Hindawi
Mathematical Problems in Engineering
Volume 2019, Article ID 3258018, 12 pages
https://doi.org/10.1155/2019/3258018

Research Article

Three-Way Decisions with Single-Valued Neutrosophic Decision Theory Rough Sets Based on Grey Relational Analysis

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Received 28 April 2019; Revised 7 August 2019; Accepted 31 August 2019; Published 3 October 2019

Academic Editor: Giuseppe D'Aniello

The single-valued neutrosophic set (SVNS) can not only depict imperfect information in the real decision system but also handle undetermined and inconformity information flexibly and effectively. Three-way decisions (3WDs) are often used as an effective method to deal with uncertainties, but the conditional probability is given by the decision maker subjectively, which makes the decision result too subjective. This paper proposes a novel model based on 3WDs to settle the multiattribute decision-making (MADM) problems, where the attribute values are described by SVNS, and the attribute weights are entirely unknown. At first, we build a single-valued neutrosophic decision theory rough set (SVNDTRS) model based on Bayesian decision process. Then, we use the analytic hierarchy process (AHP) approach to calculate the subjective weight of each attribute, the information entropy to obtain the attribute's objective weight, and the minimum total deviation approach to determine the combined weight of the attributes. After obtaining the standard weight, the grey relational analysis (GRA) method is utilized to calculate the grey correlation closeness with the ideal solution, and the conditional probability is estimated by it. In addition, we develop a decision-making method in view of the ideal solution of 3WDs with the SVNS. This approach not only considers the lowest cost but also gives a corresponding semantic explanation for the decision result of each alternative, which can supplement the decision results of GRA. At last, we illustrate the feasibility and effectiveness of 3WDs through an example of supplier selection and compare it with other methods to verify the advantages of our approach.

1. Introduction

Multiattribute decision making (MADM) is more and more momentous for modern decision science. Its essence is to use the existing decision information to sort and optimize a limited number of alternatives in a certain way. Due to the complexity and unpredictability of the external environment, the ambiguity of the object itself, the limitations of human cognition, and the subjectivity of the decision maker, decision makers usually need to provide preference information through various types of attribute values. Since Zadeh [1] introduced the concepts of the fuzzy sets (FSs), the FSs have been widely studied. Atanassov [2] put forward the intuitionistic fuzzy sets (IFSs) by adding the nonmembership degree based on the traditional FSs. IFSs consist of membership degree and nonmembership degree, and they can more easily express fuzzy information and have been rapidly developed and widely used since they were introduced. However, the degree of hesitation in IFSs cannot be defined separately. Therefore, even if IFSs can effectively describe imperfect information, they are less flexible when dealing with uncertain and inconformity information. Then, the clearly quantified neutrosophic sets (NSs) can describe the value of the proposition between true and false, which was initially proposed by Smarandache [3]. NSs are made up of membership degree, hesitancy degree, and nonmembership degree. In addition, Wang et al. [4] proposed a subcategory of NS called single-valued neutrosophic sets (SVNSs) and discussed its related rules and properties. The trait of SVNS is that membership degree, hesitancy degree, and nonmembership degree are mutually independent; all three are between 0 and 1, and the sum of them is between 0 and 3. Deli and Şubaş [5] developed a sorting method and extended it into MADM problems. Wang et al. [6, 7, 8] introduced a MADM approach in view of Maclaurin symmetric mean (MSM) operator and TODIM for SVNS. Sodenkamp et al. [9] used SVNS to process independent multisource undetermined measurements, which affected the dependability of expert evaluation in MADM problems.

In many actual MADM problems, owing to the uncertainty or imperfect of information, it is difficult to adopt a method that only accepts and rejects these two decisions. Through the expansion of the two decisions, Yao [10, 11] proposed three-way decisions (3WDs) involving acceptance, rejection, and delayed decision making [12, 13, 14]. The principle of the 3WDs is derived from the probability rough set. In light of the positive, boundary, and negative domains of the probabilistic rough set, the 3WD model including acceptance decision, delayed decision, and rejection decision is established. So far, 3WDs are widely used in some areas such as influenza emergency management, granular computing, enterprise evaluation, and social networking [13, 15, 16, 17, 18]. At the same time, many theoretical results have been achieved in the study of 3WDs. For example, Zhang et al. [19] considered a new risk measurement function by utility theory and derived a 3WD model with DTRS. Sun et al. [20] introduced a decision-theoretic rough fuzzy set model with linguistic information based on 3WDs and applied it to MADMs. Zhang et al. [21] proposed a dynamic 3WD model and proved the model is practicable and valid. According to the TODIM method, Hu et al. [22] constructed a new 3WD model and demonstrated its application in online diagnosis and medical selection.

In the 3WD model, the loss function (LF) is a key parameter. The scholars have studied a great deal of 3WD rules based on LFs of diverse forms, such as interval number [23], IFSs [24], intuitionistic uncertain linguistic variables [25], dual hesitant FSs [26], and Pythagorean FSs [27]. SVNS can handle uncertain, incomplete, and inconsistent information more flexibly. To this end, we use the SVNS to express the LF in this paper. Furthermore, how to determine the conditional probability is also the key to the 3WD method. The conditional probability in many references is subjectively given by the decision makers, which makes the decision results too subjective. Liang et al. [27] used the TOPSIS method to evaluate the conditional probability by calculating the distance between each alternative and the positive or negative ideal solution, which provides a new perspective for us to identify the conditional probability. In this paper, we measure the maximum grey correlation of each alternative with the relative neutrosophic positive ideal solution (RNPIS) and the minimum grey correlation with the relative neutrosophic negative ideal solution (RNNIS) and estimate the conditional probability using the grey relational analysis (GRA) method [28–30].

In conclusion, SVNS can handle uncertain, incomplete, and inconsistent information more flexibly, and 3WDs are often used as an effective method to deal with uncertainties, but the conditional probability in many references is given by the decision maker subjectively, which makes the decision result too subjective. Therefore, we use the GRA method to calculate conditional probability. The goal and motivation of this paper are (1) to extend 3WDs to the environment of SVNS, using SVNS to represent the LF in 3WDs; (2) to propose the SVNDTRS model and explore its properties; and (3) to use the GRA method to calculate conditional probability in 3WDs. The proposed method extends the use environment of 3WDs and provides a new idea for the determination of conditional probability in 3WDs.

The remainder of this paper is arranged as follows. In Section 2, we briefly review the basics of the NSs and SVNSs. In Section 3, we propose a method to determine the combination weight of attributes. In Section 4, we propose a single-valued neutrosophic decision theory rough set (SVNDTRS) model and its propositions. In Section 5, we estimate the conditional probability of 3WDs based on the GRA method and presented a MADM method to deal with SVNSs based on the SVNDTRS. In Section 6, we use a numerical example to demonstrate the availability, and other methods are compared and analyzed. In Section 7, we reach the conclusion.

2. SVNS

In this section, we introduce some basic concepts of the NSs and the SVNSs.

Definition 1 [3]. Let X be an object set, and the common elements of X are represented by x. A NS Y of X consists of $T_Y^+(x)$, $I_Y^+(x)$ and $F_Y^+(x)$. Y can be represented as $Y = \{T_Y^+(x), I_Y^+(x), F_Y^+(x) \mid x \in X\}$, where $T_Y^+(x)$, $I_Y^+(x)$, and $F_Y^+(x)$ represent the membership degree, hesitancy degree, and nonmembership degree, respectively. $T_Y^+(x), I_Y^+(x), F_Y^+(x) \in [^-0, 1^+]$ and satisfies $^-0 \leq T_Y^+(x) + I_Y^+(x) + F_Y^+(x) \leq 3^+$.

