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Solving multi-criteria group decision making problems under environments with a high number of alternatives using fuzzy ontologies and multi-granular linguistic modelling methods

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ABSTRACT

Classic multi-criteria group decision making models that have a high amount of alternatives are unmanageable for the experts. This is because they have to provide one value per each alternative and criteria. In this paper, we focus on solving this issue by carrying out multi-criteria group decision making methods using a different novel approach. Concretely, fuzzy ontologies reasoning procedures are used in order to automatically obtain the alternatives ranking classification. Thanks to our novel methodology, experts only need to provide the importance of a small set of criteria values making it possible for experts to perform multi-criteria group decision making procedures that have a high amount of alternatives without having to directly deal with them. Furthermore, in order to allow experts to provide their preferences in a comfortable way, multi-granular fuzzy linguistic modelling is used in order to allow each expert to choose the linguistic label set that better fits him/her.

1. Introduction

Multi-criteria group decision making has been a quite important field from its appearance in the early 2000's until nowadays. It is basically a combination between group decision making [40,41,44,50,51] and multi-criteria decision making fields [38,39,43]. Therefore, multi-criteria group decision making problems consist in creating an alternatives ranking using the preferences of a set of experts about a set of alternatives according to certain criteria values. Consequently, this area of research responds to the necessity that group decision making problems have of being able to rank alternatives according to a certain pre-specified criteria. This way, for each possible criteria value, experts create an alternatives ranking using a different set of preferences. Although experts have to provide more information, it is much easier for them to provide their preferences about a single criteria value than doing it globally for a set of them. Also, multi-criteria group decision making methods have the advantage of being able to make decisions controlling the importance that is given to each criteria value. In group decision making methods this was not possible since each expert provides his/her preferences according to his/her own criteria importance scale. Therefore, it can be affirmed that multi-criteria group decision making methods provide more means to stablish which rules will be followed for calculating the final decision making results than group decision making methods.

Multi-criteria group decision making is a field that is quite present in the recent literature. For instance, in [42], a novel framework model that is capable of dealing with multi-criteria group decision making environments is defined. For this purpose, new consensus and selection processes are proposed. In [45], a novel consensus method that is designed for its application in multi-criteria group decision making process is presented. In [19], a novel framework for multi-criteria group decision making method that uses the fuzzy Choquet integral is used. Finally, in [29], a multi-criteria group decision making method that uses neutrosophic sets for representing the provided information is presented.

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An important setting that must be established in multi-criteria group decision making methods is the determination of the criteria importance values. It is clear that not all the criteria values have the same importance but to determine the exact amount of it can become a very challenging task. One way of solving this issue that is particularly practical and adequate for multi-criteria group decision making methods is to let the experts do it. Since experts are the ones making the decision, it is logical that this decision fall on them. In the novel developed method presented, a group decision making method is used in order to carry out this task.

In a high amount of multi-criteria group decision making methods, values that each alternative holds for each criteria are well-known. For example, imagine that experts from a big company are elucidating where they should go to have dinner and the criteria that they want to discuss are the price, localization, type of food and general Internet users punctuation. In this case, it is clear that all the alternatives criteria values are well-known and, therefore, it is not necessary to ask users to provide values comparing each pair of alternatives. In this kind of decision problems, final decision results are reached when the importance given to each criteria is decided. It is important to notice that, although criteria values for each of the alternatives are well-known, the importance given to each criteria is a subjective matter that cannot be elucidated automatically.

One of the main problems that multi-criteria group decision making methods has nowadays is the impossibility of dealing with a high amount of preferences. With the appearance of Web 2.0 technologies, the amount of information and ideas that the experts can decide among have increased exponentially. In these cases, experts must choose among a high amount of alternatives and sort all of them according to a set of criteria. It can be seen that this process can become unmanageable for the experts due to the high amount of information that they have to deal with at the same time. For solving this issue, the proposed developed method uses fuzzy ontologies in order to automatize the process and allow experts to focus only on providing the importance that must be given to each criteria.

In this paper, we develop a novel multi-criteria group decision making method that uses multi-granular fuzzy linguistic modelling for improving human-computer communication and fuzzy ontologies in order to release experts from the alternatives information providing step. Thanks to fuzzy ontologies, experts can focus on deciding the importance that has to be given to each criteria value. Once that this process is done, the final ranking of alternatives is generated automatically using the criteria information associated to the alternatives that is stored using a fuzzy ontology. Because information is stored and an automatic fuzzy ontology reasoning process is carried out, the novel developed method is suitable for environments where a high amount of alternatives are dealt with. Since, in these cases, it is impossible for experts to directly manage the high amount of available alternatives, there is a clear need of methods like the presented one that automatize the process and help them to carry out this task in an organized and comfortable way.

In Section 2, basis of the used tools used to solve the dealt problem are introduced. In Section 3, a novel multi-criteria group decision making method where fuzzy ontology reasoning and multi-granular fuzzy linguistic modelling methods are used is presented. In Section 4, our method is tested in an illustrative example. In Section 5, advantages and drawbacks of this method is discussed. Finally, some conclusions are pointed out.

2. Preliminaries

In order to make this paper as self-contained as possible, this section will introduce concepts and methods to be referred to throughout this paper. In Section 2.1, how multi-criteria group decision making methods work are explained. In Section 2.2, how to deal with multi-granular linguistic information is shown. Finally, in Section 2.3, fuzzy ontologies specifications are described.

2.1. Multi-criteria group decision making

A typical multi-criteria group decision making problem that uses preference relations can be defined as follows:

Let X be a set of alternatives, C a set of criteria values and E a set of experts. A multi-criteria group decision problem consists in creating a ranking on X based on the preferences values provided by the experts: p_{lk}^{fj}. p_{lk}^{ij} refers to the preference value of the alternative x_l over x_k based on the criteria c_f that has been provided by the expert e_j.

