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Time-neutrosophic Soft Expert Sets and Its Decision Making Problem

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Abstract In this paper we present a new concept called time-neutrosophic soft expert set (T-NSESs). We also define its basic operations, namely complement, union, intersection, AND, OR and study some of their properties. We give examples for these concepts. Finally we present an application of this concept in a decision-making problem.

Keywords Soft expert set; neutrosophic soft set; neutrosophic soft expert set; time-neutrosophic soft expert sets.

Mathematics Subject Classification 03B52

1 Introduction

In some real life problems in expert system, belief system, information fusion, we must consider the truth-membership as well as the falsity-membership for proper description of an object in uncertain, ambiguous environment. Intuitionistic fuzzy sets introduced by Atanassov [1]. After Atanassov's work, Smarandache [2, 3] introduced the concept of neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. In 1999 Molodtsov [4] initiated a novel concept of soft set theory as a new mathematical tool for dealing with uncertainties. After Molodtsov's work, some different operation and application of soft sets were studied by Chen *et al.* [5] and Maji *et al.* [6]. Later, Maji [7] proposed neutrosophic soft sets with operations. Alkhazaleh *et al.* [8] generalized the concept of fuzzy soft expert sets which include the possibility of each element in the universe is attached with the parameterization of fuzzy sets while defining a fuzzy soft expert set. Alkhazaleh *et al.* [9] introduced generalized the concept of parameterized interval-valued fuzzy soft sets, where the mapping in which the approximate function are defined from fuzzy parameters set, and they gave an application of this concept in decision making. In the other study Alkhazaleh and Salleh [10] introduced the concept of soft expert sets where a user can know the opinion of all expert sets. Alkhazaleh and Salleh [11] generalized the concept of a soft expert set to fuzzy soft expert set, which is a more effective and useful. They also define its basic operations, namely complement, union, intersection, AND and OR and give an application of this concept in decision making problem. They also studied a mapping on fuzzy soft expert classes and its properties. Sahin *et al.* [12] introduced the concept of neutrosophic soft expert sets. They also define its basic operations and give an application of this concept in decision-making problem. Our objective is to introduce the concept of generalized neutrosophic soft expert set. In Section 1, from intuitionistic fuzzy sets to neutrosophic soft expert sets are mentions. In section 2, preliminaries are given. In section 3, the concept of generalized neutrosophic soft expert set and its basic operations, namely complement, union, intersection AND and OR. In section 4, we give an application of this concept in a decision-making problem. In Section 5 give conclusions.

2 Preliminary

In this section we recall some related definitions.

Definition 1 [3] *Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic sets (N-sets) A in U is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(u)$, $I_A(u)$ and $F_A(u)$ are real standard or nonstandard subsets of* $[0,1]$. *It can be written as*

$$A = \{\langle u, (T_A(u), I_A(u), F_A(u))\rangle : u \in U, T_A(u), I_A(u), F_A(u) \in [0,1]\}$$

There is no restriction on the sum of $T_A(u)$, $I_A(u)$ and $F_A(u)$ so,

$$0 \leq T_A(u) + I_A(u) + F_A(u) \leq 3$$

Definition 2 [7] *Let U be an initial universe set and E be a set of parameters. Consider $A \subseteq E$. Let $P(U)$ denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the soft neutrosophic set over U, where F is a mapping given by $F : A \to P(U)$.*

Definition 3 [6] *A neutrosophic set A is contained in another neutrosophic set B i.e. $A \subseteq B$ if for all $T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \geq F_B(x)$. Let U be a universe, E is a set of parameters, and X is a soft experts (agents). Let O be a set of opinion, $Z = E \times X \times O$ and $A \subseteq Z$.*

Definition 4 [12] *A pair (F, A) is called a soft expert set over U, where F is a mapping given by $F : A \to P(U)$ where $P(U)$ denotes the power set of U.*

Definition 5 [12] *A pair (F, A) is called a neutrosophic soft expert set over U, where F is mapping given by*

$$F : A \to P(U)$$

Where $P(U)$ denotes the power neutrosophic set of U.

Definition 6 [12] *Let (F, A) and (G, B) be two neutrosophic soft expert sets over the common universe U. (F, A) is said to be neutrosophic soft expert subset of (G, B), if $A \tilde{\subseteq} B$ and $T_{F(e)}(x) \tilde{\leq} T_G(e)(x), I_{F(e)}(x) \tilde{\leq} I_G(e)(x)$,*
$F_{F(e)}(x) \tilde{\geq} F_G(e)(x)$, for all $e \in A, x \in U$. We denote it by $(F, A) \tilde{\subseteq} (G, B)$.
(F, A) is said to be neutrosophic soft expert superset of (G, B) if (G, B) is a neutrosophic soft expert subset of (F, A). We denote by $(F, A) \tilde{\supseteq} (G, B)$.

Definition 7 [12] *Equality of two neutrosophic soft expert sets. Two (NSES), (F, A) and (G, B) over the common universe U are said to be equal if (F, A) is neutrosophic soft expert subset of (G, B) and (G, B) is neutrosophic soft expert subset of (F, A). We denote it by*

$$(F, A) = (G, B).$$

Definition 8 [12] *Complement of a neutrosophic soft expert set. The complement of a neutrosophic soft expert set (F, A) denoted by $(F, A)^c$ and is defined as $(F, A)^c = (F^c, \neg A)$ where $F^c = \neg A \to P(U)$ is mapping given by $F^c(x)$ =neutrosophic soft expert complement with $T_{F^c(x)} = F_{F(x)}, I_{F^c(x)} = I_{F(x)}, F_{F^c(x)} = T_{F(x)}$.*

Definition 9 [12] *An agree-neutrosophic soft expert set $(F, A)_1$ over U is a neutrosophic soft expert subset of (F, A) defined as follow:*

$$(F, A)_1 = \{F_1(m) : m \in E \times X \times \{1\}\}.$$

Definition 10 [12] *An disagree-neutrosophic soft expert set $(F, A)_0$ over U is a neutrosophic soft expert subset of (F, A) defined as follow:*

$$(F, A)_0 = \{F_0(m) : m \in E \times X \times \{0\}\}.$$

Definition 11 [12] *Let (H, A) and (G, B) be two NSESs over the common universe U. Then the union of (H, A) and (G, B) is denoted by "$(H, A)\tilde{\cup}(G, B)$" and is defined by $(H, A)\tilde{\cup}(G, B) = (K, C)$, where $C = A \cup B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:*

$$\begin{aligned}
T_{K(e)}(m) &= T_{H(e)}(m), if \quad e \in A - B \\
&= T_{G(e)}(m), if \quad e \in B - A \\
&= \max(T_{H(e)}(m), T_{G(e)}(m)), if \quad e \in A \cap B \\
I_{K(e)}(m) &= I_{H(e)}(m), if \quad e \in A - B \\
&= I_{G(e)}(m), if \quad e \in B - A \\
&= \frac{I_{H(e)}(m) + I_{G(e)}(m)}{2}, if \quad e \in A \cap B \\
F_{K(e)}(m) &= F_{H(e)}(m), if \quad e \in A - B \\
&= F_{G(e)}(m), if \quad e \in B - A \\
&= \min(T_{H(e)}(m), T_{G(e)}(m)), if \quad e \in A \cap B.
\end{aligned}$$

Definition 12 [12] *Let (H, A) and (G, B) be two NSESs over the common universe U. Then the intersection of (H, A) and (G, B) denoted by "$(H, A)\tilde{\cap}(G, B)$" is defined by $(H, A)\tilde{\cap}(G, B) = (K, C)$, where $C = A \cap B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:*

$$\begin{aligned}
T_{K(e)}(m) &= \min(T_{H(e)}(m), T_{G(e)}(m)) \\
I_{K(e)}(m) &= \frac{I_{H(e)}(m) + I_{G(e)}(m)}{2} \\
F_{K(e)}(m) &= \max(T_{H(e)}(m), T_{G(e)}(m))
\end{aligned}$$

Definition 13 [13] *Let U be an initial universal set and let E be a set of parameters. Let I^U denotes the power set of all fuzzy subsets of U, let $A \subseteq E$ and T be a set of time where $T = \{t_1, t_2, t_3, ..., t_n\}$. A collection of pairs $(F, E)_t$ for all $t \in T$ is called a time-fuzzy soft set $T - FSS$ over U where F is a mapping given by*

$$F_t : A \rightarrow I^U.$$

3 Time-neutrosophic Soft Expert Sets

In this section, we introduce the definition of time - neutrosophic soft expert sets and study some its properties. Throughout the paper, U is an initial universe, E is a set of parameters, T be a set of time where $T = \{t_1, t_2, t_3, ..., t_n\}$, X is a set of experts (agents), and $O = \{agree = 1, disagree = 0\}$ is a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$.