Definition 2 [4]. Let X be an object set, and the common elements of X are represented by x. A SVNS $\widetilde{Y}$ of X consists of $T_{\widetilde{Y}}(x)$, $I_{\widetilde{Y}}(x)$ and $F_{\widetilde{Y}}(x)$. $\widetilde{Y}$ can be represented as $\widetilde{Y} = \{T_{\widetilde{Y}}(x), I_{\widetilde{Y}}(x), F_{\widetilde{Y}}(x) \mid x \in X\}$, where $T_{\widetilde{Y}}(x)$, $I_{\widetilde{Y}}(x)$, and $F_{\widetilde{Y}}(x)$ represent the membership degree, hesitancy degree, and nonmembership degree, respectively. $T_{\widetilde{Y}}(x), I_{\widetilde{Y}}(x), F_{\widetilde{Y}}(x) \in [0, 1]$ and satisfies $0 \leq T_{\widetilde{Y}}(x) + I_{\widetilde{Y}}(x) + F_{\widetilde{Y}}(x) \leq 3$. Let $(T_{\widetilde{Y}}(x), I_{\widetilde{Y}}(x), F_{\widetilde{Y}}(x))$ be the single-valued neutrosophic number (SVNN) and abbreviated as $x = (T_x, I_x, F_x)$.

Definition 3 [31]. For any two SVNNs $x_m = (T_m, I_m, F_m)$ and $x_n = (T_n, I_n, F_n)$, the related operations are defined as follows:

$$
\begin{aligned}
x_m + x_n &= (T_m + T_n - T_m T_n, I_m I_n, F_m F_n),\\
x_m \times x_n &= (T_m T_n, I_m + I_n - I_m I_n, F_m + F_n - F_m F_n),\\
k x_m &= \left(1 - (1 - T_m)^k, (I_m)^k, (F_m)^k\right), \quad k > 0\\
(x_m)^k &= \left((T_m)^k, (1 - (1 - I_m))^k, (1 - (1 - F_m))^k\right), \quad k > 0.
\end{aligned}
\tag{1}
$$

Table 1: Loss functions.

	$\Gamma(P)$	$\neg\Gamma(N)$
a_P	$\lambda_{PP} = (T_{PP}, I_{PP}, F_{PP})$	$\lambda_{PN} = (T_{PN}, I_{PN}, F_{PN})$
a_B	$\lambda_{BP} = (T_{BP}, I_{BP}, F_{BP})$	$\lambda_{BN} = (T_{BN}, I_{BN}, F_{BN})$
a_N	$\lambda_{NP} = (T_{NP}, I_{NP}, F_{NP})$	$\lambda_{NN} = (T_{NN}, I_{NN}, F_{NN})$

Definition 4 [32]. The complement set of a SVNS $\widetilde{Y}$ is $\widetilde{Y}^c$, which is defined by

$$\begin{aligned} T_{\widetilde{Y}^c}(x) &= F_{\widetilde{Y}}(x); \\ I_{\widetilde{Y}^c}(x) &= 1 - I_{\widetilde{Y}}(x); \\ F_{\widetilde{Y}^c}(x) &= T_{\widetilde{Y}}(x). \end{aligned} \tag{2}$$

Definition 5 [31]. Let $x = (T_x, I_x, F_x)$ be a SVNN; the cosine similarity $S(x)$ of x is described as follows:

$$S(x) = \frac{T(x)}{\sqrt{T(x) + I(x) + F(x)}}. \tag{3}$$

Definition 6 [31]. For any two SVNNs $x_m = (T_m, I_m, F_m)$ and $x_n = (T_n, I_n, F_n)$, if $S(x_m) \le S(x_n)$, then $x_m \le x_n$.

Definition 7 [32]. Let $x_m = (T_m, I_m, F_m)$ and $x_n = (T_n, I_n, F_n)$ be any two SVNNs; the Hamming distance between two SVNNs x_m and x_n is described as follows:

$$d(x_m, x_n) = \left|T_m - T_n\right| + \left|I_m - I_n\right| + \left|F_m - F_n\right|. \tag{4}$$

The normalized Hamming distance between two SVNNs x_m and x_n is described as follows:

$$d_N(x_m, x_n) = \frac{1}{3}\left(\left|T_m - T_n\right| + \left|I_m - I_n\right| + \left|F_m - F_n\right|\right), \tag{5}$$

where $0 \le d(x_m, x_n) \le 3$, $0 \le d_N(x_m, x_n) \le 1$.

3. Basic Model of SVNDTRS

In this section, we introduce a model of SVNDTRS based on 3WDs. At first, we use SVNN to build a LF matrix. The loss of different decision schemes under different state variables is illustrated in Table 1.

For the Bayesian decision [33, 34], the decision-making process is described by the state set and action set. The state set is described by $\Omega = \{\Gamma, \neg\Gamma\}$ and indicates whether the element x is in Γ. Among them, Γ is the set of objects for correct classification and $\neg\Gamma$ is the set of objects for wrong classification. This action set is represented by $A = \{a_P, a_B, a_N\}$, where a_P, a_B, and a_N signify the decision actions that divide an object x into positive, boundary, and negative domains. The positive domain signifies that x belongs to Γ, that is, the accept event objects. The negative domain signifies that x does not belong to Γ, that is, the reject event objects. The boundary domain signifies whether the uncertainty x belongs to Γ, that is, it does not promise or delay the decision event objects. In addition, the parameters $\lambda_{mn}(m = P, B, N; n = P, N)$ describe the LFs. λ_{PP} and λ_{PN} signify the costs of correct classification and error classification of the object x in accepted decision; λ_{BP} and λ_{BN} signify the costs of correct classification and error classification of the object x in delayed decision; and λ_{NP} and λ_{NN} signify the costs of correct classification and error classification of the object x in rejected decision.

$$T_{PP} \le T_{BP} < T_{NP}, \tag{6}$$

$$I_{NP} \le I_{BP} < I_{PP}, \tag{7}$$

$$F_{NP} \le F_{BP} < F_{PP}, \tag{8}$$

$$T_{NN} \le T_{BN} < T_{PN}, \tag{9}$$

$$I_{PN} \le I_{BN} < I_{NN}, \tag{10}$$

$$F_{PN} \le F_{BN} < F_{NN}. \tag{11}$$

Moreover, new constraints should be added as follows:

$$\begin{aligned} 0 &\le T_{PP} + I_{PP} + F_{PP} \le 3, \\ 0 &\le T_{BP} + I_{BP} + F_{BP} \le 3, \\ 0 &\le T_{NP} + I_{NP} + F_{NP} \le 3, \\ 0 &\le T_{PN} + I_{PN} + F_{PN} \le 3, \\ 0 &\le T_{BN} + I_{BN} + F_{BN} \le 3, \\ 0 &\le T_{NN} + I_{NN} + F_{NN} \le 3. \end{aligned} \tag{12}$$

According to references [35, 36], (6)–(11) are the prerequisites for SVNDTRS. By taking advantage of the conclusions of Definitions 5-6, the following proposition can be depicted.

Proposition 1. *According to conditions (6)–(11), we can get the following relationship:*

$$\begin{aligned} \lambda_{PP} &\le \lambda_{BP} < \lambda_{NP}, \\ \lambda_{NN} &\le \lambda_{BN} < \lambda_{PN}. \end{aligned} \tag{13}$$

For the matter x belonging to Γ, the loss caused by demarcating it into positive domain will not be greater than the loss of dividing it into boundary domain. The loss of both is not more than that caused by dividing it into negative domain. Similarly, for the matter x which belongs to $\neg\Gamma$, the loss caused by falling it into negative domain will not be greater than that caused by bringing it to boundary domain, both of them are less than that caused by brings it to positive domain.