Steps followed by classical multi-criteria group decision making methods are exposed below:

1. **Providing user preferences:** Experts provide their preferences about each of the alternatives according to a specific criteria. In the case of using utility values, one preference value is provided for each alternative and criteria. If preference relations want to be used, a preference value for each pair of alternatives and criteria must be provided.
2. **Collective information calculation:** Once that all the experts have provided their preferences, information is aggregated into a single matrix representing all the preferences provided by the set of experts. There are several ways of carrying out this process. For instance, in [38], the collective matrix that is used by the selection process stores the collective preference value for each alternative and criteria. When using preference relations, the collective matrix generally stores information about the preference value of each pair of alternatives. In this case, the collective preference matrix can be calculated carrying out two different aggregations processes:
   - *User preferences aggregation*: Preference relation for all the experts are aggregated into a set of preferences matrices representing the value preferences of all the experts for each pair of alternatives and each criteria value.
   - *Criteria preference relation aggregation*: The elements of the set of preferences, each one representing the preferences of each criteria, can be aggregated into a single matrix representing the preferences values for each pair of alternatives and for all the criteria. In order to carry out this process in a fair way, it is important to know the importance level that should be given to each criteria value.
3. **Selection process:** Once that the collective preference information matrix has been calculated, a selection process is carried out in order to obtain the final ranking result. There is a wide range of methods that can carry out this kind of operation. For instance, in [38], comparisons among clouds operator is used [37]. In [30], the mean among the dominance choice degree (GDD) and the non-dominance choice degree (GNDD) of the preference relation is used.

A scheme of this process is shown in Fig. 1. For our designed method, preference relations have been chosen as the preference representation method.

When carrying out group decision making processes, it is desirable that experts reach a consensus before making a final decision [16]. In order to achieve this objective, consensus measures [9] can be used. A typical group decision making method that uses consensus measures is carried out as follows:

1. **Providing user preferences**: Experts provide their preferences to the system.
2. **Measuring consensus**: Consensus is calculated over the preferences provided by the experts. If the consensus is low, experts

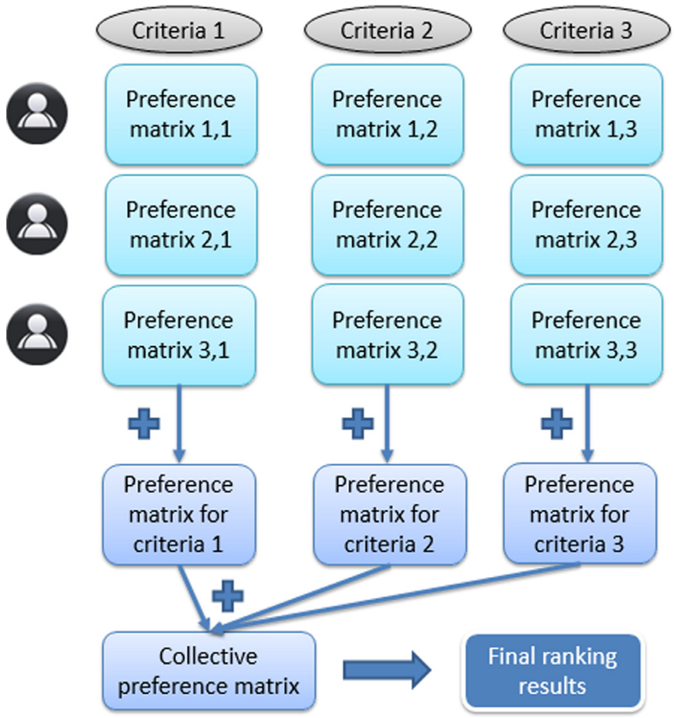

Fig. 1. Multi-criteria group decision making scheme using fuzzy preference relations.

are asked to modify their preferences and carry out more debate. If the consensus is high, experts have reached a consensus about the dealt topic. Therefore, there is no need for experts to modify their preferences or postpone the final decision.

3. **Making final decision**: Final decision results are calculated.

2.2. Multi-granular linguistic information

As we have stated in the introduction, it is extremely important to provide means in order to allow experts to communicate with the computational system in a comfortable way. Since computers carry out calculations using numbers and humans communicate among themselves using words there is a need for tools that are capable of reducing this gap.

Linguistic Modelling works under the linguistic variable concept defined by Zadeh [47–49]. A linguistic variable is a *variable whose values are not numbers but words in a natural or artificial language*. Formally, it is possible to define a linguistic variable X as a 5-tuple $\langle L, T(L), U, S, M\rangle$ where L is the variable name, $T(L)$ a finite set of labels, U the universe of discourse, S a set of syntactic rules that generate all the terms in the set $T(L)$ and M the semantic rule associating a meaning to each of the labels stored in the linguistic label set. Generally, $M(X)$ denotes a fuzzy subset of U and is defined by its membership function $\mu_{M(X)}$: $U \rightarrow [0, 1]$ where $\mu_{M(X)}(z)$ is called the membership degree of the element z to a fuzzy set $M(X)$ where $z \in U$ [46]. Now that Internet is available for a high amount of the world population, linguistic modelling is becoming more important than ever. Some recent linguistic modelling applications can be seen in [14,33,34].

When several experts have to provide information to the same system, linguistic modelling forces them to use labels from the same linguistic label set. This can become a problem since it is not probable that all the experts feel comfortable with the chosen configuration. Therefore, it would be desirable to allow each expert to choose the linguistic label set that make him/her feel more comfortable instead of fixing the granularity into an specific value. This way, if an expert knows a lot about the dealt topic, it is possible for him to choose a high granularity value and provide precise information. On the contrary, if the expert want to provide more imprecise information, he/she can do it by choosing a linguistic label set with a low granularity value. In order to solve this issue and allow each expert to use their own linguistic label set, multi-granular fuzzy linguistic modelling methods can be used.

Multi-granular fuzzy linguistic modelling methods are capable of dealing with labels belonging to different linguistic label sets. These methods transform all the heterogeneous information into labels belonging to the same linguistic label set in order to carry out computations among them. A scheme of this process can be seen in Fig. 2.

Multi-granular fuzzy linguistic modelling methods have been widely used among different areas [12,31,32,36], being group decision making one of the main ones. Some recent multi-granular fuzzy linguistic models can be found in [10,22]. A typical group decision making process that uses multi-granular fuzzy linguistic modelling methods is carried out using the following steps:

1. **Providing user preferences:** Each expert provides his/her preferences to the system using the linguistic label set that better fits his/her necessities.
2. **Information standardization:** All the labels are transformed into labels from the same linguistic label set.
3. **Applying group decision making processes:** Once that the information has been uniformed it is possible to carry out the group decision making process as exposed in Section 2.1.

