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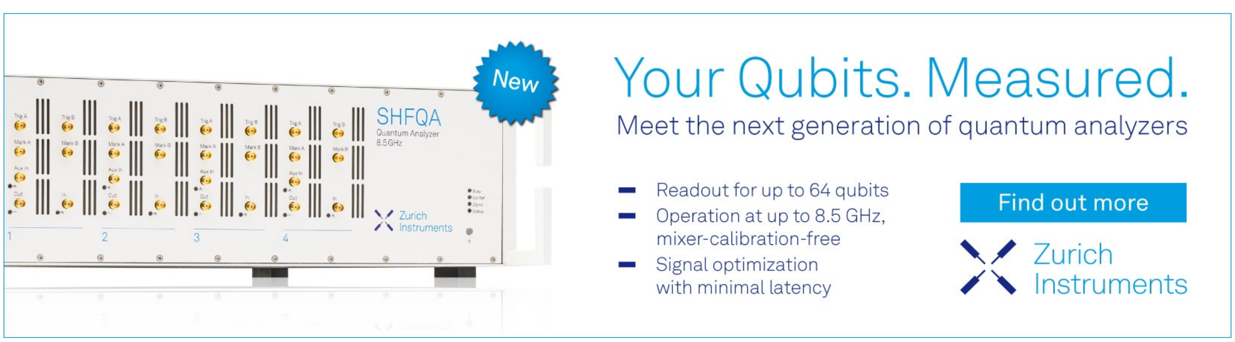

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Solving LPP with Stochastic Neutrosophic Pythagorean Z numbers

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Abstract. This document gives the idea of Neutrosophic Pythagorean Z numbers, operations on neutrosophic Pythagorean Z numbers which helps us to overcome the situation where the truth membership function, indeterminacy membership function and non- membership function is greater than one in uncertainty and reliability. Also stochastic LPP is used to solve the Numerical Example.

INTRODUCTION

In the real world, uncertainty is a pervasive phenomenon. Much of the decisions taken are based on uncertainty. Humans have a remarkable capability to make rational decisions based on information which is uncertain, imprecise and/or incomplete. Formalization of this capability, at least to some degree, is a challenge that is hard to meet. When an easily solved problem ends up with difficult optimization problems, there one may consider the new concept called Z numbers. The concept of Z numbers has been recently introduced in decision making analysis. Zadeh [5] defined Z numbers related with an uncertain variable. Smarandache [2] proposed the concept of neutrosophic set which is generalization of fuzzy set theory and intuitionistic fuzzy sets. Pythagorean set theory is a documented technique to manage uncertainty in the optimization problem. Yager [3, 4] generalized Pythagorean fuzzy set, which is a new tool to deal with vagueness considering the membership and non-membership satisfying the Pythagorean condition. It may be used to characterize the uncertain information more sufficiently and accurately than intuitionistic fuzzy set. Pythagorean fuzzy set has attracted great attention of many scholars that have been extended to new types and these extensions have been used in many areas such as decision making, aggregation operators, and information measures was given by Beliakov, James [1]. Because of such a growth, one may present an idea on Pythagorean fuzzy set with aim of offering a clear perspective on the different concepts. In particular, one may provide neutrosophic Pythagorean Z environment to deal with uncertainty and reliability. This technique is considered as a standard decision making procedure, mainly when NPZNs are functional in real decision making problems. In this paper, the researcher defines Neutrosophic Pythagorean Z numbers (NPZNs) where some of its mathematical operations are defined and various theorems are also stated to show the combination of NPZNs. The researcher proposes θ – cut of NPZN and considers the real time example in this chapter to show the value of the work. The data is collected from fifty different persons and were consolidated as neutrosophic numbers for various restrictions. And the same is formulated as LPP and solved using NPZLPP and SNPZLPP to give the suggestions for customer in choosing better bike to get maximum profit with the utilization of available resources.

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PRELIMINARIES

Neutrosophic Pythagorean Z numbers (NPZNs) and various operations on the NPZN are defined. θ – cut of Neutrosophic Pythagorean Z Numbers is defined.

Neutrosophic Pythagorean Z Numbers

A neutrosophic Pythagorean Z numbers (NPZN) is defined in a non empty set X and an unit interval I i.e., [0,1] as $NPZN = \left\{\left\langle x; \langle TR_U(x), ID_U(x), FA_U(x)\rangle \langle TR_R(x), ID_R(x), FA_R(x)\rangle : x \in X\right\rangle\right\}$ where $TR_U(x), TR_R(x) : X \to [0,1]$, $ID_U(x), ID_R(x) : X \to [0,1]$, $FA_U(x), FA_R(x) : X \to [0,1]$ denote the degree of membership, indeterminacy and non-membership function of uncertainty and reliability with $0 \le (TR_U(x))^2 + (ID_U(x))^2 + (FA_U(x))^2 \le 3$ and $0 \le (TR_R(x))^2 + (ID_R(x))^2 + (FA_R(x))^2 \le 3$ is given by

$$TR_U(x) = \begin{cases} 0 & ; for \;\; x < m \\ \frac{1}{2}\left(\frac{x-m}{n-m}\right) & ; x \in [m, n] \\ \frac{1}{2} & ; x \in [n, \alpha] \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-\alpha}{\beta-\alpha}\right) & ; x \in [\alpha, \beta] \\ \frac{1}{2} + \frac{1}{2}\left(\frac{\gamma-x}{\gamma-\beta}\right) & ; x \in [\beta, \gamma] \\ \frac{1}{2} & ; x \in [\gamma, \lambda] \\ \frac{1}{2}\left(\frac{\mu-x}{\mu-\lambda}\right), & ; x \in [\lambda, \mu] \\ 0 & ; otherwise \end{cases}$$

$$ID_U(x) = \begin{cases} 1 & ; for \; x < m'' \\ 1 - \frac{1}{2}\left(\frac{x-m''}{n''-m''}\right) & ; x \in [m'', n''] \\ \frac{1}{2} & ; x \in [n'', \alpha''] \\ \frac{1}{2}\left(\frac{x-\alpha''}{\beta''-\alpha''}\right) & ; x \in [\alpha'', \beta''] \\ 0 & ; x = \beta'' \\ \frac{1}{2}\left(\frac{\gamma''-x}{\gamma''-\beta'''}\right) & ; x \in [\beta'', \gamma''] \\ \frac{1}{2} & ; x \in [\gamma'', \lambda''] \\ 1 - \frac{1}{2}\left(\frac{\mu''-x}{\mu''-\lambda''}\right) & ; x \in [\lambda'', \mu''] \\ 1 & otherwise \end{cases}$$

$$FA_U(x) = \begin{cases} 1 & ; for \;\; x < m' \\ 1 - \frac{1}{2}\left(\frac{x-m'}{n'-m'}\right) & ; x \in [m', n'] \\ \frac{1}{2} & ; x \in [n', \alpha'] \\ \frac{1}{2}\left(\frac{x-\alpha'}{\beta'-\alpha'}\right) & ; x \in [\alpha', \beta'] \\ 0 & ; x = \beta' \\ \frac{1}{2}\left(\frac{\gamma'-x}{\gamma'-\beta'}\right) & ; x \in [\beta', \gamma'] \\ \frac{1}{2} & ; x \in [\gamma', \lambda'] \\ 1 - \frac{1}{2}\left(\frac{\mu'-x}{\mu'-\lambda'}\right) & ; x \in [\lambda', \mu'] \\ 1 & otherwise \end{cases}$$

