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SOME OPERATIONS ON NEUTROSOPHIC FUZZY GRAPHS

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ABSTRACT

The concept of neutrosophic sets can be utilized as a Mathematical tool to deal with imprecise and unspecified information. The neutrosophic set is a generalization of fuzzy sets and intutionistic fuzzy sets which is applied in Graph theory. The neutrosophic fuzzy graph is used in the problems, where the relation between vertices and hence edges are indeterminate. In this paper complement of Neutrosophic fuzzy graph is introduced and few operations on Neutrosophic fuzzy graph (union, join) are also defined.

Keywords: *Neutrosophic Fuzzy Graph (NFG), Complement, Union, Join.*

1. INTRODUCTION

Graph theory has been highly successful in certain academic fields, including natural sciences and engineering. Graph theoretic models can sometimes provide a useful structure upon which analytical techniques can be used. It is often convenient to depict the relationships between pairs of elements of a system by means of a graph or a digraph. The vertices of the graph represent the system elements and its edges or arcs represent the relationships between the elements. This approach is especially useful for transportation, scheduling, sequencing, allocation, assignment, and other problems which can be modelled as networks. Such a graph theoretic model is often useful as an aid in communicating. Zadeh [6] introduced the concept of fuzzy set. Attanassov [2] introduced the intuitionistic fuzzy sets which is a generalization of fuzzy sets. Fuzzy set theory and intuitionistic fuzzy set theory are useful models for dealing with uncertainty and incomplete information. But they may not be sufficient in modeling of indeterminate and inconsistent information encountered in real world. In order to cope with this issue, neutrosophic set theory was proposed by Smarandache [8] as a generalization of fuzzy sets and intuitionistic fuzzy sets.

Neutrosophic set proposed by Smarandache [12, 13] is a powerful tool to deal with incomplete, indeterminate and inconsistent information in real world. It is a generalization of the theory of fuzzy set [18], intuitionistic fuzzy sets [19], interval-valued fuzzy sets and interval-valued intuitionistic fuzzy sets , then the neutrosophic set is characterized by a truth-membership degree (t), an indeterminacy-membership degree (i) and a falsity-membership degree (f) independently, which are within the real standard or nonstandard unit interval $]^{-}0, 1^{+}[$.There are some difficulties in modeling of some problems in Engineering and Sciences. To overcome these difficulties, in 2010, concept of single-valued neutrosophic sets and its operations defined by Wang et al. [4] as a generalization of intuitionistic fuzzy sets.

In this research article we apply the concept of neutrosophic sets to graphs. We introduce the notion of Neutrosophic Fuzzy Graph and present few fundamental operations.

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2. PRELIMINARIES

Definition 2.1: Let X be a space of points with generic elements in X denoted by x; then the neutrosophic set A (NS A) is an object having the form

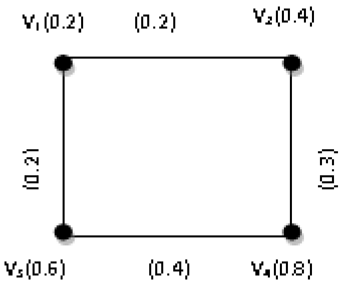
$A = \{< x: T_A(x), I_A(x), F_A(x)>, x\in X\}$

where the functions T, I, F: $X\to]^-0,1^+[$ define respectively a truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element $x \in X$ to the set A with the condition

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$$

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of$] -0, 1^+[$.

Definition 2.2: A fuzzy graph is a pair of functions G = (σ, μ) where σ is a fuzzy subset of a non empty set V and is a symmetric fuzzy relation on σ. i.e $\sigma : V\to [0,1]$ and $\mu: VxV\to[0,1]$ such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ for all u, v∈ V where uv denotes the edge between u and v and $\sigma(u) \wedge \sigma(v)$ denotes the minimum of σ(u) and σ(v). σ is called the fuzzy vertex set of V and is called the fuzzy edge set of E.



