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The generalized Dice measures for multiple attribute decision making under simplified neutrosophic environments

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Abstract. A simplified neutrosophic set (SNS) is a subclass of neutrosophic set and contains a single-valued neutrosophic set (SVNS) and an interval neutrosophic set (INS). It was proposed as a generalization of an intuitionistic fuzzy set (IFS) and an interval-valued intuitionistic fuzzy set (IVIFS) in order to deal with indeterminate and inconsistent information. The paper proposes another form of the Dice measures of SNSs and the generalized Dice measures of SNSs and indicates that the Dice measures and asymmetric measures (projection measures) are the special cases of the generalized Dice measures in some parameter values. Then, we develop the generalized Dice measures-based multiple attribute decision-making methods with simplified neutrosophic information. By the weighted generalized Dice measures between each alternative and the ideal solution (ideal alternative) corresponding to some parameter value required by decision makers' preference, all the alternatives can be ranked and the best one can be obtained as well. Finally, a real example on the selection of manufacturing schemes demonstrates the applications of the proposed decision-making methods under simplified neutrosophic environment. The effectiveness and flexibility of the proposed decision-making methods are shown by choosing different parameter values.

Keywords: Generalized Dice measure, Dice measure, decision making, simplified neutrosophic set, asymmetric measure, projection measure

1. Introduction

Multiple attribute decision making is a main branch of decision theory, where neutrosophic theory introduced by Smarandache [1] has been successfully applied in recent years. As a generalization of an intuitionistic fuzzy set (IFS) [16] and an interval-valued intuitionistic fuzzy (IVIFS) [17], a simplified neutrosophic set (SNS) introduced by Ye [10] is a subclass of a neutrosophic sets [1], including a single-valued neutrosophic set (SVNSs) [4] and an interval neutrosophic set (INSs) [3]. Hence, SNSs are very suitable for handling decision making problems with indeterminate and inconsistent information, which IFSs and IVIFSs cannot describe and deal with. Recently, many researchers have applied SNSs and the subclasses of SNSs (SVNSs and INSs) to the decision-making problems. Various methods have been developed to solve the multiple attribute decision-making problems with simplified neutrosophic information. For example, Ye [9] proposed the correlation coefficient of SVNSs and applied it to multiple attribute decision making. Chi and Liu [18] and Biswas et al. [21] extended TOPSIS method to single-valued and interval neutrosophic multiple attribute decision-making problems. Ye [11–13] presented some similarity measures of SVNSs, INSs and SNSs and applied them to decision making.

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Ye [14] put forward a cross-entropy measure of SVNSs for multiple attribute decision making problems. Ye [10], Zhang et al. [5], Liu et al. [19], Liu and Wang [20], and Peng et al. [8] developed some simplified, interval and single-valued neutrosophic number aggregation operators and applied them to multiple attribute decision-making problems. Peng et al. [7] and Zhang et al. [6] proposed outranking approaches for multicriteria decision-making problems with simplified and interval neutrosophic information. Sahin and Kucuk [22] presented a subsethood measure for SVNSs and applied it to multiple attribute decision making. Şahin and Liu [23] introduced a maximizing deviation method for neutrosophic multiple attribute decision making with incomplete weight information. Ye [15] presented a multiple attribute decision-making method based on the possibility degree ranking method and ordered weighted aggregation operators of interval neutrosophic numbers.

Since the Dice measure is one of vector similarity measures, it is a useful mathematical tool for handling decision-making problems. However, the Dice measure of SNSs [13] used for decision making lacks flexibility in decision-making process. Therefore, it is necessary to improve the Dice measure of SNSs to handle multiple attribute decision-making problems to satisfy the requirements of decision makers' preference and flexible decision making. In order to do so, the main purposes of this paper are: (1) to propose another form of the Dice measures of SNSs, (2) to present the generalized Dice measures of SNSs, and (3) to develop the generalized Dice measures-based multiple attribute decision-making methods with simplified neutrosophic information. In the decision making process, the main advantage of the proposed methods is more general and more flexible than existing decision-making methods with simplified neutrosophic information to satisfy the decision makers' preference and/or practical requirements.

The rest of the paper is organized as follows. Section 2 reviews the Dice measures of SNSs. Section 3 proposes another form of the Dice measures of SNSs. In Section 4, we propose the generalized Dice measures of SNSs and indicate the Dice measures and asymmetric measures (projection measures) as the special cases of the generalized Dice measures in some parameter values. In Section 5, the generalized Dice measures-based multiple attribute decision-making methods are developed under simplified neutrosophic environment. In Section 6, a real example on the selection of manufacturing schemes is given to show the application of the proposed methods, and then the effectiveness and flexibility of the proposed methods are indicated by choosing different parameter values. Finally, Section 7 contains conclusions and future work.

2. The Dice measures of SNSs

As a subset of a neutrosophic set [1], Ye [10] introduced a SNS and gave its definition.

Definition 1. [10] A SNS S in the universe of discourse X is defined as $S = \{\langle x, t_S(x), u_S(x), v_S(x)\rangle |x \in X\}$, where $t_S(x)$: $X \to [0, 1]$, $u_s(x) : X \to [0, 1]$, and $v_s(x) : X \to [0, 1]$ are a truth-membership function and an indeterminacy-membership function, a falsity- membership function, respectively, of the element x to the set S with the condition $0 \leq t_s(x) + u_s(x) + v_s(x) \leq 3$ for $x \in X$.

In fact, SNSs contain the concepts of SVNSs and INSs, which are the subclasses of SNSs. For convenience, a component element $< x, t_s(x), u_s(x), v_s(x) >$ in a SNS S is denoted by $s_x =< t_x, u_x, v_x >$ for short, which is called the simplified neutrosophic number (SNN), where $t_x, u_x, v_x \in [0, 1]$ and $0 \leq t_x, +u_x + v_x \leq 3$ for a single-valued neutrosophic number (SVNN), and then $t_x = [t_x^L, t_x^U] \subseteq [0, 1]$, $u_x = [u_x^L, u_x^U] \subseteq [0, 1]$, $v_x = [v_x^L, v_x^U] \subseteq [0, 1]$ and $0 \leq t_x^U + u_x^U + v_x^U \leq 3$ for an interval neutrosophic number (INN).

