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Special types of bipolar single valued neutrosophic graphs

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Abstract. Neutrosophic theory has many applications in graph theory, bipolar single valued neutrosophic graphs (BSVNGs) is the generalization of fuzzy graphs and intuitionistic fuzzy graphs, SVNGs. In this paper we introduce some types of BSVNGs, such as subdivision BSVNGs, middle BSVNGs, total BSVNGs and bipolar single valued neutrosophic line graphs (BSVNLGs), also investigate the isomorphism, co weak isomorphism and weak isomorphism properties of subdivision BSVNGs, middle BSVNGs, total BSVNGs and BSVNLGs.

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1. Introduction

Neutrosophic set theory (NS) is a part of neutrosophy which was introduced by Smarandache [43] from philosophical point of view by incorporating the degree of indeterminacy or neutrality as independent component for dealing problems with indeterminate and inconsistent information. The concept of neutrosophic set theory is a generalization of the theory of fuzzy set [50], intuitionistic fuzzy sets [5], interval-valued fuzzy sets [47] interval-valued intuitionistic fuzzy sets [6]. The concept of neutrosophic set is characterized by a truth-membership degree (T), an indeterminacy-membership degree (I) and a falsity-membership degree (f) independently, which are within the real standard or nonstandard unit interval $]^{-}0, 1^{+}[$. Therefore, if their range is restrained within the real standard unit interval $[0, 1]$: Nevertheless, NSs are hard to be apply in practical problems since the values of the functions of truth, indeterminacy and falsity lie in $]^{-}0, 1^{+}[$. The single valued neutrosophic set was introduced for the first time by Smarandache [43]. The concept of single valued neutrosophic sets is a subclass of neutrosophic sets in which the value of truth-membership, indeterminacy membership and falsity-membership degrees are intervals of numbers instead of the real numbers. Later on, Wang et al. [49] studied some properties related to single valued neutrosophic sets. The concept of neutrosophic sets and its extensions such as single valued neutrosophic sets, interval neutrosophic sets, bipolar neutrosophic sets and so on have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine and economic and can be found in [9, 15, 16, 30, 31, 32, 33, 34, 35, 36, 37, 51]. Graphs are the most powerful tool used in representing information involving relationship between objects and concepts. In a crisp graphs two vertices are either related or not related to each other, mathematically, the degree of relationship is either 0 or 1. While in fuzzy graphs, the degree of relationship takes values from $[0, 1]$. Atanassov [42] defined the concept of intuitionistic fuzzy graphs (IFGs) using five types of Cartesian products. Theconcept fuzzy graphs, intuitionistic fuzzy graphs and their extensions such interval valued fuzzy graphs, bipolar fuzzy graph, bipolar intuitionitsic fuzzy graphs, interval valued intuitionitic fuzzy graphs, hesitancy fuzzy graphs, vague graphs and so on, have been studied deeply by several researchers in the literature. When description of the object or their relations or both is indeterminate and inconsistent, it cannot be handled by fuzzy intuitionistic fuzzy, bipolar fuzzy, vague and interval valued fuzzy graphs. So, for this purpose, Smaranadache [45] proposed the concept of neutrosophic graphs based on literal indeterminacy (I) to deal with such situations. Later on, Smarandache [44] gave another definition for neutrosphic graph theory using the neutrosophic truth-values (T, I, F) without and constructed three structures of neutrosophic graphs: neutrosophic edge graphs, neutrosophic vertex graphs and neutrosophic vertex-edge graphs. Recently, Smarandache [46] proposed new version of neutrosophic graphs such as neutrosophic offgraph, neutrosophic bipolar/tripola/multipolar graph. Recently several researchers have studied deeply the concept of neutrosophic vertex-edge graphs and presented several extensions neutrosophic graphs. In [1, 2, 3]. Akram et al. introduced the concept of single valued neutrosophic hypergraphs, single valued neutrosophic planar graphs, neutrosophic soft graphs and intuitionstic neutrosophic soft graphs. Then, followed the work of Broumi et al. [7, 8, 9, 10, 11, 12, 13, 14, 15], Malik and Hassan [38] defined the concept of single valued neutrosophic trees and studied some of their properties. Later on, Hassan et Malik [17] introduced some classes of bipolar single valued neutrosophic graphs and studied some of their properties, also the authors generalized the concept of single valued neutrosophic hypergraphs and bipolar single valued neutrosophic hypergraphs in [19, 20]. In [23, 24] Hassan et Malik gave the important types of single (interval) valued neutrosophic graphs, another important classes of single valued neutrosophic graphs have been presented in [22] and in [25] Hassan et Malik introduced the concept of m-Polar single valued neutrosophic graphs and its classes. Hassan et al. [18, 21] studied the concept on regularity and total regularity of single valued neutrosophic hypergraphs and bipolar single valued neutrosophic hypergraphs. Hassan et al. [26, 27, 28] discussed the isomorphism properties on SVNHGs, BSVNHGs and IVNHGs. Nasir et al. [40] introduced a new type of graph called neutrosophic soft graphs and established a link between graphs and neutrosophic soft sets. The authors also studeied some basic operations of neutrosophic soft graphs such as union, intersection and complement. Nasir and Broumi [41] studied the concept of irregular neutrosophic graphs and investigated some of their related properties. Ashraf et al. [4], proposed some novels concepts of edge regular, partially edge regular and full edge regular single valued neutrosophic graphs and investigated some of their properties. Also the authors, introduced the notion of single valued neutrosophic digraphs (SVNDGs) and presented an application of SVNDG in multi-attribute decision making. Mehra and Singh [39] introduced a new concept of neutrosophic graph named single valued neutrosophic Signed graphs (SVNSGs) and examined the properties of this concept with suitable illustration. Ulucay et al. [48] proposed a new extension of neutrosophic graphs called neutrosophic soft expert graphs (NSEGs) and have established a link between graphs and neutrosophic soft expert sets and studies some basic operations of neutrosophic soft experts graphs such as union, intersection and complement. The neutrosophic graphs have many applications in path problems, networks and computer science. Strong BSVNG and complete BSVNG are the types of BSVNG. In this paper, we introduce others types of BSVNGs such as subdivision BSVNGs, middle BSVNGs, total BSVNGs and BSVNLGs and these are all the strong BSVNGs, also we discuss their relations based on isomorphism, co weak isomorphism and weak isomorphism.

2. Preliminaries

In this section we recall some basic concepts on BSVNG. Let G denotes BSVNG and $G^* = (V, E)$ denotes its underlying crisp graph.

Definition 2.1 ([10]). Let X be a crisp set, the single valued neutrosophic set (SVNS) Z is characterized by three membership functions $T_Z(x), I_Z(x)$ and $F_Z(x)$ which are truth, indeterminacy and falsity membership functions, $\forall x \in X$

$$T_Z(x), I_Z(x), F_Z(x) \in [0, 1].$$

Definition 2.2 ([10]). Let X be a crisp set, the bipolar single valued neutrosophic set (BSVNS) Z is characterized by membership functions $T_Z^+(x)$, $I_Z^+(x)$, $F_Z^+(x)$, $T_Z^-(x)$, $I_Z^-(x)$, and $F_Z^-(x)$. That is $\forall x \in X$

$$T_Z^+(x), I_Z^+(x), F_Z^+(x) \in [0, 1],$$
$$T_Z^-(x), I_Z^-(x), F_Z^-(x) \in [-1, 0].$$

Definition 2.3 ([10]). A bipolar single valued neutrosophic graph (BSVNG) is a pair $G = (Y, Z)$ of G^*, where Y is BSVNS on V and Z is BSVNS on E such that

$$T_Z^+(\beta\gamma) \leq \min(T_Y^+(\beta), T_Y^+(\gamma)),\ I_Z^+(\beta\gamma) \geq \max(I_Y^+(\beta), I_Y^+(\gamma)),$$
$$I_Z^-(\beta\gamma) \leq \min(I_Y^-(\beta), I_Y^-(\gamma)),\ F_Z^-(\beta\gamma) \leq \min(F_Y^-(\beta), F_Y^-(\gamma)),$$
$$F_Z^+(\beta\gamma) \geq \max(F_Y^+(\beta), F_Y^+(\gamma)),\ T_Z^-(\beta\gamma) \geq \max(T_Y^-(\beta), T_Y^-(\gamma)),$$

where

$$0 \leq T_Z^+(\beta\gamma) + I_Z^+(\beta\gamma) + F_Z^+(\beta\gamma) \leq 3$$
$$-3 \leq T_Z^-(\beta\gamma) + I_Z^-(\beta\gamma) + F_Z^-(\beta\gamma) \leq 0$$

$\forall\ \beta, \gamma \in V$.

