Computers in Industry 115 (2020) 103117

Contents lists available at ScienceDirect

Computers in Industry

journal homepage: www.elsevier.com/locate/compind

Review

# Neutrosophic fusion of rough set theory: An overview

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ARTICLE INFO

*Article history:*
Received 2 January 2019
Received in revised form 10 July 2019
Accepted 25 July 2019
Available online 25 November 2019

*Keywords:*
Neutrosophic fusion
Rough set theory
Rough neutrosophic sets
Neutrosophic rough sets
Bibliometric overview

ABSTRACT

Neutrosophic sets (NSs) and logic are one of the influential mathematical tools to manage various uncertainties. Among diverse models for analyzing neutrosophic information, rough set theory (RST) provides an effective way in the field of neutrosophic information analysis, and a multitude of scholars have focused on neutrosophic fusion of RST in recent years. At present, there are not comprehensive literature reviews and statistics of these generalized rough set theories and applications. This review study first explores a summarization of current neutrosophic fusion of RST from five basic aspects, i.e., rough neutrosophic sets (RNSs) and neutrosophic rough sets (NRSs), soft rough neutrosophic sets (SRNSs) and neutrosophic soft rough sets (NSRSs), mathematical foundations of RNSs and NRSs, RNSs and NRSs-based decision making, RNSs and NRSs-based other applications. Then, on the basis of the overview from five fundamental perspectives, a systematic bibliometric overview of current works with respect to neutrosophic fusion of RST is further conducted. Finally, in light of the results of this review, different challenging issues related to the main topics are listed, which are beneficial to future studies of NSs and logic.



**Contents**



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https://doi.org/10.1016/j.compind.2019.07.007
0166-3615/

## 1. Introduction

First established by Smarandache in 1990s, NSs and logic [1,2] involve three different membership functions (the truth one, the indeterminacy one, the falsity one) by virtue of non-standard sets and logic, in which each value of three membership functions falls within]$0^-$, $1^+$[. As a consequence, NSs can be regarded as several generalized forms [3–6]. Taking full advantages of NSs is able to describe lots of uncertain information from the perspective of three different membership functions, NSs play a key role in coping with indeterminate situations that are extensively existed in a variety of realistic scenarios [7–16].

Ever since the construction of NSs and logic, many scholars and practitioners have developed and enriched the neutrosophic set (NS) theory from different aspects such as extensions, information measures, decision making approaches, image processing, etc., and a great deal of meaningful academic works have been presented during the past two decades [17]. Among diverse soft computing models, RST [18,19] can be seen as a reasonable and effective one for neutrosophic information processing, and neutrosophic fusion of RST has become a significant research branch in NSs and logic. Compared with other theories, the motivations of integrating NSs with RST are summarized as follows:

(1) It is noted that NSs excel in depicting indeterminate information, and RST provides a framework for handling insufficient and incomplete information, they are generally accepted as related, but distinct and complementary. Hence, how to generalize RST to NSs is important for the development of both NSs and RST. To simultaneously exploit the superiorities of NSs and RST, it is meaningful to construct a hybrid model of NSs and RST.
(2) Using the idea of RST, various hidden knowledge can be unveiled from insufficient and incomplete information systems, especially for studying inconsistent data. For instance, two patients are likely to share the identical symptoms, but they actually have different medical diagnosis results. Owing to the above-mentioned unique merits owned by RST, several branches, such as cognitive science [20–22], image processing [23], fault diagnosis [24,25], medical diagnosis [26,27] and machine learning [28–30], can be extended to the context of NSs, and this extension is conducive to the processing of neutrosophic information.
(3) In the area of decision making and intelligent systems [31–41], for a general information system with attribute values, some hidden rules can be discovered by conducting attribute reductions, which may offer the most informative data for further analyzing. Moreover, RST provides viable schemes on how to make decisions under specific situations and explain decisions with respect to scenarios under which decisions have been made. In light of the above-stated advantages, RST has proved its excellent performances via attribute reductions and decision explanations. Hence, the fusion of NSs and RST is favorable to the addressing of neutrosophic decision making problems.

In classical RST, the notion of information systems is used for describing the structure of information, which is featured by a matrix with columns represented by attributes, rows represented by objects, and matrix values represented by attribute values. Afterwards, the set of entire objects is denoted by the universe of discourse, and a binary relation over the identical universe can be obtained from certain attributes of an information system. By means of binary relations, two definable sets, i.e., lower and upper approximations, can be designed for constructing a classical rough set. However, with the rapid growth of information technology, there exists growing complex data in real-world applications, especially diverse fuzzy data. Considering classical rough sets can only process binary information, integrating the superiorities of rough sets with fuzzy sets is necessary for letting RST effectively handle fuzzy information systems. Consequently, rough fuzzy sets (RFSs) along with fuzzy rough sets (FRSs) were proposed [42,43]. In specific, RFSs utilize an indiscernibility relation to describe a fuzzy notion, whereas FRSs are approximate estimations of a fuzzy or crisp set within the context of a classical fuzzy approximation space.

More recently, for the sake of broadening the application range of RFSs and FRSs, especially for the neutrosophic background, some researchers integrated rough sets with NSs and established many hybrid mathematical models. Broumi et al. [44] used an indiscernibility relation to approximate a neutrosophic concept and founded the concept of RNSs. Then taking advantages of FRSs, Sweety and Arockiarani [45] explored the approximation of a NS in terms of a neutrosophic approximation space, and constructed the notion of NRSs. The two works fill the gap between NSs and rough sets, and neutrosophic fusion of RST has turned into an interesting and promising research direction for NSs and logic. According to the existing works related to neutrosophic fusion of RST, we roughly separate them into the following five classifications (see Fig. 1):

(1) Neutrosophic fusion of RST from the aspect of basic models: the construction of approximations in terms of a corresponding target concept is an essential prerequisite in the procedure of RST-based data processing. It is noted that both NSs and RST include various extended forms to overcome the limitations of classical models. For one thing, for the sake of efficiently applying NSs to more real-world applications, several generalized NS models, such as single-valued NSs (SVNSs) [46] and interval NSs (INSs) [47], were proposed. For another, in original rough sets, since indiscernibility relations are too strict to be fully exploited, numerous extended rough set notions,

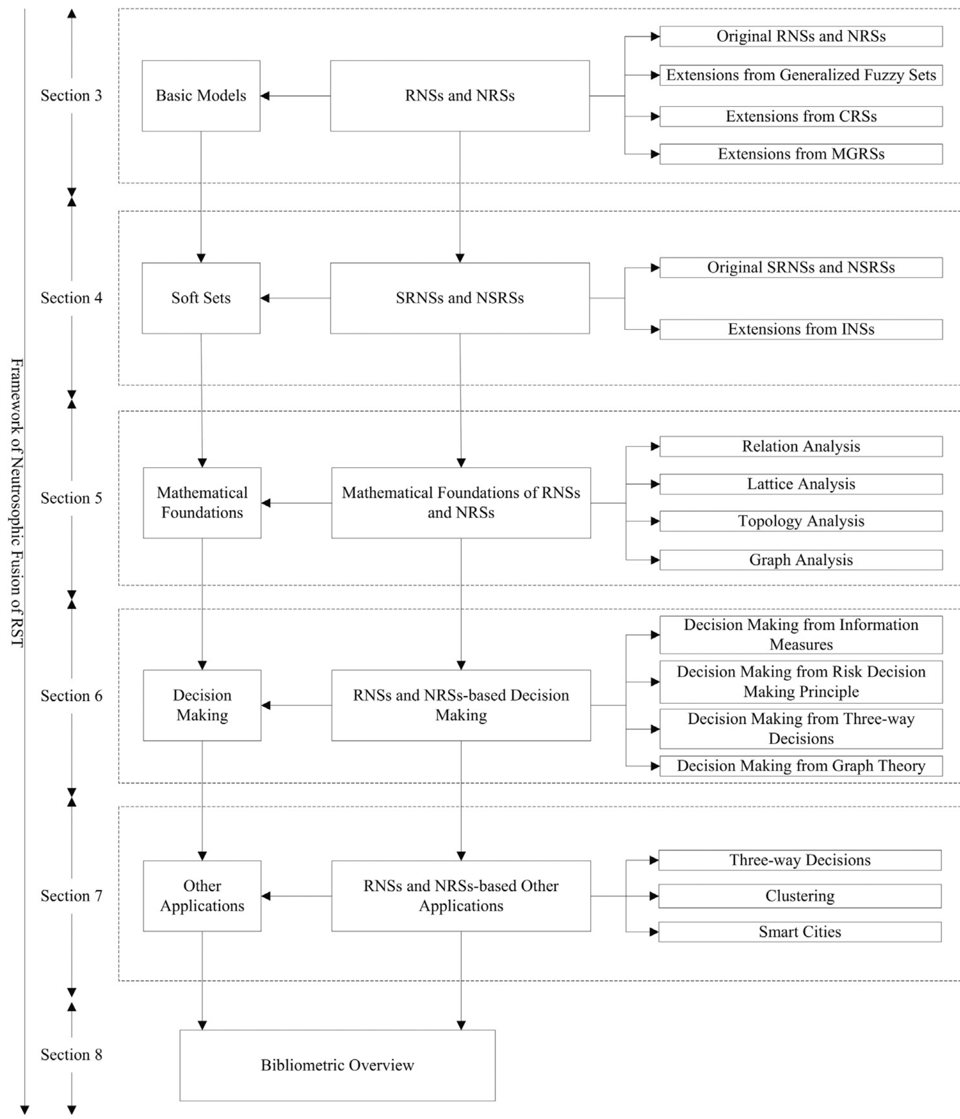


**Fig. 1.** Flow chart of the survey on neutrosophic fusion of RST.

such as dominance-based rough sets (DRSs) [48], covering-based rough sets (CRSs) [49], probabilistic rough sets (PRSs) [50] and multigranulation rough sets (MGRSs) [51–54], were developed to loosen the requirements of equivalence relations. Consequently, it is imperative to investigate basic models for the integration of RST with NSs and logic.

(2) Neutrosophic fusion of RST from the aspect of soft sets: soft sets, due to Molodotsov [55], were proposed to handle numerous uncertainties. With the introduction of soft sets, the limitations of parameterization inadequacy for rough sets, fuzzy sets and probability theory can be overcome. Thus, it is meaningful to fuse soft sets, NSs and RST for putting forward a hybrid soft computing model.

(3) Neutrosophic fusion of RST from the aspect of mathematical foundations: it is noted that mathematical foundations, such as relations, lattices, topologies and graphs, have tight connections with RST, and the study of mathematical foundations has become a significant topic within the rough set society [56]. Hence, the exploration of mathematical foundations is beneficial to the RST-based uncertainty analysis and data mining.

(4) Neutrosophic fusion of RST from the aspect of decision making: compared with other classical decision support systems, RST has proved its powerful capabilities by right of rule acquisitions and attribute reductions in the following two situations [57–59]: (a) selecting the relative significant attributes for making reasonable decisions; (b) explaining a decision by virtue of lower and upper approximations that correspond to deterministic and probabilistic decision rules. Therefore, RST-based decision making exerts a far-reaching influence on neutrosophic fusion of RST.

(5) Neutrosophic fusion of RST from the aspect of other applications: during the past decades, RST has also demonstrated its unique capabilities in numerous fields in artificial intelligence, such as clustering, classification, signal processing, image processing, and so forth [60]. Hence, the study of real-world applications has become an important research branch in neutrosophic fusion of RST.

For the remarkable performances in analyzing neutrosophic information, neutrosophic fusion of RST has received special favors of scholars from extensive areas. However, existing literature reviews have not kept in line with the fast growth of knowledge in this field. Thus, we aim to provide a systematic review of recent neutrosophic fusion of RST and illustrate future studies for better supporting the development of NSs and logic. The framework of the work is arranged below. Section 2 gathers several preliminary details of NSs and rough sets. In the next section, neutrosophic fusion of RST from the aspect of basic models is revisited. In Section 4, neutrosophic fusion of RST from the aspect of soft sets is elaborated on. Section 5 revisits neutrosophic fusion of RST from the aspect of mathematical foundations. In Section 6, we present neutrosophic fusion of RST from the aspect of decision making. Then, the following section revisits neutrosophic fusion of RST from the aspect of other applications. Section 8 discusses a bibliometric overview of current works in terms of neutrosophic fusion of RST. Finally, several concluding remarks and future works are summed up in the last section.

## 2. Preliminary details

### 2.1. NSs and logic

Smarandache [1,2] initiated the concept of neutrosophy inspired from the perspective of philosophical paradigms, which divides each notion from a certain degree of truth, indeterminacy and falsity simultaneously. Afterwards, neutrosophy has laid solid foundations for a series of new mathematical theories including NSs, neutrosophic logic, neutrosophic statistics, neutrosophic probability, etc. In specific, as an extended form of classical fuzzy logic that aims to realize some basic concepts of neutrosophy, neutrosophic logic is defined as a logic in which each proposition is assessed to own the degree of truth in a subset $\mu$, the degree of indeterminacy in a subset $\nu$, and the degree of falsity in a subset $\omega$. Moreover, neutrosophic logic provides the framework of neutrosophic connectives such as negation, conjunction and disjunction, which plays a significant role in neutrosophic fusion of RST. The mathematical formulation of neutrosophic connectives will be presented in the following part of the paper.

In light of the idea of neutrosophic logic, Smarandache [1,2] founded NSs by means of non-standard analysis. In non-standard analysis, an infinitesimal number is defined as an infinitely small number. Suppose $\epsilon$ is a such infinitesimal number, and the hyperreal number set is a generalization of the real number set, which contains classes of infinite numbers and infinitesimal numbers. Then, $1^+ = 1+\epsilon$ and $0^- = 0-\epsilon$ are two non-standard finite numbers, where "1" and "0" are standard parts, and "$\epsilon$" is a non-standard part. Obviously, $1^+$ is greater than 1 and $0^-$ is smaller than 0. Based on the above statements, we name $]0^-, 1^+[$ a non-standard unit interval.

**Definition 1.** [1,2]

Suppose $U$ is an arbitrary universe of discourse. For a general component $x$ in $U$, a NS $A$ in $U$ is in mathematical terms of:

$$A = \{\langle x, \mu_A(x), \nu_A(x), \omega_A(x)\rangle | x \in U\}, \tag{1}$$

where $\mu_A(x)$, $\nu_A(x)$ and $\omega_A(x)$ are real standard or non-standard subsets of the non-standard unit interval $]0^-, 1^+[$, i.e., $\mu$, $\nu$, $\omega : U \to ]0^-, 1^+[$ represents the truth membership, the indeterminacy membership and the falsity membership respectively for each component $x \in U$ to $A$. Hence, it is noted that a NS is constructed from philosophical standpoints which make it hard to address real-world situations. In addition, it is worth noticing that there does not exist a specific limitation for $\mu_A(x) + \nu_A(x) + \omega_A(x)$. Therefore, we have $0^- \leq \mu_A(x) + \nu_A(x) + \omega_A(x) \leq 3^+$.

