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ABSTRACT

In this article, the VIKOR method is proposed to solve the multiple criteria group decision making (MCGDM) with 2-tuple linguistic neutrosophic numbers (2TLNNs). Firstly, the fundamental concepts, operation formulas and distance calculating method of 2TLNNs are introduced. Then some aggregation operators of 2TLNNs are reviewed. Thereafter, the original VIKOR method is extended to 2TLNNs and the calculating steps of VIKOR method with 2TLNNs are proposed. In the proposed method, it's more reasonable and scientific for considering the conflicting criteria. Furthermore, the VIKOR are extended to interval-valued 2-tuple linguistic neutrosophic numbers (IV2TLNNs). Moreover, a numerical example for green supplier selection has been given to illustrate the new method and some comparisons are also conducted to further illustrate advantages of the new method.

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1. Introduction

In practical decision problems, it's difficult to present the criteria values with real values for the complexity and fuzziness of the alternatives, so it can be more useful and effective to express the criteria values by different kinds of fuzzy numbers, such as intuitionistic fuzzy set (IFSs) (Atanassov, 1986; Li, Gao, & Wei, 2018; Wu, Wei, Gao, & Wei, 2018), Pythagorean fuzzy set (PFSs) (Tang et al., 2019; Tang, Wei, & Gao, 2019a, 2019b; Yager, 2014), q-rung orthopair fuzzy sets (q-ROFSs) (Wang, Gao, Wei, & Wei, 2019; Wang, Wang, Wei, & Wei, 2019; Yager, 2017). The fuzzy set theory which initially introduced by Zadeh (1965) has been proved as a feasible mean in the

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application of MCGDM (Mahmoudi, Sadi-Nezhad, & Makui, 2016; Sharma, Kumari, & Kar, 2019; Wang, Wei, & Lu, 2018a; Wang, Wang, & Wei, 2019; Wei, 2019; Wei, Wang, Wei, Wei, & Zhang, 2019; Wei, Wang, Wang, Wei, & Zhang, 2019). Atanassov (1986) defined the IFSs which consider the membership degree and the non-membership degree. To depict the indeterminacy membership degree, Smarandache (1999) provided the neutrosophic sets (NSs). Wang, Smarandache, Zhang, and Sunderraman (2010) investigated some theories about single-valued neutrosophic sets (SVNSs) and given the definition of interval neutrosophic sets (INSs). Ye (2018) studied the MADM problems under the hesitant linguistic neutrosophic (HLN) environment. Wang, Tang, and Wei (2018) studied the dual generalized Bonferroni mean (DGBM) aggregation operators under the SVNNs environment and developed some aggregation operators based on the traditional BM operators (Deng, Wei, Gao, & Wang, 2018; Tang & Wei, 2018; Wang, Wei, & Wei, 2018; Wei, 2017; Wei & Zhang, 2019; Xu & Chen, 2011; Zhu & Xu, 2013). Liu and You (2018) proposed some linguistic neutrosophic Hamy mean (LNHM) aggregation operators. Wu, Wu, Zhou, Chen, and Guan (2018) gave the definition of SVN 2-tuple linguistic sets (SVN2TLSs) and proposed some new Hamacher aggregation operators. Ju, Ju, and Wang (2018) extended the SVN2TLSs to interval-valued environment and presented some single-valued neutrosophic interval 2-tuple linguistic Maclaurin symmetric mean (SVN-ITLMSM) operators. Wu, Wang, Wei, and Wei (2018) studied SVNNs with Hamy operators under 2-tuple linguistic varies environment. Wang, Wei et al. (2018) gave the definition of 2-tuple linguistic neutrosophic set (2TLNS) which the truth-membership degree (MD), indeterminacy-membership degree (IMD) and falsity-membership degree (FMD) are depicted by 2-tuple linguistic neutrosophic numbers (2TLNNs). Wang, Wei, and Lu (2018b) developed an extended TODIM model (Gomes & Rangel, 2009; Huang & Wei, 2018; Wang et al., 2018b; Wei, 2018) under 2-tuple linguistic neutrosophic environment and applied the new defined model in safety assessment of a construction project. Wang, Gao, and Wei (2018) studied the Muirhead mean (MM) operator and the dual Muirhead mean (DMM) operator under 2-tuple linguistic neutrosophic environment, then some 2-tuple linguistic neutrosophic Muirhead mean operators were given to deal with green supplier selection. Thereafter, the 2-tuple linguistic neutrosophic set (2TLNS) theory has been broadly used to study MCGDM problems.

For MADM problems, the way to express the assessment information is only one aspect, another vital aspect is selecting best alternative from a given alternative set. In previous document, some traditional decision making model had been applied to MADM problems, such as the EDAS model (Keshavarz Ghorabaee, Zavadskas, Olfat, & Turskis, 2015), the MABAC model (Pamucar & Cirovic, 2015), the COPRAS model (Roy, Sharma, Kar, Zavadskas, & Saparauskas, 2019), the TOPSIS model (Chen, 2000; Lai, Liu, & Hwang, 1994), The TODIM model (Gomes & Lima, 1979) and the GRA model (Li & Wei, 2014). As a powerful tool for handling MADM, The VIKOR (VIseKriterijumska Optimizacija I KOmpromisno Resenje) method (Opricovic & Tzeng, 2004), which owns precious merits of considering the compromise between group utility maximization and individual regret minimization, has been regards as a meaningful tool to apply in many decision making fields in past few years.

Comparing with these above mentioned methods, the VIKOR model has the advantage of taking the compromise between group utility maximization and individual regret minimization into consideration. Du and Liu (2011) extended the traditional VIKOR model to intuitionistic trapezoidal fuzzy environment. Park, Cho, and Kwun (2011) established the interval-valued intuitionistic fuzzy VIKOR model for MADM problems. Qin, Liu, and Pedrycz (2015) proposed an extension of VIKOR model based on interval type-2 fuzzy information. Based on extended hesitant fuzzy linguistic information, Ghadikolaei, Madhoushi, and Divsalar (2018) built new extended VIKOR model for MADM problems. Narayanamoorthy, Geetha, Rakkiyappan, and Joo (2019) developed an extended VIKOR model based on interval-valued intuitionistic hesitant fuzzy entropy for industrial robots selection. Yang, Pang, Shi, and Wang (2018) defined the linguistic hesitant intuitionistic VIKOR model for MADM. Wang, Zhang, Wang, and Li (2018) proposed the projection-based VIKOR model under picture fuzzy environment and applied it for the risk evaluation of construction project. Wu, Xu, Jiang, and Zhong (2019) presented the VIKOR model based on hesitant fuzzy linguistic term sets with possibility distributions.

According to above literature review, we can obtain that the 2-tuple linguistic neutrosophic set (2TLNS) can express the assessment information easily and reasonably, the VIKOR method can consider the conflicting criteria. Thus, to combine these two advantages, we shall propose some extended VIKOR models with 2TLNNs. The structure of our paper is organized as follows. Section 2 introduces the concepts, operation formulas, distance calculating method and some aggregation operators of 2TLNNs. Section 3 extends the original VIKOR method to 2TLNNs and introduce the calculating steps of VIKOR method with 2TLNNs. Section 4 extends the VIKOR method to IV2TLNNs and develops the calculating steps of VIKOR method with IV2TLNNs. Section 5 provides a numerical example for green supplier selection and introduces the comparison between our proposed methods with the existing method. Section 6 gives some summaries of our article.

2. Preliminaries

2.1. 2-tuple linguistic neutrosophic sets

Based on the concepts of 2-tuple linguistic fuzzy set (2TLS) and the fundamental theories of single valued neutrosophic set (SVNS), the 2-tuple linguistic neutrosophic sets (2TLNSs) which firstly defined by Wang, Wei et al. (2018) can be depicted as follows.

Definition 1. (Wang, Wei et al., 2018) Let $s_1, s_2, \ldots, s_k$ be a linguistic term set. Any label s_i shows a possible linguistic variable, and $S = \{s_0 = extremelypoor, s_1 = verypoor, s_2 = poor, s_3 = medium, \ s_4 = good, s_5 = verygood, s_6 = extremelygood.\}$, the 2TLNSs η can be depicted as:

$$\eta = \{(s_\alpha, \phi), (s_\beta, \varphi), (s_\chi, \gamma)\} \tag{1}$$

Where $\Delta^{-1}(s_\alpha,\phi), \Delta^{-1}(s_\beta,\varphi)$ *and* $\Delta^{-1}(s_\chi,\gamma) \in [0,k]$ represent the degree of the truth membership, the indeterminacy membership and the falsity membership which are expressed by 2TLNNs and satisfies the condition $0 \le \Delta^{-1}(s_\alpha,\phi) + \Delta^{-1}(s_\beta,\varphi) + \Delta^{-1}(s_\chi,\gamma) \le 3k$.

Definition 2. (Wang, Wei et al., 2018) Assume there are three 2TLNNs $\eta_1 = \{(s_{\alpha_1},\phi_1),(s_{\beta_1},\varphi_1),(s_{\chi_1},\gamma_1)\}$, $\eta_2 = \{(s_{\alpha_2},\phi_2),(s_{\beta_2},\varphi_2),(s_{\chi_2},\gamma_2)\}$ and $\eta = \{(s_\alpha,\phi),(s_\beta,\varphi),(s_\chi,\gamma)\}$, the operation laws of them can be defined:

$$(1)\ \eta_1 \oplus \eta_2 = \left\{ \begin{array}{c} \Delta\left(k\left(\frac{\Delta^{-1}(s_{\alpha_1},\phi_1)}{k} + \frac{\Delta^{-1}(s_{\alpha_2},\phi_2)}{k} - \frac{\Delta^{-1}(s_{\alpha_1},\phi_1)}{k}\cdot\frac{\Delta^{-1}(s_{\alpha_2},\phi_2)}{k}\right)\right), \\ \Delta\left(k\left(\frac{\Delta^{-1}(s_{\beta_1},\varphi_1)}{k}\cdot\frac{\Delta^{-1}(s_{\beta_2},\varphi_2)}{k}\right)\right), \Delta\left(k\left(\frac{\Delta^{-1}(s_{\chi_1},\gamma_1)}{k}\cdot\frac{\Delta^{-1}(s_{\chi_2},\gamma_2)}{k}\right)\right) \end{array} \right\};$$

$$(2)\ \eta_1 \otimes \eta_2 = \left\{ \begin{array}{l} \Delta\left(k\left(\frac{\Delta^{-1}(s_{\alpha_1},\phi_1)}{k}\cdot\frac{\Delta^{-1}(s_{\alpha_2},\phi_2)}{k}\right)\right), \\ \Delta\left(k\left(\frac{\Delta^{-1}(s_{\beta_1},\varphi_1)}{k} + \frac{\Delta^{-1}(s_{\beta_2},\varphi_2)}{k} - \frac{\Delta^{-1}(s_{\beta_1},\varphi_1)}{k}\cdot\frac{\Delta^{-1}(s_{\beta_2},\varphi_2)}{k}\right)\right), \\ \Delta\left(k\left(\frac{\Delta^{-1}(s_{\chi_1},\gamma_1)}{k} + \frac{\Delta^{-1}(s_{\gamma_2},\gamma_2)}{k} - \frac{\Delta^{-1}(s_{\chi_1},\gamma_1)}{k}\cdot\frac{\Delta^{-1}(s_{\gamma_2},\gamma_2)}{k}\right)\right) \end{array} \right\};$$

$$(3)\ \lambda\eta = \left\{\Delta\left(k\left(1-\left(1-\frac{\Delta^{-1}(s_\alpha,\phi)}{k}\right)^\lambda\right)\right), \Delta\left(k\left(\frac{\Delta^{-1}(s_\beta,\varphi)}{k}\right)^\lambda\right), \Delta\left(k\left(\frac{\Delta^{-1}(s_\chi,\gamma)}{k}\right)^\lambda\right)\right\}, \lambda>0;$$

$$(4)\ \eta^\lambda = \left\{\Delta\left(k\left(\frac{\Delta^{-1}(s_\alpha,\phi)}{k}\right)^\lambda\right), \Delta\left(k\left(1-\left(1-\frac{\Delta^{-1}(s_\beta,\varphi)}{k}\right)^\lambda\right)\right), \Delta\left(k\left(1-\left(1-\frac{\Delta^{-1}(s_\chi,\gamma)}{k}\right)^\lambda\right)\right)\right\}, \lambda>0.$$

