

An improved MULTIMOORA approach for multi-criteria decision-making based on interdependent inputs of simplified neutrosophic linguistic information

Zhang-peng Tian¹ · Jing Wang¹ · Jian-qiang Wang¹ · Hong-yu Zhang¹

Received: 30 December 2015 / Accepted: 21 May 2016
© The Natural Computing Applications Forum 2016

Abstract Multi-objective optimization by ratio analysis plus the full multiplicative form (MULTIMOORA) is a useful method to apply in multi-criteria decision-making due to the flexibility and robustness it introduces into the decision process. This paper defines several simplified neutrosophic linguistic distance measures and employs a distance-based method to determine criterion weights. Then, an improved MULTIMOORA approach is presented by integrating the simplified neutrosophic linguistic normalized weighted Bonferroni mean and simplified neutrosophic linguistic normalized geometric weighted Bonferroni mean operators as well as a simplified neutrosophic linguistic distance measure. This approach ranks alternatives according to three ordering methods, and then, uses dominance theory to combine the three rankings into a single ranking. Finally, this paper presents a practical case example and conducts a comparative analysis between the proposed approach and existing methods in order to verify the feasibility and effectiveness of the developed methodology.

Keywords Multi-criteria decision-making · Simplified neutrosophic linguistic sets · Bonferroni mean operator · MULTIMOORA

1 Introduction

Multi-criteria decision-making (MCDM) is one of the most significant research topics in decision theory, and it has been widely applied in a number of fields [1]. In some complex decision-making problems, however, decision-makers (DMs) cannot precisely express their decision information in quantitative terms, and they instead provide qualitative descriptions. Thus, Zadeh [2] originally introduced the linguistic variable, an effective tool that uses linguistic information to enhance the reliability and flexibility of classical decision models [3–5]. Recently, an increasing number of linguistic proposals associated with other theories have been studied in depth for their potential to tackle MCDM problems. One of these theories concerns intuitionistic linguistic sets [6], and their extended forms [7]. Other proposals such as gray linguistic sets [8, 9], hesitant fuzzy linguistic sets, linguistic hesitant fuzzy sets [10, 11], and hesitant fuzzy linguistic term sets [12, 13] have also been proposed, as has the 2-tuple linguistic information model [14, 15]. Other studies have addressed the transformation between linguistic information and numerical data, including the use of the membership functions of fuzzy sets (FSs) such as triangular fuzzy numbers, trapezoidal fuzzy numbers, or interval type-2 fuzzy sets [16, 17]. Researchers have constructed a model to depict uncertain and qualitative concepts based on the cloud model [18]. However, those linguistic proposals, as promising as they are, still need to be refined. The central weakness in the research is that linguistic variables can express uncertain information but not inconsistent information.

Smarandache [19] proposed neutrosophic sets (NSs), which are an extension of intuitionistic fuzzy sets (IFSs) [20]. NSs consider truth-membership, indeterminacy-

✉ Jian-qiang Wang
jqwang@csu.edu.cn

¹ School of Business, Central South University, Changsha 410083, People's Republic of China

membership and falsity-membership simultaneously, and they are more practical and flexible than FSs and IFSs in addressing uncertain, incomplete, and inconsistent information. However, without the use of special descriptions, it is difficult to apply NSs in actual engineering and economic management situations. Therefore, Wang et al. [21] introduced single-valued neutrosophic sets (SVNSs). Subsequent studies have discussed the distance measures [22, 23], similarity measures [24], correlation coefficients [25], and cross-entropy [26] of SVNSs. Ye [27] went on to define simplified neutrosophic sets (SNSs), and Peng et al. [28] defined the operations of simplified neutrosophic numbers (SNNs). Other extensions of NSs, such as interval neutrosophic sets (INSs) [29–32], multi-valued neutrosophic sets [33], trapezoidal neutrosophic sets [34, 35], interval neutrosophic hesitant sets [36], normal neutrosophic sets [37], neutrosophic cubic sets [38], and neutrosophic soft sets [39–42] have also been proposed. Studies exploring these concepts tend to focus on correlation coefficients [30], cross-entropy [31], dominance measures [32], similarity measures [34], and various aggregation operators [35–37].