In light of the structure of NSs, it is difficult to utilize NSs directly in practical applications due to the range of $\mu_A(x)$, $\nu_A(x)$ and $\omega_A(x)$ is restricted in $]0^-, 1^+[$. Instead of $]0^-, 1^+[$, it is necessary to update $]0^-, 1^+[$ to the interval $[0, 1]$. Hence, the concept of SVNSs [46] was put forward subsequently with the function $\mu, \nu, \omega : U \to [0, 1]$ for each component $x \in U$ to a SVNS $H$. Thus, we further have $0 \leq \mu_H(x) + \nu_H(x) + \omega_H(x) \leq 3$. Moreover, since interval numbers outperform crisp numbers when dealing with incomplete information systems, it is natural to propose the notion of INSs [47]. For an INS $E$, the function $\mu, \nu, \omega : U \to \mathrm{int}[0, 1]$ holds for each component $x \in U$ to $E$, where $\mathrm{int}[0, 1]$ represents the set of all closed subintervals of $[0, 1]$. It is also noted that $0 \leq \sup\mu_E(x) + \sup\nu_E(x) + \sup\omega_E(x) \leq 3$ is true, where $\mu_E(x) = [\inf\mu_E(x), \sup\mu_E(x)]$, $\nu_E(x) = [\inf\nu_E(x), \sup\nu_E(x)]$ and $\omega_E(x) = [\inf\omega_E(x), \sup\omega_E(x)]$.

With regard to specific applications of NSs, it is significant to present the mathematical formulation of neutrosophic connectives. In what follows, we present some key neutrosophic connectives defined on NSs.

**Definition 2.** [1,2]

Suppose $U$ is an arbitrary universe of discourse, $A$ and $A'$ are two NSs, then the followings are true:

(1) complement: the complement of $A$ is represented as $A^c$, where

$$A^c = \{\langle x, \omega_A(x), 1-\nu_A(x), \mu_A(x)\rangle | x \in U\}; \tag{2}$$

(2) intersection: the intersection of $A$ and $A'$ is represented as $A \cap A'$, where

$$\begin{aligned} A \cap A' = \{\langle x, &\min\left(\mu_A(x), \mu'_A(x)\right), \max\left(\nu_A(x), \nu'_A(x)\right), \\ &\max\left(\omega_A(x), \omega'_A(x)\right)\rangle | x \in U\}; \end{aligned} \tag{3}$$

(3) union: the union of $A$ and $A'$ is represented as $A \cup A'$, where

$$\begin{aligned} A \cup A' = \{\langle x, &\max\left(\mu_A(x), \mu'_A(x)\right), \min\left(\nu_A(x), \nu'_A(x)\right), \\ &\min\left(\omega_A(x), \omega'_A(x)\right)\rangle | x \in U\}; \end{aligned} \tag{4}$$

(4) inclusion: for every $x$ in $U$, $A \subseteq A'$ holds such that $\mu_A(x) \leq \mu'_A(x)$, $\nu_A(x) \geq \nu'_A(x)$ and $\omega_A(x) \geq \omega'_A(x)$;
(5) equality: for every $x$ in $U$, $A = A'$ holds such that $\mu_A(x) = \mu'_A(x)$, $\nu_A(x) = \nu'_A(x)$ and $\omega_A(x) = \omega'_A(x)$.

### 2.2. Rough sets

Prior to the introduction of original rough sets, since information systems provide a flexible framework for describing several objects with respect to their corresponding attributes, information systems lay a solid foundation for constructing rough sets [18,19]. In what follows, the concept of information systems will be presented briefly.

In a typical information system, for the representation of objects, we let $U = \{x_1, x_2, \ldots, x_n\}$ be an arbitrary universe of discourse; for the representation of attributes, we also suppose $\mathcal{A} = \{a_1, a_2, \ldots, a_m\}$ is an arbitrary set of attributes. Next, the pair $(U, \mathcal{A})$ is named an information system. In addition, it is noticed that $a : U \to V_a$ for all $a \in \mathcal{A}$, and we further represent it as $a(x) \in V_a$, where $V_a = \left\{a(x) | x \in U\right\}$ represents the domain of $a$.

Concerning a given information system, every attribute subset $\mathcal{B} \subseteq \mathcal{A}$ induces an indiscernibility relation $R_{\mathcal{B}} = \left\{a(x) = a(y) | \forall a \in \mathcal{B}, (x, y) \in U \times U\right\}$, which splits $U$ into some equivalence classes denoted by $U/R_{\mathcal{B}} = \left\{[x]_{\mathcal{B}} | x \in U\right\}$, where $[x]_{\mathcal{B}}$ represents the equivalence class induced by $x$ in terms of $\mathcal{B}$, i.e., $[x]_{\mathcal{B}} = \left\{(x, y) \in R_{\mathcal{B}} | y \in U\right\}$. In light of the above statements, for all $X \subseteq U$ and $\mathcal{B} \subseteq \mathcal{A}$, the lower approximation of $X$ is constructed as $\underline{R_{\mathcal{B}}}(X) = \left\{[x]_{\mathcal{B}} \subseteq X | x \in U\right\}$, whereas the upper approximation of $X$ is constructed as $\bar{R_{\mathcal{B}}}(X) = \left\{[x]_{\mathcal{B}} \cap X \neq \emptyset | x \in U\right\}$. Then, we name the pair $\left(\underline{R_{\mathcal{B}}}(X), \bar{R_{\mathcal{B}}}(X)\right)$ a rough set of $X$, and it is not hard to find $\underline{R_{\mathcal{B}}}(X)$ and $\bar{R_{\mathcal{B}}}(X)$ express the object set that certainly and possibly belongs to $X$ in terms of $\mathcal{B}$ respectively.

Based on $\underline{R_{\mathcal{B}}}(X)$ and $\bar{R_{\mathcal{B}}}(X)$, Pawlak et al. [18,19] further put forward the notion of accuracy and roughness to manage the uncertainty of RST. In specific, the accuracy is given by the ratio of the cardinalities of $\underline{R_{\mathcal{B}}}(X)$ and $\bar{R_{\mathcal{B}}}(X)$, which is represented by

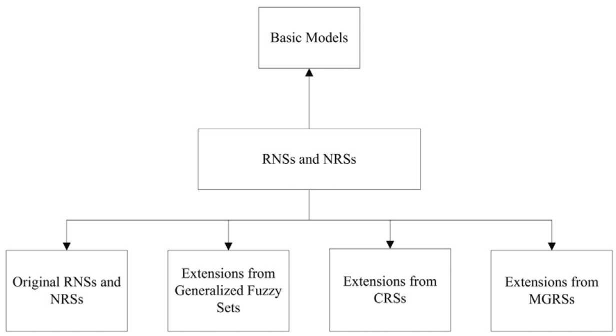


**Fig. 2.** Flow chart of the survey on neutrosophic fusion of RST from the aspect of basic models.

$\alpha_{\mathcal{B}}(X) = (|\underline{R_{\mathcal{B}}}(X)|/|\bar{R}_{\mathcal{B}}(X)|)$, whereas the roughness is denoted by $\beta_{\mathcal{B}}(X) = 1 - \alpha_{\mathcal{B}}(X)$.

## 3. Neutrosophic fusion of RST from the aspect of basic models

Considering there exists lots of extended forms of NSs and rough sets due to some limitations owned by their original forms, several models related to RNSs and NRSs have been designed during the last few years. In what follows, we plan to review neutrosophic fusion of RST from the aspect of basic models, these models are revisited in several subsections below. Moreover, we present the flowchart of this section in Fig. 2.

### *3.1. RNSs and NRSs*

In neutrosophic fusion of RST, RNSs [44] and NRSs [45] are two fundamental models by integrating NSs with rough sets. In what follows, we list specific definitions of RNSs and NRSs.

**Definition 3.** [44]

Suppose $U$ is an arbitrary universe of discourse, an equivalence relation over $U$ is denoted by $R$. For a NS $A$, the two approximations of $A$ are in mathematical terms of:

$$\underline{Q}(A) = \{\langle x, \mu_{\underline{Q}(A)}(x), \nu_{\underline{Q}(A)}(x), \omega_{\underline{Q}(A)}(x)\rangle | y \in [x]_R, x \in U\}; \tag{5}$$

$$\bar{Q}(A) = \{\langle x, \mu_{\bar{Q}(A)}(x), \nu_{\bar{Q}(A)}(x), \omega_{\bar{Q}(A)}(x)\rangle | y \in [x]_R, x \in U\}, \tag{6}$$

where $\mu_{\underline{Q}(A)}(x) = \wedge_{y\in[x]_R}\mu_A(y)$, $\nu_{\underline{Q}(A)}(x) = \vee_{y\in[x]_R}\nu_A(y)$, $\omega_{\underline{Q}(A)}(x) = \vee_{y\in[x]_R}\omega_A(y)$, $\mu_{\bar{Q}(A)}(x) = \vee_{y\in[x]_R}\mu_A(y)$, $\nu_{\bar{Q}(A)}(x) = \wedge_{y\in[x]_R}\nu_A(y)$, $\omega_{\bar{Q}(A)}(x) = \wedge_{y\in[x]_R}\omega_A(y)$. It is not difficult to find $\underline{Q}(A)$ and $\bar{Q}(A)$ are both NSs on $U$. Then, the pair $\left(\underline{Q}(A), \bar{Q}(A)\right)$ is named a RNS of $A$.

Different from RNSs that use an indiscernibility relation to approximate a neutrosophic concept, NRSs aim to design an approximation of a NS in terms of a neutrosophic approximation space. Next, we provide the mathematical formulation of NRSs below.

**Definition 4.** [45]

Suppose $U$ is an arbitrary universe of discourse, $R$ is a neutrosophic relation on $U$. Then for an arbitrary neutrosophic relation, the pair $(U, R)$ is named an approximation space related to NSs. For a NS $A$, the two approximations of $A$ in terms of $(U, R)$ are given as the following form:

$$\underline{R}(A) = \{\langle x, \mu_{\underline{R}(A)}(x), \nu_{\underline{R}(A)}(x), \omega_{\underline{R}(A)}(x)\rangle | x \in U\}; \tag{7}$$

$$\bar{R}(A) = \{\langle x, \mu_{\bar{R}(A)}(x), \nu_{\bar{R}(A)}(x), \omega_{\bar{R}(A)}(x)\rangle | x \in U\}, \tag{8}$$

**Table 1**
Distribution papers in the field of RNSs and NRSs.

| Definition | Reference | Representation | Study category |
|---|---|---|---|
| Definition 3 | Broumi et al. [44] | NSs | RNSs |
| Definition 4 | Sweety and Arockiarani [45] | NSs | NRSs |

where $\mu_{\underline{R}(A)}(x) = \wedge_{y\in U}[\omega_R(x,y) \vee \mu_A(y)]$, $\nu_{\underline{R}(A)}(x) = \vee_{y\in U}[(1-\nu_R(x,y)) \wedge \nu_A(y)]$, $\omega_{\underline{R}(A)}(x) = \vee_{y\in U}[\mu_R(x,y) \wedge \omega_A(y)]$, $\mu_{\bar{R}(A)}(x) = \vee_{y\in U}[\mu_R(x,y) \wedge \mu_A(y)]$, $\nu_{\bar{R}(A)}(x) = \wedge_{y\in U}[\nu_R(x,y) \vee \nu_A(y)]$, $\omega_{\bar{R}(A)}(x) = \wedge_{y\in U}[\omega_R(x,y) \vee \omega_A(y)]$. Then, we name the pair $\left(\underline{R}(A), \bar{R}(A)\right)$ a NRS of $A$ in terms of $(U, R)$.

In addition, the related works in terms of RNSs and NRSs are summed up in Table 1.

According to the concept of RNSs and NRSs, it is easy to know the most significant contributions of RNSs and NRSs lie in opening a brand-new research direction for both NSs and RST. Due to the limitations of NSs, RNSs and NRSs can be further studied from several following theoretical aspects in the future:

(1) Study relationships between RNSs and other types of RFSs, and relationships between NRSs and other types of FRSs.
(2) Explore axiomatic approaches to RNSs and NRSs from the aspect of both basic definitions and corresponding properties.
(3) Develop more data-driven hybrid models by integrating NSs with other generalized fuzzy sets.

### *3.2. Extensions of RNSs and NRSs from the aspect of generalized fuzzy sets*

Although classical fuzzy sets are widely adopted in plenty of practical situations for coping with fuzziness, with the increasing complexity of information contents and frameworks, classical fuzzy sets are confronted with some limitations when describing interval-valued, intuitionistic, bipolar, neutrosophic, hesitant and other complicated information. Thus, many generalized fuzzy sets have been designed one after another over the past half century [61]. In order to expand the application scope of RNSs and NRSs, it is necessary to study some reasonable extensions of RNSs and NRSs from the aspect of generalized fuzzy sets. In what follows, we list specific definitions of generalized RNSs and NRSs.

Among numerous generalized fuzzy sets, Zhang [62] constructed the concept of bipolar fuzzy sets (BFSs) in which membership degrees fall within $[-1, 1]$. In specific, the membership degree $(0, 1]$ denotes the degree of one component satisfies one characteristic, the membership degree 0 denotes one component is independent of the related characteristic, and the membership degree $[-1, 0)$ denotes the degree of one component satisfies the opposite characteristic. In 2016, Pramanik and Mondal [63] combined BFSs with RNSs for developing rough bipolar neutrosophic sets (RBNSs), and then explored some of their properties. Next, the definition of RBNSs will be discussed.

