The weighted distance measure based method to neutrosophic multi-attribute group decision making

Chunfang Liu[1,2], YueSheng Luo[1,3]
[1] *College of Science,*
Northeast Forestry University, 150040, Harbin, China
[2] *College of Automation,*
Harbin Engineering University, 150001, Harbin, China
[3] *College of Science,*
Harbin Engineering University, 150001, Harbin, China
liuchunfang1112@163.com

Abstract

Neutrosophic set (NS) is a generalization of fuzzy set (FS) that is designed for some practical situations in which each element has different truth membership function, indeterminacy membership function and falsity membership function. In this paper, we study the multi-attribute group decision making (MAGDM) problems under neutrosophic environment with the incompletely known or completely unknown attribute weight. We first define the single valued neutrosophic ideal-solution (SVNIS) and the weighted distance measure, and establish the program models to derive the attribute weights. Then we give a practical application in the framework of SVNS, the result shows that our method is reasonable and effective in dealing with decision making (DM) problems. Furthermore, we extend the method to interval valued neutrosophic set (IVNS).

Keywords : multi-attribute group decision making (MAGDM), the weighted distance measure based method; neutrosophic set (NS).

1 Introduction

Fuzzy set was introduced by Zadeh, which has been widely used in many aspects [1, 2]. On the basis of Zadeh's work, several high-order fuzzy sets have been proposed as an extension of fuzzy sets, including interval-valued fuzzy set, type-2 fuzzy set, type-n fuzzy set, soft set, rough set, intuitionistic fuzzy set, interval-valued intuitionistic fuzzy set, hesitant fuzzy set and neutrosophic set (NS) [2, 3, 4, 5, 6]. So far, the proposed high-order fuzzy sets have been successfully utilized in dealing with different uncertain problems, such as decision making [7], pattern recognition [8], etc.

As a generalization of fuzzy set, the NS was proposed by Smarandache [5] not only to deal with the decision information which is often incomplete, indeterminate and inconsistent but also include the truth membership degree, the falsity membership degree and the indeterminacy membership degree. For simplicity and practical application, Wang proposed the single valued NS (SVNS) and the interval valued NS (IVNS) which are the instances of NS and gave some operations on these sets [8, 9]. Since its appearance, many fruitful results have been appeared [23, 24]. On one hand, many researchers have proposed some aggregation operators of SVNS and INS and applied them to MADM problems [10, 11, 12, 25, 26]. On the other hand, some researchers have also proposed entropy and similarity measure of the SVNS and IVNS and applied them to MADM and pattern recognition [13, 14]. The above problems that related to the attribute weights are completely known. However, with the development of the information society and internet technology, the socio-economic environment gets more complex in many decision areas, such as capital investment decision making, medical diagnosis, personnel examination, etc. Only one decision maker cannot deal with the complex problems. Accordingly, it is necessary to gather multiple decision makers with different knowledge structures and experiences to conduct a group decision making. In some circumstances, it is difficult for the decision makers to give the information of the attribute weights correctly, which makes the attribute weights incompletely known or completely unknown. How to derive the attribute weights from the given neutrosophic information is an important topic. In intuitionistic fuzzy environments, many researchers have proposed some program models to obtain the incompletely known attribute weights or the completely unknown attribute weights, such as Xu proposed the deviation-based method [15], the ideal-solution based method [16], the group consensus-based method [17], Li proposed the consistency-based method [18], etc. Under the neutrosophic environment, Sahin proposed the maximizing deviation method [19]. Up to now, we found that there is no research of the weighted distance measure based method to neutrosophic multi-attribute group decision making. In this paper, we investigate the MAGDM problems which the information expressed by SVNS or IVNS, and the attribute weights are incompletely known or completely unknown.

The rest of the paper is organized as follows. In Section 2, we recall the concept of NS, SVNS, INS and their distance measures. In Section 3, we give the weighted distance measure based method to single valued neutrosophic set (SVNS). Furthermore, we extend the method to interval valued neutrosophic set (IVNS). Finally, a conclusion is given in Section 4.