According to the Definition 2, it's clear that the operation laws have the following properties.

$$(1)\ \eta_1 \oplus \eta_2 = \eta_2 \oplus \eta_1, \eta_1 \otimes \eta_2 = \eta_2 \otimes \eta_1, \left((\eta_1)^{\lambda_1}\right)^{\lambda_2} = (\eta_1)^{\lambda_1\lambda_2}; \tag{2}$$

$$(2)\ \lambda(\eta_1 \oplus \eta_2) = \lambda\eta_1 \oplus \lambda\eta_2, (\eta_1 \otimes \eta_2)^\lambda = (\eta_1)^\lambda \otimes (\eta_2)^\lambda; \tag{3}$$

$$(3)\ \lambda_1\eta_1 \oplus \lambda_2\eta_1 = (\lambda_1+\lambda_2)\eta_1, (\eta_1)^{\lambda_1} \otimes (\eta_1)^{\lambda_2} = (\eta_1)^{(\lambda_1+\lambda_2)}. \tag{4}$$

Definition 3. (Wang, Wei et al., 2018) Let $\eta = \{(s_\alpha, \phi), (s_\beta, \varphi), (s_\chi, \gamma)\}$ be a 2TLNN, the score and accuracy functions of η can be expressed:

$$s(\eta) = \frac{\left(2k + \Delta^{-1}(s_\alpha, \phi) - \Delta^{-1}(s_\beta, \varphi) - \Delta^{-1}(s_\chi, \gamma)\right)}{3k}, s(\eta) \in [0, 1] \tag{5}$$

$$h(\eta) = \Delta^{-1}(s_\alpha, \phi) - \Delta^{-1}(s_\chi, \gamma), h(\eta) \in [-k, k] \tag{6}$$

For two 2TLNNs η_1 and η_2, based on the Definition 3, then

$$\begin{array}{ll} (1) & \textit{if } s(\eta_1) < s(\eta_2), \textit{ then } \eta_1 < \eta_2; \\ (2) & \textit{if } s(\eta_1) > s(\eta_2), \textit{ then } \eta_1 > \eta_2; \\ (3) & \textit{if } s(\eta_1) = s(\eta_2), h(\eta_1) < h(\eta_2), \textit{ then } \eta_1 < \eta_2; \\ (4) & \textit{if } s(\eta_1) = s(\eta_2), h(\eta_1) > h(\eta_2), \textit{ then } \eta_1 > \eta_2; \\ (5) & \textit{if } s(\eta_1) = s(\eta_2), h(\eta_1) = h(\eta_2), \textit{ then } \eta_1 = \eta_2. \end{array}$$

2.2. The normalized Hamming distance

Definition 4. Let $\eta_1 = \{(s_{\alpha_1}, \phi_1), (s_{\beta_1}, \varphi_1), (s_{\chi_1}, \gamma_1)\}$ and $\eta_2 = \{(s_{\alpha_2}, \phi_2), (s_{\beta_2}, \varphi_2), (s_{\chi_2}, \gamma_2)\}$ be two 2TLNNs, then we can get the normalized Hamming distance:

$$d(\eta_1, \eta_2) = \frac{1}{3k}\left(\begin{array}{l} |\Delta^{-1}(s_{\alpha_1}, \phi_1) - \Delta^{-1}(s_{\alpha_2}, \phi_2)| + |\Delta^{-1}(s_{\beta_1}, \varphi_1) - \Delta^{-1}(s_{\beta_2}, \varphi_2)| \\ +|\Delta^{-1}(s_{\chi_1}, \gamma_1) - \Delta^{-1}(s_{\chi_2}, \gamma_2)| \end{array}\right) \tag{7}$$

Theorem 1. Assume there are three 2TLNNs $\eta_1 = \{(s_{\alpha_1}, \phi_1), (s_{\beta_1}, \varphi_1), (s_{\chi_1}, \gamma_1)\}$, $\eta_2 = \{(s_{\alpha_2}, \phi_2), (s_{\beta_2}, \varphi_2), (s_{\chi_2}, \gamma_2)\}$ and $\eta_3 = \{(s_{\alpha_3}, \phi_3), (s_{\beta_3}, \varphi_3), (s_{\chi_3}, \gamma_3)\}$, the Hamming distance d has the following properties:

$$\begin{array}{llll} (\mathrm{P1}) & 0 \le d(\eta_1, \eta_2) \le 1; & (\mathrm{P2}) & \textit{if } \ d(\eta_1, \eta_2) = 0, \ \textit{then } \eta_1 = \eta_2; \\ (\mathrm{P3}) & d(\eta_1, \eta_2) = d(\eta_2, \eta_1); & (\mathrm{P4}) & d(\eta_1, \eta_2) + d(\eta_2, \eta_3) \ge d(\eta_1, \eta_3). \end{array}$$

Proof. (P1) $0 \le d(\eta_1, \eta_2) \le 1$

Since $\Delta^{-1}(s_{\alpha_1}, \phi_1), \Delta^{-1}(s_{\alpha_2}, \phi_2) \in [0, k]$, then $0 \le |\Delta^{-1}(s_{\alpha_1}, \phi_1) - \Delta^{-1}(s_{\alpha_2}, \phi_2)| \le k$, similarly we can get $0 \le |\Delta^{-1}(s_{\beta_1}, \varphi_1) - \Delta^{-1}(s_{\beta_2}, \varphi_2)| \le k, 0 \le |\Delta^{-1}(s_{\chi_1}, \gamma_1) - \Delta^{-1}(s_{\chi_2}, \gamma_2)| \le k$, then $0 \le |\Delta^{-1}(s_{\alpha_1}, \phi_1) - \Delta^{-1}(s_{\alpha_2}, \phi_2)| + |\Delta^{-1}(s_{\beta_1}, \varphi_1) - \Delta^{-1}(s_{\beta_2}, \varphi_2)| + |\Delta^{-1}(s_{\chi_1}, \gamma_1) - \Delta^{-1}(s_{\chi_2}, \gamma_2)| \le 3k$, So $0 \le \left(|\Delta^{-1}(s_{\alpha_1}, \phi_1) - \Delta^{-1}(s_{\alpha_2}, \phi_2)| + |\Delta^{-1}(s_{\beta_1}, \varphi_1) - \Delta^{-1}(s_{\beta_2}, \varphi_2)| + |\Delta^{-1}(s_{\chi_1}, \gamma_1) - \Delta^{-1}(s_{\chi_2}, \gamma_2)|\right) \le 3k.$

Therefore $0 \le d(\eta_1, \eta_2) \le 1$, the proof is completed.

$$(\mathrm{P2}) \quad \textit{if} \quad d(\eta_1, \eta_2) = 0, \quad \textit{then} \quad \eta_1 = \eta_2$$

$$d(\eta_1,\eta_2)=\frac{1}{3k}\left(|\Delta^{-1}(s_{\alpha_1},\phi_1)-\Delta^{-1}(s_{\alpha_2},\phi_2)|+|\Delta^{-1}(s_{\beta_1},\varphi_1)-\Delta^{-1}(s_{\beta_2},\varphi_2)|+|\Delta^{-1}(s_{\chi_1},\gamma_1)-\Delta^{-1}(s_{\chi_2},\gamma_2)|\right)=0$$
$$\Rightarrow\left(|\Delta^{-1}(s_{\alpha_1},\phi_1)-\Delta^{-1}(s_{\alpha_2},\phi_2)|=0,|\Delta^{-1}(s_{\beta_1},\varphi_1)-\Delta^{-1}(s_{\beta_2},\varphi_2)|=0,|\Delta^{-1}(s_{\chi_1},\gamma_1)-\Delta^{-1}(s_{\chi_2},\gamma_2)|=0\right)$$
$$\Rightarrow\left(\Delta^{-1}(s_{\alpha_1},\phi_1)=\Delta^{-1}(s_{\alpha_2},\phi_2),\Delta^{-1}(s_{\beta_1},\varphi_1)=\Delta^{-1}(s_{\beta_2},\varphi_2),\Delta^{-1}(s_{\chi_1},\gamma_1)=\Delta^{-1}(s_{\chi_2},\gamma_2)\right)$$

That means $\eta_1=\eta_2$, so (P2) *if* $d(\eta_1,\eta_2)=0$, *then* $\eta_1=\eta_2$ is right.

(P3) $d(\eta_1,\eta_2)=d(\eta_2,\eta_1)$

$$d(\eta_1,\eta_2)=\frac{1}{3k}\left(|\Delta^{-1}(s_{\alpha_1},\phi_1)-\Delta^{-1}(s_{\alpha_2},\phi_2)|+|\Delta^{-1}(s_{\beta_1},\varphi_1)-\Delta^{-1}(s_{\beta_2},\varphi_2)|+|\Delta^{-1}(s_{\chi_1},\gamma_1)-\Delta^{-1}(s_{\chi_2},\gamma_2)|\right)$$
$$=\frac{1}{3k}\left(|\Delta^{-1}(s_{\alpha_2},\phi_2)-\Delta^{-1}(s_{\alpha_1},\phi_1)|+|\Delta^{-1}(s_{\beta_2},\varphi_2)-\Delta^{-1}(s_{\beta_1},\varphi_1)|+|\Delta^{-1}(s_{\chi_2},\gamma_2)-\Delta^{-1}(s_{\chi_1},\gamma_1)|\right)=d(\eta_2,\eta_1)$$

So we complete the proof. (P3) $d(\eta_1,\eta_2)=d(\eta_2,\eta_1)$ is hold.