In this case, D is bipolar single valued neutrosophic relation (BSVNR) on C. The BSVNG $G = (Y, Z)$ is complete (strong) BSVNG, if

$$T_Z^+(\beta\gamma) = \min(T_Y^+(\beta), T_Y^+(\gamma)),\ I_Z^+(\beta\gamma) = \max(I_Y^+(\beta), I_Y^+(\gamma)),$$

$$I_Z^-(\beta\gamma) = \min(I_Y^-(\beta), I_Y^-(\gamma)),\ F_Z^-(\beta\gamma) = \min(F_Y^-(\beta), F_Y^-(\gamma)),$$

$$F_Z^+(\beta\gamma) = \max(F_Y^+(\beta), F_Y^+(\gamma)),\ T_Z^-(\beta\gamma) = \max(T_Y^-(\beta), T_Y^-(\gamma)),$$

$\forall\ \beta, \gamma \in V (\forall\ \beta\gamma \in E)$. The order of BSVNG $G = (A, B)$ of G^*, denoted by $O(G)$, is defined by

$$O(G) = (O_T^+(G), O_I^+(G), O_F^+(G), O_T^-(G), O_I^-(G), O_F^-(G)),$$

where

$$O_T^+(G) = \sum_{\alpha\in V} T_A^+(\alpha),\ O_I^+(G) = \sum_{\alpha\in V} I_A^+(\alpha),\ O_F^+(G) = \sum_{\alpha\in V} F_A^+(\alpha),$$

$$O_T^-(G) = \sum_{\alpha\in V} T_A^-(\alpha),\ O_I^-(G) = \sum_{\alpha\in V} I_A^-(\alpha),\ O_F^-(G) = \sum_{\alpha\in V} F_A^-(\alpha).$$

The size of BSVNG $G = (A, B)$ of G^*, denoted by $S(G)$, is defined by

$$S(G) = (S_T^+(G), S_I^+(G), S_F^+(G), S_T^-(G), S_I^-(G), S_F^-(G)),$$

where

$$S_T^+(G) = \sum_{\beta\gamma\in E} T_B^+(\beta\gamma),\ S_T^-(G) = \sum_{\beta\gamma\in E} T_B^-(\beta\gamma),$$

$$S_I^+(G) = \sum_{\beta\gamma\in E} I_B^+(\beta\gamma),\ S_I^-(G) = \sum_{\beta\gamma\in E} I_B^-(\beta\gamma),$$

$$S_F^+(G) = \sum_{\beta\gamma\in E} F_B^+(\beta\gamma),\ S_F^-(G) = \sum_{\beta\gamma\in E} F_B^-(\beta\gamma).$$

The degree of a vertex β in BSVNG $G = (A, B)$ of G^*,, denoted by $d_G(\beta)$, is defined by

$$d_G(\beta) = (d_T^+(\beta), d_I^+(\beta), d_F^+(\beta), d_T^-(\beta), d_I^-(\beta), d_F^-(\beta)),$$

where

$$d_T^+(\beta) = \sum_{\beta\gamma\in E} T_B^+(\beta\gamma),\ d_T^-(\beta) = \sum_{\beta\gamma\in E} T_B^-(\beta\gamma),$$

$$d_I^+(\beta) = \sum_{\beta\gamma\in E} I_B^+(\beta\gamma),\ d_I^-(\beta) = \sum_{\beta\gamma\in E} I_B^-(\beta\gamma),$$

$$d_F^+(\beta) = \sum_{\beta\gamma\in E} F_B^+(\beta\gamma),\ d_F^-(\beta) = \sum_{\beta\gamma\in E} F_B^-(\beta\gamma).$$

3. Types of BSVNGs

In this section we introduce the special types of BSVNGs such as subdivision, middle and total and intersection BSVNGs, for this first we give the basic definitions of homomorphism, isomorphism, weak isomorphism and co weak isomorphism of BSVNGs which are very useful to understand the relations among the types of BSVNGs.

Definition 3.1. Let $G_1 = (C_1, D_1)$ and $G_2 = (C_2, D_2)$ be two BSVNGs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. Then the homomorphism $\chi : G_1 \to G_2$ is a mapping $\chi : V_1 \to V_2$ which satisfies the following conditions:

$$T^+_{C_1}(p) \leq T^+_{C_2}(\chi(p)),\ I^+_{C_1}(p) \geq I^+_{C_2}(\chi(p)),\ F^+_{C_1}(p) \geq F^+_{C_2}(\chi(p)),$$

$$T^-_{C_1}(p) \geq T^-_{C_2}(\chi(p)),\ I^-_{C_1}(p) \leq I^-_{C_2}(\chi(p)),\ F^-_{C_1}(p) \leq F^-_{C_2}(\chi(p)),$$

$\forall\ p \in V_1$,

$$T^+_{D_1}(pq) \leq T^+_{D_2}(\chi(p)\chi(q)),\ T^-_{D_1}(pq) \geq T^-_{D_2}(\chi(p)\chi(q)),$$
$$I^+_{D_1}(pq) \geq I^+_{D_2}(\chi(p)\chi(q)),\ I^-_{D_1}(pq) \leq I^-_{D_2}(\chi(p)\chi(q)),$$
$$F^+_{D_1}(pq) \geq F^+_{D_2}(\chi(p)\chi(q)),\ F^-_{D_1}(pq) \leq F^-_{D_2}(\chi(p)\chi(q)),$$

$\forall\ pq \in E_1$.

Definition 3.2. Let $G_1 = (C_1, D_1)$ and $G_2 = (C_2, D_2)$ be two BSVNGs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. Then the weak isomorphism $\upsilon : G_1 \to G_2$ is a bijective mapping $\upsilon : V_1 \to V_2$ which satisfies following conditions:

υ is a homomorphism such that

$$T^+_{C_1}(p) = T^+_{C_2}(\upsilon(p)),\ I^+_{C_1}(p) = I^+_{C_2}(\upsilon(p)),\ F^+_{C_1}(p) = F^+_{C_2}(\upsilon(p)),$$

$$T^-_{C_1}(p) = T^-_{C_2}(\upsilon(p)),\ I^-_{C_1}(p) = I^-_{C_2}(\upsilon(p)),\ F^-_{C_1}(p) = F^-_{C_2}(\upsilon(p)),$$

$\forall\ p \in V_1$.

Remark 3.3. The weak isomorphism between two BSVNGs preserves the orders.

Remark 3.4. The weak isomorphism between BSVNGs is a partial order relation.