Mathematical Operations on Neutrosophic Pythagorean Z Numbers

Let the two neutrosophic Pythagorean Z numbers are given by $NPZN_1 = \left\{ \left\langle x; \langle TR_{U1}(x), ID_{U1}(x), FA_{U1}(x) \rangle \langle TR_{R1}(x), ID_{R1}(x), FA_{R1}(x) \rangle : x \in X \right\rangle \right\}$ and $NPZN_2 = \left\{ \left\langle x; \langle TR_{U2}(x), ID_{U2}(x), FA_{U2}(x) \rangle \langle TR_{R2}(x), ID_{R2}(x), FA_{R2}(x) \rangle : x \in X \right\rangle \right\}$ then

i. $$NPZN_1 \oplus NPZN_2 = \left\{ \begin{array}{l} \left\langle \begin{array}{l} \sqrt{TR_{U1}(x)^2 + TR_{U2}(x)^2 - TR_{U1}(x)^2 . TR_{U2}(x)^2}, \\ ID_{U1}(x).ID_{U2}(x), FA_{U1}(x).FA_{U2}(x) \end{array} \right\rangle \\ \left\langle \begin{array}{l} \sqrt{TR_{R1}(x)^2 + TR_{R2}(x)^2 - TR_{R1}(x)^2 . TR_{R2}(x)^2}, \\ ID_{R1}(x).ID_{R2}(x), FA_{R1}(x).FA_{R2}(x) \end{array} \right\rangle \end{array} \right\}$$

ii. $$NPZN_1 \otimes NPZN_2 = \left\{ \begin{array}{l} \left\langle \begin{array}{l} TR_{U1}(x).TR_{U2}(x), \\ \sqrt{ID_{U1}(x)^2 + ID_{U2}(x)^2 - ID_{U1}(x)^2 . ID_{U2}(x)^2}, \\ \sqrt{FA_{U1}(x)^2 + FA_{U2}(x)^2 - FA_{U1}(x)^2 . FA_{U2}(x)^2} \end{array} \right\rangle \\ \left\langle \begin{array}{l} TR_{R1}(x).TR_{R2}(x), \\ \sqrt{ID_{R1}(x)^2 + ID_{R2}(x)^2 - ID_{R1}(x)^2 . ID_{R2}(x)^2}, \\ \sqrt{FA_{R1}(x)^2 + FA_{R2}(x)^2 - FA_{R1}(x)^2 . FA_{R2}(x)^2} \end{array} \right\rangle \end{array} \right\}$$

iii. $$\eta . NPZN_1 = \left\{ \begin{array}{l} \left\langle \sqrt{1-(1-(TR_{U1}(x))^2)^\eta}, (ID_{U1}(x))^\eta, (FA_{U1}(x))^\eta \right\rangle \\ \left\langle \sqrt{1-(1-(TR_{R1}(x))^2)^\eta}, (ID_{R1}(x))^\eta, (FA_{R1}(x))^\eta \right\rangle \end{array} \right\}$$

iv. $$NPZN_1^\eta = \left\{ \begin{array}{l} \left\langle (TR_{U1}(x))^\eta, \sqrt{1-(1-(ID_{U1}(x))^2)^\eta}, \sqrt{1-(1-(FA_{U1}(x))^2)^\eta} \right\rangle \\ \left\langle (TR_{R1}(x))^\eta, \sqrt{1-(1-(ID_{R1}(x))^2)^\eta}, \sqrt{1-(1-(FA_{R1}(x))^2)^\eta} \right\rangle \end{array} \right\}$$

VARIOUS COMBINATIONS OF NPZNS

Few theorems are stated here to show the mathematical combinations of two or three NPZNs where the proof is obvious.

Theorem (Mathematical Operations Between Two NPZNs)

Let the two neutrosophic Pythagorean Z numbers are given by $NPZN_1 = \left\{ \left\langle x; \langle TR_{U1}(x), ID_{U1}(x), FA_{U1}(x) \rangle \langle TR_{R1}(x), ID_{R1}(x), FA_{R1}(x) \rangle : x \in X \right\rangle \right\}$ and $NPZN_2 = \left\{ \left\langle x; \langle TR_{U2}(x), ID_{U2}(x), FA_{U2}(x) \rangle \langle TR_{R2}(x), ID_{R2}(x), FA_{R2}(x) \rangle : x \in X \right\rangle \right\}$ then the mathematical operation performed between these numbers are

i. $NPZN_1 \oplus NPZN_2 = NPZN_2 \oplus NPZN_1$

ii. $NPZN_1 \otimes NPZN_2 = NPZN_2 \otimes NPZN_1$

iii. $\eta(NPZN_1 \oplus NPZN_2) = \eta\, NPZN_2 \oplus \eta\, NPZN_1$

iv. $\eta_1\, NPZN_1 \oplus \eta_2\, NPZN_1 = (\eta_1 \oplus \eta_2)NPZN_1$

v. $(NPZN_1 \otimes NPZN_2)^\eta = NPZN_1^\eta \otimes NPZN_2^\eta$

vi. $NPZN_1^{\eta_1} \otimes NPZN_1^{\eta_2} = NPZN_1^{(\eta_1+\eta_2)}$

Theorem (Logical and Mathematical Operations Between Two NPZNs)

Let the two neutrosophic Pythagorean Z numbers are given by

$NPZN_1 = \left\{\left\langle x; \langle TR_{U1}(x), ID_{U1}(x), FA_{U1}(x)\rangle \langle TR_{R1}(x), ID_{R1}(x), FA_{R1}(x)\rangle : x \in X\right\rangle\right\}$ and $NPZN_2 = \left\{\left\langle x; \langle TR_{U2}(x), ID_{U2}(x), FA_{U2}(x)\rangle \langle TR_{R2}(x), ID_{R2}(x), FA_{R2}(x)\rangle : x \in X\right\rangle\right\}$ then the Logical and mathematical operation performed between these numbers are

i. $(NPZN_1 \cup NPZN_2) \oplus (NPZN_1 \cap NPZN_2) = NPZN_1 \oplus NPZN_2$
ii. $(NPZN_1 \cup NPZN_2) \otimes (NPZN_1 \cap NPZN_2) = NPZN_1 \otimes NPZN_2$

Theorem (Commutative Property Between Three NPZNs)

Let the three neutrosophic Pythagorean Z numbers are given by $NPZN_1 = \left\{\left\langle x; \langle TR_{U1}(x), ID_{U1}(x), FA_{U1}(x)\rangle \langle TR_{R1}(x), ID_{R1}(x), FA_{R1}(x)\rangle : x \in X\right\rangle\right\}$, $NPZN_2 = \left\{\left\langle x; \langle TR_{U2}(x), ID_{U2}(x), FA_{U2}(x)\rangle \langle TR_{R2}(x), ID_{R2}(x), FA_{R2}(x)\rangle : x \in X\right\rangle\right\}$ and $NPZN_3 = \left\{\left\langle x; \langle TR_{U3}(x), ID_{U3}(x), FA_{U3}(x)\rangle \langle TR_{R3}(x), ID_{R3}(x), FA_{R3}(x)\rangle : x \in X\right\rangle\right\}$ then the commutative property performed between these numbers are

i. $NPZN_1 \cup NPZN_2 \cup NPZN_3 = NPZN_3 \cup NPZN_2 \cup NPZN_1$
ii. $NPZN_1 \cap NPZN_2 \cap NPZN_3 = NPZN_3 \cap NPZN_2 \cap NPZN_1$
iii. $NPZN_1 \oplus NPZN_2 \oplus NPZN_3 = NPZN_3 \oplus NPZN_2 \oplus NPZN_1$
iv. $NPZN_1 \otimes NPZN_2 \otimes NPZN_3 = NPZN_3 \otimes NPZN_2 \otimes NPZN_1$