2 Preliminaries

Definition 2.1 [5] Assume X be a universe of discourse with a generic element in X denoted by x. A NS A on X is defined by a truth membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are defined by

$$\begin{aligned} T_A(x) &: X \to]0^-, 1^+[\\ I_A(x) &: X \to]0^-, 1^+[\\ F_A(x) &: X \to]0^-, 1^+[\end{aligned}$$

where $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

For similarity and practical application, Wang proposed the SVNS and IVN-S which are the subclasses of NS and preserve all the operations on NS. In the following part, we recall SVNS and IVNS and their distance measure, respectively.

Definition 2.2 [8] Assume X be a universe of discourse with a generic element in X denoted by x. A single valued neutrosophic set (SVNS) A on X is defined by a truth membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are defined by

$$\begin{aligned} T_A(x) &: X \to [0,1] \\ I_A(x) &: X \to [0,1] \\ F_A(x) &: X \to [0,1] \end{aligned}$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are subsets of $[0,1]$, and satisfy $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

For similarity, we utilize $A = \{T_A(x), I_A(x), F_A(x)\}$ to denote a SVNS A in the following part. If X has only one element, for convenience, we call A a single valued neutrosophic number (SVNN) and denoted by $A = \{T_A, I_A, F_A\}$.

Definition 2.3 [20] Let $A_1 = \{T_1, I_1, F_1\}$, $A_2 = \{T_2, I_2, F_2\}$ be two SVNNs, the normalized Hamming distance measure between A_1 and A_2 is defined by

$$d(A_1, A_2) = \frac{1}{3}(|T_1 - T_2| + |I_1 - I_2| + |F_1 - F_2|) \tag{1}$$

Definition 2.4 [9] Assume X be a universe of discourse with a generic element in X denoted by x, and $\mathrm{int}[0,1]$ be the set of all closed subsets of $[0,1]$. An interval valued neutrosophic set (IVNS) A on X is defined by a truth membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are defined by

$$\begin{aligned} T_A(x) &: X \to \mathrm{int}[0,1] \\ I_A(x) &: X \to \mathrm{int}[0,1] \\ F_A(x) &: X \to \mathrm{int}[0,1] \end{aligned}$$

with the condition $0 \leq \mathrm{Sup}T_A(x) + \mathrm{Sup}I_A(x) + \mathrm{Sup}F_A(x) \leq 3$.

Here we denote $T_A(x) = [T_A^-(x), T_A^+(x)]$, $I_A(x) = [I_A^-(x), I_A^+(x)]$, $F_A(x) = [F_A^-(x), F_A^+(x)]$. For convenience, we call A an interval valued neutrosophic number (IVNN) and denoted by $A = \{[T_A^-, T_A^+], [I_A^-, I_A^+], [F_A^-, F_A^+]\}$.

Definition 2.5 [14] Let $A_1 = \{[T_1^-, T_1^+], [I_1^-, I_1^+], [F_1^-, F_1^+]\}$,
$A_2 = \{[T_2^-, T_2^+], [I_2^-, I_2^+], [F_2^-, F_2^+]\}$ be two IVNNs, the normalized Hamming distance measure between A_1 and A_2 is defined by

$$\begin{aligned} d(A_1, A_2) = \ & \tfrac{1}{6}(|T_1^- - T_2^-| + |T_1^+ - T_2^+| + |I_1^- - I_2^-| \\ & + |I_1^+ - I_2^+| + |F_1^- - F_2^-| + |F_1^+ - F_2^+|) \end{aligned} \tag{2}$$

3 The weighted distance measure based method to neutrosophic set

3.1 The weighted distance measure based method to single-valued neutrosophic set

Let $X = \{X_1, X_2, \ldots, X_m\}$ be a set of alternatives, $C = \{C_1, C_2, \ldots, C_n\}$ be a set of attributes and $w = \{w_1, w_2, \ldots, w_n\}$ be the weight vector of the attribute with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Suppose that there are s decision makers $D = \{D_1, D_2, \ldots, D_s\}$, whose corresponding weighted vector is $\lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_s\}$. Let $A_k = (r_{ij}^k)_{m \times n}$ $(k = 1, 2, \ldots, s)$ be single valued neutrosophic decision matrix, where $r_{ij}^k = \{T_{ij}^k, I_{ij}^k, F_{ij}^k\}$ is the value of the attribute, expressed by SVNNs.