Definition 3.5. Let $G_1 = (C_1, D_1)$ and $G_2 = (C_2, D_2)$ be two BSVNGs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. Then the co-weak isomorphism $\kappa : G_1 \to G_2$ is a bijective mapping $\kappa : V_1 \to V_2$ which satisfies following conditions:

κ is a homomorphism such that

$$T^+_{D_1}(pq) = T^+_{D_2}(\kappa(p)\kappa(q)),\ T^-_{D_1}(pq) = T^-_{D_2}(\kappa(p)\kappa(q)),$$

$$I^+_{D_1}(pq) = I^+_{D_2}(\kappa(p)\kappa(q)),\ I^-_{D_1}(pq) = I^-_{D_2}(\kappa(p)\kappa(q)),$$

$$F^+_{D_1}(pq) = F^+_{D_2}(\kappa(p)\kappa(q)),\ F^-_{D_1}(pq) = F^-_{D_2}(\kappa(p)\kappa(q)),$$

$\forall\ pq \in E_1$.

Remark 3.6. The co-weak isomorphism between two BSVNGs preserves the sizes.

Remark 3.7. The co-weak isomorphism between BSVNGs is a partial order relation.

Table 1. BSVNSs of BSVNG.

A	T_A^+	I_A^+	F_A^+	T_A^-	I_A^-	F_A^-
a	0.2	0.1	0.4	-0.3	-0.1	-0.4
b	0.3	0.2	0.5	-0.5	-0.4	-0.6
c	0.4	0.7	0.6	-0.2	-0.6	-0.2
B	T_B^+	I_B^+	F_B^+	T_B^-	I_B^-	F_B^-
p	0.2	0.4	0.5	-0.2	-0.5	-0.6
q	0.3	0.8	0.6	-0.1	-0.7	-0.8
r	0.1	0.7	0.9	-0.1	-0.8	-0.5

Definition 3.8. Let $G_1 = (C_1, D_1)$ and $G_2 = (C_2, D_2)$ be two BSVNGs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. Then the isomorphism $\psi : G_1 \to G_2$ is a bijective mapping $\psi : V_1 \to V_2$ which satisfies the following conditions:

$$T_{C_1}^+(p) = T_{C_2}^+(\psi(p)),\ I_{C_1}^+(p) = I_{C_2}^+(\psi(p)),\ F_{C_1}^+(p) = F_{C_2}^+(\psi(p)),$$

$$T_{C_1}^-(p) = T_{C_2}^-(\psi(p)),\ I_{C_1}^-(p) = I_{C_2}^-(\psi(p)),\ F_{C_1}^-(p) = F_{C_2}^-(\psi(p)),$$

$\forall\ p \in V_1$,

$$T_{D_1}^+(pq) = T_{D_2}^+(\psi(p)\psi(q)),\ T_{D_1}^-(pq) = T_{D_2}^-(\psi(p)\psi(q)),$$
$$I_{D_1}^+(pq) = I_{D_2}^+(\psi(p)\psi(q)),\ I_{D_1}^-(pq) = I_{D_2}^-(\psi(p)\psi(q)),$$
$$F_{D_1}^+(pq) = F_{D_2}^+(\psi(p)\psi(q)),\ F_{D_1}^-(pq) = F_{D_2}^-(\psi(p)\psi(q)),$$

$\forall\ pq \in E_1$.

Remark 3.9. The isomorphism between two BSVNGs is an equivalence relation.

Remark 3.10. The isomorphism between two BSVNGs preserves the orders and sizes.

Remark 3.11. The isomorphism between two BSVNGs preserves the degrees of their vertices.

Definition 3.12. The subdivision SVNG be $sd(G) = (C, D)$ of $G = (A, B)$, where C is a BSVNS on $V \cup E$ and D is a BSVNR on C such that

(i) $C = A$ on V and $C = B$ on E,

(ii) if $v \in V$ lie on edge $e \in E$, then

$$T_D^+(ve) = \min(T_A^+(v), T_B^+(e)),\ I_D^+(ve) = \max(I_A^+(v), I_B^+(e))$$
$$I_D^-(ve) = \min(I_A^-(v), I_B^-(e)),\ F_D^-(ve) = \min(F_A^-(v), F_B^-(e))$$
$$F_D^+(ve) = \max(F_A^+(v), F_B^+(e)),\ T_D^-(ve) = \max(T_A^-(v), T_B^-(e))$$

else

$$D(ve) = O = (0, 0, 0, 0, 0, 0).$$

Example 3.13. Consider the BSVNG $G = (A, B)$ of a $G^* = (V, E)$, where $V = \{a, b, c\}$ and $E = \{p = ab, q = bc, r = ac\}$, the crisp graph of G is shown in Fig. 1. The BSVNSs A and B are defined on V and E respectively which are defined in Table 1. The SDBSVNG $sd(G) = (C, D)$ of a BSVNG G, the underlying crisp graph of $sd(G)$ is given in Fig. 2. The BSVNSs C and D are defined in Table 2.

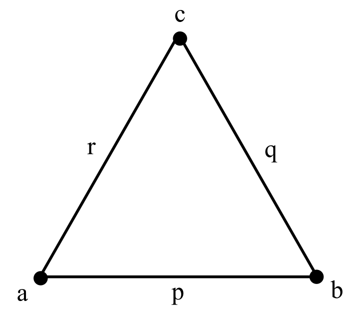

Figure 1. Crisp Graph of BSVNG.

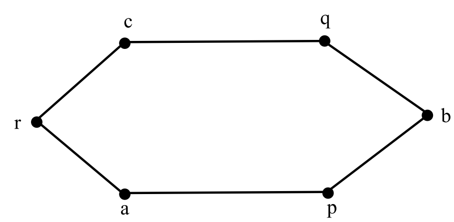

Figure 2. Crisp Graph of SDBSVNG.

Table 2. BSVNSs of SDBSVNG.

C	T_C^+	I_C^+	F_C^+	T_C^-	I_C^-	F_C^-
a	0.2	0.1	0.4	-0.3	-0.1	-0.4
p	0.2	0.4	0.5	-0.2	-0.5	-0.6
b	0.3	0.2	0.5	-0.5	-0.4	-0.6
q	0.3	0.8	0.6	-0.1	-0.7	-0.8
c	0.4	0.7	0.6	-0.2	-0.6	-0.2
r	0.1	0.7	0.9	-0.1	-0.8	-0.5
D	T_D^+	I_D^+	F_D^+	T_D^-	I_D^-	F_D^-
ap	0.2	0.4	0.5	-0.2	-0.5	-0.6
pb	0.2	0.4	0.5	-0.2	-0.5	-0.6
bq	0.3	0.8	0.6	-0.1	-0.7	-0.8
qc	0.3	0.8	0.6	-0.1	-0.7	-0.8
cr	0.1	0.7	0.9	-0.1	-0.8	-0.5
ra	0.1	0.7	0.9	-0.1	-0.8	-0.5

Proposition 3.14. *Let G be a BSVNG and $sd(G)$ be the SDBSVNG of a BSVNG G, then $O(sd(G)) = O(G) + S(G)$ and $S(sd(G)) = 2S(G)$.*

Remark 3.15. Let G be a complete BSVNG, then $sd(G)$ need not to be complete BSVNG.

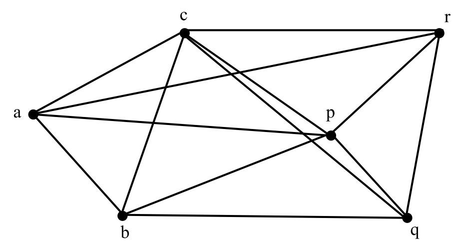

FIGURE 3. Crisp Graph of TSVNG.