Let $P(\Gamma \mid [x])$ be the conditional probability that x belongs to Γ, and $P(\neg\Gamma \mid [x])$ is the conditional probability that x belongs to $\neg\Gamma$. Therefore, we can get $P(\Gamma \mid [x]) + P(\neg\Gamma \mid [x]) = 1$. By using the Bayesian risk decision theory

[24, 33, 34], for the matter x, the expected losses $R(a_i \mid [x])\,(i = P, B, N)$ from taking different operations are indicated as follows:

$$R(a_P \mid [x]) = \lambda_{PP} P(\Gamma \mid [x]) \oplus \lambda_{PN} P(\neg\Gamma \mid [x]), \quad (14)$$

$$R(a_B \mid [x]) = \lambda_{BP} P(\Gamma \mid [x]) \oplus \lambda_{BN} P(\neg\Gamma \mid [x]), \quad (15)$$

$$R(a_N \mid [x]) = \lambda_{NP} P(\Gamma \mid [x]) \oplus \lambda_{NN} P(\neg\Gamma \mid [x]). \quad (16)$$

According to the computing principle of SVNNs, (14)–(16) can be computed as follows:

$$R(a_P \mid [x]) = \left(1-(1-T_{PP})^{P(\Gamma|[x])}, I_{PP}^{P(\Gamma|[x])}, F_{PP}^{P(\Gamma|[x])}\right) \oplus \left(1-(1-T_{PN})^{P(\neg\Gamma|[x])}, I_{PN}^{P(\neg\Gamma|[x])}, F_{PN}^{P(\neg\Gamma|[x])}\right), \quad (17)$$

$$R(a_B \mid [x]) = \left(1-(1-T_{BP})^{P(\Gamma|[x])}, I_{BP}^{P(\Gamma|[x])}, F_{BP}^{P(\Gamma|[x])}\right) \oplus \left(1-(1-T_{BN})^{P(\neg\Gamma|[x])}, I_{BN}^{P(\neg\Gamma|[x])}, F_{BN}^{P(\neg\Gamma|[x])}\right), \quad (18)$$

$$R(a_N \mid [x]) = \left(1-(1-T_{NP})^{P(\Gamma|[x])}, I_{NP}^{P(\Gamma|[x])}, F_{NP}^{P(\Gamma|[x])}\right) \oplus \left(1-(1-T_{NN})^{P(\neg\Gamma|[x])}, I_{NN}^{P(\neg\Gamma|[x])}, F_{NN}^{P(\neg\Gamma|[x])}\right). \quad (19)$$

For the expected losses of (17)–(19), we can get the following propositions.

Proposition 2. *In accordance with the algorithms of SVNSs, the expected losses $R(a_i \mid [x])\,(i = P, B, N)$ can be recounted as follows:*

$$\begin{aligned}
R(a_P \mid [x]) &= \left(1-(1-T_{PP})^{P(\Gamma|[x])}(1-T_{PN})^{P(\neg\Gamma|[x])}, I_{PP}^{P(\Gamma|[x])} I_{PN}^{P(\neg\Gamma|[x])}, F_{PP}^{P(\Gamma|[x])} F_{PN}^{P(\neg\Gamma|[x])}\right),\\
R(a_B \mid [x]) &= \left(1-(1-T_{BP})^{P(\Gamma|[x])}(1-T_{BN})^{P(\neg\Gamma|[x])}, I_{BP}^{P(\Gamma|[x])} I_{BN}^{P(\neg\Gamma|[x])}, F_{BP}^{P(\Gamma|[x])} F_{BN}^{P(\neg\Gamma|[x])}\right),\\
R(a_N \mid [x]) &= \left(1-(1-T_{NP})^{P(\Gamma|[x])}(1-T_{NN})^{P(\neg\Gamma|[x])}, I_{NP}^{P(\Gamma|[x])} I_{NN}^{P(\neg\Gamma|[x])}, F_{NP}^{P(\Gamma|[x])} F_{NN}^{P(\neg\Gamma|[x])}\right).
\end{aligned} \quad (20)$$

Proof. For the expected losses $R(a_P \mid [x])$, we first denote that $l_1 = (1-T_{PP})^{P(\Gamma|[x])}$, $l_2 = (1-T_{PN})^{P(\neg\Gamma|[x])}$, $m_1 = I_{PP}^{P(\Gamma|[x])}$, $m_2 = I_{PN}^{P(\neg\Gamma|[x])}$, $n_1 = F_{PP}^{P(\Gamma|[x])}$, and $n_2 = F_{PN}^{P(\neg\Gamma|[x])}$. According to (17)–(19) and SVNN rules of operation, $R(a_P \mid [x])$ is computed as follows:

$$\begin{aligned}
R(a_P \mid [x]) &= \left(1-(1-T_{PP})^{P(\Gamma|[x])}, I_{PP}^{P(\Gamma|[x])}, F_{PP}^{P(\Gamma|[x])}\right)\\
&\quad \oplus \left(1-(1-T_{PN})^{P(\neg\Gamma|[x])}, I_{PN}^{P(\neg\Gamma|[x])}, F_{PN}^{P(\neg\Gamma|[x])}\right)\\
&= (1-l_1, m_1, n_1) \oplus (1-l_2, m_2, n_2)\\
&= ((1-l_1)+(1-l_2)-(1-l_1)(1-l_2), m_1 m_2, n_1 n_2)\\
&= (1-l_1 l_2, m_1 m_2, n_1 n_2)\\
&= \left(1-(1-T_{PP})^{P(\Gamma[x])}(1-T_{PN})^{P(\neg\Gamma|[x])}, I_{PP}^{P(\Gamma|[x])} I_{PN}^{P(\neg\Gamma|[x])}, F_{PP}^{P(\Gamma|[x])} F_{PN}^{P(\neg\Gamma|[x])}\right).
\end{aligned} \quad (21)$$

Similarly, the expected losses $R(a_B \mid [x])$ and $R(a_N \mid [x])$ can be proved.

Proposition 3. *Assuming that $P(C \mid [x])$ is an invariant constant with $P(\Gamma \mid [x]) + P(\neg\Gamma \mid [x] = 1$, $T_{R(a_i|x)} = 1-(1-T_{iP})^{P(\Gamma|[x])}(1-T_{iN})^{P(\neg\Gamma|[x])}$ is nonmonotonically increasing as T_{iP} and $T_{iN}\,(i = P, B, N)$ increase.*

Proof. Let $x_{P_1} = (T_{iP_1}, I_{iP_1}, F_{iP_1})$ and $x_{P_2} = (T_{iP_2}, I_{iP_2}, F_{iP_2})$ be any two SVNNs, and $T_{iP_1} < T_{iP_2}$, $I_{iP_1} < I_{iP_2}$, and $F_{iP_1} < F_{iP_2}$. Let $x_{N_1} = (T_{iN_1}, I_{iN_1}, F_{iN_1})$ and $x_{N_2} = (T_{iN_2}, I_{iN_2}, F_{iN_2})$ be any two SVNNs, and $T_{iN_1} < T_{iN_2}$, $I_{iN_1} < I_{iN_2}$, and $F_{iN_1} < F_{iN_2}$.