Ye [13] presented the Dice measures of SNSs, which was defined below.

Definition 2. [13] Let $S_1 = \{s_{11}, s_{12}, \ldots, s_{1n}\}$ and $S_2 = \{s_{21}, s_{22}, \ldots, s_{2n}\}$ be two SNSs. If $s_{1j} =< t_{1j}, u_{1j}, v_{1j}, >$ and $s_{2j} =< t_{2j}, u_{2j}, v_{2j}, >$ $(j = 1, 2, \ldots, n)$ are the j-th SVNNs in S_1 and S_2 respectively, then the Dice measure between S_1 and S_2 is defined as:

$$D_{SVNN1}(S_1, S_2) = \frac{1}{n}\sum_{j=1}^{n}\frac{2s_{1j}\cdot s_{2j}}{|s_{1j}|^2 + |s_{2j}|^2}.$$

$$= \frac{1}{n}\sum_{j=1}^{n}\frac{2(t_{1j}t_{2j} + u_{1j}u_{2j} + v_{1j}v_{2j})}{(t_{1j}^2 + u_{1j}^2 + v_{1j}^2) + (t_{2j}^2 + u_{2j}^2 + v_{2j}^2)} \tag{1}$$

If $s_{1j} =< t_{1j}, u_{1j}, v_{1j} >$ and $s_{2j} =< t_{2j}, u_{2j}, v_{2j} >$ $(j = 1, 2, \ldots, n)$ are the j-th INNs in S_1 and S_2 respectively, then the Dice measure between S_1 and S_2 is defined as:

$$D_{INN1}(S_1, S_2) = \frac{1}{n}\sum_{j=1}^{n}\frac{2s_{1j}\cdot s_{2j}}{\left|s_{1j}\right|^2+\left|s_{2j}\right|^2}.$$

$$= \frac{1}{n}\sum_{j=1}^{n}\frac{2\begin{pmatrix} t_{1j}^L t_{2j}^L + t_{1j}^U t_{2j}^U + u_{1j}^L u_{2j}^L \\ +u_{1j}^U u_{2j}^U + v_{1j}^L v_{2j}^L + v_{1j}^U v_{2j}^U \end{pmatrix}}{\begin{pmatrix} (t_{1j}^L)^2 + (u_{1j}^L)^2 + (v_{1j}^L)^2 \\ +(t_{1j}^U)^2 + (u_{1j}^U)^2 + (v_{1j}^U)^2 \\ +(t_{2j}^L)^2 + (u_{2j}^L)^2 + (v_{2j}^L)^2 \\ +(t_{2j}^U)^2 + (u_{2j}^U)^2 + (v_{2j}^U)^2 \end{pmatrix}} \quad (2)$$

Then, the two Dice measures $D_{SVNN1}(S_1,\ S_2)$ and $D_{INN1}(S_1,\ S_2)$ satisfy the following properties [13]:

(P1) $D_{SVNN1}(S_1,\ S_2) = D_{SVNN1}(S_2,\ S_1)$ and $D_{INN1}(S_1,\ S_2) = D_{INN1}(S_2,\ S_1)$;

(P2) $0 \leq D_{SVNN1}(S_1,\ S_2) \leq 1$ and $0 \leq D_{INN1}(S_1,\ S_2) \leq 1$;

(P3) $D_{SVNN1}(S_1,\ S_2) = 1$ and $D_{INN1}(S_1,\ S_2) = 1$, if $S_1 = S_2$.

Especially when $t_{ij} = t_{ij}^L = t_{ij}^U,\ u_{ij} = u_{ij}^L = u_{ij}^U$, and $v_{ij} = v_{ij}^L = v_{ij}^U$ for $i = 1, 2$ and $j = 1, 2, \ldots, n$ are hold, Equation (2) is degenerated to Equation (1).

In real applications, one usually takes the important differences of each element s_{ij} ($i = 1, 2;\ j = 1, 2, \ldots, n$) into account. Let $W = (w_1,\ w_2, \ldots,\ w_n)^{\mathrm{T}}$ be the weight vector for s_{ij} ($i = 1,\ 2;\ j = 1,\ 2, \ldots,\ n$), $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$. Then, based on Equations (1) and (2), Ye [13] further introduced the weighted Dice measures of SNSs, respectively, as follows:

$$D_{WSVNN1}(S_1, S_2) = \sum_{j=1}^{n} w_j \frac{2s_{1j}\cdot s_{2j}}{\left|s_{1j}\right|^2+\left|s_{2j}\right|^2},$$

$$= \sum_{j=1}^{n} w_j \frac{2(t_{1j}t_{2j} + u_{1j}u_{2j} + v_{1j}v_{2j})}{(t_{1j}^2 + u_{1j}^2 + v_{1j}^2) + (t_{2j}^2 + u_{2j}^2 + v_{2j}^2)} \quad (3)$$

$$D_{WINN1}(S_1, S_2) = \sum_{j=1}^{n} w_j \frac{2s_{1j}\cdot s_{2j}}{\left|s_{1j}\right|^2+\left|s_{2j}\right|^2}$$

$$= \sum_{j=1}^{n} w_j \frac{2\begin{pmatrix} t_{1j}^L t_{2j}^L + t_{1j}^U t_{2j}^U + u_{1j}^L u_{2j}^L \\ +u_{1j}^U u_{2j}^U + v_{1j}^L v_{2j}^L + v_{1j}^U v_{2j}^U \end{pmatrix}}{\begin{pmatrix} (t_{1j}^L)^2 + (u_{1j}^L)^2 + (v_{1j}^L)^2 \\ +(t_{1j}^U)^2 + (u_{1j}^U)^2 + (v_{1j}^U)^2 \\ +(t_{2j}^L)^2 + (u_{2j}^L)^2 + (v_{2j}^L)^2 \\ +(t_{2j}^U)^2 + (u_{2j}^U)^2 + (v_{2j}^U)^2 \end{pmatrix}}. \quad (4)$$

3. Another form of the Dice measures of SNSs

This section proposes another form of the Dice measures of SNSs, which is defined as follows.