**Definition 5.** [63]

Suppose $U$ is an arbitrary universe of discourse, an equivalence relation over $U$ is denoted by $R$. $\mu^+(x)$, $\nu^+(x)$ and $\omega^+(x)$ record three memberships of a component $x \in U$ in terms of a bipolar neutrosophic set (BNS) $I$, whereas $\mu^-(x)$, $\nu^-(x)$ and $\omega^-(x)$ record the above three memberships respectively of a component $x \in U$ to the opposite characteristic related to a BNS $I$. Then for a BNS $I$, the two approximations of $I$ are in mathematical terms of:

$$\begin{aligned}\underline{Q}(I) = \{\langle x, &\mu^+_{\underline{Q}(I)}(x), \nu^+_{\underline{Q}(I)}(x), \omega^+_{\underline{Q}(I)}(x), \\ &\mu^-_{\underline{Q}(I)}(x), \nu^-_{\underline{Q}(I)}(x), \omega^-_{\underline{Q}(I)}(x)\rangle | y \in [x]_R, x \in U\};\end{aligned} \tag{9}$$

$$\bar{Q}(I) = \{\langle x, \mu^{+}_{\bar{Q}(I)}(x), \nu^{+}_{\bar{Q}(I)}(x), \omega^{+}_{\bar{Q}(I)}(x), \mu^{-}_{\bar{Q}(I)}(x), \nu^{-}_{\bar{Q}(I)}(x), \omega^{-}_{\bar{Q}(I)}(x)\rangle | y \in [x]_R, x \in U\}, \tag{10}$$

where $\mu^{+}_{\underline{Q}(I)}(x) = \wedge_{y \in [x]_R}\mu^{+}_{I}(y)$, $\nu^{+}_{\underline{Q}(I)}(x) = \vee_{y \in [x]_R}\nu^{+}_{I}(y)$, $\omega^{+}_{\underline{Q}(I)}(x) = \vee_{y \in [x]_R}\omega^{+}_{I}(y)$, $\mu^{-}_{\underline{Q}(I)}(x) = \wedge_{y \in [x]_R}\mu^{-}_{I}(y)$, $\nu^{-}_{\underline{Q}(I)}(x) = \vee_{y \in [x]_R}\nu^{-}_{I}(y)$, $\omega^{-}_{\underline{Q}(I)}(x) = \vee_{y \in [x]_R}\omega^{-}_{I}(y)$, $\mu^{+}_{\bar{Q}(I)}(x) = \vee_{y \in [x]_R}\mu^{+}_{I}(y)$, $\nu^{+}_{\bar{Q}(I)}(x) = \wedge_{y \in [x]_R}\nu^{+}_{I}(y)$, $\omega^{+}_{\bar{Q}(I)}(x) = \wedge_{y \in [x]_R}\omega^{+}_{I}(y)$, $\mu^{-}_{\bar{Q}(I)}(x) = \vee_{y \in [x]_R}\mu^{-}_{I}(y)$, $\nu^{-}_{\bar{Q}(I)}(x) = \wedge_{y \in [x]_R}\nu^{-}_{I}(y)$, $\omega^{-}_{\bar{Q}(I)}(x) = \wedge_{y \in [x]_R}\omega^{-}_{I}(y)$. It is not difficult to find $\underline{Q}(I)$ and $\bar{Q}(I)$ are both BNSs on $U$. Then, the pair $\left(\underline{Q}(I), \bar{Q}(I)\right)$ is named a RBNS of $I$.

As a significant extended form of NSs, SVNSs have been received an increasingly number of attentions from scholars and practitioners. Thus, it is natural to construct a brand-new hybrid model by fusing SVNSs with rough sets. In 2017, Yang's research team [64] investigated the formation of single-valued neutrosophic (SVN) rough sets (SVNRSs), along with their general propositions and axiomatic characterizations of corresponding lower and upper approximations. In the following, the mathematical formulation of SVNRSs will be elaborated on.

**Definition 6.** [64]

Suppose $U$ is an arbitrary universe of discourse, a SVN relation over $U$ is denoted by $R$, where $\mu_R, \nu_R, \omega_R : U \times U \to [0, 1]$. Then for an arbitrary SVN relation, we name the pair $(U, R)$ a SVN approximation space. For a SVNS $H$, the two approximations of $H$ related to SVN approximation space $(U, R)$ are in mathematical terms of:

$$\underline{R}(H) = \{\langle x, \mu_{\underline{R}(H)}(x), \nu_{\underline{R}(H)}(x), \omega_{\underline{R}(H)}(x)\rangle | x \in U\}; \tag{11}$$

$$\bar{R}(H) = \{\langle x, \mu_{\bar{R}(H)}(x), \nu_{\bar{R}(H)}(x), \omega_{\bar{R}(H)}(x)\rangle | x \in U\}, \tag{12}$$

where $\mu_{\underline{R}(H)}(x) = \wedge_{y \in U}[\omega_R(x, y) \vee \mu_H(y)]$, $\nu_{\underline{R}(H)}(x) = \vee_{y \in U}[(1 - \nu_R(x, y)) \wedge \nu_H(y)]$, $\omega_{\underline{R}(H)}(x) = \vee_{y \in U}[\mu_R(x, y) \wedge \omega_H(y)]$, $\mu_{\bar{R}(H)}(x) = \vee_{y \in U}[\mu_R(x, y) \wedge \mu_H(y)]$, $\nu_{\bar{R}(H)}(x) = \wedge_{y \in U}[\nu_R(x, y) \vee \nu_H(y)]$, $\omega_{\bar{R}(H)}(x) = \wedge_{y \in U}[\omega_R(x, y) \vee \omega_H(y)]$. Then, the pair $\left(\underline{R}(H), \bar{R}(H)\right)$ is named a SVNRS of $H$ in terms of SVN approximation space $(U, R)$.

According to the concept of SVNSs, in order to extend SVNSs to multiple dimensions, Ye [65,66] explored the mathematical formulation of SVN multisets (SVNMSs), and it is beneficial to the expression of dynamic decision support systems. For instance, a 4-dimension SVNMS can describe medical diagnosis data of a considered patient from different time intervals, e.g., morning, noon, afternoon and night. In accordance with Literature [67], we use both terms SVNMSs and SVN refined sets in this work, and it is noted that two terms SVNMSs and SVN refined sets can be used interchangeably.

**Definition 7.** [65–67]

Suppose $U$ is an arbitrary universe of discourse. For a general component $x$ in $U$, a SVN refined set $B$ in $U$ is in mathematical terms of:

$$B = \{\langle x, \mu_B(x), \nu_B(x), \omega_B(x)\rangle | x \in U\}, \tag{13}$$

where $\mu_B(x)$, $\nu_B(x)$ and $\omega_B(x)$ record three memberships mentioned above. Among them, we have $\mu_B(x) = \left\{\mu_{1B}(x), \mu_{2B}(x), \ldots, \mu_{pB}(x)\right\}$, $\nu_B(x) = \left\{\nu_{1B}(x), \nu_{2B}(x), \ldots, \nu_{pB}(x)\right\}$ and $\omega_B(x) = \left\{\omega_{1B}(x), \omega_{2B}(x), \ldots, \omega_{pB}(x)\right\}$, $p$ is a positive integer and represents the dimension of $B$. Moreover, for $i = 1, 2, \ldots, p$, the functions satisfy $\mu, \nu, \omega : U \to [0, 1]$ and $0 \le \mu_{iB}(x) + \nu_{iB}(x) + \omega_{iB}(x) \le 3$.

In light of various merits owned by SVN refined sets, Bao and Yang [68] explored SVN refined rough sets (SVNRRSs) and studied their related propositions and axiomatic characterizations.

**Definition 8.** [68]

Suppose $U$ is an arbitrary universe of discourse. For a general element $x$ in $U$, a SVN refined relation is denoted as the following form:

$$\mathbb{R} = \{\langle (x, y), \mu_{\mathbb{R}}(x, y), \nu_{\mathbb{R}}(x, y), \omega_{\mathbb{R}}(x, y)\rangle | (x, y) \in U \times U\}, \tag{14}$$

where $\mu_{\mathbb{R}}(x, y) = \left\{\mu_{1\mathbb{R}}(x, y), \mu_{2\mathbb{R}}(x, y), \ldots, \mu_{p\mathbb{R}}(x, y)\right\}$, $\nu_{\mathbb{R}}(x, y) = \left\{\nu_{1\mathbb{R}}(x, y), \nu_{2\mathbb{R}}(x, y), \ldots, \nu_{p\mathbb{R}}(x, y)\right\}$ and $\omega_{\mathbb{R}}(x, y) = \left\{\omega_{1\mathbb{R}}(x, y), \omega_{2\mathbb{R}}(x, y), \ldots, \omega_{p\mathbb{R}}(x, y)\right\}$, $p$ is a positive integer and represents the dimension of $\mathbb{R}$. Then for an arbitrary SVN refined relation, the pair $(U, \mathbb{R})$ is named a SVN refined approximation space. For a SVN refined set $B$, the two approximations of $B$ in terms of SVN refined approximation space $(U, \mathbb{R})$ are in mathematical terms of:

$$\underline{\mathbb{R}}(B) = \{\langle x, \mu_{\underline{\mathbb{R}}(B)}(x), \nu_{\underline{\mathbb{R}}(B)}(x), \omega_{\underline{\mathbb{R}}(B)}(x)\rangle | x \in U\}; \tag{15}$$

$$\bar{\mathbb{R}}(B) = \{\langle x, \mu_{\bar{\mathbb{R}}(B)}(x), \nu_{\bar{\mathbb{R}}(B)}(x), \omega_{\bar{\mathbb{R}}(B)}(x)\rangle | x \in U\}, \tag{16}$$

where $\mu_{\underline{\mathbb{R}}(B)}(x) = \wedge_{y \in U}[\omega_{\mathbb{R}}(x, y) \vee \mu_B(y)]$, $\nu_{\underline{\mathbb{R}}(B)}(x) = \vee_{y \in U}[(1 - \nu_{\mathbb{R}}(x, y)) \wedge \nu_B(y)]$, $\omega_{\underline{\mathbb{R}}(B)}(x) = \vee_{y \in U}[\mu_{\mathbb{R}}(x, y) \wedge \omega_B(y)]$, $\mu_{\bar{\mathbb{R}}(B)}(x) = \vee_{y \in U}[\mu_{\mathbb{R}}(x, y) \wedge \mu_B(y)]$, $\nu_{\bar{\mathbb{R}}(B)}(x) = \wedge_{y \in U}[\nu_{\mathbb{R}}(x, y) \vee \nu_B(y)]$, $\omega_{\bar{\mathbb{R}}(B)}(x) = \wedge_{y \in U}[\omega_{\mathbb{R}}(x, y) \vee \omega_B(y)]$. Then, the pair $\left(\underline{\mathbb{R}}(B), \bar{\mathbb{R}}(B)\right)$ is named a SVNRRS of $B$ in terms of SVN refined approximation space $(U, \mathbb{R})$.

It is noticed that the model of SVNRRSs is an extended form of NRSs, some researchers have also laid an emphasis on the corresponding model which originates from RNSs. To be specific, Alias et al. [69,70] explored the model of rough neutrosophic multisets (RNMSs) and several of its properties.

**Definition 9.** [69,70]

Suppose $U$ is an arbitrary universe of discourse, an equivalence relation over $U$ is denoted by $R$. For a SVNMS $B$, the two approximations of $B$ are in mathematical terms of:

$$\underline{Q}(B) = \{\langle x, \mu^{i}_{\underline{Q}(B)}(x), \nu^{i}_{\underline{Q}(B)}(x), \omega^{i}_{\underline{Q}(B)}(x)\rangle | y \in [x]_R, x \in U\}; \tag{17}$$

$$\bar{Q}(B) = \{\langle x, \mu^{i}_{\underline{Q}(B)}(x), \nu^{i}_{\underline{Q}(B)}(x), \omega^{i}_{\underline{Q}(B)}(x)\rangle | y \in [x]_R, x \in U\}, \tag{18}$$

where $\mu^{i}_{\underline{Q}(B)}(x) = \wedge_{y \in [x]_R}\mu^{i}_{B}(y)$, $\nu^{i}_{\underline{Q}(B)}(x) = \vee_{y \in [x]_R}\nu^{i}_{B}(y)$, $\omega^{i}_{\underline{Q}(B)}(x) = \vee_{y \in [x]_R}\omega^{i}_{B}(y)$, $\mu^{i}_{\bar{Q}(B)}(x) = \vee_{y \in [x]_R}\mu^{i}_{B}(y)$, $\nu^{i}_{\bar{Q}(B)}(x) = \wedge_{y \in [x]_R}\nu^{i}_{B}(y)$, $\omega^{i}_{\bar{Q}(B)}(x) = \wedge_{y \in [x]_R}\omega^{i}_{B}(y)$. It is not difficult to find $\underline{Q}(B)$ and $\bar{Q}(B)$ are both SVMSs over $U$. Then, we name the pair $\left(\underline{Q}(B), \bar{Q}(B)\right)$ a RNMS of $B$.

The notion of PRSs [71] acts as a significant generalization in rough set society. By introducing the probability theory to calculate lower and upper approximations, PRSs enable rough sets to own the ability of error tolerance when coping with noisy information, which are more robust compared with other counterparts [72]. Noticing the superiorities of coping with noisy information, Zhang et al. [73] combined PRSs with SVNMSs, and initiated two different hybrid models respectively, i.e., probabilistic rough SVN multisets (PRSVNMSs) and SVN rough multisets (SVNRMSs) under two-universe frameworks.

**Definition 10.** [73]

Suppose $U$, $V$ are arbitrary universes of discourse, a binary relation is denoted by $R \subseteq U \times V$, and a probabilistic measure is represented by $P$. Then, we name a probabilistic approximation space $(U, V, R, P)$. For a SVNMS $B$, $x \in U$, $y \in V$, the conditional probability is represented by $P(B|R(x)) = (\sum_{y \in R(x)} B(y)/|R(x)|)$. Then for any

$0 \le \beta \le \alpha \le 1$, the two approximations of $B$ related to $(U, V, R, P)$ are in mathematical terms of:

$$\underline{SVNM}_P^{\alpha}(B) = \{P(B|R(x)) \ge \alpha | x \in U, y \in V\}$$
$$= \left\{ \frac{\sum_{y \in R(x)} B(y)}{|R(x)|} \ge \alpha | x \in U, y \in V \right\}; \tag{19}$$

$$S\bar{V}\bar{N}\bar{M}_P^{\beta}(B) = \{P(B|R(x)) > \beta | x \in U, y \in V\}$$
$$= \left\{ \frac{\sum_{y \in R(x)} B(y)}{|R(x)|} > \beta | x \in U, y \in V \right\}, \tag{20}$$

then, we name the pair $(\underline{SVNM}_P^{\alpha}(B), S\bar{V}\bar{N}\bar{M}_P^{\beta}(B))$ a PRSVNMS over two universes of $B$ in terms of $(U, V, R, P)$.

Different from the above model which can be seen as a generalization of RNSs, it is also necessary to develop a similar model by virtue of RNSs. Hence, the model of SVNRMSs over two universes is further studied in detail.