In this paper, the multi-granular fuzzy linguistic modelling method exposed in [18] is the one chosen for homogenizing the information. This method uses linguistic hierarchies and 2-tuple linguistic information [17] in order to carry out the required process.

Linguistic hierarchies can be built using a set of levels where each level, $l(t, n(t))$, is represented by a different linguistic label set. t indicates the level of the hierarchy and $n(t)$ the granularity value of the associated linguistic label set. The hierarchy is built in a way that, the higher the level, the higher granularity value the associated linguistic label set has. Therefore, a linguistic hierarchy can be defined as the union of all its levels t as follows:

$$LH = \bigcup_t l(t, n(t)) \tag{1}$$

A linguistic 2-tuple is defined as a tuple (s, α) where s is a linguistic label and $\alpha \in [-0.5, 0.5)$ is called the symbolic translation. If we call β to the aggregation result of the indexes of labels that are part of the same linguistic label set and $i = round(\beta)$, then the symbolic translation is computed as $\alpha = \beta - i$. Therefore, α indicates the distance from the obtained numerical aggregation value to the closest label in the linguistic label set. In order to convert any β to the 2-tuple form (s, α), the following operator can be used:

$$\Delta : [0, g] \rightarrow S \times [-0.5, 0.5)$$
$$\Delta(\beta) = (s_i, \alpha) \ with \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-0.5, 0.5) \end{cases} \tag{2}$$

In the same way, (s, α) can be expressed in the β form as follows:

$$\Delta^{-1} : S \times [-0.5, 0.5) \rightarrow [0, g]$$
$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta \tag{3}$$

Using linguistic hierarchies and the 2-tuple representation information, it is possible to define the following multi-granular

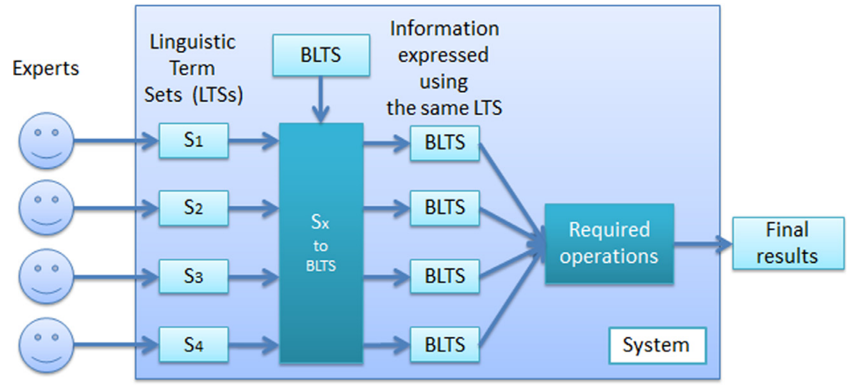

Fig. 2. Multi-granular linguistic modelling scheme.

transformation function:

$TF_t^{t'} : l(t, n(t)) \rightarrow l(t', n(t'))$

$$TF_t^{t'}\left(s_i^{n(t)}, \alpha^{n(t)}\right) = \Delta\left(\frac{\Delta^{-1}\left(s_i^{n(t)}, \alpha^{n(t)}\right) \cdot (n(t') - 1)}{n(t) - 1}\right) \tag{4}$$

Therefore, applying this expression, labels from each level t can be transformed into labels that belong to the linguistic label sets from the level t'.

2.3. Fuzzy ontologies

Ontologies are tools whose purpose is to store information in an organized way in order to analyse it and extract conclusions. An ontology is formed by the following items:

- **Individuals:** They are entities that can be described using concepts.
- **Concepts:** They are perceptions that are used to describe the individuals that conform the ontology.
- **Relations:** Their main purpose is to establish relationship connections among individuals and individuals with concepts. Thanks to them, it is possible to describe the individuals using concepts.
- **Axioms:** They are statements that establish rules that must be always fulfilled by all the elements conforming the fuzzy ontology.

Crisp ontologies are based on description logics. Therefore, they are not able to deal conveniently with imprecise information [21]. In order to improve ontologies representation capabilities and be able to provide accurate representation to imprecise information, fuzzy ontologies were developed. Bobillo [5] defines a fuzzy ontology as follows: “A fuzzy ontology is simply an ontology which uses fuzzy logic to provide a natural representation of imprecise and vague knowledge and eases reasoning over it”. Formally, a fuzzy ontology can be defined as follows:

A fuzzy ontology [2,13] is a quintuple $O_F = \{I, C, R, F, A\}$ where I represents the set of individuals, C the set of concepts, R the set of relations, F a set of fuzzy relations and A the set of axioms. A representation scheme of a fuzzy ontology is shown in Fig. 3.

Fuzzy ontologies introduce the concept of fuzzy relations, F. These kind of relations allow individuals to be related with concepts or other individuals with a certain degree, normally expressed in the interval [0, 1]. This way, a fuzzy membership function can be applied in order to establish the imprecision relation [46].

Fuzzy ontologies are becoming wide-spread and popular in the recent literature. For instance, in [8], Bobillo and Straccia extends

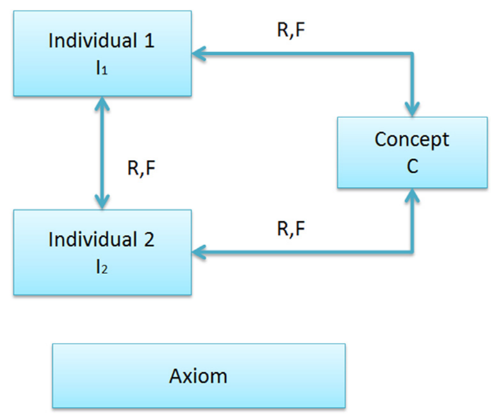

Fig. 3. Fuzzy ontology scheme.

their *fuzzyDL* software [6] with features for handling fuzzy integrals. In [15], Díaz-Rodríguez et al. use fuzzy ontologies in order to model and recognize human behaviour under certain circumstances. In [25], Morente-Molinera et al. use fuzzy ontologies in order to design an automatic process that is capable of retrieving information from users and create a knowledge database. In the decision making area, some examples can be found in [24,26,27].