(P4) $d(\eta_1,\eta_2)+d(\eta_2,\eta_3)\geq d(\eta_1,\eta_3)$

$$\begin{aligned}
d(\eta_1,\eta_2)&=\frac{1}{3k}\begin{pmatrix}|\Delta^{-1}(s_{\alpha_1},\phi_1)-\Delta^{-1}(s_{\alpha_3},\phi_3)|+|\Delta^{-1}(s_{\beta_1},\varphi_1)-\Delta^{-1}(s_{\beta_3},\varphi_3)|\\+|\Delta^{-1}(s_{\chi_1},\gamma_1)-\Delta^{-1}(s_{\chi_3},\gamma_3)|\end{pmatrix}\\
&=\frac{1}{3k}\begin{pmatrix}|\Delta^{-1}(s_{\alpha_1},\phi_1)-\Delta^{-1}(s_{\alpha_2},\phi_2)+\Delta^{-1}(s_{\alpha_2},\phi_2)-\Delta^{-1}(s_{\alpha_3},\phi_3)|\\+|\Delta^{-1}(s_{\beta_1},\varphi_1)-\Delta^{-1}(s_{\beta_2},\varphi_2)+\Delta^{-1}(s_{\beta_2},\varphi_2)-\Delta^{-1}(s_{\beta_3},\varphi_3)|\\+|\Delta^{-1}(s_{\chi_1},\gamma_1)-\Delta^{-1}(s_{\chi_2},\gamma_2)+\Delta^{-1}(s_{\chi_2},\gamma_2)-\Delta^{-1}(s_{\chi_3},\gamma_3)|\end{pmatrix}\\
&\leq\frac{1}{3k}\begin{pmatrix}|\Delta^{-1}(s_{\alpha_1},\phi_1)-\Delta^{-1}(s_{\alpha_2},\phi_2)|+|\Delta^{-1}(s_{\alpha_2},\phi_2)-\Delta^{-1}(s_{\alpha_3},\phi_3)|\\+|\Delta^{-1}(s_{\beta_1},\varphi_1)-\Delta^{-1}(s_{\beta_2},\varphi_2)|+|\Delta^{-1}(s_{\beta_2},\varphi_2)-\Delta^{-1}(s_{\beta_3},\varphi_3)|\\+|\Delta^{-1}(s_{\chi_1},\gamma_1)-\Delta^{-1}(s_{\chi_2},\gamma_2)|+|\Delta^{-1}(s_{\chi_2},\gamma_2)-\Delta^{-1}(s_{\chi_3},\gamma_3)|\end{pmatrix}\\
&=d(\eta_1,\eta_2)+d(\eta_2,\eta_3)
\end{aligned}$$

2.3. The aggregation operators of 2TLNNs

Definition 5. (Wang, Wei et al., 2018). Let $\eta_j=\{(s_{\alpha_j},\phi_j),(s_{\beta_j},\varphi_j),(s_{\chi_j},\gamma_j)\}$ $(j=1,2,\ldots,n)$ be a group of 2TLNNs, then the 2TLNNWA and 2TLNNWG operators are defined as follows.

$$\text{2TLNNWA}(\eta_1,\eta_2,\ldots,\eta_n)=\omega_1\eta_1\oplus\omega_2\eta_2\cdots\oplus\omega_n\eta_n=\mathop{\oplus}_{j=1}^{n}\omega_j\eta_j \tag{8}$$

and

$$\text{2TLNNWG}(\eta_1,\eta_2,\ldots,\eta_n)=(\eta_1)^{\omega_1}\otimes(\eta_2)^{\omega_2}\cdots\otimes(\eta_n)^{\omega_n}=\mathop{\otimes}_{j=1}^{n}(\eta_j)^{\omega_j} \tag{9}$$

where ω_j is weighting vector of $\eta_j, j=1,2,\ldots,n$. which satisfies $0\leq\omega_j\leq1$, $\sum_{j=1}^{n}\omega_j=1$.

Theorem 2. (Wang, Wei et al., 2018) Let $\eta_j = \{(s_{\alpha_j}, \phi_j), (s_{\beta_j}, \varphi_j), (s_{\chi_j}, \gamma_j)\}$ $(j = 1, 2, ..., n)$ be a group of 2TLNNs, then the operation results by 2TLNNWA and 2TLNNWG operators are also a 2TLNN where

$$2\text{TLNNWA}(\eta_1, \eta_2, ..., \eta_n) = \mathop{\oplus}_{j=1}^{n} \omega_j \eta_j$$
$$= \left\langle \begin{array}{l} \Delta\left(k\left(1 - \prod_{j=1}^{n}\left(1 - \frac{\Delta^{-1}(s_{\alpha_j}, \phi_j)}{k}\right)^{w_j}\right)\right), \Delta\left(k\prod_{j=1}^{n}\left(\frac{\Delta^{-1}(s_{\beta_j}, \varphi_j)}{k}\right)^{w_j}\right), \\ \Delta\left(k\prod_{j=1}^{n}\left(\frac{\Delta^{-1}(s_{\chi_j}, \gamma_j)}{k}\right)^{w_j}\right). \end{array} \right\rangle \tag{10}$$

and

$$2\text{TLNNWG}(\eta_1, \eta_2, ..., \eta_n) = \mathop{\otimes}_{j=1}^{n} (\eta_j)^{\omega_j}$$
$$= \left\langle \begin{array}{l} \Delta\left(k\prod_{j=1}^{n}\left(\frac{\Delta^{-1}(s_{\alpha_j}, \phi_j)}{k}\right)^{w_j}\right), \Delta\left(k\left(1 - \prod_{j=1}^{n}\left(1 - \frac{\Delta^{-1}(s_{\beta_j}, \varphi_j)}{k}\right)^{w_j}\right)\right), \\ \Delta\left(k\left(1 - \prod_{j=1}^{n}\left(1 - \frac{\Delta^{-1}(s_{\chi_j}, \gamma_j)}{k}\right)^{w_j}\right)\right). \end{array} \right\rangle \tag{11}$$

3. The VIKOR model for 2TLNNs MCGDM problems

Assume that $\{A_1, A_2, ..., A_m\}$ be a group of alternatives, $\{D_1, D_2, ..., D_\lambda\}$ be a list of experts with weighting vector be $\{v_1, v_2, ..., v_t\}$, and $\{G_1, G_2, ..., G_n\}$ be a list of criteria with weighting vector be $\{\omega_1, \omega_2, ..., \omega_n\}$, thereby satisfying$\omega_i \in [0, 1], v_i \in [0, 1]$ and $\sum_{i=1}^{n} \omega_i = 1, \sum_{i=1}^{t} v_i = 1$. Construct the evaluation matrix $\eta^\lambda = [\eta_{ij}^\lambda]_{m\times n}, i = 1, 2, ..., m, j = 1, 2, ..., n, \lambda = 1, 2, ..., t,$ where $\eta_{ij}^\lambda = \{(s_{\alpha_{ij}}, \phi_{ij})^\lambda, (s_{\beta_{ij}}, \varphi_{ij})^\lambda, (s_{\chi_{ij}}, \gamma_{ij})^\lambda\}$ means the estimate results of the alternative $A_i(i = 1, 2, ..., m)$ based on the criterion $G_j(j = 1, 2, ..., n)$ by expert $D^\lambda(\lambda = 1, 2, ..., t)$. $\Delta^{-1}(s_{\alpha_{ij}}, \phi_{ij})^\lambda \in [0, k]$ denotes the degree of truth-membership (TMD), $\Delta^{-1}(s_{\beta_{ij}}, \varphi_{ij})^\lambda \in [0, k]$ denotes the degree of indeterminacy-membership (IMD) and $\Delta^{-1}(s_{\chi_{ij}}, \gamma_{ij})^\lambda \in [0, k]$ denotes the degree of falsity-membership (FMD) $0 \le \Delta^{-1}(s_{\alpha_{ij}}, \phi_{ij})^\lambda + \Delta^{-1}(s_{\beta_{ij}}, \varphi_{ij})^\lambda + \Delta^{-1}(s_{\chi_{ij}}, \gamma_{ij})^\lambda \le 3k, (i = 1, 2, ..., m, j = 1, 2, ..., n, \lambda = 1, 2, ..., t)$.

Consider both the 2TLNNs theories and the traditional VIKOR model; we try to propose the VIKOR method with 2TLNNs to study MCGDM problems effectively. The method can be depicted as follows:

Step 1. Construct the decision matrix $\eta^\lambda = [\eta_{ij}^\lambda]_{m\times n}$, and utilize overall values of $\eta^\lambda = [\eta_{ij}^\lambda]_{m\times n}$ to $\eta = [\eta_{ij}]_{m\times n}$ by using equal (10) or (11);

Step 2. Compute the positive ideal solution (PIS) A^+ and the negative ideal solution (NIS)A^-;

$$A^{+}=\left\{\Delta^{-1}(s_{\alpha_j},\phi_j)^{+},\Delta^{-1}(s_{\beta_j},\varphi_j)^{+},\Delta^{-1}(s_{\chi_j},\gamma_j)^{+}\right\}(j=1,2,\ldots,n) \tag{12}$$

$$A=\left\{\Delta^{-1}(s_{\alpha_j},\phi_j)^{-},\Delta^{-1}(s_{\beta_j},\varphi_j)^{-},\Delta^{-1}(s_{\chi_j},\gamma_j)^{-}\right\}(j=1,2,\ldots,n) \tag{13}$$

For benefit attribute

$$\begin{aligned}A^{+}&=\left\{\Delta^{-1}(s_{\alpha_j},\phi_j)^{+},\Delta^{-1}(s_{\beta_j},\varphi_j)^{+},\Delta^{-1}(s_{\chi_j},\gamma_j)^{+}\right\}\\&=\left\{\max_i\left(\Delta^{-1}(s_{\alpha_j},\phi_j)\right),\ \min_i\left(\Delta^{-1}(s_{\beta_j},\varphi_j)\right),\ \min_i\left(\Delta^{-1}(s_{\chi_j},\gamma_j)\right)\right\}\end{aligned} \tag{14}$$

$$\begin{aligned}A^{-}&=\left\{\Delta^{-1}(s_{\alpha_j},\phi_j)^{-},\Delta^{-1}(s_{\beta_j},\varphi_j)^{-},\Delta^{-1}(s_{\chi_j},\gamma_j)^{-}\right\}\\&=\left\{\min_i\left(\Delta^{-1}(s_{\alpha_j},\phi_j)\right),\ \max_i\left(\Delta^{-1}(s_{\beta_j},\varphi_j)\right),\ \max_i\left(\Delta^{-1}(s_{\chi_j},\gamma_j)\right)\right\}\end{aligned} \tag{15}$$