Definition 3.16. The total bipolar single valued neutrosophic graph (TBSVNG) is $T(G) = (C, D)$ of $G = (A, B)$, where C is a BSVNS on $V \cup E$ and D is a BSVNR on C such that

(i) $C = A$ on V and $C = B$ on E,

(ii) if $v \in V$ lie on edge $e \in E$, then

$$T_D^+(ve) = \min(T_A^+(v), T_B^+(e)),\ I_D^+(ve) = \max(I_A^+(v), I_B^+(e))$$

$$I_D^-(ve) = \min(I_A^-(v), I_B^-(e)),\ F_D^-(ve) = \min(F_A^-(v), F_B^-(e))$$

$$F_D^+(ve) = \max(F_A^+(v), F_B^+(e)),\ T_D^-(ve) = \max(T_A^-(v), T_B^-(e))$$

else

$$D(ve) = O = (0, 0, 0, 0, 0, 0),$$

(iii) if $\alpha\beta \in E$, then

$$T_D^+(\alpha\beta) = T_B^+(\alpha\beta),\ I_D^+(\alpha\beta) = I_B^+(\alpha\beta),\ F_D^+(\alpha\beta) = F_B^+(\alpha\beta)$$

$$T_D^-(\alpha\beta) = T_B^-(\alpha\beta),\ I_D^-(\alpha\beta) = I_B^-(\alpha\beta),\ F_D^-(\alpha\beta) = F_B^-(\alpha\beta),$$

(iv) if $e, f \in E$ have a common vertex, then

$$T_D^+(ef) = \min(T_B^+(e), T_B^+(f)),\ I_D^+(ef) = \max(I_B^+(e), I_B^+(f))$$

$$I_D^-(ef) = \min(I_B^-(e), I_B^-(f)),\ F_D^-(ef) = \min(F_B^-(e), F_B^-(f))$$

$$F_D^+(ef) = \max(F_B^+(e), F_B^+(f)),\ T_D^-(ef) = \max(T_B^-(e), T_B^-(f))$$

else

$$D(ef) = O = (0, 0, 0, 0, 0, 0).$$

Example 3.17. Consider the Example 3.13 the TBSVNG $T(G) = (C, D)$ of underlying crisp graph as shown in Fig. 3. The BSVNS C is given in Example 3.13. The BSVNS D is given in Table 3.

Proposition 3.18. *Let G be a BSVNG and $T(G)$ be the TBSVNG of a BSVNG G, then $O(T(G)) = O(G) + S(G) = O(sd(G))$ and $S(sd(G)) = 2S(G)$.*

Proposition 3.19. *Let G be a BSVNG, then $sd(G)$ is weak isomorphic to $T(G)$.*

Table 3. BSVNS of TBSVNG.

D	T_D^+	I_D^+	F_D^+	T_D^-	I_D^-	F_D^-
ab	0.2	0.4	0.5	-0.2	-0.5	-0.6
bc	0.3	0.8	0.6	-0.1	-0.7	-0.8
ca	0.1	0.7	0.9	-0.1	-0.8	-0.5
pq	0.2	0.8	0.6	-0.1	-0.7	-0.8
qr	0.1	0.8	0.9	-0.1	-0.8	-0.8
rp	0.1	0.7	0.9	-0.1	-0.8	-0.6
ap	0.2	0.4	0.5	-0.2	-0.5	-0.6
pb	0.2	0.4	0.5	-0.2	-0.5	-0.6
bq	0.3	0.8	0.6	-0.1	-0.7	-0.8
qc	0.3	0.8	0.6	-0.1	-0.7	-0.8
cr	0.1	0.7	0.9	-0.1	-0.8	-0.5
ra	0.1	0.7	0.9	-0.1	-0.8	-0.5

Definition 3.20. The middle bipolar single valued neutrosophic graph (MBSVNG) $M(G) = (C, D)$ of G, where C is a BSVNS on $V \cup E$ and D is a BSVNR on C such that

(i) $C = A$ on V and $C = B$ on E, else $C = O = (0, 0, 0, 0, 0, 0)$,

(ii) if $v \in V$ lie on edge $e \in E$, then

$$T_D^+(ve) = T_B^+(e),\ I_D^+(ve) = I_B^+(e),\ F_D^+(ve) = F_B^+(e)$$

$$T_D^-(ve) = T_B^-(e),\ I_D^-(ve) = I_B^-(e),\ F_D^-(ve) = F_B^-(e)$$

else

$$D(ve) = O = (0, 0, 0, 0, 0, 0),$$

(iii) if $u, v \in V$, then

$$D(uv) = O = (0, 0, 0, 0, 0, 0),$$

(iv) if $e, f \in E$ and e and f are adjacent in G, then

$$T_D^+(ef) = T_B^+(uv),\ I_D^+(ef) = I_B^+(uv),\ F_D^+(ef) = F_B^+(uv)$$

$$T_D^-(ef) = T_B^-(uv),\ I_D^-(ef) = I_B^-(uv),\ F_D^-(ef) = F_B^-(uv).$$

Example 3.21. Consider the BSVNG $G = (A, B)$ of a G^*, where $V = \{a, b, c\}$ and $E = \{p = ab, q = bc\}$ the underlaying crisp graph is shown in Fig. 4. The BSVNSs A and B are defined in Table 4. The crisp graph of MBSVNG $M(G) = (C, D)$ is shown in Fig. 5. The BSVNSs C and D are given in Table 5.

Remark 3.22. Let G be a BSVNG and $M(G)$ be the MBSVNG of a BSVNG G, then $O(M(G)) = O(G) + S(G)$.

Remark 3.23. Let G be a BSVNG, then $M(G)$ is a strong BSVNG.

Remark 3.24. Let G be complete BSVNG, then $M(G)$ need not to be complete BSVNG.

Proposition 3.25. *Let G be a BSVNG, then $sd(G)$ is weak isomorphic with $M(G)$.*

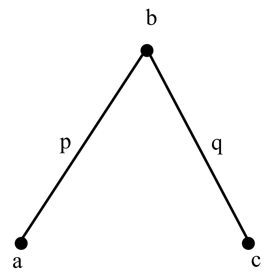

FIGURE 4. Crisp Graph of BSVNG.

TABLE 4. BSVNSs of BSVNG.

A	T_A^+	I_A^+	F_A^+	T_A^-	I_A^-	F_A^-
a	0.3	0.4	0.5	-0.2	-0.1	-0.3
b	0.7	0.6	0.3	-0.3	-0.3	-0.2
c	0.9	0.7	0.2	-0.5	-0.4	-0.6
B	T_B^+	I_B^+	F_B^+	T_B^-	I_B^-	F_B^-
p	0.2	0.6	0.6	-0.1	-0.4	-0.3
q	0.4	0.8	0.7	-0.3	-0.5	-0.6

TABLE 5. BSVNSs of MBSVNG.

C	T_C^+	I_C^+	F_C^+	T_C^-	I_C^-	F_C^-
a	0.3	0.4	0.5	-0.2	-0.1	-0.3
b	0.7	0.6	0.3	-0.3	-0.3	-0.2
c	0.9	0.7	0.2	-0.5	-0.4	-0.6
e_1	0.2	0.6	0.6	-0.1	-0.4	-0.3
e_2	0.4	0.8	0.7	-0.3	-0.5	-0.6
D	T_D^+	I_D^+	F_D^+	T_D^-	I_D^-	F_D^-
pq	0.2	0.8	0.7	-0.1	-0.5	-0.6
ap	0.2	0.6	0.6	-0.1	-0.4	-0.3
bp	0.2	0.6	0.6	-0.1	-0.4	-0.3
bq	0.2	0.6	0.6	-0.3	-0.5	-0.6
cq	0.4	0.8	0.7	-0.3	-0.5	-0.6

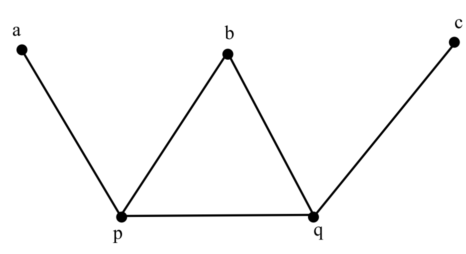

FIGURE 5. Crisp Graph of MBSVNG.

