_1v_2)\wedge\ldots\wedge T_B(v_{k-1}y) : x, v_1, v_2, \ldots, v_{k-1}, y \in V, k = 1, 2, \ldots, n\},$$

the I-strength of connectedness between x and y in V is

$$I_B^\infty(xy) = \inf\{I_B^k(xy) : k = 1, 2, \ldots, n\},$$

$$I_B^\infty(xy) = \inf\{I_B(xv_1)\vee I_B(v_1v_2)\vee\ldots\vee I_B(v_{k-1}y) : x, v_1, v_2, \ldots, v_{k-1}, y \in V, k = 1, 2, \ldots, n\}$$

and the F-strength of connectedness between x and y in V is

$$F_B^\infty(xy) = \inf\{F_B^k(xy) : k = 1, 2, \ldots, n\},$$

$$F_B^\infty(xy) = \inf\{F_B(xv_1)\vee F_B(v_1v_2)\vee\ldots\vee F_B(v_{k-1}y) : x, v_1, v_2, \ldots, v_{k-1}, y \in V, k = 1, 2, \ldots, n\}.$$

The T-strength, I-strength and F-strength between x and y in G is denoted by $T_G^\infty(xy)$, $I_G^\infty(xy)$ and $F_G^\infty(xy)$ respectively. Next $T_B^{'\infty}(xy)$, $I_B^{'\infty}(xy)$ and $F_B^{'\infty}(xy)$ denote $T_{G-\{xy\}}^\infty(xy)$, $I_{G-\{xy\}}^\infty(xy)$ and $F_{G-\{xy\}}^\infty(xy)$ where $G - \{xy\}$ is obtained from G by removing the edge xy.

Definition 3.8. *Let $G = (A, B)$ be a SVNG on the crisp graph $G^* = (V, E)$,*
(**i**) *$xy \in E$ is called bridge if xy is bridge of $G^* = (A^*, B^*)$.*
(**ii**) *$xy \in E$ is called SVN-bridge if*

$$T_B^{'\infty}(uv) < T_B^\infty(uv),\ I_B^{'\infty}(uv) > I_B^\infty(uv),\ F_B^{'\infty}(uv) > F_B^\infty(uv)$$

for some $uv \in E$, where $T_B^{'}$, $I_B^{'}$ and $F_B^{'}$ are T_B, I_B and F_B restricted to $V \times V - \{xy, yx\}$.
(**iii**) *$xy \in E$ is called a weak SVN-bridge if there exist $(\xi, \eta, \zeta) \in (0, h(B)]$ such that xy is bridge of $G^{(\xi,\eta,\zeta)}$, where $0 = (0, 0, 0)$.*
(**iv**) *$xy \in E$ is called partial SVN-bridge if xy is bridge $\forall(\xi, \eta, \zeta) \in (d(B), h(B)] \cup \{h(B)\}$.*
(**v**) *$xy \in E$ is called full SVN-bridge if xy is bridge for $G^{(\xi,\eta,\zeta)}$ for all $(\xi, \eta, \zeta) \in (0, h(B)]$, where $0 = (0, 0, 0)$.*

Example 3.1. *Consider the connected SVNG $G = (A, B)$ of a crisp graph $G^* = (V, E)$, where A and B be SVNSs of $V = \{\alpha, \beta, \gamma\}$ and $E = \{\alpha\beta, \beta\gamma, \gamma\alpha\}$ respectively defined in Table. 1. Then it shows connected SVNG has no bridges of any of five types.*

A	T_A	I_A	F_A	B	T_B	I_B	F_B
α	*1.0*	*0.0*	*0.0*	$\alpha\beta$	*0.9*	*0.1*	*0.1*
β	*1.0*	*0.0*	*0.0*	$\beta\gamma$	*0.9*	*0.1*	*0.1*
γ	*1.0*	*0.0*	*0.0*	$\gamma\alpha$	*0.9*	*0.1*	*0.1*

Table 1. SVNSs of SVNG without SVN-bridges.