Since $T_{iP_1} < T_{iP_2}$ and $T_{iN_1} < T_{iN_2}$, we have $1-T_{iP_1} > 1-T_{iP_2}$ and $1-T_{iN_1} > 1-T_{iN_2}$; then, we obtain $(1-T_{iP_1})^{P(\Gamma|[x])} > (1-T_{iP_2})^{P(\Gamma|[x])}$ and $(1-T_{iN_1})^{P(\neg\Gamma|[x])} > (1-T_{iN_2})^{P(\neg\Gamma|[x])}$.

Furthermore, we have $(1-T_{iP_1})^{P(\Gamma|[x])}(1-T_{iN_1})^{P(\neg\Gamma|[x])} > (1-T_{iP_2})^{P(\Gamma|[x])}(1-T_{iN_2})^{P(\neg\Gamma|[x])}$.

Finally, we have $1-(1-T_{iP_1})^{P(\Gamma|[x])}(1-T_{iN_1})^{P(\neg\Gamma|[x])} < 1-(1-T_{iP_2})^{P(\Gamma|[x])}(1-T_{iN_2})^{P(\neg\Gamma|[x])}$.

Proposition 4. *Assuming that $P(\Gamma \mid [x])$ is an invariant constant with $P(\Gamma \mid [x]) + P(\neg\Gamma \mid [x]) = 1$, $I_{R(a_i|x)} = I_{iP}^{P(\Gamma|[x])} I_{iN}^{P(\neg\Gamma|[x])}$ is nonmonotonically increasing as I_{iP} and I_{iN} $(i = P, B, N)$ increase.*

Proof. Since $I_{iP_1} < I_{iP_2}$ and $I_{iN_1} < I_{iN_2}$, we have $I_{iP_1}^{P(\Gamma|[x])} < I_{iP_2}^{P(\Gamma|[x])}$ and $I_{iN_1}^{P(\neg\Gamma|[x])} < I_{iN_2}^{P(\neg\Gamma|[x])}$; then, we obtain $I_{iP_1}^{P(\Gamma|[x])} I_{iN_1}^{P(\neg\Gamma|[x])} < I_{iP_2}^{P(\Gamma|[x])} I_{iN_2}^{P(\neg\Gamma|[x])}$.

Proposition 5. *Assuming that $P(\Gamma \mid [x])$ is an invariant constant with $P(\Gamma[x]) + P(\neg\Gamma \mid [x] = 1$, $F_{R(a_i|x)} = F_{iP}^{P(\Gamma[x])} F_{iN}^{P(\neg\Gamma|[x])}$ is nonmonotonically increasing as F_{iP} and F_{iN} $(i = P, B, N)$ increase.*

Proof. Since $F_{iP_1} < F_{iP_2}$ and $F_{iN_1} < F_{iN_2}$, we have $F_{iP_1}^{P(\Gamma|[x])} < F_{iP_2}^{P(\Gamma|[x])}$ and $F_{iN_1}^{P(\neg\Gamma|[x])} < F_{iN_2}^{P(\neg\Gamma|[x])}$; then, we obtain $F_{iP_1}^{P(\Gamma|[x])} F_{iN_1}^{P(\neg\Gamma|[x])} < F_{iP_2}^{P(\Gamma|[x])} F_{iN_2}^{P(\neg\Gamma|[x])}$.

Table 2: Scaling and the meaning of AHP.

Scaling	Meaning
0.1	C_k is extremely less important than C_j
0.3	$C_k$is slightly less important than C_j
0.5	C_k is as important as C_j
0.7	C_k is slightly more important than C_j
0.9	C_k is extremely more important than C_j
0.2, 0.4, 0.6, 0.8	The median of the adjacent judgments

Based on the results discussed in [15], the action plan with the lowest decision-making risk always is chosen as the first choice, and the 3WD rules can be expressed as

$$\begin{aligned}
&(P): \text{If } R(a_P \mid [x]) \le R(a_B \mid [x] \text{ and } R(a_P \mid [x]) \le R(a_N \mid [x]), \quad \text{then } x \in \mathrm{POS}(\Gamma),\\
&(B): \text{If } R(a_B \mid [x]) \le R(a_P \mid [x]) \text{ and } R(a_B \mid [x]) \le R(a_N \mid [x]), \quad \text{then } x \in \mathrm{BND}(\Gamma),\\
&(N): \text{If } R(a_N \mid [x]) \le R(a_P \mid [x]) \text{ and } R(a_N \mid [x]) \le R(a_B \mid [x]), \quad \text{then } x \in \mathrm{NEG}(\Gamma).
\end{aligned} \tag{22}$$

4. Establishing the Weights

In fact, in most MADM processes, due to the pressure of time, the limitation of expertise, and the lack of data in problem areas, we often encounter weights of each attribute that are unknown or partly known. Because many existing methods are not suitable for solving such problems, two types of methods are generally used to determine weights. One is subjective weighting, such as the point estimation method and AHP method; the other is objective weighting, such as the deviation maximization method and entropy method [28]. The subjective weight method gives the subjective weight according to the individual preferences of decision makers. While applying the objective weighting method, the subjective judgment of the decision maker is neglected, although the data are fully utilized. Therefore, these two approaches have limitations. This paper synthesized these two methods and proposed an overall merit model based on the AHP method and entropy method to optimize the integration weights.

4.1. AHP Method. Suppose that the complementary judgment matrix (the scale is 0.1~0.9) given by the decision maker for each attribute in the solution set is $Q = (q_{jk})_{n\times n}$:

$$Q = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ q_{n1} & q_{n2} & \cdots & q_{nn} \end{bmatrix}. \tag{23}$$

The judgment matrix is constructed by using the 0.1–0.9 five-scale method proposed by Du [37]. The scale values of q_{jk} are shown in Table 2.

The properties are shown as follows:

$$\begin{aligned} q_{jk} &> 0,\\ q_{jk} + q_{kj} &= 1,\\ q_{kk} &= 0.5. \end{aligned} \tag{24}$$

Then, we get the subjective weights $\mu_j\,(j = 1, 2, \ldots, n)$ by the following formulas [8]:

$$\mu_j = \frac{\sum_{k=1}^{n} q_{jk} + (n/2) - 1}{n(n-1)}. \tag{25}$$

Then, we get the subjective weight vector $\mu = (\mu_1, \mu_2, \ldots, \mu_n)^T$ of attribute $C_j\,(j = 1, 2, \ldots, n)$ with $\mu_j \ge 0$ and $\sum_{j=1}^{n} \mu_j = 1$.

Of course, (25) is right only when the complementary judgment matrix has consistency.

4.2. Entropy Method. The entropy method can dynamically mine the effective information provided by data, and its objective way of empowerment can measure the importance of evaluation index. It determines the corresponding weight by calculating the size of the entropy measure [29].