Fuzzy graph

Definition 2.3: The fuzzy subgraph H = (τ, ρ) is called a fuzzy subgraph of G = (σ, μ)

If $\tau(u) \leq \sigma(u)$ for all u∈ V and $\rho(u, v) \leq \mu(u, v)$ for all u, v ∈V.

Definition 2.4: An intuitionistic fuzzy graph (IFG) is of the form G = (V, E) where

(i) $V = \{v_1, v_2, ... ,v_n\}$ such that μ_1: $V\to [0,1]$ and γ_1: $V\to [0,1]$ denote the degree of membership and non-membership of the element $v_i\in$ Vrespectively and

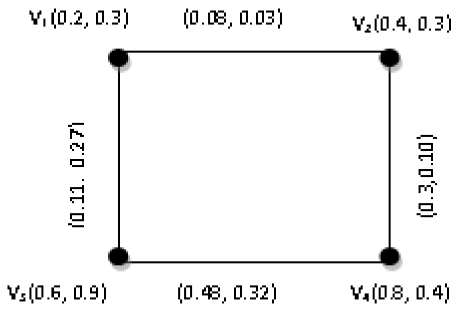
$$0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1 \text{ for every } v_i\in V, (i = 1, 2, .n),$$

(ii) $E \subseteq V\times V$ where $\mu_2 : V\times V \to[0,1]$ and $\gamma_2 : V\times V \to[0,1]$ are such that

$$\mu_2(v_i, v_j) \leq \mu_1(v_i) \cdot \mu_1(v_j)$$
$$\gamma_2(v_i, v_j) \leq \gamma_1(v_i) \cdot\gamma_1(v_j)$$

and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$

for every $(v_i ,v_j)\in E$, where · means the ordinary multiplication.



Intutionistic fuzzy graph

3. NEUTROSOPHIC FUZZY GRAPHS

Throughout this paper we denote G = (V, E) a crisp graph, N_G = (A, B) a neutrosophic fuzzy graph of Graph G.

Definition 3.1: Let X be a space of points with generic elements in X denoted by x. A neutrosophic fuzzy set A (NFS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X

$$T_A(x), I_A(x), F_A(x) \in [0, 1].$$

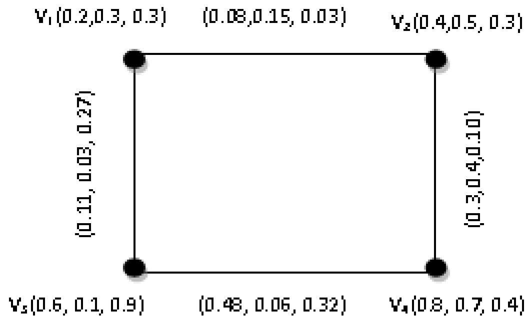
A NFS A can be written as $A = \{< x: T_A(x), I_A(x), F_A(x) >, x \in X\}$

Definition 3.2: Let A= (T_A, I_A, F_A) and B = (T_B, I_B, F_B) be neutrosophic fuzzy sets on a set X. If A = (T_A, I_A, F_A) is a aneutrosophic fuzzy relation on a set X, then A = (T_A, I_A, F_A) is called a neutrosophic fuzzy relation on B = (T_B, I_B, F_B) if

$T_B(x, y) \leq T_A(x)\cdot T_A(y)$
$I_B(x, y) \leq I_A(x) \cdot I_A(y)$
$F_B(x, y) \leq F_A(x) \cdot F_A(y)$ for all x, y ∈ X where · means the ordinary multiplication

Definition 3.3: A neutrosophic fuzzy graph (NF-graph) with underlying set V is defined to be a pair N_G= (A, B) where

