Neutrosophic MULTIMOORA: A Solution for the Standard Error in Information Sampling

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Abstract

If complete Data Mining is not possible one has to be satisfied with an information sample, as much representative as possible. The Belgian company "CIM" is doing marketing research for all Belgian newspapers, magazines and cinema. For some local newspapers, it arrives at a standard error of more than 15% or a spread of more than 30%, which is scientific nonsense but accepted by the publishers of advertisement. On the other side technical problems will ask for a much smaller standard deviation like for instance a standard error of 0.1% for the possibility that a dike is not strong enough for an eventual spring tide. Somewhat in between the usual standard error for marketing research is 5%. Is it possible to avoid this Spread by Sampling? Here Multi-Objective Optimization Methods may help. The Neutrosophic MULTIMOORA method, chosen for its robustness compared to many other competing methods, will solve the problems of normalization and of importance, whereas Fuzzy MULTIMOORA may take care of the annoying spread in the marketing samples. While an application on the construction of dwellings is given, many other applications remain possible like for Gallup polls concerning public opinion, general elections in particular.

Keywords

Neutrosophic MULTIMOORA, objectives, criteria, attributes, alternative solutions, decision matrix, weights, MOORA, Ratio System, Reference Point Method, Full Multiplicative Form, ordinal dominance, sample, standard error, spread, market research.

1 Introduction

Several solutions face different criteria expressed in different units, whereas the best outcome has to be found. Consider the following example of buying a new car. This car has to fulfill the following criteria:

1. The criterion "comfort" possesses the following attributes: excellent, medium, weak, for instance translated into the cardinal numbers: 2 for weak, 3 for medium and 4 for excellent, excellent being the double of weak (the translation of nominal words into cardinal numbers is very often exaggerated, see therefore e.g. Brauers et al. 2011).

2. The criterion "price" is expressed in $

3. The criterion "speed" is expressed in miles per hour

4. The criterion "shape" possesses the following attributes: ordinary and special, for instance translated into the cardinal numbers: 1 for ordinary and 2 for special.

In this example, the decision is made by one person. If the decision is rather coming from multi-persons it could be difficult to question the whole population concerned and one has to be satisfied by a sample representing the opinion of a group originated from face-to-face interviews till digital information. The distance between the opinion of the whole population and the sample is measured by the standard deviation in one direction and by the spread, being the double of the standard deviation, in both directions. If the publicity power of a newspaper, magazine, cinema or television would be announced by these media themselves the public, especially the publicity brokers, would have no confidence in the outcome. Therefore, a neutral institution will deliver the results by sampling. CIM is for instance the organization concerned in Belgium.

The Association for measuring the importance of Newspapers in Belgium (CIM) is only interested in the evolution of the sales of newspapers in Belgium. It means that not the size of sales is of importance but rather its evolution. Indeed, beside the national papers with high volume local or specialized newspapers are not aiming at coming on the volume of the national papers but are interested in the increase or decrease of the number of readers. Nevertheless, one has to go out

from the numbers of readers to deduct the evolution in reader's population. The fact that a higher standard deviation is noticed for the local or specialized newspapers is another remarkable fact to be taken into consideration. Results for 2013-14 are synthesized in the following table with an average spread for all newspapers together of 24% (CIM September 2014 and CIM 2013-14).

Table 1. Average readership of Belgian newspapers (2013-14)

Newspapers	Circulation	Standard deviation	Spread
National newspaper (Dutch)	250,000	5.4%	10.8%
National newspaper (French)	180,000	6%	12%
Local newspaper (French)	40,000	15%	30%
Financial newspaper	46,000	6%	12%

The results are scientifically not acceptable but the publicity brokers prefer these results above eventual statistics from the newspapers themselves.

The topic of this research is to find a method in such a multi criteria problem of sampling in order to make a choice in a rational way, to come to an optimum for the results and to interpret them. More specific it concerns here market research.

In summary, one may say that first a method is needed to compare several criteria expressed in different units, secondly how to make a sample representative and thirdly how to deal with group decisions. First a method to compare the different criteria is searched out.

2 Search for a Robust Method to Make a Choice in a Rational Way between Different Solutions Responding to Different Objectives

For the researcher in multi-objective decision making the choice between many methods is not very easy. Indeed numerous theories were developed since the forerunners: Condorcet (the Condorcet Paradox, against binary comparisons, 1785, LVIII), Gossen (law of decreasing marginal utility, 1853) Minkowski (Reference Point, 1896, 1911) and Pareto (Pareto Optimum and Indifference Curves analysis 1906, 1927) and pioneers like Kendall (ordinal scales, since 1948), Roy (ELECTRE, since 1966, with many variations in Electre since then, see therefore Schärlig, 1985; 1996), Miller and Starr (Multiplicative Form, 1969), Hwang and Yoon (TOPSIS, 1981), Saaty (AHP, since 1988), Opricovic and Tzeng (VIKOR, 2004), Brans and Mareschal (PROMETHEE, 2005).

The MULTIMOORA (Multi-Objective Optimization by a Ratio Analysis plus the Full Multiplicative Form) was proposed by Brauers and Zavadskas (2010).

The ordinary MULTIMOORA method has been proposed for usage with crisp numbers. To enable its use in solving a larger number of complex decision-making problems, several extensions have been proposed, from which there are mentioned only the most prominent: Brauers *et al*. (2011) proposed fuzzy extension of the MULTIMOORA method; Balezentis and Zeng (2013) proposed interval-valued fuzzy, Balezentis *et al*. (2014) proposed intuitionist fuzzy extension and Zavadskas *et al*. (2015) proposed interval-valued intuitionist extension of the MULTIMOORA method.