Fuzzy ontologies are a very useful tool for sorting and retrieving information. With the recent appearance of the Big Data field, it is more important than ever to develop tools that help us to carry out data mining processes [1,20,35].

3. A novel multi-criteria group decision making method using fuzzy ontologies

In this section, the steps followed by the novel multi-criteria group decision making method designed are described. The developed method sets the importance of each criteria value using the experts preferences. Afterwards, it retrieves the final alternatives ranking using an automatic reasoning process. This way, experts do not have to carry out unnecessary comparisons. The steps followed are described in more detail below:

1. **Fuzzy Ontology creation**: Fuzzy ontology is created using information about alternatives and criteria.
2. **Carrying out a Group Decision Making process over criteria values**: Importance of each criteria is discussed in an organized way using a Group Decision Making process.
3. **Calculating the criteria ranking**: The results of the Group Decision Making process are transformed into a fuzzy ontology query.

4. **Retrieving information from the ontology**: The query is perform and the preferred alternatives are retrieved.
5. **Presenting final results**: Alternatives are shown to the user. They can be rank according to the query similarity.

In the following subsections, all these steps are described in more detail.

3.1. Fuzzy ontology creation

In this first step, a fuzzy ontology must be created using the alternatives as ontology individuals and the criteria as concepts. Specifications for each of the alternatives according to each of the criteria values must be introduced. If the description needed is objective, that is, the individual is described by the criteria value in a way that cannot generate any debate, the data can be directly searched and stored in the fuzzy ontology. For instance, if a criteria value establishes the size of an alternative, this piece of information is concrete and the degree of fulfilment of each of the alternatives can be easily calculated without any experts intervention.

If the description needed is subjective, that is, the value depends on how the experts perceive the world, data cannot be provided directly and a process similar to the one exposed in [25] can be followed. In conclusion, when a criteria value is not objective, experts would need to carry out a group decision making process centered on it in order to determine the degree of fulfilment that each alternative has according to that criteria value.

In order to avoid this process, it is possible to retrieve this kind of subjective information from the Internet using users assessments webpages. There is plenty of subjective information provided by a high amount of users available on the Internet that can be used in order to fill the ontology required information. This process can be carried out by some information retrieving algorithm that is able to automatically extract the required webpage data. Internet webpages that use a social network for sharing opinions among buyers are a very interesting target for this process. For instance, if experts working in a high school are choosing which computers they should buy for the classrooms, they can use experts opinions from a computer store webpage as one of the criteria values. An automatic process capable of extracting users opinions and objective information of the computers from the webpage should be designed in order to retrieve all the information needed to fulfil the fuzzy ontology. This process just needs to retrieve the webpage information about the alternatives and parse the text in order to find the required information.

It is important to notice that the developed method is more efficient and require less experts participation when the criteria that is being discussed is objective or the information can be retrieved from the Internet since, in this case, no experts intervention is needed.

In order to create a fuzzy ontology, there are some tools that can be used. For instance, the combination of OWL and its annotations are one approach of carrying out the necessary fuzzy ontology description [7]. In combination with the FuzzyDL software [6], it is possible to build a fully working fuzzy ontology about the required topic. Thanks to this tools, it is possible to design and work in a comfortable way with fuzzy ontologies as it they were crisp ones.

3.2. Carrying out a group decision making process over criteria values

Once that the fuzzy ontology is created, experts provide their preferences about the criteria values in order to establish the importance that should be given to them in the alternatives ranking process. For this purpose, a group decision making process over the criteria values is carried out. Multi-granular fuzzy linguistic modelling methods and consensus measures are used in order to improve the human-computer communication and increase the agreement level in the decision process respectively. The next steps are followed:

1. **Providing user preferences:** Experts provide their preferences about the criteria values using a preference relation. For this process, experts can use the linguistic label set that they prefer.
2. **Collective preference matrix calculation:** Preferences of the experts are aggregated into a single collective preference matrix representing all their opinions.
3. **Preferences uniformation**: All the preferences provided by the experts are uniformed and expressed using the same linguistic label set. Transformation function exposed in (4) can be used for this purpose.
4. **Measuring consensus**: Using the preferences provided by the experts, consensus measures are calculated. Thanks to them, it is possible to observe if the experts have reach an agreement or if they have contrary opinions. If the consensus is below a certain threshold, experts are asked to modify their preferences in order to improve their agreement. If consensus is high enough, selection process is carried out in order to compute the final ranking of the criteria values. When using preference relations, consensus measures can be calculated in three different levels:
   - Among every two alternatives.
   - For each of the alternatives.
   - Overall decision making process consensus value.

   Similarity value between every two pair of experts, i and j, and their preferences for the specific alternatives x_l and x_k can be computed using the following expression [11]:

   $$sm_{ij}^{lk} = s(p_i^{lk}, p_j^{lk}) = 1 - |p_i^{lk} - p_j^{lk}| \tag{5}$$

   where p_i^{lk} indicates the preference relation value of expert i for alternatives x_l and x_k and m refers to the number of experts.
   Once that sm matrices have been calculated, the collective consensus matrix for each pair of alternatives can be computed as:

   $$cm^{lk} = \phi(sm_{ij}^{lk}) \;\; i, j = 1, \dots, m; \;\; l, k = 1, \dots, n; \;\; i < j \tag{6}$$

   where ϕ refers to the mean operator and n is the number of alternatives.
   Using the already calculated cm matrices, we can calculate the different consensus levels values as [23]:

   (a) *Pair of alternatives level*: This level represents the consensus reached between each pair of alternatives is calculated. The value, cp_{lk}, for the alternatives x_l and x_k is determined as follows:

   $$cp^{lk} = cm^{lk}, \;\; \forall l, k = 1, \dots, n; \;\; l \neq k \tag{7}$$

   (b) *Alternative level*: Consensus degree ca^l for each of the alternatives, x_l, can be calculated as follows:

   $$ca^l = \frac{\sum_{k=l, k\neq l}^{n}(cp^{lk} + cp^{lk})}{2(n-1)} \tag{8}$$

   (c) *Decision level*: Using ca^l values, the overall consensus value that has been reached by experts in the decision making process can be computed as:

   $$cr = \frac{\sum_{l=1}^{n} ca^l}{n} \tag{9}$$

   Also, proximity values [11], can be calculated in order to provide the experts with suggestions about how to modify their preferences in order to reach a consensus. Experts are not forced to follow these suggestions. Therefore, it is up to them to follow or discard them.