The entropy measure [29] of a SVNS $\widetilde{Y} = \{T_{\widetilde{Y}}(x_i), I_{\widetilde{Y}}(x_i), F_{\widetilde{Y}}(x_i) \mid x \in X\}$ is

$$E_i(\widetilde{Y}) = 1 - \frac{\sum_{i=1}^{m} \left(T_{\widetilde{Y}}(x_i) + F_{\widetilde{Y}}(x_i)\right)\left|I_{\widetilde{Y}}(x_i) - I_{\widetilde{Y}}^{c}(x_i)\right|}{m}. \tag{26}$$

The properties are as follows [29]:

(1) $E_i(\widetilde{Y}) = 0$, if $\widetilde{Y}$ is a clear set and $I_{\widetilde{Y}}(x_i) = 0, \forall x \in X$

(2) $E_i(\widetilde{Y}) = 1$ if $(T_{\widetilde{Y}}(x_i), I_{\widetilde{Y}}(x_i), F_{\widetilde{Y}}(x_i)) = (0.5, 0.5, 0.5)$, $\forall x \in X$

(3) $E_i(\widetilde{Y}) \ge E_i(\widetilde{\Upsilon})$ if $\widetilde{Y}$ is more deterministic than $\widetilde{\Upsilon}$, i.e., $T_{\widetilde{Y}}(x_i) + F_{\widetilde{Y}}(x_i) \le T_{\widetilde{\Upsilon}}(x_i) + F_{\widetilde{\Upsilon}}(x_i)$ and $|I_{\widetilde{Y}}(x_i) - I_{\widetilde{Y}^c}(x_i)| \le |I_{\widetilde{\Upsilon}}(x_i) - I_{\widetilde{\Upsilon}^c}(x_i)|$

(4) $E_i(\widetilde{Y}) = E_i(\widetilde{Y}^c)$, $\forall x \in X$

The entropy value for C_j can be denoted as

$$E_j = 1 - \frac{\sum_{i=1}^{m}\left(T_{ij}(x_i) + F_{ij}(x_i)\right)\left|I_{ij}(x_i) - I_{ij}^{c}(x_i)\right|}{m},$$

$$i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n. \tag{27}$$

The entropy weight [28] ω_j is represented by

$$\omega_j = \frac{1 - E_j}{\sum_{j=1}^{n}\left(1 - E_j\right)} = \frac{\sum_{i=1}^{m}\left(T_{ij}(x_i) + F_{ij}(x_i)\right)\left|I_{ij}(x_i) - I_{ij}^{c}(x_i)\right|}{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\left(T_{ij}(x_i) + F_{ij}(x_i)\right)\left|I_{ij}(x_i) - I_{ij}^{c}(x_i)\right|\right)}, \quad j = 1, 2, \ldots, n. \tag{28}$$

Then, we get the objective weight vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ of attribute $C_j\,(j = 1, 2, \ldots, n)$ with $\omega_j \geq 0$ and $\sum_{j=1}^{n}\omega_j = 1$.

4.3. Determining the Combination Weight. Attribute weight is of great significance to MADM, which directly affects the accuracy of decision making. In order to make an accurate and scientific decision, it is necessary to consider the subjective preferences of decision makers and strive to reduce the subjective arbitrariness of weights. Besides, the objective information of decision objects is fully utilized to achieve the unity of subjectivity and objectivity. Therefore, it is of great significance to combine the subjective and objective weights reasonably to form a combination weight. We use the method of minimum total deviation [28] to determine the combination weight. The core idea of the optimal combination weight model is that the weight deviation obtained by various weighting methods should be as small as possible [38, 39]. To this end, the following basic models can be established:

$$\begin{cases} \min \sum_{i=1}^{m}\sum_{j=1}^{n}\left\{\left[\left(w_j - \omega_j\right)S\left(\widetilde{Y}_{ij}\right)\right]^2 + \left[\left(w_j - \mu_j\right)S\left(\widetilde{Y}_{ij}\right)\right]^2\right\} \\ \text{s.t.} \\ \sum_{j=1}^{n} w_j = 1, \quad w_j \geq 0. \end{cases} \tag{29}$$

The Lagrange multiplier function is constructed by solving the model:

$$L\left(w_j, \eta\right) = \sum_{i=1}^{m}\sum_{j=1}^{n}\left\{\left[\left(w_j - \omega_j\right)S\left(\widetilde{Y}_{ij}\right)\right]^2 + \left[\left(w_j - \mu_j\right)S\left(\widetilde{Y}_{ij}\right)\right]^2\right\} + 2\eta\left(\sum_{j=1}^{n} w_j - 1\right), \tag{30}$$

and we have

$$\begin{cases} \dfrac{\partial L\left(w_j, \eta\right)}{\partial w_j} = \sum_{i=1}^{m} 2\left[2w_j - \left(\mu_j + \omega_j\right)\right]\left(S\left(\widetilde{Y}_{ij}\right)\right)^2 + 2\eta = 0, \\ \dfrac{\partial L\left(w_j, \eta\right)}{\partial \eta} = 2\left(\sum_{j=1}^{n} w_j - 1\right) = 0. \end{cases} \tag{31}$$

Then, we obtain

$$\begin{cases} \eta = \dfrac{\sum_{j=1}^{n}\left(\left(\mu_j + \omega_j\right)\sum_{i=1}^{m}\left(S\left(\widetilde{Y}_{ij}\right)\right)^2\right) - 2}{n}, \\ w_j = \dfrac{\left(\left(\mu_j + \omega_j\right)\sum_{i=1}^{m}\left(S\left(\widetilde{Y}_{ij}\right)\right)^2\right) - \left(\left(\sum_{j=1}^{n}\left(\left(\mu_j + \omega_j\right)\sum_{i=1}^{m}\left(S\left(\widetilde{Y}_{ij}\right)\right)^2\right) - 2\right)/(n)\right)}{2}. \end{cases} \tag{32}$$

Then we get the weight vector $W = (w_1, w_2, \ldots, w_n)^T$ of attribute C_j $(j = 1, 2, \ldots, n)$ with $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$.

5. Decision Analysis for 3WDs with SVNDTRS

In the existing 3WDs models, the LF is an important parameter. How to use the appropriate information form to represent the LF is very important. In addition to describing imperfect information in real decision-making system, SVNSs can also handle uncertain and inconformity information flexibly and effectively. Furthermore, how to determine the conditional probability is also the key to 3WDs. The conditional probability in many documents is subjectively given by the decision makers, making the decision results too subjective. The GRA method provides a unique perspective for the assessment of conditional probability. According to the above presentation, we constructed a new SVNDTRS model based on the GRA method. In this section, we mainly describe the decision rules of the SVNDTRS model and how to determine the conditional probability by the GRA method.

5.1. Basic Rules of SVNDTRSs. In light of the results of Definitions 5-6 and (P)-(N), we give the decision rules (P1)-(N1) as follows:

$$\text{(P1)}: \text{ if } S(R(a_P \mid [x])) \leq S(R(a_B \mid [x])) \text{ and } S(R(a_P \mid [x])) \leq S(R(a_N \mid [x])), \quad \text{then } x \in \mathrm{POS}(\Gamma), \tag{33}$$

$$\text{(B1)}: \text{ if } S(R(a_B \mid [x])) \leq S(R(a_P \mid [x])) \text{ and } S(R(a_B \mid [x])) \leq S(R(a_N \mid [x])), \quad \text{then } x \in \mathrm{BND}(\Gamma), \tag{34}$$

$$\text{(N1)}: \text{ if } S(R(a_N \mid [x])) \leq S(R(a_P \mid [x])) \text{ and } S(R(a_N \mid [x])) \leq S(R(a_B \mid [x])), \quad \text{then } x \in \mathrm{NEG}(\Gamma), \tag{35}$$

where the similar degree $S(R(a_i \mid [x]))\,(i = P, B, N)$ of the expected losses is calculated as

$$S(R(a_i \mid [x])) = \frac{T(R(a_i \mid [x]))}{\sqrt{T(R(a_i \mid [x])) + I(R(a_i \mid [x])) + F(R(a_i \mid [x]))}}, \tag{36}$$