**Definition 11.** [73]

Suppose $U$, $V$ are arbitrary finite universes of discourse, a SVN refined relation is denoted by $R$, and a probabilistic measure is denoted by $P$. Then, we name a probabilistic approximation space $(U, V, R, P)$. For a SVNMS $B$, $x \in U$, $y \in V$, the conditional probability is represented by $P(B|R(x, y)) = (\sum_{y \in V} B(y)R(x, y) / \sum_{y \in V} R(x, y))$. Then for any $0 \le \beta \le \alpha \le 1$, the two approximations of $B$ related to $(U, V, R, P)$ are in mathematical terms of:

$$\underline{SVNMR}_P^{\alpha}(B) = \{P(B|R(x, y)) \ge \alpha | x \in U, y \in V\}$$
$$= \left\{ \frac{\sum_{y \in V} B(y)R(x, y)}{\sum_{y \in V} R(x, y)} \ge \alpha | x \in U, y \in V \right\}; \tag{21}$$

$$S\bar{V}\bar{N}\bar{M}\bar{R}_P^{\beta}(B) = \{P(B|R(x, y)) > \beta | x \in U, y \in V\}$$
$$= \left\{ \frac{\sum_{y \in V} B(y)R(x, y)}{\sum_{y \in V} R(x, y)} > \beta | x \in U, y \in V \right\}, \tag{22}$$

then, we name the pair $(\underline{SVNMR}_P^{\alpha}(B), S\bar{V}\bar{N}\bar{M}\bar{R}_P^{\beta}(B))$ a SVNPRMS over two universes of $B$ in terms of $(U, V, R, P)$.

Recently, in order to effectively discover unknown knowledge from information systems with regard to INSs, Yang et al. [74] investigated interval neutrosophic rough sets (INRSs) by means of interval neutrosophic relations.

**Definition 12.** [74]

Suppose $U$ is an arbitrary universe of discourse, a relation over $U$ in terms of INSs is represented by $R$, where $\mu_R, \nu_R, \omega_R : U \times U \to \text{int}[0, 1]$. Then for an arbitrary interval neutrosophic relation, we name the pair $(U, R)$ an interval neutrosophic approximation space. For an INS $E$, the two approximations of $E$ in terms of interval neutrosophic approximation space $(U, R)$ are in mathematical terms of:

$$\underline{R}(E) = \{\langle x, \mu_{\underline{R}(E)}(x), \nu_{\underline{R}(E)}(x), \omega_{\underline{R}(E)}(x)\rangle | x \in U\}; \tag{23}$$

$$\bar{R}(E) = \{\langle x, \mu_{\bar{R}(E)}(x), \nu_{\bar{R}(E)}(x), \omega_{\bar{R}(E)}(x)\rangle | x \in U\}, \tag{24}$$

where $\mu_{\underline{R}(E)}(x) = \wedge_{y \in U}[\omega_R(x, y) \vee \mu_E(y)]$, $\nu_{\underline{R}(E)}(x) = \vee_{y \in U}[([1, 1] - \nu_R(x, y)) \wedge \nu_E(y)]$, $\omega_{\underline{R}(E)}(x) = \vee_{y \in U}[\mu_R(x, y) \wedge \omega_E(y)]$, $\mu_{\bar{R}(E)}(x) = \vee_{y \in U}[\mu_R(x, y) \wedge \mu_E(y)]$, $\nu_{\bar{R}(E)}(x) = \wedge_{y \in U}[\nu_R(x, y) \vee \nu_E(y)]$, $\omega_{\bar{R}(E)}(x) = \wedge_{y \in U}[\omega_R(x, y) \vee \omega_E(y)]$. Then, we name the pair $(\underline{R}(E), \bar{R}(E))$ an INRS of $E$ in terms of interval neutrosophic approximation space $(U, R)$.

Following the idea of SVNRSs [64], Guo et al. [75] further explored a generalized SVN rough set (GSVNRS) model by utilizing the concept of cut relations. In specific, for any $\alpha$, $\beta$, $\gamma \in (0, 1]$, the $\alpha$, $\beta$, $\gamma$-cut relation is denoted by $\tilde{R}_{\{(\alpha,\beta,\gamma)\}} = \{\langle \mu_{\tilde{R}}(x, y) \ge \alpha, \nu_{\tilde{R}}(x, y) \le \beta, \omega_{\tilde{R}}(x, y) \le \gamma \rangle | (x, y) \in U \times V\}$. Furthermore, in order to extend RNSs and NRSs to a more generalized form, Thao and Smarandache [76] and Thao et al. [77] put forward standard NRSs (SNRSs) and rough standard NSs (RSNSs) by virtue of a t-norm $\mathcal{T}$ and an implicator $\mathcal{J}$ over [0, 1].

**Definition 13.** [76]

Suppose $U$ is an arbitrary universe of discourse, $R$ is a standard neutrosophic (SN) relation over $U$, where $\mu_R, \nu_R, \omega_R : U \times U \to [0, 1]$, $G = \left\{ \langle x, \mu_G(x), \nu_G(x), \omega_G(x) \rangle | x \in U \right\}$ is a standard neutrosophic set (SNS), where $\mu, \nu, \omega : U \to [0, 1]$. Then, we name the pair $(U, R)$ a SN approximation space. For $\mathcal{T}$, $\mathcal{J}$ and $G$, the two approximations of $G$ in terms of SN approximation space $(U, R)$ are in mathematical terms of:

$$\underline{R}^{\mathcal{T}}(G)(x) = \vee_{y \in U} \mathcal{T}(R(x, y), G(y)), \forall x \in U; \tag{25}$$

$$\bar{R}_{\mathcal{J}}(G)(x) = \wedge_{y \in U} \mathcal{J}(R(x, y), G(y)), \forall x \in U, \tag{26}$$

where $\mathcal{T}$ and $\mathcal{J}$ can be represented by $\mathcal{T}_M(x, y) = (x_1 \wedge y_1, x_2 \wedge y_2, x_3 \vee y_3)$ and $\mathcal{J}(x, y) = (x_3 \vee y_1, x_2 \wedge y_2, x_1 \wedge y_3)$ for two SN numbers $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$. In light of the above statements, we name the pair $\left(\underline{R}^{\mathcal{T}}(G), \bar{R}_{\mathcal{J}}(G)\right)$ a SNRS of $G$ in terms of SN approximation space $(U, R)$.

**Definition 14.** [77]

Suppose $U$ is an arbitrary universe of discourse, an equivalence relation over $U$ is denoted by $R$. Then, we name the pair $(U, R)$ a crisp neutrosophic approximation space. For a SNS $G$, the two approximations of $G$ in terms of crisp neutrosophic approximation space $(U, R)$ are given as the following form:

$$\underline{N}(G) = \{\langle x, \mu_{\underline{N}(G)}(x), \nu_{\underline{N}(G)}(x), \omega_{\underline{N}(G)}(x)\rangle | x \in U\}; \tag{27}$$

$$\bar{N}(G) = \{\langle x, \mu_{\bar{N}(G)}(x), \nu_{\bar{N}(G)}(x), \omega_{\bar{N}(G)}(x)\rangle | x \in U\}, \tag{28}$$

where $\mu_{\underline{N}(G)}(x) = \wedge_{y \in [x]_R} \mu_G(y)$, $\nu_{\underline{N}(G)}(x) = \wedge_{y \in [x]_R} \nu_G(y)$, $\omega_{\underline{N}(G)}(x) = \vee_{y \in [x]_R} \omega_G(y)$, $\mu_{\bar{N}(G)}(x) = \vee_{y \in [x]_R} \mu_G(y)$, $\nu_{\bar{N}(G)}(x) = \wedge_{y \in [x]_R} \nu_G(y)$, $\omega_{\bar{N}(G)}(x) = \wedge_{y \in [x]_R} \omega_G(y)$. Then, the pair $\left(\underline{N}(G), \bar{N}(G)\right)$ is named a RSNS of $G$ in terms of crisp neutrosophic approximation space $(U, R)$.

In addition, the related works in terms of them are summed up in Table 2.

According to the above-mentioned various forms of generalized RNSs and NRSs, we can know that most of them put more emphasis on theoretical models and corresponding applications at the same time, which have largely enriched neutrosophic fusion of RST. In the future, several study directions are listed as follows:

(1) Investigate uncertainty measures of generalized RNSs and NRSs from the perspective of roughness measures, accuracy measures, granularity structures, etc.
(2) Put forward efficient attribute reduction algorithms for generalized RNSs and NRSs.
(3) Explore applications of generalized RNSs and NRSs for knowledge discovery by virtue of other common soft computing tools, such as three-way decisions, formal concept analysis, Dempster–Shafer (D–S) evidence theory, etc.

**Table 2**
Distribution papers in the field of generalized RNSs and NRSs.

| Definition | Reference | Representation | Study category |
|---|---|---|---|
| Definition 5 | Pramanik and Mondal [63] | BNSs | RBNSs |
| Definition 6 | Yang et al. [64] | SVNSs | SVNRSs |
| Definition 8 | Bao and Yang [68] | SVN refined sets | SVNRRSs |
| Definition 9 | Alias et al. [69,70] | SVNMSs | RNMSs |
| Definition 10 | Zhang et al. [73] | SVNMSs | PRSVNMSs |
| Definition 11 | Zhang et al. [73] | SVNMSs | SVNPRMSs |
| Definition 12 | Yang et al. [74] | INSs | INRSs |
| Definition 13 | Thao and Smarandache [76] | SNSs | SNRSs |
| Definition 14 | Thao et al. [77] | SNSs | RSNSs |

**Table 3**
Distribution papers in the field of CRSVNSs.

| Definition | Reference | Representation | Study category |
|---|---|---|---|
| Definition 15 | Ma et al. [79] | SVNSs | The first type of CRSVNSs |
| Definition 16 | Ma et al. [79] | SVNSs | The second type of CRSVNSs |
| Definition 17 | Ma et al. [79] | SVNSs | The third type of CRSVNSs |

### 3.3. Extensions of RNSs and NRSs from the aspect of covering-based rough sets (CRSs)

The notion of CRSs [49,78] is an influential rough set representative which generalizes classical rough sets to wider application scopes by replacing a division of the universe with a covering. Taking advantages of CRSs, Ma et al. [79] explored several different covering-based rough SVN sets (CRSVNSs) along with their properties. In what follows, we list different types of definitions for CRSVNSs.

**Definition 15.** [79]

Suppose $U$ is an arbitrary universe of discourse, $D$ is a covering of $U$, where $Md_D(x) = \left\{K \in D \wedge x \in T \wedge T \subseteq K \Rightarrow K = T\right\}$ is named the minimal expression of $x$, and the lowercase $D$ is omitted when the covering is clear. Then, we name the pair $(U, D)$ a covering approximation space. For a SVNS $H$, the first type of two approximations of $H$ in terms of covering approximation space $(U, D)$ is given as the following form:

$$\underline{F}(H) = \{\langle u, \mu_{\underline{F}(H)}(u), \nu_{\underline{F}(H)}(u), \omega_{\underline{F}(H)}(u)\rangle | u \in U\}; \tag{29}$$

$$\bar{F}(H) = \{\langle u, \mu_{\bar{F}(H)}(u), \nu_{\bar{F}(H)}(u), \omega_{\bar{F}(H)}(u)\rangle | u \in U\}, \tag{30}$$

where $\mu_{\underline{F}(H)}(u) = \wedge_{u\in U}\left\{\mu_H(v) | v \in \cup Md(u)\right\}$, $\nu_{\underline{F}(H)}(u) = \vee_{u\in U}\left\{\nu_H(v) | v \in \cup Md(u)\right\}$, $\omega_{\underline{F}(H)}(u) = \vee_{u\in U}\left\{\omega_H(v) | v \in \cup Md(u)\right\}$, $\mu_{\bar{F}(H)}(u) = \vee_{u\in U}\left\{\mu_H(v) | v \in \cup Md(u)\right\}$, $\nu_{\bar{F}(H)}(u) = \wedge_{u\in U}\left\{\nu_H(v) | v \in \cup Md(u)\right\}$, $\omega_{\bar{F}(H)}(u) = \wedge_{u\in U}\left\{\omega_H(v) | v \in \cup Md(u)\right\}$. Then, we name the pair $\left(\underline{F}(H), \bar{F}(H)\right)$ the first type of CRSVNSs of $H$ in terms of $(U, D)$.

**Definition 16.** [79]

Suppose $U$ is an arbitrary universe of discourse, $D$ is a covering of $U$. For a SVNS $H$, the second type of two approximations of $H$ in terms of covering approximation space $(U, D)$ is given as the following form:

$$\underline{S}(H) = \{\langle u, \mu_{\underline{S}(H)}(u), \nu_{\underline{S}(H)}(u), \omega_{\underline{S}(H)}(u)\rangle | u \in U\}; \tag{31}$$

$$\bar{S}(H) = \{\langle u, \mu_{\bar{S}(H)}(u), \nu_{\bar{S}(H)}(u), \omega_{\bar{S}(H)}(u)\rangle | u \in U\}, \tag{32}$$

where $\mu_{\underline{S}(H)}(u) = \wedge_{u\in U}\left\{\mu_H(v) | v \in \cap Md(u)\right\}$, $\nu_{\underline{S}(H)}(u) = \vee_{u\in U}\left\{\nu_H(v) | v \in \cap Md(u)\right\}$, $\omega_{\underline{S}(H)}(u) = \vee_{u\in U}\left\{\omega_H(v) | v \in \cap Md(u)\right\}$, $\mu_{\bar{S}(H)}(u) = \vee_{u\in U}\left\{\mu_H(v) | v \in \cap Md(u)\right\}$, $\nu_{\bar{S}(H)}(u) = \wedge_{u\in U}\left\{\nu_H(v) | v \in \cap Md(u)\right\}$, $\omega_{\bar{S}(H)}(u) = \wedge_{u\in U}\left\{\omega_H(v) | v \in \cap Md(u)\right\}$. Then, we name the pair $\left(\underline{S}(H), \bar{S}(H)\right)$ the second type of CRSVNSs of $H$ in terms of $(U, D)$.

**Definition 17.** [79]

Suppose $U$ is an arbitrary universe of discourse, $D$ is a covering of $U$. For a SVNS $H$, the third type of two approximations of $H$ in terms of covering approximation space $(U, D)$ is given as the following form:

$$\underline{T}(H) = \{\langle u, \mu_{\underline{T}(H)}(u), \nu_{\underline{T}(H)}(u), \omega_{\underline{T}(H)}(u)\rangle | u \in U\}; \tag{33}$$

$$\bar{T}(H) = \{\langle u, \mu_{\bar{T}(H)}(u), \nu_{\bar{T}(H)}(u), \omega_{\bar{T}(H)}(u)\rangle | u \in U\}, \tag{34}$$

where $\mu_{\underline{T}(H)}(u) = \vee_{K\in Md(u)}\{\wedge_{v\in K}\mu_H(v)\}$, $\nu_{\underline{T}(H)}(u) = \wedge_{K\in Md(u)}\{\vee_{v\in K}\nu_H(v)\}$, $\omega_{\underline{T}(H)}(u) = \wedge_{K\in Md(u)}\{\vee_{v\in K}\omega_H(v)\}$, $\mu_{\bar{T}(H)}(u) = \wedge_{K\in Md(u)}\{\vee_{v\in K}\mu_H(v)\}$, $\nu_{\bar{T}(H)}(u) = \vee_{K\in Md(u)}\{\wedge_{v\in K}\nu_H(v)\}$, $\omega_{\bar{T}(H)}(u) = \vee_{K\in Md(u)}\{\wedge_{v\in K}\omega_H(v)\}$. Then, we name the pair $\left(\underline{T}(H), \bar{T}(H)\right)$ the third type of CRSVNSs of $H$ in terms of $(U, D)$.