Theorem (Associative Property Between Three NPZNs)

Let the three neutrosophic Pythagorean Z numbers are given by $NPZN_1 = \left\{\left\langle x; \langle TR_{U1}(x), ID_{U1}(x), FA_{U1}(x)\rangle \langle TR_{R1}(x), ID_{R1}(x), FA_{R1}(x)\rangle : x \in X\right\rangle\right\}$, $NPZN_2 = \left\{\left\langle x; \langle TR_{U2}(x), ID_{U2}(x), FA_{U2}(x)\rangle \langle TR_{R2}(x), ID_{R2}(x), FA_{R2}(x)\rangle : x \in X\right\rangle\right\}$ and $NPZN_3 = \left\{\left\langle x; \langle TR_{U3}(x), ID_{U3}(x), FA_{U3}(x)\rangle \langle TR_{R3}(x), ID_{R3}(x), FA_{R3}(x)\rangle : x \in X\right\rangle\right\}$ then the associative property performed between these numbers are

i. $(NPZN_1 \cup NPZN_2) \cap NPZN_3 = (NPZN_1 \cap NPZN_3) \cup (NPZN_2 \cap NPZN_3)$
ii. $(NPZN_1 \cap NPZN_2) \cup NPZN_3 = (NPZN_1 \cup NPZN_3) \cap (NPZN_2 \cup NPZN_3)$
iii. $(NPZN_1 \cup NPZN_2) \oplus NPZN_3 = (NPZN_1 \oplus NPZN_3) \cup (NPZN_2 \oplus NPZN_3)$
iv. $(NPZN_1 \cap NPZN_2) \oplus NPZN_3 = (NPZN_1 \oplus NPZN_3) \cap (NPZN_2 \oplus NPZN_3)$
v. $(NPZN_1 \cup NPZN_2) \otimes NPZN_3 = (NPZN_1 \otimes NPZN_3) \cup (NPZN_2 \otimes NPZN_3)$
vi. $(NPZN_1 \cap NPZN_2) \otimes NPZN_3 = (NPZN_1 \otimes NPZN_3) \cap (NPZN_2 \otimes NPZN_3)$

θ – Cut of Neutrosophic Pythagorean Z Numbers

A NPZN is defined in a non empty set X and an unit interval I i.e., [0,1] as $NPZN = \left\{\left\langle x; \langle TR_U(x), ID_U(x), FA_U(x)\rangle \langle TR_R(x), ID_R(x), FA_R(x)\rangle : x \in X\right\rangle\right\}$ .Its θ – cut value is defined as $NPZN(\theta) = \left\{x \in X \mid \langle TR_U(x), ID_U(x), FA_U(x)\rangle \langle TR_R(x), ID_R(x), FA_R(x)\rangle\right\}$ where

$$NPZN(\theta)=\left\{\begin{array}{l}\left\langle\begin{array}{ll}NPZN(\theta)_{1Ul} & for\ \theta\in[m,n]\\ NPZN(\theta)_{2Ul} & for\ \theta\in[n,\alpha]\\ NPZN(\theta)_{3Ul} & for\ \theta\in[\alpha,\beta]\\ NPZN(\theta)_{3Ur} & for\ \theta\in[\beta,\gamma]\\ NPZN(\theta)_{2Ur} & for\ \theta\in[\gamma,\lambda]\\ NPZN(\theta)_{1Ur} & for\ \theta\in[\lambda,\mu]\end{array}\right\rangle\left\langle\begin{array}{ll}NPZN(\theta)_{Rl} & for\ \theta\in[m,n]\\ NPZN(\theta)_{Rr} & for\ \theta\in[n,\alpha]\end{array}\right\rangle\end{array}\right.$$

$$\left\{\begin{array}{l}\left\langle\begin{array}{ll}NPZN''(\theta)_{1Ul} & for\ \theta\in[m'',n'']\\ NPZN''(\theta)_{2Ul} & for\ \theta\in[n'',\alpha'']\\ NPZN''(\theta)_{3Ul} & for\ \theta\in[\alpha'',\beta'']\\ NPZN''(\theta)_{3Ur} & for\ \theta\in[\beta'',\gamma'']\\ NPZN''(\theta)_{2Ur} & for\ \theta\in[\gamma'',\lambda'']\\ NPZN''(\theta)_{1Ur} & for\ \theta\in[\lambda'',\mu'']\end{array}\right\rangle\left\langle\begin{array}{ll}NPZN''(\theta)_{Rl} & for\ \theta\in[m'',n]\\ NPZN''(\theta)_{Rr} & for\ \theta\in[n,\alpha'']\end{array}\right\rangle\end{array}\right.$$

$$\left\{\begin{array}{l}\left\langle\begin{array}{ll}NPZN'(\theta)_{1Ul} & for\ \theta\in[m',n']\\ NPZN'(\theta)_{2Ul} & for\ \theta\in[n',\alpha']\\ NPZN'(\theta)_{3Ul} & for\ \theta\in[\alpha',\beta']\\ NPZN'(\theta)_{3Ur} & for\ \theta\in[\beta',\gamma']\\ NPZN'(\theta)_{2Ur} & for\ \theta\in[\gamma',\lambda']\\ NPZN'(\theta)_{1Ur} & for\ \theta\in[\lambda',\mu']\end{array}\right\rangle\left\langle\begin{array}{ll}NPZN'(\theta)_{Rl} & for\ \theta\in[m',n]\\ NPZN'(\theta)_{Rr} & for\ \theta\in[n,\alpha']\end{array}\right\rangle\end{array}\right.$$

Consider the uncertain value neutrosophic Pythagorean Z numbers,

$NPZN=\left\{\left\langle x;\left\langle TR_U(x),ID_U(x),FA_U(x)\right\rangle : x\in X\right\rangle\right\}$

the θ – cut for uncertainty of the truth membership function is defined as

$$NPZN_{1Ul}=\frac{1}{2}\left(\frac{x-m}{n-m}\right)$$

Equating the above equation to θ one may get, $NPZN(\theta)_{1Ul}=2\theta(n-m)+m$

Similarly, $NPZN(\theta)_{2Ul}=\frac{1}{2}$, $NPZN(\theta)_{3Ul}=2\left(\theta-\frac{1}{2}\right)(\beta-\alpha)+\alpha$,

$NPZN(\theta)_{3Ur}=\gamma-2\left(\theta-\frac{1}{2}\right)(\gamma-\beta)$, $NPZN(\theta)_{2Ur}=\frac{1}{2}$

$NPZN(\theta)_{1Ur}=\mu-2\theta(\mu-\lambda)$

Similarly the indeterminacy membership function and falsity membership function of uncertainty and reliability with θ - cut are defined.