Definition 3. Let $S_1 = \{s_{11},\ s_{12}, \ldots,\ s_{1n}\}$ and $S_2 = \{s_{21},\ s_{22}, \ldots,\ s_{2n}\}$ be two SNSs. If $s_{1j} =< t_{1j},\ u_{1j},\ v_{1j} >$ and $s_{2j} =< t_{2j},\ u_{2j},\ v_{2j} >$ ($j = 1,\ 2, \ldots,\ n$) are the j-th SVNNs in S_1 and S_2 respectively, then the Dice measure between S_1 and S_2 is defined as:

$$D_{SVNN2}(S_1, S_2) = \frac{2(S_1\cdot S_2)}{|S_1|^2 + |S_2|^2}$$

$$= \frac{2\sum_{j=1}^{n}(t_{1j}t_{2j} + u_{1j}u_{2j} + v_{1j}v_{2j})}{\sum_{j=1}^{n}(t_{1j}^2 + u_{1j}^2 + v_{1j}^2) + \sum_{j=1}^{n}(t_{2j}^2 + u_{2j}^2 + v_{2j}^2)}. \quad (5)$$

If $s_{1j} =< t_{1j},\ u_{1j},\ v_{1j} >$ and $s_{2j} =< t_{2j},\ u_{2j},\ v_{2j} >$ ($j = 1,\ 2, \ldots,\ n$) are the j-th INNs in S_1 and S_2 respectively, then the Dice measure between S_1 and S_2 is defined as:

$$D_{INN2}(S_1, S_2) = \frac{2(S_1\cdot S_2)}{|S_1|^2 + |S_2|^2}$$

$$= \frac{2\sum_{j=1}^{n}\begin{pmatrix} t_{1j}^L t_{2j}^L + t_{1j}^U t_{2j}^U + u_{1j}^L u_{2j}^L \\ +u_{1j}^U u_{2j}^U + v_{1j}^L v_{2j}^L + v_{1j}^U v_{2j}^U \end{pmatrix}}{\begin{pmatrix} \sum_{j=1}^{n}\begin{bmatrix} (t_{1j}^L)^2 + (t_{1j}^U)^2 + (u_{1j}^L)^2 \\ +(u_{1j}^U)^2 + (v_{1j}^L)^2 + (v_{1j}^U)^2 \end{bmatrix} + \\ \sum_{j=1}^{n}\begin{bmatrix} (t_{2j}^L)^2 + (t_{2j}^U)^2 + (u_{2j}^L)^2 \\ +(u_{2j}^U)^2 + (v_{2j}^L)^2 + (v_{2j}^U)^2 \end{bmatrix} \end{pmatrix}}. \quad (6)$$

Obviously, the two Dice measures $D_{SVNN2}(S_1,\ S_2)$ and $D_{INN2}(S_1,\ S_2)$ also satisfy the following properties:

(P1) $D_{SVNN2}(S_1,\ S_2) = D_{SVNN2}(S_2,\ S_1)$ and $D_{INN2}(S_1,\ S_2) = D_{INN2}(S_2,\ S_1)$;

(P2) $0 \leq D_{SVNN2}(S_1,\ S_2) \leq 1$ and $0 \leq D_{INN2}(S_1,\ S_2) \leq 1$;

(P3) $D_{SVNN2}(S_1,\ S_2) = 1$ and $D_{INN2}(S_1,\ S_2) = 1$, if $S_1 = S_2$.

Proof:

(P1) It is obvious that the property is true.

(P2) It is obvious that the property is true according to the inequality $a^2 + b^2 \geq 2ab$ for Equations (5) and (6).

(P3) If $S_1 = S_2$, there are $s_{1j} = s_{2j}$ $(j = 1, 2, \ldots, n)$ and $|S_1| = |S_2|$. So there are $D_{SVNN2}(S_1, S_2) = 1$ and $D_{INN2}(S_1, S_2) = 1$. □

In practical applications, the elements for s_{ij} $(i = 1, 2;\ j = 1, 2, \ldots, n)$ have different weights. Let $W = (w_1, w_2, \ldots, w_n)^T$ be the weight vector for s_{ij} $(i = 1, 2;\ j = 1, 2, \ldots, n)$, $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$. Then, based on Equations (5) and (6) we further introduce the weighted Dice measures of SNSs, respectively, as follows:

$$D_{WSVNN2}(S_1, S_2) = \frac{2(S_1 \cdot S_2)_w}{|S_1|_w^2 + |S_2|_w^2} = \frac{2\sum_{j=1}^{n} w_j^2 (t_{1j}t_{2j} + u_{1j}u_{2j} + v_{1j}v_{2j})}{\sum_{j=1}^{n} w_j^2 (t_{1j}^2 + u_{1j}^2 + v_{1j}^2) + \sum_{j=1}^{n} w_j^2 (t_{2j}^2 + u_{2j}^2 + v_{2j}^2)}, \quad (7)$$

$$D_{WINN2}(S_1, S_2) = \frac{2(S_1 \cdot S_2)_w}{|S_1|_w^2 + |S_2|_w^2} = \frac{2\sum_{j=1}^{n} w_j^2 \begin{pmatrix} t_{1j}^L t_{2j}^L + t_{1j}^U t_{2j}^U + u_{1j}^L u_{2j}^L \\ + u_{1j}^U u_{2j}^U + v_{1j}^L v_{2j}^L + v_{1j}^U v_{2j}^U \end{pmatrix}}{\begin{pmatrix} \sum_{j=1}^{n} w_j^2 \begin{bmatrix} (t_{1j}^L)^2 + (t_{1j}^U)^2 + (u_{1j}^L)^2 \\ + (u_{1j}^U)^2 + (v_{1j}^L)^2 + (v_{1j}^U)^2 \end{bmatrix} + \\ \sum_{j=1}^{n} w_j^2 \begin{bmatrix} (t_{2j}^L)^2 + (t_{2j}^U)^2 + (u_{2j}^L)^2 \\ + (u_{2j}^U)^2 + (v_{2j}^L)^2 + (v_{2j}^U)^2 \end{bmatrix} \end{pmatrix}}. \quad (8)$$

4. The generalized Dice measures of SNSs

In this section, we propose the generalized Dice measures of SNSs to extend the Dice measures of SNSs.

As the generalization of the Dice measures of SNSs, the generalized Dice measures between SNSs are defined below.