A significant approach in solving complex decision-making problems was formed by adapting multiple criteria decision-making methods for the use of fuzzy numbers, proposed by Zadeh in fuzzy set theory (Zadeh, 1965).

Based on fuzzy set theory, some extensions are also proposed, such as: interval-valued fuzzy sets (Turksen, 1986), intuitionist fuzzy sets (Atanassov, 1986) and interval-valued intuitionist fuzzy sets (Atanassov, Gargov, 1989).

In addition to membership function, proposed in fuzzy sets, Atanassov (1986) introduced the non-membership function that express the non-membership to a set, and thus created the basis for solving of a much larger number of decision-making problems.

The intuitionist fuzzy set is composed of the membership (or called truth-membership) $T_A(x)$ and non-membership (or called falsity-membership) $F_A(x)$, that satisfies the conditions $T_A(x), F_A(x) \in [0,1]$ and $0 \leq T_A(x) + F_A(x) \leq 1$. Therefore, intuitionist fuzzy sets are capable to operate with incomplete information, but do not include intermediate and inconsistent information (Li *et al*., 2016).

In intuitionist fuzzy sets, the indeterminacy $\pi_A(x)$ is $1 - T_A(x) - F_A(x)$ by default. Smarandache (1999) further extended intuitionist fuzzy sets by proposing Neutrosophic, and also introduce independent indeterminacy-membership.

Such proposed neutrosophic set is composed of three independent membership functions named the truth-membership $T_A(x)$, falsity-membership $F_A(x)$ and indeterminacy-membership $I_A(x)$. *(Mohamed et al. , 2014,2015,2016a, 2016b,2017).*

Wang *et al*. (2010) further proposed a single valued neutrosophic set, by modifying the condition $T_A(x)$, $I_A(x)$ and $F_A(x)$ $\in [0,1]$ and $0 \leq T_A(x) +$

$I_A(x)+F_A(x)\leq 3$, which are more suitable for solving scientific and engineering problems (Li *et al.*, 2016).

Compared with the fuzzy set and its extensions, the single valued neutrosophic set can be identified as more flexible, for which reason an extension of the MULTIMORA method adapted for the use of single valued neutrosophic set is proposed in this approach.

3 The Neutrosophic Extension of MULTIMOORA

A Decision Matrix assembles raw data with vertically numerous objectives, criteria (a weaker form of objectives) or indicators and horizontally alternative solutions, like projects. In order to define an objective better we have to focus on the notion of Attribute. Keeney and Raiffa (1993, 32-38) present the example of the objective "reduce sulfur dioxide emissions" to be measured by the attribute "tons of sulfur dioxide emitted per year". An attribute is a common characteristic of each alternative such as its economic, social, cultural or ecological significance, whereas an objective consists in the optimization (maximization or minimization) of an attribute.

3.1. Horizontal reading of the Decision Matrix

SAW, followed by many other methods, reads the response matrix in a horizontal way. The Additive Weighting Procedure (MacCrimmon, 1968, 29-33, which was called SAW, Simple Additive Weighting Method, by Hwang and Yoon, 1981, 99) starts from:

$$Max.U_j = w_1 x_{1j} + w_2 x_{2j} + ... + w_i x_{ij} + ... + w_n x_{nj} \qquad (1)$$

Uj = overall utility of alternative j with j = 1,2,.....,m, m the number of alternatives

wi = weight of attribute i indicates as well as normalization as the level of importance of an objective, with:

$$\sum_{i=1}^{i=n} w_i = 1$$

i = 1, 2,… n; n the number of attributes or objectives
xij = response of alternative j on attribute i.

As the weights add to one a new super-objective is created and consequently it becomes difficult to speak of multiple objectives.

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With weights importance of objectives is mixed with normalization. Indeed, weights are mixtures of normalization of different units and of importance coefficients.

3.2. Vertical Reading of the Decision Matrix

Vertical reading of the Decision Matrix means that normalization is not needed as each column is expressed in the same unit. In addition, if each column is translated in ratios dimensionless measures can be created and the columns become comparable to each other. Indeed, they are no more expressed in a unit. Different kind of ratios are possible but Brauers and Zavadskas (2006) proved that the best one is based on the square root in the denominator.

Vertical reading of the decision matrix and the Brauers-Zavadskas ratios are practiced in the MOORA method.

3.3. The MOORA Method

3.3.1. Ratio System of MOORA

We go for a ratio system in which each response of an alternative on an objective is compared to a denominator, which is representative for all alternatives concerning that objective:

$$x_{ij}^{*} = \frac{x_{ij}}{\sqrt{\sum\limits_{j=i}^{m} x_{ij}^{2}}} \tag{2}$$

with:

 xij = response of alternative *j* on objective *i*
 $j = 1,2,\ldots,m;$ *m* the number of alternatives
 $i = 1,2,\ldots n;$ *n* the number of objectives
 $xij *=$ this time a dimensionless number representing the response of
   alternative *j* on objective *i*. $xij *$ is situated between 0 and 1[1].