For cost attribute

$$\begin{aligned}A^{+}&=\left\{\Delta^{-1}(s_{\alpha_j},\phi_j)^{+},\Delta^{-1}(s_{\beta_j},\varphi_j)^{+},\Delta^{-1}(s_{\chi_j},\gamma_j)^{+}\right\}\\&=\left\{\min_i\left(\Delta^{-1}(s_{\alpha_j},\phi_j)\right),\ \max_i\left(\Delta^{-1}(s_{\beta_j},\varphi_j)\right),\ \max_i\left(\Delta^{-1}(s_{\chi_j},\gamma_j)\right)\right\}\end{aligned} \tag{16}$$

$$\begin{aligned}A^{-}&=\left\{\Delta^{-1}(s_{\alpha_j},\phi_j)^{-},\Delta^{-1}(s_{\beta_j},\varphi_j)^{-},\Delta^{-1}(s_{\chi_j},\gamma_j)^{-}\right\}\\&=\left\{\max_i\left(\Delta^{-1}(s_{\alpha_j},\phi_j)\right),\ \min_i\left(\Delta^{-1}(s_{\beta_j},\varphi_j)\right),\ \min_i\left(\Delta^{-1}(s_{\chi_j},\gamma_j)\right)\right\}\end{aligned} \tag{17}$$

Step 3. Based on the Equation (7) and the attribute weighting vector ω_j, we can calculate the values of τ_i and ψ_i which express the average and the worst group scores of η_i.

$$\tau_i=\sum_{j=1}^{n}\omega_j\frac{d\left(\begin{array}{c}\left\{\Delta^{-1}(s_{\alpha_j},\phi_j)^{+},\Delta^{-1}(s_{\beta_j},\varphi_j)^{+},\Delta^{-1}(s_{\chi_j},\gamma_j)^{+}\right\},\\\left\{\Delta^{-1}(s_{\alpha_j},\phi_j),\Delta^{-1}(s_{\beta_j},\varphi_j),\Delta^{-1}(s_{\chi_j},\gamma_j)\right\}\end{array}\right)}{d\left(\begin{array}{c}\left\{\Delta^{-1}(s_{\alpha_j},\phi_j)^{+},\Delta^{-1}(s_{\beta_j},\varphi_j)^{+},\Delta^{-1}(s_{\chi_j},\gamma_j)^{+}\right\},\\\left\{\Delta^{-1}(s_{\alpha_j},\phi_j)^{-},\Delta^{-1}(s_{\beta_j},\varphi_j)^{-},\Delta^{-1}(s_{\chi_j},\gamma_j)^{-}\right\}\end{array}\right)} \tag{18}$$

$$\psi_i = \max_j \left\{ \omega_j \frac{d\begin{pmatrix} \left\{\Delta^{-1}(s_{\alpha_j}, \phi_j)^+, \Delta^{-1}(s_{\beta_j}, \varphi_j)^+, \Delta^{-1}(s_{\chi_j}, \gamma_j)^+\right\}, \\ \left\{\Delta^{-1}(s_{\alpha_j}, \phi_j), \Delta^{-1}(s_{\beta_j}, \varphi_j), \Delta^{-1}(s_{\chi_j}, \gamma_j)\right\} \end{pmatrix}}{d\begin{pmatrix} \left\{\Delta^{-1}(s_{\alpha_j}, \phi_j)^+, \Delta^{-1}(s_{\beta_j}, \varphi_j)^+, \Delta^{-1}(s_{\chi_j}, \gamma_j)^+\right\}, \\ \left\{\Delta^{-1}(s_{\alpha_j}, \phi_j)^-, \Delta^{-1}(s_{\beta_j}, \varphi_j)^-, \Delta^{-1}(s_{\chi_j}, \gamma_j)^-\right\} \end{pmatrix}} \right\} \tag{19}$$

where d is the normalized Hamming distance and $0 \leq \omega_j \leq 1$ means the weight of attributes which satisfies $\sum_{i=1}^{n} \omega_i = 1$.

Step 4. Compute the values of Q_i based on the results of τ_i and ψ_i, the calculating formula is characteristic as follows.

$$Q_i = \rho \frac{(\tau_i - \tau^+)}{(\tau^- - \tau^+)} + (1 - \rho) \frac{(\psi_i - \psi^+)}{(\psi^- - \psi^+)} \tag{20}$$

where

$$\tau^+ = \min_i \tau_i, \tau^- = \max_i \tau_i \tag{21}$$

$$\psi^+ = \min_i \psi_i, \psi^- = \max_i \psi_i \tag{22}$$

where ρ means the coefficient of decision making strategic. $\rho > 0.5$ depicts "the maximum group utility", $\rho = 0.5$ depicts equality and $\rho < 0.5$ depicts the minimum regret.

Step 5. To choose the best alternative by rank the values of Q_i, the alternative with minimum value is the best choice.

4. The VIKOR method for IV2TLNNs MCDM problems

4.1. The IV2TLNSs

To solve MCDM problems more effectively, we extend the 2TLNSs to interval-valued environment to propose the IV2TLNSs as follows.

Definition 6. Let $s_1, s_2, \ldots, s_k$ be a linguistic term set. Any label s_i shows a possible linguistic variable, the IV2TLNSs $\tilde{\eta}$ can be depicted as:

$$\tilde{\eta} = \left\{ \left[(s_\alpha, \phi)^L, (s_\alpha, \phi)^U\right], \left[(s_\beta, \varphi)^L, (s_\beta, \varphi)^U\right], \left[(s_\chi, \gamma)^L, (s_\chi, \gamma)^U\right] \right\} \tag{23}$$

where $\left[\Delta^{-1}(s_\alpha, \phi)^L, \Delta^{-1}(s_\alpha, \phi)^U\right]$, $\left[\Delta^{-1}(s_\beta, \varphi)^L, \Delta^{-1}(s_\beta, \varphi)^U\right]$ *and* $\left[\Delta^{-1}(s_\chi, \gamma)^L, \Delta^{-1}(s_\chi, \gamma)^U\right]$ $\in [0, k]$ represent the degree of the truth membership, the indeterminacy membership and the falsity membership which are expressed by IV2TLNNs and satisfies the condition $0 \le \Delta^{-1}(s_\alpha, \phi)^U + \Delta^{-1}(s_\beta, \varphi)^U + \Delta^{-1}(s_\chi, \gamma)^U \le 3k$.

Definition 7. Let $\tilde{\eta} = \{[(s_\alpha, \phi)^L, (s_\alpha, \phi)^U], [(s_\beta, \varphi)^L, (s_\beta, \varphi)^U], [(s_\chi, \gamma)^L, (s_\chi, \gamma)^U]\}$ be an IV2TLNN, the score and accuracy functions of $\tilde{\eta}$ can be expressed:

$$s(\tilde{\eta}) = \frac{1}{6k}\left\{\begin{array}{l} \left(2k + \Delta^{-1}(s_\alpha, \phi)^L - \Delta^{-1}(s_\beta, \varphi)^L - \Delta^{-1}(s_\chi, \gamma)^L\right) \\ +\left(2k + \Delta^{-1}(s_\alpha, \phi)^U - \Delta^{-1}(s_\beta, \varphi)^U - \Delta^{-1}(s_\chi, \gamma)^U\right) \end{array}\right\}, s(\tilde{\eta}) \in [0, 1] \quad (24)$$

$$h(\tilde{\eta}) = \frac{\left(\Delta^{-1}(s_\alpha, \phi)^L - \Delta^{-1}(s_\chi, \gamma)^L\right) + \left(\Delta^{-1}(s_\alpha, \phi)^U - \Delta^{-1}(s_\chi, \gamma)^U\right)}{2}, h(\tilde{\eta}) \in [-k, k] \quad (25)$$

For two IV2TLNNs $\tilde{\eta}_1$ and $\tilde{\eta}_2$, based on the Definition 7, then

$$\begin{array}{ll} (1) & if\ \ s(\tilde{\eta}_1) \prec s(\tilde{\eta}_2),\ \ then\ \ \tilde{\eta}_1 \prec \tilde{\eta}_2; \\ (2) & if\ \ s(\tilde{\eta}_1) \succ s(\tilde{\eta}_2),\ \ then\ \ \tilde{\eta}_1 \succ \tilde{\eta}_2; \\ (3) & if\ \ s(\tilde{\eta}_1) = s(\tilde{\eta}_2),\ \ h(\tilde{\eta}_1) \prec h(\tilde{\eta}_2),\ \ then\ \ \tilde{\eta}_1 \prec \tilde{\eta}_2; \\ (4) & if\ \ s(\tilde{\eta}_1) = s(\tilde{\eta}_2),\ \ h(\tilde{\eta}_1) \succ h(\tilde{\eta}_2),\ \ then\ \ \tilde{\eta}_1 \succ \tilde{\eta}_2; \\ (5) & if\ \ s(\tilde{\eta}_1) = s(\tilde{\eta}_2),\ \ h(\tilde{\eta}_1) = h(\tilde{\eta}_2),\ \ then\ \ \tilde{\eta}_1 = \tilde{\eta}_2. \end{array}$$

Definition 8. Let $\tilde{\eta}_1 = \{[(s_{\alpha_1}, \phi_1)^L, (s_{\alpha_1}, \phi_1)^U], [(s_{\beta_1}, \varphi_1)^L, (s_{\beta_1}, \varphi_1)^U], [(s_{\chi_1}, \gamma_1)^L, (s_{\chi_1}, \gamma_1)^U]\}$ and $\tilde{\eta}_2 = \{[(s_{\alpha_2}, \phi_2)^L, (s_{\alpha_2}, \phi_2)^U], [(s_{\beta_2}, \varphi_2)^L, (s_{\beta_2}, \varphi_2)^U], [(s_{\chi_2}, \gamma_2)^L, (s_{\chi_2}, \gamma_2)^U]\}$ be two IV2TLNNs, then we can get the normalized Hamming distance:

$$d(\tilde{\eta}_1, \tilde{\eta}_2) = \frac{1}{6k}\left(\begin{array}{l} |\Delta^{-1}(s_{\alpha_1}, \phi_1)^L - \Delta^{-1}(s_{\alpha_2}, \phi_2)^L| + |\Delta^{-1}(s_{\beta_1}, \varphi_1)^L - \Delta^{-1}(s_{\beta_2}, \varphi_2)^L| \\ +|\Delta^{-1}(s_{\chi_1}, \gamma_1)^L - \Delta^{-1}(s_{\chi_2}, \gamma_2)^L| + |\Delta^{-1}(s_{\alpha_1}, \phi_1)^U - \Delta^{-1}(s_{\alpha_2}, \phi_2)^U| \\ +|\Delta^{-1}(s_{\beta_1}, \varphi_1)^U - \Delta^{-1}(s_{\beta_2}, \varphi_2)^U| + |\Delta^{-1}(s_{\chi_1}, \gamma_1)^U - \Delta^{-1}(s_{\chi_2}, \gamma_2)^U| \end{array}\right) \quad (26)$$