Example 3.2. *Consider the connected SVNG $G = (A, B)$ of a crisp graph $G^* = (V, E)$, where A and B be SVNSs of $V = \{\alpha, \beta, \gamma, \delta\}$ and $E = \{\alpha\beta, \beta\gamma, \gamma\delta, \delta\alpha\}$ respectively defined in Table. 2. Then, $d(B) = (0.1, 0.5, 0.5)$ and $h(B) = (0.9, 0.1, 0.1)$. Thus $(\xi, \eta, \zeta) \in (0, h(B)]$ which means for $0 < \xi \leq 0.1$, $0 < \eta \leq 0.5$ and $0 < \zeta \leq 0.5$, we obtain $G^{(\xi,\eta,\zeta)} = (V, \{\alpha\beta, \beta\gamma, \gamma\delta, \delta\alpha\})$, for $0.1 < \xi \leq 0.9$, $0 < \eta \leq 0.1$ and $0 < \zeta \leq 0.1$, we obtain $G^{(\xi,\eta,\zeta)} = (V, \{\alpha\delta, \gamma\delta\})$. Thus $\gamma\delta$ is full SVN-bridge and $\delta\alpha$ is partial SVN-bridge but not full SVN-bridge.*

A	T_A	I_A	F_A	B	T_B	I_B	F_B
α	*1.0*	*0.0*	*0.0*	$\alpha\beta$	*0.1*	*0.5*	*0.5*
β	*1.0*	*0.0*	*0.0*	$\beta\gamma$	*0.1*	*0.5*	*0.5*
γ	*1.0*	*0.0*	*0.0*	$\gamma\delta$	*0.9*	*0.1*	*0.1*
δ	*1.0*	*0.0*	*0.0*	$\delta\alpha$	*0.9*	*0.1*	*0.1*

Table 2. SVNSs of SVNG without full SVN-bridge.

Remark 3.1. *Let xy be a bridge in G^* then xy is SVN-bridge if and only if*

$$T_B(xy) > T_B^{'\infty}(xy),\ I_B(xy) < I_B^{'\infty}(xy),\ F_B(xy) < F_B^{'\infty}(xy).$$

Remark 3.2. *The xy is SVN bridge if and only if xy is not weakest bridge of any cycle.*

Proposition 3.1. *The edge xy is SVN-bridge if and only if xy is bridge for G^* and*

$$T_B(xy) = h(T_B),\ I_B(xy) = h(I_B),\ F_B(xy) = h(F_B).$$

Proof. Suppose that xy is full bridge then xy is bridge of $G^{(\xi,\eta,\zeta)}$ for all $(\xi,\eta,\zeta) \in (0,h(B)] = (0,h(T_B)] \times (0,h(I_B)] \times (0,h(F_B)]$. Hence $xy \in B^{h(B)}$ and so

$$T_B(xy) = h(T_B),\ I_B(xy) = h(I_B),\ F_B(xy) = h(F_B)$$

since xy is bridge for $G^{(\xi,\eta,\zeta)}$ for all $(\xi,\eta,\zeta) \in (0,h(B)]$. It follows that xy is bridge for G^*, since $V = A^{d(B)}$ and $E = B^{d(B)}$.
Conversely: Suppose xy is bridge for G^* and

$$T_B(xy) = h(T_B),\ I_B(xy) = h(I_B),\ F_B(xy) = h(F_B).$$

Then $xy \in B^{(\xi,\eta,\zeta)}$ for all $(\xi,\eta,\zeta) \in (0,h(B)]$, thus since xy is bridge for G^*, xy is bridge for $G^{(\xi,\eta,\zeta)}$ for all $(\xi,\eta,\zeta) \in (0,h(B)]$, since each $G^{(\xi,\eta,\zeta)}$ is subgraph of G^*. Hence xy is a full SVN-bridge. □

Proposition 3.2. *If an arc xy is not in the cycle of crisp graph G^*, then the following conditions are equivalent.*
(**i**) $T_B(xy) = h(T_B),\ I_B(xy) = h(I_B),\ F_B(xy) = h(F_B)$.
(**ii**) *xy is partial SVN-bridge.*
(**iii**) *xy is full SVN-bridge.*

Proof. Since xy is not contained in a cycle of G^* and xy is bridge of G^*. Hence by proposition 3.1, $(i) \Leftrightarrow (iii)$ obvious $(iii) \Leftrightarrow (ii)$. Next suppose that (ii) holds, then xy is bridge for $G^{(\xi,\eta,\zeta)}$ for all $(\xi,\eta,\zeta) \in (d(B),h(B)]$ and so $xy \in B^{h(B)}$. Thus $T_B(xy) = h(T_B),\ I_B(xy) = h(I_B),\ F_B(xy) = h(F_B)$, thus (i) holds. □