5. **Calculating the resulting ranking**: When the consensus is high enough, final ranking of the criteria values is calculated. For this purpose, the mean between the GDD and GNDD operators can be used [30]. GDD operator can be calculated from the collective preference matrix as follows:

$$GDD_i = \phi(c_{i1}, c_{i2}, \ldots, c_{i(i-1)}, c_{i(i+1)}, \ldots, c_{in}) \quad (10)$$

Alternatively, GNDD operator can be calculated using the following expression:

$$GNDD_i = \phi(c^s_{1i}, c^s_{2i}, \ldots, c^s_{(i-1)i}, c^s_{(i+1)i}, \ldots, c^s_{ni}) \quad (11)$$

where

$$c^s_{ji} = \max\{c_{ji} - c_{ij}, 1\}$$

Therefore, the final ranking values, RV, are calculated as follows:

$$RV = (GDD_i + GNDD_i)/2, \forall i \in [0, m] \quad (12)$$

Finally, criteria values are sorted using the RV values calculated.

3.3. Calculating the criteria ranking

Once that criteria values have been ranked, a weighting vector is computed in order to use it in the fuzzy ontology searching step. The most voted criteria should be the one having the highest weighting value while the least voted criteria should be the one having the lowest one. In this paper, we propose to use the next formula in order to calculate the weight vector:

$$W_j = \frac{j}{\sum_{i=1}^{n} i}, j \in \{1, n\} \quad (13)$$

where n is the number of criteria values and j indicates the position in the ranking being 1 the least voted option and n the most voted one. It is important to point out that expression (13) generates equidistant weighting values for the criteria values according to their position in the ranking.

Example 2. Having a set of sorted criteria values $F = \{f_1, \ldots, f_5\}$ according to the preference values given by a set of experts, the associated weighting vector for the fuzzy ontology searching step can be calculated as follows:

$$W = \left\{\frac{1}{15}, \frac{2}{15}, \frac{3}{15}, \frac{4}{15}, \frac{5}{15}\right\} = \{0, 0.06, 0.13, 0.2, 0.26, 0.33\} \quad (14)$$

It can be seen that $\sum W = 1$ which is the important restriction that all weighting vectors must fulfil.

3.4. Retrieving information from the ontology

Once that the weighting vector values have been calculated from the experts ranking, the ontology reasoner is used in order to retrieve the alternatives that better fulfill the criteria according to the criteria importance calculated. In order to carry out this process, a fuzzy ontology query is performed using all the criteria values and the weights that the group decision making process have elucidated. Only the λ best alternatives are retrieved where λ is a predefined value establishing how many results will be showed to the experts.

The steps that are followed in order to perform a query are exposed below:

1. **Providing user query**: The expert provides the query to the ontology management system. All the information that he/she wants to obtain is specified on it. Different weight values can be given to each of the concepts.
2. **Searching in ontology**: Each individual of the ontology is compared with the data from the query that has been provided by the user. Individuals that fulfill the query over a certain previously established degree are stored and ordered in a ranking.

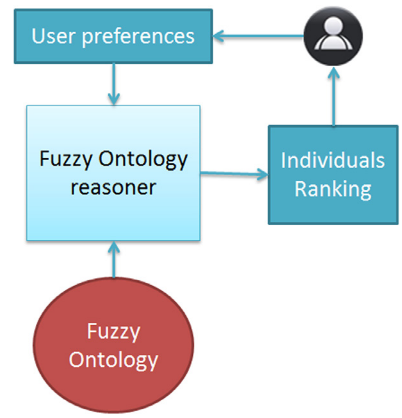

Fig. 4. Fuzzy ontology query process.

Table 1
Smartphones similarity calculation example.

Smartphone	Low_SS	Medium_C	High_M	Similarity value
s_1	0.9455	0.7244	0.9647	0.8782
s_2	0.25	0.5	0.25	0.33
s_3	0	0	0.8258	0.2752

3. **Presenting results**: The ranking generated in the previous step is showed to the user.

In Fig. 4, a graphical scheme of this process is showed.
Example. Having the following ontology:

- **Set of individuals**: Conformed by 200 different smartphones.
- **Set of concepts**: The concepts used are the screen size, capacity and microprocessor speed. Their values are all specified using the linguistic label set $S = \{Low, Medium, High\}$.
- **Fuzzy relationships**: All the individuals are related to each concept. There is no relation among individuals.

A user can select a smartphone from the ontology following the next steps:

1. The user provides the features that he/she wants the smartphone to has. The user indicates that he/she is looking for an smartphone that has a *high* screen size, a *medium* capacity and a *high* microprocessor speed.
2. Once that the query has been provided, similarity measures are applied over the query and all the 200 smartphones located in the ontology. In Table 1, similarity values are calculated for some of the smartphones in the ontology. Membership values for each of the smartphones to each label of the concepts selected by the users are also shown. Low_SS, Medium_C and High_M refers to low screen size, medium capacity and high microprocessor respectively. For instance, for s_1, similarity value is calculated using the weighted mean operator as follows:

$$similarity_{s1} = 0.9455 \cdot 0.33 + 0.7244 \cdot 0.33 + 0.9647 \cdot 0.33$$
$$= 0.8782$$

It is important to notice that the same importance has been given to each concept.
3. Smartphones are sorted and a ranking of the most similar smartphones is presented to the user. In the presented example, s_1 is the best choice since it has the highest ranking value.