Theorem 3. Let $\tilde{\eta}_1 = \{[(s_{\alpha_1}, \phi_1)^L, (s_{\alpha_1}, \phi_1)^U], [(s_{\beta_1}, \varphi_1)^L, (s_{\beta_1}, \varphi_1)^U], [(s_{\chi_1}, \gamma_1)^L, (s_{\chi_1}, \gamma_1)^U]\}$, $\tilde{\eta}_2 = \{[(s_{\alpha_2}, \phi_2)^L, (s_{\alpha_2}, \phi_2)^U], [(s_{\beta_2}, \varphi_2)^L, (s_{\beta_2}, \varphi_2)^U], [(s_{\chi_2}, \gamma_2)^L, (s_{\chi_2}, \gamma_2)^U]\}$ and $\tilde{\eta}_3 = \{[(s_{\alpha_3}, \phi_3)^L, (s_{\alpha_3}, \phi_3)^U], [(s_{\beta_3}, \varphi_3)^L, (s_{\beta_3}, \varphi_3)^U], [(s_{\chi_3}, \gamma_2)^L, (s_{\chi_3}, \gamma_3)^U]\}$, the Hamming distance d also has the following properties:

$$\begin{array}{llll} (P1) & 0 \le d(\tilde{\eta}_1, \tilde{\eta}_2) \le 1; & (P2) & if\ d(\tilde{\eta}_1, \tilde{\eta}_2) = 0,\ then\ \tilde{\eta}_1 = \tilde{\eta}_2; \\ (P3) & d(\tilde{\eta}_1, \tilde{\eta}_2) = d(\tilde{\eta}_2, \tilde{\eta}_1); & (P4) & d(\tilde{\eta}_1, \tilde{\eta}_2) + d(\tilde{\eta}_2, \tilde{\eta}_3) \ge d(\tilde{\eta}_1, \tilde{\eta}_3). \end{array}$$

4.2. The aggregation operators of IV2TLNNs

Definition 9. Let $\tilde{\eta}_j = \{[(s_{\alpha_j}, \phi_j)^L, (s_{\alpha_j}, \phi_j)^U], [(s_{\beta_j}, \varphi_j)^L, (s_{\beta_j}, \varphi_j)^U], [(s_{\chi_j}, \gamma_j)^L, (s_{\chi_j}, \gamma_j)^U]\}$ be a group of IV2TLNNs, then the IV2TLNNWA and IV2TLNNWG operators can be defined as follows.

$$\mathrm{IV2TLNNWA}(\tilde{\eta}_1, \tilde{\eta}_2, \ldots, \tilde{\eta}_n) = \overset{n}{\underset{j=1}{\oplus}} \omega_j \tilde{\eta}_j$$
$$= \left\{ \begin{array}{c} \left[\Delta\left(k\left(1 - \prod_{j=1}^{n} \left(1 - \frac{\Delta^{-1}(s_{\alpha_j}, \phi_j)^L}{k} \right)^{w_j} \right) \right), \Delta\left(k\left(1 - \prod_{j=1}^{n} \left(1 - \frac{\Delta^{-1}(s_{\alpha_j}, \phi_j)^U}{k} \right)^{w_j} \right) \right) \right], \\ \left[\Delta\left(k \prod_{j=1}^{n} \left(\frac{\Delta^{-1}(s_{\beta_j}, \varphi_j)^L}{k} \right)^{w_j} \right), \Delta\left(k \prod_{j=1}^{n} \left(\frac{\Delta^{-1}(s_{\beta_j}, \varphi_j)^U}{k} \right)^{w_j} \right) \right], \\ \left[\Delta\left(k \prod_{j=1}^{n} \left(\frac{\Delta^{-1}(s_{\chi_j}, \gamma_j)^L}{k} \right)^{w_j} \right), \Delta\left(k \prod_{j=1}^{n} \left(\frac{\Delta^{-1}(s_{\chi_j}, \gamma_j)^U}{k} \right)^{w_j} \right) \right]. \end{array} \right\} \tag{27}$$

and

$$\mathrm{IV2TLNNWG}(\tilde{\eta}_1, \tilde{\eta}_2, \ldots, \tilde{\eta}_n) = \overset{n}{\underset{j=1}{\otimes}} (\tilde{\eta}_j)^{\omega_j}$$
$$= \left\{ \begin{array}{c} \left[\Delta\left(k \prod_{j=1}^{n} \left(\frac{\Delta^{-1}(s_{\alpha_j}, \phi_j)^L}{k} \right)^{w_j} \right), \Delta\left(k \prod_{j=1}^{n} \left(\frac{\Delta^{-1}(s_{\alpha_j}, \phi_j)^U}{k} \right)^{w_j} \right) \right], \\ \left[\Delta\left(k\left(1 - \prod_{j=1}^{n} \left(1 - \frac{\Delta^{-1}(s_{\beta_j}, \varphi_j)^L}{k} \right)^{w_j} \right) \right), \Delta\left(k\left(1 - \prod_{j=1}^{n} \left(1 - \frac{\Delta^{-1}(s_{\beta_j}, \varphi_j)^U}{k} \right)^{w_j} \right) \right) \right], \\ \left[\Delta\left(k\left(1 - \prod_{j=1}^{n} \left(1 - \frac{\Delta^{-1}(s_{\chi_j}, \gamma_j)^L}{k} \right)^{w_j} \right) \right), \Delta\left(k\left(1 - \prod_{j=1}^{n} \left(1 - \frac{\Delta^{-1}(s_{\chi_j}, \gamma_j)^U}{k} \right)^{w_j} \right) \right) \right]. \end{array} \right\} \tag{28}$$

4.3. Computing steps for MCGDM problems with IV2TLNNs

Assume that $\{A_1, A_2, \ldots A_m\}$ be a group of alternatives and $\{G_1, G_2, \ldots G_n\}$ be a list of criteria with weighting vector be $\{\omega_1, \omega_2, \ldots \omega_n\}$, thereby satisfying $\omega_i \in [0, 1]$ and $\sum_{i=1}^{n} \omega_i = 1$. Construct the evaluation matrix $\tilde{\eta} = [\tilde{\eta}_{ij}]_{m \times n}, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$ where $\tilde{\eta}_{ij} = \{[(s_{\alpha_{ij}}, \phi_{ij})^L, (s_{\alpha_{ij}}, \phi_{ij})^U], [(s_{\beta_{ij}}, \varphi_{ij})^L, (s_{\beta_{ij}}, \varphi_{ij})^U], [(s_{\chi_{ij}}, \gamma_{ij})^L, (s_{\chi_{ij}}, \gamma_{ij})^U]\}$ means the estimate results of the alternative $A_i (i = 1, 2, \ldots, m)$ based on the criterion $G_j (j = 1, 2, \ldots, n)$. The calculating steps also can be depicted as follows:

Step 1. Construct the decision matrix $\tilde{\eta} = [\tilde{\eta}_{ij}]_{m \times n}$;
Step 2. Compute the positive ideal solution A^+ and the negative ideal solution A^-;

$$A^+ = \left\{ \begin{array}{c} \left[\left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^L \right)^+, \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^U \right)^+ \right], \\ \left[\left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^L \right)^+, \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^U \right)^+ \right], \\ \left[\left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^L \right)^+, \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^U \right)^+ \right] \end{array} \right\} (j = 1, 2, \ldots, n) \tag{29}$$

$$A^{-} = \left\{ \begin{array}{c} \left[\left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^L\right)^{-}, \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^U\right)^{-} \right], \\ \left[\left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^L\right)^{-}, \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^U\right)^{-} \right], \\ \left[\left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^L\right)^{-}, \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^U\right)^{-} \right] \end{array} \right\} (j = 1, 2, \ldots, n) \tag{30}$$

For benefit attribute

$$\left\{ \begin{array}{c} \left[\left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^L\right)^{+}, \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^U\right)^{+} \right], \\ \left[\left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^L\right)^{+}, \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^U\right)^{+} \right], \\ \left[\left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^L\right)^{+}, \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^U\right)^{+} \right] \end{array} \right\} = \left\{ \begin{array}{c} \left[\max_i \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^L\right), \max_i \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^U\right) \right], \\ \left[\min_i \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^L\right), \min_i \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^U\right) \right], \\ \left[\min_i \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^L\right), \min_i \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^U\right) \right] \end{array} \right\} \tag{31}$$

$$\left\{ \begin{array}{c} \left[\left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^L\right)^{-}, \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^U\right)^{-} \right], \\ \left[\left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^L\right)^{-}, \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^U\right)^{-} \right], \\ \left[\left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^L\right)^{-}, \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^U\right)^{-} \right] \end{array} \right\} = \left\{ \begin{array}{c} \left[\min_i \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^L\right), \min_i \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^U\right) \right], \\ \left[\max_i \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^L\right), \max_i \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^U\right) \right], \\ \left[\max_i \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^L\right), \max_i \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^U\right) \right] \end{array} \right\} \tag{32}$$

For cost attribute

$$\left\{ \begin{array}{c} \left[\left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^L\right)^{+}, \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^U\right)^{+} \right], \\ \left[\left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^L\right)^{+}, \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^U\right)^{+} \right], \\ \left[\left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^L\right)^{+}, \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^U\right)^{+} \right] \end{array} \right\} = \left\{ \begin{array}{c} \left[\min_i \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^L\right), \min_i \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^U\right) \right], \\ \left[\max_i \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^L\right), \max_i \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^U\right) \right], \\ \left[\max_i \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^L\right), \max_i \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^U\right) \right] \end{array} \right\} \tag{33}$$

$$\left\{ \begin{array}{c} \left[\left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^L\right)^{-}, \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^U\right)^{-} \right], \\ \left[\left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^L\right)^{-}, \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^U\right)^{-} \right], \\ \left[\left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^L\right)^{-}, \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^U\right)^{-} \right] \end{array} \right\} = \left\{ \begin{array}{c} \left[\max_i \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^L\right), \max_i \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^U\right) \right], \\ \left[\min_i \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^L\right), \min_i \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^U\right) \right], \\ \left[\min_i \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^L\right), \min_i \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^U\right) \right] \end{array} \right\} \tag{34}$$

Step 3. Based on the Equation (26) and the attribute weighting vector ω_j, we can calculate the values of $\tilde{\tau}_i$ and $\tilde{\psi}_i$ which express the average and the worst group scores of $\tilde{\eta}_i$.