Remark 3.3. *If xy is a bridge, then xy is weak SVN-bridge and SVN-bridge.*

Proposition 3.3. *An arc xy is SVN-bridge if and only if xy is weak SVN-bridge.*

Proof. Suppose that xy is a weak SVN-bridge, then there exists $(\xi,\eta,\zeta) \in (0,h(B)]$ such that xy is bridge for $G^{(\xi,\eta,\zeta)}$. Hence by removing xy it disconnects $G^{(\xi,\eta,\zeta)}$, thus any path from x to y in G has an edge uv with $T_B(uv) < \xi,\ I_B(uv) > \eta,\ F_B(uv) > \zeta$. Hence by removal of arc xy implies that

$$T_B^{'\infty}(xy) < \xi \leq T_B^{\infty}(xy),\ I_B^{'\infty}(xy) > \eta \geq I_B^{\infty}(xy),\ F_B^{'\infty}(xy) > \zeta \geq F_B^{\infty}(xy).$$

Hence xy is SVN-bridge.
Conversely: Suppose that xy is SVN-bridge, then there is an arc uv such that by removing of xy implies that

$$T_B^{'\infty}(uv) < T_B^{\infty}(uv),\ I_B^{'\infty}(uv) > I_B^{\infty}(uv),\ F_B^{'\infty}(uv) > F_B^{\infty}(uv).$$

Hence xy is on every strongest path joining u and v and in fact $T_B(uv) \geq,\ I_B(uv) \leq$ and $F_B(uv) \leq$ this value. Thus there does not exist a path other than xy connecting x and y in $G^{(T_B(xy),I_B(xy),F_B(xy))}$, else this other path without xy would be of strength $\geq T_B(xy)$, $\leq I_B(xy)$ and $\leq F_B(xy)$ and would be part of a path connecting u and v of strongest length, contrary to fact that xy is on every such path. Hence xy is on every such path. Hence xy is a bridge of $G^{(T_B(xy),I_B(xy),F_B(xy))}$ and

$$0 < T_B(xy) \leq h(T_B),\ 0 < I_B(xy) \leq h(I_B),\ 0 < F_B(xy) \leq h(F_B).$$

Thus $(T_B(xy), I_B(xy), F_B(xy))$ are the desired (ξ, η, ζ). □

Definition 3.9. *A vertex $x \in V$ in G is called T-cut vertex if by removing it decreases the T-strength of connectivity between some pair of nodes. A vertex $x \in V$ in G is called I-cut vertex if by removing it increases the I-strength of connectivity between some pair of nodes. A vertex $x \in V$ in G is called F-cut vertex if by removing it increases the F-strength of connectivity between some pair of nodes. A vertex $x \in V$ is a SVN-cut vertex if it is T-cut vertex, I-cut vertex and F-cut vertex.*

Definition 3.10. *Let $x \in V$,*
(**i**) *The vertex $x \in V$ is called a cut vertex, if x is a cut vertex of $G^* = (A^*, B^*)$.*
(**ii**) *The vertex $x \in V$ is called SVN-cut vertex if $T_B^{'\infty}(uv) < T_B^{\infty}(uv)$, $I_B^{'\infty}(uv) > I_B^{\infty}(uv)$, $F_B^{'\infty}(uv) > F_B^{\infty}(uv)$ for some $u, v \in V$, where $T_B^{'}$, $I_B^{'}$ and $F_B^{'}$ are T_B, I_B and F_B restricted to $V \times V - \{xz, zx : z \in V\}$.*
(**iii**) *The vertex $x \in V$ is called a partial single valued neutrosophic cut vertex if x is a cut vertex for $G^{(\xi,\eta,\zeta)}$ $\forall(\xi, \eta, \zeta) \in (d(B), h(B)] \cup \{h(B)\}$.*
(**iv**) *The vertex $x \in V$ is called a weak SVN-cut vertex if there exists $(\xi, \eta, \zeta) \in (0, h(B)]$ such that x is a cut vertex of $G^{(\xi,\eta,\zeta)}$.*
(**v**) *The vertex $x \in V$ is called a full SVN-cut vertex if x is a cut vertex for $G^{(\xi,\eta,\zeta)}$ if there exists $(\xi, \eta, \zeta) \in (0, h(B)]$.*

