Research Article

The Neutro-Stability Analysis of Neutrosophic Cubic Sets with Application in Decision Making Problems

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The neutrosophic cubic sets (NCSs) attained attraction of many researchers in the current time, so the need to discuss and study their stability was felt. Thus, in this article, we discuss the three types of stability of NCSs such as truth-stability, indeterminacy-stability, and falsity-stability. We define the left (resp., right) truth-left evaluative set, left (resp., right) indeterminacy-evaluative set, and left (resp., right) falsity-evaluative set. A new notion of stable NCSs, partially stable NCSs, and unstable NCSs is defined. We observe that every NCS needs not to be a stable NCS but each stable NCS must be an NCS, i.e., every internal NCS is a stable NCS but an external NCS may or may not be a stable NCS. We also discuss some conditions under which the left and right evaluative points of an external NCS becomes a neutrosophic bipolar fuzz set. We have provided the condition under which an external NCS becomes stable. Moreover, we discuss the truth-stable degree, indeterminacy-stable degree, and falsity-stable degree of NCSs. We have also defined an almost truth-stable set, almost indeterminacy-stable set, almost falsity-stable set, almost partially stable set, and almost stable set with examples. Application of stable NCSs is given with a numerical example at the end.

1. Introduction

The crisp set lost the stability as it covers the extremes only, which is not the ideal situation in every problem. To cover this gap, Zadeh [1] presented the idea of the fuzzy set (FS) in 1965 which is stable as compared to the crisp set. But, when there is a case to handle the negative characteristics, the fuzzy set (FS) too lost its stability. To cover this gap, Atanassov [2], in 1986, gave the idea of intutionistic fuzzy sets (IFSs) which are more stable than the fuzzy set. But, the problem with Atanassov's idea is that indeterminacy is lost and no proper attraction is given to it. Then, Smarandache [3] covered this gap by giving a new idea of a neutrosophic set which is a stable version other than the fuzzy set and intutionistic fuzzy sets. The neutrosophic set (NS) is the extension of the FS, IVFS, and IFS. In the NS, we deal with its three components, that is, truthfulness, indeterminate, and untruthfulness, and these three functions are independent completely. Neutrosophy gives us a support for a whole

family of new mathematical theories with the abstraction of both classical and fuzzy counterparts. In real life and in scientific problems to apply the neutrosophic set, Wang et al. [4] introduced the new idea of a single-valued neutrosophic set (SVNS) and interval neutrosophic set (INS). These are subclasses of the NS, in which truthfulness, indeterminate, and untruthfulness were taken in a closed interval [0, 1], see also [5]. On the other side, Zadeh [6] made another extension which is known as the interval-valued fuzzy set (IVFS), in which he described interval membership function. There are many real-life applications of the IVFS, i.e., Sambuc [7] in medical diagnosis in thyroidian, Gorzalczany in approximate reasoning, and Turksen [8, 9] in interval-valued logic. In 2012, the theme of the cubic set (CS) was used by Jun et al. [10]. CS is the combination of the IVFS and FS in the form of an ordered pair. These all are mathematical tools to determine the complications in our daily life. Jun et al. [11] gave the idea of the NCS. For application of NCSs, we refer to [12–17]. In 2017, the concept of stable cubic sets

was introduced by Muhiuddin et al. [18]. In 2019 and 2020, Smarandache [19–21] generalized the classical algebraic structures to neutroalgebraic structures (or neutroalgebras) (whose operations and axioms are partially true, partially indeterminate, and partially false) as extensions of partial algebra and to antialgebraic structures (or antialgebras) (whose operations and axioms are totally false). Also, in general, he extended any classical structure, in no matter what field of knowledge, to a neutrostructure and an anti-structure. Similarly, as alternatives to a classical theorem (that is true for all sets' elements) are the neutrotheorem (partially true, partially indeterminate, and partially false) and antitheorem (false for all sets' elements), respectively.

In this paper, we define different types of the stable neutrosophic cubic set with examples and some basic results. We also define the concept of almost stable neutrosophic cubic sets. At the end, we have provided an application of the presented theory.

2. Preliminaries

This section mainly recalls some basic concepts related to fuzzy sets [1], cubic sets [10], neutrosophic sets [3, 4], neutrosophic cubic sets [11], and evaluative structure of cubic sets [18]. For more detail of these sets, we refer the reader to [1, 3, 4, 10, 11, 18].

Definition 1 (see [1]). A mapping $p: U \longrightarrow [0, 1]$ is called an FS, and $\widetilde{p}$ (ů) is a membership function and denoted by p.

Definition 2 (see [10]). A structure $C = \{(\mathring{u}; \widetilde{p}(\mathring{u}), p(\mathring{u}) | \mathring{u} \in U)\}$ is a cubic set in U in which $\widetilde{p}(\mathring{u})$ is IVF in U, and $p(\mathring{u})$ is an FS in U. This is simply denoted by $C = (\widetilde{p}, p)$. $C^{\mathring{u}}$ denotes the collection of cubic sets in U.

Definition 3 (see [3, 4]). A neutrosophic set is a structure

$$N = \{(\mathring{u}; T_N(\mathring{u}), I_N(\mathring{u}), F_N(\mathring{u}) | \mathring{u} \in U)\}, \tag{1}$$

in U. Here, $(T_N(\mathring{u}), I_N(\mathring{u}), F_N(\mathring{u}) \in [0, 1])$ are three functions, known as truthfulness, indeterminate, and untruthfulness, respectively, simply denoted by $N = (T_N, I_N, F_N)$.

Definition 4 (see [11]). A structure

$$N_C = \left\{\left(\mathring{u}; \widetilde{T}_{N_C}(\mathring{u}), \widetilde{I}_{N_C}(\mathring{u}), \widetilde{F}_{N_C}(\mathring{u}), T_{N_C}(\mathring{u}), I_{N_C}(\mathring{u}), F_{N_C}(\mathring{u}) | \mathring{u} \in U\right)\right\}, \tag{2}$$

is an NCS in X. Here,

$$\left(\widetilde{T}_{N_C} = \left[T_{N_C}^L, T_{N_C}^U\right], \widetilde{I}_{N_C} = \left[I_{N_C}^L, I_{N_C}^U\right], \widetilde{F}_{N_C}\left[F_{N_C}^L, F_{N_C}^U\right]\right), \tag{3}$$

is an interval NS and $(T_{N_C}, I_{N_C}, F_{N_C})$ is an NS in X simply denoted by

$$N_C = \left(\widetilde{T}_{N_C}, \widetilde{I}_{N_C}, \widetilde{F}_{N_C}, T_{N_C}, I_{N_C}, F_{N_C}\right),$$
$$[0, 0] \leq \widetilde{T}_{N_C} + \widetilde{I}_{N_C} + \widetilde{F}_{N_C} \leq [3, 3], \tag{4}$$
$$0 \leq T_{N_C} + I_{N_C} + F_{N_C} \leq 1.$$

Definition 5 (see [18]). A structure $C = \{(\mathring{u}; \widetilde{p}(\mathring{u}), p(\mathring{u}) | \mathring{u} \in U)\}$ is a CS in U in which $C(\mathring{u})$ is the evaluative structure defined as follows:

$$E_C = \{(\mathring{u}; E_C(\mathring{u}) | \mathring{u} \in U)\}, \tag{5}$$

where $E_C(\mathring{u}) = \langle l(E_C(\mathring{u})), r(E_C(\mathring{u})) \rangle$ with left evaluative point $l(E_C(\mathring{u})) = p(\mathring{u}) - \widetilde{p}(\mathring{u})$ and right evaluative point $r(E_C(\mathring{u})) = p(\mathring{u})^+ - p(\mathring{u})$ at $\mathring{u} \in U$. We say that $E_C(\mathring{u})$ is the evaluative point of $C = (\widetilde{p}, p)$ at $\mathring{u} \in U$.

3. Neutrostable Neutrosophic Cubic Sets

In this section, we provide the concepts of the truth-evaluative set, indeterminacy-evaluative set, falsity-evaluative set, stable truth-element, stable indeterminacy-element, stable falsity-element, and unstable element of the NCS. We also discuss some interesting results.

Definition 6. Let $p = \langle T_p, I_p, F_p, t_p, i_p, f_p \rangle$ be an NCS in U. Then,

(1) The truth-evaluative set of $p = \langle T_p, I_p, F_p, t_p, i_p, f_p \rangle$ is represented as

$$\begin{aligned} E_{Tp} &= \left\{\left(\mathring{u}, E_{Tp}(\mathring{u})\right) | \mathring{u} \in U\right\} \\ &= (\text{left truth} - \text{evaluative point}, \text{right truth} - \text{evaluative point}) \\ &= \left(l\left(E_{Tp}(\mathring{u})\right), r\left(E_{Tp}(\mathring{u})\right)\right) \\ &= \left(t(\mathring{u}) - T^-(\mathring{u}), T^+(\mathring{u}) - t(\mathring{u})\right). \end{aligned} \tag{6}$$

(2) The indeterminacy-evaluative set of $p = \langle T_p, I_p, F_p, t_p, i_p, f_p \rangle$ is represented as

$$
\begin{aligned}
E_{Ip} &= \left\{ \left(\mathring{u}, E_{Ip}(\mathring{u}) \right) | \mathring{u} \in U \right\} \\
&= (\text{left indeterminacy} - \text{evaluative point}, \text{right indeterminacy} - \text{evaluative point}) \\
&= \left\langle l\left(E_{Ip}(\mathring{u}) \right), r\left(E_{Ip}(\mathring{u}) \right) \right\rangle \\
&= \left(i(\mathring{u}) - I^-(\mathring{u}), I^+(\mathring{u}) - i(\mathring{u}) \right).
\end{aligned}
\tag{7}
$$

(3) The falsity-evaluative set of $p = \langle T_p, I_p, F_p, t_p, i_p, f_p \rangle$ is represented as

$$
\begin{aligned}
E_{Fp} &= \left\{ \left(\mathring{u}, E_{Fp}(\mathring{u}) \right) | \mathring{u} \in U \right\} \\
&= (\text{left falsity} - \text{evaluative point}, \text{right falsity} - \text{evaluative point}) \\
&= \left\langle l\left(E_{Fp}(\mathring{u}) \right), r\left(E_{Fp}(\mathring{u}) \right) \right\rangle \\
&= \left(f(\mathring{u}) - F^-(\mathring{u}), F^+(\mathring{u}) - f(\mathring{u}) \right).
\end{aligned}
\tag{8}
$$

The collection

$$
E_{Lp}(\mathring{u}) = \left(l\left(E_{Tp}(\mathring{u}) \right), l\left(E_{Ip}(\mathring{u}) \right), l\left(E_{Fp}(\mathring{u}) \right) \right),
\tag{9}
$$

is called the left evaluative point and the collection

$$
E_{Rp}(\mathring{u}) = \left(r\left(E_{Tp}(\mathring{u}) \right), r\left(E_{Ip}(\mathring{u}) \right), r\left(E_{Fp}(\mathring{u}) \right) \right),
\tag{10}
$$

is called the right evaluative point. We say that $E_\beta(\mathring{u}) = (E_{L\beta}(\mathring{u}), E_{R\beta}(\mathring{u}))$ is the evaluative point.