In order to overcome the defects involved in using linguistic variables associated with IFSs, gray sets and hesitant fuzzy sets, Ye [43] introduced single-valued neutrosophic linguistic sets, which are based on linguistic term sets and SVNSs. Ye [44] also defined interval neutrosophic linguistic sets and developed a MCDM method based on the interval neutrosophic linguistic weighted arithmetic average and interval neutrosophic linguistic weighted geometric average operators. In Ref. [44], Tian et al. [45] pointed out that additive and multiplicative operations of interval neutrosophic linguistic numbers might produce unreasonable results. Accordingly, Tian et al. [45] defined simplified neutrosophic linguistic sets (SNLSs) and modified the operations of simplified neutrosophic linguistic numbers (SNLNs) based on the linguistic scale function. Furthermore, numerous researchers have explored extensions of neutrosophic linguistic sets (NLSs). For example, Broumi and Smarandache [46] defined single-valued neutrosophic trapezoid linguistic sets and developed a MCDM method based on single-valued neutrosophic trapezoid linguistic aggregation operators. Broumi et al. [47] extended the order of preference by similarity to ideal solution (TOPSIS) method to deal with MCDM with interval neutrosophic linguistic variables. Ye [48] developed two kinds of interval neutrosophic uncertain linguistic aggregation operators and applied them in managing multi-criteria group decision-making problems in the context of interval neutrosophic uncertain linguistic environments. Ma et al. [49] developed an interval neutrosophic linguistic MCDM method

based on prioritized weighted harmonic mean operators and applied it in evaluating medical treatment options.

The aforementioned applications of NLSs and their extensions can be effective in dealing with neutrosophic linguistic MCDM problems. However, they also have some limitations which are outlined below:

1. MCDM methods [43, 44, 46–48] conduct operations by directly obtaining the labels of linguistic terms. In this way, it can be applied in a single semantic situation, and ignoring the influence of different semantics on the decision-making result.
2. The methods [43–48] can solve neutrosophic linguistic MCDM problems that require the criterion weights to be completely known. However, due to the increasing complexity of decision-making circumstances, it can be difficult and subjective to provide exact criterion weight information.
3. All of these MCDM methods are based on aggregation operators [44–48] and TOPSIS [43, 47] with neutrosophic linguistic information. They all have a single decision-making structure, and not enough attention is paid to improving robustness when processing the assessment information.

The above analysis describes the motivation behind proposing a comprehensive approach for tackling MCDM with SNLNs. This study develops an approach that takes advantage of linguistic scale functions, aggregation operators, and distance-based methods to overcome these disadvantages. The main contributions of this paper are summarized below:

1. This paper adopts linguistic scale functions to conduct the transformation from qualitative information to quantitative data.
2. In order to obtain criterion weights, this paper defines a series of distance measures for SNLNs and develops a distance-based method to derive the optimal criterion weight information objectively.
3. This paper develops an improved multi-objective optimization by ratio analysis plus the full multiplicative form (MULTIMOORA). This method integrates the simplified neutrosophic linguistic normalized weighted Bonferroni mean (SNLNWBM) and simplified neutrosophic linguistic normalized geometric weighted Bonferroni mean (SNLNWGBM) operators as well as the distance-based method, and establishes a synthetic structure. Furthermore, the proposed method employs a dominance measure to rank all alternatives.

The rest of the paper is organized as follows. Section 2 briefly reviews the concepts of linguistic term sets, NSs, SNSs, and SNLSs as well as the traditional MULTIMOORA method. Section 3 introduces two kinds of

simplified neutrosophic linguistic aggregation operators, the SNLNWBM and SNLNWGBM operators. This section also defines a family of distance measures for SNLNs. Section 4 proposes a simplified neutrosophic linguistic MULTIMOORA approach based on the SNLNWBM and SNLNWGBM operators and the distance measure to deal with MCDM problems. Section 5 provides an illustrative example to demonstrate the feasibility and applicability of the proposed approach. In addition, a comparative analysis is carried out between the proposed approach and the existing methods. Section 6 presents the conclusions of this study.

2 Preliminaries

This section reviews several relevant definitions of SNLNs, including linguistic term sets, linguistic scale functions, NSs, SNSs, and operations and comparison rules for SNLNs.

Let $H = \{h_1, h_2, \dots, h_{2t+1}\}$ represent a finite and totally ordered discrete term set, where t is a nonnegative integer. It is required that h_i and h_j must satisfy the following characteristics [3, 4].

1. The set is ordered: $h_i < h_j$ if and only if $i < j$;
2. Negation operator: $neg(h_i) = h_{(2t+2-i)}$.

To preserve all the given information, the discrete linguistic set H is extended to a continuous set $\bar{H} = \{h_i | 1 \leq i \leq L\}$, in which L ($L > 2t+1$) is a sufficiently large positive integer. If $h_i \in \bar{H}$, then h_i is called the original linguistic term; otherwise, h_i is called the virtual linguistic term [50].