3.5. Presenting final results

The retrieved alternatives are sorted according to their importance and provided to the experts. The first one on the list is the

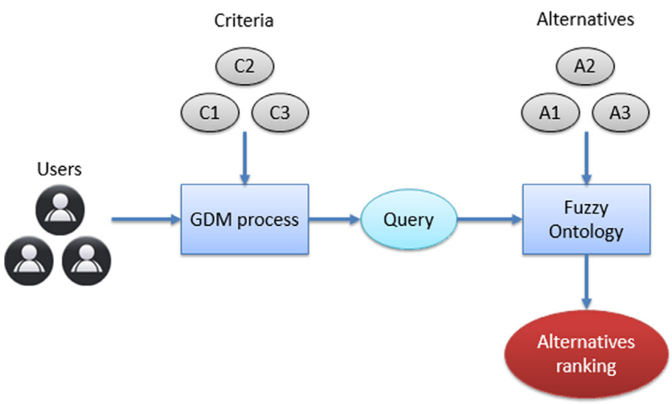

Fig. 5. Developed method working scheme.

best alternative. Nevertheless, optionally, experts could decide to carry out another group decision making process over the retrieved alternatives if they want to refine personally the automatically retrieved results.

A brief graphical scheme of this process can be seen in Fig. 5.

4. Illustrative example

In this section, we will apply the designed method in order to solve a real world problem. Imagine that a set of experts want to buy a wine in order to bring it to a formal dinner. In this example, experts have to choose the specific wine according to three different characteristics:

- **Alcohol**: The level of alcohol of the wine. Information is represented using a linguistic label set of 3 labels. Therefore, each wine is related to each of the labels using a value indicating the membership degree in the interval [0,1].
- **Acidity**: Indicates the degree of acidity of the wine. In order to represent the information, the alcohol approach has been followed.
- **Suitability for a formal dinner**: Indicates the adequateness of the wine for being used in a formal dinner. Since there is not specific parameters, this concept relation values are defined subjectively extracting opinions of users from the Internet. One way of achieving this goal is following the guidelines exposed in [25].

In the fuzzy ontology used, there are 623 wines that the experts can choose among. In order to carry out the multi-criteria group decision making process, the following steps can be performed:

1. **Group decision making process over the criteria**: Experts carry out a group decision making process over the criteria values. In this case, the criteria values conforms the following set:

   $$C = \{LA, MA, HA, LAC, MAC, HAC, LFD, MFD, HFD\}$$

   where $\{LA, MA, HA\}$, $\{LAC, MAC, HAC\}$ and $\{LFD, MFD, HFD\}$ are the linguistic label sets representing the different levels of alcohol, acidity and price respectively. LX represents low values, MX medium values and HX high values and X in the values name indicates the represented concept. It should be noticed that the three concepts taken into account in this example are objective, that is, each alternative can be described using then without any expert advice. Also, since they represent gradual information, there is a need to store several criteria values for each concept. Each one represents different quantity levels of the original concept. It should be pointed out that this process is not always necessary. For example, if we were talking about smartphones and high definition screens, an unique criteria value, $\{HD\_Screen\}$, will be sufficient to expose the concept.
2. **Calculating the weight of the criteria**: Once that the experts have ranked the criteria values, weights must be assigned to each of them. In this example, two steps must be followed:
   (a) *Selecting the criteria values:* Since there are several criteria values representing different degrees for the same concept, only the most voted criteria value for each category should remain. This is because it is not logical to search for a wine that, for example, has a low and a high alcohol level at the same time. For instance, if the following criteria ranking is obtained:

   $$R = \{HA, MAC, LA, LFD, HAC, MFD, MA, LAC, HFD\}$$

   The weights are calculated over the following ranking:

   $$R_* = \{HA, MAC, LFD\} \tag{15}$$

   Since they are the criteria values most voted for each concept.
   (b) *Assigning weights:* Once that the reduced set of alternatives needed for this example is calculated, weights are assigned. For this purpose, expression exposed in (13) can be used. Afterwards, a fuzzy ontology query is generated using the ranking and the weight information. In the previous example the query could be expressed as follows:

   $$QFO = \{0.5 \cdot HA, 0.33 \cdot MAC, 0.166 \cdot LFD\}$$
3. **Calculating the alternatives ranking**: The query generated in the previous step is performed over the fuzzy ontology and alternatives that better fulfil it are returned to the experts. In this example, the query QFO is run over the 623 wines from the ontology and best results are presented to the experts. As mentioned in the previous section, it is optional to carry out a group decision making process over the returned alternatives.

In order to provide better understanding of this process, we are going to show a brief numerical example below. For the sake of clarity and comprehensibility, we are going to suppose that four experts, $E = \{e_1, e_2, e_3, e_4\}$, want to select a wine with a high alcohol level, high acidity and high suitability for a formal dinner. These parameters have been chosen for the example to remain brief and clear. The first step that have to be carried out is a group decision making process over the set of criteria $C = \{HA, HAC, HFD\}$. Preferences provided by the experts are showed below:

$$P_1 = \begin{pmatrix} - & s^3_3 & s^3_3 \\ s^3_2 & - & s^3_1 \\ s^3_2 & s^3_1 & - \end{pmatrix} P_2 = \begin{pmatrix} - & s^5_1 & s^5_3 \\ s^5_5 & - & s^5_5 \\ s^5_3 & s^5_1 & - \end{pmatrix}$$

$$P_3 = \begin{pmatrix} - & s^7_2 & s^7_1 \\ s^7_7 & - & s^7_6 \\ s^7_3 & s^7_3 & - \end{pmatrix} P_4 = \begin{pmatrix} - & s^7_3 & s^7_4 \\ s^7_7 & - & s^7_7 \\ s^7_1 & s^7_1 & - \end{pmatrix}$$

Each expert has used a different linguistic label set in order to provide their preferences. Experts e_1, e_2 and $\{e_3, e_4\}$ have provided their preferences using linguistic label sets $S_3 = \{s_1, \ldots, s_3\}$, $S_5 = \{s_1, \ldots, s_5\}$ and $S_7 = \{s_1, \ldots, s_7\}$ respectively. The first step consists in transforming the experts preferences in order to be able to operate with them. Since S_7 is the most informative linguistic label set, transformations will be performed in order to express all the preferences using labels from that linguistic label set. After applying expression (4), the experts preferences are represented inside the system as follows:

$$P_1^7 = \begin{pmatrix} - & s_7^7 & s_7^7 \\ s_4^7 & - & s_1^7 \\ s_4^7 & s_1^7 & - \end{pmatrix} P_2^7 = \begin{pmatrix} - & s_1^7 & s_4^7 \\ s_7^7 & - & s_7^7 \\ s_4^7 & s_1^7 & - \end{pmatrix}$$

$$P_3^7 = \begin{pmatrix} - & s_2^7 & s_1^7 \\ s_7^7 & - & s_6^7 \\ s_3^7 & s_3^7 & - \end{pmatrix} P_4^7 = \begin{pmatrix} - & s_3^7 & s_4^7 \\ s_7^7 & - & s_7^7 \\ s_1^7 & s_1^7 & - \end{pmatrix}$$