Definition 14 *A pair $(F, A)_t$ is called a time- neutrosophic soft expert set over U, where F is a mapping given by where $N(U)$ be the set of all neutrosophic soft expert subsets of U. Let $A \subseteq Z$ and T be a set of time where $T = \{t_1, t_2, t_3, ..., t_n\}$. A time-neutrosophic soft expert set F_t^μ over U. A time- neutrosophic soft expert set F_t^μ over U is defined by the set of ordered pairs*

$$F_t^\mu = \{F(e), \mu(e) : e \in A, F(e) \in N(U), \mu(e) \in [0, 1]\}$$

where F is a mapping given by $F : A \rightarrow N(U)$ and μ is a fuzzy set such that $\mu : A \rightarrow I = [0, 1]$. Here F_t^μ is a mapping defined by $F_t^\mu : A \rightarrow N(U)$. For any parameter $e \in A, F(e)$ is referred as the neutrosophic value set of parameter e, i.e,

$$F_t^\mu(e) = \{\langle u^t/(T_{F(e)}(u), I_{F(e)}(u), F_{F(e)}(u)\rangle\},$$

Where $T, I, F : U \rightarrow [0, 1]$ are the membership function of truth, indeterminacy and falsity of the element $u \in U$ respectively. For any $u \in U$ and $e \in A$

$$0 \leq T_A(u) + I_A(u) + F_A(u) \leq 3.$$

In fact F_t^μ is a parameterized family of neutrosophic soft expert sets on U, which has the degree of possibility of the approximate value set which is prepresented by $\mu(e)$ for each parameter e. So we can write it as follows:

$$F_t^\mu(e) = \{(\frac{u_1^t}{F(e)(u_1)}, \frac{u_2^t}{F(e)(u_2)}, \frac{u_3^t}{F(e)(u_3)}, ..., \frac{u_n^t}{F(e)(u_n)}), \mu(e)\}.$$

Example 1 Suppose that a company produces new types of smart phones and wants to take the opinion of some experts about these phones. Let $U = \{u_1, u_2, u_3\}$ be set of phones, $E = \{e_1, e_2, e_3\}$ is a set of decision parameters where $(e_i, i = 1, 2, 3)$ denotes the parameters e_1 =screen, e_2 = GHz, e_3 = price and $T = \{t_1, t_2, t_3\}$ be set of time. Let $X = \{p, q, r\}$ be set of experts. Suppose that;

$$F_1^\mu(e_1, p, 1) = \{(\frac{u_1^{t_1}}{0.4, 0.3, 0.2}, \frac{u_2^{t_1}}{0.6, 0.1, 0.8}, \frac{u_3^{t_1}}{0.5, 0.7, 0.2}), 0.5\}$$

$$F_2^{\mu}(e_1, q, 1) = \{(\frac{u_1^{t_2}}{0.3, 0.2, 0.5}, \frac{u_2^{t_2}}{0.5, 0.6, 0.2}, \frac{u_3^{t_2}}{0.8, 0.1, 0.4}), 0.3\}$$

$$F_3^{\mu}(e_1, r, 1) = \{(\frac{u_1^{t_3}}{0.8, 0.4, 0.3}, \frac{u_2^{t_3}}{0.7, 0.3, 0.5}, \frac{u_3^{t_3}}{0.2, 0.6, 0.5}), 0.7\}$$

$$F_1^{\mu}(e_2, p, 1) = \{(\frac{u_1^{t_1}}{0.7, 0.3, 0.6}, \frac{u_2^{t_1}}{0.5, 0.1, 0.4}, \frac{u_3^{t_1}}{0.8, 0.6, 0.3}), 0.1\}$$

$$F_2^{\mu}(e_2, q, 1) = \{(\frac{u_1^{t_2}}{0.6, 0.7, 0.1}, \frac{u_2^{t_2}}{0.8, 0.4, 0.7}, \frac{u_3^{t_2}}{0.5, 0.1, 0.7}), 0.8\}$$

$$F_3^{\mu}(e_2, r, 1) = \{(\frac{u_1^{t_3}}{0.5, 0.1, 0.8}, \frac{u_2^{t_3}}{0.9, 0.3, 0.6}, \frac{u_3^{t_3}}{0.4, 0.1, 0.7}), 0.6\}$$

$$F_1^{\mu}(e_3, p, 1) = \{(\frac{u_1^{t_1}}{0.6, 0.3, 0.2}, \frac{u_2^{t_1}}{0.5, 0.6, 0.7}, \frac{u_3^{t_1}}{0.8, 0.1, 0.4}), 0.7\}$$

$$F_2^{\mu}(e_3, q, 1) = \{(\frac{u_1^{t_2}}{0.7, 0.3, 0.4}, \frac{u_2^{t_2}}{0.6, 0.2, 0.5}, \frac{u_3^{t_2}}{0.7, 0.4, 0.6}), 0.6\}$$

$$F_3^{\mu}(e_3, r, 1) = \{(\frac{u_1^{t_3}}{0.7, 0.4, 0.6}, \frac{u_2^{t_3}}{0.5, 0.3, 0.6}, \frac{u_3^{t_3}}{0.1, 0.4, 0.2}), 0.4\}$$

$$F_1^{\mu}(e_1, p, 0) = \{(\frac{u_1^{t_1}}{0.4, 0.1, 0.2}, \frac{u_2^{t_1}}{0.7, 0.3, 0.5}, \frac{u_3^{t_1}}{0.4, 0.1, 0.6}), 0.3\}$$

$$F_2^{\mu}(e_1, q, 0) = \{(\frac{u_1^{t_2}}{0.7, 0.3, 0.5}, \frac{u_2^{t_2}}{0.6, 0.2, 0.4}, \frac{u_3^{t_2}}{0.4, 0.5, 0.1}), 0.2\}$$

$$F_3^{\mu}(e_1, r, 0) = \{(\frac{u_1^{t_3}}{0.6, 0.4, 0.3}, \frac{u_2^{t_3}}{0.7, 0.2, 0.6}, \frac{u_3^{t_3}}{0.4, 0.1, 0.3}), 0.1\}$$

$$F_1^{\mu}(e_2, p, 0) = \{(\frac{u_1^{t_1}}{0.5, 0.1, 0.7}, \frac{u_2^{t_1}}{0.4, 0.1, 0.5}, \frac{u_3^{t_1}}{0.7, 0.1, 0.4}), 0.3\}$$

$$F_2^{\mu}(e_2, q, 0) = \{(\frac{u_1^{t_2}}{0.4, 0.3, 0.6}, \frac{u_2^{t_2}}{0.7, 0.2, 0.5}, \frac{u_3^{t_2}}{0.8, 0.1, 0.4}), 0.7\}$$

$$F_3^{\mu}(e_2, r, 0) = \{(\frac{u_1^{t_3}}{0.3, 0.2, 0.6}, \frac{u_2^{t_3}}{0.4, 0.3, 0.5}, \frac{u_3^{t_3}}{0.5, 0.1, 0.4}), 0.2\}$$

$$F_1^{\mu}(e_3, p, 0) = \{(\frac{u_1^{t_1}}{0.4, 0.3, 0.6}, \frac{u_2^{t_1}}{0.5, 0.1, 0.6}, \frac{u_3^{t_1}}{0.6, 0.2, 0.5}), 0.8\}$$

$$F_2^{\mu}(e_3, q, 0) = \{(\frac{u_1^{t_2}}{0.6, 0.2, 0.7}, \frac{u_2^{t_2}}{0.8, 0.1, 0.4}, \frac{u_3^{t_2}}{0.5, 0.3, 0.4}), 0.5\}$$

$$F_3^{\mu}(e_3, r, 0) = \{(\frac{u_1^{t_3}}{0.5, 0.4, 0.6}, \frac{u_2^{t_3}}{0.6, 0.4, 0.3}, \frac{u_3^{t_3}}{0.7, 0.2, 0.1}), 0.1\}$$

Definition 15 *For two time-neutrosophic soft expert sets $\{T-NSESs\}$ (F_t^{μ}, A) and (G_t^{η}, B) over U, (F_t^{μ}, A) is called a time- neutrosophic soft expert subset of (G_t^{η}, B) if,*
(i) $B \subseteq A$,
(ii) *for all $t \in T$, $\varepsilon \in B, G_t^{\eta}(\varepsilon)$ is time-neutrosophic soft expert subset $F_t^{\mu}(\varepsilon)$.*

Example 2 Recall Example 1 such that;

$$A = \{(e_1, p, 1), (e_2, p, 1), (e_2, q, 0), (e_3, r, 1)\}, \quad B = \{(e_1, p, 1), (e_2, p, 1), (e_3, r, 1)\}$$

Since B is neutrosophic soft expert subset of A, clearly $B \subset A$. Let (G_t^{η}, B) and (F_t^{μ}, A) be defined as follows:

$$\begin{aligned}
(F_t^{\mu}, A) = \quad & \{[(e_1, p, 1), (\tfrac{u_1^{t_1}}{0.4,0.3,0.2}, \tfrac{u_2^{t_1}}{0.6,0.1,0.8}, \tfrac{u_3^{t_1}}{0.5,0.7,0.2}), 0.5],\\
& [(e_2, p, 1), (\tfrac{u_1^{t_1}}{0.7,0.3,0.6}, \tfrac{u_2^{t_1}}{0.5,0.1,0.4}, \tfrac{u_3^{t_1}}{0.8,0.6,0.3}), 0.1],\\
& [(e_2, q, 0), (\tfrac{u_1^{t_2}}{0.4,0.3,0.6}, \tfrac{u_2^{t_2}}{0.7,0.2,0.5}, \tfrac{u_3^{t_2}}{0.8,0.1,0.4}), 0.7],\\
& [(e_3, r, 1), (\tfrac{u_1^{t_3}}{0.7,0.4,0.6}, \tfrac{u_2^{t_3}}{0.5,0.3,0.6}, \tfrac{u_3^{t_3}}{0.1,0.4,0.2}), 0.4]\}.\\
(G_t^{\eta}, B) = \quad & \{[(e_1, p, 1), (\tfrac{u_1^{t_1}}{0.4,0.3,0.2}, \tfrac{u_2^{t_1}}{0.6,0.1,0.8}, \tfrac{u_3^{t_1}}{0.5,0.7,0.2}), 0.5],\\
& [(e_2, p, 1), (\tfrac{u_1^{t_1}}{0.7,0.3,0.6}, \tfrac{u_2^{t_1}}{0.5,0.1,0.4}, \tfrac{u_3^{t_1}}{0.8,0.6,0.3}), 0.1],\\
& [(e_3, r, 1), (\tfrac{u_1^{t_3}}{0.7,0.4,0.6}, \tfrac{u_2^{t_3}}{0.5,0.3,0.6}, \tfrac{u_3^{t_3}}{0.1,0.4,0.2}), 0.4]\}.
\end{aligned}$$

Therefore $(G_t^{\eta}, B) \subseteq (F_t^{\mu}, A)$.