In addition, the related works in terms of CRSVNSs are summed up in Table 3.

In light of the above three different definitions of CRSVNSs, we can know that Literature [79] mainly focuses on some theoretical aspects of the presented models. Recently, a series of application-oriented works by means of CRSs were proposed by Zhan's research team [80–83], both some generalized CRSs models and their related decision making methods were put forward. Inspired by literatures [80–83], some future trends of CRSVNSs are listed as follows:

(1) Develop theoretical conclusions of CRSVNSs in depth from the aspect of attribute reduction algorithms, matroidal structures, topological properties, and so forth.
(2) Extend CRSVNSs to more generalized fuzzy contexts to construct complicated hybrid RST models.
(3) Propose decision making approaches of CRSVNSs and utilize them in plenty of real-world situations.

### 3.4. Extensions of RNSs and NRSs from the aspect of multigranulation rough sets (MGRSs)

From the above generalized rough set models, it is not difficult to see that lower and upper approximations are designed based on a single relation on one or two universes, which may lead to a difficulty when solving some practical multigranulation contexts, such as group decision making situations. Thus, Qian et al. [51,52] extended single granulations to multiple granulations by considering several binary relations simultaneously, and then proposed the model of MGRSs. The merits of MGRSs are mainly reflected in two aspects, one is enhancing the computational efficiency of discovering unknown knowledge for multi-dimension or multi-source information systems, the other one is adding the analysis of decision risks from risk-seeking and risk-averse tactics. Hence, MGRSs play a key role in handling various complex multigranulation problems [53,54]. Recently, Zhang et al. [84,85] introduced MGRSs to the context of SVNSs and INSs respectively, and further developed SVN MGRSs (SVNMGRSs) and interval neutrosophic MGRSs (INMGRSs) under two-universe frameworks. The mathematical formulation of similar models on a single universe [86] can be seen as reduced forms of the above models over two universes. In what follows, we list the mathematical formulation of SVNMGRSs and INMGRSs over two universes respectively.

**Definition 18.** [84]

Suppose $U, V$ are arbitrary universes of discourse, a SVN relation over $U \times V$ is denoted by $R_i\,(i = 1, 2, \ldots, m)$. Then, we name the pair $(U, V, R_i)$ a SVN multigranulation approximation space. For a SVNS $H$, the optimistic and pessimistic two approximations of $H$ related to SVN multigranulation approximation space $(U, V, R_i)$ are given as the following form:

$$\underline{\sum_{i=1}^{m} R_i}^{O}(H)=\{\langle x,\mu_{\underline{\sum_{i=1}^{m}R_i}^{O}(H)}(x),\nu_{\underline{\sum_{i=1}^{m}R_i}^{O}(H)}(x),\omega_{\underline{\sum_{i=1}^{m}R_i}^{O}(H)}(x)\rangle \mid x\in U\} \tag{35}$$

$$\sum_{i=1}^{\bar{m}} R_i^{O}(H)=\{\langle x,\mu_{\sum_{i=1}^{\bar{m}}R_i^{O}(H)}(x),\nu_{\sum_{i=1}^{\bar{m}}R_i^{O}(H)}(x),\omega_{\sum_{i=1}^{\bar{m}}R_i^{O}(H)}(x)\rangle \mid x\in U\} \tag{36}$$

$$\underline{\sum_{i=1}^{m} R_i}^{P}(H)=\{\langle x,\mu_{\underline{\sum_{i=1}^{m}R_i}^{P}(H)}(x),\nu_{\underline{\sum_{i=1}^{m}R_i}^{P}(H)}(x),\omega_{\underline{\sum_{i=1}^{m}R_i}^{P}(H)}(x)\rangle \mid x\in U\} \tag{37}$$

$$\sum_{i=1}^{\bar{m}} R_i^{P}(H)=\{\langle x,\mu_{\sum_{i=1}^{\bar{m}}R_i^{P}(H)}(x),\nu_{\sum_{i=1}^{\bar{m}}R_i^{P}(H)}(x),\omega_{\sum_{i=1}^{\bar{m}}R_i^{P}(H)}(x)\rangle \mid x\in U\}, \tag{38}$$

where $\mu_{\underline{\sum_{i=1}^{m}R_i}^{O}(H)}(x) = \vee_{i=1}^{m}\wedge_{y\in V}\left[\omega_{R_i}(x,y)\vee\mu_H(y)\right]$, $\nu_{\underline{\sum_{i=1}^{m}R_i}^{O}(H)}(x) = \wedge_{i=1}^{m}\vee_{y\in V}\left[\left(1-\nu_{R_i}(x,y)\right)\wedge\nu_H(y)\right]$, $\omega_{\underline{\sum_{i=1}^{m}R_i}^{O}(H)}(x) = \wedge_{i=1}^{m}\vee_{y\in V}\left[\mu_{R_i}(x,y)\wedge\omega_H(y)\right]$, $\mu_{\sum_{i=1}^{\bar{m}}R_i^{O}(H)}(x) = \wedge_{i=1}^{m}\vee_{y\in V}\left[\mu_{R_i}(x,y)\wedge\mu_H(y)\right]$, $\nu_{\sum_{i=1}^{\bar{m}}R_i^{O}(H)}(x) = \vee_{i=1}^{m}\wedge_{y\in V}\left[\nu_{R_i}(x,y)\vee\nu_H(y)\right]$, $\omega_{\sum_{i=1}^{\bar{m}}R_i^{O}(H)}(x) = \vee_{i=1}^{m}\wedge_{y\in V}\left[\omega_{R_i}(x,y)\vee\omega_H(y)\right]$, $\mu_{\underline{\sum_{i=1}^{m}R_i}^{P}(H)}(x) = \wedge_{i=1}^{m}\wedge_{y\in V}\left[\omega_{R_i}(x,y)\vee\mu_H(y)\right]$, $\nu_{\underline{\sum_{i=1}^{m}R_i}^{P}(H)}(x) = \vee_{i=1}^{m}\vee_{y\in V}\left[\left(1-\nu_{R_i}(x,y)\right)\wedge\nu_H(y)\right]$, $\omega_{\underline{\sum_{i=1}^{m}R_i}^{P}(H)}(x) = \vee_{i=1}^{m}\vee_{y\in V}\left[\mu_{R_i}(x,y)\wedge\omega_H(y)\right]$, $\mu_{\sum_{i=1}^{\bar{m}}R_i^{P}(H)}(x) = \vee_{i=1}^{m}\vee_{y\in V}\left[\mu_{R_i}(x,y)\wedge\mu_H(y)\right]$, $\nu_{\sum_{i=1}^{\bar{m}}R_i^{P}(H)}(x) = \wedge_{i=1}^{m}\wedge_{y\in V}\left[\nu_{R_i}(x,y)\vee\nu_H(y)\right]$, $\omega_{\sum_{i=1}^{\bar{m}}R_i^{P}(H)}(x) = \wedge_{i=1}^{m}\wedge_{y\in V}\left[\omega_{R_i}(x,y)\vee\omega_H(y)\right]$. Then, the pair $(\underline{\sum_{i=1}^{m}R_i}^{O}(H), \sum_{i=1}^{\bar{m}}R_i^{O}(H))$ is named an optimistic SVNMGRS of $H$ in terms of SVN multigranulation approximation space $(U, V, R_i)$, whereas the pair $(\underline{\sum_{i=1}^{m}R_i}^{P}(H), \sum_{i=1}^{\bar{m}}R_i^{P}(H))$ is named a pessimistic SVNMGRS of $H$ in terms of SVN multigranulation approximation space $(U, V, R_i)$.

**Definition 19.** [85]

Suppose $U, V$ are arbitrary universes of discourse, an interval neutrosophic relation over $U \times V$ is denoted by $R_i\,(i = 1, 2, \ldots, m)$. Then, we name the pair $(U, V, R_i)$ an interval neutrosophic multigranulation approximation space. For an INS $E$, the optimistic and pessimistic two approximations of $E$ related to interval neutrosophic multigranulation approximation space $(U, V, R_i)$ are given as the following form:

$$\underline{\sum_{i=1}^{m} R_i}^{O}(E)=\{\langle x,\mu_{\underline{\sum_{i=1}^{m}R_i}^{O}(E)}(x),\nu_{\underline{\sum_{i=1}^{m}R_i}^{O}(E)}(x),\omega_{\underline{\sum_{i=1}^{m}R_i}^{O}(E)}(x)\rangle \mid x\in U\}; \tag{39}$$

$$\sum_{i=1}^{\bar{m}} R_i^{O}(E)=\{\langle x,\mu_{\sum_{i=1}^{\bar{m}}R_i^{O}(E)}(x),\nu_{\sum_{i=1}^{\bar{m}}R_i^{O}(E)}(x),\omega_{\sum_{i=1}^{\bar{m}}R_i^{O}(E)}(x)\rangle \mid x\in U\}; \tag{40}$$

$$\underline{\sum_{i=1}^{m} R_i}^{P}(E)=\{\langle x,\mu_{\underline{\sum_{i=1}^{m}R_i}^{P}(E)}(x),\nu_{\underline{\sum_{i=1}^{m}R_i}^{P}(E)}(x),\omega_{\underline{\sum_{i=1}^{m}R_i}^{P}(E)}(x)\rangle \mid x\in U\}; \tag{41}$$

$$\sum_{i=1}^{\bar{m}} R_i^{P}(E)=\{\langle x,\mu_{\sum_{i=1}^{\bar{m}}R_i^{P}(E)}(x),\nu_{\sum_{i=1}^{\bar{m}}R_i^{P}(E)}(x),\omega_{\sum_{i=1}^{\bar{m}}R_i^{P}(E)}(x)\rangle \mid x\in U\}, \tag{42}$$

where $\mu_{\underline{\sum_{i=1}^{m}R_i}^{O}(E)}(x) = \vee_{i=1}^{m}\wedge_{y\in V}\left[\omega_{R_i}(x,y)\vee\mu_E(y)\right]$, $\nu_{\underline{\sum_{i=1}^{m}R_i}^{O}(E)}(x) = \wedge_{i=1}^{m}\vee_{y\in V}\left[\left([1,1]-\nu_{R_i}(x,y)\right)\wedge\nu_E(y)\right]$, $\omega_{\underline{\sum_{i=1}^{m}R_i}^{O}(E)}(x) = \wedge_{i=1}^{m}\vee_{y\in V}\left[\mu_{R_i}(x,y)\wedge\omega_E(y)\right]$, $\mu_{\sum_{i=1}^{\bar{m}}R_i^{O}(E)}(x) = \wedge_{i=1}^{m}\vee_{y\in V}\left[\mu_{R_i}(x,y)\wedge\mu_E(y)\right]$, $\nu_{\sum_{i=1}^{\bar{m}}R_i^{O}(E)}(x) = \vee_{i=1}^{m}\wedge_{y\in V}\left[\nu_{R_i}(x,y)\vee\nu_E(y)\right]$, $\omega_{\sum_{i=1}^{\bar{m}}R_i^{O}(E)}(x) = \vee_{i=1}^{m}\wedge_{y\in V}\left[\omega_{R_i}(x,y)\vee\omega_E(y)\right]$, $\mu_{\underline{\sum_{i=1}^{m}R_i}^{P}(E)}(x) = \wedge_{i=1}^{m}\wedge_{y\in V}\left[\omega_{R_i}(x,y)\vee\mu_E(y)\right]$, $\nu_{\underline{\sum_{i=1}^{m}R_i}^{P}(E)}(x) = \vee_{i=1}^{m}\vee_{y\in V}\left[\left(1-\nu_{R_i}(x,y)\right)\wedge\nu_E(y)\right]$, $\omega_{\underline{\sum_{i=1}^{m}R_i}^{P}(E)}(x) = \vee_{i=1}^{m}\vee_{y\in V}\left[\mu_{R_i}(x,y)\wedge\omega_E(y)\right]$, $\mu_{\sum_{i=1}^{\bar{m}}R_i^{P}(E)}(x) = \vee_{i=1}^{m}\vee_{y\in V}\left[\mu_{R_i}(x,y)\wedge\mu_E(y)\right]$, $\nu_{\sum_{i=1}^{\bar{m}}R_i^{P}(E)}(x) = \wedge_{i=1}^{m}\wedge_{y\in V}\left[\nu_{R_i}(x,y)\vee\nu_E(y)\right]$, $\omega_{\sum_{i=1}^{\bar{m}}R_i^{P}(E)}(x) = \wedge_{i=1}^{m}\wedge_{y\in V}\left[\omega_{R_i}(x,y)\vee\omega_E(y)\right]$. Then, the pair $(\underline{\sum_{i=1}^{m}R_i}^{O}(E), \sum_{i=1}^{\bar{m}}R_i^{O}(E))$ is named an optimistic INMGRS of $E$ in terms of interval neutrosophic multigranulation approximation space $(U, V, R_i)$, whereas the pair $(\underline{\sum_{i=1}^{m}R_i}^{P}(E), \sum_{i=1}^{\bar{m}}R_i^{P}(E))$ is named a pessimistic INMGRS of $E$ in terms of interval neutrosophic multigranulation approximation space $(U, V, R_i)$.

In addition, the related works in terms of SVNMGRSs and INMGRSs over two universes are summed up in Table 4.

**Table 4**
Distribution papers in the field of SVNMGRSs and INMGRSs over two universes.

| Definition | Reference | Representation | Study category |
|---|---|---|---|
| Definition 18 | Zhang et al. [84] | SVNSs | SVNMGRSs over two universes |
| Definition 19 | Zhang et al. [85] | INSs | INMGRSs over two universes |

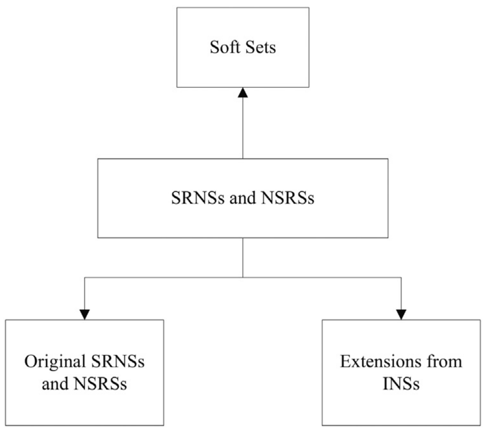


**Fig. 3.** Flow chart of the survey on neutrosophic fusion of RST from the aspect of soft sets.

Literatures [84,85] provide detailed mathematical structures of SVNMGRSs over two universes and INMGRSs over two universes, and further establish related decision making algorithms for realistic scenarios. In the future, there still exist several challenging issues to be considered:

(1) Extend SVNMGRSs over two universes and INMGRSs over two universes to multi-scale information systems [87] to model specific granular information transformations. Both corresponding rough approximations and some main properties are worth studying.
(2) Construct viable and efficient algorithms for large-scale of alternatives, high-dimensional attributes and dynamic decision making problems.
(3) Explore existing theoretical models within the context of neutrosophic duplets, neutrosophic triplets and neutrosophic multisets.