Example 1. Let $\beta = \{\langle \mathring{u}, T(\mathring{u}), I(\mathring{u}), F(\mathring{u}), t(\mathring{u}), i(\mathring{u}), f(\mathring{u}) \rangle | \mathring{u} \in I \}$ be an NCS in U. If

$$
\begin{aligned}
&\langle T(\mathring{u}), I(\mathring{u}), F(\mathring{u}), t(\mathring{u}), i(\mathring{u}), f(\mathring{u}) \rangle \\
&\quad = \langle [0.2, 0.4], [0.4, 0.6], [0.5, 0.7], (0.3, 0.2, 0.8) \rangle, \\
&\qquad \text{for all } \mathring{u} \in U,
\end{aligned}
\tag{11}
$$

then $E_{T\beta} = \{0.1, 0.1\}, E_{I\beta} = \{-0.2, 0.4\}, E_{F\beta} = \{0.3, -0.1\}$. Thus,

$$
\begin{aligned}
E_\beta(\mathring{u}) &= \left(E_{L\beta}(\mathring{u}), E_{R\beta}(\mathring{u}) \right) \\
&= \{ (\mathring{u}, \langle 0.1, -0.2, 0.3, 0.1, 0.4, -0.1 \rangle) | \mathring{u} \in U \}.
\end{aligned}
\tag{12}
$$

Remark 1. In Example 1, we observe that the left or right evaluative point of the NCS is not necessarily an NS. This motivates us to define the following terminologies.

Definition 7. Let $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ be an NCS in U with the evaluative set

$$
E_\beta = \left\{ \left(\mathring{u}; \left(E_{L\beta}(\mathring{u}), E_{R\beta}(\mathring{u}) \right) \right) | \mathring{u} \in U \right\}.
\tag{13}
$$

An element $\mathring{u} \in U$ is called

(1) Truth stable element of U if

$$
\begin{aligned}
E_{T\beta} &= \left\{ \mathring{u}; t\left(l\left(S_{T\beta}(\mathring{u}) \right), r\left(S_{T\beta}(\mathring{u}) \right) \right) \right\} \\
&= \{ \mathring{u}; t \, (\text{left truth stable} - \text{element}, \text{right truth stable} - \text{element}) \} \\
&= \{ \mathring{u}; t \, (t(\mathring{u}) - T^-(\mathring{u} T^+(\mathring{u}) - t(\mathring{u}))n \geq q0 \}.
\end{aligned}
\tag{14}
$$

(2) Indeterminacy stable element of U if

$$
\begin{aligned}
E_{I\beta} &= \left\{ \mathring{u}; t\left(l\left(S_{I\beta}(\mathring{u}) \right), r\left(S_{I\beta}(\mathring{u}) \right) \right) \right\} \\
&= \{ \mathring{u}; t \, (\text{left indeterminacy stable} - \text{element}, \text{right indeterminacy stable} - \text{element}) \}, \\
&= \{ \mathring{u}; t \, (i(\mathring{u}) - I^-(\mathring{u}), I^+(\mathring{u}) - i(\mathring{u}))n \geq q0 \}
\end{aligned}
\tag{15}
$$

(3) Falsity stable element of U if

$$E_{F\beta} = \left(l\left(S_{F\beta}(\mathring{u}) \right), r\left(S_{F\beta}(\mathring{u}) \right) \right)$$
$$= \{\mathring{u}; t \text{ (left stable falsity − element, right stable falsity − element)}\} \tag{16}$$
$$= \left(f(\mathring{u}) - F^-(\mathring{u}), F^+(\mathring{u}) - f(\mathring{u}) \right) \geq 0.$$

An element $\mathring{u} \in U$ is called stable if it satisfies conditions (1–3). The set of all stable elements of U is called stable cut of $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ in U and is denoted by S_β. We say that $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ is a stable neutrosophic set if $S_\beta = U$.

An element $\mathring{u} \in U$ is called partially stable if it partially satisfies conditions (1–3). The set of all partially stable elements of U is called partially stable cut of $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ in U and is denoted by P_β. We say that $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ is a partially stable neutrosophic set if $P_\beta \subset U$.

An element $\mathring{u} \in U$ is called antistable (unstable) if it does not satisfy conditions (1–3). The set of all unstable stable elements of U is called unstable stable cut of $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ in U and is denoted by U_β. We say that $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ is a unstable stable neutrosophic set if $U_{\backslash ss} \subseteq U$.

Thus, $U = S_\beta \cup P_\beta \cup U_\beta$.

Example 2. Let $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ be an NCS in $U = \{0, a, b, c\}$ given by Table 1.

Clearly, $\{0, a\}$ are stable elements of U and $\{b, c\}$ are unstable elements of U. Thus,

$$U = \{a, b, c, d\}$$
$$= S_\beta = \{0, a\} \cup P_\beta = \Phi \cup U_\beta = \{b, c\}. \tag{17}$$

Example 3. Let $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ be an NCS in $U = \{a, b\}$ given by Table 2.

Clearly, a and b are stable elements of U. Thus,

$$U = \{a, b\}$$
$$= S_\beta = \{a, b\} \cup P_\beta = \Phi \cup U_\beta = \Phi. \tag{18}$$

Remark 2. Every internal NCS is a stable NCS, as shown in example 3. If an NCS is neither internal nor external, then we may have some stable elements with respect to the internal portion and some unstable elements with respect to the external portion as given in the Example 2. Thus, an external NCS may or may not be a stable NCS, as shown in Examples 4 and 5.

Example 4. Let $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ be an external NCS in $U = \{a, b\}$ given by Table 3.

Then, clearly, a, b are unstable elements of U. Thus,

$$U = \{a, b\}$$
$$= S_\beta = \Phi \cup P_\beta = \{a, b\} \cup U_\beta = \Phi. \tag{19}$$

Example 5. Let $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ be an external NCS in $U = \{a, b\}$ given by Table 4.

Then, clearly, a, b are stable elements of U. Thus,

$$U = \{a, b\}$$
$$= S_\beta = \{a, b\} \Phi \cup P_\beta = \Phi \cup U_\beta = \Phi. \tag{20}$$

Example 6. Let $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ be an external NCS in $U = \{a\}$ given by Table 5.

Clearly, a is an unstable element of U. Thus, $U_\beta = \{a\} = U$. Hence, $U = S_\beta = \Phi \cup P_\beta = \Phi \cup U_\beta = \{a\}$.

Example 7. Let $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ be an external NCS in $UU = \{a\}$ given by Table 6.

Clearly, a is an unstable element of U. Thus, $U_\beta = \{a\} = U$. Hence, $U = S_\beta = \Phi \cup P_\beta = \Phi \cup U_\beta = \{a\}$.

Example 8. Let $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ be an NCS in $U = \{a, b, c\}$ given by Table 7.

Clearly, a and b are partially stable elements of U, so $P_\beta = \{a, b\} \subset U$ and c is the only stable element of U, so $S_\beta = \{c\}$. Also, there is no element which is unstable, so $U_\beta = \Phi$. Hence, $U = S_\beta \cup P_\beta \cup U_\beta$.

Remark 3

(1) If we have an external NCS which is unstable like in Example 6 such that

$$t(\mathring{u}) > \left[T^-(\mathring{u}), T^+(\mathring{u}) \right], i(\mathring{u})$$
$$> \left[I^-(\mathring{u}), I^+(\mathring{u}) \right], f(\mathring{u}) > \left[F^-(\mathring{u}), F^+(\mathring{u}) \right], \tag{21}$$

then its right evaluative point becomes a neutrosophic bipolar fuzzy set.

(2) If we have an external NCS which is unstable like in example 7 such that

$$t(\mathring{u}) < \left[T^-(\mathring{u}), T^+(\mathring{u}) \right], i(\mathring{u})$$
$$< \left[I^-(\mathring{u}), I^+(\mathring{u}) \right], f(\mathring{u}) < \left[F^-(\mathring{u}), F^+(\mathring{u}) \right], \tag{22}$$

then its left evaluative point becomes a neutrosophic bipolar fuzzy set.

TABLE 1: Neutrosophic cubic set β of U.

U	$T_\beta(\mathring{u})$	$I_\beta(\mathring{u})$	$F_\beta(\mathring{u})$	$t_\beta(\mathring{u})$	$i_\beta(\mathring{u})$	$f_\beta(\mathring{u})$
0	[0.3, 0.5]	[0.2, 0.4]	[0.2, 0.5]	0.4	0.3	0.4
a	[0.3, 0.5]	[0.3, 0.5]	[0.3, 0.6]	0.4	0.4	0.5
b	[0.6, 0.8]	[0.5, 0.6]	[0.4, 0.5]	0.5	0.4	0.3
c	[0.4, 0.8]	[0.5, 0.6]	[0.6, 0.7]	0.9	0.7	0.8

TABLE 2: Neutrosophic cubic set β of U.

U	$T_\beta(\mathring{u})$	$I_\beta(\mathring{u})$	$F_\beta(\mathring{u})$	$t_\beta(\mathring{u})$	$i_\beta(\mathring{u})$	$f_\beta(\mathring{u})$
a	[0.1, 0.7]	[0.1, 0.6]	[0.2, 0.8]	0.6	0.5	0.7
b	[0.6, 0.8]	[0.6, 0.9]	[0.5, 0.7]	0.7	0.8	0.6

TABLE 3: Neutrosophic cubic set β of U.

U	$T_\beta(\mathring{u})$	$I_\beta(\mathring{u})$	$F_\beta(\mathring{u})$	$t_\beta(\mathring{u})$	$i_\beta(\mathring{u})$	$f_\beta(\mathring{u})$
a	[0.1, 0.3]	[0.1, 0.4]	[0.3, 0.6]	0.4	0.5	0.7
b	[0.5, 0.8]	[0.6, 0.8]	[0.4, 0.6]	0.4	0.5	0.3

TABLE 4: External neutrosophic cubic set β of U.

U	$T_\beta(\mathring{u})$	$I_\beta(\mathring{u})$	$F_\beta(\mathring{u})$	$t_\beta(\mathring{u})$	$i_\beta(\mathring{u})$	$f_\beta(\mathring{u})$
a	[0.2, 0.4]	[0.3, 0.5]	[0.3, 0.6]	0.2	0.3	0.3
b	[0.4, 0.8]	[0.6, 0.7]	[0.4, 0.5]	0.8	0.7	0.5

TABLE 5: External neutrosophic cubic set β of U.

U	$T_\beta(\mathring{u})$	$I_\beta(\mathring{u})$	$F_\beta(\mathring{u})$	$t_\beta(\mathring{u})$	$i_\beta(\mathring{u})$	$f_\beta(\mathring{u})$
a	[0.3, 0.5]	[0.1, 0.4]	[0.4, 0.6]	0.8	0.5	0.7

TABLE 6: External neutrosophic cubic set β of U.

U	$T_\beta(\mathring{u})$	$I_\beta(\mathring{u})$	$F_\beta(\mathring{u})$	$t_\beta(\mathring{u})$	$i_\beta(\mathring{u})$	$f_\beta(\mathring{u})$
a	[0.5, 0.6]	[0.3, 0.5]	[0.7, 0.9]	0.4	0.2	0.6

TABLE 7: Neutrosophic cubic set β of U.

U	$T_\beta(\mathring{u})$	$I_\beta(\mathring{u})$	$F_\beta(\mathring{u})$	$t_\beta(\mathring{u})$	$i_\beta(\mathring{u})$	$f_\beta(\mathring{u})$
a	[0.7, 0.8]	[0.3, 0.5]	[0.6, 0.9]	0.7	0.8	0.2
b	[0.1, 0.5]	[0.6, 0.9]	[0.3, 0.8]	0.2	0.7	0.1
c	[0.1, 0.4]	[0.2, 0.5]	[0.3, 0.7]	0.3	0.4	0.5

(3) Every NCS needs not to be a stable NCS, but each stable NCS must be an NCS.