Definition 16 *Two $\{T-NSESs\}$, (F_t^{μ}, A) and (G_t^{η}, B) over U, are said to be equal if (F_t^{μ}, A) is a $\{T-NSESs\}$ subset of (G_t^{η}, B) and (G_t^{η}, B) is a $\{T-NSESs\}$ subset of (F_t^{μ}, A).*

Definition 17 *Agree-$\{T-NSESs\}$, $(F_t^{\mu}, A)_1$ over U is a $\{T-NSESs\}$ subset of (F_t^{μ}, A) defined as $(F_t^{\mu}, A)_1 = \{F_1(\alpha) : \alpha \in E \times X \times \{1\}\}$.*

Example 3 Recall Example 1. Then the agree- time- neutrosophic soft expert sets $(F_t^{\mu}, Z)_1$ over U is

$$\begin{aligned}
(F_t^{\mu}, Z)_1 = \quad & \{[(\tfrac{u_1^{t_1}}{0.4,0.3,0.2}, \tfrac{u_2^{t_1}}{0.6,0.1,0.8}, \tfrac{u_3^{t_1}}{0.5,0.7,0.2}), 0.5],\\
& [(\tfrac{u_1^{t_2}}{0.3,0.2,0.5}, \tfrac{u_2^{t_2}}{0.5,0.6,0.2}, \tfrac{u_3^{t_2}}{0.8,0.1,0.4}), 0.3],\\
& [(\tfrac{u_1^{t_3}}{0.8,0.4,0.3}, \tfrac{u_2^{t_3}}{0.7,0.3,0.5}, \tfrac{u_3^{t_3}}{0.2,0.6,0.5}), 0.7],\\
& [(\tfrac{u_1^{t_1}}{0.7,0.3,0.6}, \tfrac{u_2^{t_1}}{0.5,0.1,0.4}, \tfrac{u_3^{t_1}}{0.8,0.6,0.3}), 0.1],\\
& [(\tfrac{u_1^{t_2}}{0.6,0.7,0.1}, \tfrac{u_2^{t_2}}{0.8,0.4,0.7}, \tfrac{u_3^{t_2}}{0.5,0.1,0.7}), 0.8],\\
& [(\tfrac{u_1^{t_3}}{0.5,0.1,0.8}, \tfrac{u_2^{t_3}}{0.9,0.3,0.6}, \tfrac{u_3^{t_3}}{0.4,0.1,0.7}), 0.6],\\
& [(\tfrac{u_1^{t_1}}{0.6,0.3,0.2}, \tfrac{u_2^{t_1}}{0.5,0.6,0.7}, \tfrac{u_3^{t_1}}{0.8,0.1,0.4}), 0.7],\\
& [(\tfrac{u_1^{t_2}}{0.7,0.3,0.4}, \tfrac{u_2^{t_2}}{0.6,0.2,0.5}, \tfrac{u_3^{t_2}}{0.7,0.4,0.6}), 0.6],\\
& [(\tfrac{u_1^{t_3}}{0.7,0.4,0.6}, \tfrac{u_2^{t_3}}{0.5,0.3,0.6}, \tfrac{u_3^{t_3}}{0.1,0.4,0.2}), 0.4]\}.
\end{aligned}$$

Definition 18 *A disagree-$\{T-NSESs\}$, $(F_t^{\mu}, A)_0$ over U is a $\{T-NSESs\}$ subset of (F_t^{μ}, A) is defined as $(F_t^{\mu}, A)_0 = \{F_0(\alpha) : \alpha \in E \times X \times \{0\}\}$.*

Example 4 Consider Example 1. Then the agree-time-neutrosophic soft expert sets $(F_t^{\mu}, Z)_0$ over U is

$$\begin{aligned}(F_t^{\mu}, Z)_0 = \; & \{[(\tfrac{u_1^{t_1}}{0.4,0.1,0.2}, \tfrac{u_2^{t_1}}{0.7,0.3,0.5}, \tfrac{u_3^{t_1}}{0.4,0.1,0.6}), 0.3],\\
& [(\tfrac{u_1^{t_2}}{0.7,0.3,0.5}, \tfrac{u_2^{t_2}}{0.6,0.2,0.4}, \tfrac{u_3^{t_2}}{0.4,0.5,0.1}), 0.2],\\
& [(\tfrac{u_1^{t_3}}{0.6,0.4,0.3}, \tfrac{u_2^{t_3}}{0.7,0.2,0.6}, \tfrac{u_3^{t_3}}{0.4,0.1,0.3}), 0.1],\\
& [(\tfrac{u_1^{t_1}}{0.5,0.1,0.7}, \tfrac{u_2^{t_1}}{0.4,0.1,0.5}, \tfrac{u_3^{t_1}}{0.7,0.1,0.4}), 0.3],\\
& [(\tfrac{u_1^{t_2}}{0.4,0.3,0.6}, \tfrac{u_2^{t_2}}{0.7,0.2,0.5}, \tfrac{u_3^{t_2}}{0.8,0.1,0.4}), 0.7],\\
& [(\tfrac{u_1^{t_3}}{0.3,0.2,0.6}, \tfrac{u_2^{t_3}}{0.4,0.3,0.5}, \tfrac{u_3^{t_3}}{0.5,0.1,0.4}), 0.2],\\
& [(\tfrac{u_1^{t_1}}{0.4,0.3,0.6}, \tfrac{u_2^{t_1}}{0.5,0.1,0.6}, \tfrac{u_3^{t_1}}{0.6,0.2,0.5}), 0.8],\\
& [(\tfrac{u_1^{t_2}}{0.6,0.2,0.7}, \tfrac{u_2^{t_2}}{0.8,0.1,0.4}, \tfrac{u_3^{t_2}}{0.5,0.3,0.4}), 0.5],\\
& [(\tfrac{u_1^{t_3}}{0.5,0.4,0.6}, \tfrac{u_2^{t_3}}{0.6,0.4,0.3}, \tfrac{u_3^{t_3}}{0.7,0.2,0.1}), 0.1]\}.\end{aligned}$$

Definition 19 *The complement of a time-neutrosophic soft expert set (F_t^{μ}, A) is denoted by $(F_t^{\mu}, A)^c$ for all $t \in T$ and is defined by $(F_t^{\mu}, A)^c = (F_t^{\mu(c)}, \neg A)$ where $F_t^{\mu(c)} : \neg A \to I^{N(U)}$ is mapping given by $F_t^{\mu(c)}(\alpha) = \{T_{F(\alpha)^c} = F_{F(\alpha)}, I_{F(\alpha)^c} = \overline{1} - I_{F(\alpha)}, F_{F(\alpha)^c} = T_{F(\alpha)}$ and $\mu^c(\alpha) = \overline{1} - \mu(\alpha) \;\; for \;\; each \;\; \alpha \in E.\}$*

Example 5 Recall Example 1. Complement of the time- neutrosophic soft expert set F_t^{μ} denoted by $F_t^{\mu(c)}$ is given as follows:

$$\begin{aligned}(F_t^{\mu(c)}, Z) = \; & \{[(\neg e_1, p, 1), (\tfrac{u_1^{t_1}}{0.2,0.7,0.4}, \tfrac{u_2^{t_1}}{0.8,0.9,0.6}, \tfrac{u_3^{t_1}}{0.2,0.3,0.5}), 0.5],\\
& [(\neg e_2, p, 1), (\tfrac{u_1^{t_2}}{0.5,0.8,0.3}, \tfrac{u_2^{t_2}}{0.2,0.4,0.5}, \tfrac{u_3^{t_2}}{0.4,0.9,0.8}), 0.7],\\
& [(\neg e_3, p, 1), (\tfrac{u_1^{t_3}}{0.2,0.6,0.8}, \tfrac{u_2^{t_3}}{0.5,0.7,0.7}, \tfrac{u_3^{t_3}}{0.5,0.4,0.2}), 0.3],\\
& [(\neg e_1, q, 1), (\tfrac{u_1^{t_1}}{0.6,0.7,0.7}, \tfrac{u_2^{t_1}}{0.4,0.9,0.5}, \tfrac{u_3^{t_1}}{0.3,0.4,0.8}), 0.9],\\
& [(\neg e_2, q, 1), (\tfrac{u_1^{t_2}}{0.1,0.3,0.6}, \tfrac{u_2^{t_2}}{0.7,0.6,0.8}, \tfrac{u_3^{t_2}}{0.7,0.9,0.5}), 0.2],\\
& [(\neg e_3, q, 1), (\tfrac{u_1^{t_3}}{0.8,0.9,0.5}, \tfrac{u_2^{t_3}}{0.6,0.7,0.9}, \tfrac{u_3^{t_3}}{0.7,0.9,0.4}), 0.4],\\
& [(\neg e_1, r, 1), (\tfrac{u_1^{t_1}}{0.2,0.7,0.6}, \tfrac{u_2^{t_1}}{0.7,0.4,0.5}, \tfrac{u_3^{t_1}}{0.4,0.9,0.8}), 0.3],\\
& [(\neg e_2, r, 1), (\tfrac{u_1^{t_2}}{0.4,0.7,0.7}, \tfrac{u_2^{t_2}}{0.5,0.8,0.6}, \tfrac{u_3^{t_2}}{0.6,0.6,0.7}), 0.4],\\
& [(\neg e_3, r, 1), (\tfrac{u_1^{t_3}}{0.6,0.6,0.7}, \tfrac{u_2^{t_3}}{0.6,0.7,0.5}, \tfrac{u_3^{t_3}}{0.2,0.6,0.1}), 0.6],\\
& [(\neg e_1, p, 0), (\tfrac{u_1^{t_1}}{0.2,0.9,0.4}, \tfrac{u_2^{t_1}}{0.5,0.7,0.7}, \tfrac{u_3^{t_1}}{0.6,0.9,0.4}), 0.7],\\
& [(\neg e_2, p, 0), (\tfrac{u_1^{t_2}}{0.5,0.7,0.7}, \tfrac{u_2^{t_2}}{0.4,0.8,0.6}, \tfrac{u_3^{t_2}}{0.1,0.5,0.4}), 0.8],\\
& [(\neg e_3, p, 0), (\tfrac{u_1^{t_3}}{0.3,0.6,0.6}, \tfrac{u_2^{t_3}}{0.6,0.8,0.7}, \tfrac{u_3^{t_3}}{0.3,0.9,0.4}), 0.9],\\
& [(\neg e_1, q, 0), (\tfrac{u_1^{t_1}}{0.7,0.9,0.5}, \tfrac{u_2^{t_1}}{0.5,0.9,0.4}, \tfrac{u_3^{t_1}}{0.4,0.9,0.7}), 0.7],\\
& [(\neg e_2, q, 0), (\tfrac{u_1^{t_2}}{0.6,0.7,0.4}, \tfrac{u_2^{t_2}}{0.5,0.8,0.7}, \tfrac{u_3^{t_2}}{0.4,0.9,0.1}), 0.3],\\
& [(\neg e_3, q, 0), (\tfrac{u_1^{t_3}}{0.6,0.8,0.3}, \tfrac{u_2^{t_3}}{0.5,0.7,0.4}, \tfrac{u_3^{t_3}}{0.4,0.9,0.5}), 0.8],\\
& [(\neg e_1, r, 0), (\tfrac{u_1^{t_1}}{0.6,0.7,0.4}, \tfrac{u_2^{t_1}}{0.6,0.9,0.5}, \tfrac{u_3^{t_1}}{0.5,0.8,0.6}), 0.2],\\
& [(\neg e_2, r, 0), (\tfrac{u_1^{t_2}}{0.7,0.8,0.6}, \tfrac{u_2^{t_2}}{0.4,0.9,0.8}, \tfrac{u_3^{t_2}}{0.4,0.7,0.5}), 0.5],\\
& [(\neg e_3, r, 0), (\tfrac{u_1^{t_3}}{0.6,0.6,0.5}, \tfrac{u_2^{t_3}}{0.3,0.6,0.6}, \tfrac{u_3^{t_3}}{0.1,0.8,0.7}), 0.9]\}.\end{aligned}$$