Proposition 3.26. *Let G be a BSVNG, then $M(G)$ is weak isomorphic with $T(G)$.*

Proposition 3.27. *Let G be a BSVNG, then $T(G)$ is isomorphic with $G \cup M(G)$.*

Definition 3.28. Let $P(X) = (X, Y)$ be the intersection graph of a G^*, let C_1 and D_1 be BSVNSs on V and E, respectively and C_2 and D_2 be BSVNSs on X and Y respectively. Then bipolar single valued neutrosophic intersection graph (BSVNIG) of a BSVNG $G = (C_1, D_1)$ is a BSVNG $P(G) = (C_2, D_2)$ such that,

$$T^+_{C_2}(X_i) = T^+_{C_1}(v_i),\ I^+_{C_2}(X_i) = I^+_{C_1}(v_i),\ F^+_{C_2}(X_i) = F^+_{C_1}(v_i)$$
$$T^-_{C_2}(X_i) = T^-_{C_1}(v_i),\ I^-_{C_2}(X_i) = I^-_{C_1}(v_i),\ F^-_{C_2}(X_i) = F^-_{C_1}(v_i)$$
$$T^+_{D_2}(X_iX_j) = T^+_{D_1}(v_iv_j),\ T^-_{D_2}(X_iX_j) = T^-_{D_1}(v_iv_j),$$
$$I^+_{D_2}(X_iX_j) = I^+_{D_1}(v_iv_j),\ I^-_{D_2}(X_iX_j) = I^-_{D_1}(v_iv_j),$$
$$F^+_{D_2}(X_iX_j) = F^+_{D_1}(v_iv_j),\ F^-_{D_2}(X_iX_j) = F^-_{D_1}(v_iv_j)$$

$\forall\ X_i, X_j \in X$ and $X_iX_j \in Y$.

Proposition 3.29. *Let $G = (A_1, B_1)$ be a BSVNG of $G^* = (V, E)$, and let $P(G) = (A_2, B_2)$ be a BSVNIG of $P(S)$. Then BSVNIG is a also BSVNG and BSVNG is always isomorphic to BSVNIG.*

Proof. By the definition of BSVNIG, we have

$$\begin{aligned}
T^+_{B_2}(S_iS_j) &= T^+_{B_1}(v_iv_j) \le \min(T^+_{A_1}(v_i), T^+_{A_1}(v_j)) = \min(T^+_{A_2}(S_i), T^+_{A_2}(S_j)),\\
I^+_{B_2}(S_iS_j) &= I^+_{B_1}(v_iv_j) \ge \max(I^+_{A_1}(v_i), I^+_{A_1}(v_j)) = \max(I^+_{A_2}(S_i), I^+_{A_2}(S_j)),\\
F^+_{B_2}(S_iS_j) &= F^+_{B_1}(v_iv_j) \ge \max(F^+_{A_1}(v_i), F^+_{A_1}(v_j)) = \max(F^+_{A_2}(S_i), F^+_{A_2}(S_j)),\\
T^-_{B_2}(S_iS_j) &= T^-_{B_1}(v_iv_j) \ge \max(T^-_{A_1}(v_i), T^-_{A_1}(v_j)) = \max(T^-_{A_2}(S_i), T^-_{A_2}(S_j)),\\
I^-_{B_2}(S_iS_j) &= I^-_{B_1}(v_iv_j) \le \min(I^-_{A_1}(v_i), I^-_{A_1}(v_j)) = \min(I^-_{A_2}(S_i), I^-_{A_2}(S_j)),\\
F^-_{B_2}(S_iS_j) &= F^-_{B_1}(v_iv_j) \le \min(F^-_{A_1}(v_i), F^-_{A_1}(v_j)) = \min(F^-_{A_2}(S_i), F^-_{A_2}(S_j)).
\end{aligned}$$

This shows that BSVNIG is a BSVNG.

Next define $f : V \to S$ by $f(v_i) = S_i$ for $i = 1, 2, 3, \ldots, n$ clearly f is bijective. Now $v_iv_j \in E$ if and only if $S_iS_j \in T$ and $T = \{f(v_i)f(v_j) : v_iv_j \in E\}$. Also

$$T^+_{A_2}(f(v_i)) = T^+_{A_2}(S_i) = T^+_{A_1}(v_i),\ I^+_{A_2}(f(v_i)) = I^+_{A_2}(S_i) = I^+_{A_1}(v_i),$$
$$F^+_{A_2}(f(v_i)) = F^+_{A_2}(S_i) = F^+_{A_1}(v_i),\ T^-_{A_2}(f(v_i)) = T^-_{A_2}(S_i) = T^-_{A_1}(v_i),$$
$$I^-_{A_2}(f(v_i)) = I^-_{A_2}(S_i) = I^-_{A_1}(v_i),\ F^-_{A_2}(f(v_i)) = F^-_{A_2}(S_i) = F^-_{A_1}(v_i),$$

$\forall\ v_i \in V$,

$$T^+_{B_2}(f(v_i)f(v_j)) = T^+_{B_2}(S_iS_j) = T^+_{B_1}(v_iv_j),$$
$$I^+_{B_2}(f(v_i)f(v_j)) = I^+_{B_2}(S_iS_j) = I^+_{B_1}(v_iv_j),$$
$$F^+_{B_2}(f(v_i)f(v_j)) = F^+_{B_2}(S_iS_j) = F^+_{B_1}(v_iv_j),$$
$$T^-_{B_2}(f(v_i)f(v_j)) = T^-_{B_2}(S_iS_j) = T^-_{B_1}(v_iv_j),$$
$$I^-_{B_2}(f(v_i)f(v_j)) = I^-_{B_2}(S_iS_j) = I^-_{B_1}(v_iv_j),$$
$$F^-_{B_2}(f(v_i)f(v_j)) = F^-_{B_2}(S_iS_j) = F^-_{B_1}(v_iv_j),$$

$\forall\ v_iv_j \in E$. □

Table 6. BSVNSs of BSVNG.

