

Vikor Method for Decision Making Problem Using Octagonal Neutrosophic Soft Matrix

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Abstract: The scope of this paper is to introduce a new concept of buying a car on the basis of choice parameter by using vikor method. Here octagonal fuzzy number plays a vital role for solving many decision making problem involving uncertainty. In this paper, the defuzzification is done using the area of region, the fuzzified value can be converted into neutrosophic soft matrix and vikor method is applied to the defuzzified data to find the best solution. Finally, an illustrative example gives the effectiveness and feasibility of the proposed approach.

Key words: Fuzzy soft set, Neutrosophic soft set, Octagonal fuzzy number, Vikor method.

1. Introduction

The abstract idea of fuzzy set was given by L.A. Zadeh[21] in 1965. On fuzzy soft sets, Advances in fuzzy system was given by B. Ahmed and A. Kharal[1] (2009). The neutrosophic set was developed by F. Smarandache[19] it applied in the field of decision making problems in mass scale of vague, contingency and incompative data. Neutrosophic is related to actual membership function, unstable membership function and not actual membership function. Molodtsov[13] put many research in the field of soft set theory in the application of computer and mathematics in 1999. The octagonal fuzzy numbers are proposed by Malini.S.U and Kennedy Felbin.C[12] in 2013. The octagonal fuzzy number is defuzzified into singleton (crisps) value by area of region. The vikor method is proposed by Opricovis.S and Tzeng[16], in 2007 the extended vikor method in comparison with outranking methods. Compromise solution by multi criteria decision making methods and a comparative analysis of vikor method is developed by Opricovis.S and Tzeng[17], in 2004. The Vikor algorithm is remarkably successful method for solving the uncertainty. The importance feature of the proposed pattern is its level of commiserate are measured by a fixed remark and other possibilities. This gives a successful in producing a desired or intended result.

2. Preliminaries

2.1 Definition: (Fuzzy Soft Set)[1,2]

Let U be an initial universal set and E be a set of parameters. Let $A \subseteq E$. A pair (\widetilde{F}_A, E) is called a fuzzy soft set over U.

Where \widetilde{F}_A is a mapping given by $\widetilde{F}_A: E \rightarrow I^U$. Where I^U denotes the collection of fuzzy subsets U.

2.2 Definition: (Fuzzy Soft Matrices)[4,5]

Let (\widetilde{F}_A, E) be fuzzy soft set over U. Then a subset of $U \times E$ is uniquely defined by $R_A = \{(u, e): e \in A, u \in F_A(e)\}$ which is called relation form of (\widetilde{F}_A, E) . The characteristic function of R_A is written by, $\mu_{R_A}: U \times E \rightarrow [0, 1]$. Where μ_{R_A} is the membership value of $u \in U$.

If $\mu_{ij} = \mu_{R_A}(u_i, e_j)$. $[\mu_{ij}]_{m \times n} = \begin{bmatrix} \mu_{11} & \cdots & \mu_{1n} \\ \vdots & \ddots & \vdots \\ \mu_{m1} & \cdots & \mu_{mn} \end{bmatrix}$

$m \times n$ is soft matrix of the soft set (\widetilde{F}_A, E) over U.

2.3 Definition: (Neutrosophic Soft Set)[3]

Let $U = \{f_1, f_2, \dots, f_m\}$ be the universal set and E be the set of parameters given by $E = \{e_1, e_2, \dots, e_n\}$. Let $A \subseteq E$ a pair (F, A) be a fuzzy neutrosophic soft set. Then the fuzzy neutrosophic soft set (F, A) in a matrix form as $A = [a_{ij}]$ where $i=1, 2, \dots, m$ and $j=1, 2, \dots, n$.

Let (T_{ij}, I_{ij}, F_{ij}) be the neutrosophic soft matrix. We can define a matrix in the form

$$(T_{ij}, I_{ij}, F_{ij})_{m \times n} = \begin{bmatrix} (T_{11}, I_{11}, F_{11}) & \dots & (T_{1n}, I_{1n}, F_{1n}) \\ \vdots & \ddots & \vdots \\ (T_{m1}, I_{m1}, F_{m1}) & \dots & (T_{mn}, I_{mn}, F_{mn}) \end{bmatrix}$$

Where T_{ij} is the truth membership function, I_{ij} is the indeterminacy-membership and F_{ij} is a false membership function.