Since, in all the cases, the symbolic translation is 0, we have omitted it in the matrices in order to make them easier to read. After all the information is expressed using the same means, consensus measures are applied in order to determine if experts have reached a consensus. 0.70 is set as the level of global consensus that experts have to reach in order to stop debate and calculate final decision making results. Expressions (7)–(9) are applied for calculating global consensus and consensus reached in each alternative. Consensus reached by experts in each alternative is showed below:

$$C_a = \{0.6111, 0.6319, 0.6319\} \tag{16}$$

Global decision consensus result is $C_g = 0.625$. It should be noticed that, in this example, consensus values are located in the [0,1] interval being 1 the highest consensus value.

Since global consensus is not high enough, experts are asked to modify their preferences. After another debate session, experts provide the following preferences:

$$P_1 = \begin{pmatrix} - & s_1^3 & s_2^3 \\ s_3^3 & - & s_3^3 \\ s_2^3 & s_1^3 & - \end{pmatrix} P_2 = \begin{pmatrix} - & s_1^5 & s_3^5 \\ s_5^5 & - & s_5^5 \\ s_3^5 & s_1^5 & - \end{pmatrix}$$

$$P_3 = \begin{pmatrix} - & s_2^7 & s_1^7 \\ s_7^7 & - & s_6^7 \\ s_3^7 & s_3^7 & - \end{pmatrix} P_4 = \begin{pmatrix} - & s_3^7 & s_4^7 \\ s_7^7 & - & s_7^7 \\ s_1^7 & s_1^7 & - \end{pmatrix}$$

These preferences relations are transformed in order to use the same linguistic label set. Results are shown below:

$$P_1^7 = \begin{pmatrix} - & s_1^7 & s_4^7 \\ s_7^7 & - & s_7^7 \\ s_4^7 & s_1^7 & - \end{pmatrix} P_2^7 = \begin{pmatrix} - & s_1^7 & s_4^7 \\ s_7^7 & - & s_7^7 \\ s_4^7 & s_1^7 & - \end{pmatrix}$$

$$P_3^7 = \begin{pmatrix} - & s_2^7 & s_1^7 \\ s_7^7 & - & s_6^7 \\ s_3^7 & s_3^7 & - \end{pmatrix} P_4^7 = \begin{pmatrix} - & s_3^7 & s_4^7 \\ s_7^7 & - & s_7^7 \\ s_1^7 & s_1^7 & - \end{pmatrix}$$

Consensus values for each alternative are recalculated using the new information. Results are shown below:

$$C_a = \{0.8194, 0.88888, 0.80555\} \tag{17}$$

Global decision consensus result is $C_g = 0.8379$ this time.

Since global consensus is above the specified threshold, 0.70, it means that experts have reached an agreement. Therefore, final decision making results are calculated. First, preferences of all the experts are aggregated into a single collective preference matrix:

$$P_C = \begin{pmatrix} - & (s_2^7, -0.25) & (s_3^7, 0.25) \\ s_7^7 & - & (s_7, -0.25) \\ s_3^7 & (s_1^7, 0.5) & - \end{pmatrix}$$

Table 2
Similarity calculation example.

Wine	HAC	HFD	HA	Similarity value
Zenato_veneto_roso	0	0.612	0.5	0.2665
Abadal_Cabernet_Sauvignon_R	1	0.18	1	0.8598
Chateau_dArmailhac	0	0	1	0.33

Finally, GDD and GNDD operators are applied in order to get a ranking of the criteria values. Expression (10) is applied in order to calculate the GDD values for each alternative:

$$GDD = \{0.3333, 0.8194, 0.3055\}$$

Alternatively, expression (11) is applied for calculating GNDD values:

$$GNDD = \{0.7083, 1, 0.6944\}$$

As it happened with consensus values, GDD and GNDD values are expressed in the interval [0,1] being 1 the value indicating the highest level of importance. After aggregating GDD and GNDD values, the following results are obtained:

$$DR = \{0.5208, 0.9097, 0.5\}$$

Therefore, ranking of the criteria values is as follows: $R = \{HAC, HA, HFD\}$ where HAC is the most voted criteria and HFD the least voted one.

In order to increase experts comprehensibility of the decision making results and consensus values, it is possible to show all this information using linguistic values. In order to carry out this process, the next two steps can be followed:

1. First, information expressed in the interval [0, 1] is expressed in the interval [0, g] where g indicates the granularity of the linguistic label set that we want to use to show the information.
2. Second, expression (2) is used in order to convert the numeric value into a 2-tuple value. Depending on the precision that wants to be provided to the experts, 2-tuple values or just the label can be provided.

In the case of the group decision making process carried out, the obtained information is presented to the users in a linguistic manner using 2-tuple representation and the linguistic label set S_7 as follows:

$$C_a = \{(s_6^7, -0.09), (s_6^7, 0.33), (s_6^7, -0.17)\}$$

$$C_g = (s_6^7, 0.274)$$

$$DR = \{(s_4^7, 0.12), (s_6^7, 0.4582), s_4^7\}$$

Now that experts have ranked the criteria values, the system builds a fuzzy ontology query in order to determine the alternatives ranking. Using expression (13) and the criteria values ranking $R = \{HAC, HA, HFD\}$, importance of each criteria value is established in a fuzzy ontology query as follows:

$$Q = \{0.5 \cdot HAC, 0.33 \cdot HA, 0.166 \cdot HFD\}$$

After establishing the query, similarity among the query and each of the 623 wines stored in the ontology is calculated. In Table 2, three different examples of how this value is calculated are shown. The next three calculations have been carried out respectively for each of the wines:

$$0 \cdot 0.5 + 0.612 \cdot 0.166 + 0.5 \cdot 0.33 = 0.2665$$

$$1 \cdot 0.5 + 0.18 \cdot 0.166 + 1 \cdot 0.33 = 0.8598$$

$$0 \cdot 0.5 + 0 \cdot 0.166 + 1 \cdot 0.33 = 0.33$$

Table 3
Best 4 alternatives.