$$\tilde{\tau}_i = \sum_{j=1}^{n} \omega_j \frac{d\left(\left\{ \begin{array}{l} \left[\left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^L \right)^+, \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^U \right)^+ \right], \\ \left[\left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^L \right)^+, \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^U \right)^+ \right], \\ \left[\left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^L \right)^+, \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^U \right)^+ \right] \end{array} \right\}, \left\{ \begin{array}{l} \left[\Delta^{-1}(s_{\alpha_j}, \phi_j)^L, \Delta^{-1}(s_{\alpha_j}, \phi_j)^U \right], \\ \left[\Delta^{-1}(s_{\beta_j}, \varphi_j)^L, \Delta^{-1}(s_{\beta_j}, \varphi_j)^U \right], \\ \left[\Delta^{-1}(s_{\chi_j}, \gamma_j)^L, \Delta^{-1}(s_{\chi_j}, \gamma_j)^U \right] \end{array} \right\} \right)}{d\left(\left\{ \begin{array}{l} \left[\left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^L \right)^+, \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^U \right)^+ \right], \\ \left[\left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^L \right)^+, \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^U \right)^+ \right], \\ \left[\left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^L \right)^+, \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^U \right)^+ \right] \end{array} \right\}, \left\{ \begin{array}{l} \left[\left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^L \right)^-, \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^U \right)^- \right], \\ \left[\left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^L \right)^-, \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^U \right)^- \right], \\ \left[\left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^L \right)^-, \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^U \right)^- \right] \end{array} \right\} \right)} \tag{35}$$

$$\tilde{\psi}_i = \max_j \left\{ \omega_j \frac{d\left(\left\{ \begin{array}{l} \left[\left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^L \right)^+, \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^U \right)^+ \right], \\ \left[\left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^L \right)^+, \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^U \right)^+ \right], \\ \left[\left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^L \right)^+, \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^U \right)^+ \right] \end{array} \right\}, \left\{ \begin{array}{l} \left[\Delta^{-1}(s_{\alpha_j}, \phi_j)^L, \Delta^{-1}(s_{\alpha_j}, \phi_j)^U \right], \\ \left[\Delta^{-1}(s_{\beta_j}, \varphi_j)^L, \Delta^{-1}(s_{\beta_j}, \varphi_j)^U \right], \\ \left[\Delta^{-1}(s_{\chi_j}, \gamma_j)^L, \Delta^{-1}(s_{\chi_j}, \gamma_j)^U \right] \end{array} \right\} \right)}{d\left(\left\{ \begin{array}{l} \left[\left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^L \right)^+, \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^U \right)^+ \right], \\ \left[\left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^L \right)^+, \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^U \right)^+ \right], \\ \left[\left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^L \right)^+, \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^U \right)^+ \right] \end{array} \right\}, \left\{ \begin{array}{l} \left[\left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^L \right)^-, \left(\Delta^{-1}(s_{\alpha_j}, \phi_j)^U \right)^- \right], \\ \left[\left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^L \right)^-, \left(\Delta^{-1}(s_{\beta_j}, \varphi_j)^U \right)^- \right], \\ \left[\left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^L \right)^-, \left(\Delta^{-1}(s_{\chi_j}, \gamma_j)^U \right)^- \right] \end{array} \right\} \right)} \right\} \tag{36}$$

where d is the normalized Hamming distance and $0 \leq \omega_j \leq 1$ means the weight of attributes which satisfies $\sum_{i=1}^{n} \omega_i = 1$.

Step 4. Compute the values of Q_i based on the results of $\tilde{\tau}_i$ and $\tilde{\psi}_i$, the calculating formula is characteristic as follows.

$$Q_i = \rho \frac{(\tilde{\tau}_i - \tilde{\tau}^+)}{(\tilde{\tau}^- - \tilde{\tau}^+)} + (1 - \rho) \frac{(\tilde{\psi}_i - \tilde{\psi}^+)}{(\tilde{\psi}^- - \tilde{\psi}^+)} \tag{37}$$

where

$$\tilde{\tau}^+ = \min_i \tilde{\tau}_i, \tau^- = \max_i \tilde{\tau}_i \tag{38}$$

$$\tilde{\psi}^+ = \min_i \tilde{\psi}_i, \tilde{\psi}^- = \max_i \tilde{\psi}_i \tag{39}$$

where ρ means the coefficient of decision making strategic. $\rho > 0.5$ depicts "the maximum group utility", $\rho = 0.5$ depicts equality and $\rho < 0.5$ depicts the minimum regret.

Step 5. To choose the best alternative by rank the values of Q_i, the alternative with minimum value is the best choice.

5. The numerical example

5.1. Numerical for 2TLNNs MCGDM problems

After China's entering into the WTO, the economy has developed rapidly and it has held a high rate of economic growth. But the stamina of development is confronted with severe challenges: On the one hand, the international economic situation is continuously changing and many enterprises in China are limited by international green barriers; On the other hand, while enjoying great economic development achievements, people also realized that our country's environment and resources are becoming more and more serious. While China's economic development is growing at a high speed, the ecological environment and natural resources have been seriously injured and the contradiction between natural resource environment and social economic development has become increasingly obvious. Under the background of people's urgent need for environmental protection and healthy living, many enterprises in China are aware of the necessity and importance of green health and low carbon environmental protection for the survival and development of enterprises. Green suppliers selection is a classical MADM problem. In this chapter, we provide a numerical example to select best green suppliers selection by using VIKOR method with 2TLNNs. Assume that five possible green suppliers $A_i (i = 1, 2, 3, 4, 5)$ to be selected and four criteria to assess these green suppliers: ① G_1 is the product quality factor; ② G_2 is environmental factors; ③ G_3 is delivery factor; ④ G_4 is price factor. The five possible green suppliers $A_i (i = 1, 2, 3, 4, 5)$ are to be evaluated with 2TLNNs with the four criteria by three experts (criteria weight $\omega = (0.32, 0.13, 0.35, 0.20)$, experts weight $v = (0.25, 0.35, 0.40)$.), which are given in Tables 1–3.

Step 1. Utilize overall values of $\eta^{\lambda} = \left[\eta_{ij}^{\lambda}\right]_{m \times n}$ to $\eta = [\eta_{ij}]_{m \times n}$ by using 2TLNNWA operator, the aggregation results are listed in Table 4.

Table 1. 2TLNNs evaluation matrix by the first expert.

	G_1	G_2	G_3	G_4
A_1	$\{(s_4,0), (s_2,0), (s_1,0)\}$	$\{(s_5,0), (s_3,0), (s_2,0)\}$	$\{(s_4,0), (s_1,0), (s_1,0)\}$	$\{(s_3,0), (s_2,0), (s_2,0)\}$
A_2	$\{(s_5,0), (s_4,0), (s_4,0)\}$	$\{(s_3,0), (s_4,0), (s_2,0)\}$	$\{(s_2,0), (s_1,0), (s_3,0)\}$	$\{(s_3,0), (s_1,0), (s_2,0)\}$
A_3	$\{(s_5,0), (s_4,0), (s_3,0)\}$	$\{(s_2,0), (s_4,0), (s_5,0)\}$	$\{(s_3,0), (s_3,0), (s_4,0)\}$	$\{(s_2,0), (s_1,0), (s_4,0)\}$
A_4	$\{(s_3,0), (s_2,0), (s_3,0)\}$	$\{(s_4,0), (s_3,0), (s_2,0)\}$	$\{(s_3,0), (s_3,0), (s_4,0)\}$	$\{(s_2,0), (s_1,0), (s_1,0)\}$
A_5	$\{(s_1,0), (s_4,0), (s_5,0)\}$	$\{(s_2,0), (s_3,0), (s_1,0)\}$	$\{(s_3,0), (s_4,0), (s_5,0)\}$	$\{(s_2,0), (s_4,0), (s_3,0)\}$

Table 2. 2TLNNs evaluation matrix by the second expert.

	G_1	G_2	G_3	G_4
η_1	$\{(s_5,0), (s_1,0), (s_2,0)\}$	$\{(s_4,0), (s_3,0), (s_1,0)\}$	$\{(s_4,0), (s_2,0), (s_1,0)\}$	$\{(s_5,0), (s_1,0), (s_2,0)\}$
η_2	$\{(s_4,0), (s_3,0), (s_3,0)\}$	$\{(s_3,0), (s_2,0), (s_4,0)\}$	$\{(s_2,0), (s_1,0), (s_3,0)\}$	$\{(s_5,0), (s_4,0), (s_2,0)\}$
η_3	$\{(s_3,0), (s_4,0), (s_3,0)\}$	$\{(s_2,0), (s_4,0), (s_5,0)\}$	$\{(s_5,0), (s_1,0), (s_2,0)\}$	$\{(s_2,0), (s_1,0), (s_2,0)\}$
η_4	$\{(s_4,0), (s_5,0), (s_4,0)\}$	$\{(s_2,0), (s_3,0), (s_2,0)\}$	$\{(s_3,0), (s_3,0), (s_4,0)\}$	$\{(s_4,0), (s_4,0), (s_5,0)\}$
η_5	$\{(s_2,0), (s_4,0), (s_5,0)\}$	$\{(s_3,0), (s_1,0), (s_5,0)\}$	$\{(s_2,0), (s_3,0), (s_4,0)\}$	$\{(s_2,0), (s_1,0), (s_3,0)\}$

Table 3. 2TLNNs evaluation matrix by the third expert.

	G_1	G_2	G_3	G_4
A_1	$\{(s_5,0), (s_1,0), (s_1,0)\}$	$\{(s_4,0), (s_1,0), (s_2,0)\}$	$\{(s_3,0), (s_3,0), (s_1,0)\}$	$\{(s_4,0), (s_2,0), (s_2,0)\}$
A_2	$\{(s_5,0), (s_4,0), (s_5,0)\}$	$\{(s_3,0), (s_2,0), (s_1,0)\}$	$\{(s_2,0), (s_1,0), (s_4,0)\}$	$\{(s_4,0), (s_5,0), (s_3,0)\}$
A_3	$\{(s_2,0), (s_1,0), (s_4,0)\}$	$\{(s_5,0), (s_1,0), (s_3,0)\}$	$\{(s_4,0), (s_3,0), (s_4,0)\}$	$\{(s_5,0), (s_2,0), (s_3,0)\}$
A_4	$\{(s_2,0), (s_2,0), (s_3,0)\}$	$\{(s_4,0), (s_1,0), (s_2,0)\}$	$\{(s_5,0), (s_3,0), (s_2,0)\}$	$\{(s_1,0), (s_4,0), (s_5,0)\}$
A_5	$\{(s_1,0), (s_4,0), (s_5,0)\}$	$\{(s_2,0), (s_4,0), (s_4,0)\}$	$\{(s_3,0), (s_4,0), (s_3,0)\}$	$\{(s_2,0), (s_4,0), (s_4,0)\}$

Table 4. The aggregation values by 2TLNNWA operator.