A_1	$T^+_{A_1}$	$I^+_{A_1}$	$F^+_{A_1}$	$T^-_{A_1}$	$I^-_{A_1}$	$F^-_{A_1}$
α_1	0.2	0.5	0.5	-0.1	-0.4	-0.5
α_2	0.4	0.3	0.3	-0.2	-0.3	-0.2
α_3	0.4	0.5	0.5	-0.3	-0.2	-0.6
α_4	0.3	0.2	0.2	-0.4	-0.1	-0.3
B_1	$T^+_{B_1}$	$I^+_{B_1}$	$F^+_{B_1}$	$T^-_{B_1}$	$I^-_{B_1}$	$F^-_{B_1}$
x_1	0.1	0.6	0.7	-0.1	-0.4	-0.5
x_2	0.3	0.6	0.7	-0.2	-0.3	-0.6
x_3	0.2	0.7	0.8	-0.3	-0.2	-0.6
x_4	0.1	0.7	0.8	-0.1	-0.4	-0.5

Definition 3.30. Let $G^* = (V, E)$ and $L(G^*) = (X, Y)$ be its line graph, where A_1 and B_1 be BSVNSs on V and E, respectively. Let A_2 and B_2 be BSVNSs on X and Y, respectively. The bipolar single valued neutrosophic line graph (BSVNLG) of BSVNG $G = (A_1, B_1)$ is BSVNG $L(G) = (A_2, B_2)$ such that,

$$T^+_{A_2}(S_x) = T^+_{B_1}(x) = T^+_{B_1}(u_x v_x),\ I^+_{A_2}(S_x) = I^+_{B_1}(x) = I^+_{B_1}(u_x v_x),$$
$$I^-_{A_2}(S_x) = I^-_{B_1}(x) = I^-_{B_1}(u_x v_x),\ F^-_{A_2}(S_x) = F^-_{B_1}(x) = F^-_{B_1}(u_x v_x),$$
$$F^+_{A_2}(S_x) = F^+_{B_1}(x) = F^+_{B_1}(u_x v_x),\ T^-_{A_2}(S_x) = T^-_{B_1}(x) = T^-_{B_1}(u_x v_x),$$

$\forall\ S_x, S_y \in X$ and

$$T^+_{B_2}(S_x S_y) = \min(T^+_{B_1}(x), T^+_{B_1}(y)),\ I^+_{B_2}(S_x S_y) = \max(I^+_{B_1}(x), I^+_{B_1}(y)),$$
$$I^-_{B_2}(S_x S_y) = \min(I^-_{B_1}(x), I^-_{B_1}(y)),\ F^-_{B_2}(S_x S_y) = \min(F^-_{B_1}(x), F^-_{B_1}(y)),$$
$$F^+_{B_2}(S_x S_y) = \max(F^+_{B_1}(x), F^+_{B_1}(y)),\ T^-_{B_2}(S_x S_y) = \max(T^-_{B_1}(x), T^-_{B_1}(y)),$$

$\forall\ S_x S_y \in Y$.

Remark 3.31. Every BSVNLG is a strong BSVNG.

Remark 3.32. The $L(G) = (A_2, B_2)$ is a BSVNLG corresponding to BSVNG $G = (A_1, B_1)$.

Example 3.33. Consider the $G^* = (V, E)$ where $V = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ and $E = \{x_1 = \alpha_1\alpha_2, x_2 = \alpha_2\alpha_3, x_3 = \alpha_3\alpha_4, x_4 = \alpha_4\alpha_1\}$ and $G = (A_1, B_1)$ is BSVNG of $G^* = (V, E)$ which is defined in Table 6. Consider the $L(G^*) = (X, Y)$ such that $X = \{\Gamma_{x_1}, \Gamma_{x_2}, \Gamma_{x_3}, \Gamma_{x_4}\}$ and $Y = \{\Gamma_{x_1}\Gamma_{x_2}, \Gamma_{x_2}\Gamma_{x_3}, \Gamma_{x_3}\Gamma_{x_4}, \Gamma_{x_4}\Gamma_{x_1}\}$. Let A_2 and B_2 be BSVNSs of X and Y respectively, then BSVNLG $L(G)$ is given in Table 7.

Proposition 3.34. *The $L(G) = (A_2, B_2)$ is a BSVNLG of some BSVNG $G = (A_1, B_1)$ if and only if*

$$T^+_{B_2}(S_x S_y) = \min(T^+_{A_2}(S_x), T^+_{A_2}(S_y)),$$
$$T^-_{B_2}(S_x S_y) = \max(T^-_{A_2}(S_x), T^-_{A_2}(S_y)),$$
$$I^+_{B_2}(S_x S_y) = \max(I^+_{A_2}(S_x), I^+_{A_2}(S_y)),$$
$$F^-_{B_2}(S_x S_y) = \min(F^-_{A_2}(S_x), F^-_{A_2}(S_y)),$$

TABLE 7. BSVNSs of BSVNLG.

A_1	$T^+_{A_1}$	$I^+_{A_1}$	$F^+_{A_1}$	$T^-_{A_1}$	$I^-_{A_1}$	$F^-_{A_1}$
Γ_{x_1}	0.1	0.6	0.7	-0.1	-0.4	-0.5
Γ_{x_2}	0.3	0.6	0.7	-0.2	-0.3	-0.6
Γ_{x_3}	0.2	0.7	0.8	-0.3	-0.2	-0.6
Γ_{x_4}	0.1	0.7	0.8	-0.1	-0.4	-0.5
B_1	$T^+_{B_1}$	$I^+_{B_1}$	$F^+_{B_1}$	$T^-_{B_1}$	$I^-_{B_1}$	$F^-_{B_1}$
$\Gamma_{x_1}\Gamma_{x_2}$	0.1	0.6	0.7	-0.1	-0.4	-0.6
$\Gamma_{x_2}\Gamma_{x_3}$	0.2	0.7	0.8	-0.2	-0.3	-0.6
$\Gamma_{x_3}\Gamma_{x_4}$	0.1	0.7	0.8	-0.1	-0.4	-0.6
$\Gamma_{x_4}\Gamma_{x_1}$	0.1	0.7	0.8	-0.1	-0.4	-0.5

$$I^-_{B_2}(S_xS_y) = \min(I^-_{A_2}(S_x), I^-_{A_2}(S_y)),$$
$$F^+_{B_2}(S_xS_y) = \max(F^+_{A_2}(S_x), F^+_{A_2}(S_y)),$$

$\forall\ S_xS_y \in Y$.

Proof. Assume that,

$$T^+_{B_2}(S_xS_y) = \min(T^+_{A_2}(S_x), T^+_{A_2}(S_y)),$$
$$T^-_{B_2}(S_xS_y) = \max(T^-_{A_2}(S_x), T^-_{A_2}(S_y)),$$
$$I^+_{B_2}(S_xS_y) = \max(I^+_{A_2}(S_x), I^+_{A_2}(S_y)),$$
$$F^-_{B_2}(S_xS_y) = \min(F^-_{A_2}(S_x), F^-_{A_2}(S_y)),$$
$$I^-_{B_2}(S_xS_y) = \min(I^-_{A_2}(S_x), I^-_{A_2}(S_y)),$$
$$F^+_{B_2}(S_xS_y) = \max(F^+_{A_2}(S_x), F^+_{A_2}(S_y)),$$

$\forall\ S_xS_y \in Y$. Define

$$T^+_{A_1}(x) = T^+_{A_2}(S_x),\ I^+_{A_1}(x) = I^+_{A_2}(S_x),\ F^+_{A_1}(x) = F^+_{A_2}(S_x),$$
$$T^-_{A_1}(x) = T^-_{A_2}(S_x),\ I^-_{A_1}(x) = I^-_{A_2}(S_x),\ F^-_{A_1}(x) = F^-_{A_2}(S_x)$$

$\forall\ x \in E$. Then

$$I^+_{B_2}(S_xS_y) = \max(I^+_{A_2}(S_x), I^+_{A_2}(S_y)) = \max(I^+_{A_2}(x), I^+_{A_2}(y)),$$
$$I^-_{B_2}(S_xS_y) = \min(I^-_{A_2}(S_x), I^-_{A_2}(S_y)) = \min(I^-_{A_2}(x), I^-_{A_2}(y)),$$
$$T^+_{B_2}(S_xS_y) = \min(T^+_{A_2}(S_x), T^+_{A_2}(S_y)) = \min(T^+_{A_2}(x), T^+_{A_2}(y)),$$
$$T^-_{B_2}(S_xS_y) = \max(T^-_{A_2}(S_x), T^-_{A_2}(S_y)) = \max(T^-_{A_2}(x), T^-_{A_2}(y)),$$
$$F^-_{B_2}(S_xS_y) = \min(F^-_{A_2}(S_x), F^-_{A_2}(S_y)) = \min(F^-_{A_2}(x), F^-_{A_2}(y)),$$
$$F^+_{B_2}(S_xS_y) = \max(F^+_{A_2}(S_x), F^+_{A_2}(S_y)) = \max(F^+_{A_2}(x), F^+_{A_2}(y)).$$