2.4 Definition: (Octagonal Fuzzy Number)[12]

A fuzzy number \tilde{A} denoted by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ is a normal octagonal fuzzy number whose membership function $\mu_{\tilde{A}}(x)$, where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$ are real numbers is given as

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{if } x < a_1 \\ k \left(\frac{x-a_1}{a_2-a_1} \right), & \text{if } a_1 \leq x \leq a_2 \\ k, & \text{if } a_2 \leq x \leq a_3 \\ k + (1-k) \left(\frac{x-a_3}{a_4-a_3} \right), & \text{if } a_3 \leq x \leq a_4 \\ 1, & \text{if } a_4 \leq x \leq a_5 \\ k + (1-k) \left(\frac{a_6-x}{a_6-a_5} \right), & \text{if } a_5 \leq x \leq a_6 \\ k, & \text{if } a_6 \leq x \leq a_7 \\ k + (1-k) \left(\frac{a_8-x}{a_8-a_7} \right), & \text{if } a_7 \leq x \leq a_8 \end{cases}$$

Where $0 < k < 1$

3. VIKOR Method

The vikor method is widely classified for multi criteria decision making problem. This method is to derive on ranking and selecting a set of possibilities and solve consolation solution for a problem with belligerent criteria. Opricovic[16] introduced the concept of vikor method in 1998. The vikor method is related with positive and negative ideal solution, its sorts the variable into two or more available alternatives to find out the best compromise solution. By vikor method, we can put new ideas for group decision making problem under certain criteria and few clear define calculation is done as follows.

The consolation ranking procedure of the vikor method has the following steps:

1. Find out the positive ideal solution q_j^+ and negative q_j^- ideal solution as

$$q_j^+ = \max_i q_{ij}, q_j^- = \min_i q_{ij} \text{ where } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n$$

2. Calculate the values for S_i and T_i

$$S_i = \sum_{j=1}^n w_j (q_j^+ - q_{ij}) / (q_j^+ - q_j^-),$$

$$T_i = \max_j w_j (q_j^+ - q_{ij}) / (q_j^+ - q_j^-)$$

Where w_j is the weight.

3. Calculate the value for E_i ,

$$E_i = v (S_i - S^+) / (S^+ - S^-) + (1-v) (T_i - T^+) / (T^+ - T^-)$$

Where $S^+ = \min_i S_i, S^- = \max_i S_i,$

$$T^+ = \min_i T_i, T^- = \max_i T_i$$

Here v is referred as weight.

4. The values of E_i is arranged in increasing order according to the parameters.

4. Algorithm

Step1. Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ be an octagonal fuzzy number such that it defuzzified by $\frac{1}{2} h [(a_8 - a_1) + (a_7 - a_2)] + \frac{1}{2} h [(a_6 - a_3) + (a_5 - a_4)]$ and it converted neutrosophic soft matrix into crisp value.

Step2. The positive and negative solution is:

$$q^+ = \{\tilde{r}_1^+, \dots, \tilde{r}_n^+\}, \text{ where } \tilde{r}_j^+ = \max \{S(\tilde{r}_{1j}), \dots, S(\tilde{r}_{mj})\}, j=1,2,\dots,n$$

$$q^- = \{\tilde{r}_1^-, \dots, \tilde{r}_n^-\}, \text{ where } \tilde{r}_j^- = \min \{S(\tilde{r}_{1j}^-), \dots, S(\tilde{r}_{mj}^-)\}, j=1,2,\dots,n$$

Step3. Calculate the values for S_i and T_i

$$S_i = \sum_{j=1}^n w_j \|\tilde{r}_j^+ - \tilde{r}_i^+\| / \|\tilde{r}_j^+ - \tilde{r}_j^+\|, i=1,2,\dots,m$$

$$T_i = \max_j w_j \|\tilde{r}_j^+ - \tilde{r}_i^+\| / \|\tilde{r}_j^+ - \tilde{r}_j^+\|, i=1,2,\dots,m$$

Step4. Calculate the value for

$$E_i = v(S_i - S^+) / (S^+ - S^+) + (1-v)(T_i - T^+) / (T^+ - T^+)$$

$$\text{Where, } S^+ = \min_i S_i, S^- = \max_i S_i,$$

$$T^+ = \min_i T_i, T^- = \max_i T_i. \text{ Here } v=0.5$$

Step5. Arrange the values for E_i , according to their choice parameter in increasing order.