In MADM environments, the ideal point is used to help the identification of the best alternative in the decision set. Although the ideal point does not exist in real world, it does provide an effective way to evaluate the best alternative. Now we suppose the ideal SVNN as $\alpha_j^* = \{t^*, i^*, f^*\} = \{1, 0, 0\}$. Based on the ideal SVNN, we define the single valued neutrosophic positive ideal-solution (SVNPIS).

Definition 3.1 Let $\alpha_j^* = \{1, 0, 0\}$ $(j = 1, 2, \ldots, n)$ be n ideal SVNNs, then a SVNPIS is defined by

$$A^* = \{\alpha_1^*, \alpha_2^*, \ldots, \alpha_n^*\}.$$

Definition 3.2 Let $A_i^k = \{r_{i1}^k, r_{i2}^k, \ldots, r_{in}^k\}$ $(i = 1, 2, \ldots, m)$ be the i th alternative of the kth decision makers $(k = 1, 2, \ldots, s)$, $A^* = \{\alpha_1^*, \alpha_2^*, \ldots, \alpha_n^*\}$ be the SVNPIS, then the weighted Hamming distance measure (WHDM) between A_i and A^* is defined by

$$d(A_i, A^*) = \sum_{k=1}^{s} \lambda_k \sum_{j=1}^{n} w_j d(r_{ij}^k, \alpha_j^*). \tag{3}$$

3.1.1 Incompletely known attribute weights

In the decision making process, the incomplete information of the attribute weight provided by the decision makers can usually be constructed using several basic ranking forms [21]. Let H be the set of information about the incompletely known attribute weights, which may be constructed in the following forms [22], for $i \neq j$:
(a) A weak ranking:$\{w_i \geq w_j\}$;
(b) A strict ranking: $\{w_i - w_j \geq \delta_i (> 0)\}$;
(c) A ranking with multiples: $\{w_i \geq \delta_i w_j\}, 0 \leq \delta_i \leq 1$;
(d) An interval form: $\{\delta_i \leq w_i \leq \delta_i + \varepsilon_i\}, 0 \leq \delta_i \leq \delta_i + \varepsilon_i$;
(e) A ranking of differences: $\{w_i - w_j \geq w_k - w_l\}$, for $j \neq k \neq l$.
We now establish the following single-objective programming model based on the weighted distance measure method:

$$(\mathrm{M1}) \left\{ \begin{array}{l} \mathrm{Min} f(w) = \sum_{k=1}^{s} \lambda_k \sum_{i=1}^{m} \sum_{j=1}^{n} w_j d(r_{ij}^k, \alpha_j^*) \\ s.t. w_j \in H, \sum_{j=1}^{n} w_j = 1, w_j \geq 0, j = 1, 2, \ldots, n. \end{array} \right.$$

where λ_k is the weight of the decision maker $D_k(k = 1, 2, \ldots, s)$ and

$$d(r_{ij}^k, \alpha_j^*) = \frac{1}{3}(|T_{ij}^k - 1| + I_{ij}^k + F_{ij}^k) \tag{4}$$

$d(r_{ij}^k, \alpha_j^*)$ represents the weighted distance measure between the attribute value r_{ij}^k and the SVNPIS α_j^*. The desirable weight vector $w = (w_1, w_2, \ldots, w_n)$ should make the sum of all the weighted distance measure (3) small. So we construct this model to make the overall distance small.
By solving the model (M1) with Matlab software, we get the optimal solution $w^* = (w_1^*, w_2^*, \ldots, w_n^*)$, which is considered as the weight of the attributes $C_1, C_2, \ldots, C_n$. Then we utilize $d(A_i, A^*)$ to rank all the alternatives. The smaller the weighted distance measure, the better the alternative.

3.1.2 Completely unknown attribute weights

If the information about the attribute weight is completely unknown, we establish the following programming model:

$$(\mathrm{M2}) \left\{ \begin{array}{l} \mathrm{Min} f(w) = \sum_{k=1}^{s} \lambda_k \sum_{i=1}^{m} \sum_{j=1}^{n} w_j d(r_{ij}^k, \alpha_j^*) \\ s.t. \sum_{j=1}^{n} w_j^2 = 1, w_j \geq 0, j = 1, 2, \ldots, n. \end{array} \right.$$