2.1 Linguistic scale functions

To use data more efficiently and to express semantics more flexibly, linguistic scale functions assign different semantic values to linguistic terms under different situations [45]. This method is preferable in practice because the functions are flexible and can give more deterministic results according to different semantics. For linguistic term h_i in set H , where $H = \{h_i | i = 1, 2, \dots, 2t + 1\}$, the relationship between the element h_i and its subscript i is strictly monotonically increasing [45].

Definition 1 [45] If $\theta_i \in [0, 1]$ is a numerical value, then the linguistic scale function f that conducts the mapping from h_i to θ_i ($i = 1, 2, \dots, 2t+1$) is defined as follows:

$$f : h_i \rightarrow \theta_i (i = 1, 2, \dots, 2t + 1),$$

where $0 \leq \theta_1 < \theta_2 < \dots < \theta_{2t+1} \leq 1$.

Clearly, function f is strictly monotonically increasing with respect to the subscript i . The symbol θ_i ($i = 1, 2, \dots, 2t + 1$) reflects the preference of DMs when they are using the linguistic term $h_i \in H$ ($i = 1, 2, \dots, 2t + 1$). Therefore, the function or value in fact denotes the semantics of linguistic terms.

1.

$$f_1(h_i) = \theta_i = \frac{i-1}{2t} (i = 1, 2, \dots, 2t + 1).$$

The evaluation scale of the linguistic information given above is divided on average.

2.

$$f_2(h_i) = \theta_i = \begin{cases} \frac{\alpha^t - \alpha^{t-i+1}}{2\alpha^t - 2} (i = 1, 2, \dots, t + 1) \\ \frac{\alpha^t + \alpha^{i-t-1} - 2}{2\alpha^t - 2} (i = t + 2, t + 3, \dots, 2t + 1) \end{cases}.$$

The value of α can be determined using a subjective approach. Let A and B be two indicators. Assume that A is far more significant than B and the importance ratio is m . Then $\alpha^k = m$, where k represents the scale level and $\alpha = \sqrt[k]{m}$. At present, the vast majority of researchers believe that $m = 9$ is the upper limit for an importance ratio. If the scale level is 7, then $\alpha = \sqrt[7]{9} \approx 1.37$ can be calculated [51]. With the extension from the middle of the given linguistic term set to both ends, the absolute deviation between adjacent linguistic subscripts also increases.

3.

$$f_3(h_i) = \theta_i = \begin{cases} \frac{t^\beta - (t-i+1)^\beta}{2t^\beta} (i = 1, 2, \dots, t + 1) \\ \frac{t^\gamma + (i-t-1)^\gamma}{2t^\gamma} (i = t + 2, t + 3, \dots, 2t + 1) \end{cases}.$$

The values $\beta, \gamma \in [0, 1]$ denote the curvature of the subjective value function for gains and losses, respectively [52]. Kahneman and Tversky [52] experimentally determined that $\beta = \gamma = 0.88$, which is consistent with empirical data. With the extension from the middle of the given linguistic term set to both ends, the absolute deviation between adjacent linguistic subscripts will decrease.

To preserve all the given information and facilitate calculations, the above function can be expanded to $f^* : \bar{H} \rightarrow R^+ (R^+ = \{r | r \geq 0, r \in R\})$, which satisfies $f^*(h_i) = \theta_i$, and is a strictly monotonically increasing and continuous function. Because of its monotonicity, the

mapping from \bar{H} to R^+ is one-to-one, and the inverse function of f^* exists and is denoted by f^{*-1} .

2.2 NSs, SNSs and SNLSs

Definition 2 [19] Let X be a space of points or objects with a generic element in X , denoted by x . A NS A in X is characterized by a truth-membership function $t_A(x)$, an indeterminacy-membership function $i_A(x)$, and a falsity-membership function $f_A(x)$. Furthermore, $t_A(x)$, $i_A(x)$, and $f_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$, that is, $t_A(x) : X \rightarrow]0^-, 1^+[$, $i_A(x) : X \rightarrow]0^-, 1^+[$, and $f_A(x) : X \rightarrow]0^-, 1^+[$. There is no restriction on the sum of $t_A(x)$, $i_A(x)$ and $f_A(x)$; therefore, $0^- \leq \sup t_A(x) + \sup i_A(x) + \sup f_A(x) \leq 3^+$.

