



A SVN S A can be written as  $A = \{ \langle x: T_A(x), I_A(x), F_A(x) \rangle \mid x \in X \}$  (2)

**C. Definition: 2.3**

Let  $A_1 = (T_1, I_1, F_1)$  and  $A_2 = (T_2, I_2, F_2)$  be two single valued neutrosophic numbers. Then the operations for SVN S are defined as follows:

- 1)  $\tilde{A}_1 + \tilde{A}_2 = \langle T_1 + T_2 - T_1 T_2, I_1 I_2, F_1 F_2 \rangle$
- 2)  $\tilde{A}_1 \times \tilde{A}_2 = \langle T_1 + T_2, I_1 + I_2 - I_1 I_2, F_1 + F_2 - F_1 F_2 \rangle$
- 3)  $\lambda \tilde{A}_1 = \langle 1 - (1 - T_1)^\lambda, I_1^\lambda, F_1^\lambda \rangle$
- 4)  $\tilde{A}_1^\lambda = \langle T_1^\lambda, 1 - (1 - I_1)^\lambda, 1 - (1 - F_1)^\lambda \rangle$ , where  $\lambda > 0$

**D. Definition: 2.4**

The empty set  $O_n$  may be defined as  $O_n = \{ \langle x, (0, 1, 1) \rangle \mid x \in X \}$

A convenient method for converting single valued neutrosophic number is by use of score function.

**E. Definition: 2.5**

Let  $\tilde{A}_1 = (T_1, I_1, F_1)$  be a single valued neutrosophic number. Then, the score function  $S(\tilde{A}_1)$  accuracy function  $a(\tilde{A}_1)$  and certainty function  $C(\tilde{A}_1)$  of SVN S are defined as follows.

$$S(\tilde{A}_1) = \frac{2+T_1-I_1-F_1}{3}; \text{ (ii) } a(\tilde{A}_1) = T_1 - F_1; \text{ (iii) } C(\tilde{A}_1) = T_1$$

**F. Definition: 2.6**

Suppose that  $\tilde{A}_1 = (T_1, I_1, F_1)$  and  $\tilde{A}_2 = (T_2, I_2, F_2)$  are two single values neutrosophic numbers. Then we define ranking method as follows.

- 1) If  $S(\tilde{A}_1) > S(\tilde{A}_2)$ , then is  $\tilde{A}_1$  greater than  $\tilde{A}_2$  that is,  $\tilde{A}_1$  is superior to  $\tilde{A}_2$ , denoted by  $\tilde{A}_1 > \tilde{A}_2$ .
- 2) If  $S(\tilde{A}_1) = S(\tilde{A}_2)$  and  $a(\tilde{A}_1) > a(\tilde{A}_2)$  then  $\tilde{A}_1$  is greater than  $\tilde{A}_2$ , that is  $\tilde{A}_1$  is superior of  $\tilde{A}_2$ , denoted by  $\tilde{A}_1 > \tilde{A}_2$ .
- 3) If  $S(\tilde{A}_1) = S(\tilde{A}_2)$ ,  $a(\tilde{A}_1) = a(\tilde{A}_2)$  and  $C(\tilde{A}_1) = C(\tilde{A}_2)$  then  $\tilde{A}_1$  is greater than  $\tilde{A}_2$  that is,  $\tilde{A}_1$  is superior to  $\tilde{A}_2$  denoted by  $\tilde{A}_1 > \tilde{A}_2$ .
- 4) If  $S(\tilde{A}_1) = S(\tilde{A}_2)$ ,  $a(\tilde{A}_1) = a(\tilde{A}_2)$  and  $C(\tilde{A}_1) = C(\tilde{A}_2)$  then  $\tilde{A}_1$  is equal to  $\tilde{A}_2$  that is  $\tilde{A}_1$  is indifferent to  $\tilde{A}_2$  denoted by  $\tilde{A}_1 = \tilde{A}_2$ .

**III. ILLUSTRATIVE EXAMPLE**

Now we will solve a problem to verify the proposed approach.

- 1) There are 6 jobs, each of which has to go through 3 machines  $M_1, M_2$  and  $M_3$  in the order  $M_1 M_2 M_3$ . Find the minimum elapsed time if no passing of jobs is permitted. Also determine the idle time for each machine. Here each job has been assigned to single valued neutrosophic number as follows.