(4) Observing Example 5, we reached at Theorem 1.

Theorem 1. *If an external NCS $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ in U satisfies the condition*

$$(\forall \mathring{u} \in U) \begin{pmatrix} (T_\beta^-(\mathring{u}) = t_\beta(\mathring{u}), T_\beta^+(\mathring{u}) = t(\mathring{u})), \\ (I^-(\mathring{u}) = i(\mathring{u}), I^+(\mathring{u}) = i(\mathring{u})), \\ (F^-(\mathring{u}) = f(\mathring{u}), F^+(\mathring{u}) = f(\mathring{u})) \end{pmatrix}, \qquad (23)$$

then $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ is a stable NCS.

Proof. Straightforward.

Remark 4. We observe that if β is both an internal and external NCS, then β is a stable NCS.

Theorem 2. *The complement of a stable NCS is also a stable NCS.*

Proof. Let $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ be a stable NCS in U. Then,

$$U = S_\beta = \{ \mathring{u} \in \mathring{u} \mid l(E_\beta(\mathring{u})) \geq 0, r(E_\beta(\mathring{u})) \geq 0 \}. \tag{24}$$

Hence,

$$\begin{pmatrix} t(\mathring{u}) - T^-(\mathring{u}) \geq 0, \\ T^+(\mathring{u}) - t(\mathring{u}) \geq 0 \end{pmatrix}, \begin{pmatrix} i(\mathring{u}) - I^-(\mathring{u}) \geq 0, \\ I^+(\mathring{u}) - i(\mathring{u}) \geq 0 \end{pmatrix}, \begin{pmatrix} f(\mathring{u}) - F^-(\mathring{u}) \geq 0, \\ F^+(\mathring{u}) - f(\mathring{u}) \geq 0 \end{pmatrix}, \quad \forall \mathring{u} \in U. \tag{25}$$

It follows that

$$l(E_{\beta^C}(\mathring{u})) = (1 - t(\mathring{u})) - (1 - T^+(\mathring{u})) = T^+(\mathring{u}) - t(\mathring{u}) \geq 0,$$

$$l(E_{\beta^C}(\mathring{u})) = (1 - i(\mathring{u})) - (1 - I^+(\mathring{u})) = I^+(\mathring{u}) - i(\mathring{u}) \geq 0,$$

$$l(E_{\beta^C}(\mathring{u})) = (1 - f(\mathring{u})) - (1 - F^+(\mathring{u})) = F^+(\mathring{u}) - f(\mathring{u}) \geq 0,$$

$$r(E_{\beta^C}(\mathring{u})) = (1 - T^-(\mathring{u})) - (1 - t(\mathring{u})) = t(\mathring{u}) - T^-(\mathring{u}) \geq 0,$$

$$r(E_{\beta^C}(\mathring{u})) = (1 - I^-(\mathring{u})) - (1 - i(\mathring{u})) = i(\mathring{u}) - I^-(\mathring{u}) \geq 0,$$

$$r(E_{\beta^C}(\mathring{u})) = (1 - F^-(\mathring{u})) - (1 - f(\mathring{u})) = f(\mathring{u}) - F^-(\mathring{u}) \geq 0. \tag{26}$$

Therefore, $\beta^C = \langle T_\beta^c, I_\beta^c, F_\beta^c, t_\beta^c, i_\beta^c, f_\beta^c \rangle$ is a stable NCS.

Theorem 3. *The complement of an unstable NCS is also an unstable NCS.*

Proof. Let $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ be an unstable NCS in U. Then,

$$U = U_\beta = \{ \mathring{u} \in \mathring{u} \mid l(E_\beta(\mathring{u})) < 0 \} \cup \{ \mathring{u} \in \mathring{u} \mid r(E_\beta(\mathring{u})) < 0 \} \neq \Phi, \tag{27}$$

and so, there exist $\mathring{u} \in U$ such that

$$(t(\mathring{u}) - T^-(\mathring{u}) < 0), (i(\mathring{u}) - I^-(\mathring{u}) < 0),$$
$$(f(\mathring{u}) - F^-(\mathring{u}) < 0), \quad \forall \mathring{u} \in U, \tag{28}$$

or

$$(T^+(\mathring{u}) - t(\mathring{u}) < 0), (I^+(\mathring{u}) - i(\mathring{u}) < 0),$$
$$(F^+(\mathring{u}) - f(\mathring{u}) < 0), \quad \forall \mathring{u} \in U. \tag{29}$$

It follows that

$$l(E_{\beta^C}(\mathring{u})) = (1 - t(\mathring{u})) - (1 - T^+(\mathring{u})) = T^+(\mathring{u}) - t(\mathring{u}) < 0,$$

$$l(E_{\beta^C}(\mathring{u})) = (1 - i(\mathring{u})) - (1 - I^+(\mathring{u})) = I^+(\mathring{u}) - i(\mathring{u}) < 0,$$

$$l(E_{\beta^C}(\mathring{u})) = (1 - f(\mathring{u})) - (1 - F^+(\mathring{u})) = F^+(\mathring{u}) - f(\mathring{u}) < 0, \tag{30}$$

or

$$r(E_{\beta^C}(\mathring{u})) = (1 - T^-(\mathring{u})) - (1 - t(\mathring{u})) = t(\mathring{u}) - T^-(\mathring{u}) < 0,$$

$$r(E_{\beta^C}(\mathring{u})) = (1 - I^-(\mathring{u})) - (1 - i(\mathring{u})) = i(\mathring{u}) - I^-(\mathring{u}) < 0,$$

$$r(E_{\beta^C}(\mathring{u})) = (1 - F^-(\mathring{u})) - (1 - f(\mathring{u})) = f(\mathring{u}) - F^-(\mathring{u}) < 0. \tag{31}$$

Hence, $U_{\beta^C} \neq \Phi$, and therefore, $\beta^C = \langle T_\beta^c, I_\beta^c, F_\beta^c, t_\beta^c, i_\beta^c, f_\beta^c \rangle$ is an unstable NCS.

Example 9 illustrates Theorem 3.

Example 9. Let $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ be an NCS in $U = \{a, b\}$ given by Table 8.

Clearly, a and b are unstable elements of U and their complements are represented by Table 9.

Then, $\beta^c = \langle T_\beta^c, I_\beta^c, F_\beta^c, t_\beta^c, i_\beta^c, f_\beta^c \rangle$ is unstable since $a \in U_{\beta^c}$.

Theorem 4. *The P-union and P-intersection of two stable NCSs in U are stable cubic sets in U.*

Proof. Let $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ and $\beta_2 = \langle T_{\beta_2}, I_{\beta_2}, F_{\beta_2}, t_{\beta_2}, i_{\beta_2}, f_{\beta_2} \rangle$ be two NCSs in U. Then,

$$S_\beta = \{ \mathring{u} \in U \mid l(\beta_\beta(\mathring{u})) \geq 0, r(\beta_{5\beta}(\mathring{u})) \geq 0 \} = U,$$
$$S_{\beta_2} = \{ \mathring{u} \in U \mid l(\beta_{5\beta_2}(\mathring{u})) \geq 0, r(\beta_{5\beta_2}(\mathring{u})) \geq 0 \} = U. \tag{32}$$

It follows that

TABLE 8: Neutrosophic cubic set β of U.

U	$T_\beta(\mathring{u})$	$I_\beta(\mathring{u})$	$F_\beta(\mathring{u})$	$t_\beta(\mathring{u})$	$i_\beta(\mathring{u})$	$f_\beta(\mathring{u})$
a	$[0.1, 0.5]$	$[0.3, 0.6]$	$[0.2, 0.4]$	0.4	0.5	0.3
b	$[0.6, 0.9]$	$[0.1, 0.9]$	$[0.1, 0.6]$	0.7	0.6	0.5

TABLE 9: Complement of neutrosophic cubic set β of U provided in Table 8.

U	$T_\beta^c(\mathring{u})$	$I_\beta^c(\mathring{u})$	$F_\beta^c(\mathring{u})$	$t_\beta^c(\mathring{u})$	$i_\beta^c(\mathring{u})$	$f_\beta^c(\mathring{u})$
a	$[0.5, 0.9]$	$[0.4, 0.7]$	$[0.6, 0.8]$	0.6	0.5	0.7
b	$[0.1, 0.4]$	$[0.1, 0.9]$	$[0.4, 0.9]$	0.3	0.4	0.5

$$\begin{pmatrix} t_\beta(\mathring{u}) - T_\beta^-(\mathring{u}) \geq 0, \\ T_\beta^+(\mathring{u}) - t_\beta(\mathring{u}) \geq 0 \end{pmatrix}, \begin{pmatrix} i_\beta(\mathring{u}) - I_\beta^-(\mathring{u}) \geq 0, \\ I_\beta^+(\mathring{u}) - i_\beta(\mathring{u}) \geq 0 \end{pmatrix}, \begin{pmatrix} f_\beta(\mathring{u}) - F_\beta^-(\mathring{u}) \geq 0, \\ F_\beta^+(\mathring{u}) - f_\beta(\mathring{u}) \geq 0 \end{pmatrix}, \quad \forall \mathring{u} \in U,$$

$$\begin{pmatrix} t_{\beta_2}(\mathring{u}) - T_{\beta_2}^-(\mathring{u}) \geq 0, \\ T_{\beta_2}^+(\mathring{u}) - t_{\beta_2}(\mathring{u}) \geq 0 \end{pmatrix}, \begin{pmatrix} i_{\beta_2}(\mathring{u}) - I_{\beta_2}^-(\mathring{u}) \geq 0, \\ I_{\beta_2}^+(\mathring{u}) - i_{\beta_2}(\mathring{u}) \geq 0 \end{pmatrix}, \begin{pmatrix} f_{\beta_2}(\mathring{u}) - F_{\beta_2}^-(\mathring{u}) \geq 0, \\ F_{\beta_2}^+(\mathring{u}) - f_{\beta_2}(\mathring{u}) \geq 0 \end{pmatrix}, \quad \forall \mathring{u} \in U. \tag{33}$$

Assume that $t_{\beta_1}(\mathring{u}) \geq t_{\beta_2}(\mathring{u}), i_{\beta_1}(\mathring{u}) \geq i_{\beta_2}(\mathring{u}), f_{\beta_1}(\mathring{u}) \geq f_{\beta_2}(\mathring{u})$ and consider the following cases:

(i) $\begin{matrix} (T_{\beta_1}^-(\mathring{u}) \geq T_{\beta_2}^-(\mathring{u}), T_{\beta_1}^+(\mathring{u}) \geq T_{\beta_2}^+(\mathring{u})) \\ (I_{\beta_1}^-(\mathring{u}) \geq I_{\beta_2}^-(\mathring{u}), I_{\beta_1}^+(\mathring{u}) \geq I_{\beta_2}^+(\mathring{u})) \\ (F_{\beta_1}^-(\mathring{u}) \geq F_{\beta_2}^-(\mathring{u}), F_{\beta_1}^+(\mathring{u}) \geq F_{\beta_2}^+(\mathring{u})) \end{matrix}$