Because of the challenge inherent in applying NSs to practical problems, Ye [27] reduced the NS of nonstandard interval numbers into a kind of SNS of standard interval numbers.

Definition 3 [27] Let X be a space of points or objects with a generic element in X , denoted by x . NS A in X is characterized by $t_A(x)$, $i_A(x)$ and $f_A(x)$, which are single subintervals or subsets in the real standard $[0, 1]$, that is, $t_A(x) : X \rightarrow [0, 1]$, $i_A(x) : X \rightarrow [0, 1]$, and $f_A(x) : X \rightarrow [0, 1]$. The sum of $t_A(x)$, $i_A(x)$, and $f_A(x)$ satisfies the condition $0 \leq t_A(x) + i_A(x) + f_A(x) \leq 3$. A simplification of A is denoted by $A = \{(x, t_A(x), i_A(x), f_A(x)) | x \in X\}$, which is called a SNS and is a subclass of NS. If $\|X\|=1$, a SNS will degenerate to a simplified neutrosophic number, denoted by $A = (t_A, i_A, f_A)$.

Definition 4 [45] Let X be a space of points or objects with a generic element in X , denoted by x and let $H = \{h_1, h_2, \dots, h_{2t+1}\}$ be a finite and totally ordered discrete term set, where t is a nonnegative integer. SNLS A in X is characterized as $A = \{(x, h_{\theta(x)}, (t(x), i(x), f(x))) | x \in X\}$, where $h_{\theta(x)} \in H$, $t(x) \in [0, 1]$, $i(x) \in [0, 1]$, and $f(x) \in [0, 1]$, with the condition $0 \leq t(x) + i(x) + f(x) \leq 3$ for any $x \in X$. And $t(x)$, $i(x)$ and $f(x)$ represent, respectively, the degree of truth-membership, indeterminacy-membership, and falsity-membership of element x in X to the linguistic term $h_{\theta(x)}$. In addition, if $\|X\|=1$, a SNLS will degenerate to a SNLN, denoted by $A = \langle h_{\theta}, (t, i, f) \rangle$. A will degenerate to a linguistic term if $t = 1$, $i = 0$, and $f = 0$.

2.3 Operations and comparison rules for SNLNs

Definition 5 [45] Let $a_i = \langle h_{\theta_i}, (t_i, i_i, f_i) \rangle$ and $a_j = \langle h_{\theta_j}, (t_j, i_j, f_j) \rangle$ be any two SNLNs, f^* be a linguistic scale function, and let $\lambda \geq 0$. Then, the following operations of SNLNs can be defined.

1. $a_i \oplus a_j = \langle f^{*-1}(f^*(h_{\theta_i}) + f^*(h_{\theta_j})), \left(\frac{f^*(h_{\theta_i})t_i + f^*(h_{\theta_j})t_j}{f^*(h_{\theta_i}) + f^*(h_{\theta_j})}, \frac{f^*(h_{\theta_i})i_i + f^*(h_{\theta_j})i_j}{f^*(h_{\theta_i}) + f^*(h_{\theta_j})}, \frac{f^*(h_{\theta_i})f_i + f^*(h_{\theta_j})f_j}{f^*(h_{\theta_i}) + f^*(h_{\theta_j})} \right) \rangle$;
2. $a_i \otimes a_j = \langle f^{*-1}(f^*(h_{\theta_i})f^*(h_{\theta_j})), (t_i t_j, i_i + i_j - i_i i_j, f_i + f_j - f_i f_j) \rangle$;
3. $\lambda a_i = \langle f^{*-1}(\lambda f^*(h_{\theta_i})), (t_i, i_i, f_i) \rangle$;
4. $a_i^\lambda = \langle f^{*-1}((f^*(h_{\theta_i}))^\lambda), (t_i^\lambda, 1 - (1 - i_i)^\lambda, 1 - (1 - f_i)^\lambda) \rangle$.

Definition 6 [45] Let $a_i = \langle h_{\theta_i}, (t_i, i_i, f_i) \rangle$ be a SNLN, and let f^* be a linguistic scale function. Then, the score function, accuracy function, and certainty function for a_i can be defined, respectively, as follows:

1. $S(a_i) = f^*(h_{\theta_i})(t_i + 1 - i_i + 1 - f_i)$;
2. $A(a_i) = f^*(h_{\theta_i})(t_i - f_i)$;
3. $C(a_i) = f^*(h_{\theta_i})t_i$.