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[1] However, sometimes the interval could be *[-1; 1]*. Indeed, for instance in the case of productivity growth some sectors, regions or countries may show a decrease instead of an increase in productivity i.e. a negative dimensionless number. Instead of a normal increase in productivity growth a decrease remains possible. At that moment, the interval becomes *[-1, 1]*. Take the example of productivity, which has to increase (positive). Consequently, we look for a maximization of productivity e.g. in European and American countries. What if the opposite does occur? For instance, take the original transition from the USSR to Russia. Contrary to the other European countries productivity decreased. It means that in formula (2) the numerator for Russia was negative with the whole ratio becoming negative. Consequently, the interval changes to: *[-1, +1]* instead of *[0, 1]*.

For optimization, these responses are added in case of maximization and subtracted in case of minimization:

$$y_j^* = \sum_{i=1}^{i=g} x_{ij}^* \quad - \quad \sum_{i=g+1}^{i=n} x_{ij}^* \tag{3}$$

with:

$i = 1, 2, \ldots, g$ as the objectives to be maximized.

$i = g+1, g+2, \ldots, n$ as the objectives to be minimized

$y_j* =$ the total assessment of alternative j with respect to all objectives.

y_j* can be positive or negative depending of the totals of its maxima and minima.

An ordinal ranking of the y_j* in a descending order shows the final preference. Indeed, cardinal scales can be compared in an ordinal ranking after Kenneth J. Arrow (1974): "Obviously, a cardinal utility implies an ordinal preference but not *vice versa*".

A second part of MOORA consists of the Reference Point Method which uses the ratios found in the Ratio System of MOORA.

3.3.2. Reference Point method of MOORA

A second Method in MOORA is the Reference Point Approach which will use the ratios found earlier and whereby also a Maximal Objective Reference Point is used. The Maximal Objective Reference Point approach is called realistic and non-subjective as the co-ordinates (r_i), which are selected for the reference point, are realized in one of the candidate alternatives. In the example, A (10;100), B (100;20) and C (50;50), the maximal objective reference point R_m results in: (100;100). Per objective the coordinates of the corresponding ratio are subtracted from the coordinates of the Reference Point.

Then these results are subject to the *Metric of Tchebycheff* (Karlin and Studden, 1966, 280)[2]:

$$\underset{(j)}{Min}\left\{\underset{(i)}{max}\sqrt[2]{(r_i - x_{ij}^*)^2}\right\} \tag{4}$$

r_i = the ith co-ordinate of the reference point

$x_{ij}* =$ the dimensionless measurement of objective i for alternative j

$i = 1, 2 \ldots \ldots n;$ n the number of objectives

$j = 1, 2 \ldots \ldots m;$ m the number of alternatives

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[2] Previously the following formula was used as Tchebycheff has done:

$$\underset{(j)}{Min}\left\{\underset{(i)}{max}\left| r_i - x_{ij}^* \right|\right\}$$

with $\left| r_i - x_{ij}^* \right|$ the absolute value necessary if x_{ij}^* is larger than r_i

The outcome is the same, but the square presentation (4) is more in accordance with formula (2).

An ordinal ranking of the results in an ascending order shows the final preference.

3.3.3. The problem of importance

With weights importance of objectives is mixed with normalization. On the contrary the dimensionless measures of MOORA do not need external normalization. However, the problem of importance remains. Therefore, in MOORA to give more importance to an objective its response on an alternative under the form of a dimensionless number could be multiplied with a significance coefficient. However, if this would be done the outcome will not change. Therefore, another approach has to be followed. Replacement of an objective by some sub-objectives, as valuable as the original objectives, will solve the problem of importance for the original objective. For instance, employment is replaced separately by direct and indirect employment or pollution is divided into three different forms of pollution.

3.3.4. MOORA can it be called Robust? Characteristics of Robustness in Multi-Objective Optimization

(Brauers, 2010; Brauers and Zavadskas; 2012; Brauers and Zavadskas, 2010; Brauers and Zavadskas, 2009; Brauers and Ginevičius, 2010; Brauers and Ginevičius, 2009)

1. All stakeholders are involved (see: Brauers and Lepkova, 2003 and 2002).
2. Respect for Consumer Sovereignty (Brauers, 2008b)
3. All non-correlated objectives are involved, as much as possible (see Brauers et al. 2008)
4. All interrelations between objectives and alternatives are considered at the same time and for instance not two by two (otherwise a victim of the Condorcet-Arrow Paradox, see: Brauers, 2004, 118-124).
5. Non-subjective as much as possible:
   - In the choice of the objectives (assistance can be given by the Ameliorated Nominal Group Technique, see Brauers, 2008a; Brauers and Lepkova 2003 and 2002)
   - To give importance to an objective either in a direct way or by substitution (assistance can be given by the Delphi Method, see Brauers, 2008a; Brauers, 1976; Dalkey and Helmer, 1963)
   - Omitting Normalization. Dimensionless Measurements as used here are preferred to weights, which need normalization (for normalization, see: Brauers and Zavadskas 2007; Brauers 2007a and b).

6. Based on Cardinal Numbers is more robust than on Ordinal Numbers. The Rank Correlation Method of Kendall is based on ordinal numbers. He argues (Kendall, 1948, 1): "we shall often operate with these numbers as if they were the cardinals of ordinary arithmetic, adding them, subtracting them and even multiplying them", but he never gave a proof of this statement. In his later work this statement is dropped (Kendall and Gibbons, 1990).
7. Uses the most recent available data.
8. The use of two different methods of MOO is more robust than using a single one.