Hence

$$NPZN(\theta)=\left\{\begin{array}{l}\left\langle\begin{array}{ll}2\theta(n-m)+m & for\ \theta\in[m,n]\\ \frac{1}{2} & for\ \theta\in[n,\alpha]\\ 2\left(\theta-\frac{1}{2}\right)(\beta-\alpha)+\alpha & for\ \theta\in[\alpha,\beta]\\ \gamma-2\left(\theta-\frac{1}{2}\right)(\gamma-\beta) & for\ \theta\in[\beta,\gamma]\\ \frac{1}{2} & for\ \theta\in[\gamma,\lambda]\\ \mu-2\theta(\mu-\lambda) & for\ \theta\in[\lambda,\mu]\end{array}\right\rangle\\ \left\langle\begin{array}{ll}\theta(n-m)+m & for\ \theta\in[m,n]\\ \alpha-\theta(\alpha-n) & for\ \theta\in[n,\alpha]\end{array}\right\rangle\end{array}\right.$$

$$
\left\{
\begin{array}{ll}
\left\langle
\begin{array}{ll}
2(\theta+1)(n''-m'')+m'' & for\ \theta\in[m'',n''] \\
\frac{1}{2} & for\ \theta\in[n'',\alpha''] \\
2\theta(\beta''-\alpha'')+\alpha'' & for\ \theta\in[\alpha'',\beta''] \\
\gamma''-2\theta(\gamma''-\beta'') & for\ \theta\in[\beta'',\gamma''] \\
\frac{1}{2} & for\ \theta\in[\gamma'',\lambda''] \\
\mu''-2(\theta+1)(\mu''-\lambda'') & for\ \theta\in[\lambda'',\mu'']
\end{array}
\right\rangle
&
\left\langle
\begin{array}{ll}
2(\theta+1)(n'-m')+m' & for\ \theta\in[m',n'] \\
\frac{1}{2} & for\ \theta\in[n',\alpha'] \\
2\theta(\beta'-\alpha')+\alpha' & for\ \theta\in[\alpha',\beta'] \\
\gamma'-2\theta(\gamma'-\beta') & for\ \theta\in[\beta',\gamma'] \\
\frac{1}{2} & for\ \theta\in[\gamma',\lambda'] \\
\mu'-2(\theta+1)(\mu'-\lambda') & for\ \theta\in[\lambda',\mu']
\end{array}
\right\rangle
\\
\left\langle
\begin{array}{ll}
n-\theta(n-m'') & for\ \theta\in[m'',n] \\
\theta(\alpha''-n)+n & for\ \theta\in[n,\alpha'']
\end{array}
\right\rangle
&
\left\langle
\begin{array}{ll}
n-\theta(n-m') & for\ \theta\in[m',n] \\
\theta(\alpha'-n)+n & for\ \theta\in[n,\alpha']
\end{array}
\right\rangle
\end{array}
\right.
$$

RANKING OF NPZN

Value Based Ranking - Uncertainty

Let $(PZN)(\theta_1)$, $(PZN)(\theta_2)$ and $(PZN)(\theta_3)$ are any θ_1 - cut , θ_2 - cut and θ_3 - cut set of an uncertainty in NPZN respectively. The truth membership function indeterminacy membership function and falsity membership function values of the uncertain value are defined as

$$V_{TR_U} = \int_0^1 \left(NPZN_{1Ul}^{\theta}(\theta_1) + NPZN_{1Ur}^{\theta}(\theta_1)\right) f(\theta_1)\, d\theta_1 \longrightarrow \quad (4.1)$$

$$V_{ID_U} = \int_0^1 \left(NPZN''^{\theta}_{1Ul}(\theta_2) + NPZN''^{\theta}_{1Ur}(\theta_2)\right) g(\theta_2)\, d\theta_2 \longrightarrow \quad (4.2)$$

$$V_{FA_U} = \int_0^1 \left(NPZN'^{\theta}_{1Ul}(\theta_3) + NPZN'^{\theta}_{1Ur}(\theta_3)\right) h(\theta_3)\, d\theta_3 \longrightarrow \quad (4.3)$$

respectively.

The function $f(\theta_1) = \theta_1$, $g(\theta_2) = 1 - \theta_2$ and $h(\theta_3) = 1 - \theta_3$ give different mass elements in different θ_1 , θ_2 and θ_3 cut sets. The value of the truth membership function of an uncertain value of NPZN is calculated from the equation (4.1) as

$NPZN_{1Ul}^{\theta}(\theta_1) = 2\theta_1(n-m)+m$ and $NPZN_{1Ur}^{\theta}(\theta_1) = \mu - 2\theta_1(\mu-\lambda)$

Substituting $NPZN_{1Ul}^{\theta}(\theta_1)$ and $NPZN_{1Ur}^{\theta}(\theta_1)$ in (4.1) one may get

$$V_{TR_U} = \frac{4n - m - \mu + 4\lambda}{6} \longrightarrow \quad (4.4)$$

In a similar way, according to the equation (4.2) and (4.3) the value of indeterminacy membership function and falsity membership function of the uncertain value of NPZN is calculated as follows:

$$V_{ID_U} = \frac{4n'' - m'' - \mu'' + 4\lambda''}{6} \longrightarrow \quad (4.5)$$

$$V_{FA_U} = \frac{4n' - m' - \mu' + 4\lambda'}{6} \longrightarrow \quad (4.6)$$

A new ranking is proposed from the equations (4.4), (4.5) and (4.6) in this section.

A value - index for NPZN is defined as:

$$V_{\chi} = \chi V_{TR_U} + (1-\chi)V_{ID_U} + (1-\chi)V_{FA_U} \longrightarrow \quad (4.7)$$

the uncertain neutrosophic heptagonal numbers is converted into crisp value using this ranking.

Ranking of NPZN - Reliability

The area for triangular truth membership (m, n, α) of the reliability is obtained as (1/2) x base x height = $\frac{1}{2}(\alpha - m)$. Similarly, the indeterminacy and falsity membership is attained as $\frac{1}{2}(\alpha'' - m'')$ and $\frac{1}{2}(\alpha' - m')$. The ranking for the reliability value is obtained by taking the average of the area of truth membership function, indeterminacy membership function and falsity membership function.

i.e, $\frac{1}{2}[(\alpha - m) + (\alpha'' - m'') + (\alpha' - m')]$ ⟶ (4.8)

NPZ LINEAR PROGRAMMING PROBLEM

The general form of optimization problem with NPZ numbered objective function and NPZ numbered constraints are given by

$$Max\ or\ Min\ (\tilde{\varpi}) = \sum_{j=1}^{n} (\tilde{\tau}_{Uj}^{k}, \tilde{\tau}_{Rj}^{k}) x_j, \text{ where } k = 1,2,3,.....,K$$

subject to

$$\sum_{i=1}^{n} (\tilde{\sigma}_{Uij}, \tilde{\sigma}_{Rij}) x_j \geq / \leq / = (\tilde{k}_{Ui}, \tilde{k}_{Ri}),\ i = 1,2,3,.....,m; \quad j = 1,2,3,.....,n$$

$$x_j \geq 0,\ j = 1,2,3,....,n.$$

Stochastic NPZLPP With Burr Type II Distribution

Stochastic or probabilistic model is formulated when the parameters of linear programming problem is considered as random values. In this chapter, the probability value is considered for the right hand side vector and the problem is solved to get the best optimum solution.

Formulation of Stochastic NPZLPP

The mathematical model of a SFLPP may be given as:

$$Min\ \tilde{\varpi} = \sum_{j=1}^{n} (\tau_j\ or\ \tilde{\tau}_j) x_j\ or\ \tilde{x}_j$$

subject to,

$$\sum_{j=1}^{n} (\tilde{\sigma}_{ij}\ or\ \sigma_{ij}) x_j\ or\ \hat{x}_j \leq \sinh^{-1}\left(\frac{-1}{\lambda}\right) \log\left[\frac{p_i^{-1/\lambda_i} - 1}{\theta}\right], \quad i = 1,2,3,.....,m$$

$$x_j\ or\ \tilde{x}_j \geq 0,\ j = 1,2,3,....,n.$$

Algorithm for Solving NPZLPP

Step 1: The collected data is consolidated into a table and the necessary data is chosen for the problem. Formulate them into a LPP.

Step 2: The LPP is further formulated into a heptagonal neutrosophic Pythagorean Z numbers LPP.