Definition 7 [45] Let $a_i = \langle h_{\theta_i}, (t_i, i_i, f_i) \rangle$ and $a_j = \langle h_{\theta_j}, (t_j, i_j, f_j) \rangle$ be two SNLNs, and let f^* be a linguistic scale function. Then, the comparison method can be defined as follows:

1. If $S(a_i) > S(a_j)$, then $a_i > a_j$;
2. If $S(a_i) = S(a_j)$ and $A(a_i) > A(a_j)$, then $a_i > a_j$;
3. If $S(a_i) = S(a_j)$, $A(a_i) = A(a_j)$, and $C(a_i) > C(a_j)$, then $a_i > a_j$;
4. If $S(a_i) = S(a_j)$, $A(a_i) = A(a_j)$, and $C(a_i) = C(a_j)$, then $a_i = a_j$.

2.4 The MULTIMOORA method

Brauers and Zavadskas [53] developed the MULTIMOORA method based on the multi-objective optimization by ratio analysis (MOORA) method [54]. Because it consists of three different decision-making methods, MULTIMOORA is more robust than MOORA [55]. Recently the MULTIMOORA method has been employed for various purposes, including robot selection [56], supplier selection [57], healthcare waste treatment technology evaluation and selection [58], and personnel selection [59].

The MOORA method consists of two parts: the ratio system and the reference point approach. The MULTIMOORA method includes internal normalization and treats all objectives as equally significant. In principle, stakeholders' interest in the issue can only give more importance to an objective. They can either multiply the dimensionless number denoting the response on an objective by a significance coefficient or they can decide beforehand to split an objective into different sub-objectives.

Consider matrix $X = [x_{ij}]_{m \times n}$, where x_{ij} denotes the i th alternative of the j th objective ($i = 1, 2, \dots, m; j = 1, 2, \dots, n$).

The ratio system of MOORA defines data normalization by comparing alternative of an objective to all values of the objective, as follows:

$$x_{ij}^* = \frac{x_{ij}}{\sqrt{\sum_i^m x_{ij}^2}}, \tag{1}$$

where x_{ij}^* represents the i th alternative of the j th objective ($i = 1, 2, \dots, m; j = 1, 2, \dots, n$). These numbers usually belong to the interval $[0, 1]$. Indicators are added (if a desirable value of the indicator is a maximum) or subtracted (if a desirable value of the indicator is a minimum); thus, the summarizing index of each alternative is derived in this way:

$$y_i^* = \sum_{j=1}^g x_{ij}^* - \sum_{j=g+1}^n x_{ij}^*, \tag{2}$$

where $g(g = 1, 2, \dots, n)$ denotes the number of objectives to be maximized. Then, according to the summarizing index, the height of the alternative rankings increases with increasing y_i^* .

The reference point for MOORA is on the basis of the ratio system. The maximal objective reference point is established in accordance with the ratio found by employing Eq. (1). The j th coordinate of the reference point or vector can be denoted by $r_j = \max_i x_{ij}^*$ in case of maximization. Every coordinate of this vector denotes the maximum or minimum of a certain objective or indicator. Then, every element of the normalized response matrix is recalculated, and the final ranks are given according to the deviation from the reference point and the Min–Max metric of Tchebycheff:

$$\min_i \left(\max_j |r_j - x_{ij}^*| \right). \tag{3}$$

Brauers and Zavadskas [53] proposed that MOORA be updated by the full multiplicative form method that embodies maximization as well as minimization of the purely multiplicative utility function. This became the full multiplicative form of MULTIMOORA. The overall utility of the i th alternative is represented as dimensionless numbers:

$$U_i = \frac{A_i}{B_i}, \tag{4}$$

where $A_i = \prod_{j=1}^g x_{ij}$ ($i = 1, 2, \dots, m$) represents the product of the objectives of the i th alternative to be maximized with $g = 1, 2, \dots, n$ being the number of objectives to be maximized; meanwhile, $B_i = \prod_{j=g+1}^n x_{ij}$ represents the product of objectives of the i th alternative to be minimized with $n - g$ being the number of objectives to be minimized. In this way, MULTIMOORA summarizes MOORA’s ratio system and reference point in the full multiplicative form. The dominance theory was then proposed to unite the three ranks provided by the respective parts of MULTIMOORA into a single one [60].

3 BM aggregation operators and distance measures for SNLNs

This section presents two kinds of BM aggregation operators, the SNLNWBM operator and the SNLNWGBM operator. This section also defines a family of distance measures for SNLNs.