1. The functions T_A: V→ [0, 1], I_A:V→ [0, 1] and F_A:V→ [0, 1] denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element v_i∈V, respectively, and
$$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3 \tag{1}$$
2. E ⊆V x V where, functions T_B: V x V→ [0, 1], I_B: V x V→ [0, 1] and F_B: V x V→ [0, 1] are defined by
$$T_B(v_i, v_j) \leq T_A(v_i) \cdot T_A(v_j) \tag{2}$$
$$I_B(v_i, v_j) \leq I_A(v_i) \cdot I_A(v_j) \tag{3}$$
$$F_B(v_i, v_j) \leq F_A(v_i) \cdot F_A(v_j) \tag{4}$$
for all v_i, v_j∈ V where · means the ordinary multiplication Denotes the degree of truth–membership, indeterminacy–membership and falsity-membership of the edge (v_i ,v_j)∈ E respectively, where
$$0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3 \tag{5}$$
for all (v_i, v_j) ∈ E (i, j = 1,2,.....,n)



Neutrosophic Fuzzy Graph

We call "A" the neutrosophic fuzzy vertex set of V, "B" the neutrosophic fuzzy edge set of E, respectively.

Notation:

- μ, λ, γ denote the truth membership value, indeterminancy membership value and falsity membership value respectively
- $(v_i, \mu_i, \lambda_i, \gamma_i)$ denotes the degree of truth membership, indeterminancy membership and falsity membership of the vertex V_i.
- $(e_{ij}, \mu'_{ij}, \lambda'_{ij}, \gamma'_{ij})$ denotes the degree of truth membership, indeterminancy membership and falsity membership of the edge e_{ij}.

Note 3.4:

(i) When $\mu'_{ij} = \lambda'_{ij} = \gamma'_{ij} = 0$, for some i and j, then there is no edge between v_i and v_j. Otherwise there exists edge between v_i and v_j.
(ii) If one of the inequalities (1) or (2) or (3) or (4) or (5) is not satisfied, then G is not a NFG**.**

Definition 3.5: A NF-graph N_H = (A′ , B′) of H = (V′ , E′) is partial NF-subgraph of NF-graph N_G = (A,B) of G = (V, E) if

(i) V′ ⊆ V, where $T_A'(v_i) \leq T_A(v_i)$, $I_A'(v_i) \leq I_A(v_i)$, $F_A'(v_i) \leq F_A(v_i)$ for all v_i∈ V′ .
(ii) E′ ⊆ E, where $T_B'(v_i , v_j) \leq T_B(v_i , v_j)$, $I_B'(v_i , v_j) \leq I_B(v_i , v_j)$, $F_B'(v_i , v_j) \leq F_B(v_i , v_j)$ for all (v_i , v_j) ∈ E′

Definition 3.6: A NF-graph N_H = (A′, B′) of H = (V′ , E′) is NF-subgraph of NF-graph N_G = (A,B)of G = (V, E) if

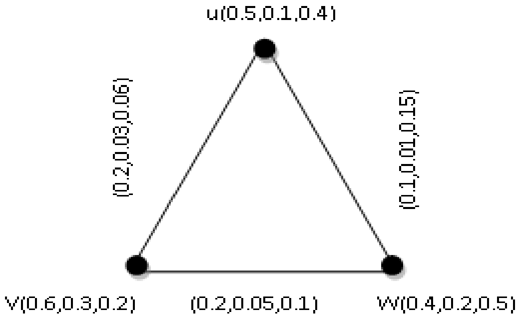
(i) V′ ⊆ V, where $T_A'(v_i) = T_A(v_i)$, $I_A'(v_i) = I_A(v_i)$, $F_A'(v_i) = F_A(v_i)$ for all v_i∈ V′ .
(i) E′ ⊆ E, where $T_B'(v_i , v_j) = T_B(v_i, v_j)$, $I_B'(v_i, v_j) = I_B(v_i, v_j)$, $F_B'(v_i, v_j) = F_B(v_i, v_j)$ for all (v_i, v_j) ∈ E′