Example 3.3. *Consider the connected SVNG $G = (A, B)$ of a crisp graph $G^* = (V, E)$, where A and B be SVNSs of $V = \{\alpha, \beta, \gamma\}$ and $E = \{\alpha\beta, \beta\gamma, \gamma\alpha\}$ respectively defined in Table. 3. Then $d(B) = (0.5, 0.4, 0.4)$ and $h(B) = (0.9, 0.1, 0.1)$. Thus $(\xi, \eta, \zeta) \in (0, h(B)]$ which means for $0 < \xi \leq 0.5$, $0 < \eta \leq 0.4$ and $0 < \zeta \leq 0.4$, we obtain $G^{(\xi,\eta,\zeta)} = (V, \{\alpha\beta, \beta\gamma, \gamma\alpha\})$, for $0.5 < \xi \leq 0.9$, $0 < \eta \leq 0.1$ and $0 < \zeta \leq 0.1$, we obtain $G^{(\xi,\eta,\zeta)} = (V, \{\alpha\beta, \gamma\alpha\})$. Hence α is SVN-cut vertex and a partial SVN-cut vertex but neither a cut vertex nor a full cut vertex.*

A	T_A	I_A	F_A	B	T_B	I_B	F_B
α	*1.0*	*0.0*	*0.0*	$\alpha\beta$	*0.9*	*0.1*	*0.1*
β	*1.0*	*0.0*	*0.0*	$\beta\gamma$	*0.5*	*0.4*	*0.4*
γ	*1.0*	*0.0*	*0.0*	$\gamma\alpha$	*0.9*	*0.1*	*0.1*

Table 3. SVNSs of SVNG with partial SVN-cut vertex.

Remark 3.4. *Let G be a SVNG such that G^* is a cycle, then a vertex is a SVN-cut vertex of G if and only if it is a same vertex of two SVN-bridges.*

Remark 3.5. *If $z \in V$ is a same vertex of at least two SVN-bridges, then z is a SVN cut vertex.*

Remark 3.6. *If G is a complete SVNG, then $T_B^\infty(uv) = T_B(uv)$, $I_B^\infty(uv) = I_B(uv)$ and $F_B^\infty(uv) = F_B(uv)$.*

Remark 3.7. *The complete SVNG has no SVN-cut vertex.*

Definition 3.11. (**i**) *The SVNG G is called a block if G^* is a block.*
(**ii**) *The SVNG G is called a block if it has no single valued neutrosophic cut vertices.*
(**iii**) *The SVNG G is called a weak block if there exists $(\xi, \eta, \zeta) \in (0, h(B)]$, such that $G^{(\xi,\eta,\zeta)}$ is a block.*
(**iv**) *The SVNG G is called a partial SVN-block if $G^{(\xi,\eta,\zeta)}$ is a block $\forall(\xi, \eta, \zeta) \in (d(B), h(B)] \cup \{h(B)\}$.*
(**v**) *The SVNG G is called a full SVN-block if $G^{(\xi,\eta,\zeta)}$ is block $\forall(\xi, \eta, \zeta) \in (0, h(B)]$.*

Example 3.4. *Consider the connected SVNG $G = (A, B)$ of a crisp graph $G^* = (V, E)$, where A and B be SVNSs of $V = \{l, m, n\}$ and $E = \{lm, mn, nl\}$ respectively defined in Table. 4. Then by routine calculations $d(B) = (0.5, 0.4, 0.4)$ and $h(B) = (0.9, 0.1, 0.1)$. Thus $(\xi, \eta, \zeta) \in (0, h(B)]$ which means for $0 < \xi \leq 0.5$, $0 < \eta \leq 0.4$ and $0 < \zeta \leq 0.4$, we obtain $G^{(\xi,\eta,\zeta)} = (V, \{lm, mn, nl\})$, for $0.5 < \xi \leq 0.9$, $0 < \eta \leq 0.1$ and $0 < \zeta \leq 0.1$, we obtain $G^{(\xi,\eta,\zeta)} = (V, \{lm, ln\})$. Hence G is block and a weak block SVN-block, however G is not SVN block since l is SVN-cut vertex of G, also G is not a partial SVN block, since l is cut vertex for $0.5 < \xi \leq 0.9$, $0 < \eta \leq 0.1$ and $0 < \zeta \leq 0.1$.*

A	T_A	I_A	F_A	B	T_B	I_B	F_B
l	1.0	0.0	0.0	lm	0.9	0.1	0.1
m	1.0	0.0	0.0	mn	0.5	0.4	0.4
n	1.0	0.0	0.0	nl	0.9	0.1	0.1

Table 4. SVNSs of SVN-Block.