3.1 Simplified neutrosophic linguistic BM aggregation operators

Definition 8 [45] Let $p, q \geq 0$, $a_i = \langle h_{\theta_i}, (t_i, i_i, f_i) \rangle$ ($i = 1, 2, \dots, n$) be a collection of SNLNs, and let $\text{SNLNWB}_{\omega}^{p,q} : \Omega^n \rightarrow \Omega$. If

$$\text{SNLNWB}_{\omega}^{p,q}(a_1, a_2, \dots, a_n) = \left(\bigoplus_{\substack{i,j=1 \\ i \neq j}}^n \frac{\omega_i \omega_j}{1 - \omega_i} (a_i^p \otimes a_j^q) \right)^{\frac{1}{p+q}}, \tag{5}$$

where Ω is the set of all SNLNs and $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ is the weight vector of a_i ($i = 1, 2, \dots, n$), $\omega_i \in [0, 1]$ and $\sum_{i=1}^n \omega_i = 1$. Then $\text{SNLNWB}_{\omega}^{p,q}$ is called the SNLNWBM operator.

According to the operations of SNLNs described in Definition 6, the following results can be obtained.

Theorem 1 [45] Let $p, q \geq 0$, $a_i = \langle h_{\theta_i}, (t_i, i_i, f_i) \rangle$ ($i = 1, 2, \dots, n$) be a collection of SNLNs, and let $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ be the weight vector of a_i ($i = 1, 2, \dots, n$), $\omega_i \in [0, 1]$ and $\sum_{i=1}^n \omega_i = 1$. Then the aggregated result by using Eq. (5) is also a SNLN.

$$\begin{aligned}
 \text{SNLNWB}_{\omega}^{p,q}(a_1, a_2, \dots, a_n) &= \left(\bigoplus_{\substack{i,j=1 \\ i \neq j}}^n \frac{\omega_i \omega_j}{1 - \omega_i} (a_i^p \otimes a_j^q) \right)^{\frac{1}{p+q}} \\
 &= \left\langle f^{*-1} \left(\left(\prod_{\substack{i,j=1 \\ i \neq j}}^n \left(\frac{\omega_i \omega_j}{1 - \omega_i} (f^*(h_{\theta_i}))^p (f^*(h_{\theta_j}))^q \right) \right)^{\frac{1}{p+q}} \right), \left(\left(\frac{\sum_{\substack{i,j=1 \\ i \neq j}}^n \left(\frac{\omega_i \omega_j}{1 - \omega_i} (f^*(h_{\theta_i}))^p (f^*(h_{\theta_j}))^q t_i^p t_j^q \right)}{\sum_{\substack{i,j=1 \\ i \neq j}}^n \left(\frac{\omega_i \omega_j}{1 - \omega_i} (f^*(h_{\theta_i}))^p (f^*(h_{\theta_j}))^q \right)} \right)^{\frac{1}{p+q}} \right. \right. \\
 &\quad \left. \left. 1 - \left(1 - \frac{\sum_{\substack{i,j=1 \\ i \neq j}}^n \left(\frac{\omega_i \omega_j}{1 - \omega_i} (f^*(h_{\theta_i}))^p (f^*(h_{\theta_j}))^q (1 - (1 - i_i)^p (1 - i_j)^q \right)}{\sum_{\substack{i,j=1 \\ i \neq j}}^n \left(\frac{\omega_i \omega_j}{1 - \omega_i} (f^*(h_{\theta_i}))^p (f^*(h_{\theta_j}))^q \right)} \right)^{\frac{1}{p+q}} \right. \right. \right. \\
 &\quad \left. \left. 1 - \left(1 - \frac{\sum_{\substack{i,j=1 \\ i \neq j}}^n \left(\frac{\omega_i \omega_j}{1 - \omega_i} (f^*(h_{\theta_i}))^p (f^*(h_{\theta_j}))^q (1 - (1 - f_i)^p (1 - f_j)^q \right)}{\sum_{\substack{i,j=1 \\ i \neq j}}^n \left(\frac{\omega_i \omega_j}{1 - \omega_i} (f^*(h_{\theta_i}))^p (f^*(h_{\theta_j}))^q \right)} \right)^{\frac{1}{p+q}} \right) \right) \right\rangle. \tag{6}
 \end{aligned}$$

Equation (6) can be proven true through mathematical induction on n referring to Ref. [45]. Moreover, the SNLNWBM operator can satisfy the following properties. The detailed proofs are shown in [45].