(ii) $\begin{matrix} (T_{\beta_1}^-(\mathring{u}) \leq T_{\beta_2}^-(\mathring{u}), T_{\beta_1}^+(\mathring{u}) \geq T_{\beta_2}^+(\mathring{u})) \\ (I_{\beta_1}^-(\mathring{u}) \geq I_{\beta_2}^-(\mathring{u}), I_{\beta_1}^+(\mathring{u}) \geq I_{\beta_2}^+(\mathring{u})) \\ (F_{\beta_1}^-(\mathring{u}) \geq F_{\beta_2}^-(\mathring{u}), F_{\beta_1}^+(\mathring{u}) \geq F_{\beta_2}^+(\mathring{u})) \end{matrix}$

(iii) $\begin{matrix} (T_{\beta_1}^-(\mathring{u}) \leq T_{\beta_2}^-(\mathring{u}), T_{\beta_1}^+(\mathring{u}) \leq T_{\beta_2}^+(\mathring{u})) \\ (I_{\beta_1}^-(\mathring{u}) \geq I_{\beta_2}^-(\mathring{u}), I_{\beta_1}^+(\mathring{u}) \geq I_{\beta_2}^+(\mathring{u})) \\ (F_{\beta_1}^-(\mathring{u}) \geq F_{\beta_2}^-(\mathring{u}), F_{\beta_1}^+(\mathring{u}) \geq F_{\beta_2}^+(\mathring{u})) \end{matrix}$

(iv) $\begin{matrix} (T_{\beta_1}^-(\mathring{u}) \leq T_{\beta_2}^-(\mathring{u}), T_{\beta_1}^+(\mathring{u}) \geq T_{\beta_2}^+(\mathring{u})) \\ (I_{\beta_1}^-(\mathring{u}) \leq I_{\beta_2}^-(\mathring{u}), I_{\beta_1}^+(\mathring{u}) \geq I_{\beta_2}^+(\mathring{u})) \\ (F_{\beta_1}^-(\mathring{u}) \geq F_{\beta_2}^-(\mathring{u}), F_{\beta_1}^+(\mathring{u}) \geq F_{\beta_2}^+(\mathring{u})) \end{matrix}$

(v) $\begin{matrix} (T_{\beta_1}^-(\mathring{u}) \leq T_{\beta_2}^-(\mathring{u}), T_{\beta_1}^+(\mathring{u}) \leq T_{\beta_2}^+(\mathring{u})) \\ (I_{\beta_1}^-(\mathring{u}) \leq I_{\beta_2}^-(\mathring{u}), I_{\beta_1}^+(\mathring{u}) \leq I_{\beta_2}^+(\mathring{u})) \\ (F_{\beta_1}^-(\mathring{u}) \geq F_{\beta_2}^-(\mathring{u}), F_{\beta_1}^+(\mathring{u}) \geq F_{\beta_2}^+(\mathring{u})) \end{matrix}$

(vi) $\begin{matrix} (T_{\beta_1}^-(\mathring{u}) \leq T_{\beta_2}^-(\mathring{u}), T_{\beta_1}^+(\mathring{u}) \leq T_{\beta_2}^+(\mathring{u})) \\ (I_{\beta_1}^-(\mathring{u}) \leq I_{\beta_2}^-(\mathring{u}), I_{\beta_1}^+(\mathring{u}) \leq I_{\beta_2}^+(\mathring{u})) \\ (F_{\beta_1}^-(\mathring{u}) \leq F_{\beta_2}^-(\mathring{u}), F_{\beta_1}^+(\mathring{u}) \geq F_{\beta_2}^+(\mathring{u})) \end{matrix}$

(vii) $\begin{matrix} (T_{\beta_1}^-(\mathring{u}) \leq T_{\beta_2}^-(\mathring{u}), T_{\beta_1}^+(\mathring{u}) \leq T_{\beta_2}^+(\mathring{u})) \\ (I_{\beta_1}^-(\mathring{u}) \leq I_{\beta_2}^-(\mathring{u}), I_{\beta_1}^+(\mathring{u}) \leq I_{\beta_2}^+(\mathring{u})) \\ (F_{\beta_1}^-(\mathring{u}) \leq F_{\beta_2}^-(\mathring{u}), F_{\beta_1}^+(\mathring{u}) \leq F_{\beta_2}^+(\mathring{u})) \end{matrix}$

(viii) $\begin{matrix} (T_{\beta_1}^-(\mathring{u}) \geq T_{\beta_2}^-(\mathring{u}), T_{\beta_1}^+(\mathring{u}) \leq T_{\beta_2}^+(\mathring{u})) \\ (I_{\beta_1}^-(\mathring{u}) \leq I_{\beta_2}^-(\mathring{u}), I_{\beta_1}^+(\mathring{u}) \leq I_{\beta_2}^+(\mathring{u})) \\ (F_{\beta_1}^-(\mathring{u}) \leq F_{\beta_2}^-(\mathring{u}), F_{\beta_1}^+(\mathring{u}) \leq F_{\beta_2}^+(\mathring{u})) \end{matrix}$

(ix) $\begin{matrix} (T_{\beta_1}^-(\mathring{u}) \geq T_{\beta_2}^-(\mathring{u}), T_{\beta_1}^+(\mathring{u}) \geq T_{\beta_2}^+(\mathring{u})) \\ (I_{\beta_1}^-(\mathring{u}) \leq I_{\beta_2}^-(\mathring{u}), I_{\beta_1}^+(\mathring{u}) \leq I_{\beta_2}^+(\mathring{u})) \\ (F_{\beta_1}^-(\mathring{u}) \leq F_{\beta_2}^-(\mathring{u}), F_{\beta_1}^+(\mathring{u}) \leq F_{\beta_2}^+(\mathring{u})) \end{matrix}$

(x) $\begin{matrix} (T_{\beta_1}^-(\mathring{u}) \geq T_{\beta_2}^-(\mathring{u}), T_{\beta_1}^+(\mathring{u}) \geq T_{\beta_2}^+(\mathring{u})) \\ (I_{\beta_1}^-(\mathring{u}) \geq I_{\beta_2}^-(\mathring{u}), I_{\beta_1}^+(\mathring{u}) \leq I_{\beta_2}^+(\mathring{u})) \\ (F_{\beta_1}^-(\mathring{u}) \leq F_{\beta_2}^-(\mathring{u}), F_{\beta_1}^+(\mathring{u}) \leq F_{\beta_2}^+(\mathring{u})) \end{matrix}$

(xi) $\begin{matrix} (T_{\beta_1}^-(\mathring{u}) \geq T_{\beta_2}^-(\mathring{u}), T_{\beta_1}^+(\mathring{u}) \geq T_{\beta_2}^+(\mathring{u})) \\ (I_{\beta_1}^-(\mathring{u}) \geq I_{\beta_2}^-(\mathring{u}), I_{\beta_1}^+(\mathring{u}) \geq I_{\beta_2}^+(\mathring{u})) \\ (F_{\beta_1}^-(\mathring{u}) \leq F_{\beta_2}^-(\mathring{u}), F_{\beta_1}^+(\mathring{u}) \leq F_{\beta_2}^+(\mathring{u})) \end{matrix}$

(xii) $\begin{matrix} (T_{\beta_1}^-(\mathring{u}) \geq T_{\beta_2}^-(\mathring{u}), T_{\beta_1}^+(\mathring{u}) \geq T_{\beta_2}^+(\mathring{u})) \\ (I_{\beta_1}^-(\mathring{u}) \geq I_{\beta_2}^-(\mathring{u}), I_{\beta_1}^+(\mathring{u}) \geq I_{\beta_2}^+(\mathring{u})) \\ (F_{\beta_1}^-(\mathring{u}) \geq F_{\beta_2}^-(\mathring{u}), F_{\beta_1}^+(\mathring{u}) \leq F_{\beta_2}^+(\mathring{u})) \end{matrix}$

The first case implies that

$$\max\left\{\left(t_{\beta_1}(\mathring{u}), i_{\beta_1}(\mathring{u}), f_{\beta_1}(\mathring{u})\right), \left(t_{\beta_2}(\mathring{u}), i_{\beta_2}(\mathring{u}), f_{\beta_2}(\mathring{u})\right)\right\}$$

$$= \left(t_{\beta_1}(\mathring{u}) \geq T_{\beta_1}^-(\mathring{u}), i_{\beta_1}(\mathring{u}) \geq I_{\beta_1}^-(\mathring{u}), i_{\beta_1}(\mathring{u}) \geq I_{\beta_1}^-(\mathring{u})\right)$$

$$= \max\left\{T_{\beta_1}^-(\mathring{u}), I_{\beta_1}^-(\mathring{u}), F_{\beta_1}^-(\mathring{u}), T_{\beta_2}^-(\mathring{u}), I_{\beta_2}^-(\mathring{u}), F_{\beta_2}^-(\mathring{u})\right\},$$

$$\max\left\{\left(t_{\beta_1}(\mathring{u}), i_{\beta_1}(\mathring{u}), f_{\beta_1}(\mathring{u})\right), \left(t_{\beta_2}(\mathring{u}, i_{\beta_2}(\mathring{u}), f_{\beta_2}(\mathring{u})\right)\right\}$$

$$= \left(t_{\beta_1}(\mathring{u}) \geq T_{\beta_1}^+(\mathring{u}), i_{\beta_1}(\mathring{u}) \geq I_{\beta_1}^+(\mathring{u}), i_{\beta_1}(\mathring{u}) \geq I_{\beta_1}^+(\mathring{u})\right)$$

$$= \max\left\{T_{\beta_1}^+(\mathring{u}), I_{\beta_1}^+(\mathring{u}), F_{\beta_1}^+(\mathring{u}), T_{\beta_2}^{+-}(\mathring{u}), I_{\beta_2}^+(\mathring{u}), F_{\beta_2}^+(\mathring{u})\right\}. \tag{34}$$

It follows that

$$\left(t_{\beta_1}(\mathring{u}) - T_{\beta_1}^-(\mathring{u}), i_{\beta_1}(\mathring{u}) - I_{\beta_1}^-(\mathring{u}), f_{\beta_1}(\mathring{u}) - F_{\beta_1}^-(\mathring{u})\right) \geq 0,$$

$$\left(T_{\beta_1}(\mathring{u})^+ - t_{\beta_1}(\mathring{u}), I_{\beta_1}(\mathring{u})^+ - i_{\beta_1}(\mathring{u}), F_{\beta_1}(\mathring{u})^+ - f_{\beta_1}(\mathring{u})\right) \geq 0. \tag{35}$$

The result of the remaining cases can be obtained in the same way. Therefore, $\beta_1 \cup {}_P\beta_2$ is a stable CS in U. By the same way, we also know that $\beta_1 \cup {}_P\beta_2$ is a stable CS in U.

Example 10 shows that the $\dot{R}$-union and the $\dot{R}$-intersection of two stable NCSs in U may not be a stable NCS in U.

Example 10. Let $\beta_1 = \langle T_{\beta_1}, I_{\beta_1}, F_{\beta_1}, t_{\beta_1}, i_{\beta_1}, f_{\beta_1} \rangle$ and $\beta_2 = \langle T_{\beta_2}, I_{\beta_2}, F_{\beta_2}, t_{\beta_2}, i_{\beta_2}, f_{\beta_2} \rangle$ be two NCSs in $U = \{a, b\}$ defined by Tables 10 and 11, respectively.