Definition 3.7: A neutrosophic fuzzy graph N_G = (A, B) is called complete if

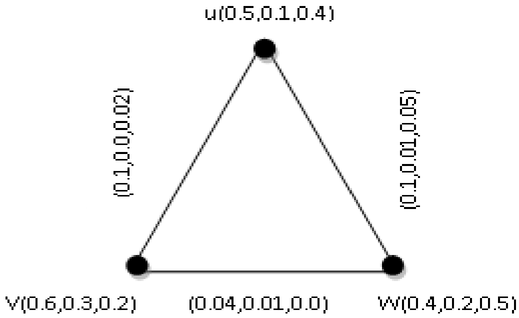
$T_B(v_i, v_j) = T_A(v_i) \cdot T_A(v_j)$
$I_B(v_i, v_j) = I_A(v_i) \cdot I_A(v_j)$
$F_B(v_i, v_j) = F_A(v_i) \cdot F_A(v_j)$ for all v_i, v_j∈ V

Definition 3.8: The complement of an NFG $N_G = (A, B)$ is a NFG, $\overline{N_G} = (\overline{A}, \overline{B})$, where

- $\overline{V} = V$
- $\overline{\mu_i} = \mu_i$, $\overline{\lambda_i} = \lambda_i$, $\overline{\gamma_i} = \gamma_i$
- $\overline{\mu'_{ij}} = \mu_i \cdot \mu_j - \mu_{ij}$, $\overline{\lambda'_{ij}} = \lambda_i \cdot \lambda_j - \lambda_{ij}$, $\overline{\gamma'_{ij}} = \gamma_i \cdot \gamma_j - \gamma_{ij}$



***N_G:** Neutrosophicfuzzy graph*



***$\overline{N_G}$* : Complement of N_G**

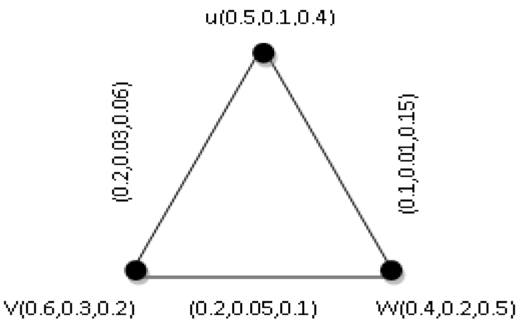
Note: We can easily verify that $\overline{\overline{N_G}} = N_G$.

4. OPERATIONS ON NEUTROSOPHIC FUZZY GRAPH

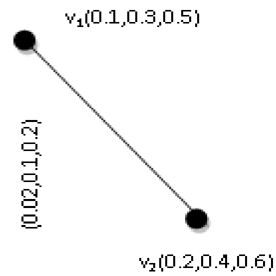
Definition 4.1: Let G1 = (V1, E1) and G2=(V2, E2) be two graphs, $N_{G1} = (A_1, B_1)$ and $N_{G2} = (A_2, B_2)$ be the neutrosophic fuzzy graphs of the crisp graphs G1 and G2 respectively with $V_1 \cap V_2 = \phi$. G = G1 ∪ G2 = (V1 ∪V2, E1 ∪ E2) be the union of G1 and G2.