5. Numerical example

Let U be the universe set. Let Mr.Raj want to buy a car on the basis of choice parameter $\{C_1, C_2, C_3\}$. Let 'A' be a BMW, 'B' be a Benz and 'C' be Audi. Let us find out the best car to buy by comparing three cars A, B and C which their characteristic are 'vehicle front structure and profile', 'stiffness and geometry of front and side structure', 'frontal overhang ahead of front wheels', 'front and rear suspension characteristic', 'vehicle door rocker geometry', 'door latch and structure geometry', 'wheelbase' and 'static stability factor'. Here C_1 represents guide1, C_2 represents guide2 and C_3 represents guide3. Here all the choice parameter has consider as octagonal fuzzy number.

$\{E_1(0.10,0.11,0.12,0.13,0.14,0.15,0.16,0.17)$ (0.11,0.14,0.15,0.21,0.22,0.23,0.24,0.25) (0.22,0.24,0.31,0.32,0.46,0.48,0.51,0.54)}	$\{E_2(0.13,0.14,0.15,0.16,0.17,0.18,0.20,0.21)$ (0.17,0.21,0.24,0.27,0.31,0.32,0.33,0.34) (0.20,0.21,0.23,0.33,0.41,0.42,0.43,0.51)}	$\{E_3(0.21,0.23,0.24,0.31,0.42,0.42,0.43,0.45)$ (0.31,0.32,0.34,0.37,0.41,0.43,0.51,0.53) (0.33,0.41,0.47,0.49,0.51,0.53,0.57,0.61)}
$\{E_1(0.21,0.23,0.27,0.31,0.34,0.39,0.41,0.42)$ (0.51,0.53,0.57,0.61,0.67,0.71,0.72,0.75) (0.33,0.41,0.45,0.49,0.51,0.57,0.62,0.65)}	$\{E_2(0.23,0.27,0.31,0.32,0.37,0.52,0.57,0.61)$ (0.17,0.19,0.21,0.25,0.32,0.36,0.39,0.42) (0.27,0.31,0.42,0.45,0.47,0.52,0.54,0.57)}	$\{E_3(0.31,0.34,0.37,0.43,0.47,0.51,0.53,0.57)$ (0.21,0.23,0.31,0.33,0.42,0.47,0.51,0.53) (0.11,0.13,0.22,0.24,0.34,0.37,0.41,0.45)}
$\{E_1(0.14,0.17,0.20,0.21,0.24,0.31,0.34,0.41)$ (0.21,0.27,0.32,0.41,0.52,0.61,0.63,0.67) (0.27,0.29,0.34,0.47,0.51,0.63,0.67,0.72)}	$\{E_2(0.12,0.17,0.19,0.21,0.22,0.31,0.33,0.41)$ (0.15,0.21,0.23,0.31,0.34,0.37,0.41,0.45) (0.23,0.27,0.31,0.34,0.37,0.42,0.49,0.51)}	$\{E_3(0.31,0.34,0.41,0.52,0.57,0.59,0.61,0.63)$ (0.14,0.21,0.26,0.30,0.37,0.41,0.45,0.51) (0.24,0.27,0.31,0.37,0.43,0.48,0.52,0.58)}

[In this matrix Guide 1,2,3 is mentioned in the row and Car A,B,C is mentioned in the column]

Step 1

The octagonal Fuzzy numbers is defuzzied by

$\frac{1}{2} h \{ \langle a_8 - a_1 \rangle + \langle a_7 - a_2 \rangle \} + \frac{1}{2} h \{ \langle a_6 - a_3 \rangle + \langle a_5 - a_4 \rangle \}$ where $h=0.5$ into singleton crisps value. Then the neutrosophic soft matrix is

	A	B	C
C_1	(0.04,0.09,0.23)	(0.05,0.10,0.20)	(0.18,0.14,0.13)
C_2	(0.14,0.16,0.17)	(0.24,0.17,0.20)	(0.16,0.21,0.22)
C_3	(0.15,0.31,0.29)	(0.15,0.17,0.16)	(0.21,0.21,0.72)

Step 2

By vikor method the positive and negative solution is:

$$X^+ = \{\tilde{r}_1^+, \tilde{r}_2^+, \tilde{r}_3^+\} = \{(0.15,0.31,0.29) (0.24,0.17,0.20) (0.21,0.21,0.72)\}$$

$$X^- = \{\tilde{r}_1^-, \tilde{r}_2^-, \tilde{r}_3^-\} = \{(0.04,0.09,0.23) (0.05,0.10,0.20) (0.16,0.21,0.22)\}$$