$$S(R(a_P \mid [x])) = \frac{1-(1-T_{PP})^{P(\Gamma|[x])}(1-T_{PN})^{P(\neg\Gamma|[x])}}{\sqrt{\left(1-(1-T_{PP})^{P(\Gamma|[x])}(1-T_{PN})^{P(\neg\Gamma|[x])}\right)^2+\left(I_{PP}^{P(\Gamma|[x])}I_{PN}^{P(\neg\Gamma|[x])}\right)^2+\left(F_{PP}^{P(\Gamma|[x])}F_{PN}^{P(\neg\Gamma|[x])}\right)^2}}, \tag{37}$$

$$S(R(a_B \mid [x])) = \frac{1-(1-T_{BP})^{P(\Gamma|[x])}(1-T_{BN})^{P(\neg\Gamma|[x])}}{\sqrt{\left(1-(1-T_{BP})^{P(\Gamma|[x])}(1-T_{BN})^{P(\neg\Gamma|[x])}\right)^2+\left(I_{BP}^{P(\Gamma|[x])}I_{BN}^{P(\neg\Gamma|[x])}\right)^2+\left(F_{BP}^{P(\Gamma|[x])}F_{BN}^{P(\neg\Gamma|[x])}\right)^2}}, \tag{38}$$

$$S(R(a_N \mid [x])) = \frac{1-(1-T_{NP})^{P(\Gamma|[x])}(1-T_{NN})^{P(\neg\Gamma|[x])}}{\sqrt{\left(1-(1-T_{NP})^{P(\Gamma|[x])}(1-T_{NN})^{P(\neg\Gamma|[x])}\right)^2+\left(I_{NP}^{P(\Gamma|[x])}I_{NN}^{P(\neg\Gamma|[x])}\right)^2+\left(F_{NP}^{P(\Gamma|[x])}F_{NN}^{P(\neg\Gamma|[x])}\right)^2}}. \tag{39}$$

5.2. Computing Conditional Probability of SVNDTRS with GRA Method. Let $\widetilde{Y} = (\widetilde{Y}_{ij})_{m\times n} = (T_{ij}, I_{ij}, F_{ij})_{m\times n}$ be a SVNS-based decision matrix, where T_{ij}, I_{ij}, and F_{ij} represent the membership degree, hesitancy degree, and nonmembership degree of evaluation for the attribute C_j with respect to the alternative A_i.

Firstly, we determine the relative RNPIS X^+ and the RNNIS X^-.

The RNPIS X^+ can be defined as

$$X^+ = \{x_1^+, x_2^+, \ldots, x_n^+\}, \tag{40}$$

where $x_j^+ = (T_j^+, I_j^+, F_j^+) = (\max_{1\leq i\leq m} T_{ij}, \min_{1\leq i\leq m} I_{ij}, \min_{1\leq i\leq m} F_{ij})$ for $j = 1, 2, \ldots, n$.

And the RNNIS X^- can be defined as

$$X^- = \{x_1^-, x_2^-, \ldots, x_n^-\}, \tag{41}$$

where $x_j^- = (T_j^-, I_j^-, F_j^-) = (\min_{1\leq i\leq m} T_{ij}, \min_{1\leq i\leq m} I_{ij}, \min_{1\leq i\leq m} F_{ij})$ for $j = 1, 2, \ldots, n$.

The grey relational coefficient (GRC) between x_i and the RNPIS X^+ on the $j-$th attribute is

$$g_{ij}^+ = \frac{\min_{1\leq i\leq m}\min_{1\leq j\leq n}\Delta_{ij}^+ + \xi\max_{1\leq i\leq n}\max_{1\leq j\leq n}\Delta_{ij}^+}{\Delta_{ij}^+ + \xi\max_{1\leq i\leq m}\max_{1\leq j\leq n}\Delta_{ij}^+}, \tag{42}$$

where $\Delta_{ij}^+ = d_N(x_{ij}, x_j^+)$, for $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n$, $\xi = 0.5$.

The GRC between x_i and the RNPIS X^+ is

$$G_i^+ = \sum_{j=1}^{n} w_j g_{ij}^+, \quad i = 1, 2, \ldots, m. \tag{43}$$

The GRC between x_i and the RNNIS X^-on the $j-$th attribute is

$$g_{ij}^- = \frac{\min_{1\le i\le m}\min_{1\le j\le n}\Delta_{ij}^- + \xi\max_{1\le i\le m}\max_{1\le j\le n}\Delta_{ij}^-}{\Delta_{ij}^- + \xi\max_{1\le i\le m}\max_{1\le j\le n}\Delta_{ij}^-}, \tag{44}$$

where $\Delta_{ij}^- = d_N(x_{ij}, x_j^-)$, for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, $\xi = 0.5$.

The GRC between x_i and the RNNIS X^- is

$$G_i^- = \sum_{j=1}^{n} w_j g_{ij}^-, \quad i = 1, 2, \ldots, m. \tag{45}$$

Compute the neutrosophic relative relational degree (NRRD) H_i:

$$\begin{aligned} H_i &= \frac{G_i^+}{G_i^+ + G_i^-} \\ &= \frac{\sum_{j=1}^n w_j\left(\left(\min_{1\le i\le m}\min_{1\le j\le n}\Delta_{ij}^+ + \xi\max_{1\le i\le m}\max_{1\le j\le n}\Delta_{ij}^+\right)/\left(\Delta_{ij}^+ + \xi\max_{1\le i\le m}\max_{1\le j\le n}\Delta_{ij}^+\right)\right)}{\sum_{j=1}^n w_j\left(\left(\min_{1\le i\le m}\min_{1\le j\le n}\Delta_{ij}^+ + \xi\max_{1\le i\le m}\max_{1\le j\le n}\Delta_{ij}^+\right)/\left(\Delta_{ij}^+ + \xi\max_{1\le i\le m}\max_{1\le j\le n}\Delta_{ij}^+\right)\right) + \sum_{j=1}^n w_j\left(\min_{1\le i\le m}\min_{1\le j\le n}\Delta_{ij}^- + \xi\max_{1\le i\le m}\max_{1\le j\le n}\Delta_{ij}^-\right)/\left(\Delta_{ij}^- + \xi\max_{1\le i\le m}\max_{1\le j\le n}\Delta_{ij}^-\right)}, \end{aligned} \tag{46}$$

where $\Delta_{ij}^+ = d_N(x_{ij}, x_j^+) = (1/3)(|T_{ij} - T_j^+| + |I_{ij} - I_j^+| + |F_{ij} - F_j^+|)$, $\Delta_{ij}^- = d_N(x_{ij}, x_j^-) = (1/3)\ (|T_{ij} - T_j^-| + |I_{ij} - I_j^-| + |F_{ij} - F_j^-|)$.

Here, NRRD H_i represents the probability of x_i in the state Γ. Therefore, we define H_i as the conditional probability of object x_i.

5.3. Approaches for SVBDTRS Model Based on 3WDs. In this section, we utilize the SVNDTRS model to deal with a MADM problem. For a MADM problem, let $A = \{A_1, A_2, \ldots, A_m\}$ be the set of alternatives and $C = \{C_1, C_2, \ldots, C_n\}$ be the set of attributes. Suppose that $Y = (\widetilde{Y}_{ij})_{m\times n}$ is the SVNS decision matrix, which is shown as follows:

$$Y = \begin{bmatrix} \widetilde{Y}_{11} & \widetilde{Y}_{12} & \cdots & \widetilde{Y}_{1n} \\ \widetilde{Y}_{21} & \widetilde{Y}_{22} & \cdots & \widetilde{Y}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \widetilde{Y}_{m1} & \widetilde{Y}_{m2} & \cdots & \widetilde{Y}_{mn} \end{bmatrix}, \tag{47}$$

where $\widetilde{Y}_{ij}$ is the preference value given by the decision maker for the alternatives $A_i \in A\,(i = 1, 2, \ldots, m)$ to the attribute $C_j \in C\,(j = 1, 2, \cdots, n)$. Based on these necessary conditions, the decision results are required.