The union of NFGs N_{G1} and N_{G2} is a NFG defined by

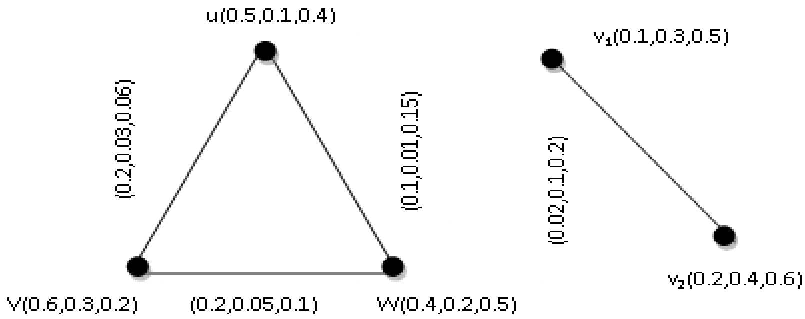
$$(\mu_1 \cup \mu_2)(v) = \begin{cases} \mu_1(v) & \text{if } v \in v_1 - v_2 \\ \mu_2(v) & \text{if } v \in v_2 - v_1 \end{cases}$$
$$(\lambda_1 \cup \lambda_2)(v) = \begin{cases} \lambda_1(v) & \text{if } v \in v_1 - v_2 \\ \lambda_2(v) & \text{if } v \in v_2 - v_1 \end{cases}$$
$$(\gamma_1 \cup \gamma_2)(v) = \begin{cases} \gamma_1(v) & \text{if } v \in v_1 - v_2 \\ \gamma_2(v) & \text{if } v \in v_2 - v_1 \end{cases}$$
$$(\mu'_1 \cup \mu'_2)(e_{ij}) = \begin{cases} \mu'_1(e_{ij}) & \text{if } e_{ij} \in E_1 - E_2 \\ \mu'_2(e_{ij}) & \text{if } e_{ij} \in E_2 - E_1 \end{cases}$$
$$(\lambda'_1 \cup \lambda'_2)(e_{ij}) = \begin{cases} \lambda'_1(e_{ij}) & \text{if } e_{ij} \in E_1 - E_2 \\ \lambda'_2(e_{ij}) & \text{if } e_{ij} \in E_2 - E_1 \end{cases}$$
$$(\gamma'_1 \cup \gamma'_2)(e_{ij}) = \begin{cases} \gamma'_1(e_{ij}) & \text{if } e_{ij} \in E_1 - E_2 \\ \gamma'_2(e_{ij}) & \text{if } e_{ij} \in E_2 - E_1 \end{cases}$$



$[N_{G1}]$



$[N_{G2}]$



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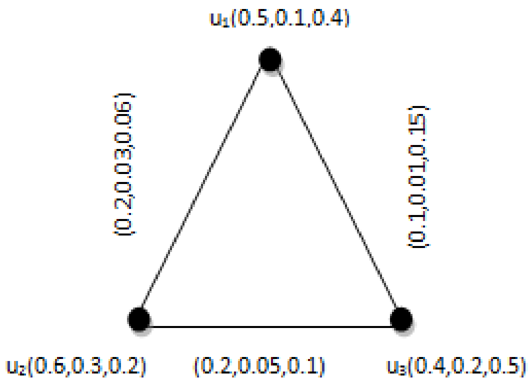
Neutrosophic fuzzy graph: $N_{G1} \cup N_{G2}$

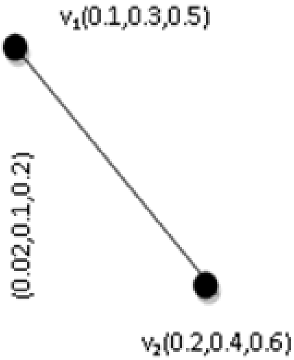
Definition 4.2: Let G1 = (V1, E1) and G2 = (V2, E2) be two graphs, $N_{G1}= (A_1, B_1)$ and $N_{G2} = (A_2, B_2)$ be the neutrosophic fuzzy graphs of the crisp graphs G1 and G2 respectively with $V_1 \cap V_2 = \phi$. G = G1 ⊕ G2 = (V1 ∪V2, E1 ∪ E2 ∪ E′) be the join of G1 and G2.