Step 3

Compute S_i and T_i from the following steps:

$$S_1 = \frac{w_1 \|\tilde{r}_1^+ - r_{11}\|}{\|\tilde{r}_1^+ - \tilde{r}_1^+\|} + \frac{w_2 \|\tilde{r}_2^+ - r_{12}\|}{\|\tilde{r}_2^+ - \tilde{r}_2^+\|} + \frac{w_3 \|\tilde{r}_3^+ - r_{13}\|}{\|\tilde{r}_3^+ - \tilde{r}_3^+\|} = 1.13$$

$$S_2 = \frac{w_1 \|\tilde{r}_1^+ - r_{21}\|}{\|\tilde{r}_1^+ - \tilde{r}_1^+\|} + \frac{w_2 \|\tilde{r}_2^+ - r_{22}\|}{\|\tilde{r}_2^+ - \tilde{r}_2^+\|} + \frac{w_3 \|\tilde{r}_3^+ - r_{23}\|}{\|\tilde{r}_3^+ - \tilde{r}_3^+\|} = 0.64$$

$$S_3 = \frac{w_1 \|\tilde{r}_1^+ - r_{31}\|}{\|\tilde{r}_1^+ - \tilde{r}_1^+\|} + \frac{w_2 \|\tilde{r}_2^+ - r_{32}\|}{\|\tilde{r}_2^+ - \tilde{r}_2^+\|} + \frac{w_3 \|\tilde{r}_3^+ - r_{33}\|}{\|\tilde{r}_3^+ - \tilde{r}_3^+\|} = 0.15$$

$$T_1 = \max_3 \left\{ \frac{w_1 \|\tilde{r}_1^+ - r_{11}\|}{\|\tilde{r}_1^+ - \tilde{r}_1^+\|}, \frac{w_2 \|\tilde{r}_2^+ - r_{12}\|}{\|\tilde{r}_2^+ - \tilde{r}_2^+\|}, \frac{w_3 \|\tilde{r}_3^+ - r_{13}\|}{\|\tilde{r}_3^+ - \tilde{r}_3^+\|} \right\} = 0.63$$

$$T_2 = \max_3 \left\{ \frac{w_1 \|\tilde{r}_1^+ - r_{21}\|}{\|\tilde{r}_1^+ - \tilde{r}_1^+\|}, \frac{w_2 \|\tilde{r}_2^+ - r_{22}\|}{\|\tilde{r}_2^+ - \tilde{r}_2^+\|}, \frac{w_3 \|\tilde{r}_3^+ - r_{23}\|}{\|\tilde{r}_3^+ - \tilde{r}_3^+\|} \right\} = 0.50$$

$$T_3 = \frac{\max}{3} \left\{ \frac{w_1 \| r_1^+ - r_{31} \|}{\| r_1^+ - r_1^- \|}, \frac{w_2 \| r_2^+ - r_{32} \|}{\| r_2^+ - r_2^- \|}, \frac{w_3 \| r_3^+ - r_{33} \|}{\| r_3^+ - r_3^- \|} \right\} = 0.15$$

Step 4

Compute value for

$$E_i = v (S_i - S^+) / (S^+ - S^-) + (1-v) (T_i - T^+) / (T^+ - T^-)$$

Let v=0.5, we get $E_1 = 1$, $E_2 = 0.61$ and $E_3 = 0$.

Step 5

The value of E in increasing order. We get

Cars	A	B	C	Ranking	Compromise Solution
E(Guides)	0.997	0.61	0.003	A < B < C	A

Step 6

The ranking of E in increasing order, the alternative with first position is A with E (A) = 0.997.

We conclude that car A received **maximum positive commend**. So Mr.Raj can select BMW car.

Customer	Cars	Solution
Mr.Raj	BMW	0.997
	Benz	0.61
	Audi	0.003

From this table this shows that Mr.Raj can choose BMW car while compared to Benz and Audi. In case there is a tie, then the process is repeated for Mr.Raj by reassessing the Guides.

6. Conclusion

In this paper, we have introduced the vikor method for solving the octagonal fuzzy number in decision making problem under uncertainty. The octagonal fuzzy number is defuzzified neutrosophic soft matrix into crisp value and vikor method is applied to get a fruitful result for choosing a best car by comparing the three alternative parameters. Thus vikor is successful method for solving MCDM problem.

7. References

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