Proposition 1 *If (F_t^{μ}, A) is a time-neutrosophic soft expert set over U, then*

(i) $((F_t^{\mu}, A)^c)^c = (F_t^{\mu}, A)$
(ii) $((F_t^{\mu}, A)_1)^c = (F_t^{\mu}, A)_0$
(iii) $((F_t^{\mu}, A)_0)^c = (F_t^{\mu}, A)_1$

Proof (i) From Definition 19 we have $(F_t^{\mu}, A)^c = (F_t^{\mu(c)}, \neg A)$ where, $F_t^{\mu(c)}(\alpha) = T_{F(\alpha)^c} = F_{F(\alpha)}, I_{(F(\alpha)^c} = \overline{1} - I_{F(\alpha)}, F_{F(\alpha)^c} = T_{F(\alpha)}$ and $\mu^c(\alpha) = \overline{1} - \mu(\alpha) \quad \forall\alpha \in E, \forall t \in T$. Now $((F_t^{\mu}, A)^c)^c = ((F_t^{\mu(c)})^c, \neg A)$ where

$$\begin{aligned} F_t^{\mu(c)}(\alpha) &= [T_{F(\alpha)^c} = F_{F(\alpha)}, I_{F(\alpha)^c} = \overline{1} - I_{F(\alpha)}, F_{F(\alpha)^c} = T_{F(\alpha)}, \mu^c(\alpha) = \overline{1} - \mu(\alpha)]^c \\ &= [T_{F(\alpha)} = F_{F(\alpha)^c}, I_{F(\alpha)} = \overline{1} - I_{F(\alpha)^c}, F_{F(\alpha)} = T_{F(\alpha)^c}, \mu(\alpha) = \overline{1} - \mu^c(\alpha)] \\ &= [T_{F(\alpha)} = F_{F(\alpha)^c}, I_{F(\alpha)} = \overline{1} - (\overline{1} - I_{F(\alpha)}), F_{F(\alpha)} = T_{F(\alpha)^c}, \mu(\alpha) = \overline{1} - (\overline{1} - \mu(\alpha))] \\ &= [T_{F(\alpha)} = F_{F(\alpha)^c}, I_{F(\alpha)} = I_{F(\alpha)}, F_{F(\alpha)} = T_{F(\alpha)^c}, \mu(\alpha) = \mu(\alpha)] \\ &= (F_t^{\mu}, A), \forall\alpha \in E, \forall t \in T. \end{aligned}$$

The proof of the propositions (ii)-(iii) are obvious. □

Definition 20 *The union of two entities $\{T-NSESs\}$ (F_t^{μ}, A) and (G_t^{η}, B) over U, denoted by "$(F_t^{\mu}, A)\tilde{\cup}(G_t^{\eta}, B)$" is the $\{$T-NSESs$\}(F_t^{\mu}, A)\tilde{\cup}(G_t^{\eta}, B) = (H_t^{\Omega}, C)$, where $C = A \cup B$ and the truth-membership, indeterminacy-membership and falsity-membership of (H_t^{Ω}, C) are as follows:*

$$T_{H_t^{\Omega}(e)}(m) = \begin{cases} T_{F_t^{\mu}(e)}(m), & \text{if } e \in A - B\ ; \\ T_{G_t^{\eta}(e)}(m), & \text{if } e \in B - A; \\ \max(T_{F_t^{\mu}(e)}(m), T_{G_t^{\eta}(e)}(m)), & \text{if } e \in A \cap B. \end{cases}$$

$$I_{H_t^{\Omega}(e)}(m) = \begin{cases} I_{F_t^{\mu}(e)}(m), & \text{if } e \in A - B\ ; \\ I_{G_t^{\eta}(e)}(m), & \text{if } e \in B - A; \\ \min(I_{F_t^{\mu}(e)}(m), I_{G_t^{\eta}(e)}(m)), & \text{if } e \in A \cap B. \end{cases}$$

$$F_{H_t^{\Omega}(e)}(m) = \begin{cases} F_{F_t^{\mu}(e)}(m), & \text{if } e \in A - B\ ; \\ F_{G_t^{\eta}(e)}(m), & \text{if } e \in B - A; \\ \min(F_{F_t^{\mu}(e)}(m), F_{G_t^{\eta}(e)}(m)), & \text{if } e \in A \cap B. \end{cases}$$

and where $\Omega(m) = \max(\mu_{(e)}(m), \eta_{(e)}(m))$.

Example 6 Suppose that (F_t^{μ}, A) and (G_t^{η}, B) are two $\{T-NSESs\}$ over U, such that

$$\begin{aligned} (F_t^{\mu}, A) = \quad & \{[(e_1, p, 1), (\tfrac{u_1^{t_1}}{0.4,0.3,0.2}, \tfrac{u_2^{t_1}}{0.6,0.1,0.8}, \tfrac{u_3^{t_1}}{0.5,0.7,0.2}), 0.3], \\ & [(e_2, p, 1), (\tfrac{u_1^{t_1}}{0.7,0.3,0.6}, \tfrac{u_2^{t_1}}{0.5,0.1,0.4}, \tfrac{u_3^{t_1}}{0.8,0.6,0.3}), 0.2], \\ & [(e_2, q, 0), (\tfrac{u_1^{t_2}}{0.4,0.3,0.6}, \tfrac{u_2^{t_2}}{0.7,0.2,0.5}, \tfrac{u_3^{t_2}}{0.8,0.1,0.4}), 0.6], \\ & [(e_3, r, 1), (\tfrac{u_1^{t_3}}{0.7,0.4,0.6}, \tfrac{u_2^{t_3}}{0.5,0.3,0.6}, \tfrac{u_3^{t_3}}{0.1,0.4,0.2}), 0.5]\}. \end{aligned}$$

$$\begin{aligned} (G_t^{\eta}, B) = \quad & \{[(e_1, p, 1), (\tfrac{u_1^{t_1}}{0.6,0.5,0.1}, \tfrac{u_2^{t_1}}{0.8,0.2,0.3}, \tfrac{u_3^{t_1}}{0.9,0.2,0.3}), 0.1], \\ & [(e_2, p, 1), (\tfrac{u_1^{t_1}}{0.6,0.7,0.1}, \tfrac{u_2^{t_1}}{0.8,0.4,0.7}, \tfrac{u_3^{t_1}}{0.5,0.1,0.7}), 0.4], \\ & [(e_3, r, 1), (\tfrac{u_1^{t_3}}{0.4,0.1,0.2}, \tfrac{u_2^{t_3}}{0.5,0.4,0.2}, \tfrac{u_3^{t_3}}{0.3,0.6,0.4}), 0.8]\}. \end{aligned}$$

Then $(F_t^\mu, A)\tilde{\cup}(G_t^\eta, B) = (H_t^\Omega, C)$ where

$$\begin{aligned}(H_t^\Omega, C) = \quad & \{[(e_1,p,1),(\tfrac{u_1^{t1}}{0.6,0.3,0.1},\tfrac{u_2^{t1}}{0.8,0.1,0.3},\tfrac{u_3^{t1}}{0.9,0.2,0.2}),0.3],\\ & [(e_2,p,1),(\tfrac{u_1^{t1}}{0.6,0.3,0.1},\tfrac{u_2^{t1}}{0.8,0.2,0.5},\tfrac{u_3^{t1}}{0.7,0.1,0.4}),0.4],\\ & [(e_2,q,0),(\tfrac{u_1^{t2}}{0.4,0.3,0.6},\tfrac{u_2^{t2}}{0.7,0.2,0.5},\tfrac{u_3^{t2}}{0.8,0.1,0.4}),0.6],\\ & [(e_3,r,1),(\tfrac{u_1^{t3}}{0.8,0.1,0.2},\tfrac{u_2^{t3}}{0.5,0.3,0.2},\tfrac{u_3^{t3}}{0.3,0.4,0.2}),0.8]\}.\end{aligned}$$

Proposition 2 *If $(F_t^\mu, A), (G_t^\eta, B)$and (H_t^Ω, C) are three $\{T-NSESs\}$ over U, then*
(i) $(F_t^\mu, A)\tilde{\cup}(G_t^\eta, B)\tilde{\cup}(H_t^\Omega, C) = (F_t^\mu, A)\tilde{\cup}((G_t^\eta, B)\tilde{\cup}(H_t^\Omega, C))$
(ii) $(F_t^\mu, A)\tilde{\cup}(F_t^\mu, A)\tilde{\subseteq}(F_t^\mu, A)$.