Definition 4. Let $S_1 = \{s_{11}, s_{12}, \ldots, s_{1n}\}$ and $S_2 = \{s_{21}, s_{22}, \ldots, s_{2n}\}$ be two SNSs, where $s_{1j} = (t_{1j}, u_{1j}, v_{1j})$ and $s_{2j} = (t_{2j}, u_{2j}, v_{2j})$ $(j = 1, 2, \ldots, n)$ are considered as the j-th SVNNs in the SNSs S_1 and S_2. Then the generalized Dice measures between S_1 and S_2 are defined, respectively, as follows:

$$G_{SVNN1}(S_1, S_2) = \frac{1}{n}\sum_{j=1}^{n} \frac{s_{1j} \cdot s_{2j}}{\lambda \left|s_{1j}\right|^2 + (1-\lambda)\left|s_{2j}\right|^2} = \frac{1}{n}\sum_{j=1}^{n} \frac{(t_{1j}t_{2j} + u_{1j}u_{2j} + v_{1j}v_{2j})}{\lambda(t_{1j}^2 + u_{1j}^2 + v_{1j}^2) + (1-\lambda)(t_{2j}^2 + u_{2j}^2 + v_{2j}^2)}, \quad (9)$$

$$G_{SVNN2}(S_1, S_2) = \frac{S_1 \cdot S_2}{\lambda |S_1|^2 + (1-\lambda)|S_2|^2} = \frac{\sum_{j=1}^{n} (t_{1j}t_{2j} + u_{1j}u_{2j} + v_{1j}v_{2j})}{\lambda \sum_{j=1}^{n} (t_{1j}^2 + u_{1j}^2 + v_{1j}^2) + (1-\lambda)\sum_{j=1}^{n} (t_{2j}^2 + u_{2j}^2 + v_{2j}^2)}, \quad (10)$$

where λ is a positive parameter for $0 \leq \lambda \leq 1$.

Then, the generalized Dice measures imply some special cases by choosing some values of the parameter λ. If $\lambda = 0.5$, the two generalized Dice measures (9) and (10) are degenerated to the Dice measures (1) and (5); if $\lambda = 0, 1$, the two generalized Dice measures are degenerated to the following asymmetric measures respectively:

$$G_{SVNN1}(S_1, S_2) = \frac{1}{n}\sum_{j=1}^{n} \frac{s_{1j} \cdot s_{2j}}{\left|s_{2j}\right|^2} = \frac{1}{n}\sum_{j=1}^{n} \frac{t_{1j}t_{2j} + u_{1j}u_{2j} + v_{1j}v_{2j}}{t_{2j}^2 + u_{2j}^2 + v_{2j}^2} \quad \text{for } \lambda = 0, \quad (11)$$

$$G_{SVNN1}(S_1, S_2) = \frac{1}{n}\sum_{j=1}^{n} \frac{s_{1j} \cdot s_{2j}}{\left|s_{1j}\right|^2} = \frac{1}{n}\sum_{j=1}^{n} \frac{t_{1j}t_{2j} + u_{1j}u_{2j} + v_{1j}v_{2j}}{t_{1j}^2 + u_{1j}^2 + v_{1j}^2} \quad \text{for } \lambda = 1, \quad (12)$$

$$G_{SVNN2}(S_1, S_2) = \frac{S_1 \cdot S_2}{|S_2|^2} = \frac{\sum_{j=1}^{n} (t_{1j}t_{2j} + u_{1j}u_{2j} + v_{1j}v_{2j})}{\sum_{j=1}^{n} (t_{2j}^2 + u_{2j}^2 + v_{2j}^2)} \quad \text{for } \lambda = 0, \quad (13)$$

$$G_{SVNN2}(S_1, S_2) = \frac{S_1 \cdot S_2}{|S_1|^2} = \frac{\sum_{j=1}^{n} (t_{1j}t_{2j} + u_{1j}u_{2j} + v_{1j}v_{2j})}{\sum_{j=1}^{n} (t_{1j}^2 + u_{1j}^2 + v_{1j}^2)} \quad \text{for } \lambda = 1. \quad (14)$$

Obviously, the four asymmetric measures are the extension of the relative projection measure (the improved projection measure) of interval numbers [2], hence the four asymmetric measures can be considered as the projection measures of SNSs.

For practical applications, the elements of s_{ij} $(i = 1, 2;\ j = 1, 2, \ldots, n)$ imply different weights. Assume that $W = (w_1, w_2 \ldots, w_n)^T$ is the weight vector for s_{ij} $(i = 1, 2;\ j = 1, 2, \ldots, n)$, $w_j \geq 0$

and $\sum_{j=1}^{n} w_j = 1$. Thus, based on Equations (9) and (10) we further introduce the following weighted generalized Dice measures of SNSs, respectively, as follows:

$$G_{WSVNN1}(S_1, S_2) = \sum_{j=1}^{n} w_j \frac{s_{1j} \cdot s_{2j}}{\lambda \left|s_{1j}\right|^2 + (1-\lambda)\left|s_{2j}\right|^2} = \sum_{j=1}^{n} w_j \frac{t_{1j}t_{2j} + u_{1j}u_{2j} + v_{1j}v_{2j}}{\lambda(t_{1j}^2 + u_{1j}^2 + v_{1j}^2) + (1-\lambda)(t_{2j}^2 + u_{2j}^2 + v_{2j}^2)}, \tag{15}$$

$$G_{WSVNN2}(S_1, S_2) = \frac{(S_1 \cdot S_2)_w}{\lambda |S_1|_w^2 + (1-\lambda)|S_2|_w^2} = \frac{\sum_{j=1}^{n} w_j^2(t_{1j}t_{2j} + u_{1j}u_{2j} + v_{1j}v_{2j})}{\left(\begin{array}{l}\lambda \sum_{j=1}^{n} w_j^2(t_{1j}^2 + u_{1j}^2 + v_{1j}^2) + \\ (1-\lambda)\sum_{j=1}^{n} w_j^2(t_{2j}^2 + u_{2j}^2 + v_{2j}^2)\end{array}\right)}. \tag{16}$$