A BSVNS A_1 that yields the property

$$T^+_{B_1}(xy) \leq \min(T^+_{A_1}(x), T^+_{A_1}(y)),\ I^+_{B_1}(xy) \geq \max(I^+_{A_1}(x), I^+_{A_1}(y)),$$
$$I^-_{B_1}(xy) \leq \min(I^-_{A_1}(x), I^-_{A_1}(y)),\ F^-_{B_1}(xy) \leq \min(F^-_{A_1}(x), F^-_{A_1}(y)),$$
$$F^+_{B_1}(xy) \geq \max(F^+_{A_1}(x), F^+_{A_1}(y)),\ T^-_{B_1}(xy) \geq \max(T^-_{A_1}(x), T^-_{A_1}(y))$$

will suffice. Converse is straight forward. □

Proposition 3.35. *If $L(G)$ be a BSVNLG of BSVNG G, then $L(G^*) = (X, Y)$ is the crisp line graph of G^*.*

Proof. Since $L(G)$ is a BSVNLG,

$$T^+_{A_2}(S_x) = T^+_{B_1}(x),\ I^+_{A_2}(S_x) = I^+_{B_1}(x),\ F^+_{A_2}(S_x) = F^+_{B_1}(x),$$

$$T^-_{A_2}(S_x) = T^-_{B_1}(x),\ I^-_{A_2}(S_x) = I^-_{B_1}(x),\ F^-_{A_2}(S_x) = F^-_{B_1}(x)$$

$\forall\ x \in E$, $S_x \in X$ if and only if $x \in E$, also

$$T^+_{B_2}(S_xS_y) = \min(T^+_{B_1}(x), T^+_{B_1}(y)),\ I^+_{B_2}(S_xS_y) = \max(I^+_{B_1}(x), I^+_{B_1}(y)),$$

$$I^-_{B_2}(S_xS_y) = \min(I^-_{B_1}(x), I^-_{B_1}(y)),\ F^-_{B_2}(S_xS_y) = \min(F^-_{B_1}(x), F^-_{B_1}(y)),$$

$$F^+_{B_2}(S_xS_y) = \max(F^+_{B_1}(x), F^+_{B_1}(y)),\ T^-_{B_2}(S_xS_y) = \max(T^-_{B_1}(x), T^-_{B_1}(y)),$$

$\forall\ S_xS_y \in Y$. Then $Y = \{S_xS_y : S_x \cap S_y \neq \phi, x, y \in E, x \neq y\}$. □

Proposition 3.36. *The $L(G) = (A_2, B_2)$ be a BSVNLG of BSVNG G if and only if $L(G^*) = (X, Y)$ is the line graph and*

$$T^+_{B_2}(xy) = \min(T^+_{A_2}(x), T^+_{A_2}(y)),\ I^+_{B_2}(xy) = \max(I^+_{A_2}(x), I^+_{A_2}(y)),$$

$$I^-_{B_2}(xy) = \min(I^-_{A_2}(x), I^-_{A_2}(y)),\ F^-_{B_2}(xy) = \min(F^-_{A_2}(x), F^-_{A_2}(y)),$$

$$F^+_{B_2}(xy) = \max(F^+_{A_2}(x), F^+_{A_2}(y)),\ T^-_{B_2}(xy) = \max(T^-_{A_2}(x), T^-_{A_2}(y)),$$

$\forall\ xy \in Y$.

Proof. It follows from propositions 3.34 and 3.35. □

Proposition 3.37. *Let G be a BSVNG, then $M(G)$ is isomorphic with $sd(G) \cup L(G)$.*

Theorem 3.38. *Let $L(G) = (A_2, B_2)$ be BSVNLG corresponding to BSVNG $G = (A_1, B_1)$.*

(1) *If G is weak isomorphic onto $L(G)$ if and only if $\forall\ v \in V,\ x \in E$ and G^* to be a cycle, such that*

$$T^+_{A_1}(v) = T^+_{B_1}(x),\ I^+_{A_1}(v) = T^+_{B_1}(x),\ F^+_{A_1}(v) = T^+_{B_1}(x),$$

$$T^-_{A_1}(v) = T^-_{B_1}(x),\ I^-_{A_1}(v) = T^-_{B_1}(x),\ F^-_{A_1}(v) = T^-_{B_1}(x).$$

(2) *If G is weak isomorphic onto $L(G)$, then G and $L(G)$ are isomorphic.*

Proof. By hypothesis, G^* is a cycle. Let $V = \{v_1, v_2, v_3, \ldots, v_n\}$ and $E = \{x_1 = v_1v_2, x_2 = v_2v_3, \ldots, x_n = v_nv_1\}$, where $P : v_1v_2v_3 \ldots v_n$ is a cycle, characterize a BSVNS A_1 by $A_1(v_i) = (p_i, q_i, r_i, p^{'}_i, q^{'}_i, r^{'}_i)$ and B_1 by $B_1(x_i) = (a_i, b_i, c_i, a^{'}_i, b^{'}_i, c^{'}_i)$ for $i = 1, 2, 3, \ldots, n$ and $v_{n+1} = v_1$. Then for $p_{n+1} = p_1$, $q_{n+1} = q_1$, $r_{n+1} = r_1$,

$$a_i \leq \min(p_i, p_{i+1}),\ b_i \geq \max(q_i, q_{i+1}),\ c_i \geq \max(r_i, r_{i+1}),$$

$$a^{'}_i \geq \max(p^{'}_i, p^{'}_{i+1}),\ b^{'}_i \leq \min(q^{'}_i, q^{'}_{i+1}),\ c^{'}_i \leq \min(r^{'}_i, r^{'}_{i+1}),$$

for $i = 1, 2, 3, \ldots, n$.

Now let $X = \{\Gamma_{x_1}, \Gamma_{x_2}, \ldots, \Gamma_{x_n}\}$ and $Y = \{\Gamma_{x_1}\Gamma_{x_2}, \Gamma_{x_2}\Gamma_{x_3}, \ldots, \Gamma_{x_n}\Gamma_{x_1}\}$. Then for $a_{n+1} = a_1$, we obtain

$$A_2(\Gamma_{x_i}) = B_1(x_i) = (a_i, b_i, c_i, a^{'}_i, b^{'}_i, c^{'}_i)$$

and $B_2(\Gamma_{x_i}\Gamma_{x_{i+1}}) = (\min(a_i, a_{i+1}), \max(b_i, b_{i+1}), \max(c_i, c_{i+1}), \max(a_i^{'}, a_{i+1}^{'}), \min(b_i^{'}, b_{i+1}^{'}), \min(c_i^{'}, c_{i+1}^{'}))$ for $i = 1, 2, 3, \ldots, n$ and $v_{n+1} = v_1$. Since f preserves adjacency, it induce permutation π of $\{1, 2, 3, \ldots, n\}$,

$$f(v_i) = \Gamma_{v_{\pi(i)}v_{\pi(i)+1}}$$

and

$$v_iv_{i+1} \to f(v_i)f(v_{i+1}) = \Gamma_{v_{\pi(i)}v_{\pi(i)+1}}\Gamma_{v_{\pi(i+1)}v_{\pi(i+1)+1}},$$

for $i = 1, 2, 3, \ldots, n-1$. Thus

$$p_i = T^+_{A_1}(v_i) \leq T^+_{A_2}(f(v_i)) = T^+_{A_2}(\Gamma_{v_{\pi(i)}v_{\pi(i)+1}}) = T^+_{B_1}(v_{\pi(i)}v_{\pi(i)+1}) = a_{\pi(i)}.$$