To solve this model, we construct the Lagrange function as follows:

$$L(w, \lambda) = \sum_{k=1}^{s} \lambda_k \sum_{i=1}^{m} \sum_{j=1}^{n} w_j d(r_{ij}^k, \alpha_j^*) + \frac{\lambda}{2}(\sum_{j=1}^{n} w_j^2 - 1) \tag{5}$$

where λ is the Lagrange multiplier.
Differentiating (5) with respect to $w_j(j = 1, 2, \ldots, n)$ and λ, setting these partial derivatives equal to zero, the following set of the equations are obtained:

$$\left\{ \begin{array}{l} \frac{\partial L}{\partial w_j} = \sum_{k=1}^{s} \lambda_k \sum_{i=1}^{m} d(r_{ij}^k, \alpha_j^*) + w_j \lambda = 0 \\ \frac{\partial L}{\partial \lambda} = \sum_{j=1}^{n} w_j^2 = 1 \end{array} \right. \tag{6}$$

By solving Eq.(6), we obtain the weight w_j and normalize it with $w_j^* = \frac{w_j}{\sum_{j=1}^{n} w_j}$, then we get

$$w_j^* = \frac{\sum_{k=1}^{s} \lambda_k \sum_{i=1}^{m} d(r_{ij}^k, \alpha_j^*)}{\sum_{j=1}^{n} \sum_{k=1}^{s} \lambda_k \sum_{i=1}^{m} d(r_{ij}^k, \alpha_j^*)} \tag{7}$$

we get the optimal solution $w^* = (w_1^*, w_2^*, \ldots, w_n^*)$, which is considered as the weight of the attributes $C_1, C_2, \ldots, C_n$. Later, we calculate the distance measure (3) and get the most desirable one.

3.1.3 Ilustrative example

Example 1. Here we choose the decision making problem adapted from [19]. An automotive company is desired to select the most appropriate supplier for one of the key elements in its manufacturing process. After pre-evaluation, four suppliers have remained as alternatives for further evaluation. In order to evaluate alternative suppliers, a committee composed of four decision makers has been formed. The committee selects four attributes to evaluate the alternatives: (1)C_1: product quality, (2)C_2: relationship closeness, (3)C_3: delivery performance, (4)C_4: price. Suppose that there are four decision-makers, denoted by d_1, d_2, d_3, d_4, whose corresponding weight vector is $\lambda = (0.25, 0.25, 0.25, 0.25)$. The four possible alternatives are to be evaluated under these four attributes and are in the form of SVNNs for each decision-maker, as shown in the following single valued neutrosophic decision matrix:

$$D_1 = \begin{bmatrix} \{0.4,0.2,0.3\} & \{0.4,0.2,0.3\} & \{0.2,0.2,0.5\} & \{0.7,0.2,0.3\} \\ \{0.6,0.1,0.2\} & \{0.6,0.1,0.2\} & \{0.5,0.2,0.3\} & \{0.5,0.1,0.2\} \\ \{0.3,0.2,0.3\} & \{0.5,0.2,0.3\} & \{0.1,0.5,0.2\} & \{0.1,0.4,0.5\} \\ \{0.7,0.2,0.1\} & \{0.6,0.1,0.2\} & \{0.4,0.3,0.2\} & \{0.4,0.5,0.1\} \end{bmatrix}$$

$$D_2 = \begin{bmatrix} \{0.1,0.3,0.5\} & \{0.5,0.1,0.5\} & \{0.3,0.1,0.6\} & \{0.4,0.1,0.4\} \\ \{0.2,0.5,0.4\} & \{0.3,0.4,0.3\} & \{0.2,0.3,0.1\} & \{0.2,0.3,0.5\} \\ \{0.5,0.2,0.6\} & \{0.2,0.4,0.3\} & \{0.5,0.2,0.5\} & \{0.1,0.5,0.3\} \\ \{0.2,0.4,0.2\} & \{0.1,0.1,0.3\} & \{0.1,0.5,0.4\} & \{0.5,0.3,0.1\} \end{bmatrix}$$

$$D_3 = \begin{bmatrix} \{0.3,0.2,0.1\} & \{0.3,0.1,0.3\} & \{0.1,0.4,0.5\} & \{0.2,0.3,0.5\} \\ \{0.6,0.1,0.4\} & \{0.6,0.4,0.2\} & \{0.5,0.4,0.1\} & \{0.5,0.2,0.4\} \\ \{0.3,0.3,0.6\} & \{0.4,0.2,0.4\} & \{0.2,0.3,0.2\} & \{0.3,0.5,0.1\} \\ \{0.3,0.6,0.1\} & \{0.5,0.3,0.2\} & \{0.3,0.3,0.6\} & \{0.4,0.3,0.2\} \end{bmatrix}$$