Already in 1983 at least 96 methods for Multi-Objective Optimization existed (Despontin et al., 1983). Since then numerous other methods appeared. Therefore, we only cite the probably most used methods for Multi-Objective Optimization.

First Schärlig (1985, 1996) gives the name of Methods of Partial Aggregation to the Electre Group (Electre I, Electre Iv, Electre Is, Electre TRI, Electre II, Electre III and Electre IV) and to Prométhée. As the study under consideration asks for total aggregation methods based on partial aggregation cannot be used.

The Analytic Hierarchy Process (AHP of Saaty, 1988), followed by the Analytic Network Process (ANP, Saaty & Kulakowski, 2016), compare in pairs and are based on weights. The use of weights in operational research was introduced by Churchman and Ackoff (1954) and Churchman et al. (1957). The Additive Weighting Procedure called SAW was already mentioned. Also, the methods of partial aggregation use weights. In addition, all these methods are expert oriented with qualitative statements as a basis.

Reference Point Methods like TOPSIS (Hwang and Yoon (1981) and VIKOR (Opricovic, Tzeng 2004) do not use weights but rather dimensionless measures but they are overtaken by MOORA which is composed of two different dimensionless based methods, each controlling each other.

An interesting example of MOORA compared with other methods is what Chakraborty has done for industrial management. Chakraborty (2011) checked six famous methods of Multi-Objective Decision Making for decision making in manufacturing. Next Table 1 shows the results.

Table 2. Comparative performance of some MODM methods

MODM	Computational time	Simplicity	Mathematical Calculations	Stability	Information Type
		Very simple			
MOORA	Very less	Very critical	Minimum	Good	Quantitative
AHP	Very high	Moderately critical	Maximum	Poor	Mixed
TOPSIS	Moderate		Moderate	Medium	Quantitative
VIKOR	Less	Simple	Moderate	Medium	Quantitative
ELECTRE	High	Moderately critical	Moderate	Medium	Mixed
PROMETHEE	High	Moderately critical	Moderate	Medium	Mixed

Karuppanna & Sekar (2016, 61) studied the several approaches not only towards Manufacturing but also to the Service Sectors, which is extremely important for the underlying study.

Table 3. Comparison of MOORA with other Approaches for application in the Service Sectors

MADM method	Computational Time	Simplicity	Mathematical calculations
MOORA	very less	very simple	Minimum
AHP	very high	very critical	Maximum
ANP	Moderate	Moderately critical	Moderate
VIKOR	Less	Simple	Moderate
TOPSIS	Moderate	Moderately critical	Moderate
ELECTRE	High	Moderately critical	Moderate
PROMOTHEE	High	Moderately critical	Moderate

3.3.5. MOORA and Market Research

Market research works mostly with a confidence level of 95%, which means a 5% probability that outside conditions will interfere. On the other side for instance a dam against flooding has to have a confidence level of 99.9%, i.e. a probability of 1 on 1,000 that the dam will be too low or will collapse.

On the other side, the size of the sample is important. Marketing accepts for instance 100 interviews with a standard error of: se $= \sqrt{\dfrac{pq}{n}} = \sqrt{\dfrac{0.25}{100}} =$ 0.05 which means 5% under or 5% above the real percentage (p = expected probability; q the opposite q = 1 − p).

In a normal distribution: $q = p = 0.5$. The sum of the 5% under plus the 5% above the real percentage or the sum of the standard errors is called the Spread. Hoel (1971, 101) speaks of the extent of the spread, whereas Hays (1973, 236) calls it spread or dispersion. Mueller et al. (1970) speak rather of "Range".

3.3.6. Consumer's Attitude on Contractor's Ranking: a Presentation of a Case Study

This example is taken from: Brauers et al., 2008. Construction, taking off, maintenance and facilities management of a building are typical examples of consumer sovereignty: the new owner likes to have a reasonable price to pay, to have confidence in the contractor, to know about the duration of the works, the service after completion and the quality of the work. On the other side, the contractor has his objectives too, like the satisfaction of the client, diminishing of external costs and annoyances and the management cost per employee as low as possible. In other words, it concerns a problem of multi-objectives. Therefore, a final ranking will show the best performing contractor from the point of view of the clients but also from the point of view of the contractors.

The largest maintenance contractors of dwellings in Vilnius, the capital of Lithuania, were approached, of which 15 agreed to fix and estimate their main objectives, namely 9 objectives as given in Table 4.

Table 4**.** Main attributes and objectives of maintenance contractors of dwellings in Vilnius

1. Cost of building management Lt/m2 *min*
2. Cost of common assets management Lt/m2 *min*
3. HVAC system maintenance cost (mean) Lt/m2 *min*
4. Courtyard territory cleaning (in summer) Lt/m2 *min*
5. Total service cost Lt/m2 *min*
6. Length of time in maintenance business experience in years *max*
7. Market share for each contractor % *max*
8. Number of projects per executive units/person *max*
9. Evaluation of management cost (Cmin / Cp) *max*

Table 5 summarizes the reaction of the contractors on the proposed objectives.