Table - 1

Jobs \ Machines	A	B	C	D	E	F
$M_1$	(.1, .2, .4)	(.2, .2, 0)	(.5, .5, .5)	(.2, 1, 0)	(.1, .2, 0)	(.2, .2, .2)
$M_2$	(.1, .8, .6)	(.1, .7, .5)	(.1, .7, .6)	(.2, .5, .7)	(.2, .7, .6)	(.2, .8, .6)
$M_3$	(.2, .2, .1)	(.8, .7, .2)	(.2, .2, .3)	(.3, .1, .2)	(.1, .2, .5)	(.1, .1, .1)

**A. Solution**

The neutrosophic set of values is converted by means of a score function is as follows: The above problem transformed into

Table - 2

Jobs \ Machines	A	B	C	D	E	F
$M_1$	.5	.67	.50	.4	.63	.60
$M_2$	.233	.3	.267	.33	.3	.27
$M_3$	.63	.63	.567	.67	.467	.63

Minimum  $M_{1j} = .4, J = 1, 2, \dots, 6$

Maximum  $M_{2j} = .33, J = 1, 2, \dots, 6$

Minimum  $M_{3j} = .467, J = 1, 2, \dots, 6$

Min  $M_{1j} \geq$  Max  $M_{2j}$  and

Min  $M_{3j} \geq$  Max  $M_{2j}$

The problem can be converted into that of 6 jobs and 2 machines respectively. These two fictitious machines are denoted by G and H, where each

$G = M_{1j} + M_{2j}, J = 1, 2, \dots, 6$

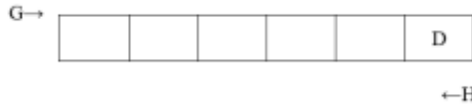
And  $H = M_{2j} + M_{3j}, J = 1, 2, \dots, 6$

The equivalent problem involving 6 jobs and 2 fictitious machines G and H becomes

Table - 3

	A	B	C	D	E	F
G	.733	.97	.767	.73	.93	.87
H	.863	.93	.834	.10	.767	.90

By examining, we find the smallest value. It is .1 hour for H in 4<sup>th</sup> column. Then we schedule job D in the last as shown below.

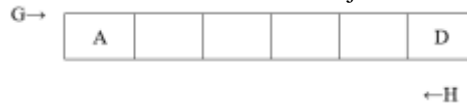


The scheduled set of processing time is

Table - 4

	A	B	C	E	F
G	.733	.97	.767	.93	.87
H	.863	.93	.834	.767	.9

The smallest value is .73. it is for machine G for Job A. Then we schedule job A in first column as given below.

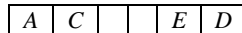


Then the reduced set of processing time becomes

Table - 5

Jobs Machines	B	C	E	F
G	.97	.767	.93	.87
H	.93	.834	.767	.9

There are two equal minimal values .762: Job C for machine G and Job E for machine H. According to the rules Job C is scheduled next to A and Job E is scheduled next to Job D as shown below:

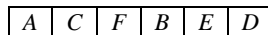


The reduced set of processing time becomes

Table - 6

Jobs Machines	B	F
G	.97	.87
H	.93	.9

Next smallest value is .87 hours. It is for machine G for Job F. Therefore, we schedule Job F next to C and we get the optimal sequence as



Now we can calculate the elapsed time corresponding to the optimal sequence, using the individual processing time given in the problem. The details are shown in the table below:

Table - 7

Machines Jobs	$M_1$		$M_2$		$M_3$	
	In	Out	In	Out	In	Out
A	0	.5	.5	.733	.733	1.363
C	.5	1	1	1.267	1.363	1.93
F	1	1.6	1.6	1.87	1.93	2.56
B	1.6	2.27	2.27	2.57	2.57	3.2
E	2.27	2.9	2.9	3.2	3.2	3.667
D	2.9	3.3	3.3	3.63	3.667	4.337

Then the minimum elapsed time is 4.337 hours.

Idle time for machine  $M_1 = 1.037$  hours

For machine  $M_2 = .5 + .267 + .333 + .4 + .33 + .1 + .707$   
 $= 2.637$  hours

And for machine  $M_3 = .733 + .01$   
 $= .743$  hours

#### IV. CONCLUSION

This paper is introduced to solve sequencing problem for  $m$  machines and  $n$  jobs on neutrosophic set ie, under uncertainty environment by converting it in to 2 machines and  $n$  jobs problem. We can also solve the same sequencing problem by converting it into  $m$  machines and two jobs sequencing problem.

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