$$D_4 = \begin{bmatrix} \{0.2,0.2,0.3\} & \{0.3,0.2,0.3\} & \{0.2,0.3,0.5\} & \{0.4,0.2,0.5\} \\ \{0.4,0.1,0.2\} & \{0.6,0.3,0.5\} & \{0.1,0.2,0.2\} & \{0.5,0.1,0.2\} \\ \{0.3,0.5,0.1\} & \{0.2,0.2,0.3\} & \{0.5,0.4,0.3\} & \{0.5,0.3,0.2\} \\ \{0.3,0.1,0.1\} & \{0.2,0.1,0.4\} & \{0.2,0.3,0.2\} & \{0.3,0.1,0.6\} \end{bmatrix}$$

Case 1. Incompletely known attribute weights

Suppose the incompletely known information of the attribute weight is given as follows:

$$H = \{0.18 \le w_1 \le 0.2, 0.15 \le w_2 \le 0.25, 0.30 \le w_3 \le 0.35, 0.3 \le w_4 \le 0.4, \sum_{j=1}^{4} w_j = 1\}.$$

Step 1. By model (M1), we establish the following model:

$$\begin{cases} \text{Min} f(w) = 1.5833w_1 + 1.5038w_2 + 1.825w_3 + 1.625w_4 \\ s.t. w \in H \end{cases}$$

Step 2. By solving this model with Matlab software, we get the weight vector:

$$w_1 = 0.18, w_2 = 0.22, w_3 = 0.30, w_4 = 0.30.$$

Step 3. Use the distance measure (3), we have

$$d(A_1, A^*) = 0.4365, d(A_2, A^*) = 0.3618, d(A_3, A^*) = 0.4502, d(A_4, A^*) = 0.4033.$$

Step 4. Rank the alternatives.

Since $d(A_3, A^*)$ is the biggest, and $d(A_2, A^*)$ is the smallest, we rank the alternatives as follows:

$$A_2 \succ A_4 \succ A_1 \succ A_3,$$

where $\succ$ indicates the relationship superior or preferred to, and A_2 is the best alternative.
Case 2. Completely unknown attribute weights

Step 1. By model (M2), we establish the following model:

$$\begin{cases} \text{Min} f(w) = 1.5833w_1 + 1.5038w_2 + 1.825w_3 + 1.625w_4 \\ s.t. \sum_{j=1}^{4} w_j^2 = 1, w_j \geq 0, j = 1, 2, 3, 4. \end{cases}$$

Step 2. Use Eq. (7) to obtain the weight vector of attributes:

$$w_1^* = 0.18, w_2^* = 0.22, w_3^* = 0.30, w_4^* = 0.30.$$

Step 3. Use the distance measure (4) and (5), we have

$$d(A_1, A^*) = 0.4352, d(A_2, A^*) = 0.3613, d(A_3, A^*) = 0.4482, d(A_4, A^*) = 0.3984.$$

Step 4. Rank the alternatives. Since $d(A_3, A^*)$ is the biggest, and $d(A_2, A^*)$ is the smallest, we rank the alternatives as follows:

$$A_2 \succ A_4 \succ A_1 \succ A_3,$$

where $\succ$ indicates the relationship superior or preferred to, and A_2 is the best alternative.

3.2 The weighted distance measure based method to interval valued neutrosophic set

Let $X = \{X_1, X_2, \ldots, X_m\}$ be a set of alternatives, $C = \{C_1, C_2, \ldots, C_n\}$ be a set of attributes and $w = \{w_1, w_2, \ldots, w_n\}$ be the weight vector of the attribute with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Suppose that there are s decision makers $D = \{D_1, D_2, \ldots, D_s\}$, whose corresponding weighted vector is $\lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_s\}$. Let $A_k = (r_{ij}^k)_{m\times n}$ $(k = 1, 2, \ldots, s)$ be interval valued neutrosophic decision matrix, where $r_{ij}^k = \{T_{ij}^k, I_{ij}^k, F_{ij}^k\}$ is the value of the attribute, expressed by IVNNs.