Table 5. Initial decision making matrix of 15 contractors of dwellings in Vilnius

Objectives ↓	1	2	3	4	5	6	7	8	9
Alternatives ↔	MIN.	MIN.	MIN.	MIN.	MIN.	MAX.	MAX.	MAX.	MAX.
a_1	0.064	0.11	0.18	0.31	0.67	12	11.75	4.6	0.83
a_2	0.06	0.14	0.37	0.12	0.5	3	0.39	0.33	0.885
a_3	0.057	0.11	0.18	0.15	0.69	12	5.25	1.47	0.935
a_4	0.06	0.12	0.10	0.15	0.57	12	7.1	2.78	0.9
a_5	0.058	0.1	0.18	0.2	0.45	12	5.56	1.39	0.9
a_6	0.071	0.3	0.18	0.26	0.82	13	26.62	5.67	0.746
a_7	0.11	0.14	0.18	0.12	0.55	5	2.82	1.2	0.483
a_8	0.058	0.18	0.37	0.19	0.61	11	9.48	3.03	0.916
a_9	0.053	0.14	0.16	0.23	0.8	11	2.23	0.8	1
a_{10}	0.07	0.26	0.29	0.2	0.7	11	13.5	9.05	0.75
a_{11}	0.12	0.2	0.09	0.2	0.81	4	4.7	1.5	0.443
a_{12}	0.071	0.28	0.18	0.28	0.73	12	2.35	0.86	0.746
a_{13}	0.078	0.2	0.18	0.3	0.76	8	5.6	3.25	0.681
a_{14}	0.056	0.14	0.18	0.12	0.5	11	2.66	1.7	0.948
a_{15}	0.12	0.14	0.09	0.21	0.56	3	0.04	0.03	0.531

Brauers et al, 2008, 250.

From information of the Dwelling Owners Association, a panel of 30 owners of dwellings chosen at random agreed with these 9 objectives, but they increased the objectives with 11 other ones (These additional objectives were: standard of management services, maintenance of common property, work organization, effectiveness of information use, certification of company, range of services, reliability of company, company reputation, staff qualification and past experience, communication skills, geographical market restrictions.). However, these additional objectives were only expressed in qualitative points showing some overlapping and after their rating represented only 25.9% importance of the total. If these opinions are only taken as indicative these qualitative objectives can be dropped[3].

For the 9 objectives with 30 interviews even chosen at random mean a confidence level of: standard error se = $\sqrt{\dfrac{pq}{n}} = \sqrt{\dfrac{0.25}{30}} = 0.09$, which means 9% under or 9% above the real percentage or a Spread of 18%.

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[3] Another approach would be that the corresponding ordinal qualifications are transformed into cardinal numbers, which has to be done under severe reservations. See therefore: Brauers, 2004, 97-99 and Brauers et al. 2011, 268-271.

Beside this formula: one has to be aware of the *Universe* or *Population* around the sample (Mueller et al. 1970, 343) which is not directly quantitative:

- Only the Vilnius population above the age of 18 has to be taken into consideration and in addition only households;
- an advance payment for buying property of 15 to 30% is needed in Lithuania (Swedbank, 2012);
- In addition, only 13% of the Vilnius population have a mortgage (SEB, 2013,6). From this 13% has to be excluded: existing mortgages, buying an existing property, buying a social apartment or people not interested in the location in question;
- Saving rate in Lithuania was only 1.92% in 2008, which is extremely low. In 2009 there was even dissaving (Statistics Lithuania, 2014).

Accepting the 18% spread for a limited universe one may conclude that the 30 respondents are representative for the potential buyers of the proposed property in Vilnius.

The nature of the construction industry involves that the total number of the minima is mostly larger than the total number of the maxima, which is the case here. Instead of attributing significance coefficients the contractors and the small sample of owners preferred the *Attribution of Sub-Objectives*. Indeed, five objectives on nine concern the super objective minimization of costs. Even, the last maximization forms in fact a cost consideration.

The following table 6 presents the ranking of the contractors.

Table 6. Ranking Contractors after MOORA with 18% spread (a)

	Ratio method	Reference Point Method	MOORA
a6	1	1	1
a10	2	2	2
a1	4	3	3
a4	3	5	4

Brauers et al, 2008, 251.

The other 10 contractors have a low rate and are unclear about their ranking.

A summary of the two methods in order to come to MOORA is made on view.

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The problem remains of the high spread of 18%. How to solve the failure of:

- The high spread of 18%
- An unclear ranking?

Therefore, we look at MULTIMOORA and Fuzzy MULTIMOORA.

4 How to make a Sample representative without Spread?

4.1. The MULTIMOORA Method

To the two methods of MOORA a third method is added: the Full Multiplicative Form. The use of three different methods of MOO is more robust than using of two, making MULTIMOORA superior to all existing methods of Multiple Objectives Optimization.

In the Full Multiplicative Form per row of an alternative all objectives are simply multiplied, but the objectives to be minimized are parts of the multiplication process as denominators.

A problem may arise for a single zero or for a negative number for one of the objectives making the final product zero or entirely negative. In order to escape of this nonsense solution 0.001 replaces zero, if the lowest number present is 0.01. For a negative number, which will be very exceptional, see a case in footnote 1 above, -1 becomes 0.0001 and -2 becomes 0.00001 etc. but only for the objective under consideration.

In MOORA a summary of the two methods was made on view, impossible for MULTIMOORA. Adding of ranks, ranks mean an ordinal scale (1st, 2nd, 3rd etc.) signifies a return to a cardinal operation $(1 + 2 + 3 + …)$. Is this allowed? The answer is "no" following the Noble Prize Winner Arrow:

The Impossibility Theorem of Arrow

"Obviously, a cardinal utility implies an ordinal preference but not *vice versa*" (Arrow 1974).