The join of NFGs N_{G1} and N_{G2} is a defined by

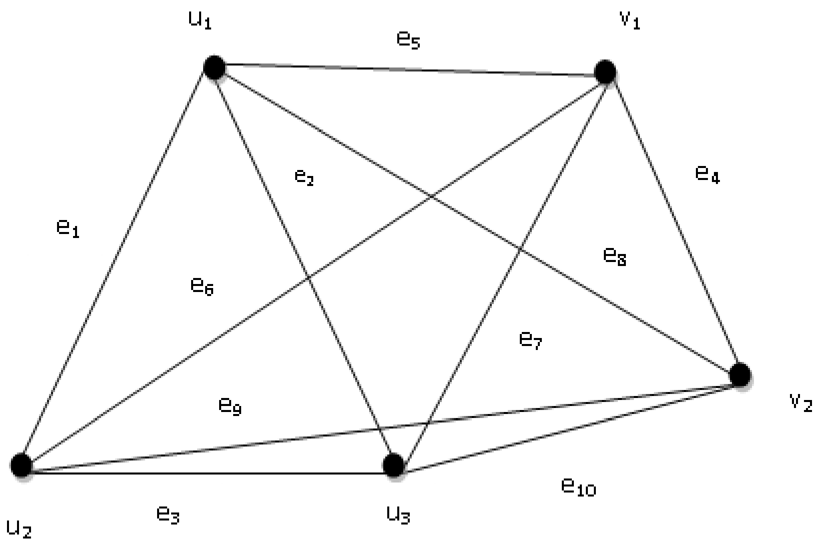
$$(\mu_1 \oplus \mu_2)(v) = (\mu_1 \cup \mu_2)(v) \qquad \text{if } v \in V_1 \cup V_2$$
$$(\lambda_1 \oplus \lambda_2)(v) = (\lambda_1 \cup \lambda_2)(v) \qquad \text{if } v \in V_1 \cup V_2$$
$$(\gamma_1 \oplus \gamma_2)(v) = (\gamma_1 \cup \gamma_2)(v) \qquad \text{if } v \in V_1 \cup V_2$$
$$(\mu_1' \oplus \mu_2')(e_{ij}) = (\mu_1' \cup \mu_2')(e_{ij}) \qquad \text{if } e_{ij} \in E_1 \cup E_2$$
$$(\mu_1' \oplus \mu_2')(e_{ij}) = \left(\mu_1(v_i) \cdot \mu_2(v_j)\right) \qquad \text{if } e_{ij} \in E'$$
$$(\lambda_1' \oplus \lambda_2')(e_{ij}) = (\lambda_1' \cup \lambda_2')(e_{ij}) \qquad \text{if } e_{ij} \in E_1 \cup E_2$$
$$(\lambda_1' \oplus \lambda_2')(e_{ij}) = \left(\lambda_1(v_i) \cdot \lambda_2(v_j)\right) \qquad \text{if } e_{ij} \in E'$$
$$(\gamma_1' \oplus \gamma_2')(e_{ij}) = (\gamma_1' \cup \gamma_2')(e_{ij}) \qquad \text{if } e_{ij} \in E_1 \cup E_2$$
$$(\gamma_1' \oplus \gamma_2')(e_{ij}) = \left(\gamma_1(v_i) \cdot \gamma_2(v_j)\right) \qquad \text{if } e_{ij} \in E'$$



[N_{G1}]



[N_{G2}]



NFG: $N_{G1} \oplus N_{G2}$

where, $u_1 = (0.5, 0.1, 0.4)$, $u_2 = (0.6, 0.3, 0.2)$, $u_3 = (0.4, 0.2, 0.5)$, $v_1 = (0.2, 0.3, 0.4)$,
$v_2 = (0.3, 0.5, 0.7)$, $e_1 = (0.2, 0.03, 0.06)$, $e_2 = (0.1, 0.01, 0.15)$, $e_3 = (0.2, 0.05, 0.1)$,
$e_4 = (0.06, 0.10, 0.2)$, $e_5 = (0.1, 0.03, 0.16)$, $e_6 = (0.12, 0.09, 0.08)$, $e_7 = (0.08, 0.05, 0.28)$,
$e_8 = (0.15, 0.05, 0.28)$, $e_9 = (0.18, 0.15, 0.14)$, $e_{10} = (0.12, 0.10, 0.35)$

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