Then,

$$\beta_1 \cup_R \beta_2 = \left\{ \begin{array}{l} \langle a, [0.4, 0.5], [0.3, 0.9], [0.7, 0.9], 0.15, 0.35, 0.6 \rangle, \\ \langle b, [0.6, 0.9], [0.8, 0.9], [0.5, 0.6], 0.6, 0.8, 0.25 \rangle \end{array} \right\},$$

$$\beta_1 \cap_R \beta_2 = \left\{ \begin{array}{l} \langle a, [0.1, 0.3], [0.1, 0.4], [0.3, 0.7], 0.4, 0.8, 0.80 \rangle, \\ \langle b, [0.6, 0.9], [0.1, 0.9], [0.2, 0.4], 0.7, 0.8, 0.56 \rangle \end{array} \right\}.$$

$$(36)$$

Hence, we know that

$$(\forall \mathring{u} \in U) \left(\max \left\{ \begin{array}{l} (T_{\beta_1}(\mathring{u})^-, I_{\beta_1}(\mathring{u})^-, F_{\beta_1}(\mathring{u})^-), \\ (T_{\beta_2}(\mathring{u})^-, I_{\beta_2}(\mathring{u})^-, F_{\beta_2}(\mathring{u})^-) \end{array} \right\} \le \left((t_{\beta_1}, i_{\beta_1}, f_{\beta_1}) \wedge (t_{\beta_2}, i_{\beta_2}, f_{\beta_2}) \right) (\mathring{u}) \right). \qquad (38)$$

Then, the $\dot{\text{R}}$-union of β_1 and β_2 is a stable NCS in U.

Proof. Let $\beta_1 = \langle T_{\beta_1}, I_{\beta_1}, F_{\beta_1}, t_{\beta_1}, i_{\beta_1}, f_{\beta_1} \rangle$ and $\beta_2 = \langle T_{\beta_2}, I_{\beta_2}, F_{\beta_2}, t_{\beta_2}, i_{\beta_2}, f_{\beta_2} \rangle$ be two internal NCSs in U. Then, $(T_{\beta_1}$

$$\max \left\{ \left(T_{\beta_1}(\mathring{u})^-, I_{\beta_1}^-(\mathring{u}) \le F_{\beta_1}(\mathring{u})^- \right), \left(T_{\beta_1}(\mathring{u})^-, I_{\beta_1}(\mathring{u})^- \le F_{\beta_1}(\mathring{u})^- \right) \right\}$$
$$\le \left((t_{\beta_1}, i_{\beta_1}, f_{\beta_1}) \wedge (t_{\beta_2}, i_{\beta_2}, f_{\beta_2}) \right) (\mathring{u}) \qquad (39)$$
$$\le \max \left\{ \left(T_{\beta_1}(\mathring{u})^+, I_{\beta_1}(\mathring{u})^+ \le F_{\beta_1}(\mathring{u})^+ \right), \left(T_{\beta_1}(\mathring{u})^+, I_{\beta_1}(\mathring{u})^+ \le F_{\beta_1}(\mathring{u})^+ \right) \right\},$$

for all $\mathring{u} \in U$. Hence, the $\dot{\text{R}}$-union of β_1 and β_2 is an internal NCS, and so it is stable by the fact that every internal NCS is stable.

$$(\forall \mathring{u} \in U) \left(\max \left\{ \begin{array}{l} (T_{\beta_1}(\mathring{u})^+, I_{\beta_1}(\mathring{u})^+, F_{\beta_1}(\mathring{u})^+), \\ (T_{\beta_2}(\mathring{u})^+, I_{\beta_2}(\mathring{u})^+, F_{\beta_2}(\mathring{u})^+) \end{array} \right\} \le \left((t_{\beta_1}, i_{\beta_1}, f_{\beta_1}) \vee (t_{\beta_2}, i_{\beta_2}, f_{\beta_2}) \right) (\mathring{u}) \right). \qquad (40)$$

Then, the $\dot{\text{R}}$-intersection of β_1 and β_2 is a stable NCS in U.

Proof. Straightforward.

$$E_{\beta_1 \cup_R \beta_2}(a) = \langle (-0.25, 0.35), (0.05, 0.55), (-0.1, 0.3) \rangle,$$
$$E_{\beta_1 \cup \beta_2}(b) = \langle (0, 0.3), (0, 0.1), (-0.25, 0.35) \rangle,$$
$$E_{\beta_1 \cap \beta_2}(a) = \langle (0.3, -0.1), (0.7, -0.4), (0.5, -0.1) \rangle,$$
$$E_{\beta_1 \cap \beta_2}(b) = \langle (0.1, 0.2), (0.7, 0.1), (0.36, -0.16) \rangle.$$

$$(37)$$

Theorem 5. *Let $\beta_1 = \langle T_{\beta_1}, I_{\beta_1}, F_{\beta_1}, t_{\beta_1}, i_{\beta_1}, f_{\beta_1} \rangle$ and $\beta_2 = \langle T_{\beta_2}, I_{\beta_2}, F_{\beta_2}, t_{\beta_2}, i_{\beta_2}, f_{\beta_2} \rangle$ be two internal NCSs in U such that*

$(\mathring{u})^- \le t_{\beta_1}(\mathring{u}) \le T_{\beta_1}(\mathring{u})^+)$, $(I_{\beta_1}(\mathring{u})^- \le i_{\beta_1}(\mathring{u}) \le I_{\beta_1}(\mathring{u})^+)$, and $(F_{\beta_1}(\mathring{u})^- \le f_{\beta_1}(\mathring{u}) \le F_{\beta_1}(\mathring{u})^+)$ and $(T_{\beta_2}(\mathring{u})^- \le t_{\beta_2}(\mathring{u}) \le T_{\beta_2}(\mathring{u})^+)$, $(I_{\beta_2}(\mathring{u})^- \le i_{\beta_2}(\mathring{u}) \le I_{\beta_2}(\mathring{u})^+)$, and $(F_{\beta_2}(\mathring{u})^- \le f_{\beta_2}(\mathring{u}) \le F_{\beta_2}(\mathring{u})^+)$, $\forall \mathring{u} \in U$. We know that

Theorem 6. *Let $\beta_1 = \langle T_{\beta_1}, I_{\beta_1}, F_{\beta_1}, t_{\beta_1}, i_{\beta_1}, f_{\beta_1} \rangle$ and $\beta_2 = \langle T_{\beta_2}, I_{\beta_2}, F_{\beta_2}, t_{\beta_2}, i_{\beta_2}, f_{\beta_2} \rangle$ be two internal NCSs in U such that*

(1) The truth-stable degree of β in U is denoted by $\text{Tru}(\text{SD}_\beta)$ and is defined as

$$\text{Tru}(\text{SD}_\beta) = \left(\sum_{\mathring{u} \in U} l(E_{T\beta}(\mathring{u})), r(E_{T\beta}(\mathring{u})) \right). \qquad (41)$$

(2) The indeterminacy-stable degree of β in U is denoted by $\text{Ind}(\text{SD}_\beta)$ and is defined as

$$\text{Ind}(\text{SD}_\beta) = \left(\sum_{\mathring{u} \in U} l(E_{I\beta}(\mathring{u})), r(E_{I\beta}(\mathring{u})) \right). \qquad (42)$$

(3) The falsity-stable degree of β in U is denoted by $\text{Fal}(\text{SD}_\beta)$ and is defined as

4. Neutro-Almost-Stable Neutrosophic Cubic Set

In this section, we introduce a new class of the stable neutrosophic cubic set, namely, the neutro-almost-stable neutrosophic cubic set.

Definition 8. Let $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ be an NCS with the evaluative set $E_\beta = \left\{ (\mathring{u}, E_\beta(\mathring{u})) | \mathring{u} \in U \right\}$ in U. Then,

TABLE 10: Neutrosophic cubic set β_1 of U.

U	$T_{\beta_1}(\mathring{u})$	$I_{\beta_1}(\mathring{u})$	$F_{\beta_1}(\mathring{u})$	$t_{\beta_1}(\mathring{u})$	$i_{\beta_1}(\mathring{u})$	$f_{\beta_1}(\mathring{u})$
a	[0.4, 0.5]	[0.3, 0.4]	[0.3, 0.7]	0.4	0.35	0.60
b	[0.3, 0.7]	[0.8, 0.9]	[0.5, 0.6]	0.60	0.8	0.56

TABLE 11: Neutrosophic cubic set β_2 of U.

U	$T_{\beta_2}(\mathring{u})$	$I_{\beta_2}(\mathring{u})$	$F_{\beta_2}(\mathring{u})$	$t_{\beta_2}(\mathring{u})$	$i_{\beta_2}(\mathring{u})$	$f_{\beta_2}(\mathring{u})$
a	[0.1, 0.3]	[0.1, 0.9]	[0.7, 0.9]	0.15	0.8	0.8
b	[0.6, 0.9]	[0.1, 0.9]	[0.2, 0.4]	0.7	0.8	0.25

$$\mathrm{Fal}\left(\mathrm{SD}_\beta\right) = \left(\sum_{\mathring{u} \in U} l\left(E_{F\beta}(\mathring{u})\right), r\left(E_{F\beta}(\mathring{u})\right)\right). \tag{43}$$

(4) The stable degree of β in U is denoted by SD_β and is defined as $\mathrm{SD}_\beta = (\mathrm{Tru}(\mathrm{SD}_\beta), \mathrm{Ind}(\mathrm{SD}_\beta), \mathrm{Fal}(\mathrm{SD}_\beta))$.

Definition 9. An NCS with the evaluative set $E_\beta = \left\{(\mathring{u}, E_\beta(\mathring{u})) | \mathring{u} \in U\right\}$ in U is said to be

(1) Almost truth-stable if $\mathrm{Tru}(\mathrm{SD}_\beta) \geq 0$

(2) Almost indeterminacy-stable if $\mathrm{Ind}(\mathrm{SD}_\beta) \geq 0$

(3) Almost falsity-stable if $\mathrm{Fal}(\mathrm{SD}_\beta) \geq 0$

(4) Almost stable if it is almost truth-stable, almost indeterminacy-stable, and almost falsity-stable, i.e., $\mathrm{Tru}(\mathrm{SD}_\beta) \geq 0, \mathrm{Ind}(\mathrm{SD}_\beta) \geq 0, \mathrm{Fal}(\mathrm{SD}_\beta) \geq 0$.

(5) Almost partially stable if it is almost partially truth-stable, almost partially indeterminacy-stable, and almost partially falsity-stable.

(6) Almost unstable if it is almost truth-unstable, almost indeterminacy-unstable, and almost falsity-unstable, i.e., $\mathrm{Tru}(\mathrm{SD}_\beta) < 0, \mathrm{Ind}(\mathrm{SD}_\beta) < 0, \mathrm{Fal}(\mathrm{SD}_\beta) < 0$.