Now we suppose the ideal IVNN as $\beta_j^* = \{t^*, i^*, f^*\} = \{[1, 1], [0, 0], [0, 0]\}$. Based on the ideal IVNN, we define the interval valued neutrosophic positive ideal-solution (IVNPIS).

Definition 3.3 Let $\beta_j^* = \{[1, 1], [0, 0], [0, 0]\}$ $(j = 1, 2, \ldots, n)$ be n ideal IVNNs, then a IVNPIS is defined by

$$A^* = \{\beta_1^*, \beta_2^*, \ldots, \beta_n^*\}.$$

Definition 3.4 Let $A_i = \{r_{i1}, r_{i2}, \ldots, r_{in}\}$ $(i = 1, 2, \ldots, m)$ be the i th alternative, $A^* = \{\beta_1^*, \beta_2^*, \ldots, \beta_n^*\}$ be the IVNPIS, then the weighted Hamming distance measure (WHDM) between A_i and A^* is defined by

$$d(A_i, A^*) = \sum_{k=1}^{s} \lambda_k \sum_{j=1}^{n} w_j d(r_{ij}^k, \beta_j^*) \tag{8}$$

3.2.1 Incompletely known attribute weights

We now establish the following single-objective programming model based on the weighted distance measure method:

$$(\mathrm{M3})\begin{cases}\mathrm{Min}f(w)=\sum_{k=1}^{s}\lambda_k\sum_{i=1}^{m}\sum_{j=1}^{n}w_j d(r_{ij}^k,\beta_j^*)\\ s.t. w_j\in H,\sum_{j=1}^{n}w_j=1,w_j\geq 0,j=1,2,\ldots,n.\end{cases}$$

where λ_k is the weight of the decision maker D_k $(k=1,2,\ldots,s)$ and

$$d(r_{ij}^k,\beta_j^*)=\frac{1}{6}(|T_{ij}^{-k}-1|+|T_{ij}^{+k}-1|+|I_{ij}^{-k}|+|I_{ij}^{+k}|+|F_{ij}^{-k}|+|F_{ij}^{+k}|) \tag{9}$$

$d(r_{ij}^k,\beta_j^*)$ represents the distance measure between the attribute value r_{ij}^k and the IVNPIS β_j^*. The desirable weight vector $w=(w_1,w_2,\ldots,w_n)$ should make the sum of all the weighted distance (8) small. So we construct this model to make the overall distances small. The smaller the WHD, the better the alternative. We use (8) to rank the alternative.

By solving the model (M3) with Matlab software, we get the optimal solution $w^*=(w_1^*,w_2^*,\ldots,w_n^*)$, which is considered as the weight of the attributes $C_1,C_2,\ldots,C_n$. Then we utilize $d(A_i,A^*)$ to rank all the alternatives. The smaller the distance, the better the alternative.

3.2.2 Completely unknown attribute weights

If the information about the attribute weight is completely unknown, we establish the following programming model:

$$(\mathrm{M4})\begin{cases}\mathrm{Min}f(w)=\sum_{k=1}^{s}\lambda_k\sum_{i=1}^{m}\sum_{j=1}^{n}w_j d(r_{ij}^k,\beta_j^*)\\ s.t. \sum_{j=1}^{n}w_j^2=1,w_j\geq 0,j=1,2,\ldots,n.\end{cases}$$

By Lagrange multiple method, we get the completely unknown weight w_j and normalize it with $w_j^*=\frac{w_j}{\sum_{j=1}^{n}w_j}$ as follows:

$$w_j^*=\frac{\sum_{k=1}^{s}\lambda_k\sum_{i=1}^{m}d(r_{ij}^k,\beta_j^*)}{\sum_{j=1}^{n}\sum_{k=1}^{s}\lambda_k\sum_{i=1}^{m}d(r_{ij}^k,\beta_j^*)} \tag{10}$$

which is considered as the weight of the attributes C_j. Later, we calculate the distance measure $d(A_i,A^*)$, and then get the most desirable one.