## 4. Neutrosophic fusion of RST from the aspect of soft sets

By introducing the scheme of parameterization, soft sets [55] act as a reasonable and effective soft computing tool for addressing the limitations triggered by randomness, hence studies about soft sets have been grown rapidly in many realistic areas [88]. Moreover, we present the flowchart of this section in Fig. 3.

### 4.1. Fusion of rough sets and soft sets under the NSs background

Akram et al. [89] put forward the model of SRNSs and NSRSs by fusing NSs, rough sets and soft sets, and then they further generalized NSRSs and SRNSs to graphs. In what follows, we list the concept of NSRSs and SRNSs.

**Definition 20.** [89]

Suppose $C$ is a Boolean set, $D$ is an attribute set. For an arbitrary full soft set $S$ over $C$ such that $S_S(a) \subset C$ and $a \in D$. For all $a \in D$, we define $S_S : D \to P(C)$ where $S_S(a) = \left\{ b \in C | (a, b) \in S \right\}$. Then, we name the pair $(C, S)$ a full soft approximation space. For a NS $N = \left\{ \langle b, \mu_N(b), \nu_N(b), \omega_N(b) \rangle | b \in C \right\} \in \mathcal{N}(C)$, where $\mathcal{N}(C)$ denotes the neutrosophic power set of $C$, the two soft approximations of $N$ in terms of full soft approximation space $(C, S)$ are given as the following form:

$$\underline{S}(N) = \{\langle b, \mu_{\underline{S}(N)}(b), \nu_{\underline{S}(N)}(b), \omega_{\underline{S}(N)}(b)\rangle | b \in C\}; \tag{43}$$

$$\bar{S}(N) = \{\langle b, \mu_{\bar{S}(N)}(b), \nu_{\bar{S}(N)}(b), \omega_{\bar{S}(N)}(b)\rangle | b \in C\}, \tag{44}$$

where $\mu_{\underline{S}(N)}(b) = \vee_{a \in D}(S(a, b) \wedge (\wedge_{t \in C}((1 - S(a, t)) \vee \mu_N(t))))$, $\mu_{\bar{S}(N)}(b) = \wedge_{a \in D}((1 - S(a, b)) \vee (\vee_{t \in C}(S(a, t) \wedge \mu_N(t))))$, $\nu_{\underline{S}(N)}(b) = \wedge_{a \in D}((1 - S(a, b)) \vee (\vee_{t \in C}(S(a, t) \wedge \nu_N(t))))$, $\nu_{\bar{S}(N)}(b) = \vee_{a \in D}(S(a, b) \wedge (\wedge_{t \in C}((1 - S(a, t)) \vee \nu_N(t))))$, $\omega_{\underline{S}(N)}(b) = \wedge_{a \in D}((1 - S(a, b)) \vee (\vee_{t \in C}(S(a, t) \wedge \omega_N(t))))$, $\omega_{\bar{S}(N)}(b) = \vee_{a \in D}(S(a, b) \wedge (\wedge_{t \in C}((1 - S(a, t)) \vee \omega_N(t))))$. Then, the pair $(\underline{S}(N), \bar{S}(N))$ is named a SRNS of $N$ in terms of $(C, S)$.

According to the model of SRNSs, the mathematical formulation of soft rough neutrosophic graphs (SRNGs) is further explored below.

**Definition 21.** [90]

Suppose $V$ is an arbitrary universe of discourse, then a SRNG on $V$ is represented by a 5-ordered tuple $G = (D, S, SN, R, RM)$, where $D$ is an attribute set, $S$ is an arbitrary full soft set on $V$, $R$ is an arbitrary full soft set over $E \subseteq V$, $SN = \left(\underline{S}(N), \bar{S}(N)\right)$ is a NSRS on $V$, $RM = \left(\underline{R}(M), \bar{R}(M)\right)$ is a NSRS on $E \subset V$.

**Definition 22.** [89]

Suppose $Y$ is an arbitrary finite universe of discourse, $P$ is a universal set of parameters, and we have $M \subseteq P$. For an arbitrary neutrosophic soft relation $W$ over $Y \times M$, we name the pair $(Y, M, W)$ a neutrosophic soft approximation space. For a NS $A \in \mathcal{N}(M)$, where $\mathcal{N}(M)$ denotes the neutrosophic power set of $M$, the two neutrosophic soft rough approximations of $A$ in terms of neutrosophic soft approximation space $(Y, M, W)$ are given as the following form:

$$\underline{W}(A) = \{\langle u, \mu_{\underline{W}(A)}(u), \nu_{\underline{W}(A)}(u), \omega_{\underline{W}(A)}(u)\rangle | u \in Y\}; \tag{45}$$

$$\bar{W}(A) = \{\langle u, \mu_{\bar{W}(A)}(u), \nu_{\bar{W}(A)}(u), \omega_{\bar{W}(A)}(u)\rangle | u \in Y\}, \tag{46}$$

where $\mu_{\underline{W}(A)}(u) = \wedge_{e \in M}\left(\omega_{W(A)}(u, e) \vee \mu_A(e)\right)$, $\nu_{\underline{W}(A)}(u) = \vee_{e \in M}\left(\left(1 - \nu_{W(A)}(u, e)\right) \wedge \nu_A(e)\right)$, $\omega_{\underline{W}(A)}(u) = \vee_{e \in M}\left(\mu_{W(A)}(u, e) \wedge \omega_A(e)\right)$, $\mu_{\bar{W}(A)}(u) = \vee_{e \in M}\left(\mu_{W(A)}(u, e) \wedge \mu_A(e)\right)$, $\nu_{\bar{W}(A)}(u) = \wedge_{e \in M}\left(\nu_{W(A)}(u, e) \vee \nu_A(e)\right)$, $\omega_{\bar{W}(A)}(u) = \wedge_{e \in M}\left(\omega_{W(A)}(u, e) \vee \omega_A(e)\right)$. Then, we name the pair $\left(\underline{W}(A), \bar{W}(A)\right)$ a NSRS of $A$ in terms of $(Y, M, W)$.

According to the model of NSRSs, the notion of neutrosophic soft rough graphs (NSRGs) is further explored below.

**Definition 23.** [91]

Suppose $V$ is an arbitrary universe of discourse, then a NSRG on $V$ is represented by a 4-ordered tuple $G = (V, M, W(A), S(\tilde{A}))$, where $M$ is a parameter set, $W$ and $S$ are arbitrary neutrosophic soft relations over $V \times M$, $W(A) = \left(\underline{W}(A), \bar{W}(A)\right)$ is a NSRS of $A$, $S(\tilde{A}) = (\underline{S}(\tilde{A}), \bar{S}(\tilde{A}))$ is a NSRS on $V$.

### 4.2. Fusion of rough sets and soft sets under the INSs background

Considering the significance of INSs when dealing with incomplete information systems, Broumi and Smarandache [92–94]

**Table 5**
Distribution papers in the field of NSRSs, SRNSs, SINRSs and INSRSs.

| Definition | Reference | Representation | Study category |
|---|---|---|---|
| Definitions 20 and 22 | Akram et al. [89] | NSs | NSRSs and SRNSs |
| Definition 21 | Malik et al. [90] | NSs | SRNGs |
| Definition 23 | Akram et al. [91] | NSs | NSRGs |
| Definition 24 | Broumi and Smarandache [93] | INSs | SINRSs |
| Definition 25 | Broumi and Smarandache [94] | INSs | INSRSs |

integrated INSs, soft sets and rough sets for establishing a novel comprehensive model named soft INRSs (SINRSs) and interval neutrosophic soft rough sets (INSRSs). In what follows, we list specific definitions of SINRSs and INSRSs.

**Definition 24.** [93]

Suppose a soft set in terms of INSs over $U$ is represented by $\Omega = (f, C)$, and we name the pair $\left(U, \Omega\right)$ the soft approximation space in terms of INSs, where for each $b \in C$, we define a function $f(b) = \left\{\langle x, \mu_{f(b)}(x), \nu_{f(b)}(x), \omega_{f(b)}(x)\rangle | x \in U\right\}$, $\mu_{f(b)}, \nu_{f(b)}, \omega_{f(b)} : C \to \text{int}[0, 1]$. Then for an INS $\sigma$, the two soft interval neutrosophic rough approximations of $\sigma$ in terms of soft interval neutrosophic approximation space $\left(U, \Omega\right)$ are in mathematical terms of:

$$\downarrow Apr_{SIVN}(\sigma) = \left\{ \begin{array}{l} x, \underset{b \in C}{\wedge} \left(\mu_{f(b)}(x) \wedge \mu_\sigma(x)\right), \\ \underset{b \in C}{\wedge} \left(\nu_{f(b)}(x) \vee \nu_\sigma(x)\right), \\ \underset{b \in C}{\wedge} \left(\omega_{f(b)}(x) \vee \omega_\sigma(x)\right) | x \in U \end{array} \right\}; \tag{47}$$

$$\uparrow Apr_{SIVN}(\sigma) = \left\{ \begin{array}{l} x, \underset{b \in C}{\wedge} \left(\mu_{f(b)}(x) \vee \mu_\sigma(x)\right), \\ \underset{b \in C}{\wedge} \left(\nu_{f(b)}(x) \wedge \nu_\sigma(x)\right), \\ \underset{b \in C}{\wedge} \left(\omega_{f(b)}(x) \wedge \omega_\sigma(x)\right) | x \in U \end{array} \right\}, \tag{48}$$

then, the pair $(\downarrow Apr_{SIVN}(\sigma), \uparrow Apr_{SIVN}(\sigma))$ is named a SINRS of $\sigma$ in terms of $\left(U, \Omega\right)$.

**Definition 25.** [94]

Suppose a full soft set on $U$ is denoted by $\Omega = (f, C)$, and we name the pair $S = \left(U, \Omega\right)$ the soft approximation space. For an INS $\sigma$, the two soft rough approximations of $\sigma$ in terms of $S$ are in mathematical terms of:

$$\underline{N}_S(\sigma) = \left\{ \begin{array}{l} x, \wedge \left\{\mu_\sigma(y) | \exists b \in C(\{x, y\} \subseteq f(b))\right\}, \\ \vee \left\{\nu_\sigma(y) | \exists b \in C(\{x, y\} \subseteq f(b))\right\}, \\ \vee \left\{\omega_\sigma(y) | \exists b \in C(\{x, y\} \subseteq f(b))\right\} | x \in U \end{array} \right\}; \tag{49}$$

$$\bar{N}_S(\sigma) = \left\{ \begin{array}{l} x, \vee \left\{\mu_\sigma(y) | \exists b \in C(\{x, y\} \subseteq f(b))\right\}, \\ \wedge \left\{\nu_\sigma(y) | \exists b \in C(\{x, y\} \subseteq f(b))\right\}, \\ \wedge \left\{\omega_\sigma(y) | \exists b \in C(\{x, y\} \subseteq f(b))\right\} | x \in U \end{array} \right\}, \tag{50}$$

then, the pair $\left(\underline{N}_S(\sigma), \bar{N}_S(\sigma)\right)$ is named an INSRS of $\sigma$ in terms of $\left(U, \Omega\right)$.

In addition, the related works in terms of NSRSs, SRNSs, SINRSs and INSRSs are shown in Table 5.

This section mainly presents the fusion of rough sets and soft sets under the NSs and INSs background, which includes both fundamental definitions and related graphs. In the future, some study directions are summarized below:

(1) Establish several extended models for NSRSs, SRNSs, SINRSs and INSRSs from the aspect of generalized fuzzy sets, such as type-2 fuzzy sets, hesitant fuzzy sets, pythagorean fuzzy sets, etc.
(2) Extend NSRSs, SRNSs, SINRSs and INSRSs to multi-granularity and multi-scale frameworks.
(3) Design more convincing decision making methods for the above-stated models and apply them to various complicated situations.

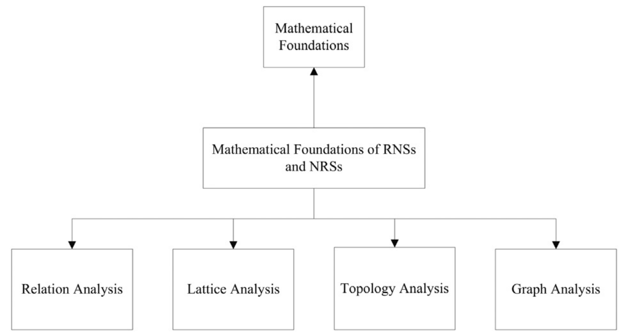


**Fig. 4.** Flow chart of the survey on neutrosophic fusion of RST from the aspect of mathematical foundations.

## 5. Neutrosophic fusion of RST from the aspect of mathematical foundations

The exploration of mathematical foundations is significant in rough set society, which is conducive to discovering more interesting theoretical results and utilizing them in various real-world situations. This section plans to sum up neutrosophic fusion of RST from the aspect of mathematical foundations [95–100], i.e., relations, lattices, topologies, graphs, etc. Moreover, we present the flowchart of this section in Fig. 4.

### 5.1. Relation analysis of hybrid models

Arockiarani and Sweety [95] studied the composition of rough neutrosophic relations and inverse rough neutrosophic relations respectively. In what follows, we list some main results of the relation analysis of hybrid models.