Example 11. Let $\beta_1 = \langle T_{\beta_1}, I_{\beta_1}, F_{\beta_1}, t_{\beta_1}, i_{\beta_1}, f_{\beta_1} \rangle$ and $\beta_2 = \langle T_{\beta_2}, I_{\beta_2}, F_{\beta_2}, t_{\beta_2}, i_{\beta_2}, f_{\beta_2} \rangle$ be two NCSs in $U = \{a, b\}$ defined by Tables 12 and 13, respectively, with the evaluative set

$$E_{\beta_1} = \langle(a; \langle 0, 0.1 \rangle, \langle 0.05, 0.05 \rangle, \langle 0.3, 0.1 \rangle),$$
$$(b; \langle 0.3, 0.1 \rangle, \langle 0, 0.1 \rangle, \langle 0.06, 0.04 \rangle)\rangle. \tag{44}$$

Then, $\mathrm{Tru}(\mathrm{SD}_{\beta_1}) = (0.3, 0.2) \geq 0$, $\mathrm{Ind}(\mathrm{SD}_{\beta_1}) = (0.05, 0.15) \geq 0, \mathrm{Fal}(\mathrm{SD}_{\beta_2}) = (0.36, 0.14) \geq 0$. Thus,

$$\mathrm{SD}_{\beta_1} = (0.3, 0.2, 0.05, 0.15, 0.36, 0.14) \geq 0, \tag{45}$$

alsowith the evaluative set

$$E_{\beta_2} = \langle(a\langle 0.05, 0.15 \rangle, \langle 0.7, 0.1 \rangle, \langle 0.1, 0.1 \rangle),$$
$$(b\langle 0.1, 0.2 \rangle, \langle 0.7, 0.1 \rangle, \langle 0.05, 0.15 \rangle)\rangle. \tag{46}$$

Then, $\mathrm{Tru}(\mathrm{SD}_{\beta_2}) = (0.15, 0.25) \geq 0$, $\mathrm{Ind}(\mathrm{SD}_{\beta_2}) = (0.14, 0.2) \geq 0, \mathrm{Fal}(\mathrm{SD})_{\beta_2} = (0.15, 0.25) \geq 0$. Thus,

$$\mathrm{SD}_{\beta_2} = (0.15, 0.35, 0.14, 0.2, 0.15, 0.25) \geq 0. \tag{47}$$

So, β_1 and β_2 both are almost stable NCSs.

Example 12. Let $\beta_3 = \langle T_{\beta_3}, I_{\beta_3}, F_{\beta_3}, t_{\beta_3}, i_{\beta_3}, f_{\beta_3} \rangle$ be an NCS in $U = \{a, b\}$ defined by Table 14.

The evaluative set is

$$E_{\beta_3} = \langle(a\langle -0.1, 0.2 \rangle, \langle 0.3, -0.1 \rangle, \langle -0.1, 0.3 \rangle),$$
$$(b\langle -0.1, 0.5 \rangle, \langle -0.2, 0.3 \rangle, \langle 0.2, -0.1 \rangle)\rangle. \tag{48}$$

Then, $\mathrm{Tru}(\mathrm{SD}_{\beta_3}) = (-0.2, 0.7) < 0$. Thus, the NCS $\beta_3 = \langle T_{\beta_3}, I_{\beta_3}, F_{\beta_3}, t_{\beta_3}, i_{\beta_3}, f_{\beta_3} \rangle$ in U is not almost truth-stable as $\mathrm{Tru}(\mathrm{SD}_{\beta_3}) < 0$. Also, $\mathrm{Ind}(\mathrm{SD}_{\beta_3}) = (0.1, 0.2) \geq 0$. Thus, the NCS $\beta_3 = \langle T_{\beta_3}, I_{\beta_3}, F_{\beta_3}, t_{\beta_3}, i_{\beta_3}, f_{\beta_3} \rangle$ in U is almost indeterminacy-stable as $\mathrm{Ind}(\mathrm{SD}_{\beta_3}) \geq 0$. Similarly $\beta_3 = \langle T_{\beta_3}, I_{\beta_3}, F_{\beta_3}, t_{\beta_3}, i_{\beta_3}, f_{\beta_3} \rangle$ in U is almost falsity-stable as $\mathrm{Fal}(\mathrm{SD}_{\beta_3}) \geq 0$. So, finally, we can say that β_3 is an almost partially stable NCS.

Example 13. Let $\beta_4 = \langle T_{\beta_4}, I_{\beta_4}, F_{\beta_4}, t_{\beta_4}, i_{\beta_4}, f_{\beta_4} \rangle$ be an NCS in $U = \{a, b\}$ defined by Table 15

The evaluative set is

$$E_{\beta_4} = \langle(a; \langle 0.2, -0.1 \rangle, \langle 0.3, -0.1 \rangle, \langle -0.1, 0.3 \rangle),$$
$$(b; \langle -0.1, 0.5 \rangle, \langle -0.2, 0.3 \rangle, \langle 0.2, -0.1 \rangle)\rangle. \tag{49}$$

Then, $\mathrm{Tru}(\mathrm{SD}_{\beta_4}) = (0.1, 0.4) \geq 0, \mathrm{Ind}(\mathrm{SD}_{\beta_4}) = (0.1, 0.2) \geq 0, \mathrm{Fal}(\mathrm{SD}_{\beta_4}) = (0.1, 0.2) \geq 0$. So, β_4 is an almost-stable NCS, but it is not a stable NCS, as from Definition 7; $S_\beta = \Phi, P_\beta = \Phi, U_\beta = \{a, b\}$.

Remark 5. From Examples 11, 12, and 13, we have the following results.

Theorem 7

(1) Every stable NCS $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ in U is an almost-stable NCS, but the converse is not true

(2) Every internal NCS is almost stable

(3) Every external NCS may or may not be stable

(4) The P-union and P-intersection of two stable NCSs are almost stable

(5) The complement of an almost-stable NC is also an almost-stable NCS

Proof. Straightforward.

TABLE 12: Neutrosophic cubic set β_1 of U.

U	$T_{\beta_1}(\mathring{u})$	$I_{\beta_1}(\mathring{u})$	$F_{\beta_1}(\mathring{u})$	$t_{\beta_1}(\mathring{u})$	$i_{\beta_1}(\mathring{u})$	$f_{\beta_1}(\mathring{u})$
a	[0.4, 0.5]	[0.3, 0.4]	[0.3, 0.7]	0.4	0.35	0.60
b	[0.3, 0.7]	[0.8, 0.9]	[0.5, 0.6]	0.60	0.8	0.56

TABLE 13: Neutrosophic cubic set β_2 of U.

U	$T_{\beta_2}(\mathring{u})$	$I_{\beta_2}(\mathring{u})$	$F_{\beta_2}(\mathring{u})$	$t_{\beta_2}(\mathring{u})$	$i_{\beta_2}(\mathring{u})$	$f_{\beta_2}(\mathring{u})$
a	[0.1, 0.3]	[0.1, 0.9]	[0.7, 0.9]	0.15	0.8	0.8
b	[0.6, 0.9]	[0.1, 0.9]	[0.2, 0.4]	0.7	0.8	0.25

TABLE 14: Neutrosophic cubic set β_3 of U.

U	$T_{\beta_3}(\mathring{u})$	$I_{\beta_3}(\mathring{u})$	$F_{\beta_3}(\mathring{u})$	$t_{\beta_3}(\mathring{u})$	$i_{\beta_3}(\mathring{u})$	$f_{\beta_3}(\mathring{u})$
a	[0.2, 0.3]	[0.3, 0.5]	[0.4, 0.6]	0.1	0.6	0.3
b	[0.3, 0.7]	[0.8, 0.9]	[0.5, 0.6]	0.2	0.6	0.7

TABLE 15: Neutrosophic cubic set β_4 of U.

U	$T_{\beta_4}(\mathring{u})$	$I_{\beta_4}(\mathring{u})$	$F_{\beta_4}(\mathring{u})$	$t_{\beta_4}(\mathring{u})$	$i_{\beta_4}(\mathring{u})$	$f_{\beta_4}(\mathring{u})$
a	[0.2, 0.3]	[0.3, 0.5]	[0.4, 0.6]	0.4	0.6	0.3
b	[0.3, 0.7]	[0.8, 0.9]	[0.5, 0.6]	0.2	0.6	0.7

5. Application in Decision Making

In this section, we shall define a new approach to multiple attribute group decision making with the help of stable neutrosophic cubic sets. We also provide a numerical example. Suppose $H = \{H_1, H_2, \ldots, H_m\}$. Each alternative H_i respects n criteria $G_j = \{G_1, G_2, \ldots, G_n\}$ which are expressed by a stable NCS $q_{i_j} = ((\tilde{q}_{\mathrm{Tru}\,ij}, \tilde{q}_{\mathrm{Ind}\,ij}, \tilde{q}_{\mathrm{Fal}\,ij})(q_{\mathrm{Tru}\,ij}, q_{\mathrm{Ind}\,ij}, q_{\mathrm{Fal}\,ij}))$, $(j = 1, 2, \ldots n, i = 1, 2, \ldots, m)$. The criteria $G_1, \ldots, G_k$ are benefit and criteria $G_{k+1}, \ldots, G_n$ are non-benefit criteria, and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ is the weighted vector of the criteria, where, $\omega_i \varepsilon [0, 1]$ and $\sum \omega_i = 1$. So, the decision matrix is obtained as $D = (q_{ij})_{m \times n}$. The steps of the decision making based on stable NCSs are given as follows:

Step 1: we standardize the decision matrix.

Step 2: we construct the normalized decision matrix. Normalize score or data are as follows:

$$r_{i_j} = \frac{\mathring{u}_{i_j}}{\left(\sum \mathring{u}_{i_j}^2\right)}, \quad \text{for } i = 1, \ldots, m; j = 1, \ldots, n. \quad (50)$$

Step 3: we construct the weighted normalized decision matrix:

$$v_{i_j} = w_j \cdot r_{i_j}. \quad (51)$$

Step 4: we determine the ideal and negative ideal solutions. Ideal solution $A^* = \{v_1, \ldots, v_n\}$, where

$$v_j^* = \left\{ \max\left(v_{i_j}\right), \text{ if } j \in J; \min\left(v_{i_j}\right), \text{ if } j \in J' \right\}. \quad (52)$$

Negative ideal solution is

$$A' = \{v_1', \ldots, v_n'\}, \quad (53)$$

where

$$v_j' \left\{ \max\left(v_{i_j}\right), \quad \text{if } j \in J; \min\left(v_{i_j}\right), \text{ if } j \in J' \right\}. \quad (54)$$

Step 5: we calculate the separation measures for each alternative. Separation from the ideal alternatives is

$$S_i^* = \sqrt{\left[\sum \left(v_j^* - v_{i_j}\right)^2\right]}, \quad i = 1, \ldots, m. \quad (55)$$

Similarly, separation from negative ideal alternatives is

$$S_i' = \sqrt{\left[\sum \left(v_j' - v_{i_j}\right)^2\right]}, \quad i = 1, \ldots, m. \quad (56)$$

Step 6: we calculate the relative closeness to the ideal solution C_i^* where

$$C_i^* = \frac{S_i'}{\left(S_i^* + S_i'\right)}, \quad 0 \le C_i^* \le 1. \quad (57)$$

We select the option with C_i^* closest to 1.

5.1. Numerical Application. At the end of December 2019 [22], in Wuhan, the China Health Commission reported a cluster of pneumonia cases of unknown etiology. The pathogen was identified as novel coronavirus 2019. Later, the World Health Organization named it Coronavirus Disease

2019 (COVID-19). After the discovery of COVID-19, it spread in more than 200 countries. COVID-19 has zoonotic basis, which was then spread through the human interaction to human population [23]. Common signs of COVID-19 infection are similar to those of common cold and include respiratory symptoms such as dry cough, fever, shortness of breath, and breathing difficulties. Initially its etiology was unknown. Later on, it was studied thoroughly and found that it has an incubation period of 14 days, during which some individuals show all the symptoms while others show mild symptoms. It is sensitive to know that someone have the disease due to the dual nature (same as common flu) of COVID-19 symptoms [24]. In this section, we use the TOPSIS method to rank the COVID-19 in four provinces of Pakistan. A numerical example which is solved using the TOPSIS method is presented to demonstrate the applicability and effectiveness of the proposed method.