Next, we use the SVNDTRS model to solve this MADM problem. The method comprises the following procedures:

Step 1: according to the nonempty object sets and nonempty attribute sets, establish the LF matrix.

Step 2: with Section 4, determine the weight $W = (w_1, w_2, \ldots, w_n)^T$ of all attributes.

Step 2-1: according to the complementary judgment matrix given by experts, the subjective weight of attributes is calculated by formula (25):

$$\mu_j = \frac{\sum_{k=1}^n q_{jk} + (n/2) - 1}{n(n-1)}. \tag{48}$$

Step 2-2: according to the entropy method, the objective weight of attributes is computed by formula (28):

$$\omega_j = \frac{1 - E_j}{\sum_{j=1}^n \left(1 - E_j\right)} = \frac{\sum_{i=1}^m \left(T_{ij}(x_i) + F_{ij}(x_i)\right)\left|I_{ij}(x_i) - I_{ij}^c(x_i)\right|}{\sum_{j=1}^n \left(\sum_{i=1}^m \left(T_{ij}(x_i) + F_{ij}(x_i)\right)\left|I_{ij}(x_i) - I_{ij}^c(x_i)\right|\right)}. \tag{49}$$

Step 2-3: based on the optimal combination weighting model with minimum total deviation, the combination weights of attributes are calculated by formula (32):

$$w_j = \frac{\left(\left(\mu_j + \omega_j\right)\sum_{i=1}^m \left(S\left(\widetilde{Y}_{ij}\right)\right)^2\right) - \left(\sum_{j=1}^n \left(\left(\mu_j + \omega_j\right)\sum_{i=1}^m \left(S\left(\widetilde{Y}_{ij}\right)\right)^2\right) - 2\right)/(n)}{2}. \tag{50}$$

Step 3: for (40) and (41), identify the RNPIS X^+ and the RNNIS X^-.

Step 4: in light of (46), calculate the NRRD of x_i to the RNPIS X^+, expressed as H_i. Furthermore, reckon the value of conditional probability of the x_i as $P(\Gamma \mid x_i) = H_i$ $(i = 1, 2, \ldots, m)$.

Step 5: on the basis of the LFs and the conditional probability, use (37)–(39) to compute cosine similarity $S(R(a_i \mid [x]))(i = P, B, N)$ of the expected losses.

Step 6: in line with the decision rules (P1)-(N1), further determine the decision results of each alternative.

6. A Numerical Example

A company is preparing to expand its production scale to cope with the complexity of the market and the diversity of demand. At present, it is planning to select long-term suppliers from six suppliers. For core enterprises, how to select suppliers scientifically and reasonably plays a crucial role in cost control, supply chain stability, and risk management. The six suppliers form the sets $A = \{A_1, A_2, A_3, A_4, A_5, A_6\}$. The company considers the four attributes of the supplier in terms of quality, production capacity, after-sales service, and management ability. The four attributes constitute the attribute sets $C = \{C_1, C_2, C_3, C_4\}$, and the weight of each attribute is unknown. The six alternatives $A = \{A_1, A_2, A_3, A_4, A_5, A_6\}$ are appraised by the decision maker under the four attributes $C = \{C_1, C_2, C_3, C_4\}$, and the decision matrices $Y = (\widetilde{Y}_{ij})_{6\times 4}$ are built , as listed in Table 3.

6.1. Decision-Making Steps

Step 1: according to (6)–(11) and the alternative evaluation, the LF is given as follows: $\lambda_{PP} = (T_{PP}, I_{PP}, F_{PP}) = (0, 0.8, 1)$, $\lambda_{PN} = (T_{PN}, I_{PN}, F_{PN}) = (0.7, 0.2, 0.15)$, $\lambda_{BP} = (T_{BP}, I_{BP}, F_{BP}) = (0.35, 0.4, 0.3)$, $\lambda_{BN} = (T_{BN}, I_{BN}, F_{BN}) = (0.5, 0.3, 0.4)$, $\lambda_{NP} = (T_{NP}, I_{NP}, F_{NP}) = (0.9, 0.3, 0.2)$, $\lambda_{NN} = (T_{NN}, I_{NN}, F_{NN}) = (0.05, 0.95, 0.9)$.

Step 2: with Section 4, determine the weight $W = (w_1, w_2, \ldots, w_n)^T$ of all attributes.

Step 2-1: the complementary judgment matrix given by experts is as follows:

$$Q = \begin{bmatrix} 0.5 & 0.3 & 0.5 & 0.1 \\ 0.7 & 0.5 & 0.7 & 0.3 \\ 0.5 & 0.3 & 0.5 & 0.1 \\ 0.9 & 0.7 & 0.9 & 0.5 \end{bmatrix}. \tag{51}$$

According to formula (25), we can calculate the subjective weight of attributes $\mu = (0.3, 0.2333, 0.3, 0.1667)^T$.

Step 2-2: according to the entropy method, we can compute the objective weight of attributes by formula (28):

Table 3: SVNS decision matrix.

	C_1	C_2	C_3	C_4
A_1	(0.4, 0.3, 0.2)	(0.5, 0.1, 0.4)	(0.3, 0.2, 0.3)	(0.5, 0.1, 0.3)
A_2	(0.5, 0.0, 0.1)	(0.3, 0.2, 0.3)	(0.4, 0.2, 0.5)	(0.6, 0.1, 0.1)
A_3	(0.5, 0.1, 0.4)	(0.5, 0.0, 0.4)	(0.6, 0.2, 0.1)	(0.7, 0.2, 0.1)
A_4	(0.7, 0.2, 0.1)	(0.4, 0.3, 0.2)	(0.6, 0.1, 0.3)	(0.6, 0.1, 0.2)
A_5	(0.4, 0.3, 0.2)	(0.5, 0.2, 0.2)	(0.7, 0.2, 0.1)	(0.7, 0.2, 0.1)
A_6	(0.6, 0.1, 0.1)	(0.9, 0.1, 0.1)	(0.6, 0.2, 0.3)	(0.5, 0.2, 0.2)

$$\omega = (0.2250, 0.2773, 0.2425, 0.2552)^T. \tag{52}$$

Step 2-3: based on the optimal combination weighting model with minimum total deviation, the combination weights of attributes are calculated by formula (32): $w = (0.2650, 0.2344, 0.2476, 0.2530)^T$.

Step 3: for (40) and (41), we identify the RNPIS X^+ and the RNNIS X^-:

$$\begin{aligned} X^+ &= [(0.7, 0.0, 0.1), (0.9, 0.0, 0.1), (0.7, 0.1, 0.1), \\ &\quad (0.7, 0.1, 0.1)], \\ X^- &= [(0.4, 0.3, 0.4), (0.4, 0.3, 0.4), (0.3, 0.2, 0.5), \\ &\quad (0.5, 0.2, 0.3)]. \end{aligned} \tag{53}$$

Step 4: on the basis of (46), we compute the NRRD of x_i with respect to the RNPIS X^+, expressed as H_i. Furthermore, we reckon the value of conditional probability of x_i as $P(\Gamma \mid x_i) = H_i$ $(i = 1, 2, \ldots, 6)$. The results are shown in Table 4.

Step 5: on the basis of the LFs and the conditional probability, we can calculate cosine similarity of the expected losses by using (37)–(39). The results are listed in Table 5.

In order to more intuitively express the 3WD results of each alternative, we present the results of Table 5 in Figure 1.