3.2.3 Ilustrative example

Example 2. The decision making problem is adapted from [19]. Suppose an organization plans to implement ERP system. The first step is to form a project team that consists of CIO and two senior representatives from user departments. By collecting all information about ERP vendors and systems, project team chooses four potential ERP systems $A_i(i=1,2,3,4)$ as candidates. The company employs some external professional organizations (experts) to aid this decision making. The project team selects four attributes to evaluate the alternatives: (1)C_1: function and technology, (2)C_2: strategic fitness, (3)C_3: vendors ability, (4)C_4: vendors reputation. Suppose that there are three decision-makers, denoted by D_1,D_2,D_3, whose corresponding weight vector is $\lambda=(\frac{1}{3},\frac{1}{3},\frac{1}{3})$. The four possible alternatives are to be evaluated under these four attributes and are in the form of IVNNs for each decision-maker, as shown in the following interval valued neutrosophic decision matrix:

$$D_1 = \begin{bmatrix} \{\{[0.4,0.5],[0.2,0.3],[0.3,0.5]\} & \{[0.3,0.4],[0.3,0.6],[0.2,0.4]\} & \{[0.2,0.5],[0.2,0.6],[0.3,0.5]\} \\ \{[0.5,0.6],[0.3,0.5],[0.2,0.5]\}\} & & \\ \{\{[0.6,0.7],[0.1,0.2],[0.2,0.3]\} & \{[0.1,0.3],[0.1,0.4],[0.2,0.5]\} & \{[0.4,0.5],[0.2,0.5],[0.3,0.7]\} \\ \{[0.2,0.4],[0.1,0.4],[0.3,0.3]\}\} & & \\ \{\{[0.3,0.4],[0.2,0.3],[0.3,0.4]\} & \{[0.3,0.6],[0.2,0.3],[0.2,0.5]\} & \{[0.2,0.7],[0.2,0.4],[0.3,0.6]\} \\ \{[0.2,0.6],[0.4,0.7],[0.2,0.7]\}\} & & \\ \{\{[0.2,0.6],[0.1,0.2],[0.1,0.2]\} & \{[0.2,0.5],[0.4,0.5],[0.1,0.6]\} & \{[0.3,0.5],[0.1,0.3],[0.2,0.2]\} \\ \{[0.4,0.4],[0.1,0.6],[0.1,0.5]\}\} & & \end{bmatrix}$$

$$D_2 = \begin{bmatrix} \{\{[0.4,0.6],[0.1,0.3],[0.2,0.4]\} & \{[0.3,0.5],[0.1,0.4],[0.3,0.4]\} & \{[0.4,0.5],[0.2,0.4],[0.1,0.3]\} \\ \{[0.3,0.6],[0.3,0.6],[0.3,0.6]\}\} & & \\ \{\{[0.3,0.5],[0.1,0.2],[0.2,0.3]\} & \{[0.3,0.4],[0.2,0.2],[0.1,0.3]\} & \{[0.2,0.7],[0.3,0.5],[0.3,0.6]\} \\ \{[0.2,0.5],[0.2,0.7],[0.1,0.2]\}\} & & \\ \{\{[0.5,0.6],[0.2,0.3],[0.3,0.4]\} & \{[0.1,0.4],[0.1,0.3],[0.3,0.5]\} & \{[0.5,0.5],[0.4,0.6],[0.3,0.4]\} \\ \{[0.1,0.2],[0.1,0.4],[0.5,0.6]\}\} & & \\ \{\{[0.3,0.4],[0.1,0.2],[0.1,0.3]\} & \{[0.3,0.3],[0.1,0.5],[0.2,0.4]\} & \{[0.2,0.3],[0.4,0.5],[0.5,0.6]\} \\ \{[0.3,0.3],[0.2,0.3],[0.1,0.4]\}\} & & \end{bmatrix}$$

$$D_3 = \begin{bmatrix} \{\{[0.1,0.3],[0.2,0.3],[0.4,0.5]\} & \{[0.3,0.3],[0.1,0.3],[0.3,0.4]\} & \{[0.2,0.6],[0.3,0.5],[0.3,0.5]\} \\ \{[0.4,0.6],[0.3,0.4],[0.2,0.3]\}\} & & \\ \{\{[0.3,0.6],[0.3,0.5],[0.3,0.5]\} & \{[0.3,0.4],[0.3,0.4],[0.3,0.5]\} & \{[0.3,0.5],[0.2,0.4],[0.1,0.5]\} \\ \{[0.1,0.2],[0.3,0.5],[0.3,0.4]\}\} & & \\ \{\{[0.4,0.5],[0.2,0.4],[0.2,0.4]\} & \{[0.2,0.3],[0.1,0.1],[0.3,0.4]\} & \{[0.1,0.4],[0.2,0.6],[0.3,0.6]\} \\ \{[0.4,0.5],[0.2,0.6],[0.1,0.3]\}\} & & \\ \{\{[0.2,0.4],[0.3,0.4],[0.1,0.3]\} & \{[0.1,0.4],[0.2,0.5],[0.1,0.5]\} & \{[0.3,0.6],[0.2,0.4],[0.2,0.2]\} \\ \{[0.2,0.4],[0.3,0.3],[0.2,0.6]\}\} & & \end{bmatrix}$$