Similarly, $p_i^{'} \geq a_{\pi(i)}^{'}$, $q_i \geq b_{\pi(i)}$, $r_i \geq c_{\pi(i)}$, $q_i^{'} \leq b_{\pi(i)}^{'}$, $r_i^{'} \leq c_{\pi(i)}^{'}$ and

$$\begin{aligned} a_i &= T^+_{B_1}(v_iv_{i+1}) \leq T^+_{B_2}(f(v_i)f(v_{i+1})) \\ &= T^+_{B_2}(\Gamma_{v_{\pi(i)}v_{\pi(i)+1}}\Gamma_{v_{\pi(i+1)}v_{\pi(i+1)+1}}) \\ &= \min(T^+_{B_1}(v_{\pi(i)}v_{\pi(i)+1}), T^+_{B_1}(v_{\pi(i+1)}v_{\pi(i+1)+1})) \\ &= \min(a_{\pi(i)}, a_{\pi(i)+1}). \end{aligned}$$

Similarly, $b_i \geq \max(b_{\pi(i)}, b_{\pi(i)+1})$, $c_i \geq \max(c_{\pi(i)}, c_{\pi(i)+1})$, $a_i^{'} \geq \max(a_{\pi(i)}^{'}, a_{\pi(i)+1}^{'})$, $b_i^{'} \leq \min(b_{\pi(i)}^{'}, b_{\pi(i)+1}^{'})$ and $c_i^{'} \leq \min(c_{\pi(i)}^{'}, c_{\pi(i)+1}^{'})$ for $i = 1, 2, 3, \ldots, n$. Therefore

$$p_i \leq a_{\pi(i)},\ q_i \geq b_{\pi(i)},\ r_i \geq c_{\pi(i)},\ p_i^{'} \geq a_{\pi(i)}^{'},\ q_i^{'} \leq b_{\pi(i)}^{'},\ r_i^{'} \leq c_{\pi(i)}^{'}$$

and

$$\begin{gathered} a_i \leq \min(a_{\pi(i)}, a_{\pi(i)+1}),\ a_i^{'} \geq \max(a_{\pi(i)}^{'}, a_{\pi(i)+1}^{'}), \\ b_i \geq \max(b_{\pi(i)}, b_{\pi(i)+1}),\ b_i^{'} \leq \min(b_{\pi(i)}^{'}, b_{\pi(i)+1}^{'}), \\ c_i \geq \max(c_{\pi(i)}, c_{\pi(i)+1}),\ c_i \leq \min(c_{\pi(i)}^{'}, c_{\pi(i)+1}^{'}) \end{gathered}$$

thus

$$a_i \leq a_{\pi(i)},\ b_i \geq b_{\pi(i)},\ c_i \geq c_{\pi(i)},\ a_i^{'} \geq a_{\pi(i)}^{'},\ b_i^{'} \leq b_{\pi(i)}^{'},\ c_i^{'} \leq c_{\pi(i)}^{'}$$

and so

$$\begin{gathered} a_{\pi(i)} \leq a_{\pi(\pi(i))},\ b_{\pi(i)} \geq b_{\pi(\pi(i))},\ c_{\pi(i)} \geq c_{\pi(\pi(i))} \\ a_{\pi(i)}^{'} \geq a_{\pi(\pi(i))}^{'},\ b_{\pi(i)}^{'} \leq b_{\pi(\pi(i))}^{'},\ c_{\pi(i)}^{'} \leq c_{\pi(\pi(i))}^{'} \end{gathered}$$

$\forall\ i = 1, 2, 3, \ldots, n$. Next to extend,

$$\begin{gathered} a_i \leq a_{\pi(i)} \leq \ldots \leq a_{\pi^j(i)} \leq a_i,\ a_i^{'} \geq a_{\pi(i)}^{'} \geq \ldots \geq a_{\pi^j(i)}^{'} \geq a_i^{'} \\ b_i \geq b_{\pi(i)} \geq \ldots \geq b_{\pi^j(i)} \geq b_i,\ b_i^{'} \leq b_{\pi(i)}^{'} \leq \ldots \leq b_{\pi^j(i)}^{'} \leq b_i^{'} \\ c_i \geq c_{\pi(i)} \geq \ldots \geq c_{\pi^j(i)} \geq c_i,\ c_i^{'} \leq c_{\pi(i)}^{'} \leq \ldots \leq c_{\pi^j(i)}^{'} \leq c_i^{'} \end{gathered}$$

where π^{j+1} identity. Hence

$$a_i = a_{\pi(i)},\ b_i = b_{\pi(i)},\ c_i = c_{\pi(i)},\ a_i^{'} = a_{\pi(i)}^{'},\ b_i^{'} = b_{\pi(i)}^{'},\ c_i^{'} = c_{\pi(i)}^{'}$$

$\forall\ i = 1, 2, 3, \ldots, n$. Thus we conclude that

$$\begin{gathered} a_i \leq a_{\pi(i+1)} = a_{i+1},\ b_i \geq b_{\pi(i+1)} = b_{i+1},\ c_i \geq c_{\pi(i+1)} = c_{i+1} \\ a_i^{'} \geq a_{\pi(i+1)}^{'} = a_{i+1}^{'},\ b_i^{'} \leq b_{\pi(i+1)}^{'} = b_{i+1}^{'},\ c_i^{'} \leq c_{\pi(i+1)}^{'} = c_{i+1}^{'} \end{gathered}$$

which together with

$$a_{n+1} = a_1,\ b_{n+1} = b_1,\ c_{n+1} = c_1,\ a^{'}_{n+1} = a^{'}_1,\ b^{'}_{n+1} = b^{'}_1,\ c^{'}_{n+1} = c^{'}_1$$

which implies that

$$a_i = a_1,\ b_i = b_1,\ c_i = c_1,\ a^{'}_i = a^{'}_1,\ b^{'}_i = b^{'}_1,\ c^{'}_i = c^{'}_1$$

$\forall\ i = 1, 2, 3, \ldots, n$. Thus we have

$$\begin{aligned}
a_1 = a_2 = \ldots = a_n = p_1 = p_2 = \ldots = p_n \\
a^{'}_1 = a^{'}_2 = \ldots = a^{'}_n = p^{'}_1 = p^{'}_2 = \ldots = p^{'}_n \\
b_1 = b_2 = \ldots = b_n = q_1 = q_2 = \ldots = q_n \\
b^{'}_1 = b^{'}_2 = \ldots = b^{'}_n = q^{'}_1 = q^{'}_2 = \ldots = q^{'}_n \\
c_1 = c_2 = \ldots = c_n = r_1 = r_2 = \ldots = r_n \\
c^{'}_1 = c^{'}_2 = \ldots = c^{'}_n = r^{'}_1 = r^{'}_2 = \ldots = r^{'}_n
\end{aligned}$$

Therefore (a) and (b) holds, since converse of result (a) is straight forward. □

4. Conclusion

The neutrosophic graphs have many applications in path problems, networks and computer science. Strong BSVNG and complete BSVNG are the types of BSVNG. In this paper, we discussed the special types of BSVNGs, subdivision BSVNGs, middle BSVNGs, total BSVNGs and BSVNLGs of the given BSVNGs. We investigated isomorphism properties of subdivision BSVNGs, middle BSVNGs, total BSVNGs and BSVNLGs.

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