Example 3.5. *Consider the connected SVNG $G = (A, B)$ of a crisp graph $G^* = (V, E)$, where A and B be SVNSs of $V = \{p, q, r\}$ and $E = \{pq, qr, rp\}$ respectively defined in Table. 5. Then by routine calculations $d(B) = (0.9, 0.1, 0.1)$ and $h(B) = (0.9, 0.1, 0.1)$. Thus $(\xi, \eta, \zeta) \in (0, h(B)]$ which means for $0 < \xi \leq 0.9$, $0 < \eta \leq 0.1$ and $0 < \zeta \leq 0.1$, we obtain $G^{(\xi,\eta,\zeta)} = (V, \{pq, qr, rp\})$, for $0.5 < \xi \leq 0.9$, $0 < \eta \leq 0.1$ and $0 < \zeta \leq 0.1$, we obtain $G^{(\xi,\eta,\zeta)} = (V, \{pq, rp\})$. Hence G is block, a SVN-block and a full SVN-block.*

A	T_A	I_A	F_A	B	T_B	I_B	F_B
p	1.0	0.0	0.0	pq	0.9	0.1	0.1
q	1.0	0.0	0.0	qr	0.5	0.4	0.4
r	1.0	0.0	0.0	rp	0.9	0.1	0.1

Table 5. SVNSs of full SVN-Block.

Definition 3.12. *The connected SVNG G is said to be a firm if*

$$\min\{T_A(x) : x \in V\} \geq \max\{T_B(xy) : xy \in E\},$$

$$\max\{I_A(x) : x \in V\} \leq \min\{I_B(xy) : xy \in E\},$$
$$\max\{F_A(x) : x \in V\} \leq \min\{F_B(xy) : xy \in E\}.$$

Definition 3.13. *Let G be a connected SVNG, then*
(**i**) *The SVNG G is said to be a cycle whenever G^* is a cycle.*
(**ii**) *The SVNG G is said to be a SVN-cycle whenever G^* is a cycle and there is a unique $pq \in E$ such that*

$$T_B(pq) = \min\{T_B(uv) : uv \in E\},$$
$$I_B(pq) = \max\{I_B(uv) : uv \in E\},$$
$$F_B(pq) = \max\{F_B(uv) : uv \in E\}.$$

(**iii**) *The SVNG G is said to be a weak SVN-cycle if there exists $(\xi, \eta, \zeta) \in (0, h(B)]$ such that $G^{(\xi,\eta,\zeta)}$ is a cycle.*
(**iv**) *The SVNG G is called a partial SVN-cycle if $G^{(\xi,\eta,\zeta)}$ is a cycle $\forall(\xi, \eta, \zeta) \in (d(B), h(B)] \cup \{h(B)\}$.*
(**v**) *The SVNG G is called a full SVN-cycle if $G^{(\xi,\eta,\zeta)}$ is cycle $\forall(\xi, \eta, \zeta) \in (0, h(B)]$.*

Remark 3.8. *The SVN-cycle G is partial SVN-cycle if and only if G is a full SVN-cycle.*

Remark 3.9. *The SVNG G is a full SVN-cycle if and only if B is constant on E. and G is a cycle.*

Definition 3.14. *A connected SVNG $G = (A, B)$ is said to be a SVN-tree if it has a SVN spanning subgraph $H = (A, C)$ which is a tree, where for all edges xy not in H satisfying $T_B(xy) < T_C^\infty(xy)$, $I_B(xy) > I_C^\infty(xy)$, $F_B(xy) > F_C^\infty(xy)$.*

Definition 3.15. (**i**) *The SVNG G is called a forest if G^* is a forest.*
(**ii**) *The SVNG $G = (A, B)$ is said to be a SVN-forest if G has a SVN spanning subgraph forest $H = (A, C)$, where all arcs $uv \in E - W$, satisfying $T_B(uv) < T_C^\infty(uv)$, $I_B(uv) > I_C^\infty(uv)$, $F_B(uv) > F_C^\infty(uv)$.*
(**iii**) *The SVNG G is called a weak SVN-forest if $\forall(\xi, \eta, \zeta) \in (0, h(B)]$ such that $G^{(\xi,\eta,\zeta)}$ is a forest.*
(**iv**) *The SVNG G is called a partial SVN-forest if $G^{(\xi,\eta,\zeta)}$ is a forest $\forall(\xi, \eta, \zeta) \in (d(B), h(B)] \cup \{h(B)\}$.*
(**v**) *The SVNG G is called a full SVN-forest if $G^{(\xi,\eta,\zeta)}$ is forest for all $(\xi, \eta, \zeta) \in (0, h(B)]$.*