	G_1	G_2
A_1	$\{(s_5,-0.1892), (s_1,0.1892), (s_1,0.2746)\}$	$\{(s_4,0.3182), (s_2,-0.0668), (s_2,-0.4308)\}$
A_2	$\{(s_5,-0.2746), (s_4,-0.3831), (s_4,-0.0455)\}$	$\{(s_3,0.0000), (s_2,0.3784), (s_2,-0.0681)\}$
A_3	$\{(s_3,0.4425), (s_2,0.2974), (s_3,0.3659)\}$	$\{(s_4,-0.2974), (s_2,0.2974), (s_4,0.0760)\}$
A_4	$\{(s_3,0.0794), (s_3,-0.2438), (s_3,0.3178)\}$	$\{(s_3,0.4509), (s_2,-0.0668), (s_2,0.0000)\}$
A_5	$\{(s_1,0.3756), (s_4,0.0000), (s_5,0.0000)\}$	$\{(s_2,0.3831), (s_2,0.2914), (s_3,0.0582)\}$
	G_3	G_4
A_1	$\{(s_4,-0.3522), (s_2,-0.0221), (s_1,0.0000)\}$	$\{(s_4,0.2634), (s_2,-0.4308), (s_2,0.0000)\}$
A_2	$\{(s_2,0.0000), (s_1,0.0000), (s_3,0.3659)\}$	$\{(s_4,0.2634), (s_3,0.0925), (s_2,0.3522)\}$
A_3	$\{(s_4,0.2634), (s_2,0.0423), (s_3,0.1383)\}$	$\{(s_4,-0.2974), (s_1,0.3195), (s_3,-0.2028)\}$
A_4	$\{(s_4,0.0668), (s_3,0.0000), (s_3,0.0314)\}$	$\{(s_3,-0.4313), (s_3,-0.1716), (s_3,0.3437)\}$
A_5	$\{(s_3,-0.3178), (s_4,-0.3831), (s_4,-0.2303)\}$	$\{(s_2,0.0000), (s_2,0.4623), (s_3,0.3659)\}$

Step 2. Compute the values of A^+ (PIS) and A^-(NIS), for all attributes are benefit and based on the formula (16) and (17), we can obtain the (PIS) A^+ and (NIS)A^- as follows.

$$A^+ = \left\{ \begin{array}{l} \{(s_5,\ -0.1892), (s_1, 0.1892), (s_1, 0.2746)\}, \\ \{(s_4, 0.3182), (s_2,\ -0.0668), (s_2,\ -0.4308)\}, \\ \{(s_4, 0.2634), (s_1, 0.0000), (s_1, 0.0000)\}, \\ \{(s_4, 0.2634), (s_1, 0.3195), (s_2, 0.0000)\} \end{array} \right\}$$

$$A^- = \left\{ \begin{array}{l} \{(s_1, 0.3756), (s_4, 0.0000), (s_5, 0.0000)\}, \\ \{(s_2, 0.3831), (s_2, 0.3784), (s_4, 0.0760)\}, \\ \{(s_2, 0.0000), (s_4,\ -0.3831), (s_4,\ -0.2303)\}, \\ \{(s_2, 0.0000), (s_3, 0.0925), (s_3, 0.3659)\} \end{array} \right\}$$

Step 3. Based on the Equation (7) and the attribute weighting vector ω_j, calculate the values of τ_i and ψ_i.

$$\tau_1 = 0.0821, \tau_2 = 0.5137, \tau_3 = 0.4351, \tau_4 = 0.5677, \tau_5 = 0.9161,$$

$$\psi_1 = 0.0729, \psi_2 = 0.2118, \psi_3 = 0.1466, \psi_4 = 0.1934, \psi_5 = 0.3200.$$

Table 5. Ordering by the VIKOR method with 2TLNNs.

Parameter	Q_1	Q_2	Q_3	Q_4	Q_5	Ordering
$\rho = 0.0$	0.0000	0.5621	0.2982	0.4878	1.0000	$Q_1 > Q_3 > Q_4 > Q_2 > Q_5$
$\rho = 0.1$	0.0000	0.5576	0.3107	0.4973	1.0000	$Q_1 > Q_3 > Q_4 > Q_2 > Q_5$
$\rho = 0.3$	0.0000	0.5487	0.3357	0.5162	1.0000	$Q_1 > Q_3 > Q_4 > Q_2 > Q_5$
$\rho = 0.5$	0.0000	0.5398	0.3607	0.5351	1.0000	$Q_1 > Q_3 > Q_4 > Q_2 > Q_5$
$\rho = 0.7$	0.0000	0.5309	0.3858	0.5540	1.0000	$Q_1 > Q_3 > Q_4 > Q_2 > Q_5$
$\rho = 1.0$	0.0000	0.5175	0.4233	0.5823	1.0000	$Q_1 > Q_3 > Q_4 > Q_2 > Q_5$

Step 4. Compute the values of Q_i based on the results of τ_i and ψ_i, the calculating values are listed as follows. (Let $\rho = 0.4$)

$$Q_1 = 0.0000, Q_2 = 0.5442, Q_3 = 0.3482, Q_4 = 0.5256, Q_5 = 1.0000.$$

Step 5. To choose the best alternative by rank the values of Q_i, the ranking of Q_i is $Q_1 > Q_3 > Q_4 > Q_2 > Q_5$, and the best choice is η_1.

By altering the parameter ρ, we can derive the following results which listed in Table 5.

From Table 5, we can easily find that the ordering of alternatives are same, which indicates our developed method has the robustness and can be applied to deal with practical decision making problems.

5.2. Comparative analyses

In this section, we compare our proposed VIKOR method under 2TLNNs with the 2TLNNWA and 2TLNNWG operators defined by Wang, Wei et al. (2018). Based on the values of Table 4 and attributes weighting vector $\omega = (0.32, 0.13, 0.35, 0.20)^T$, we can compute overall value η_i by 2TLNNWA and 2TLNNWG operators.

We can get calculating results η_i by 2TLNNWA operator:

$$\begin{aligned}
\eta_1 &= \{(s_4, 0.2963), (s_2, -0.4001), (s_1, 0.3163)\} \\
\eta_2 &= \{(s_4, -0.2615), (s_2, 0.1166), (s_3, 0.0692)\} \\
\eta_3 &= \{(s_4, -0.1558), (s_2, -0.0267), (s_3, 0.2448)\} \\
\eta_4 &= \{(s_3, 0.4351), (s_3, -0.2747), (s_3, 0.0146)\} \\
\eta_5 &= \{(s_2, 0.1265), (s_3, 0.2595), (s_4, -0.0744)\}
\end{aligned}$$

We can get calculating results η_i by 2TLNNWG operator:

$$\begin{aligned}
\eta_1 &= \{(s_4, 0.2030), (s_2, -0.3488), (s_1, 0.3771)\} \\
\eta_2 &= \{(s_3, 0.2297), (s_3, -0.3938), (s_3, 0.2565)\} \\
\eta_3 &= \{(s_4, -0.1997), (s_2, 0.0282), (s_3, 0.2931)\} \\
\eta_4 &= \{(s_3, 0.3222), (s_3, -0.2358), (s_3, 0.0783)\} \\
\eta_5 &= \{(s_2, 0.0115), (s_3, 0.4173), (s_4, 0.1509)\}
\end{aligned}$$

Table 6. Alternative scores $s(\eta_i)$ by 2TLNNWA and 2TLNNWG operators.

2TLNNWA operator	2TLNNWG operator
$s(\eta_1) = 0.7433, s(\eta_2) = 0.5863,\ s(\eta_3) = 0.5903,\ s(\eta_4) = 0.5386,\ s(\eta_5) = 0.3856.$	$s(\eta_1) = 0.7319, s(\eta_2) = 0.5204,\ s(\eta_3) = 0.5822,\ s(\eta_4) = 0.5267,\ s(\eta_5) = 0.3580.$

Table 7. Rank of Alternatives by 2TLNNWA and 2TLNNWG operators.

	order
2TLNNWA	$A_1 > A_3 > A_2 > A_4 > A_5$
2TLNNWG	$A_1 > A_3 > A_4 > A_2 > A_5$
2TLNNs VIKOR	$A_1 > A_3 > A_4 > A_2 > A_5$

Then, we calculate the alternative scores $s(\eta_i)$ by score functions of 2TLNNs which are listed in Table 6.

The ranking of alternatives by 2TLNNWA and 2TLNNWG operators are listed in Table 7.

Compare the values of our proposed VIKOR method under 2TLNNs with 2TLNNWA and 2TLNNWG operators, the results are slightly different in ranking of alternatives and the best alternatives are same, VIKOR method with 2TLNNs can consider the conflicting attributes and can be more reasonable and scientific in the application of MCGDM problems.

5.3. Numerical case for MCDM problems with IV2TLNNs

In this chapter, if the evaluation values of five green suppliers are depicted by IV2TLNNs, then we can study the MCDM problems by using the VIKOR method with IV2TLNNs, the decision matrix are listed in Table 8 (attribute weighting vector $\omega = (0.4, 0.2, 0.1, 0.3)^T$).

Step 1. Construct the decision matrix (See Table 8)

Step 2. Compute the values of A^+ (PIS) and A^-(NIS), for all attributes are benefit and based on the formula (18) and (19), we can obtain the (PIS) A^+ and (NIS)A^- as follows.

$$A^+ = \left\{ \begin{array}{c} \{[(s_4,0),(s_6,0)],[(s_1,0),(s_2,0)],[(s_2,0),\ (s_3,0)]\}, \\ \{[(s_3,0),(s_5,0)],[(s_2,0),(s_3,0)],[(s_1,0),\ (s_2,0)]\}, \\ \{[(s_5,0),(s_6,0)],[(s_1,0),(s_3,0)],[(s_1,0),\ (s_2,0)]\}, \\ \{[(s_5,0),(s_6,0)],[(s_2,0),(s_3,0)],[(s_1,0),\ (s_2,0)]\} \end{array} \right\}$$

$$A^- = \left\{ \begin{array}{c} \{[(s_1,0),(s_2,0)],[(s_4,0),(s_5,0)],[(s_5,0),\ (s_6,0)]\}, \\ \{[(s_1,0),(s_3,0)],[(s_5,0),(s_6,0)],[(s_4,0),\ (s_5,0)]\}, \\ \{[(s_1,0),(s_2,0)],[(s_4,0),(s_5,0)],[(s_3,0),\ (s_4,0)]\}, \\ \{[(s_1,0),(s_2,0)],[(s_4,0),(s_6,0)],[(s_2,0),\ (s_3,0)]\} \end{array} \right\}$$

Table 8. IV2TLNNs evaluation matrix.

	G_1	G_2
A_1	$\{[(s_4,0),(s_5,0)],[(s_1,0),(s_2,0)],[(s_2,0),(s_3,0)]\}$	$\{[(s_3,0),(s_5,0)],[(s_2,0),(s_3,0)],[(s_1,0),(s_2,0)]\}$
A_2	$\{[(s_1,0),(s_2,0)],[(s_2,0),(s_3,0)],[(s_3,0),(s_4,0)]\}$	$\{[(s_1,0),(s_3,0)],[(s_2,0),(s_4,0)],[(s_3,0),(s_5,0)]\}$
A_3	$\{[(s_4,0),(s_5,0)],[(s_3,0),(s_4,0)],[(s_4,0),(s_5,0)]\}$	$\{[(s_2,0),(s_3,0)],[(s_2,0),(s_4,0)],[(s_1,0),(s_2,0)]\}$
A_4	$\{[(s_4,0),(s_6,0)],[(s_2,0),(s_4,0)],[(s_5,0),(s_6,0)]\}$	$\{[(s_3,0),(s_4,0)],[(s_2,0),(s_3,0)],[(s_1,0),(s_2,0)]\}$
A_5	$\{[(s_1,0),(s_2,0)],[(s_4,0),(s_5,0)],[(s_5,0),(s_6,0)]\}$	$\{[(s_2,0),(s_3,0)],[(s_5,0),(s_6,0)],[(s_4,0),(s_5,0)]\}$
	G_3	G_4
A_1	$\{[(s_5,0),(s_6,0)],[(s_1,0),(s_3,0)],[(s_2,0),(s_3,0)]\}$	$\{[(s_5,0),(s_6,0)],[(s_3,0),(s_4,0)],[(s_1,0),(s_2,0)]\}$
A_2	$\{[(s_5,0),(s_6,0)],[(s_4,0),(s_5,0)],[(s_1,0),(s_2,0)]\}$	$\{[(s_3,0),(s_4,0)],[(s_2,0),(s_3,0)],[(s_1,0),(s_2,0)]\}$
A_3	$\{[(s_4,0),(s_5,0)],[(s_3,0),(s_4,0)],[(s_2,0),(s_3,0)]\}$	$\{[(s_1,0),(s_2,0)],[(s_4,0),(s_5,0)],[(s_2,0),(s_3,0)]\}$
A_4	$\{[(s_3,0),(s_4,0)],[(s_4,0),(s_5,0)],[(s_2,0),(s_3,0)]\}$	$\{[(s_1,0),(s_2,0)],[(s_2,0),(s_3,0)],[(s_1,0),(s_2,0)]\}$
A_5	$\{[(s_1,0),(s_3,0)],[(s_2,0),(s_5,0)],[(s_3,0),(s_4,0)]\}$	$\{[(s_2,0),(s_3,0)],[(s_4,0),(s_6,0)],[(s_2,0),(s_3,0)]\}$