Axioms on Ordinal and Cardinal Scales

1. A deduction of an Ordinal Scale, a ranking, from cardinal data is always possible.
2. An Ordinal Scale can never produce a series of cardinal numbers.
3. An Ordinal Scale of a certain kind, a ranking, can be translated in an ordinal scale of another kind.

In application of axiom 3 the rankings of three methods of MULTIMOORA are translated into another ordinal scale based on *Dominance*, *being Dominated, Transitivity and Equability*.

4.2. Ordinal Dominance Theory

4.2.1 Dominance

Absolute Dominance means that an alternative, solution or project is dominating in ranking all other alternatives, solutions or projects which are all being dominated. This absolute dominance shows as rankings for MULTIMOORA: $(1-1-1)$. *General Dominance in two of the three methods* is of the form with a < b < c <d:

$(d-a-a)$ is generally dominating $(c-b-b)$;

$(a-d-a)$ is generally dominating $(b-c-b)$;

$(a-a-d)$ is generally dominating $(b-b-c)$;

And further transitiveness plays fully.

Transitiveness. If *a* dominates *b* and *b* dominates *c* than also *a* will dominate *c*.

Overall Dominance of one alternative on the next one. For instance $(a-a-a)$ is overall dominating $(b-b-b)$ which is overall being dominated.

1.2 2 Equability

Absolute Equabilit*y* has the form**:** for instance (e–e–e) for 2 alternatives. *Partial Equability* of 2 on 3 exists e. g. (5–e–7) and (6–e–3).

4.3. MULTIMOORA with spread

Table 7. Ranking Contractors after MULTIMOORA with 18% spread (a)

Original study: MOORA with 18% spread		MULTIMOORA with 18% spread	
a6	1	a6	**1**
a10	2	a4	**2**
a1	3	a10	**3**
a4	4	a1	**4**
		a5	**5**
		a3	**6**
		a8	**7**
		a14	**8**
		a13	**9**
		a9	**10**
		a7	**11**
		a11	**12**
		a12	**13**
		a2	**14**
		a15	**15**

Calculations available from the authors

The spread still remains in MULTIMOORA. Fuzzy MULTIMOORA will try to remove the spread by extending the numbers until the standard deviation on both sides as given in next table 8.

Table 8. Ranking Contractors with 9% less and 9% more for each objective

Obj. 1			Obj. 2.... Obj. 3 ...Obj. 4...Obj. 5....Obj. 6....Obj. 7....Obj. 8...	Obj. 9		
0.058	0.064	0.070	---	0.76	0.83	0.90
0.055	0.060	0.065	---	0.81	0.89	0.96
0.052	0.057	0.062	---	0.85	0.94	1.02
0.053	0.058	0.063	---	0.83	0.91	0.99
0.053	0.058	0.063	---	0.83	0.91	0.99
0.065	0.071	0.077	---	0.68	0.75	0.81
0.100	0.110	0.120	---	0.44	0.48	0.53
0.053	0.058	0.063	---	0.83	0.92	1.00
0.048	0.053	0.058	---	0.91	1.00	1.09
0.065	0.071	0.077	---	0.68	0.75	0.81
0.109	0.120	0.131	---	0.40	0.44	0.48
0.065	0.071	0.077	---	0.68	0.75	0.81
0.071	0.078	0.085	---	0.62	0.68	0.74
0.051	0.056	0.061	---	0.86	0.95	1.03
0.109	0.120	0.131	---	0.48	0.53	0.58

27 objectives and sub-objectives replace the 9 objectives.

Consumer Sovereignty will play by giving to each objective a minus value or a max value of 9% deviation corresponding with the confidence level. For instance, input of contractor a1 into objective 6 being 12 is replaced by 10.92, 12 and 13.08 (see table 5).

In taking rows and columns in table 8 the numbers will have more or less the form of an upside-down Gauss Curve, however not standard normal or symmetrical (Hoel, 1971, 100-104) but skewed (Hays, 1973, 317) and with the restriction that the solutions are not continuous but discrete. Fuzzy means also that all points on a line linking all values of an alternative solution, here a contractor, are also possible.

In the given example, it is not certain that a contractor will accept the changes, proposed by the client, as it means a change in his offer.

4.4. The Fuzzy MULTIMOORA Method

Being a special case of the fuzzy sets, fuzzy numbers express uncertain quantities. Among various instances of fuzzy numbers, the triangular fuzzy numbers are often used for multi-criteria decision making. A triangular fuzzy

number $\tilde{x}$ can be represented by a tripet: $\tilde{x} = (a, b, c)$, where a and c are the minimum and maximum bounds, respectively, and b is the modal value or kernel (Kaufmann and Gupta, 1991).