**Definition 26.** [95]

Suppose $U, V, W$ are three arbitrary universes of discourse, rough neutrosophic relations over $U \times V$ and $V \times W$ related to $X \times Y$ and $Y \times Z$ are denoted by $R_1$ and $R_2$. Then, the composition of $R_1$ and $R_2$, represented by $R_1 \circ R_2$, which is expressed on $U \times W$ in terms of $X \times Z$, where

$$\mu_{R_1 \circ R_2}(x, y) = \underset{y \in V}{\vee} \{\wedge \left(\mu_{R_1}(x, y), \mu_{R_2}(y, z)\right)\}; \tag{51}$$

$$\nu_{R_1 \circ R_2}(x, y) = \underset{y \in V}{\vee} \{\wedge \left(\nu_{R_1}(x, y), \nu_{R_2}(y, z)\right)\}; \tag{52}$$

$$\omega_{R_1 \circ R_2}(x, y) = \underset{y \in V}{\wedge} \{\vee \left(\omega_{R_1}(x, y), \omega_{R_2}(y, z)\right)\}. \tag{53}$$

**Definition 27.** [95]

Suppose $X$ and $Y$ are two RNSs over $U$ and $V$, a rough neutrosophic relation over $U \times V$ in terms of $X \times Y$ is denoted by $R \subset U \times V$. Next, we name $R^{-1} \subset V \times U$ a rough neutrosophic relation over $V \times U$ in terms of $Y \times X$, where for every $(x, y) \in V \times U$, we have $\mu_{R^{-1}}(y, x) = \mu_R(x, y)$, $\nu_{R^{-1}}(y, x) = \nu_R(x, y)$, $\omega_{R^{-1}}(y, x) = \omega_R(x, y)$. In light of the above statements, the relation $R^{-1}$ is named an inverse rough neutrosophic relation of $R$.

### 5.2. Lattice analysis of hybrid models

Recently, many scholars have put their emphasis on studying NSs from the perspective of lattice theory. In what follows, we list some main results of the lattice analysis of hybrid models.

**Definition 28.** [96,97]

Suppose $L$ is a lattice, $A$ is a NS. Then, $A$ is named a neutrosophic sublattice of $L$ if the following requirements exist

(1) $\mu_A(u \vee v) \geq \min\{\mu_A(u), \mu_A(v)\}$; $\mu_A(u \wedge v) \geq \max\{\mu_A(u), \mu_A(v)\}$;
(2) $\nu_A(u \vee v) \geq \min\{\nu_A(u), \nu_A(v)\}$; $\nu_A(u \wedge v) \geq \max\{\nu_A(u), \nu_A(v)\}$;
(3) $\omega_A(u \vee v) \leq \max\{\mu_A(u), \mu_A(v)\}$; $\omega_A(u \wedge v) \leq \min\{\omega_A(u), \omega_A(v)\}$.

The set of all neutrosophic lattices of $L$ is denoted by $NL(L)$. Moreover, a RNS of $L$ is named a rough neutrosophic lattice, and both $\underline{Q}(A)$ and $\bar{Q}(A)$ are neutrosophic lattices of $L$.

**Definition 29.** [96,97]

Suppose $Q(U)$ is a rough lattice, a NRS in $Q(U)$ is denoted by $Q(A) = \left(\underline{Q}(A), \bar{Q}(A)\right)$. Then, we name $Q(A)$ a neutrosophic rough sublattice if the followings are true

(1) $\mu_A(u \vee v) \geq \min\{\mu_A(u), \mu_A(v)\}$; $\mu_A(u \wedge v) \geq \min\{\mu_A(u), \mu_A(v)\}$;
(2) $\nu_A(u \vee v) \geq \min\{\nu_A(u), \nu_A(v)\}$; $\nu_A(u \wedge v) \geq \min\{\nu_A(u), \nu_A(v)\}$;
(3) $\omega_A(u \vee v) \geq \max\{\mu_A(u), \mu_A(v)\}$; $\omega_A(u \wedge v) \geq \max\{\omega_A(u), \omega_A(v)\}$.

### 5.3. Topology analysis of hybrid models

The study of properties of topological frameworks and rough sets are significant issues in RST. Liu and Yang [98] constructed the notion of SVN topologies. In what follows, we list some main results of the topology analysis of hybrid models.

**Definition 30.** [98]

Suppose $V$ is an arbitrary universe of discourse, a SVN topology over $V$ is a family $\varsigma$ of SVNSs if the following requirements exist

(1) $\emptyset, V \in \varsigma$;
(2) $F \cap G \in \varsigma$ for any $F, G \in \varsigma$;
(3) $\cup_{j \in J} F_i \in \varsigma$ for each $F_j \in \varsigma, j \in J$, $J$ denotes an index set.

Then, we name $(V, \varsigma)$ a SVN topological space, and every SVNS $F$ in $\varsigma$ is named a SVN open set in $(V, \varsigma)$. In light of the above statements, for any SVNS $F$, the SVN interior and closure of $F$ are given as the following form:

$$\mathrm{Int}(F) = \cup\{P | P \in \varsigma \text{ and } P \subseteq F\}; \tag{54}$$

$$Clo(F) = \cap\{Q | Q^c \in \varsigma \text{ and } F \subseteq Q\}. \tag{55}$$

### 5.4. Graph analysis of hybrid models

The graph theory is a powerful tool to express information involving relations between elements, and relations are denoted by edges and elements are denoted by vertices. In order to handle diverse fuzzy information systems, Rosenfeld [101] generalized classical graph theory to fuzzy contexts, and put forward the notion of fuzzy graphs. Recently, Sayed et al. [99] established the concept of rough neutrosophic digraphs (RNDs). Then, Ishfaq et al. [100] further explored some properties and applications by virtue of RNDs. Afterwards, Akram et al. [102] developed the concept of neutrosophic rough digraphs (NRDs) along with some important properties. In what follows, we list some main results of RNDs and NRDs.

**Definition 31.** [99]

Suppose $X$ is an arbitrary universe of discourse, a RND on $X$ is a 4-ordered tuple $G = (R, RX, S, SY)$, where $R$ is an equivalence relation over $X$, $S$ is an equivalence relation on $Y \subseteq X \times X$, $RX = \left(\underline{R}(X), \bar{R}(X)\right)$ is a RNS on $X$, $SY = \left(\underline{S}(Y), \bar{S}(Y)\right)$ is a rough neutrosophic relation on $X$, $(RX, SY)$ is a RND where $\underline{G} = (\underline{R}(X), \underline{S}(Y))$ and $\bar{G} = \left(\bar{R}(X), \bar{S}(Y)\right)$ are two approximations of $G$ such that

$$\mu_{\underline{S}(Y)}(xy) \leq \min\{\mu_{\underline{R}(X)}(x), \mu_{\underline{R}(X)}(y)\};$$
$$\nu_{\underline{S}(Y)}(xy) \leq \min\{\nu_{\underline{R}(X)}(x), \nu_{\underline{R}(X)}(y)\};$$
$$\omega_{\underline{S}(Y)}(xy) \leq \max\{\omega_{\underline{R}(X)}(x), \omega_{\underline{R}(X)}(y)\};$$
$$\mu_{\bar{S}(Y)}(xy) \leq \min\{\mu_{\bar{R}(X)}(x), \mu_{\bar{R}(X)}(y)\};$$
$$\nu_{\bar{S}(Y)}(xy) \leq \min\{\nu_{\bar{R}(X)}(x), \nu_{\bar{R}(X)}(y)\};$$
$$\omega_{\bar{S}(Y)}(xy) \leq \max\{\omega_{\bar{R}(X)}(x), \omega_{\bar{R}(X)}(y)\}.$$

**Definition 32.** [102]

Suppose $U$ is an arbitrary universe of discourse, a NRD on $U$ is a 4-ordered tuple $G = (R, RU, S, SV)$, where $R$ is a neutrosophic tolerance relation over $U$, $S$ is a neutrosophic tolerance relation over $V \subseteq U \times U$, $RU = \left(\underline{R}(U), \bar{R}(U)\right)$ is a NRS over $U$, $(RU, SV)$ is a NRD where $\underline{G} = (\underline{R}(U), \underline{S}(V))$ and $\bar{G} = \left(\bar{R}(U), \bar{S}(V)\right)$ are two approximations of $G$ such that

$$\mu_{\underline{S}(V)}(xy) \leq \min\{\mu_{\underline{R}(U)}(x), \mu_{\underline{R}(U)}(y)\};$$
$$\nu_{\underline{S}(V)}(xy) \leq \min\{\nu_{\underline{R}(U)}(x), \nu_{\underline{R}(U)}(y)\};$$
$$\omega_{\underline{S}(V)}(xy) \leq \max\{\omega_{\underline{R}(U)}(x), \omega_{\underline{R}(U)}(y)\};$$
$$\mu_{\bar{S}(V)}(xy) \leq \min\{\mu_{\bar{R}(U)}(x), \mu_{\bar{R}(U)}(y)\};$$
$$\nu_{\bar{S}(V)}(xy) \leq \min\{\nu_{\bar{R}(U)}(x), \nu_{\bar{R}(U)}(y)\};$$
$$\omega_{\bar{S}(V)}(xy) \leq \max\{\omega_{\bar{R}(U)}(x), \omega_{\bar{R}(U)}(y)\}.$$

In addition, the related works in terms of mathematical foundations of hybrid models are shown in Table 6.

This section mainly presents neutrosophic fusion of RST from the aspect of mathematical foundations i.e., relations, lattices, topologies, graphs, etc. Future works need to be done by combining with other application fields, such as complex networks, formal concept analysis, feature selections, knowledge acquisitions, and so forth.

## 6. Neutrosophic fusion of RST from the aspect of decision making

It is acknowledged that RST can be regarded as a powerful approach to handle plenty of decision making situations. By means of lower approximations which correspond to affirmatory decision

**Table 6**
Distribution papers in the field of mathematical foundations of hybrid models.

| Definition | Reference | Representation | Study category |
|---|---|---|---|
| Definitions 26 and 27 | Arockiarani and Sweety [95] | NSs | Rough neutrosophic relations |
| Definitions 28 and 29 | Arockiarani and Sweety [96,97] | NSs | Rough neutrosophic lattices |
| Definition 30 | Liu and Yang [98] | SVNSs | Rough neutrosophic topologies |
| Definition 31 | Sayed et al. [99] | NSs | RNDs |
| Definition 32 | Akram et al. [102] | NSs | NRDs |

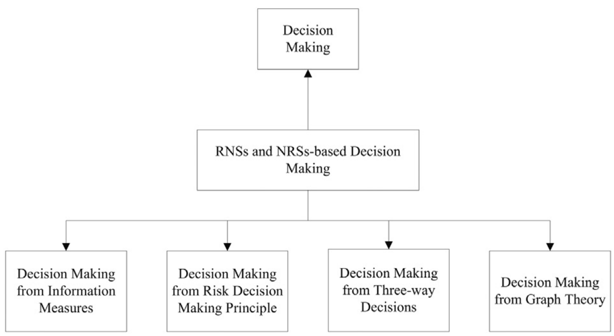


**Fig. 5.** Flow chart of the survey on neutrosophic fusion of RST from the aspect of decision making.

tactics and upper approximations which correspond to probabilistic decision tactics, many scholars have designed numerous decision making models from the perspective of information measures, risk decision making principle, three-way decisions, graph theory, etc. Moreover, NSs and their extensions provide preferences of decision makers over attribute sets and alternatives under the background of diverse applications. In what follows, we present the flowchart of this section in Fig. 5.

In addition, the related works in terms of decision making of hybrid models are shown in Table 7.

According to Table 7, neutrosophic fusion of RST from the aspect of decision making has been developed and enriched by lots of scholars during the past five years. In the future, scholars may need to resolve the following challenges:

(1) Identify practical decision making scenarios in which neutrosophy produces uncertainties and apply them to neutrosophic models.
(2) Explore viable schemes to prove that results outperform the ones obtained with other decision making approaches.
(3) Put forward decision making algorithms that combine multiple types of uncertainties including neutrosophy in order to obtain reasonable and better decision conclusions.
(4) New decision making algorithms should be more creative than simple and direct extensions of existing ones by using NSs and RST.

## 7. Neutrosophic fusion of RST from the aspect of other applications

In previous sections, we have revisited the notion of NSs and rough sets, neutrosophic fusion of RST from the aspect of basic models, soft sets, mathematical foundations, and decision making, our goal in the current section is to present other applications by means of RNSs and NRSs. In general, there are three kinds of typical applications by means of RNSs and NRSs, i.e., three-way decisions, clustering and smart cities, some detailed illustrations of them are shown in Fig. 6. Moreover, we also list them as follows:

(1) Three-way decisions: three-way decisions, proposed by Yao [50,111–115], aim to split a universal set into three different disjointed parts by using the semantics of acceptance, noncommitment and rejection in ternary classifications. Recently, Yao [116] further expressed three-way decisions as a trisecting-acting-outcome (TAO) model, which plays a significant role in the process of granular computing-based human thinking. In recent years, some researchers have introduced three-way decisions to the background of NSs. Abdel-Basset et al. [117] investigated three-way decisions based on NSs and analytic hierarchy process (AHP)-quality function deployment (QFD) paradigm, and then applied the presented three-way decisions framework to supplier selection problems. Singh [118,119] studied three-way neutrosophic concept lattice representations in detail. Zhang et al. [73] put forward the notion of SVNRMSs and further constructed a three-way decisions approach for medical diagnosis. In the future, neutrosophic fusion of RST from the aspect of three-way decisions should be further enriched.
(2) Clustering: according to the characteristics of the data, clustering generally means the procedure of combining a set of objects into clusters [120]. Since it is well known that data used for clustering may be neutrosophic, it is meaningful to develop related clustering algorithms for better handling this type of data. Thao et al. [121] extended the notion of fuzzy equivalences to the standard RNSs and NRSs, then a clustering analysis based on the datasets of NSs was conducted by utilizing these definitions. In the future, efficient clustering algorithms based on other generalized NSs and RST are worth looking into.
(3) Smart cities: a smart city is a modern urban area that combines diverse electronic data collection sensors to provide information which is used to efficiently make use of all aspects of assets and resources, and maximize the quality of citizen's services [122,123]. In order to reasonably handle incomplete information systems existed in smart cities by virtue of SVNSs and rough sets, Abdel-Basset and Mohamed [124] integrated the two soft computing tools for processing all aspects of incompleteness in smart city information systems, this integration aimed to promote the quality of various creative services from smart cities to their citizens. Afterwards, in order to cope with incomplete information systems in healthcare fields, a realistic experiment was performed to test the validation of the proposed method. In the future, it is necessary to apply existing tools in numerous fields such as industry and waste management for serving smart cities.