Step 3: (i) Check whether the objective function of the given heptagonal NPZLPP is to be maximized or minimized.

(ii) Convert all the inequations of the constraints into less than constraint for Maximizing the objective function and greater than constraint for Minimizing the objective function.

Step 4: (i) The uncertain value of NPZN is converted into crisp value using the ranking function $V_{\chi} = \chi V_{TR_U} + (1-\chi) V_{ID_U} + (1-\chi) V_{FA_U}$

(ii) The reliability value of NPZN is converted into crisp value by $\frac{1}{2}[(\alpha - m) + (\alpha'' - m'') + (\alpha' - m')]$

(iii) The ZLPP is formed from the above ranking which is converted into crisp LPP by taking the average for the uncertain value and reliability value.

Step 5: The net evaluation is computed using the relation $\tilde{\varpi}_j - \tilde{\tau}_j = \tilde{\tau}_B \tilde{\sigma}_j - \tilde{\tau}_j$ and examine the sign of $\tilde{\varpi}_j - \tilde{\tau}_j$

(a) If all $\tilde{\varpi}_j - \tilde{\tau}_j \geq 0$, the solution is optimal.

(b) If atleast one $\tilde{\varpi}_j - \tilde{\tau}_j < 0$, the solution is not optimal.

Stochastic Model for Solving NPZLPP Using Burr Type II Distribution

Step 1: The problem used for solving NPZLPP is considered here.

Step 2: Take the right hand side resource vector of constraint as burr type II distribution and convert the NPZLPP as stochastic NPZLPP.

Step 3: Consider any values for p_i in the below equation to formulate in stochastic NPZLPP.

$$\sum_{j=1}^{n} (\tilde{\sigma}_{ij} \ or \ \sigma_{ij}) x_j \ or \ \hat{x}_j \leq \sinh^{-1}\left(\frac{-1}{\lambda}\right) \log\left[\frac{p_i^{-1/\lambda_i} - 1}{\theta}\right]$$

Step 4: By taking different λ and θ values, one may get the probabilistic constraint.

Step 5: The net evaluation is computed using the relation $\tilde{\varpi}_j - \tilde{\tau}_j = \tilde{\tau}_B \tilde{\sigma}_j - \tilde{\tau}_j$ and examine the sign of $\tilde{\varpi}_j - \tilde{\tau}_j$

(a) If all $\tilde{\varpi}_j - \tilde{\tau}_j \geq 0$, the solution is optimal.

(b) If atleast one $\tilde{\varpi}_j - \tilde{\tau}_j < 0$, the solution is not optimal.

Numerical Example

The information about two wheelers was collected from fifty different persons on their own satisfaction towards the vehicle on various parameters like age, speed, mileage, design, brand image, income, period, maintenance cost etc., sample of three person datas are given below

TABLE 1. Data's collected different persons

S. No	Name	Age	Income	Brand mage	Speed	Design	Mileage	Period	Maintenance Cost	Vehicle
1	Karthikeyan	31-40	10k-20k	VH	H	VH	H	2-3	below 3K	Bajaj
2	Saravan	21-30	10k-20k	H	H	H	H	4-5	3K-5K	Bajaj
3	Nallasamy	31-40	10k-20k	VH	H	H	H	2-3	below 5K	Bajaj

According to the data collected from fifty different persons, 24 persons suggested for brand image i.e. 48%, 8 persons suggested for speed i.e. 16%, 14 persons suggested for design i.e. 28% and 4 persons suggested for mileage i.e. 8%, the average value of four categories is ranked as BI > D > S > M.

Based on the ranking, the problem is further developed and formulated as LPP and solved to obtain the optimum solution by two different methods.

The vehicle wise split up of data for various parameters is formed as neutrosophic numbers and is given in the below table. For example, The age of the Honda user is categorized as (21-30, 31-40, 41-50) and it is given by (5,5,1) which represents the membership, non-membership and indeterminant value. Similarly, the other parameters are also mentioned in the table.

TABLE 2. Vehicle wise neutrosophic data

Vehicle / Criteria	Age	Speed	Mileage	Design	Brand Image	Income	Period	Maintenance Cost
Bajaj	(0,2,3) (0.75,0.5,0.25)	(0,4,1) (0.75,0.5,0.25)	(3,2,0) (0.75,0.5,0.25)	(0,4,1) (0.75,0.5,0.25)	(0,4,1) (0.75,0.5,0.25)	(0,0,5) (0.75,0.5,0.25)	(0,2,3) (0.75,0.5,0.25)	(2,3,0) (0.75,0.5,0.25)
Hero Splendor	(2,5,1) (0.75,0.5,0.25)	(2,2,4) (0.75,0.5,0.25)	(6,2,0) (0.75,0.5,0.25)	(1,4,3) (0.75,0.5,0.25)	(0,6,2) (0.75,0.5,0.25)	(2,5,0) (0.75,0.5,0.25)	(4,2,2) (0.75,0.5,0.25)	(0,7,1) (0.75,0.5,0.25)
Honda	(5,5,1) (0.75,0.5,0.25)	(6,10,4) (0.75,0.5,0.25)	(9,8,3) (0.75,0.5,0.25)	(4,14,2) (0.75,0.5,0.25)	(5,14,1) (0.75,0.5,0.25)	(5,15,0) (0.75,0.5,0.25)	(1,2,15) (0.75,0.5,0.25)	(7,10,3) (0.75,0.5,0.25)
Suzuki	(3,1,5) (0.75,0.5,0.25)	(1,5,3) (0.75,0.5,0.25)	(7,3,0) (0.75,0.5,0.25)	(2,4,4) (0.75,0.5,0.25)	(4,6,0) (0.75,0.5,0.25)	(2,7,1) (0.75,0.5,0.25)	(2,3,5) (0.75,0.5,0.25)	(2,8,0) (0.75,0.5,0.25)
TVS	(1,4,0) (0.75,0.5,0.25)	(1,5,1) (0.75,0.5,0.25)	(4,2,1) (0.75,0.5,0.25)	(1,4,2) (0.75,0.5,0.25)	(2,5,0) (0.75,0.5,0.25)	(0,2,5) (0.75,0.5,0.25)	(0,1,3) (0.75,0.5,0.25)	(2,5,0) (0.75,0.5,0.25)

Neutrosophic Pythagorean ZLPP

Step 1: The above problem is formulated as heptagonal NPZLPP. Where x_1= Honda, x_2= Suzuki, x_3 = TVS, x_4= Hero Splendor, x_5= Bajaj, BI = brand image, D = design and MC = maintenance cost.