The following arithmetic operations are available for the fuzzy numbers (Wang, Chang, 2007):

1. Addition $\oplus$:

$$\tilde{A} \oplus \tilde{B} = (a, b, c) \oplus (d, e, f) = (a + d, b + e, c + f); \tag{5}$$

2. Subtraction $\ominus$:

$$\tilde{A} \ominus \tilde{B} = (a, b, c) \ominus (d, e, f) = (a - f, b - e, c - d); \tag{6}$$

3. Multiplication $\otimes$:

$$\tilde{A} \otimes \tilde{B} = (a, b, c) \otimes (d, e, f) = (a \times d, b \times e, c \times f) \tag{7}$$

4. Division $\oslash$:

$$\tilde{A} \oslash \tilde{B} = (a, b, c) \oslash (d, e, f) = (a \setminus f, b \setminus e, c \setminus d). \tag{8}$$

The vertex method can be applied to measure the distance between two fuzzy numbers. Let $\tilde{A} = (a, b, c)$ and $\tilde{B} = (d, e, f)$ be the two triangular fuzzy numbers. Then, the vertex method can be applied:

$$d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{3}[(a-d)^2 + (b-e)^2 + (c-f)^2]}. \tag{9}$$

Fuzzy MULTIMOORA Method

Fuzzy MULTIMOORA was introduced by Brauers et al. (2011). In this study, we employ the modified version as reported by Balezentiene et al. (2013). The fuzzy MULTIMOORA begins with fuzzy decision matrix $\tilde{X}$, where $\tilde{x}_{ij} = (x_{ij1}, x_{ij2}, x_{ij3})$ are aggregated responses of alternatives on objectives.

The Fuzzy Ratio System

The Ratio System defines normalization of the fuzzy numbers $\tilde{x}_{ij}$ resulting in matrix of dimensionless numbers. The normalization is performed by comparing appropriate values of fuzzy numbers:

$$\tilde{x}_{ij}^{*} = (x_{ij1}^{*}, x_{ij2}^{*}, x_{ij3}^{*}) = \begin{cases} x_{ij1}^{*} = x_{ij1} \bigg/ \sqrt{\dfrac{1}{3}\sum_{i=1}^{m}\Big[(x_{ij1})^2 + (x_{ij2})^2 + (x_{ij3})^2\Big]} \\[4mm] x_{ij2}^{*} = x_{ij2} \bigg/ \sqrt{\dfrac{1}{3}\sum_{i=1}^{m}\Big[(x_{ij1})^2 + (x_{ij2})^2 + (x_{ij3})^2\Big]}, \forall i, j. \\[4mm] x_{ij3}^{*} = x_{ij3} \bigg/ \sqrt{\dfrac{1}{3}\sum_{i=1}^{m}\Big[(x_{ij1})^2 + (x_{ij2})^2 + (x_{ij3})^2\Big]} \end{cases} \tag{10}$$

The normalization is followed by computation of the overall utility scores, $\tilde{y}_i^{*}$, for each ith alternative. The normalized ratios are added or subtracted with respect to the type of criteria:

$$\tilde{y}_i^{*} = \sum_{j=1}^{g} \tilde{x}_{ij}^{*} \ominus \sum_{j=g+1}^{n} \tilde{x}_{ij}^{*}, \tag{11}$$

where $g = 1, 2, \ldots, n$ stands for number of criteria to be maximized. Then each ratio $\tilde{y}_i^{*} = (y_{i1}^{*}, y_{i2}^{*}, y_{i3}^{*})$ is defuzzified:

$$BNP_i = \frac{y_{i1}^{*} + y_{i2}^{*} + y_{i3}^{*}}{3} \tag{12}$$

BNP_i denotes the best non-fuzzy performance value of the ith alternative. Consequently, the alternatives with higher BNP values are attributed with higher ranks.

The Fuzzy Reference Point

The fuzzy Reference Point approach is based on the fuzzy Ratio System. The Maximal Objective Reference Point (vector) $\tilde{r}$ is found according to ratios found in Eq. 10. The jth coordinate of the reference point resembles the fuzzy maximum or minimum of the jth criterion $\tilde{x}_j^{+}$, where

$$\begin{cases} \tilde{x}_j^{+} = \left(\max_i x_{ij1}^{*}, \max_i x_{ij2}^{*}, \max_i x_{ij3}^{*} \right), j \le g; \\[3mm] \tilde{x}_j^{+} = \left(\min_i x_{ij1}^{*}, \min_i x_{ij2}^{*}, \min_i x_{ij3}^{*} \right), j > g. \end{cases} \tag{13}$$

Then every element of normalized responses matrix is recalculated and final rank is given according to deviation from the reference point (Eq. 13) and the Min-Max Metric of Tchebycheff:

$$\min_i \left(\max_j d(\tilde{r}_j, \tilde{x}_{ij}^*) \right). \tag{14}$$

The Fuzzy Full Multiplicative Form

Overall utility of the ith alternative can be expressed as a dimensionless number by employing Eq. 8:

$$\tilde{U}_i' = \tilde{A}_i \oslash \tilde{B}_i, \tag{15}$$

$$\tilde{A}_i = (A_{i1}, A_{i2}, A_{i3}) = \prod_{j=1}^{g} \tilde{x}_{ij}, \quad i = 1, 2, \cdots, m$$ denotes the product of objectives of the ith alternative to be maximized with $g = 1, \cdots, n$ being the number of criteria to be maximized.

$$\tilde{B}_i = (B_{i1}, B_{i2}, B_{i3}) = \prod_{j=g+1}^{n} \tilde{x}_{ij}$$ denotes the product of objectives of the ith alternative to be minimized with $n - g$ as the number of criteria to be minimized. Since the overall utility $\tilde{U}_i'$ is a fuzzy number, one needs to defuzzify it to rank the alternatives (cf. Eq. 12). The higher the best non-fuzzy performance value (BNP), the higher will be the rank of a certain alternative.