Definition 5. Let $S_1 = \{s_{11}, s_{12}, \ldots, s_{1n}\}$ and $S_2 = \{s_{21}, s_{22}, \ldots, s_{2n}\}$ be two SNSs, where $s_{1j} = (t_{1j}, u_{1j}, v_{1j})$ and $s_{2j} = (t_{2j}, u_{2j}, v_{2j})$ ($j = 1, 2, \ldots, n$) are considered as the j-th INNs in the SNSs S_1 and S_2. Then the generalized Dice measures between S_1 and S_2 are defined, respectively, as follows:

$$G_{INN3}(S_1, S_2) = \frac{1}{n}\sum_{j=1}^{n} \frac{s_{1j} \cdot s_{2j}}{\lambda \left|s_{1j}\right|^2 + (1-\lambda)\left|s_{2j}\right|^2} = \frac{1}{n} \sum_{j=1}^{n} \frac{t_{1j}^L t_{2j}^L + t_{1j}^U t_{2j}^U + u_{1j}^L u_{2j}^L + u_{1j}^U u_{2j}^U + v_{1j}^L v_{2j}^L + v_{1j}^U v_{2j}^U}{\left(\begin{array}{l}\lambda \left[\begin{array}{l}(t_{1j}^L)^2 + (u_{1j}^L)^2 + (v_{1j}^L)^2 \\ +(t_{1j}^U)^2 + (u_{1j}^U)^2 + (v_{1j}^U)^2\end{array}\right] \\ +(1-\lambda)\left[\begin{array}{l}(t_{2j}^L)^2 + (u_{2j}^L)^2 + (v_{2j}^L)^2 \\ +(t_{2j}^U)^2 + (u_{2j}^U)^2 + (v_{2j}^U)^2\end{array}\right]\end{array}\right)}, \tag{17}$$

$$G_{INN4}(S_1, S_2) = \frac{S_1 \cdot S_2}{\lambda |S_1|^2 + (1-\lambda)|S_2|^2} = \frac{\sum_{j=1}^{n} (t_{1j}^L t_{2j}^L + t_{1j}^U t_{2j}^U + u_{1j}^L u_{2j}^L + u_{1j}^U u_{2j}^U + v_{1j}^L v_{2j}^L + v_{1j}^U v_{2j}^U)}{\left(\begin{array}{l}\lambda \sum_{j=1}^{n} \left[\begin{array}{l}(t_{1j}^L)^2 + (t_{1j}^U)^2 + (u_{1j}^L)^2 \\ +(u_{1j}^U)^2 + (v_{1j}^L)^2 + (v_{1j}^U)^2\end{array}\right] + \\ (1-\lambda)\sum_{j=1}^{n} \left[\begin{array}{l}(t_{2j}^L)^2 + (t_{2j}^U)^2 + (u_{2j}^L)^2 \\ +(u_{2j}^U)^2 + (v_{2j}^L)^2 + (v_{2j}^U)^2\end{array}\right]\end{array}\right)}, \tag{18}$$

where λ is a positive parameter for $0 \le \lambda \le 1$. Especially, when $t_{ij} = t_{ij}^L = t_{ij}^U$, $u_{ij} = u_{ij}^L = u_{ij}^U$, and $v_{ij} = v_{ij}^L = v_{ij}^U$ for $i = 1, 2$ and $j = 1, 2, \ldots, n$ are hold, Equations (17) and (18) are degenerated to Equations (9) and (10).

Similarly, if $\lambda = 0.5$, the two generalized Dice measures (17) and (18) are degenerated to the Dice measures (2) and (6); if $\lambda = 0, 1$, then the two generalized Dice measures are degenerated to the following asymmetric measures respectively:

$$G_{INN3}(S_1, S_2) = \frac{1}{n}\sum_{j=1}^{n} \frac{s_{1j} \cdot s_{2j}}{\left|s_{2j}\right|^2} = \frac{1}{n}\sum_{j=1}^{n} \frac{t_{1j}^L t_{2j}^L + t_{1j}^U t_{2j}^U + u_{1j}^L u_{2j}^L + u_{1j}^U u_{2j}^U + v_{1j}^L v_{2j}^L + v_{1j}^U v_{2j}^U}{(t_{2j}^L)^2 + (u_{2j}^L)^2 + (v_{2j}^L)^2 + (t_{2j}^U)^2 + (u_{2j}^U)^2 + (v_{2j}^U)^2} \quad \text{for } \lambda = 0, \tag{19}$$

$$G_{INN3}(S_1, S_2) = \frac{1}{n}\sum_{j=1}^{n} \frac{s_{1j} \cdot s_{2j}}{\left|s_{1j}\right|^2} = \frac{1}{n}\sum_{j=1}^{n} \frac{t_{1j}^L t_{2j}^L + t_{1j}^U t_{2j}^U + u_{1j}^L u_{2j}^L + u_{1j}^U u_{2j}^U + v_{1j}^L v_{2j}^L + v_{1j}^U v_{2j}^U}{(t_{1j}^L)^2 + (u_{1j}^L)^2 + (v_{1j}^L)^2 + (t_{1j}^U)^2 + (u_{1j}^U)^2 + (v_{1j}^U)^2} \quad \text{for } \lambda = 1, \tag{20}$$

$$G_{INN4}(S_1, S_2) = \frac{S_1 \cdot S_2}{|S_2|^2} = \frac{\sum_{j=1}^{n} (t_{1j}^L t_{2j}^L + t_{1j}^U t_{2j}^U + u_{1j}^L u_{2j}^L + u_{1j}^U u_{2j}^U + v_{1j}^L v_{2j}^L + v_{1j}^U v_{2j}^U)}{\sum_{j=1}^{n} [(t_{2j}^L)^2 + (t_{2j}^U)^2 + (u_{2j}^L)^2 + (u_{2j}^U)^2 + (v_{2j}^L)^2 + (v_{2j}^U)^2]} \quad \text{for } \lambda = 0, \tag{21}$$

$$G_{INN4}(S_1, S_2) = \frac{S_1 \cdot S_2}{|S_1|^2} = \frac{\sum_{j=1}^{n} (t_{1j}^L t_{2j}^L + t_{1j}^U t_{2j}^U + u_{1j}^L u_{2j}^L + u_{1j}^U u_{2j}^U + v_{1j}^L v_{2j}^L + v_{1j}^U v_{2j}^U)}{\sum_{j=1}^{n} [(t_{1j}^L)^2 + (t_{1j}^U)^2 + (u_{1j}^L)^2 + (u_{1j}^U)^2 + (v_{1j}^L)^2 + (v_{1j}^U)^2]} \quad \text{for } \lambda = 1. \tag{22}$$

Then, the four asymmetric measures are also considered as the extension of the relative projection measure (the improved projection measure) of interval numbers [2], which are also called the projection measures of SNSs.