Wine	Similarity value
Langa_Trilogia	0.996
La_Caliera_Moscato_dAsti	0.9693335
Montalto_Nero_dAvola	0.8582225
Remonte_Chardonnay	0.8435555

After performing this operation with all the wines in the fuzzy ontology, an alternative ranking is showed to the users with the wines having the highest similarity values. For this example, the top 4 wines are chosen. They are specified in Table 3.

Therefore, it is possible to conclude that *Langa_Trilogia* is the most desirable alternative for the experts. Since the four wines have very high similarity values, it is possible to carry out another group decision making process over the results in order to make a final choice.

Table 4
Developed method characteristics.

Desirable characteristics	Fulfilled?
It is able to work with a high number of alternatives.	Yes
Allow the use of preference relation matrices.	Yes
Do not bother experts with endless participation.	Yes
Experts only provide preferences on the criteria.	Only when no subjective information is needed.
Experts can select the linguistic label set that they prefer.	Yes
Best alternatives are selected automatically.	Yes, unless a group decision making process want to be carried out over the results.
Fuzzy Ontologies provides a good framework in order to manage the alternatives information.	Yes
Experts provide the alternatives information.	No, it should be extracted from somewhere else.

5. Discussion

In this paper, a novel method that is capable of working in multi-criteria group decision making environments is presented. Fuzzy Ontologies have been used in order to carry out part of the process automatically and allow experts to focus only on the importance of each of the available criteria values. Also, they provide a good framework in order to store alternatives related information and characteristics. If traditional multi-criteria and group decision making methods are used in order to carry out decision making process in this environment, there is a need of experts to rate all the alternatives according to each of the criteria values. When a high number of alternatives are available, this is too much work for experts to carry out. Again, fuzzy ontologies avoid experts of having to provide that much information by using ontology reasoning processes in order to extract alternatives fulfilling a specific criteria. Thanks to this, there is not an specific limit of alternatives that our method can handle.

When using pairwise comparisons, the system is capable of knowing the alternatives that experts prefer the most even if the alternatives set is big. In multi-criteria group decision making environments that have a high number of alternatives, this representation method becomes unmanageable. This is because there is a need for experts to provide information of $(n \cdot n - n) \cdot c$ preference values. c indicates the number of criteria values and n the number of alternatives. For example, in an environment with 10 alternatives and 5 criteria values, experts would have to provide 450 preference values which is too much information. Thanks to our method, the number of criteria values that experts have to provide is reduced to $c \cdot c$. In the previous example, 25 values were needed. It is important to notice that, thanks to the fuzzy ontologies application, the number of preference values that experts have to provide is independent of the number of alternatives. For instance, in a traditional multi-criteria group decision making, if 100 alternatives are available with 5 criteria values, 49,500 comparisons are needed. Our method still requires the same 25 preference values.

It is important to notice that previous computations are only valid when criteria values are objective and they can be obtained without performing any experts debate. For instance, this is the case of criteria values that describe alternatives using measures to describe their features. In the case of subjective criteria values, there is a need for experts to rate each alternative according to the criteria. For instance, this is the case when a criteria value refers to subjective concepts like popularity or suitability which cannot be easily measured using objective information. Therefore, for subjective criteria values, there is a need for experts to carry out a rating process. One way of accomplishing this issue is to carry out a group decision making process. The main advantage of this approach is that experts are the one providing the missing information needed for the fuzzy ontology to carry out the reasoning process. The main drawback of this approach is that experts need to provide the information increasing the overall amount of preferences that they have to provide. If there is a high amount of criteria values being subjective, this process can become tedious. Another solution to this issue that focus in avoiding the need of carrying out this experts rating process consists in obtaining the required information from the Internet. It is well known that Internet is filled with users subjective information about a high amount of topics [3,4,21,28]. Therefore, it is possible to retrieve the subjective needed information from users opinion websites. The main advantage of this approach is that no extra experts preferences are needed. The main disadvantage is that there is a need for experts to rely in the obtained information since the group decision making process is going to use it in order to compute the final ranking results.

A good example of a potential straightforward application area of this novel developed method would be its application to existing collaborative and distributed User eXperience (UX) design processes. In effect, being these processes of an intrinsically iterative nature, where an increased precision of the results is pursued over and over, the applied use of group decision making methods over the best results obtained from a fuzzy ontology reasoning process would be natural.

On the other hand, the presented proposal implies, within the decision making processes applied to UX design, an improved UX of the own processes, by making experts feel more comfortable when having to deal with less details within the overall process.

In order to highlight the advantages of our method, a list of features and their level of fulfilment for the novel developed method are shown in Table 4.

6. Conclusions and future work

In this paper, a novel approach for solving multi-criteria group decision making problems in environments that have a high amount of alternatives have been exposed. Our main goal has been to design a method that allows experts to deal only with criteria values automatizing how alternatives are ranked. The less the experts need to deal with a high amount of elements, the more comfortable the method is for them to use. Thanks to this, experts only need to discuss about the importance of the criteria values. Knowing the importance given to criteria values, the best alternative that fulfils the experts expectations is retrieved automatically using a fuzzy ontology reasoning process. A posterior group decision making method over the best results can be performed if experts feel like they have to increase results precision using their own knowledge.

The novel developed method is perfectly suitable on environments where a high amount of alternatives are available and it is not affordable for experts to compare and provide information about all of them. Also, carrying out most of the process automatically makes the system more comfortable for experts to use since they do not have to deal with every detail of the process. In order to prove the usefulness of our approach, a use example using a wine ontology have been presented.

The main disadvantage of our method is that subjective information about the alternatives is not easy to obtain. Two different solutions have been given. The first one consists on carrying out a group decision making method over the alternatives and the specific subjective criteria value. The second one tries to avoid experts participation and retrieves the information from the Internet using users opinion webpages.

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