Theorem 2 (Commutativity) *Let $(a'_1, a'_2, \dots, a'_n)$ be any permutation of (a_1, a_2, \dots, a_n) . If $p = q$, then*

$$\text{SNLNWB}_{\omega}^{p,q}(a'_1, a'_2, \dots, a'_n) = \text{SNLNWB}_{\omega}^{p,q}(a_1, a_2, \dots, a_n).$$

Theorem 3 (Idempotency) *Let $a_i = a (i = 1, 2, \dots, n)$. Then $\text{SNLNWB}_{\omega}^{p,q}(a_1, a_2, \dots, a_n) = a$.*

Theorem 4 (Monotonicity) *Let $a_i = \langle h_{a_i}, (t_{a_i}, i_{a_i}, f_{a_i}) \rangle (i = 1, 2, \dots, n)$ and $b_i = \langle h_{b_i}, (t_{b_i}, i_{b_i}, f_{b_i}) \rangle$ be two collections of SNLNs. If $h_{a_i} \geq h_{b_i}, t_{a_i} \geq t_{b_i}, i_{a_i} \leq i_{b_i}$ and $f_{a_i} \leq f_{b_i}$ for all i , then $\text{SNLNWB}_{\omega}^{p,q}(a_1, a_2, \dots, a_n) \geq \text{SNLNWB}_{\omega}^{p,q}(b_1, b_2, \dots, b_n)$.*

Theorem 5 (Boundedness) *Let $a_i = \langle h_{a_i}, (t_{a_i}, i_{a_i}, f_{a_i}) \rangle (0, 1, 2, \dots, n)$ be a collection of SNLNs, and let $a = \left\langle \min_i \{h_{a_i}\}, \left(\min_i \{t_{a_i}\}, \max_i \{i_{a_i}\}, \max_i \{f_{a_i}\} \right) \right\rangle$, $b = \left\langle \max_i \{h_{a_i}\}, \left(\max_i \{t_{a_i}\}, \min_i \{i_{a_i}\}, \min_i \{f_{a_i}\} \right) \right\rangle$. Then $a \leq \text{SNLNWB}_{\omega}^{p,q}(a_1, a_2, \dots, a_n) \leq b$.*

Definition 9 Let $p, q \geq 0, a_i = \langle h_{\theta_i}, (t_i, i_i, f_i) \rangle (i = 1, 2, \dots, n)$ be a collection of SNLNs, and let $\text{SNLNWGB}_{\omega}^{p,q} : \Omega^n \rightarrow \Omega$. If

$$\text{SNLNWGB}_{\omega}^{p,q}(a_1, a_2, \dots, a_n) = \frac{1}{p+q} \bigotimes_{\substack{i,j=1 \\ i \neq j}}^n (pa_i \oplus qa_j)^{\frac{\omega_i \omega_j}{1 - \omega_i}}, \tag{7}$$

where Ω is the set of all SNLNs and $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ is the weight vector of $a_i (i = 1, 2, \dots, n)$, $\omega_i \in [0, 1]$ and $\sum_{i=1}^n \omega_i = 1$. Then $\text{SNLNWGB}_{\omega}^{p,q}$ is called the SNLNWGBM operator.

Theorem 6 *Let $p, q \geq 0, a_i = \langle h_{\theta_i}, (t_i, i_i, f_i) \rangle (i = 1, 2, \dots, n)$ be a collection of SNLNs, and let $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ be the weight vector of $a_i (i = 1, 2, \dots, n)$, $\omega_i \in [0, 1]$ and $\sum_{i=1}^n \omega_i = 1$. Then the aggregated result by using Eq. (7) is also a SNLN.*

$$\begin{aligned}
 \text{SNLNWGB}_{\omega}^{p,q}(a_1, a_2, \dots, a_n) &= \frac{1}{p+q} \bigotimes_{\substack{i,j=1 \\ i \neq j}}^n (pa_i \oplus qa_j)^{\frac{\omega_i \omega_j}{1 - \omega_i}} \\
 &= \left\langle f^{*-1} \left(\frac{1}{p+q} \prod_{\substack{i,j=1 \\ i \neq j}}^n (pf^*(h_{\theta_i}) + qf^*(h_{\theta_j}))^{\frac{\omega_i \omega_j}{1 - \omega_i}} \right), \right. \\
 &\quad \left(\prod_{\substack{i,j=1 \\ i \neq j}}^n \left(\frac{pf^*(h_{\theta_i})t_i + qf^*(h_{\theta_j})t_j}{pf^*(h_{\theta_i}) + qf^*(h_{\theta_j})} \right)^{\frac{\omega_i \omega_j}{1 - \omega_i}} \right. \\
 &\quad \left. 1 - \prod_{\substack{i,j=1 \\ i \neq j}}^n \left(1 - \frac{pf^*(h_{\theta_i})i_i + qf^*(h_{\theta_j})i_j}{pf^*(h_{\theta_i}) + qf^*(h_{\theta_j})} \right)^{\frac{\omega_i \omega_j}{1 - \omega_i}} \right. \\
 &\quad \left. 1 - \prod_{\substack{i,j=1 \\ i \neq j}}^n \left(1 - \frac{pf^*(h_{\theta_i})f_i + qf^*(h_{\theta_j})f_j}{pf^*(h_{\theta_i}) + qf^*(h_{\theta_j})} \right)^{\frac{\omega_i \omega_j}{1 - \omega_i}} \right) \right\rangle. \tag{8}
 \end{aligned}$$