For practical applications, the elements of s_{ij} ($i = 1, 2$; $j = 1, 2, \ldots, n$) imply different weights. Assume that $W = (w_1, w_2, \ldots, w_n)^T$ is the weight vector for s_{ij} ($i = 1, 2$; $j = 1, 2, \ldots, n$), $w_j \ge 0$ and $\sum_{j=1}^{n} w_j = 1$. Similarly, based on Equations (17) and (18) we also further introduce the weighted generalized Dice measures of SNSs, respectively, as follows:

$$G_{WINN3}(S_1, S_2) = \sum_{j=1}^{n} w_j \frac{s_{1j} \cdot s_{2j}}{\lambda \left|s_{1j}\right|^2 + (1-\lambda)\left|s_{2j}\right|^2}$$

$$= \sum_{j=1}^{n} w_j \frac{\begin{bmatrix} t_{1j}^L t_{2j}^L + t_{1j}^U t_{2j}^U + u_{1j}^L u_{2j}^L \\ +u_{1j}^U u_{2j}^U + v_{1j}^L v_{2j}^L + v_{1j}^U v_{2j}^U \end{bmatrix}}{\begin{pmatrix} \lambda \begin{bmatrix} (t_{1j}^L)^2 + (u_{1j}^L)^2 + (v_{1j}^L)^2 \\ +(t_{1j}^U)^2 + (u_{1j}^U)^2 + (v_{1j}^U)^2 \end{bmatrix} \\ +(1-\lambda) \begin{bmatrix} (t_{2j}^L)^2 + (u_{2j}^L)^2 + (v_{2j}^L)^2 \\ +(t_{2j}^U)^2 + (u_{2j}^U)^2 + (v_{2j}^U)^2 \end{bmatrix} \end{pmatrix}}, \quad (23)$$

$$G_{WINN4}(S_1, S_2) = \frac{(S_1 \cdot S_2)_w}{\lambda |S_1|_w^2 + (1-\lambda)|S_2|_w^2}$$

$$= \frac{\sum_{j=1}^{n} w_j^2 \begin{pmatrix} t_{1j}^L t_{2j}^L + t_{1j}^U t_{2j}^U + u_{1j}^L u_{2j}^L \\ +u_{1j}^U u_{2j}^U + v_{1j}^L v_{2j}^L + v_{1j}^U v_{2j}^U \end{pmatrix}}{\begin{pmatrix} \lambda \sum_{j=1}^{n} w_j^2 \begin{bmatrix} (t_{1j}^L)^2 + (t_{1j}^U)^2 + (u_{1j}^L)^2 \\ +(u_{1j}^U)^2 + (v_{1j}^L)^2 + (v_{1j}^U)^2 \end{bmatrix} + \\ (1-\lambda) \sum_{j=1}^{n} w_j^2 \begin{bmatrix} (t_{2j}^L)^2 + (t_{2j}^U)^2 + (u_{2j}^L)^2 \\ +(u_{2j}^U)^2 + (v_{2j}^L)^2 + (v_{2j}^U)^2 \end{bmatrix} \end{pmatrix}}. \quad (24)$$

5. Decision making-methods based on the generalized Dice measures

In this section, we propose multiple attribute decision-making methods by using the generalized Dice measures of SNSs under simplified neutrosophic environment.

For multiple attribute decision-making problems, let $S = \{S_1, S_2, \ldots, S_m\}$ be a set of alternatives and $R = \{R_1, R_2, \ldots, R_n\}$ be a set of attributes. Then, the weight of the attribute R_j $(j = 1, 2, \ldots, n)$ is $w_j, w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Thus, the fit judgment (satisfaction evaluation) of an attribute R_j $(j = 1, 2, \ldots, n)$ for an alternative S_i $(i = 1, 2, \ldots, m)$ is represented by a SNS $S_i = \{s_{i1}, s_{i2}, \ldots, s_{in}\}$, where $s_{ij} =< t_{ij}, u_{ij}, v_{ij} >$ is a SVNN for $0 \leq t_{ij} + u_{ij} + v_{ij} \leq 3$ or an INN for $0 \leq t_{ij}^U + u_{ij}^U + v_{ij}^U \leq 3$ $(j = 1, 2, \ldots, n$ and $i = 1, 2, \ldots, m)$. Therefore, we can establish a simplified neutrosophic decision matrix $D = (s_{ij})_{m \times n}$.

In the multiple attribute decision-making problem, the concept of an ideal solution (ideal alternative) can be used to help identify the best alternative in the decision set [13]. Hence, by an ideal SVNN

$$s_j^* =< t_j^*, u_j^*, v_j^* >=< \max_i(t_{ij}), \min_i(u_{ij}), \min_i(v_{ij}) >$$

or an ideal INN

$$s_j^* =< t_j^*, u_j^*, v_j^* >=< [\max_i(t_{ij}^L), \max_i(t_{ij}^U)], [\min_i(u_{ij}^L), \min_i(u_{ij}^U)], [\min_i(v_{ij}^L), \min_i(v_{ij}^U)] >$$

for $j = 1, 2, \ldots, n$ and $i = 1, 2, \ldots, m$, we can determine a simplified neutrosophic ideal solution (ideal alternative) $S^* = \{s_1^*, s_2^*, \ldots, s_n^*\}$, where $s_j^* =< t_j^*, u_j^*, v_j^* >$ is the j-th ideal SNN.

In the decision-making process, decision makers take some value of the parameter $\lambda \in [0, 1]$ according to their preference and/or real requirements, the weighted generalized Dice measure between S_i $(i = 1, 2, \ldots, m)$ and S^* is obtained by using one of Equations (15), (16), (23) and (24) to rank the alternatives.

Thus, the greater the value of the weighted generalized Dice measure between S_i $(i = 1, 2, \ldots, m)$ and S^* is, the better the alternative S_i is.

6. Decision-making example of manufacturing schemes

A real example about the decision-making problem of manufacturing schemes with simplified neutrosophic information is given to demonstrate the applications and effectiveness of the proposed decision-making methods in realistic scenarios.