5.2. Example. Let us consider the decision making problem. Suppose that there is a panel and they selected four possible alternatives (H_1, H_2, H_3, H_4) to find out the spreading of COVID-19 in provinces of Pakistan: H_1 is KPK, H_2 is Sindh,

H_3 is Punjab, and H_4 is Balochistan. A group of doctors intends to choose one province be the most affected area from four provinces, to be further evaluated according to the four attributes, which are shown as G_1 effected people, G_2 recovered people, G_3 admitted people, and G_4 number of deaths. By this method, we can find out which province is more affected. Then, we must take some action to stop the cases in that province. The experts give them advice for quarantine. Also, they suggest them treatment and say that the treatment will be continued until the transmission of virus stops. By using the stable neutrosophic cubic information, the alternatives are evaluated by the decision maker and the results are presented in the decision matrix.

The decided steps of the TOPSIS method are presented as follows:

Step 1

(a) The decision makers take their analysis of each alternatives based on each criterion and the performance of each alternative H_i with respect to each criterion G_j (Tex translation failed).

$$
D = \begin{array}{c} \\ H_1 \\ \\ \\ H_2 \\ \\ \\ H_3 \\ \\ \\ H_4 \\ \end{array}
\begin{array}{cccc}
G_1 & G_2 & G_3 & G_4 \\
\left\{ \begin{matrix} [0.1,0.4], \\ [0.2,0.6], \\ [0.1,0.4], \\ (0.2,0.5,0.2) \end{matrix} \right\} &
\left\{ \begin{matrix} [0.1,0.3], \\ [0.1,0.3], \\ [0.1,0.3], \\ (0.2,0.2,0.2) \end{matrix} \right\} &
\left\{ \begin{matrix} [0.1,0.4], \\ [0.1,0.4], \\ [0.1,0.4], \\ (0.2,0.2,0.2) \end{matrix} \right\} &
\left\{ \begin{matrix} [0.1,0.4], \\ [0.1,0.3], \\ [0.1,0.4], \\ (0.2,0.2,0.2) \end{matrix} \right\} \\
\left\{ \begin{matrix} [0.1,0.3], \\ [0.1,0.4], \\ [0.2,0.5], \\ (0.2,0.2,0.3) \end{matrix} \right\} &
\left\{ \begin{matrix} [0.1,0.4], \\ [0.1,0.6], \\ [0.1,0.4], \\ (0.3,0.4,0.2) \end{matrix} \right\} &
\left\{ \begin{matrix} [0.1,0.4], \\ [0.1,0.4], \\ [0.1,0.4], \\ (0.2,0.2,0.3) \end{matrix} \right\} &
\left\{ \begin{matrix} [0.1,0.3], \\ [0.2,0.6], \\ [0.1,0.4], \\ (0.2,0.4,0.3) \end{matrix} \right\} \\
\left\{ \begin{matrix} [0.2,0.5], \\ [0.2,0.5], \\ [0.1,0.4], \\ (0.3,0.3,0.3) \end{matrix} \right\} &
\left\{ \begin{matrix} [0.1,0.4], \\ [0.1,0.3], \\ [0.2,0.6], \\ (0.3,0.2,0.4) \end{matrix} \right\} &
\left\{ \begin{matrix} [0.3,0.6], \\ [0.3,0.6], \\ [0.1,0.5], \\ (0.4,0.4,0.3) \end{matrix} \right\} &
\left\{ \begin{matrix} [0.1,0.4], \\ [0.1,0.4], \\ [0.2,0.6], \\ (0.2,0.3,0.4) \end{matrix} \right\} \\
\left\{ \begin{matrix} [0.1,0.4], \\ [0.1,0.4], \\ [0.1,0.4], \\ (0.2,0.2,0.2) \end{matrix} \right\} &
\left\{ \begin{matrix} [0.1,0.4], \\ [0.3,0.6], \\ [0.1,0.5], \\ (0.3,0.4,0.4) \end{matrix} \right\} &
\left\{ \begin{matrix} [0.1,0.3], \\ [0.1,0.5], \\ [0.1,0.3], \\ (0.2,0.3,0.2) \end{matrix} \right\} &
\left\{ \begin{matrix} [0.1,0.4], \\ [0.2,0.4], \\ [0.3,0.6], \\ (0.3,0.3,0.4) \end{matrix} \right\}
\end{array} \tag{58}
$$

(b) Then, the decision makers present their analysis in the form of a stable neutrosophic cubic set, according to Definitions 6 and 7 and Example 3:

$$D = \begin{array}{c} & G_1 & G_2 & G_3 & G_4 \\ H_1 & \left\{\begin{array}{c}(0.1,0.2),\\(0.3,0.1),\\(0.1,0.2)\end{array}\right\} & \left\{\begin{array}{c}(0.1,0.1),\\(0.1,0.1),\\(0.1,0.1)\end{array}\right\} & \left\{\begin{array}{c}(0.1,0.2),\\(0.1,0.2),\\(0.1,0.2)\end{array}\right\} & \left\{\begin{array}{c}(0.1,0.2),\\(0.1,0.1),\\(0.1,0.2)\end{array}\right\} \\ H_2 & \left\{\begin{array}{c}(0.1,0.1),\\(0.1,0.2),\\(0.1,0.3)\end{array}\right\} & \left\{\begin{array}{c}(0.2,0.1),\\(0.3,0.2),\\(0.1,0.2)\end{array}\right\} & \left\{\begin{array}{c}(0.1,0.2),\\(0.1,0.2),\\(0.2,0.1)\end{array}\right\} & \left\{\begin{array}{c}(0.1,0.1),\\(0.2,0.2),\\(0.2,0.1)\end{array}\right\} \\ H_3 & \left\{\begin{array}{c}(0.1,0.2),\\(0.1,0.2),\\(0.2,0.1)\end{array}\right\} & \left\{\begin{array}{c}(0.1,0.2),\\(0.1,0.1),\\(0.2,0.2)\end{array}\right\} & \left\{\begin{array}{c}(0.1,0.2),\\(0.1,0.2),\\(0.2,0.2)\end{array}\right\} & \left\{\begin{array}{c}(0.1,0.2),\\(0.2,0.1),\\(0.2,0.2)\end{array}\right\} \\ H_4 & \left\{\begin{array}{c}(0.1,0.2),\\(0.1,0.2),\\(0.1,0.2)\end{array}\right\} & \left\{\begin{array}{c}(0.2,0.1),\\(0.1,0.2),\\(0.3,0.1)\end{array}\right\} & \left\{\begin{array}{c}(0.1,0.1),\\(0.2,0.2),\\(0.1,0.1)\end{array}\right\} & \left\{\begin{array}{c}(0.2,0.1),\\(0.1,0.1),\\(0.1,0.2)\end{array}\right\} \end{array}. \tag{59}$$

Step 2. The normalized decision matrix is

$$\begin{array}{c} & G_1 & G_2 & G_3 & G_4 \\ H_1 & \left\{\begin{array}{c}(0.25,0.29),\\(0.5,0.143),\\(0.2,0.25)\end{array}\right\} & \left\{\begin{array}{c}(0.17,0.1),\\(0.17,0.17),\\(0.143,0.17)\end{array}\right\} & \left\{\begin{array}{c}(0.25,0.29),\\(0.2,0.25),\\(0.17,0.33)\end{array}\right\} & \left\{\begin{array}{c}(0.20,0.33),\\(0.17,0.20),\\(0.17,0.29)\end{array}\right\} \\ H_2 & \left\{\begin{array}{c}(0.25,0.143),\\(0.17,0.29),\\(0.2,0.38)\end{array}\right\} & \left\{\begin{array}{c}(0.33,0.1),\\(0.5,0.33),\\(0.143,0.33)\end{array}\right\} & \left\{\begin{array}{c}(0.25,0.29),\\(0.2,0.25),\\(0.33,0.17)\end{array}\right\} & \left\{\begin{array}{c}(0.20,0.17),\\(0.33,0.4),\\(0.33,0.143)\end{array}\right\} \\ H_3 & \left\{\begin{array}{c}(0.25,0.29),\\(0.17,0.29),\\(0.5,0.125)\end{array}\right\} & \left\{\begin{array}{c}(0.17,0.2),\\(0.17,0.17),\\(0.29,0.33)\end{array}\right\} & \left\{\begin{array}{c}(0.25,0.29),\\(0.2,0.25),\\(0.33,0.33)\end{array}\right\} & \left\{\begin{array}{c}(0.20,0.33),\\(0.33,0.20),\\(0.33,0.29)\end{array}\right\} \\ H_4 & \left\{\begin{array}{c}(0.25,0.29),\\(0.17,0.29),\\(0.2,0.25)\end{array}\right\} & \left\{\begin{array}{c}(0.33,0.1),\\(0.17,0.33),\\(0.43,0.17)\end{array}\right\} & \left\{\begin{array}{c}(0.25,0.143),\\(0.4,0.25),\\(0.17,0.17)\end{array}\right\} & \left\{\begin{array}{c}(0.40,0.17),\\(0.17,0.20),\\(0.17,0.29)\end{array}\right\} \end{array}. \tag{60}$$

Step 3. The weighted normalized decision matrix where $w = (0.3, 0.1, 0.2, 0.4)$ is

$$
\begin{array}{cccc}
G_1 & G_2 & G_3 & G_4 \\
H_1 \left\{\begin{array}{c} (0.075, 0.087), \\ (0.15, 0.043), \\ (0.06, 0.075) \end{array}\right\} & \left\{\begin{array}{c} (0.017, 0.01), \\ (0.017, 0.017), \\ (0.0143, 0.017) \end{array}\right\} & \left\{\begin{array}{c} (0.05, 0.06), \\ (0.04, 0.05), \\ (0.034, 0.066) \end{array}\right\} & \left\{\begin{array}{c} (0.08, 0.132), \\ (0.07, 0.08), \\ (0.07, 0.12) \end{array}\right\} \\
H_2 \left\{\begin{array}{c} (0.075, 0.043), \\ (0.051, 0.087), \\ (0.06, 0.114) \end{array}\right\} & \left\{\begin{array}{c} (0.033, 0.01), \\ (0.05, 0.033), \\ (0.0143, 0.033) \end{array}\right\} & \left\{\begin{array}{c} (0.05, 0.06), \\ (0.04, 0.05), \\ (0.066, 0.034) \end{array}\right\} & \left\{\begin{array}{c} (0.08, 0.07), \\ (0.132, 0.16), \\ (0.132, 0.06) \end{array}\right\} \\
H_3 \left\{\begin{array}{c} (0.075, 0.087), \\ (0.051, 0.087), \\ (0.15, 0.038) \end{array}\right\} & \left\{\begin{array}{c} (0.017, 0.02), \\ (0.017, 0.017), \\ (0.029, 0.033) \end{array}\right\} & \left\{\begin{array}{c} (0.05, 0.06), \\ (0.04, 0.05), \\ (0.066, 0.066) \end{array}\right\} & \left\{\begin{array}{c} (0.08, 0.132), \\ (0.132, 0.08), \\ (0.132, 0.12) \end{array}\right\} \\
H_4 \left\{\begin{array}{c} (0.075, 0.087), \\ (0.051, 0.087), \\ (0.06, 0.075) \end{array}\right\} & \left\{\begin{array}{c} (0.033, 0.01), \\ (0.017, 0.033), \\ (0.043, 0.017) \end{array}\right\} & \left\{\begin{array}{c} (0.05, 0.143), \\ (0.08, 0.05), \\ (0.034, 0.034) \end{array}\right\} & \left\{\begin{array}{c} (0.40, 0.07), \\ (0.07, 0.08), \\ (0.07, 0.12) \end{array}\right\}
\end{array} \tag{61}
$$