Step 3. Based on the Equation (11) and the attribute weighting vector ω_j, calculate the values of $\tilde{\tau}_i$ and $\tilde{\psi}_i$.

$$\tilde{\tau}_1 = 0.0736, \tilde{\tau}_2 = 0.4678, \tilde{\tau}_3 = 0.5632, \tilde{\tau}_4 = 0.4307, \tilde{\tau}_5 = 0.9350,$$

$$\tilde{\psi}_1 = 0.0400, \tilde{\psi}_2 = 0.2316, \tilde{\psi}_3 = 0.2800, \tilde{\psi}_4 = 0.1895, \tilde{\psi}_5 = 0.4000.$$

Step 4. Compute the values of Q_i based on the results of $\tilde{\tau}_i$ and $\tilde{\psi}_i$, the calculating values are listed as follows. (Let $\rho = 0.4$)

$$Q_1 = 0.0000, Q_2 = 0.5024\ , Q_3 = 0.6274, Q_4 = 0.4150, Q_5 = 1.0000.$$

Step 5. To choose the best alternative by rank the values of Q_i, the ranking of Q_i is $Q_1 > Q_4 > Q_2 > Q_3 > Q_5$, and the best choice is $\tilde{\eta}_1$.

By altering the parameter ρ, we can derive the following results which listed in Table 9.

From Table 9, we can easily find that the ordering of alternatives are same, which indicates our developed method has the robustness and can be applied to deal with practical decision making problems.

5.4. Comparative analyses

In this section, we compare our proposed the extend VIKOR method under IV2TLNNs with the IV2TLNNWA and IV2TLNNWG operators. Based on the values of Table 4 and attributes weighting vector $\omega = (0.4, 0.2, 0.1, 0.3)^T$, we can utilize overall $\tilde{\eta}_{ij}$ by IV2TLNNWA and IV2TLNNWG operators.

Table 9. Ordering by the VIKOR method with IV2TLNNs.

Parameter	Q_1	Q_2	Q_3	Q_4	Q_5	Ordering
$\rho = 0.0$	0.0000	0.5322	0.6667	0.4152	1.0000	$Q_1 > Q_4 > Q_2 > Q_3 > Q_5$
$\rho = 0.1$	0.0000	0.5247	0.6568	0.4151	1.0000	$Q_1 > Q_4 > Q_2 > Q_3 > Q_5$
$\rho = 0.3$	0.0000	0.5098	0.6372	0.4150	1.0000	$Q_1 > Q_4 > Q_2 > Q_3 > Q_5$
$\rho = 0.5$	0.0000	0.4949	0.6175	0.4149	1.0000	$Q_1 > Q_4 > Q_2 > Q_3 > Q_5$
$\rho = 0.7$	0.0000	0.4800	0.5979	0.4148	1.0000	$Q_1 > Q_4 > Q_2 > Q_3 > Q_5$
$\rho = 1.0$	0.0000	0.4577	0.5684	0.4146	1.0000	$Q_1 > Q_4 > Q_2 > Q_3 > Q_5$

We can get calculating results $\tilde{\eta}_i$ by IV2TLNNWA operator:

$$\tilde{\eta}_1 = \{[(s_4, 0.3562), (s_6, 0.0000)], [(s_2, -0.4029), (s_3, -0.2192)], [(s_1, 0.4142), (s_2, 0.4495)]\}$$
$$\tilde{\eta}_2 = \{[(s_2, 0.3481), (s_6, 0.0000)], [(s_2, 0.1435), (s_3, 0.3442)], [(s_2, -0.0668), (s_3, 0.1698)]\}$$
$$\tilde{\eta}_3 = \{[(s_3, -0.0243), (s_4, 0.1118)], [(s_3, 0.0157), (s_4, 0.2769)], [(s_2, 0.2974), (s_3, 0.3935)]\}$$
$$\tilde{\eta}_4 = \{[(s_3, 0.0267), (s_6, 0.0000)], [(s_2, 0.1435), (s_4, -0.4577)], [(s_2, 0.0403), (s_3, 0.2321)]\}$$
$$\tilde{\eta}_5 = \{[(s_2, -0.4721), (s_3, -0.3659)], [(s_4, -0.0975), (s_5, 0.4772)], [(s_3, 0.4516), (s_5, -0.4877)]\}$$

We can get calculating results $\tilde{\eta}_i$ by IV2TLNNWG operator:

$$\tilde{\eta}_1 = \{[(s_4, 0.1289), (s_5, 0.3783)], [(s_2, -0.1024), (s_3, 0.0196)], [(s_2, -0.4721), (s_3, -0.4641)]\}$$
$$\tilde{\eta}_2 = \{[(s_2, -0.3668), (s_3, -0.0196)], [(s_2, 0.2679), (s_4, -0.4785)], [(s_2, 0.3199), (s_4, -0.2974)]\}$$
$$\tilde{\eta}_3 = \{[(s_2, 0.2974), (s_3, 0.4294)], [(s_3, 0.1863), (s_4, 0.3755)], [(s_3, -0.1698), (s_4, -0.0477)]\}$$
$$\tilde{\eta}_4 = \{[(s_2, 0.4208), (s_4, -0.1789)], [(s_2, 0.2679), (s_4, -0.2855)], [(s_3, 0.4314), (s_6, 0.0000)]\}$$
$$\tilde{\eta}_5 = \{[(s_1, 0.4142), (s_3, -0.4492)], [(s_4, 0.1339), (s_6, 0.0000)], [(s_4, 0.0567), (s_6, 0.0000)]\}$$

Calculating the alternative scores $s(\tilde{\eta}_i)$ by score functions of IV2TLNNs which listed in Table 10.

The ranking of alternatives by IV2TLNNWA and IV2TLNNWG operators are listed in Table 11.

Compare the values of our proposed VIKOR method under IV2TLNNs with IV2TLNNWA and IV2TLNNWG operators, the results are slightly different in ranking of alternatives and the best alternatives are same, IV2TLNNs VIKOR method can consider the conflicting attributes and can be more reasonable and scientific in the application of MCGDM problems.

5.5. Discussion

Based on above two numerical examples, we can easily find our proposed methods can express more fuzzy information and apply broadly situations in real MCGDM problems. Based on 2-tuple linguistic neutrosophic fuzzy set (2TLNS) and traditional VIKOR method, we develop the 2-tuple linguistic neutrosophic VIKOR method and the interval-valued 2-tuple linguistic neutrosophic VIKOR method; our research results can be more suitable for MCGDM problems than single-valued neutrosophic VIKOR method depicted in literature (Huang, Wei, & Wei, 2017). For the single-valued neutrosophic VIKOR method can't deal with MCGDM problems which the assessment results are depicted with 2TLNNs.

Table 10. Alternative scores $s(\tilde{\eta}_i)$ by IV2TLNNWA and IV2TLNNWG operators.

IV2TLNNWA operator	IV2TLNNWG operator
$s(\tilde{\eta}_1) = 0.7254, s(\tilde{\eta}_2) = 0.6044,\ s(\tilde{\eta}_3) = 0.5029,$ $s(\tilde{\eta}_4) = 0.6130,\ s(\tilde{\eta}_5) = 0.3005.$	$s(\tilde{\eta}_1) = 0.6813, s(\tilde{\eta}_2) = 0.4667,\ s(\tilde{\eta}_3) = 0.4273,$ $s(\tilde{\eta}_4) = 0.4119,\ s(\tilde{\eta}_5) = 0.2160.$

Table 11. Rank of Alternatives by IV2TLNNWA and IV2TLNNWG operators.

	Order
IV2TLNNWA	$A_1 > A_4 > A_2 > A_3 > A_5$
IV2TLNNWG	$A_1 > A_2 > A_3 > A_4 > A_5$
IV2TLNNs VIKOR	$A_1 > A_4 > A_2 > A_3 > A_5$

Furthermore, in complicated decision-making environment, the decision maker's risk attitude is an important factor to think about. the VIKOR methods, which consider the compromise between group utility maximization and individual regret minimization, can make this come true by altering the parameters whereas other decision making ways such as the 2TLNNWA operator, the 2TLNNWG operator, the IV2TLNNWA operator and the IV2TLNNWG operator don't have the ability that dynamic adjust to the parameter according to the decision maker's risk attitude, so it is difficult to solve the risk multiple attribute decision making in real practice.

6. Conclusion

The 2-tuple linguistic neutrosophic set (2TLNS), which is the generalized form of 2-tuple linguistic set (2TLS) and single-valued neutrosophic set (SVNS), can express the assessment information more easily and reasonably. The VIKOR method, which can consider the compromise between group utility maximization and individual regret minimization, can derive more accuracy decision making results. In this paper, based on traditional VIKOR method and the 2-tuple linguistic neutrosophic set, we develop the 2-tuple linguistic neutrosophic VIKOR method. Furthermore, we extend the 2TLNSs to interval-valued environment and propose the VIKOR method with IV2TLNNs. Moreover, a numerical example for green supplier selection has been proposed to illustrate the new method and some comparisons are also conducted to further illustrate advantages of the new method. In the future, our proposed VIKOR method with 2TLNNs and VIKOR method with IV2TLNNs can be applied to the risk analysis (Wei, Qin, Li, Zhu, & Wei, 2019; Wei, Yu, Liu, & Cao, 2018), the MCGDM problems (Hashemi, Mousavi, Zavadskas, Chalekaee, & Turskis, 2018; Yazdani, Zarate, Coulibaly, & Zavadskas, 2017; Zavadskas, Turskis, Vilutienė, & Lepkova, 2017) and many other uncertain and fuzzy environments (Deng & Gao, 2019; Gao, 2018; Gupta, Mehlawat, & Grover, 2019; Li & Lu, 2019; Lu & Wei, 2019; Wang, Gao, & Lu, 2019; Wang, 2019; Wu, Gao, & Wei, 2019; Wu, Wang, & Gao, 2019; Xian, Chai, & Guo, 2019).

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