To select the best manufacturing scheme (alternative) for the flexible manufacturing system in a manufacturing company, the technique department of the company provides four manufacturing schemes (alternatives) with respect to some product as a set of the alternatives $S = \{S_1, S_2, S_3, S_4\}$ for the flexible manufacturing system. A decision must be made according to the four attributes: (1) R_1 is the improvement of quality; (2) R_2 is the market response; (3) R_3 is the manufacturing cost; (4) R_4 is the manufacturing complexity. The weight vector of the four attributes $W = (0.3, 0.25, 0.25, 0.2)^T$ is given by decision makers.

In the decision-making problem, the decision makers are required to make the fit judgment (satisfaction evaluation) of an attribute R_j $(j = 1, 2, 3, 4)$ for an alternative S_i $(i = 1, 2, 3, 4)$ and to give simplified neutrosophic evaluation information, which is shown in the following decision matrix with SVNNs:

$$D = \begin{bmatrix} (0.75, 0.2, 0.3) & (0.7, 0.2, 0.3) & (0.65, 0.2, 0.25) & (0.75, 0.2, 0.1) \\ (0.8, 0.1, 0.2) & (0.75, 0.2, 0.1) & (0.75, 0.2, 0.1) & (0.85, 0.1, 0.2) \\ (0.7, 0.2, 0.2) & (0.78, 0.2, 0.1) & (0.85, 0.15, 0.1) & (0.76, 0.2, 0.2) \\ (0.8, 0.2, 0.1) & (0.85, 0.2, 0.2) & (0.7, 0.2, 0.2) & (0.86, 0.1, 0.2) \end{bmatrix}.$$

Then, the developed decision-making methods can be used for the decision making problem.

According to

$$s_j^* = < t_j^*, u_j^*, v_j^* > = < \max_i(t_{ij}), \min_i(u_{ij}), \min_i(v_{ij}) >$$

for $j = 1, 2, 3, 4$ and $i = 1, 2, 3, 4$, we can obtain an ideal solution (ideal alternative) as follows:

$$S^* = \{s_1^*, s_2^*, s_3^*, s_4^*\} = \left\{ \begin{array}{l} < 0.8, 0.1, 0.1 >, < 0.85, 0.2, 0.1 >, \\ < 0.85, 0.15, 0.1 >, < 0.86, 0.1, 0.1 > \end{array} \right\}.$$

By using Equation (15) or (16) and different values of the parameter λ, the weighted generalized Dice measure values between S_i $(i = 1, 2, 3, 4)$ and S^* can be obtained, which are shown in Tables 1 and 2 respectively.

From Tables 1 and 2, we can see that different ranking orders are indicated by taking different values of the parameter λ and different generalized Dice measures. Then we can obtain that the best alternative is S_2 or S_3 or S_4.

Furthermore, for the special cases of the two generalized Dice measures we obtain the following results:

(1) When $\lambda = 0$, the two weighted generalized Dice measures are reduced to the weighted projection measures of S_i on S^*. Thus, the alternative S_2 is the best choice among all the alternatives.

(2) When $\lambda = 0.5$, the two weighted generalized Dice measures are reduced to the weighted Dice similarity measures of S_i and S^*. Thus, the alternative S_2 is the best choice among all the alternatives.

(3) When $\lambda = 1$, the two weighted generalized Dice measures are reduced to the weighted projection measures of S^* on S_i. Thus, the alternative S_3 or S_4 is the best choice among all the alternatives.

Table 1
The measure values of Equation (15) and ranking orders

λ	$G_{WSVNN1}(S_1, S^*)$	$G_{WSVNN1}(S_2, S^*)$	$G_{WSVNN1}(S_3, S^*)$	$G_{WSVNN1}(S_4, S^*)$	Ranking order
0	0.8895	0.9517	0.9361	0.9287	$S_2 \succ S_3 \succ S_4 \succ S_1$
0.2	0.9157	0.9667	0.9558	0.9472	$S_2 \succ S_3 \succ S_4 \succ S_1$
0.5	0.9612	0.9924	0.9876	0.9816	$S_2 \succ S_3 \succ S_4 \succ S_1$
0.7	0.9966	1.0119	1.0104	1.0100	$S_2 \succ S_3 \succ S_4 \succ S_1$
1	1.0594	1.0455	1.0475	1.0641	$S_4 \succ S_1 \succ S_3 \succ S_2$

Table 2
The measure values of Equation (16) and ranking orders

λ	$G_{WSVNN2}(S_1, S^*)$	$G_{WSVNN2}(S_2, S^*)$	$G_{WSVNN2}(S_3, S^*)$	$G_{WSVNN2}(S_4, S^*)$	Ranking order
0	0.8908	0.9503	0.9387	0.9372	$S_2 \succ S_3 \succ S_4 \succ S_1$
0.2	0.9175	0.9667	0.9587	0.9557	$S_2 \succ S_3 \succ S_4 \succ S_1$
0.5	0.9605	0.9924	0.9903	0.9849	$S_2 \succ S_3 \succ S_4 \succ S_1$
0.7	0.9915	1.0102	1.0126	1.0054	$S_3 \succ S_2 \succ S_4 \succ S_1$
1	1.0419	1.0383	1.0479	1.0378	$S_3 \succ S_1 \succ S_2 \succ S_4$

Obviously, according to different values of the parameter λ and different measures, ranking orders may be different. Thus the proposed decision-making methods can be assigned some value of λ and some measure to satisfy the decision makers' preference and/or real requirements.