Proof
(i) We want to prove that

$$(F_t^\mu, A)\tilde{\cup}(G_t^\eta, B)\tilde{\cup}(H_t^\Omega, C) = (F_t^\mu, A)\tilde{\cup}((G_t^\eta, B)\tilde{\cup}(H_t^\Omega, C))$$

by using Definition 20. We consider the case when $e \in A \cap B$ as other cases trivial. Then we have

$$\begin{aligned}(F_t^\mu, A)\tilde{\cup}(G_t^\eta, B) = \{&(u^t \max(T_{F_t^\mu(e)}(m), T_{G_t^\eta(e)}(m)), \min(I_{F_t^\mu(e)}(m), I_{G_t^\eta(e)}(m)),\\ &\min(F_{F_t^\mu(e)}(m), F_{G_t^\eta(e)}(m))), \max(\mu_{(e)}(m), \eta_{(e)}(m))u \in U\}.\end{aligned}$$

We also consider the case when $e \in H$ as the other cases are trivial. Then we have

$$\begin{aligned}&(F_t^\mu, A)\tilde{\cup}(G_t^\eta, B)\tilde{\cup}(H_t^\Omega, C)\\ &= \{(u^t/\max(T_{F_t^\mu(e)}(m), T_{G_t^\eta(e)}(m)), \min(I_{F_t^\mu(e)}(m), I_{G_t^\eta(e)}(m)), \min(F_{F_t^\mu(e)}(m), F_{G_t^\eta(e)}(m))),\\ &\quad \max(\mu_{(e)}(m), \eta_{(e)}(m)), (u^t/T_{H_t^\Omega(e)}(m), I_{H_t^\Omega(e)}(m), F_{H_t^\Omega(e)}(m), \max(\mu_{(e)}(m), \eta_{(e)}(m), \Omega_{(e)}(m))u \in U\}\\ &= \{u^t/T_{F_t^{\mu(e)}}(m), I_{F_t^{\mu(e)}}(m), F_{F_t^{\mu(e)}}(m), (u^t/\max(T_{H_t^\Omega(e)}(m), T_{G_t^\eta(e)}(m)), \min(I_{H_t^\Omega(e)}(m), I_{G_t^\eta(e)}(m)),\\ &\quad \min(F_{H_t^\Omega(e)}(m), F_{G_t^\eta(e)}(m))), \max(\mu_{(e)}(m), \eta_{(e)}(m)), \max(\mu_{(e)}(m), \eta_{(e)}(m), \Omega_{(e)}(m))u \in U\}\\ &= (F_t^\mu, A)\tilde{\cup}((G_t^\eta, B)\tilde{\cup}(H_t^\Omega, C)).\end{aligned}$$

(ii) The proof is straightforward. □

Definition 21 *The intersection of two $\{T\text{-}NSESs\}$ (F_t^μ, A) and (G_t^η, B) over U, denoted by "$(F_t^\mu, A)\tilde{\cap}(G_t^\eta, B)$" is the $\{T\text{-}NSESs\}(F_t^\mu, A)\tilde{\cap}(G_t^\eta, B) = (K_t^\delta, C)$, where $C = A \cap B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K_t^δ, C) are as follows:*

$$T_{K_t^\delta(e)}(m) = \begin{cases} T_{F_t^\mu(e)}(m), & \text{if } e \in A - B\ ;\\ T_{G_t^\eta(e)}(m), & \text{if } e \in B - A;\\ \min(T_{F_t^\mu(e)}(m), T_{G_t^\eta(e)}(m)), & \text{if } e \in A \cap B.\end{cases}$$

$$I_{K_t^\delta(e)}(m) = \begin{cases} I_{F_t^\mu(e)}(m), & \text{if } e \in A - B\ ;\\ I_{G_t^\eta(e)}(m), & \text{if } e \in B - A;\\ \max(I_{F_t^\mu(e)}(m), I_{G_t^\eta(e)}(m)), & \text{if } e \in A \cap B.\end{cases}$$

$$F_{K_t^\delta(e)}(m) = \begin{cases} F_{F_t^\mu(e)}(m), & \text{if } e \in A - B\ ; \\ F_{G_t^\eta(e)}(m), & \text{if } e \in B - A; \\ \max(F_{F_t^\mu(e)}(m), F_{G_t^\eta(e)}(m)), & \text{if } e \in A \cap B. \end{cases}$$

and where $\delta(m) = \min(\mu_{(e)}(m), \eta_{(e)}(m))$.

Example 7 Suppose that (F_t^μ, A) and (G_t^η, B) are two $\{T-NSESs\}$ over U, such that

$$\begin{aligned}(F_t^\mu, A) = \ & \{[(e_1, p, 1), (\tfrac{u_1^{t_1}}{0.4,0.3,0.2}, \tfrac{u_2^{t_1}}{0.6,0.1,0.8}, \tfrac{u_3^{t_1}}{0.5,0.7,0.2}), 0.3], \\ & [(e_2, q, 1), (\tfrac{u_1^{t_1}}{0.7,0.3,0.6}, \tfrac{u_2^{t_1}}{0.5,0.1,0.4}, \tfrac{u_3^{t_1}}{0.8,0.6,0.3}), 0.2], \\ & [(e_2, q, 0), (\tfrac{u_1^{t_2}}{0.4,0.3,0.6}, \tfrac{u_2^{t_2}}{0.7,0.2,0.5}, \tfrac{u_3^{t_2}}{0.8,0.1,0.4}), 0.6].\end{aligned}$$

$$\begin{aligned}(G_t^\eta, B) = \ & \{[(e_1, p, 1), (\tfrac{u_1^{t_1}}{0.6,0.5,0.1}, \tfrac{u_2^{t_1}}{0.8,0.2,0.3}, \tfrac{u_3^{t_1}}{0.9,0.2,0.3}), 0.1], \\ & [(e_3, r, 1), (\tfrac{u_1^{t_3}}{0.4,0.1,0.2}, \tfrac{u_2^{t_3}}{0.5,0.4,0.2}, \tfrac{u_3^{t_3}}{0.3,0.6,0.4}), 0.8]\}.\end{aligned}$$

Then $(F_t^\mu, A)\tilde{\cap}(G_t^\eta, B) = (K_t^\delta, C)$ where

$$(K_t^\delta, C) = \ \{[(e_1, p, 1), (\tfrac{u_1^{t_1}}{0.4,0.3,0.2}, \tfrac{u_2^{t_1}}{0.6,0.1,0.8}, \tfrac{u_3^{t_1}}{0.5,0.2,0.3}), 0.1]\}.$$

Proposition 3 *If* $(F_t^\mu, A), (G_t^\eta, B)$ *and* (K_t^δ, C) *are three* $\{T-NSESs\}$ *over* U, *then*

(i) $(F_t^\mu, A)\tilde{\cap}(G_t^\eta, B)\tilde{\cap}(K_t^\delta, C) = (F_t^\mu, A)\tilde{\cap}((G_t^\eta, B)\tilde{\cap}(K_t^\delta, C))$

(ii) $(F_t^\mu, A)\tilde{\cap}(F_t^\mu, A)\tilde{\subseteq}(F_t^\mu, A)$.

Proof

(i) We want to prove that $(F_t^\mu, A)\tilde{\cap}(G_t^\eta, B)\tilde{\cap}(K_t^\delta, C) = (F_t^\mu, A)\tilde{\cap}((G_t^\eta, B)\tilde{\cap}(K_t^\delta, C))$ by using Definition 21. We consider the case when $e \in A \cap B$ as other cases trivial. Then we have

$$\begin{aligned}(F_t^\mu, A)\tilde{\cap}(G_t^\eta, B) = \{&(u^t/\min(T_{F_t^\mu(e)}(m), T_{G_t^\eta(e)}(m)), \max(I_{F_t^\mu(e)}(m), I_{G_t^\eta(e)}(m)), \\ &\max(F_{F_t^\mu(e)}(m), F_{G_t^\eta(e)}(m))), \min(\mu_{(e)}(m), \eta_{(e)}(m))u \in U\}.\end{aligned}$$

We also consider here the case when $e \in K$ as the other cases are trivial. Then we have

$$\begin{aligned}&(F_t^\mu, A)\tilde{\cap}(G_t^\eta, B)\tilde{\cap}(K_t^\delta, C) \\ &= \{(u^t/\min(T_{F_t^\mu(e)}(m), T_{G_t^\eta(e)}(m)), \max(I_{F_t^\mu(e)}(m), I_{G_t^\eta(e)}(m)), \max(F_{F_t^\mu(e)}(m), F_{G_t^\eta(e)}(m))), \\ &\min(\mu_{(e)}(m), \eta_{(e)}(m)), (u^t/T_{K_t^\delta(e)}(m), I_{K_t^\delta(e)}(m), F_{K_t^\delta(e)}(m), \min(\mu_{(e)}(m), \eta_{(e)}(m), \delta_{(e)}(m))u \in U\} \\ &= \{u^t/T_{F_t^{\mu(e)}}(m), I_{F_t^{\mu(e)}}(m), F_{F_t^{\mu(e)}}(m), (u^t/\max(T_{K_t^\delta(e)}(m), T_{G_t^\eta(e)}(m)), \max(I_{K_t^\delta(e)}(m), I_{G_t^\eta(e)}(m)), \\ &\max(F_{K_t^\delta(e)}(m), F_{G_t^\eta(e)}(m))), \min(\mu_{(e)}(m), \eta_{(e)}(m)), \min(\mu_{(e)}(m), \eta_{(e)}(m), \delta_{(e)}(m))u \in U\} \\ &= (F_t^\mu, A)\tilde{\cap}((G_t^\eta, B)\tilde{\cap}(K_t^\delta, C)).\end{aligned}$$

(ii) The proof is straightforward. □

Proposition 4 *If* $(F_t^\mu, A), (G_t^\eta, B)$ *and* (K_t^δ, C) *are three* $\{T-NSESs\}$ *over* U, *then*

(i) $(F_t^\mu, A)\tilde{\cup}(G_t^\eta, B)\tilde{\cap}(K_t^\delta, C) = ((F_t^\mu, A)\tilde{\cap}(K_t^\delta, C))\tilde{\cup}((G_t^\eta, B)\tilde{\cap}(K_t^\delta, C))$.