Example 3.6. *Consider the connected SVNG $G = (A, B)$ of a crisp graph $G^* = (V, E)$, where A and B be SVNSs of $V = \{\alpha, \beta, \gamma, \delta\}$ and $E = \{\alpha\beta, \beta\gamma, \gamma\delta, \delta\alpha\}$ respectively defined in Table. 6. Then $d(B) = (0.5, 0.4, 0.4)$ and $h(B) = (0.9, 0.1, 0.1)$, for $0 < \xi \leq 0.5$, $0 < \eta \leq 0.4$ and $0 < \zeta \leq 0.4$, we obtain $G^{(\xi,\eta,\zeta)} = (V, \{\alpha\beta, \beta\gamma, \gamma\delta, \delta\alpha\})$, for $0.5 < \xi \leq 0.9$, $0 < \eta \leq 0.1$ and $0 < \zeta \leq 0.1$, we obtain $G^{(\xi,\eta,\zeta)} = (V, \{\alpha\beta, \gamma\delta\})$. Hence G is a partial SVN-forest but neither SVN-forest nor full SVN forest.*

A	T_A	I_A	F_A	B	T_B	I_B	F_B
α	1.0	0.0	0.0	$\alpha\beta$	0.9	0.1	0.1
β	1.0	0.0	0.0	$\beta\gamma$	0.5	0.4	0.4
γ	1.0	0.0	0.0	$\delta\alpha$	0.5	0.4	0.4
δ	1.0	0.0	0.0	$\gamma\delta$	0.9	0.1	0.1

Table 6. SVNSs of partial SVN-forest.

Proposition 3.4. *The SVNG G is full SVN-forest if and only if G is forest.*

Proof. Suppose that G is a full SVN-forest, then G^* is a forest.
Conversely: Suppose that G is forest, then G^* is a forest and so must be $G^{(\xi,\eta,\zeta)}$ for all $(\xi,\eta,\zeta) \in (0, h(B)]$, since each $G^{(\xi,\eta,\zeta)}$ is a subgraph of G^*, this completes the proof. □

Proposition 3.5. *The SVNG G is weak SVN-forest if and only if G does not contain a cycle whose edges are of strength $h(B)$.*

Proof. Suppose that G contains a cycle whose edges are of strength $h(B)$, then $G^{(\xi,\eta,\zeta)}$ for $(\xi,\eta,\zeta) \in (0, h(B)]$ that contains this cycle and so is not a forest, thus G is not a weak SVN-forest.
Conversely: Suppose G does not contain a cycle whose edges are of strength $h(B)$, then $G^{h(B)}$ does not contain a cycle and so it is forest. □

Remark 3.10. *If G is a SVN-forest, then G is a weak SVN-forest.*

Theorem 3.1. *Let G be a forest and B is a constant on E if and only if G is a full SVN-forest, G^* and $G^{h(B)}$ have the same number of connected components, and G is a firm.*

Proof. Suppose that G is a forest and B is constant on E, then for all $(\xi,\eta,\zeta) \in (0, h(B)]$, then $G^{(\xi,\eta,\zeta)} = G^*$ and so G is full SVN-forest also G^* and $G^{h(B)}$ have the same number of connected components, clearly G is a firm, since B is constant on E.
Converse part is obvious. □

Corollary 3.1. *The SVNG G is a tree and B is constant on E if and only if G is a full SVN-tree and G is a firm.*

Definition 3.16. (**i**) *The SVNG G is called a tree if G^* is a tree.*
(**ii**) *The SVNG $G = (A, B)$ is said to be a SVN-tree if it has a SVN spanning subgraph $H = (A, C)$ which is a tree, where for all edges $uv \in E - W$, satisfying $T_B(uv) < T_C^{\infty}(uv)$, $I_B(uv) > I_C^{\infty}(uv)$, $F_B(uv) > F_C^{\infty}(uv)$.*
(**iii**) *The SVNG G is called a weak SVN-tree if $\forall(\xi,\eta,\zeta) \in (0, h(B)]$ such that $G^{(\xi,\eta,\zeta)}$ is a tree.*
(**iv**) *The SVNG G is called a partial SVN-tree if $G^{(\xi,\eta,\zeta)}$ is a tree $\forall(\xi,\eta,\zeta) \in (d(B), h(B)] \cup \{h(B)\}$.*
(**v**) *The SVNG G is called a full SVN-tree if $G^{(\xi,\eta,\zeta)}$ is tree for all $(\xi,\eta,\zeta) \in (0, h(B)]$.*