$$Max\ BI = \begin{Bmatrix} \langle [0,1,4,5,7,10,11],[9,11,13,14,17,18,20],[0,0.5,1,1,2,3,3] \rangle \\ \langle [0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5] \rangle \end{Bmatrix} x_1 + \begin{Bmatrix} \langle [0,1,3,4,7,9,11],[2,4,5,6,8,10,11],[-2,-1,0,1,3,5,6] \rangle \\ \langle [0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5] \rangle \end{Bmatrix} x_2 +$$
$$\begin{Bmatrix} \langle [-1,0,1,2,4,5,7],[1,3,4,5,7,8,10],[-2,-1,0,1,3,5,6] \rangle \\ \langle [0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5] \rangle \end{Bmatrix} x_3 + \begin{Bmatrix} \langle [-2,-1,0,1,3,5,6],[2,4,5,6,8,10,11],[-1,0,1,2,4,5,7] \rangle \\ \langle [0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5] \rangle \end{Bmatrix} x_4 +$$
$$\begin{Bmatrix} \langle [-2,-1,0,1,3,5,6],[0,1,3,4,7,9,11],[0,0.5,1,1,2,3,3] \rangle \\ \langle [0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5] \rangle \end{Bmatrix} x_5$$

$$Max\ D=\left\{\begin{matrix}\langle[0,1,3,4,7,9,11],[9,11,13,14,17,18,20],[-1,0,1,2,4,5,7]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_1+\left\{\begin{matrix}\langle[-1,0,1,2,4,5,7],[0,1,3,4,7,9,11],[0,1,3,4,7,9,11]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_2+$$

$$\left\{\begin{matrix}\langle[0,0.5,1,1,2,3,3],[0,1,3,4,7,9,11],[-2,0,1,2,4,6,8]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_3+\left\{\begin{matrix}\langle[0,0.5,1,1,2,3,3],[0,1,3,4,7,9,11],[0,1,2,3,5,6,9]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_4+$$

$$\left\{\begin{matrix}\langle[-2,-1,0,1,3,5,6],[0,1,3,4,7,9,11],[0,0.5,1,1,2,3,3]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_5$$

$$Min\ MC=\left\{\begin{matrix}\langle[2,3,5,7,8,11,12],[6,7,8,10,12,14,15],[0,1,2,3,5,6,9]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_1+\left\{\begin{matrix}\langle[-2,0,1,2,4,6,8],[4,5,7,8,11,13,16],[-2,-1,0,1,3,5,6]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_2+$$

$$\left\{\begin{matrix}\langle[-2,0,1,2,4,6,8],[1,2,4,5,7,9,11],[-2,-1,0,1,3,5,6]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_3+\left\{\begin{matrix}\langle[-2,-1,0,1,3,5,6],[2,3,5,7,8,11,12],[0,0.5,1,1,2,3,3]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_4+$$

$$\left\{\begin{matrix}\langle[-2,0,1,2,4,6,8],[0,1,2,3,5,6,9],[-2,-1,0,1,3,5,6]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_5$$

subject to,

$$\left\{\begin{matrix}\langle[2,3,5,6,8,10,11],[6,8,9,10,12,15,16],[0,1,3,4,7,9,11]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_1+\left\{\begin{matrix}\langle[0,0.5,1,1,2,3,3],[0,1,4,5,7,10,11],[0,1,2,3,5,6,9]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_2+$$

$$\left\{\begin{matrix}\langle[0,0.5,1,1,2,3,3],[0,1,4,5,7,10,11],[0,0.5,1,1,2,3,3]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_3+\left\{\begin{matrix}\langle[-2,0,1,2,4,6,8],[-2,0,1,2,4,6,8],[0,1,3,4,7,9,11]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_4+$$

$$\left\{\begin{matrix}\langle[-2,-1,0,1,3,5,6],[0,1,3,4,7,9,11],[0,0.5,1,1,2,3,3]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_5\le/\ge\left\{\begin{matrix}\langle[2,3,5,6,8,10,11],[6,8,9,10,12,15,16],[0,1,3,4,7,9,11]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}$$

$$\left\{\begin{matrix}\langle[1,5,7,9,11,15,17],[4,5,7,8,11,13,16],[0,1,2,3,5,6,9]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_1+\left\{\begin{matrix}\langle[2,3,5,7,8,11,12],[0,1,2,3,5,6,9],[-2,-1,0,1,3,5,6]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_2+$$

$$\left\{\begin{matrix}\langle[0,1,3,4,7,9,11],[-2,0,1,2,4,6,8],[0,0.5,1,1,2,3,3]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_3+\left\{\begin{matrix}\langle[2,3,5,6,8,10,11],[-2,0,1,2,4,6,8],[-2,-1,0,1,3,5,6]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_4+$$

$$\left\{\begin{matrix}\langle[0,1,2,3,5,6,9],[-2,0,1,2,4,6,8],[-2,-1,0,1,3,5,6]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_5\le/\ge\left\{\begin{matrix}\langle[1,5,7,9,11,15,17],[4,5,7,8,11,13,16],[0,1,2,3,5,6,9]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}$$

$$\left\{\begin{matrix}\langle[0,1,4,5,7,10,11],[0,1,4,5,7,10,11],[2,3,5,7,8,11,12]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_1+\left\{\begin{matrix}\langle[0,1,2,3,5,6,9],[0,0.5,1,1,2,3,3],[0,1,4,5,7,10,11]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_2+$$

$$\left\{\begin{matrix}\langle[0,0.5,1,1,2,3,3],[0,1,3,4,7,9,11],[-2,-1,0,1,3,5,6]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_3+\left\{\begin{matrix}\langle[-2,0,1,2,4,6,8],[0,1,4,5,7,10,11],[0,0.5,1,1,2,3,3]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_4+$$

$$\left\{\begin{matrix}\langle[-2,-1,0,1,3,5,6],[-2,0,1,2,4,6,8],[0,1,2,3,5,6,9]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_5\le/\ge\left\{\begin{matrix}\langle[0,1,4,5,7,10,11],[0,1,4,5,7,10,11],[2,3,5,7,8,11,12]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}$$

$$\left\{\begin{matrix}\langle[0,1,4,5,7,10,11],[0,1,4,5,7,10,11],[-2,-1,0,1,3,5,6]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_1+\left\{\begin{matrix}\langle[-2,0,1,2,4,6,8],[2,3,5,7,8,11,12],[0,0.5,1,1,2,3,3]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_2+$$

$$\left\{\begin{matrix}\langle[-2,-1,0,1,3,5,6],[-2,0,1,2,4,6,8],[0,1,4,5,7,10,11]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_3+\left\{\begin{matrix}\langle[-2,0,1,2,4,6,8],[0,1,4,5,7,10,11],[-2,-1,0,1,3,5,6]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_4+$$

$$\left\{\begin{matrix}\langle[-2,-1,0,1,3,5,6],[-2,-1,0,1,3,5,6],[0,1,4,5,7,10,11]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}x_5\le/\ge\left\{\begin{matrix}\langle[0,1,4,5,7,10,11],[0,1,4,5,7,10,11],[0,1,4,5,7,10,11]\rangle\\ \langle[0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5]\rangle\end{matrix}\right\}$$

$$\begin{Bmatrix} \langle [0,0.5,1,1,2,3,3],[-2,0,1,2,4,6,8],[9,10,13,15,17,19,20] \rangle \\ \langle [0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5] \rangle \end{Bmatrix} x_1 + \begin{Bmatrix} \langle [-2,0,1,2,4,6,8],[0,1,2,3,5,6,9],[0,1,4,5,7,10,11] \rangle \\ \langle [0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5] \rangle \end{Bmatrix} x_2 +$$

$$\begin{Bmatrix} \langle [-2,-1,0,1,3,5,6],[0,0.5,1,1,2,3,3],[0,1,2,3,5,6,9] \rangle \\ \langle [0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5] \rangle \end{Bmatrix} x_3 + \begin{Bmatrix} \langle [0,1,3,4,7,9,11],[-2,0,1,2,4,6,8],[-2,0,1,2,4,6,8] \rangle \\ \langle [0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5] \rangle \end{Bmatrix} x_4 +$$

$$\begin{Bmatrix} \langle [-2,-1,0,1,3,5,6],[-2,0,1,2,4,6,8],[0,1,2,3,5,6,9] \rangle \\ \langle [0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5] \rangle \end{Bmatrix} x_5 \leq / \geq \begin{Bmatrix} \langle [0,1,3,4,7,9,11],[0,1,2,3,5,6,9],[9,10,13,15,17,19,20] \rangle \\ \langle [0.5,0.75,1],[0.25,0.5,0.75],[0,0.25,0.5] \rangle \end{Bmatrix}$$

and $x_j \geq 0$.