(ii) $(F_t^\mu, A)\tilde{\cap}(G_t^\eta, B)\tilde{\cup}(K_t^\delta, C) = ((F_t^\mu, A)\tilde{\cup}(K_t^\delta, C))\tilde{\cap}((G_t^\eta, B)\tilde{\cup}(K_t^\delta, C))$.

Proof The proofs can be easily obtained from relative definitions. □

Definition 22 *If* (F_t^μ, A) *and* (G_t^η, B) *are two* $\{$*T-NSESs*$\}$ *over* U*, then "*(F_t^μ, A) *AND* (G_t^η, B)*" denoted by* $(F_t^\mu, A) \wedge (G_t^\eta, B)$ *is denoted by*

$$(F_t^\mu, A) \wedge (G_t^\eta, B) = (H_t^\Omega, A \times B)$$

such that $H_t^\Omega(\alpha, \beta) = F_t^\mu(\alpha) \cap G_t^\eta(\beta)$ *and the truth-membership, indeterminacy-membership and falsity-membership of* $(H_t^\Omega, A \times B)$ *are as follows:*

$$T_{H_t^\Omega(\alpha,\beta)}(m) = \min(T_{F_t^\mu(\alpha)}(m), T_{G_t^\eta(\beta)}(m)),$$

$$I_{H_t^\Omega(\alpha,\beta)}(m) = \max(I_{F_t^\mu(\alpha)}(m), I_{G_t^\eta(\beta)}(m)),$$

$$F_{H_t^\Omega(\alpha,\beta)}(m) = \max(F_{F_t^\mu(\alpha)}(m), F_{G_t^\eta(\beta)}(m)),$$

and $\Omega(m) = \min(\mu_{(e)}(m), \eta_{(e)}(m)), \forall \alpha \in A, \forall \beta \in B,$ *for all* $t \in T$.

Example 8 Suppose that (F_t^μ, A) and (G_t^η, B) are two $\{T - NSESs\}$ over U, such that

$$\begin{aligned}(F_t^\mu, A) = \ & \{[(e_1, p, 1), (\tfrac{u_1^{t_1}}{0.2,0.3,0.5}, \tfrac{u_2^{t_1}}{0.4,0.1,0.2}, \tfrac{u_3^{t_1}}{0.6,0.3,0.7}), 0.4],\\ & [(e_3, r, 0), (\tfrac{u_1^{t_1}}{0.5,0.2,0.1}, \tfrac{u_2^{t_1}}{0.6,0.3,0.7}, \tfrac{u_3^{t_1}}{0.2,0.1,0.8}), 0.3]\}.\end{aligned}$$

$$\begin{aligned}(G_t^\eta, B) = \ & \{[(e_1, p, 1), (\tfrac{u_1^{t_1}}{0.3,0.2,0.6}, \tfrac{u_2^{t_1}}{0.6,0.3,0.2}, \tfrac{u_3^{t_1}}{0.8,0.1,0.2}), 0.5],\\ & [(e_2, q, 0), (\tfrac{u_1^{t_1}}{0.1,0.3,0.5}, \tfrac{u_2^{t_1}}{0.7,0.1,0.6}, \tfrac{u_3^{t_1}}{0.4,0.3,0.6}), 0.6]\}.\end{aligned}$$

Then $(F_t^\mu, A) \wedge (G_t^\eta, B) = (H_t^\Omega, A \times B)$ where

$$\begin{aligned}(H_t^\Omega, A \times B) = \ & \{[(e_1, p, 1), (e_1, p, 1), (\tfrac{u_1^{t_1}}{0.2,0.2,0.6}, \tfrac{u_2^{t_1}}{0.4,0.1,0.2}, \tfrac{u_3^{t_1}}{0.6,0.1,0.7}), 0.4],\\ & [(e_1, p, 1), (e_2, q, 0), (\tfrac{u_1^{t_1}}{0.1,0.3,0.5}, \tfrac{u_2^{t_1}}{0.4,0.1,0.6}, \tfrac{u_3^{t_1}}{0.4,0.3,0.7}), 0.4],\\ & [(e_3, r, 0), (e_1, p, 1), (\tfrac{u_1^{t_2}}{0.3,0.2,0.6}, \tfrac{u_2^{t_2}}{0.6,0.3,0.7}, \tfrac{u_3^{t_2}}{0.2,0.1,0.8}), 0.3],\\ & [(e_3, r, 0), (e_2, q, 0), (\tfrac{u_1^{t_3}}{0.1,0.2,0.5}, \tfrac{u_2^{t_3}}{0.6,0.1,0.7}, \tfrac{u_3^{t_3}}{0.2,0.1,0.8}), 0.3]\}.\end{aligned}$$

Definition 23 *If* (F_t^μ, A) *and* (G_t^η, B) *are two* $\{$*T-NSESs*$\}$ *over* U*, then "*(F_t^μ, A) *OR* (G_t^η, B)*" denoted by* $(F_t^\mu, A) \vee (G_t^\eta, B)$ *is denoted by*

$$(F_t^\mu, A) \vee (G_t^\eta, B) = (K_t^\delta, A \times B)$$

such that $K_t^\delta(\alpha, \beta) = F_t^\mu(\alpha) \cup G_t^\eta(\beta)$ *and the truth-membership, indeterminacy-membership and falsity-membership of* $(K_t^\delta, A \times B)$ *are as follows:*

$$T_{K_t^\delta(\alpha,\beta)}(m) = \max(T_{F_t^\mu(\alpha)}(m), T_{G_t^\eta(\beta)}(m)),$$

$$I_{K_t^\delta(\alpha,\beta)}(m) = \min(I_{F_t^\mu(\alpha)}(m), I_{G_t^\eta(\beta)}(m)),$$

$$F_{K_t^\delta(\alpha,\beta)}(m) = \min(F_{F_t^\mu(\alpha)}(m), F_{G_t^\eta(\beta)}(m)),$$

and $\delta(m) = \max(\mu_{(e)}(m), \eta_{(e)}(m)), \forall \alpha \in A, \forall \beta \in B, forallt \in T.$

Example 9 Suppose that (F_t^μ, A) and (G_t^η, B) are two $\{T-NSESs\}$ over U, such that

$$\begin{aligned}(F_t^\mu, A) = \ & \{[(e_1,p,1),(\tfrac{u_1^{t_1}}{0.2,0.3,0.5},\tfrac{u_2^{t_1}}{0.4,0.1,0.2},\tfrac{u_3^{t_1}}{0.6,0.3,0.7}),0.4],\\ & [(e_3,r,0),(\tfrac{u_1^{t_1}}{0.5,0.2,0.1},\tfrac{u_2^{t_1}}{0.6,0.3,0.7},\tfrac{u_3^{t_1}}{0.2,0.1,0.8}),0.3]\}.\\ (G_t^\eta, B) = \ & \{[(e_1,p,1),(\tfrac{u_1^{t_1}}{0.3,0.2,0.6},\tfrac{u_2^{t_1}}{0.6,0.3,0.2},\tfrac{u_3^{t_1}}{0.8,0.1,0.2}),0.5],\\ & [(e_2,q,0),(\tfrac{u_1^{t_1}}{0.1,0.3,0.5},\tfrac{u_2^{t_1}}{0.7,0.1,0.6},\tfrac{u_3^{t_1}}{0.4,0.3,0.6}),0.6]\}.\end{aligned}$$

Then $(F_t^\mu, A) \vee (G_t^\eta, B) = (K_t^\delta, A\times B)$ where

$$\begin{aligned}(K_t^\delta, A\times B) = \ & \{[(e_1,p,1),(e_1,p,1),(\tfrac{u_1^{t_1}}{0.3,0.2,0.5},\tfrac{u_2^{t_1}}{0.6,0.1,0.2},\tfrac{u_3^{t_1}}{0.8,0.1,0.2}),0.5],\\ & [(e_1,p,1),(e_2,q,0),(\tfrac{u_1^{t_1}}{0.2,0.3,0.5},\tfrac{u_2^{t_1}}{0.7,0.1,0.2},\tfrac{u_3^{t_1}}{0.6,0.3,0.6}),0.6],\\ & [(e_3,r,0),(e_1,p,1),(\tfrac{u_1^{t_2}}{0.5,0.2,0.1},\tfrac{u_2^{t_2}}{0.7,0.3,0.6},\tfrac{u_3^{t_2}}{0.8,0.1,0.2}),0.5],\\ & [(e_3,r,0),(e_2,q,0),(\tfrac{u_1^{t_3}}{0.5,0.2,0.1},\tfrac{u_2^{t_3}}{0.7,0.1,0.6},\tfrac{u_3^{t_3}}{0.4,0.1,0.6}),0.6]\}.\end{aligned}$$

Proposition 5 *If (F_t^μ, A) and (G_t^η, B) are time- neutrosophic soft expert sets over U. Then*

(i) $((F_t^\mu, A)\wedge(G_t^\eta, B))^c = (F_t^\mu, A)^c \vee (G_t^\eta, B)^c$.

(ii) $((F_t^\mu, A)\vee(G_t^\eta, B))^c = (F_t^\mu, A)^c \wedge (G_t^\eta, B)^c$.

Proof The proofs can be easily obtained from relative definitions. □

4 An Application of Time-Neutrosophic Soft Expert Set

In this section, we present an application of time-neutrosophic soft expert set theory in a decision-making problem which demonstrates that this method can be successfully applied to problems of many fields that contain uncertainty. We suggest the following algorithm to solve time-neutrosophic soft expert based decision making problems.

Suppose you want to get workplace worker. Five alternatives are as follows: $U = \{u_1, u_2, u_3, u_4, u_5\}$, suppose there are four parameters $E = \{e_1, e_2, e_3, e_4\}$ where the parameters $e_i (i = 1, 2, 3, 4)$ stand for "education," "age," "capability" and "experience" respectively and $T = \{t_1, t_2, t_3\}$ be set of time. Let X={p,q,r} be a set of experts. From those findings we can find the most suitable choice for the decision. After a serious discussion, the experts construct the following time-nuetrosophic soft expert set (F_t^μ, Z) given in the next page.