Step 6: according to the decision rules (P1)-(N1), we can further determine the result of each alternative: $A_6 \in \mathrm{POS}(\Gamma)$, $A_5, A_3, A_4, A_2 \in \mathrm{BND}(\Gamma)$, and $A_1 \in \mathrm{NEG}(\Gamma)$. From the above results, we can see that A_6 should be invested, A_1 should not be invested, and A_5, A_3, A_4, and A_2 whether or not should be invested should be further studied.

6.2. Comparative Analysis. For thoroughly testing the availability and significance of our development method, two methods are adopted for comparative analysis, including the grey relation analysis method based on Biswas et al. [32] and the extended TOPSIS method proposed by Zhang and Wu [40]. Furthermore, we apply these two methods to the same example as in this paper.

(1) The decision result based on the GRA method proposed by Biswas et al. [32].

Utilizing the GRA method, the GRC and the NRRD are indicated in Table 6.

Table 4: The GRC and the NRRD and its conditional probability.

	A_1	A_2	A_3	A_4	A_5	A_6
G_i^+	0.444	0.579	0.606	0.608	0.629	0.663
G_i^-	0.868	0.750	0.647	0.658	0.666	0.628
H_i	0.339	0.436	0.483	0.480	0.484	0.514
$P(\Gamma \mid A_i)$	0.339	0.436	0.483	0.480	0.484	0.514

Table 5: The cosine similarity degree of the expected losses.

	A_1	A_2	A_3	A_4	A_5	A_6
$S(R(a_P \mid A_i))$	0.5111	0.4497	0.4177	0.4198	0.4172	0.3968
$S(R(a_B \mid A_i))$	0.4235	0.4130	0.4076	0.4080	0.4070	0.4042
$S(R(a_N \mid A_i))$	0.4219	0.4959	0.5279	0.5260	0.5284	0.5470

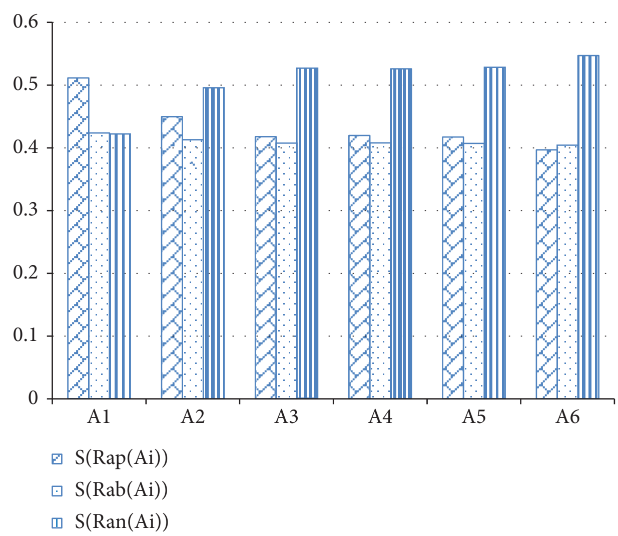

Figure 1: The cosine similarity degree of the expected losses.

Table 6: The GRC and the NRRD.

	A_1	A_2	A_3	A_4	A_5	A_6
G_i^+	0.443	0.566	0.604	0.594	0.624	0.668
G_i^-	0.864	0.763	0.645	0.674	0.660	0.624
H_i	0.339	0.426	0.483	0.468	0.486	0.517

From Table 6, we can get $H_6 > H_5 > H_3 > H_4 > H_2 > H_1$, so $A_6 > A_5 > A_3 > A_4 > A_2 > A_1$. Thus the best alternative is A_6.

(2) The decision result based on the extended TOPSIS method proposed by Zhang and Wu [40].

The weights of attributes are determined by maximizing deviation method as follows:

$$w = [0.2561, 0.3069, 0.2709, 0.1661]^T. \qquad (54)$$

We can get the relative closeness RC_i of each alternative to the RNPIS X^+ (Table 7).

From Table 7, we can get $RC_6 > RC_3 > RC_5 > RC_4 > RC_2 > RC_1$, so $A_6 > A_3 > A_5 > A_4 > A_2 > A_1$. Thus the best option is A_6.

Table 7: The relative closeness of each alternative to the RNPIS X^+.

	A_1	A_2	A_3	A_4	A_5	A_6
RC_i	0.3027	0.3682	0.5254	0.4889	0.5246	0.7016

From the above results, we can see that the GRA method proposed by Biswas et al. [32] has the same ranking results as that of our proposed method and has the same best option A_6. In this way, we can demonstrate the effectiveness of our proposed method. Nevertheless, the decision-making method in [32] is the attribute weight obtained by the entropy method, without considering the subjective preferences of decision makers. In this paper, the objective weight is obtained by the entropy method, while the subjective weight is obtained by the AHP method, and the combination weight is obtained based on the method of minimum total deviation. Moreover, the GRA method based on [32] has a drawback, which can only give the ranking. The decision results based on SVNDTRS model not only gives the ranking but also provides the relevant semantic explanation for the selection of each alternative, which is more scientific and flexible than the GRA method [32].

Comparing the ranking obtained by the MADM method in [40] and the proposed method, we get the same best choice A_6, but the positions of alternatives A_3 and A_5 are not the same. Among the rankings obtained in this paper, alternative A_5 is better than alternative A_3. But in [40], alternative A_3 is better than alternative A_5. The reason for this inconsistency is that the attribute weights of the two methods are different. In [40], attribute weights are obtained by maximizing deviation, ignoring the subjective preferences of decision makers, and C_2 is the most important attribute. This paper combines the subjective weights obtained by the AHP method and finds C_1 as the most important attribute. This paper not only considers the subjective preferences of decision makers but also takes full advantage of the evaluative information of decision-making objects, reduces the subjective randomness of weight, and achieves the unity of subjective and objective. Therefore, the ranking of our proposed method is more reasonable.

In a word, the 3WD method based on the SVNDTRS model proposed in this paper has the following advantages: (1) It not only considers the subjective preferences of decision makers but also makes the best of objective information of decision objects to realize the unity of subjectivity and objectivity. (2) Using the GRA method to obtain the conditional probability of 3WDs provides a new viewpoint for obtaining the conditional probability of 3WDs. (3) It not only gives the ranking results of each alternative but also provides the corresponding semantic explanation for the selection of alternatives.

7. Conclusion

In this paper, we extended 3WDs to the environment of SVNSs and used SVNSs to express the evaluation values and LFs given by decision makers. We also proposed a method to obtain the attribute weights. We applied the AHP method to get the subjective weight of attributes and entropy method to obtain the objective weight and established the combination weight of attributes based on the principle of minimum total deviation. It not only considers the subjective preferences of decision makers but also takes most advantage of the evaluative information of decision makers to realize the unity of subjectivity and objectivity. Based on this, we proposed a SVNDTRS model based on the GRA method. The NRRD calculated by the GRA method is defined as the conditional probability of 3WDs. Finally, we extended the proposed method to a numerical example of supplier selection and compared it with other methods to demonstrate the effectiveness of our proposed method. We presented a new idea for the determination of conditional probability in 3WDs and a new solution for the LF expressed by SVNSs in 3WDs. In future, we will study the application of 3WDs to settle the MADM problem composed of fuzzy information.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This study was supported by the National Natural Science Foundation of China (nos. 71771140 and 71471172), 文化名家暨"四个一批"人才 (Cultural Masters and "the Four Kinds of a Batch" Talents), and the Special Funds of Taishan Scholars Project of Shandong Province (no. ts201511045).

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