If the fit judgment (satisfaction evaluation) of an attribute R_j $(j = 1, 2, 3, 4)$ for an alternative S_i $(i = 1, 2, 3, 4)$ is given in the decision making problem by the following decision matrix with INNs:

$$D = \begin{bmatrix} <[0.7, 0.8], [0.1, 0.2], [0.2, 0.3]> & <[0.7, 0.8], [0.1, 0.2], [0.2, 0.3]> & <[0.6, 0.7], [0.1, 0.2], [0.2, 0.4]> & <[0.7, 0.8], [0.1, 0.2], [0.1, 0.2]> \\ <[0.7, 0.9], [0.1, 0.2], [0.2, 0.3]> & <[0.7, 0.8], [0.1, 0.3], [0.1, 0.2]> & <[0.7, 0.8], [0.2, 0.3], [0.1, 0.2]> & <[0.8, 0.9], [0.0, 0.1], [0.2, 0.3]> \\ <[0.7, 0.8], [0.1, 0.3], [0.2, 0.3]> & <[0.8, 0.9], [0.1, 0.2], [0.1, 0.2]> & <[0.8, 0.9], [0.1, 0.2], [0.1, 0.1]> & <[0.7, 0.8], [0.1, 0.2], [0.1, 0.2]> \\ <[0.8, 0.9], [0.2, 0.3], [0.1, 0.2]> & <[0.8, 0.9], [0.2, 0.3], [0.2, 0.3]> & <[0.7, 0.8], [0.1, 0.2], [0.1, 0.3]> & <[0.7, 0.9], [0.0, 0.1], [0.1, 0.3]> \end{bmatrix}.$$

Then, according to

$$s_j^* = < t_j^*, u_j^*, v_j^* > = < [\max_i(t_{ij}^L), \max_i(t_{ij}^U)], [\min_i(u_{ij}^L), \min_i(u_{ij}^U)], [\min_i(v_{ij}^L), \min_i(v_{ij}^U)] >$$

for $j = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$, we can obtain the ideal solution (ideal alternative) as follows:

$$S^* = \{s_1^*, s_2^*, \ldots, s_n^*\} = \left\{ \begin{array}{l} < [0.8, 0.9], [0.1, 0.2], [0.1, 0.2] >, \\ < [0.7, 0.8], [0.1, 0.2], [0.1, 0.2] >, \\ < [0.8, 0.9], [0.1, 0.2], [0.1, 0.1] >, \\ < [0.8, 0.9], [0.0, 0.1], [0.1, 0.2] > \end{array} \right\}.$$

According to Equations (23) or (24) and different values of the parameter λ, the weighted generalized Dice measure values between S_i $(i = 1, 2, 3, 4)$ and S^* can be obtained, which are shown in Tables 3 and 4 respectively.

From Tables 3 and 4, different ranking orders are shown by taking different values of λ and different measures. Then we can obtain that the best alternative is S_1 or S_2 or S_4.

Furthermore, for the special cases of the two generalized Dice measures we obtain the following results:

(1) When $\lambda = 0$, the two weighted generalized Dice measures are reduced to the weighted projection measures of S_i on S^*. Thus, the alternative S_4 is the best choice among all the alternatives.
(2) When $\lambda = 0.5$, the two weighted generalized Dice measures are reduced to the weighted Dice similarity measures of S_i and S^*. Thus, the alternative S_2 is the best choice among all the alternatives.
(3) When $\lambda = 1$, the two weighted generalized Dice measures are reduced to the weighted projection measures of S^* on S_i. Thus, the alternative S_1 is the best choice among all the alternatives.

Therefore, according to different values of the parameter λ and different measures, ranking orders may be also different. Thus the proposed decision-making methods can be assigned some value of λ and some measure to satisfy the decision makers' preference and/or real requirements.

Obviously, the decision-making methods based the Dice measures and the projection measures are the special cases of the proposed decision-making methods based on generalized Dice measures. Therefore, in the decision-making process, the decision-making methods developed in this paper are more general and more flexible than existing decision-making methods under simplified neutrosophic environment.

Table 3
The measure values of Equation (23) and ranking orders

λ	$G_{WINN3}(S_1, S^*)$	$G_{WINN3}(S_2, S^*)$	$G_{WINN3}(S_3, S^*)$	$G_{WINN3}(S_4, S^*)$	Ranking order
0	0.9085	0.9770	0.9861	1.0159	$S_4 \succ S_3 \succ S_2 \succ S_1$
0.2	0.9325	0.9819	0.9860	1.0015	$S_4 \succ S_3 \succ S_2 \succ S_1$
0.5	0.9737	0.9903	0.9901	0.9863	$S_2 \succ S_3 \succ S_4 \succ S_1$
0.7	1.0053	0.9966	0.9955	0.9797	$S_1 \succ S_2 \succ S_3 \succ S_4$
1	1.0601	1.0072	1.0075	0.9746	$S_1 \succ S_3 \succ S_2 \succ S_4$

Table 4
The measure values of Equation (24) and ranking orders

λ	$G_{WINN4}(S_1, S^*)$	$G_{WINN4}(S_2, S^*)$	$G_{WINN4}(S_3, S^*)$	$G_{WINN4}(S_4, S^*)$	Ranking order
0	0.9035	0.9726	0.9790	1.0109	$S_4 \succ S_3 \succ S_2 \succ S_1$
0.2	0.9306	0.9795	0.9833	1.0011	$S_4 \succ S_3 \succ S_2 \succ S_1$
0.5	0.9743	0.9901	0.9900	0.9867	$S_2 \succ S_3 \succ S_4 \succ S_1$
0.7	1.0059	0.9972	0.9945	0.9774	$S_1 \succ S_2 \succ S_3 \succ S_4$
1	1.0572	1.0082	1.0013	0.9637	$S_1 \succ S_2 \succ S_3 \succ S_4$

7. Conclusion

This paper proposed another form of the Dice measures between SNSs and the generalized Dice measures of SNSs and indicated the Dice measures of SNSs and the projection measures (asymmetric measures) of SNSs are the special cases of the generalized Dice measures of SNSs corresponding to some parameter values. Then, we developed multiple attribute decision-making methods based on the generalized Dice measures of SNSs under simplified neutrosophic environment. According to different parameter values and some measure preferred by decision makers, by the weighted generalized Dice measure between each alternative and the ideal solution (ideal alternative), all alternatives can be ranked and the best alternative can be selected as well. Finally, a real example about the selection of manufacturing schemes (alternatives) demonstrated the applications of the developed methods under simplified neutrosophic environment, and then the effectiveness and flexibility of the developed decision-making methods were shown corresponding to different parameter values. In the decision-making process under simplified neutrosophic environment, the main advantage is more general and more flexible than existing decision-making methods to satisfy the decision makers' preference and/or practical requirements.

In the future work, we shall extend the generalized Dice measures of SNSs to other areas such as pattern recognition, fault diagnosis, and image processing.

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