In Tables 1 and 2 we present the agree-time neutrosophic soft expert set and disagree-neutrosophic soft expert set. Now to determine the best choices, we first mark the highest numerical degree underline in each row in agree-time-neutrosophic soft expert set and disagree-time-neutrosophic soft expert set excluding the last column which is the degree of such belongingness of an expert against of parameters. Then we calculate the score of each of such expert in agree-time-neutrosophic soft expert set and disagree-time-neutrosophic soft expert set by taking the sum of the products of these numerical degrees with the corresponding values of λ. Then we calculate the final score by subtracting the score of expert in the agree-time-neutrosophic soft expert set from the scores of expert in disagree-time-neutrosophic soft expert set. The expert with the highest score is the desired expert.

$$
\begin{aligned}
(F_t^{\mu}, Z) = \; & \{[(e_1,p,1),(\frac{u_1^{t_1}}{0.2,0.3,0.4},\frac{u_2^{t_1}}{0.8,0.2,0.6},\frac{u_3^{t_1}}{0.6,0.3,0.5},\frac{u_4^{t_1}}{0.4,0.2,0.3},\frac{u_5^{t_1}}{0.6,0.3,0.1}),0.8],\\
& [(e_1,q,1),(\frac{u_1^{t_2}}{0.3,0.1,0.4},\frac{u_2^{t_2}}{0.2,0.1,0.5},\frac{u_3^{t_2}}{0.4,0.3,0.2},\frac{u_4^{t_2}}{0.4,0.2,0.3},\frac{u_5^{t_2}}{0.7,0.2,0.5}),0.5],\\
& [(e_1,r,1),(\frac{u_1^{t_3}}{0.3,0.5,0.1},\frac{u_2^{t_3}}{0.6,0.2,0.5},\frac{u_3^{t_3}}{0.1,0.4,0.2},\frac{u_4^{t_3}}{0.5,0.2,0.3},\frac{u_5^{t_3}}{0.4,0.3,0.2}),0.3],\\
& [(e_2,p,1),(\frac{u_1^{t_1}}{0.6,0.2,0.3},\frac{u_2^{t_1}}{0.4,0.2,0.5},\frac{u_3^{t_1}}{0.3,0.4,0.1},\frac{u_4^{t_1}}{0.7,0.3,0.6},\frac{u_5^{t_1}}{0.5,0.2,0.4}),0.6],\\
& [(e_2,q,1),(\frac{u_1^{t_2}}{0.1,0.3,0.6},\frac{u_2^{t_2}}{0.7,0.3,0.1},\frac{u_3^{t_2}}{0.6,0.2,0.5},\frac{u_4^{t_2}}{0.3,0.1,0.6},\frac{u_5^{t_2}}{0.4,0.3,0.2}),0.5],\\
& [(e_2,r,1),(\frac{u_1^{t_3}}{0.6,0.3,0.5},\frac{u_2^{t_3}}{0.7,0.2,0.6},\frac{u_3^{t_3}}{0.5,0.3,0.4},\frac{u_4^{t_3}}{0.2,0.1,0.3},\frac{u_5^{t_3}}{0.6,0.2,0.5}),0.4],\\
& [(e_3,p,1),(\frac{u_1^{t_1}}{0.2,0.4,0.6},\frac{u_2^{t_1}}{0.7,0.4,0.2},\frac{u_3^{t_1}}{0.4,0.1,0.2},\frac{u_4^{t_1}}{0.8,0.4,0.3},\frac{u_5^{t_1}}{0.7,0.3,0.4}),0.1],\\
& [(e_3,q,1),(\frac{u_1^{t_2}}{0.4,0.2,0.6},\frac{u_2^{t_2}}{0.5,0.3,0.6},\frac{u_3^{t_2}}{0.6,0.2,0.7},\frac{u_4^{t_2}}{0.8,0.2,0.4},\frac{u_5^{t_2}}{0.6,0.2,0.3}),0.7],\\
& [(e_3,r,1),(\frac{u_1^{t_3}}{0.3,0.6,0.5},\frac{u_2^{t_3}}{0.6,0.2,0.5},\frac{u_3^{t_3}}{0.2,0.1,0.4},\frac{u_4^{t_3}}{0.5,0.3,0.2},\frac{u_5^{t_3}}{0.4,0.1,0.5}),0.4],\\
& [(e_4,p,1),(\frac{u_1^{t_1}}{0.2,0.3,0.6},\frac{u_2^{t_1}}{0.7,0.1,0.5},\frac{u_3^{t_1}}{0.4,0.2,0.8},\frac{u_4^{t_1}}{0.9,0.2,0.4},\frac{u_5^{t_1}}{0.3,0.4,0.6}),0.5],\\
& [(e_4,q,1),(\frac{u_1^{t_2}}{0.5,0.2,0.3},\frac{u_2^{t_2}}{0.2,0.3,0.4},\frac{u_3^{t_2}}{0.4,0.1,0.5},\frac{u_4^{t_2}}{0.6,0.3,0.2},\frac{u_5^{t_2}}{0.7,0.3,0.4}),0.5],\\
& [(e_4,r,1),(\frac{u_1^{t_3}}{0.5,0.2,0.1},\frac{u_2^{t_3}}{0.6,0.3,0.5},\frac{u_3^{t_3}}{0.2,0.5,0.3},\frac{u_4^{t_3}}{0.5,0.1,0.4},\frac{u_5^{t_3}}{0.3,0.2,0.5}),0.4],\\
& [(e_1,p,0),(\frac{u_1^{t_1}}{0.2,0.3,0.4},\frac{u_2^{t_1}}{0.5,0.3,0.1},\frac{u_3^{t_1}}{0.6,0.3,0.4},\frac{u_4^{t_1}}{0.6,0.2,0.4},\frac{u_5^{t_1}}{0.7,0.5,0.6}),0.8],\\
& [(e_1,q,0),(\frac{u_1^{t_2}}{0.5,0.1,0.7},\frac{u_2^{t_2}}{0.4,0.2,0.3},\frac{u_3^{t_2}}{0.8,0.5,0.4},\frac{u_4^{t_2}}{0.7,0.3,0.6},\frac{u_5^{t_2}}{0.5,0.3,0.4}),0.6],\\
& [(e_1,r,0),(\frac{u_1^{t_3}}{0.3,0.1,0.6},\frac{u_2^{t_3}}{0.6,0.3,0.7},\frac{u_3^{t_3}}{0.3,0.2,0.4},\frac{u_4^{t_3}}{0.8,0.1,0.4},\frac{u_5^{t_3}}{0.6,0.4,0.5}),0.7],\\
& [(e_2,p,0),(\frac{u_1^{t_1}}{0.7,0.3,0.5},\frac{u_2^{t_1}}{0.6,0.2,0.4},\frac{u_3^{t_1}}{0.4,0.3,0.5},\frac{u_4^{t_1}}{0.3,0.2,0.5},\frac{u_5^{t_1}}{0.4,0.3,0.5}),0.9],\\
& [(e_2,q,0),(\frac{u_1^{t_2}}{0.6,0.2,0.4},\frac{u_2^{t_2}}{0.5,0.3,0.7},\frac{u_3^{t_2}}{0.8,0.1,0.3},\frac{u_4^{t_2}}{0.2,0.3,0.6},\frac{u_5^{t_2}}{0.6,0.2,0.4}),0.3],\\
& [(e_2,r,0),(\frac{u_1^{t_3}}{0.6,0.3,0.4},\frac{u_2^{t_3}}{0.5,0.2,0.4},\frac{u_3^{t_3}}{0.7,0.4,0.5},\frac{u_4^{t_3}}{0.5,0.2,0.4},\frac{u_5^{t_3}}{0.4,0.3,0.5}),0.1],\\
& [(e_3,p,0),(\frac{u_1^{t_1}}{0.6,0.2,0.4},\frac{u_2^{t_1}}{0.6,0.1,0.5},\frac{u_3^{t_1}}{0.5,0.4,0.6},\frac{u_4^{t_1}}{0.8,0.3,0.6},\frac{u_5^{t_1}}{0.7,0.2,0.4}),0.4],\\
& [(e_3,q,0),(\frac{u_1^{t_2}}{0.7,0.1,0.6},\frac{u_2^{t_2}}{0.4,0.5,0.8},\frac{u_3^{t_2}}{0.4,0.3,0.5},\frac{u_4^{t_2}}{0.6,0.2,0.5},\frac{u_5^{t_2}}{0.4,0.3,0.1}),0.2],\\
& [(e_3,r,0),(\frac{u_1^{t_3}}{0.2,0.3,0.6},\frac{u_2^{t_3}}{0.7,0.4,0.5},\frac{u_3^{t_3}}{0.4,0.2,0.8},\frac{u_4^{t_3}}{0.9,0.1,0.4},\frac{u_5^{t_3}}{0.6,0.3,0.2}),0.3],\\
& [(e_4,p,0),(\frac{u_1^{t_1}}{0.4,0.2,0.6},\frac{u_2^{t_1}}{0.5,0.2,0.6},\frac{u_3^{t_1}}{0.9,0.5,0.1},\frac{u_4^{t_1}}{0.3,0.2,0.6},\frac{u_5^{t_1}}{0.4,0.3,0.5}),0.6],\\
& [(e_4,q,0),(\frac{u_1^{t_2}}{0.3,0.2,0.1},\frac{u_2^{t_2}}{0.6,0.1,0.5},\frac{u_3^{t_2}}{0.6,0.2,0.5},\frac{u_4^{t_2}}{0.8,0.3,0.2},\frac{u_5^{t_2}}{0.2,0.3,0.4}),0.5],\\
& [(e_4,r,0),(\frac{u_1^{t_3}}{0.6,0.2,0.5},\frac{u_2^{t_3}}{0.7,0.1,0.6},\frac{u_3^{t_3}}{0.5,0.3,0.1},\frac{u_4^{t_3}}{0.3,0.2,0.6},\frac{u_5^{t_3}}{0.4,0.3,0.1}),0.1]\}.
\end{aligned}
$$

The following algorithm may be followed by you want to get workplace worker.