## 8. A bibliometric overview of current works

The notion of bibliometric, first originated by Pritchard [125] in 1969, generally aims to process the cross-science of all knowledge carriers quantitatively by utilizing some proposed statistical approaches [126–128]. Nowadays, bibliometric has been paying an increasing number of attentions from scholars and practitioners, and it has been utilized in diverse areas such as fuzzy sets [129], rough sets [130], decision making [131], etc. Considering the significance of neutrosophic fusion of RST, it is necessary to perform a bibliometric overview of current works related to neutrosophic fusion of RST. In what follows, a bibliometric overview

**Table 7**
Distribution papers in the field of RNSs and NRSs-based decision making.

| Reference | Representation | Study category | Application field | Study contribution |
|---|---|---|---|---|
| Broumi and Smarandache [93] | INSs | SIVNRSs | Staff selections | Designed a multiple attribute group decision making approach by using the model of SIVNRSs |
| Broumi and Smarandache [94] | INSs | IVNSRSs | Staff selections | Presented a multiple attribute group decision making approach by using the model of IVNSRSs |
| Akram et al. [89] | NSs | NSRSs | Selection of generic version of brand name medicine | Developed a multiple attribute decision making approach by using the model of NSRSs |
| Pramanik and Mondal [103] | NSs | RNSs | Medical diagnosis | Constructed a multiple attribute decision making approach by virtue of cosine similarity measures |
| Pramanik and Mondal [104] | NSs | RNSs | Medical diagnosis | Put forward a multiple attribute decision making approach by virtue of cotangent similarity measures |
| Pramanik and Mondal [105] | NSs | RNSs | Medical diagnosis | Proposed a multiple attribute decision making approach by virtue of dice and Jaccard similarity measures |
| Mondal and Pramanik [106] | NSs | RNSs | Select schools | Studied a multiple attribute decision making approach by virtue of grey relational analysis |
| Mondal and Pramanik [107] | NSs | RNSs | Purchase SIM (Subscriber Identification Module) cards | Designed a multiple attribute decision making approach by virtue of rough accuracy score functions |
| Mondal et al. [108] | NSs | RNSs | Investment problems | Presented a multiple attribute decision making approach by virtue of rough variation coefficient similarity measures |
| Pramanik et al. [109] | NSs | RNSs | Medical diagnosis | Explored a multiple attribute group decision making approach by virtue of correlation coefficient measures |
| Mondal et al. [110] | NSs | RNSs | Logistic center location selection problems | Put forward a multiple attribute group decision making approach by virtue of TOPSIS methods |
| Yang et al. [64] | SVNSs | SVNRSs | Medical diagnosis | Constructed a multiple attribute decision making approach by virtue of risk decision making principle |
| Bao and Yang [68] | SVN refined sets | SVNRRSs | Medical diagnosis | Established a multiple attribute decision making approach by virtue of risk decision making principle |
| Alias et al. [70] | SVNMSs | RNMSs | Marketing strategy problems | Discussed a multiple attribute decision making approach by virtue of risk decision making principle |
| Yang et al. [74] | INSs | INRSs | Medical diagnosis | Investigated a multiple attribute decision making approach by virtue of risk decision making principle |
| Guo et al. [75] | SVNSs | GSVNRSs | Medical diagnosis | Proposed a multiple attribute decision making approach by virtue of three-way decisions |
| Zhang et al. [84] | SVNSs | SVNMGRSs | Steam turbine fault diagnosis | Designed a multiple attribute group decision making approach by virtue of risk decision making principle |
| Zhang et al. [85] | INSs | INMGRSs | Selection of merger and acquisition targets | Studied a multiple attribute group decision making approach by virtue of risk decision making principle |
| Bo et al. [86] | SVNSs | SVNMGRSs | Selection of houses | Researched a multiple attribute group decision making approach by virtue of risk decision making principle |
| Zhang et al. [73] | SVNMSs | SVNRMSs | Medical diagnosis | Presented a multiple attribute decision making approach by virtue of three-way decisions |
| Sayed et al. [99] | NSs | RNSs | Investment problems | Constructed a multiple attribute decision making approach by virtue of RNDs |
| Akram et al. [102] | NSs | NRSs | Online ratings and recruitment problems | Developed a multiple attribute decision making approach by virtue of NRDs |
| Malik et al. [90] | NSs | SRNSs | Selection of the right path for transferring goods | Studied a multiple attribute decision making approach by virtue of SRNGs |
| Akram et al. [91] | NSs | NSRSs | Selection of generic version of brand name medicine | Put forward a multiple attribute decision making approach by virtue of NSRGs |

of current works will be shown from the aspect of the most productive authors, annual trends, main research topics, country level, and publishing journals or conferences.

### 8.1. The most productive authors

According to the collected papers related to neutrosophic fusion of RST, over 35 authors have published works by combining NSs with rough sets from different areas. We list the summary of top 6 most productive scholars below, which is shown in Fig. 7.

It is noted that the most productive author is Florentin Smarandache, who founded the theory of NSs and logic. Then followed by Surapati Pramanik, Kalyan Mondal, Said Broumi, Hailong Yang and Muhammad Akram.

### 8.2. Annual trends

The pioneering papers for the integration of RST with NSs and logic were put forward formally in 2014, Broumi and Smarandache [44,92] established the model of RNSs and IVNSRSs respectively. In light of the above works originated by Broumi and Smarandache, the fusion research shows a rapid growth trend. According to Fig. 8, 5% of total works were published in 2014. In 2015 and 2016, the percentage increased to 15%. Then 23% of the fusion studies were proposed in 2017. Finally in 2018, this year witnessed 42% of all publications. With the high growth rate, it is believed that an increasing number of fusion works will be further explored in the coming years.

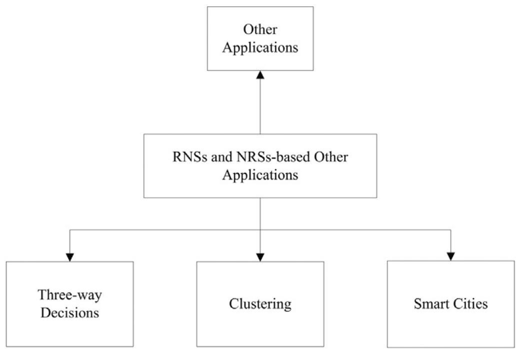


**Fig. 6.** Flow chart of the survey on neutrosophic fusion of RST from the aspect of other applications.

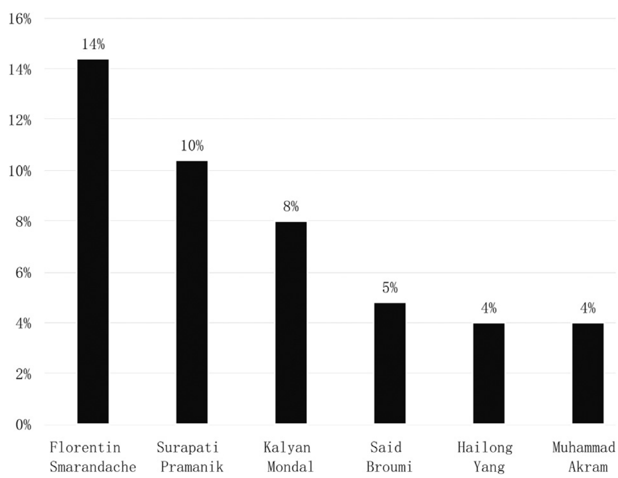


**Fig. 7.** The top 6 most productive authors.

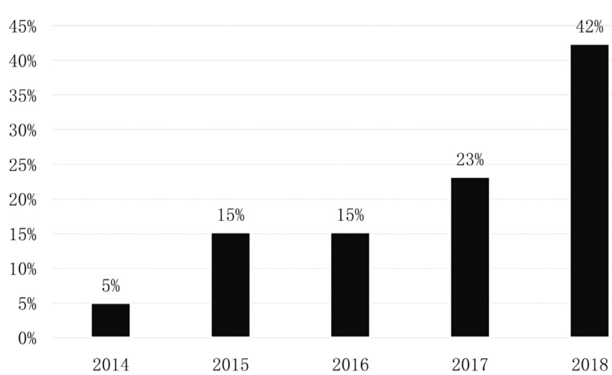


**Fig. 8.** Annual trends.

### 8.3. Main research topics

One of the primary tasks in the paper is to sum up different aspects of neutrosophic fusion of RST. To facilitate the whole overview procedure, we roughly divide the main research topics into several aspects, i.e., rough sets, soft sets, relations, lattices, topologies, graphs, similarity measures, grey sets, decision making, clustering analysis, etc., and we just list top 5 research topics below. According to Fig. 9, it is noticed that 46% of total fusion works are located in the field of rough sets. Next 29% of the fusion studies focus on decision making. Then followed by soft sets, similarity measures and clustering analysis. With the trend of main research topics, rough sets and decision making still exert a significant influence on neutrosophic fusion of RST, and other research topics are also suggested to be further discussed in the future.

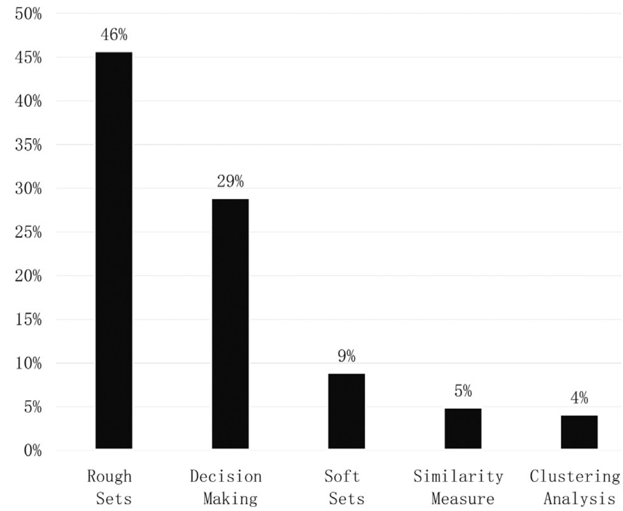


**Fig. 9.** Main research topics.

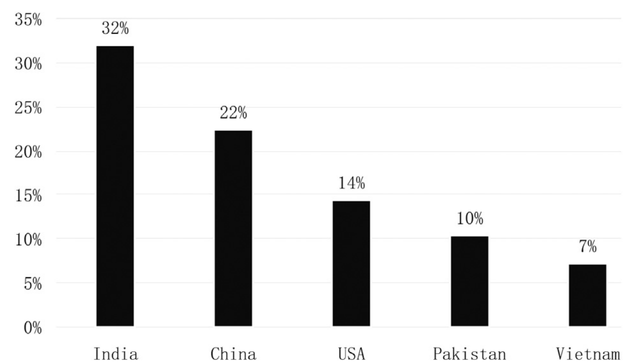


**Fig. 10.** Country level.

### 8.4. Country level

After introducing the fusion research from the most productive authors, annual trends and main research topics, we further investigate the fusion works from the country level (top 5 countries). According to Fig. 10, Indian scholars act as the largest contributor of neutrosophic fusion of RST with a share of 32%. Then China is the second contributor in the same area with 22% of the total publications. Following India and China, USA, Pakistan and Vietnam also contribute plenty of excellent fusion works. In light of the scenario, it is hoped that more international scholars can participate in the field actively and publish more meaningful works.

### 8.5. Publishing journals or conferences

Our data shows over 20 journals or conferences have published papers with respect to neutrosophic fusion of RST. We summarize the top 5 publication outlets for the above topics in Fig. 11. According to Fig. 11, a professional NS periodical titled *Neutrosophic Sets and Systems*, founded by Smarandache, is the largest outlet for the fusion works. Then followed by *Symmetry*, two special issues named “Neutrosophic Theories Applied in Engineering” and “Algebraic Structures of Neutrosophic Triplets, Neutrosophic Duplets, or Neutrosophic Multisets” provide the second largest outlet for the fusion works. In addition, there are only 2 science citation index expanded (SCIE) journals in the top 5 list, i.e., *Symmetry* and *Jour-*

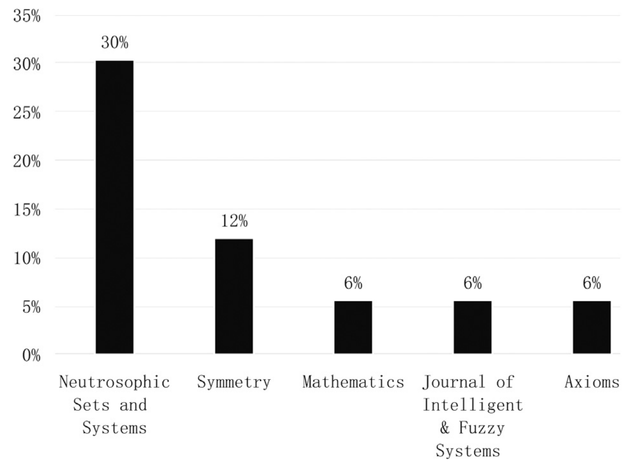


**Fig. 11.** Publishing journals or conferences.

*nal of Intelligent & Fuzzy Systems*. In the future, we believe that *Neutrosophic Sets and Systems* can be included by SCIE in every neutrosophic scholar's efforts, and it is meaningful to publish more fusion studies in top journals or conferences.

## 9. Conclusions

NSs and logic act as a powerful tool for handling indeterminate information existed widely in lots of practical scenarios, and RST offers a powerful scheme to the exploitation of complicated neutrosophic information systems. In order to make a comprehensive overview for neutrosophic fusion of RST, the work concentrates on five perspectives of the existing fusion works, i.e., basic models, soft sets, mathematical foundations, decision making and other applications. Then on the basis of the above review aspects, a bibliometric overview of current works in terms of neutrosophic fusion of RST is also conducted from the aspect of the most productive authors, annual trends, main research topics, country level, and publishing journals or conferences. In light of the presented discussions, neutrosophic fusion of RST has achieved a considerable development in different research areas.

In the future, there is still much work to be further considered to enrich the analysis of neutrosophic information systems by virtue of RST. To be specific, it is desirable to explore other significant aspects of RNSs and NRSs-based models, such as attribute reductions, rule acquisitions, uncertainty measures, etc. Moreover, the development of novel neutrosophic fusion approaches by means of RST is also worthy for attention, such as the fusion from the aspect of D–S evidence theory, the fusion from the aspect of formal concept analysis, the fusion under the background of other uncertain information systems, the fusion under the background of big data situations. Besides, it is meaningful to expand the current application scope of the fusion models, and it is hoped that more real-world applications such as intelligent business and urban computing can be studied in depth by utilizing the proposed fusion models.

## Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this work.

## Acknowledgments

The authors thank the editors and the anonymous referees for their insightful and constructive comments that have led to an improved version of this paper. This research was partially supported by the National Natural Science Foundation of China (Nos. 61806116, 61672331, 61972238, 61802237 and 61603278),Training Program for Young Scientific Researchers of Higher Education Institutions in Shanxi, the Natural Science Foundation of Shanxi (No. 201801D221175), Cultivate Scientific Research Excellence Programs of Higher Education Institutions in Shanxi (CSREP) (2019SK036), Scientific and Technological Innovation Programs of Higher Education Institutions in Shanxi (STIP) (No. 201802014,2019L0066), and Industry-University-Research Collaboration Program Between Shanxi University and Xiaodian District. Key R&D program of Shanxi Province(International Cooperation, 201903D421041).

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