Example 3.7. *Consider the connected SVNG $G = (A, B)$ of a crisp graph $G^* = (V, E)$, where A and B be SVNSs of $V = \{\alpha, \beta, \gamma\}$ and $E = \{\alpha\beta, \beta\gamma, \gamma\alpha\}$ respectively defined in Table. 7. Then $d(B) = (0.5, 0.4, 0.4)$ and $h(B) = (0.9, 0.1, 0.1)$, for $0 < \xi \leq 0.5$, $0 < \eta \leq 0.4$ and $0 < \zeta \leq 0.4$, we obtain $G^{(\xi,\eta,\zeta)} = (V, \{\alpha\beta, \beta\gamma, \gamma\alpha\})$, for $0.5 < \xi \leq 0.9$, $0 < \eta \leq 0.1$ and $0 < \zeta \leq 0.1$, we obtain $G^{(\xi,\eta,\zeta)} = (\{\alpha, \beta\}, \{\alpha\beta\})$. Hence G is a partial SVN-tree but neither SVN-tree nor full SVN-tree.*

A	T_A	I_A	F_A	B	T_B	I_B	F_B
α	1.0	0.0	0.0	$\alpha\beta$	0.9	0.1	0.1
β	1.0	0.0	0.0	$\beta\gamma$	0.5	0.4	0.4
γ	0.5	0.2	0.2	$\gamma\alpha$	0.5	0.4	0.4

Table 7. SVNSs partial SVN-tree.

Remark 3.11. *If G is a SVN-tree, then G is not complete SVNG.*

Remark 3.12. *If G is a SVN-tree, then arcs of spanning subgraph H are the SVN-bridges of G.*

Remark 3.13. *If G is a SVN-tree, then internal vertices of spanning subgraph H are the SVN-cut vertices of G.*

Remark 3.14. *If G is a SVN-tree, then xy is SVN-bridge if and only if $T_B^{\infty}(xy) = T_B(xy)$, $I_B^{\infty}(xy) = I_B(xy)$, $F_B^{\infty}(xy) = F_B(xy)$.*

Remark 3.15. *The SVNG G is a SVN-tree if and only if there is a unique maximum spanning tree of G.*

Remark 3.16. *Let G is a firm, if G is a weak SVN-tree, then G is a SVN-tree.*

Definition 3.17. (**i**) *The SVNG G is called a connected if G^* is a connected.*
(**ii**) *The SVNG $G = (A, B)$ is said to be a SVN connected if G is SVN-block.*
(**iii**) *The SVNG G is called a weak SVN connected if there exists $(\xi, \eta, \zeta) \in (0, h(B)]$ such that $G^{(\xi,\eta,\zeta)}$ is a connected.*
(**iv**) *The SVNG G is called a partial SVN connected if $G^{(\xi,\eta,\zeta)}$ is a connected $\forall(\xi, \eta, \zeta) \in (d(B), h(B)] \cup \{h(B)\}$.*
(**v**) *The SVNG G is called a full SVN connected if $G^{(\xi,\eta,\zeta)}$ is tree $\forall(\xi, \eta, \zeta) \in (0, h(B)]$.*

Proposition 3.6. *If G is connected then G is weakly connected.*

Proof. Since G is connected implies that G^* is connected. Now $G^* = G^{h(B)}$ and so G is weak connected. □

Proposition 3.7. *If G is firm and weak connected then G is connected.*

Proof. If $G^{(\xi,\eta,\zeta)}$ is connected for some $(\xi, \eta, \zeta) \in (0, h(B)]$, then G^* is connected, since G is firm. □

Proposition 3.8. (**i**) *If G is a weak SVN-tree, then G is weak connected and G is a weak SVN-forest, conversely if there are $(\xi_1, \eta_1, \zeta_1), (\xi_2, \eta_2, \zeta_2) \in (0, h(B)]$, with $\xi_1 < \xi_2$, $\eta_1 < \eta_2$ and $\zeta_1 < \zeta_2$ such that $G^{(\xi_1,\eta_1,\zeta_1)}$ is a forest and $G^{(\xi_2,\eta_2,\zeta_2)}$ is connected, then G is weak SVN-tree.*
(**ii**) *The SVNG G is a tree if and only if G is a forest and G is connected.*
(**iii**) *The SVNG G is partial SVN-tree if and only if G is a partial SVN-forest and G is partially connected SVNG.*
(**iv**) *The SVNG G is full SVN-tree if and only if G is a full SVN-forest and G is fully connected SVNG.*