In the above formulation, Brand Image (BI) and Design (D) is taken as Maximizing function and Maintenance Cost (MC) is taken as minimizing function.

Step 2: The formulated NPZLPP is converted into θ – cut of PZLPP by the following steps.

(i) The uncertain value of NPZN is converted into crisp value using the ranking function $V_\chi = \chi V_{TR_U} + (1-\chi) V_{ID_U} + (1-\chi) V_{FA_U}$

(ii) The reliability value of NPZN is converted into crisp value by $\frac{1}{2}\left[(\alpha - m) + (\alpha'' - m'') + (\alpha' - m')\right]$

$$\begin{aligned} Max\ BI = &\{\langle \chi(5.5) + (1-\chi)(14.5) + (1-\chi)(1.5) \rangle \langle 0.25 \rangle\} x_1 + \{\langle \chi(4.8) + (1-\chi)(7.3) + (1-\chi)(2) \rangle \langle 0.25 \rangle\} x_2 + \\ &\{\langle \chi(2.3) + (1-\chi)(5.5) + (1-\chi)(2) \rangle \langle 0.25 \rangle\} x_3 + \{\langle \chi(2) + (1-\chi)(7.2) + (1-\chi)(2.3) \rangle \langle 0.25 \rangle\} x_4 + \\ &\{\langle \chi(2) + (1-\chi)(4.8) + (1-\chi)(1.8) \rangle \langle 0.25 \rangle\} x_5 \end{aligned}$$

$$\begin{aligned} Max\ D = &\{\langle \chi(4.8) + (1-\chi)(14.5) + (1-\chi)(2.3) \rangle \langle 0.25 \rangle\} x_1 + \{\langle \chi(2.3) + (1-\chi)(4.8) + (1-\chi)(4.8) \rangle \langle 0.25 \rangle\} x_2 + \\ &\{\langle \chi(1.8) + (1-\chi)(4.8) + (1-\chi)(3) \rangle \langle 0.25 \rangle\} x_3 + \{\langle \chi(1.8) + (1-\chi)(4.8) + (1-\chi)(3.2) \rangle \langle 0.25 \rangle\} x_4 + \\ &\{\langle \chi(2) + (1-\chi)(4.8) + (1-\chi)(1.8) \rangle \langle 0.25 \rangle\} x_5 \end{aligned}$$

$$\begin{aligned} Min\ MC = &\{\langle \chi(7) + (1-\chi)(10.5) + (1-\chi)(3.2) \rangle \langle 0.25 \rangle\} x_1 + \{\langle \chi(3) + (1-\chi)(8.7) + (1-\chi)(2) \rangle \langle 0.25 \rangle\} x_2 + \\ &\{\langle \chi(3) + (1-\chi)(4.5) + (1-\chi)(2) \rangle \langle 0.25 \rangle\} x_3 + \{\langle \chi(2) + (1-\chi)(7) + (1-\chi)(1.8) \rangle \langle 0.25 \rangle\} x_4 + \\ &\{\langle \chi(1.3) + (1-\chi)(3.2) + (1-\chi)(2) \rangle \langle 0.25 \rangle\} x_5 \end{aligned}$$

subject to,

$$\begin{aligned} &\{\langle \chi(6.5) + (1-\chi)(11.7) + (1-\chi)(4.8) \rangle \langle 0.25 \rangle\} x_1 + \{\langle \chi(1.8) + (1-\chi)(5.5) + (1-\chi)(3.2) \rangle \langle 0.25 \rangle\} x_2 + \\ &\{\langle \chi(1.8) + (1-\chi)(5.5) + (1-\chi)(1.8) \rangle \langle 0.25 \rangle\} x_3 + \{\langle \chi(3) + (1-\chi)(5.5) + (1-\chi)(4.8) \rangle \langle 0.25 \rangle\} x_4 + \\ &\{\langle \chi(2) + (1-\chi)(4.8) + (1-\chi)(1.8) \rangle \langle 0.25 \rangle\} x_5 \leq | \geq \{\langle \chi(6.5) + (1-\chi)(11.7) + (1-\chi)(4.8) \rangle \langle 0.25 \rangle\} \end{aligned}$$

$$\begin{aligned} &\{\langle \chi(10.3) + (1-\chi)(8.7) + (1-\chi)(3.2) \rangle \langle 0.25 \rangle\} x_1 + \{\langle \chi(7) + (1-\chi)(3.2) + (1-\chi)(2) \rangle \langle 0.25 \rangle\} x_2 + \\ &\{\langle \chi(4.8) + (1-\chi)(3) + (1-\chi)(1.8) \rangle \langle 0.25 \rangle\} x_3 + \{\langle \chi(7) + (1-\chi)(3) + (1-\chi)(2) \rangle \langle 0.25 \rangle\} x_4 + \\ &\{\langle \chi(3.2) + (1-\chi)(3) + (1-\chi)(2) \rangle \langle 0.25 \rangle\} x_5 \leq | \geq \{\langle \chi(10.3) + (1-\chi)(8.7) + (1-\chi)(3.2) \rangle \langle 0.25 \rangle\} \end{aligned}$$

$$\begin{aligned} &\{\langle \chi(5.5) + (1-\chi)(5.5) + (1-\chi)(7) \rangle \langle 0.25 \rangle\} x_1 + \{\langle \chi(3.2) + (1-\chi)(1.8) + (1-\chi)(5.5) \rangle \langle 0.25 \rangle\} x_2 + \\ &\{\langle \chi(1.8) + (1-\chi)(5.5) + (1-\chi)(2) \rangle \langle 0.25 \rangle\} x_3 + \{\langle \chi(3) + (1-\chi)(5.5) + (1-\chi)(1.8) \rangle \langle 0.25 \rangle\} x_4 + \\ &\{\langle \chi(2) + (1-\chi)(3) + (1-\chi)(3.2) \rangle \langle 0.25 \rangle\} x_5 \leq | \geq \{\langle \chi(5.5) + (1-\chi)(5.5) + (1-\chi)(7) \rangle \langle 0.25 \rangle\} \end{aligned}$$

$$\begin{aligned} &\{\langle \chi(5.5) + (1-\chi)(5.5) + (1-\chi)(2) \rangle \langle 0.25 \rangle\} x_1 + \{\langle \chi(3) + (1-\chi)(7) + (1-\chi)(1.8) \rangle \langle 0.25 \rangle\} x_2 + \\ &\{\langle \chi(2) + (1-\chi)(3) + (1-\chi)(5.5) \rangle \langle 0.25 \rangle\} x_3 + \{\langle \chi(3) + (1-\chi)(5.5) + (1-\chi)(2) \rangle \langle 0.25 \rangle\} x_4 + \\ &\{\langle \chi(2) + (1-\chi)(2) + (1-\chi)(5.5) \rangle \langle 0.25 \rangle\} x_5 \leq | \geq \{\langle \chi(5.5) + (1-\chi)(5.5) + (1-\chi)(5.5) \rangle \langle 0.25 \rangle\} \end{aligned}$$

$$\begin{aligned} &\{\langle \chi(1.8) + (1-\chi)(3) + (1-\chi)(14.5) \rangle \langle 0.25 \rangle\} x_1 + \{\langle \chi(3) + (1-\chi)(2) + (1-\chi)(5.5) \rangle \langle 0.25 \rangle\} x_2 + \\ &\{\langle \chi(2) + (1-\chi)(1.8) + (1-\chi)(2) \rangle \langle 0.25 \rangle\} x_3 + \{\langle \chi(3.2) + (1-\chi)(3) + (1-\chi)(3) \rangle \langle 0.25 \rangle\} x_4 + \\ &\{\langle \chi(2) + (1-\chi)(3) + (1-\chi)(2) \rangle \langle 0.25 \rangle\} x_5 \leq | \geq \{\langle \chi(3.2) + (1-\chi)(2) + (1-\chi)(14.5) \rangle \langle 0.25 \rangle\} \end{aligned}$$