Now calculate the score of u_i by using the data in Table 3:

Score $(u_1) = 0$

Score $(u_2) = (0.8 * 0.7) + (0.7 * 0.4) + (0.6 * 0.2) + (0.7 * 0.5) + (0.6 * 0.5) + (0.6 * 0.3) = 1.79$

Score $(u_3) = 0$

Score $(u_4) = (0.7 * 0.8) + (0.8 * 0.3) + (0.9 * 0.6) + (0.8 * 0.4) = 1.66$

Score $(u_5) = (0.7 * 0.6) + (0.7 * 0.6) = 0.84$

Table 1: Agree-time-neutrosophic Soft Expert Set

U	u_1	u_2	u_3	u_4	u_5	λ
(e_1, p)	0.2,0.3,0.4	0.8,0.2,0.6	0.6,0.3,0.5	0.4,0.2,0.3	0.6,0.3,0.1	0.8
(e_2, p)	0.3,0.1,0.4	0.2,0.1,0.5	0.4,0.3,0.2	0.4,0.3,0.2	0.7,0.2,0.5	0.5
(e_3, p)	0.3,0.5,0.1	0.6,0.2,0.5	0.1,0.4,0.2	0.5,0.2,0.3	0.4,0.3,0.2	0.3
(e_4, p)	0.6,0.2,0.3	0.4,0.2,0.5	0.3,0.4,0.1	0.7,0.3,0.6	0.5,0.2,0.4	0.6
(e_1, q)	0.1,0.3,0.6	0.7,0.3,0.1	0.6,0.2,0.5	0.3,0.1,0.6	0.4,0.3,0.2	0.5
(e_2, q)	0.6,0.3,0.5	0.7,0.2,0.6	0.5,0.3,0.4	0.2,0.1,0.3	0.6,0.2,0.5	0.4
(e_3, q)	0.2,0.4,0.6	0.7,0.4,0.2	0.4,0.1,0.2	0.8,0.4,0.3	0.7,0.3,0.4	0.1
(e_4, q)	0.4,0.2,0.6	0.5,0.3,0.6	0.6,0.2,0.7	0.8,0.2,0.4	0.6,0.2,0.3	0.7
(e_1, r)	0.3,0.6,0.5	0.6,0.2,0.5	0.2,0.1,0.4	0.5,0.3,0.2	0.4,0.1,0.5	0.4
(e_2, r)	0.2,0.3,0.6	0.2,0.3,0.4	0.4,0.2,0.8	0.2,0.5,0.3	0.3,0.4,0.6	0.5
(e_3, r)	0.5,0.2,0.3	0.6,0.3,0.5	0.4,0.1,0.5	0.6,0.3,0.2	0.7,0.3,0.4	0.5
(e_4, r)	0.5,0.2,0.1	0.5,0.3,0.1	0.2,0.5,0.3	0.5,0.1,0.4	0.3,0.2,0.5	0.1

Table 2: Disagree-time-neutrosophic Soft Expert Set

U	u_1	u_2	u_3	u_4	u_5	λ
(e_1, p)	0.2,0.3,0.4	0.8,0.2,0.6	0.6,0.3,0.5	0.4,0.2,0.3	0.6,0.3,0.1	0.8
(e_2, p)	0.3,0.1,0.4	0.2,0.1,0.5	0.4,0.3,0.2	0.4,0.3,0.2	0.7,0.2,0.5	0.5
(e_3, p)	0.3,0.5,0.1	0.6,0.2,0.5	0.1,0.4,0.2	0.5,0.2,0.3	0.4,0.3,0.2	0.3
(e_4, p)	0.6,0.2,0.3	0.4,0.2,0.5	0.3,0.4,0.1	0.7,0.3,0.6	0.5,0.2,0.4	0.6
(e_1, q)	0.1,0.3,0.6	0.7,0.3,0.1	0.6,0.2,0.5	0.3,0.1,0.6	0.4,0.3,0.2	0.5
(e_2, q)	0.6,0.3,0.5	0.7,0.2,0.6	0.5,0.3,0.4	0.2,0.1,0.3	0.6,0.2,0.5	0.4
(e_3, q)	0.2,0.4,0.6	0.7,0.4,0.2	0.4,0.1,0.2	0.8,0.4,0.3	0.7,0.3,0.4	0.1
(e_4, q)	0.4,0.2,0.6	0.5,0.3,0.6	0.6,0.2,0.7	0.8,0.2,0.4	0.6,0.2,0.3	0.7
(e_1, r)	0.3,0.6,0.5	0.6,0.2,0.5	0.2,0.1,0.4	0.5,0.3,0.2	0.4,0.1,0.5	0.4
(e_2, r)	0.2,0.3,0.6	0.2,0.3,0.4	0.4,0.2,0.8	0.2,0.5,0.3	0.3,0.4,0.6	0.5
(e_3, r)	0.5,0.2,0.3	0.6,0.3,0.5	0.4,0.1,0.5	0.6,0.3,0.2	0.7,0.3,0.4	0.5
(e_4, r)	0.5,0.2,0.1	0.5,0.3,0.1	0.2,0.5,0.3	0.5,0.1,0.4	0.3,0.2,0.5	0.1

Now calculate the score of u_i by using the data in Table 4:
Score $(u_1) = (0.7 * 0.8) + (0.7 * 0.3) = 0.77$
Score $(u_2) = (0.7 * 0.1) = 0.07$
Score $(u_3) = (0.9 * 0.6) + (0.8 * 0.7) + (0.8 * 0.4) + (0.7 * 0.2) = 1.56$
Score $(u_4) = (0.8 * 0.5) + (0.8 * 0.5) + (0.8 * 0.6) + (0.9 * 0.3) = 1.55$
Score $(u_5) = (0.7 * 0.9) = 0.63$.
The final score of u_i is as follows:
Score $(u_1) = 0 - 0.77 = -0.77$,
Score $(u_2) = 1.79 - 0.07 = 1.72$,
Score $(u_3) = 0 - 1.56 = -1.56$,
Score $(u_4) = 1.66 - 1.55 = 0.11$,
Score $(u_5) = 0.84 - 0.63 = 0.21$.

Table 3: Degree Table of Agree-time- neutrosophic Soft Expert Set

R	u_i	Highest numerical degree	λ
(e_1, p)	u_2	0.8	0.8
(e_2, p)	u_4	0.7	0.5
(e_3, p)	u_4	0.8	0.3
(e_4, p)	u_4	0.9	0.6
(e_1, q)	u_5	0.7	0.5
(e_2, q)	u_2	0.7	0.4
(e_3, q)	u_4	0.8	0.1
(e_4, q)	u_5	0.7	0.7
(e_1, r)	u_2	0.6	0.4
(e_2, r)	u_2	0.7	0.5
(e_3, r)	u_2	0.6	0.5
(e_4, r)	u_2	0.6	0.1

Table 4: Degree Table of Disagree-time-neutrosophic Soft Expert Set

R	u_i	Highest numerical degree	λ
(e_1, p)	u_5	0.7	0.8
(e_2, p)	u_1	0.7	0.5
(e_3, p)	u_4	0.8	0.3
(e_4, p)	u_3	0.9	0.6
(e_1, q)	u_3	0.8	0.5
(e_2, q)	u_3	0.8	0.4
(e_3, q)	u_1	0.7	0.1
(e_4, q)	u_4	0.8	0.7
(e_1, r)	u_4	0.8	0.4
(e_2, r)	u_3	0.7	0.5
(e_3, r)	u_4	0.9	0.5
(e_4, r)	u_2	0.7	0.1

Clearly, the maximum score is the score 1.72, shown in the above for the u_2. Hence the best decision for experts are to select u_2, followed by u_5.

5 Conclusion

The aim of our work, "time-neutrosophic soft expert set" aid in all areas, will give the best decision without the need for experts in the field. In this study, we add the indeterminacy parameters and differently previous studies we have shown in this study, could increase the number of specialists. We have introduced the concept of time-neutrosophic soft expert set which is more effective and useful and studied some of its properties. Also the basic operations on neutrosophic soft expert set namely complement, union, intersection, AND and OR have been defined. Finally, we present an application of T-NSESs in a decision making problem.

References

[1] Atanassov, K. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems.* 1986. 20: 87–96.

[2] Smarandache, F. Neutrosophic set, a generalization of the intuitionistic fuzzy sets. *Int. J. Pure Appl. Math.* 2005. 24: 287–297.

[3] Smarandache, F. *A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic*. Rehoboth: American Research Press. 1998.

[4] Molodtsov, D. A. Soft set theory-first results. *Computers and Mathematics with Applications.* 1999 37: 19–31.

[5] Chen, D., Tsang, E. C. C., Yeung, D. S. and Wang, X. The parameterization reduction of soft sets and its application. *Computers and Mathematics with Applications.* 2005. 49: 757–763

[6] Maji, P. K., Roy, A. R. and Biswas, R. Soft set theory. *Computers and Mathematics with Applications.* 2003. 45(4-5): 555–562.

[7] Maji, P. K. Neutrosophic soft set. *Computers and Mathematics with Applications.* 2013. 45: 555–562

[8] Alkhazaleh, S., Salleh, A. R. and Hassan, N. *Possibility Fuzzy Soft Set. Advances in Decision Sciences.* 2011. 18 pages.

[9] Alkhazaleh, S., A. R. Salleh and Hassan, N. Fuzzy parameterized interval-valued fuzzy soft set. *Applied Mathematical Sciences.* 2011. 5(67): 3335–3346.

[10] Alkhazaleh, S. and Salleh, A. R. Soft expert sets. Advances in Decision Sciences, 15 pages

[11] Alkhazaleh, S. and Salleh, A. R. Fuzzy soft expert set and its application. *Applied Mathematics.* 2014. 5: 1349–1368. http://dx.doi.org/10.4236/am. 2014.59127

[12] Şahin, M., Alkhazaleh, S., Uluçay, V. Neutrosophic soft expert sets. *Applied Mathematics.* 2015. 6(01): 116–127.

[13] Hazaymeh, A. A. *Fuzzy Soft Set and Fuzzy Soft Expert Set: Some Generalizations and Hypothetical Applications.* Universiti Sains Islam Malaysia, Nilai, Negeri Sembilan. Ph.D. Thesis. 2013.