Step 4. Positive and negative ideal solution: the positive ideal solution $A^* = (a_1, a_2, a_3, a_4)$ contains the greatest numbers of the first, second, and third column and smallest numbers of the fourth column. The negative ideal solution $A\prime = (a_1', a_2', a_3', a_4')$ contains the smallest numbers of the first, second, and third column and greatest numbers of the fourth column.

$$
\begin{array}{l}
A^* \left\{\begin{array}{c} (0.075, 0.087), \\ (0.15, 0.087), \\ (0.15, 0.114) \end{array}\right\} \left\{\begin{array}{c} (0.033, 0.02), \\ (0.05, 0.033), \\ (0.029, 0.033) \end{array}\right\} \left\{\begin{array}{c} (0.05, 0.143), \\ (0.08, 0.05), \\ (0.066, 0.066) \end{array}\right\} \left\{\begin{array}{c} (0.08, 0.07), \\ (0.07, 0.07), \\ (0.07, 0.06) \end{array}\right\}, \\
A' \left\{\begin{array}{c} (0.075, 0.043), \\ (0.051, 0.043), \\ (0.06, 0.038) \end{array}\right\} \left\{\begin{array}{c} (0.017, 0.01), \\ (0.017, 0.017), \\ (0.0143, 0.017) \end{array}\right\} \left\{\begin{array}{c} (0.06, 0.05), \\ (0.04, 0.05), \\ (0.034, 0.034) \end{array}\right\} \left\{\begin{array}{c} (0.40, 0.132), \\ (0.132, 0.16), \\ (0.132, 0.12) \end{array}\right\}.
\end{array} \tag{62}
$$

Step 5. Separation measures for the positive and negative ideal solution are

$$
\begin{aligned}
a_1^* &= 0.3694, \\
a_2^* &= 0.2133, \\
a_3^* &= 0.0409, \\
a_4^* &= 0.1292, \\
a_1' &= 0.1308, \\
a_2' &= 0.1206, \\
a_3' &= 0.1236, \\
a_4' &= 0.0349.
\end{aligned} \tag{63}
$$

Step 6. Ranking order of the alternatives is shown by (Figures 1–4). Ranking of COVID-19 is obtained by completing the TOPSIS calculation.

$$
\begin{aligned}
H_1 &= 0.2615, \\
H_2 &= 0.3612, \\
H_3 &= 0.7514, \\
H_4 &= 0.2127, \\
H_3 &> H_2 > H_1 > H_4.
\end{aligned} \tag{64}
$$

Thus, we concluded that H_3 is the most effected province of Pakistan till April 12, 2020. Here, we used stable neutrosophic cubic sets, but we may use other versions of stable neutrosophic cubic sets.
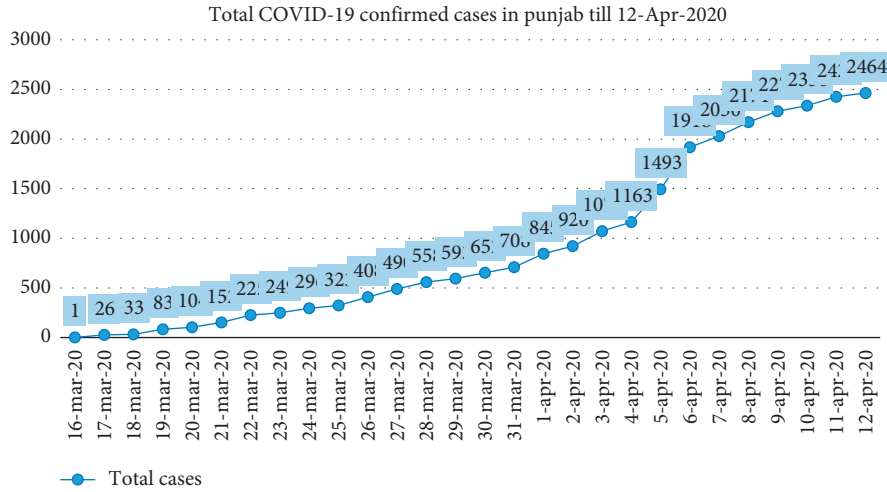
Total COVID-19 confirmed cases in punjab till 12-Apr-2020

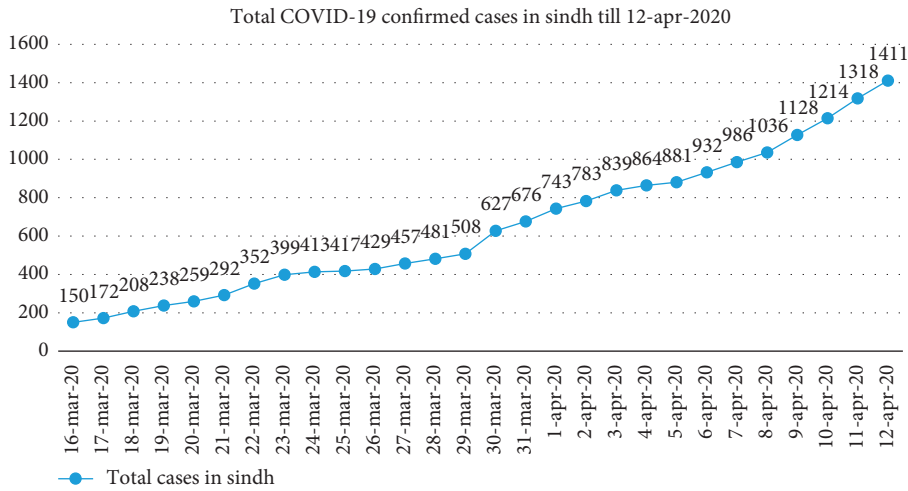Figure 1: Total COVID-19 confirmed cases in Punjab till 12 Apr 2020.

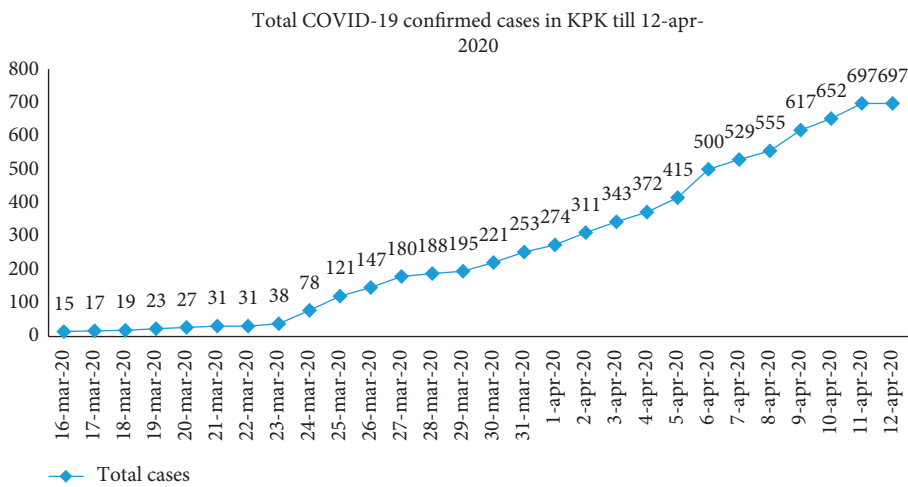Total COVID-19 confirmed cases in sindh till 12-apr-2020

Figure 2: Total COVID-19 confirmed cases in Sindh till 12 Apr 2020.
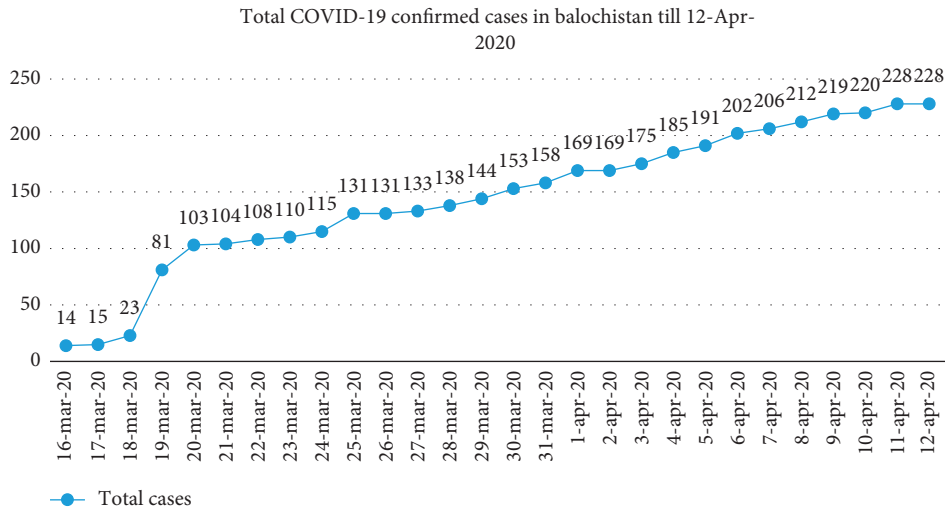
Total COVID-19 confirmed cases in KPK till 12-apr-2020

Figure 3: Total COVID-19 confirmed cases in KPK till 12 Apr 2020.

Total COVID-19 confirmed cases in balochistan till 12-Apr-2020

FIGURE 4: Total COVID-19 confirmed cases in Balochistan till 12 Apr 2020.

6. Conclusions

In this article, we work out with the idea of stable NCSs and internal and external stable NCSs. Also, we define their union, intersection, and complement with examples. After that, we demonstrate the application of the TOPSIS method to find out the ranking of COVID-19. For this purpose, we used a numerical example to find out the most affected area. We reached at the following key points:

Every stable NCS $\beta = \langle T_\beta, I_\beta, F_\beta, t_\beta, i_\beta, f_\beta \rangle$ in U is an almost-stable NCS, which is, of course, an NCS which turns into a cubic set with three different parts as truth, indeterminacy, and falsity, but the converse of this chain is not true always.

If we have an external NCS which is unstable such that

$$
\begin{aligned}
t(\mathring{u}) &> [T^-(\mathring{u}), T^+(\mathring{u})], i(\mathring{u}) \\
&> [I^-(\mathring{u}), I^+(\mathring{u})], f(\mathring{u}) > [F^-(\mathring{u}), F^+(\mathring{u})],
\end{aligned}
\tag{65}
$$

then its right evaluative point becomes a neutrosophic bipolar fuzzy set.

If we have an external NCS which is unstable such that

$$
\begin{aligned}
t(\mathring{u}) &< [T^-(\mathring{u}), T^+(\mathring{u})], i(\mathring{u}) < [I^-(\mathring{u}), I^+(\mathring{u})], f(\mathring{u}) \\
&< [F^-(\mathring{u}), F^+(\mathring{u})],
\end{aligned}
\tag{66}
$$

then its left evaluative point becomes a neutrosophic bipolar fuzzy set.

We used the idea of stable neutrosophic cubic sets in the application section, so results are within the range; otherwise, we may have results which lie outside the domain of neutrosophic cubic sets. This is the main advantage of stable neutrosophic cubic sets.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

Acknowledgments

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