Step 3: The ZLPP is formed from the above ranking which is converted into crisp LPP by taking the average for uncertain value and reliability value.
Therefore, the heptagonal NPZLPP is converted into crisp LPP for different χ values.
When χ = 0

$$Max\ BI = 8.13\,x_1 + 4.78x_2 + 3.88x_3 + 4.88x_4 + 3.43x_5$$
$$Max\ D = 8.53\,x_1 + 4.98x_2 + 4.03x_3 + 4.13x_4 + 3.43x_5$$
$$Min\ MC = 6.9\,x_1 + 5.48x_2 + 3.38x_3 + 4.53x_4 + 2.73x_5$$

subject to,

$$8.38\,x_1 + 4.48\,x_2 + 3.78\,x_3 + 4.03\,x_4 + 3.43\,x_5 \le / \ge 8.38$$
$$6.08\,x_1 + 2.73\,x_2 + 2.53\,x_3 + 2.63\,x_4 + 2.63\,x_5 \le / \ge 6.08$$
$$6.38x_1 + 3.78\,x_2 + 3.88\,x_3 + 3.78\,x_4 + 3.23\,x_5 \le / \ge 6.38$$
$$3.88\,x_1 + 4.53\,x_2 + 4.38x_3 + 3.88x_4 + 3.88\,x_5 \le / \ge 5.63$$
$$8.88\,x_1 + 3.88\,x_2 + 2.03x_3 + 3.13x_4 + 2.63\,x_5 \le / \ge 8.38$$

and $x_j \ge 0$.

Similarly, one may formulate for different χ values.
Step 4: By solving the above crisp LPP, one may get an optimum solution for different χ values and average for all the values of χ is given in table (6.5.3).

TABLE 3. Optimum solution for vehicle

Max BI = 8.20	**Max D = 8.49**	**Min MC = 7.06**
$x_1 = 0.34$	$x_1 = 0.79$	$x_1 = 0.73$
$x_2 = 0$	$x_2 = 0.36$	$x_2 = 0$
$x_3 = 0$	$x_3 = 0$	$x_3 = 0$
$x_4 = 1.11$	$x_4 = 0$	$x_4 = 0$
$x_5 = 0$	$x_5 = 0$	$x_5 = 0.72$

Hence, from the optimum solution, it is observed that the company may concentrate more in design.
Stochastic NPZLPP
Step 1: Convert the problem mentioned in (6.5.4) into stochastic LPP by taking the right hand side resource vector as burr type II distribution and considering different probability value. Thus the formulation is

$$Max\ BI = 8.13\,x_1 + 4.78x_2 + 3.88x_3 + 4.88x_4 + 3.43x_5$$
$$Max\ D = 8.53\,x_1 + 4.98x_2 + 4.03x_3 + 4.13x_4 + 3.43x_5$$
$$Min\ MC = 6.9\,x_1 + 5.48x_2 + 3.38x_3 + 4.53x_4 + 2.73x_5$$

subject to,

$$P(8.38\,x_1 + 4.48\,x_2 + 3.78\,x_3 + 4.03\,x_4 + 3.43\,x_5 \le \tilde{\upsilon}_1) \ge 0.98$$
$$P(6.08\,x_1 + 2.73\,x_2 + 2.53\,x_3 + 2.63\,x_4 + 2.63\,x_5 \le \tilde{\upsilon}_1) \ge 0.96$$
$$P(6.38x_1 + 3.78\,x_2 + 3.88\,x_3 + 3.78\,x_4 + 3.23\,x_5 \le \tilde{\upsilon}_1) \ge 0.95$$
$$P(3.88\,x_1 + 4.53\,x_2 + 4.38x_3 + 3.88x_4 + 3.88\,x_5 \le \tilde{\upsilon}_1) \ge 0.9$$
$$P(8.88\,x_1 + 3.88\,x_2 + 2.03x_3 + 3.13x_4 + 2.63\,x_5 \le \tilde{\upsilon}_1) \ge)0.98$$

and $x_j \ge 0$

Step 2: By taking different values for λ and θ the formulation when χ = 0 is

$$Max\ BI = 8.13\,x_1 + 4.78x_2 + 3.88x_3 + 4.88x_4 + 3.43x_5$$
$$Max\ D = 8.53\,x_1 + 4.98x_2 + 4.03x_3 + 4.13x_4 + 3.43x_5$$
$$Min\ MC = 6.9\,x_1 + 5.48x_2 + 3.38x_3 + 4.53x_4 + 2.73x_5$$

subject to,

$$8.38\,x_1 + 4.48\,x_2 + 3.78\,x_3 + 4.03\,x_4 + 3.43\,x_5 \le / \ge 8.51$$
$$6.08\,x_1 + 2.73\,x_2 + 2.53\,x_3 + 2.63\,x_4 + 2.63\,x_5 \le / \ge 6.7$$
$$6.38x_1 + 3.78\,x_2 + 3.88\,x_3 + 3.78\,x_4 + 3.23\,x_5 \le / \ge 7$$
$$3.88\,x_1 + 4.53\,x_2 + 4.38x_3 + 3.88x_4 + 3.88\,x_5 \le / \ge 6.3$$
$$8.88\,x_1 + 3.88\,x_2 + 2.03x_3 + 3.13x_4 + 2.63\,x_5 \le / \ge 8.51$$

and $x_j \ge 0$.

Similarly, one may formulate for different χ values.

Step 4: By solving the above crisp LPP for different χ values, one may get the optimum solution. By taking average for all the χ values, one may get the result which is given in table below

TABLE 4. Stochastic solution of brand image, design and maintenance cost

Max BI = 9	Max D = 9.02	Min MC = 7.39
$x_1 = 0.33$	$x_1 = 0.5$	$x_1 = 0.65$
$x_2 = 0$	$x_2 = 0.96$	$x_2 = 0$
$x_3 = 0$	$x_3 = 0$	$x_3 = 0$
$x_4 = 1.29$	$x_4 = 0$	$x_4 = 0$
$x_5 = 0$	$x_5 = 0$	$x_5 = 1.05$

Hence, from the stochastic solution, it is observed that the company may concentrate more on Designing.

The high network individuals prefer the superbikes irrespective of the cost of bike. On analyzing the further development of survey, it has been found that brand image and design have the maximum value as ascertained. With this stochastic NPZLPP gives the value as 9 and the normal solution is 8.20. Similarly, the value for design obtained is 9.02 and 8.49 respectively by the two methods. Hence, the researcher concludes that brand image and design are main criteria whereas it is also found that the customers are much inclined to lower the maintenance cost which shows that value is 7.39 and 7.06 for the two methods.

CONCLUSION

In this chapter NPZNs is defined and stated some theorems based on logical and mathematical operations. θ – cut of NPZNs are defined and theorems on operations of θ – cut of NPZNs are proved. The researcher has also undergone survey to get the details of two wheelers among fifty different persons. The findings of this research enable the optimum utilization of available resources which increases the efficiency of production provided by the company. This research work may be extended for the other works like production in various other fields in order to get better profit.

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