Thus, the fuzzy MULTIMOORA summarizes fuzzy MOORA (i. e. fuzzy Ratio System and fuzzy Reference Point) and the fuzzy Full Multiplicative Form. Employing this theory and as said before to each objective a minus value or a max value of 9% corresponding with the confidence level will be given. For instance, input of contractor a1 into criterion 6 being 12 is replaced in a fuzzy reasoning by 10.92, 12 and 13.08. A voter can give more importance to contractor a1 and to criterion 6 by preferring 13.8 above 12.

The three parts of Fuzzy MULTIMOORA presents the following results as given in table 9. The summary of the three parts is made by the Ordinal Dominance Theory as explained earlier.

Table 9. Ranking by Fuzzy MULTIMOORA after its three parts and with the application of Ordinal Dominance Theory (a)

	Fuzzy Ratio System	Fuzzy Reverence Point Method	Fuzzy Multiplicative Form	Fuzzy MULTIMOORA
a6	1	1	3	1
a1	2	3	2	2
a10	3	2	4	3
a4	4	5	1	4
a5	5	7	5	5
a3	6	8	6	6
a8	8	4	7	7
a14	7	11	8	8
a13	10	6	9	9
a9	9	13	10	10
a7	12	10	11	11
a11	13	9	12	12
a12	11	12	13	13
a2	14	14	14	14
a15	15	15	15	15

(a) Calculations available from the authors

Table 10 ranks the three possibilities for refining market analysis.

Table 10. Ranking Contractors after the three Possibilities (a)

MOORA with 18% spread		MULTIMOORA with 18% spread		Fuzzy MULTIMOORA no spread	
a6	1	a6	1	a6	1
a10	2	a4	2	a1	2
a1	3	a10	3	a10	3
a4	4	a1	4	a4	4
		a5	5	a5	5
		a3	6	a3	6
		a8	7	a8	7
		a14	8	a14	8
		a13	9	a13	9
		a9	10	a9	10

a7	**11**	a7	11
a11	**12**	a11	12
a12	**13**	a12	13
a2	**14**	a2	14
a15	**15**	a15	15

(a) Calculations available from the authors. To make it easier to understand MULTIMOORA or in particular to apply MULTIMOORA for marketing research the software is made in Excel style: first in numbers and then in control modus for formulas. For Excel applications, see: Herkenhoff and Fogli, 2013; Quirk, 2011.

Contractor a6 is preferred overall, which brings much certainty on this solution. Contrary to MULTIMOORA with 18% spread Contractor a1 is the second best as the method without spread shows its domination on the remaining other ones.

Nevertheless, one has to be aware about the real outcome. In the worst case, it could be that a client asks for a 9% additional effort from the side of the contractor. Can the winning contractor not anticipate this situation? Of course, he can, however with the danger that the winning contractor would become one of his colleagues.

On the other side, the contractor will be quasi certain that the client will buy his constructions, unless outside influences would interfere.

The theory is of general use each time a sample replaces total data mining around a certain phenomenon. Application on Gallup polls concerning public opinion, general elections in particular, form another example of information sampling.

5 Conclusion

The Belgian society called CIM is doing marketing research for all Belgian newspapers, magazines and cinema arriving at a spread of 24% as an average for all newspapers and even for some local newspapers at a spread of 30%, which is scientific nonsense but accepted by the publishers of advertisement. On the other side technical problems will ask for a much smaller standard deviation like for instance a standard error of 0.1% for the possibility that a dike is not strong enough for an eventual spring tide. Something in between the usual standard error for marketing research accepted is 5%.

Is it possible to avoid this Spread by Sampling? Here Multi-Objective Optimization Methods may be helpful with the additional question: which methods of MOO are useful in this case? It could not be methods based on the SAW principle as the choice of weights is another point of uncertainty. Neither

can be thought of methods comparing objectives or alternative solutions two by two with in this way being a victim of the Condorcet-Arrow Paradox. Rather have to be thought of methods based on dimensionless measurements like in the MOORA and MULTIMOORA Methods.

To the Ratio Method and the Reference Point Method of MOORA a third method is added in MULTIMOORA: The Full Multiplicative Form. The use of three different methods of MOO is more robust than using one or two.

Compared to crisp, fuzzy, interval-valued and intuitionistic fuzzy numbers, the neutrosophic set provides provide significantly greater flexibility, which can be conducive for solving decision-making problems associated with uncertainty, estimations and predictions.

Decision Making can be quantified by setting up a Decision Matrix with for instance Objectives or Criteria as columns and alternative solutions like Projects as rows. In this study Decision Making is quantified in its objectives, with the problem of normalization, due to the different units of the objectives and with the problem of importance. A MULTIMOORA method, chosen for its robustness instead of many other competing methods, will solve the problems of normalization and of importance, whereas Fuzzy MULTIMOORA will take care of the annoying spread in the samples.

Beside this method one has to be aware of the Universe around the sample, which is not directly quantitative. The Universe has not to be a disturbing factor.

It was Fuzzy MULTIMOORA which brought the solution to the Spread Problem by considering all the possible extreme positions delivered by the standard error. The outcome would have the form of an upside-down Gauss curve however not symmetrical but skewed and with the restriction that the solutions are not continuous but discrete.

Finally, a correction was made by the introduction of the Neutrosophic Extension of MULTIMOORA.

The example of disclosing the desiderata of potential buyers of property in Lithuania presents an illustration of the theory. However, the theory is of general use each time a sample replaces total mining of all data around a certain phenomenon like for Gallup polls concerning public opinion, general elections in particular.

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