Case 1. Incompletely known attribute weights
Suppose the incompletely known information of the attribute weight is given as follows:
$H = \{0.18 \le w_1 \le 0.2, 0.15 \le w_2 \le 0.25, 0.30 \le w_3 \le 0.35, 0.3 \le w_4 \le 0.4, \sum_{j=1}^{4} w_j = 1\}$

Step 1. By model (M3), we establish the following model:

$$\begin{cases} \text{Min} f(w) = 1.4278w_1 + 1.7278w_2 + 1.8278w_3 + 1.7667w_4 \\ s.t. w \in H \end{cases}$$

Step 2. By solving this model with Matlab software, we get the weight vector:

$$w_1 = 0.18, w_2 = 0.25, w_3 = 0.20, w_4 = 0.37.$$

Step 3. Use the distance measure (9) and (10), we have

$$d(A_1, A^*) = 0.4204, d(A_2, A^*) = 0.4182, d(A_3, A^*) = 0.4471, d(A_4, A^*) = 0.41.$$

Step 4. Rank the alternatives.
Since $d(A_3, A^*)$ is the biggest, and $d(A_4, A^*)$ is the smallest, we rank the alternatives as follows:

$$A_4 \succ A_2 \succ A_1 \succ A_3,$$

where $\succ$ indicates the relationship superior or preferred to, and A_4 is the best alternative.
Case 2. Completely unknown attribute weights

Step 1. By model (M4), we establish the following model:

$$\begin{cases} \text{Min} f(w) = 1.4278w_1 + 1.7278w_2 + 1.8278w_3 + 1.7667w_4 \\ s.t. \sum_{j=1}^{4} w_j^2 = 1, w_j \geq 0, j = 1, 2, 3, 4. \end{cases}$$

Step 2. Use Eq. (10) to obtain the weight vector of attributes:

$$w_1^* = 0.2115, w_2^* = 0.2560, w_3^* = 0.2708, w_4^* = 0.2617.$$

Step 3. Use the distance measure (9) and (10), we have

$$d(A_1, A^*) = 0.4200, d(A_2, A^*) = 0.3776, d(A_3, A^*) = 0.4421, d(A_4, A^*) = 0.4054.$$

Step 4. Rank the alternatives.
Since $d(A_3, A^*)$ is the biggest, and $d(A_2, A^*)$ is the smallest, we rank the alternatives as follows:

$$A_2 \succ A_4 \succ A_1 \succ A_3,$$

where $\succ$ indicates the relationship superior or preferred to, and A_2 is the best alternative.

3.3 Comparative analysis

Considering the proposed method and the maximizing deviation method proposed by Sahin, there exsit some differences. In Sahins method, they calculated the distance measure of all the attributes and assign a small weight to the attribute which has a similar effect among the alternatives, then they used the weighted aggregation operators and the score functions to rank the alternatives; while, the proposed method calculates the distance measure between the attributes and the ideal solution, and obtain the weight that make the weighted distance measure small, we then use the weighted distance measure to rank the alternatives which avoid the complex calculation of aggregation operators processing. The two methods are all effective to deal with the incompletely known or completely unknown attribute weight by solve the program models. The advantage of the proposed method is that calculation is simple and convenient, which can deal with the MAGDM problem effectively.

4 Conclusion

In this paper, we investigate the multi-attribute group decision making problems expressed with neutrosophic set and the attribute weights are incompletely known or completely unknown. We first define the single valued neutrosophic ideal solution (SVNIS), and then establish the optimal models to derive the attribute weight. Furthermore, an approach to MAGDM within the framework of SVNS is developed, and the result shows that our approach is reasonable and effective in dealing with decision making problems. Finally, we extend the method to IVNS.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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