Proof. **(i)** If $G^{(\xi,\eta,\zeta)}$ is a tree for some $(\xi, \eta, \zeta) \in (0, h(B)]$, then $G^{(\xi,\eta,\zeta)}$ is connected and is a forest. For converse, note that $G^{(\xi_2,\eta_2,\zeta_2)}$ must also be a forest, since also $G^{(\xi_2,\eta_2,\zeta_2)}$ is connected, $G^{(\xi_2,\eta_2,\zeta_2)}$ is a tree.
(ii), **(iii)** and **(iv)** are obvious. □

Proposition 3.9. *The SVNG G is firm if and only if $G^{(\xi,\eta,\zeta)}$ is firm for all $(\xi, \eta, \zeta) \in (0, h(B)]$.*

Proof. Suppose G is firm, let $(\xi, \eta, \zeta) \in (0, h(B)]$, for $xy \in T^{(\xi,\eta,\zeta)}$ then

$$\xi \leq T_B(xy) \leq \min\{T_A(x) : x \in V\} \leq \min\{T_A(x) : x \in T_A^{\xi}\}$$
$$\eta \geq I_B(xy) \geq \max\{I_A(x) : x \in V\} \geq \max\{I_A(x) : x \in I_A^{\eta}\}$$
$$\zeta \geq F_B(xy) \geq \max\{F_A(x) : x \in V\} \geq \max\{F_A(x) : x \in F_A^{\zeta}\}$$

therefore

$$\max\{T_B(xy) : xy \in T_B^{\xi}\} \leq \min\{T_A(x) : x \in T_A^{\xi}\}$$
$$\min\{I_B(xy) : xy \in I_B^{\eta}\} \leq \max\{I_A(x) : x \in I_A^{\eta}\}$$
$$\min\{F_B(xy) : xy \in F_B^{\zeta}\} \leq \max\{F_A(x) : x \in F_A^{\zeta}\}$$

thus we conclude that $B^{(\xi,\eta,\zeta)*} = B^{(\xi,\eta,\zeta)}$, $A^{(\xi,\eta,\zeta)*} = A^{(\xi,\eta,\zeta)}$ and $G^{(\xi,\eta,\zeta)}$ is a firm.
Conversely: Suppose that $G^{(\xi,\eta,\zeta)}$ is a firm for all $(\xi, \eta, \zeta) \in (0, h(B)]$. Let

$$\min\{T_A(x) : x \in V\} = \xi_0 > 0$$
$$\max\{I_A(x) : x \in V\} = \eta_0 > 0$$
$$\max\{F_A(x) : x \in V\} = \zeta_0 > 0$$

next

$$\max\{T_B(xy) : xy \in T_B^{\xi_0}\} \leq \xi_0$$
$$\min\{I_B(xy) : xy \in I_B^{\eta_0}\} \geq \eta_0$$
$$\min\{F_B(xy) : xy \in F_B^{\zeta_0}\} \geq \zeta_0$$

since $G^{(\xi_0,\eta_0,\zeta_0)}$ is firm and $V = A^{(\xi_0,\eta_0,\zeta_0)} = A^{(\xi_0,\eta_0,\zeta_0)*}$ Let $xy \in E - B^{(\xi,\eta,\zeta)*}$, then $T_B(xy) < \xi_0,\ I_B(xy) > \eta_0$ and $F_B(xy) > \zeta_0$. Thus

$$\max\{T_B(xy) : xy \in E\} \leq \xi_0 = \min\{T_A(x) : x \in V\},$$
$$\min\{I_B(xy) : xy \in E\} \geq \eta_0 = \max\{I_A(x) : x \in V\},$$
$$\min\{F_B(xy) : xy \in E\} \geq \zeta_0 = \max\{F_A(x) : x \in V\}.$$

Hence G is firm. □

4. Conclusion

The neutrosophic graphs have many applications in path problems, networks and computer science. The edge connectivity in SVNG is basic concept to understand the connections of connectedness between two systems of computers. The SVN-bridges, cycles, trees, cut-Vertices and Levels are introduced here, also the SVN-Blocks and firms are introduced with its properties and criteria to prove the SVNG to be firm or Block.

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