Similar to Theorem 1, Theorem 6 can be proven true using mathematical induction on n . Moreover, the SNLNWGBM operator can also satisfy the properties of

commutativity, idempotency, monotonicity, and boundedness. The proofs are omitted here.

The parameters p and q have no influence on the aggregation results using the SNLNWGBM operator when $p = q$.

3.2 Distance measures between two SNLNs

Definition 10 [61] Let $a_i = \langle h_{\theta_i}, (t_i, i_i, f_i) \rangle$ and $a_j = \langle h_{\theta_j}, (t_j, i_j, f_j) \rangle$ be two SNLNs, and let f^* be a linguistic scale function. Then, the generalized distance measure between a_i and a_j can be defined as follows:

$$d(a_i, a_j) = \left(\frac{1}{3} \left(|f^*(h_{\theta_i})t_i - f^*(h_{\theta_j})t_j|^\lambda + |f^*(h_{\theta_i})(1 - i_i) - f^*(h_{\theta_j})(1 - i_j)|^\lambda + |f^*(h_{\theta_i})(1 - f_i) - f^*(h_{\theta_j})(1 - f_j)|^\lambda \right) \right)^{\frac{1}{\lambda}} \tag{9}$$

When $\lambda = 1, 2$, Eq. (5) is reduced to the Hamming distance and Euclidean distance, respectively.

Theorem 7 [61] Let $a_i = \langle h_{\theta_i}, (t_i, i_i, f_i) \rangle$, $a_j = \langle h_{\theta_j}, (t_j, i_j, f_j) \rangle$, and $a_k = \langle h_{\theta_k}, (t_k, i_k, f_k) \rangle$ be any three SNLNs, and let f^* be a linguistic scale function. Then the distance measure in Definition 10 satisfies the following properties:

1. $d(a_i, a_j) \geq 0$ (If and only if $a_i = a_j$, then $d(a_i, a_j) = 0$);
2. $d(a_i, a_j) = d(a_j, a_i)$;
3. If $h_{\theta_i} \leq h_{\theta_j} \leq h_{\theta_k}$, $t_i \leq t_j \leq t_k$, $i_i \geq i_j \geq i_k$ and $f_i \geq f_j \geq f_k$, then $d(a_i, a_j) \leq d(a_i, a_k)$ and $d(a_j, a_k) \leq d(a_i, a_k)$.

Proof Clearly, $d(a_i, a_j)$ satisfies Properties (1) and (2). The proof of Property (3) is shown below.

Since $h_{\theta_i} \leq h_{\theta_j} \leq h_{\theta_k}$, $t_i \leq t_j \leq t_k$, $i_i \geq i_j \geq i_k$, $f_i \geq f_j \geq f_k$, and f^* is a strictly monotonically increasing and continuous function, then $f^*(h_{\theta_i}) \leq f^*(h_{\theta_j}) \leq f^*(h_{\theta_k})$, and the following inequalities can be obtained.

$$\begin{aligned} f^*(h_{\theta_i})t_i &\leq f^*(h_{\theta_j})t_j \leq f^*(h_{\theta_k})t_k \\ \Rightarrow |f^*(h_{\theta_i})t_i - f^*(h_{\theta_j})t_j|^\lambda &\leq |f^*(h_{\theta_i})t_i - f^*(h_{\theta_k})t_k|^\lambda, \\ f^*(h_{\theta_i})(1 - i_i) &\leq f^*(h_{\theta_j})(1 - i_j) \leq f^*(h_{\theta_k})(1 - i_k) \\ \Rightarrow |f^*(h_{\theta_i})(1 - i_i) - f^*(h_{\theta_j})(1 - i_j)|^\lambda &\leq |f^*(h_{\theta_i})(1 - i_i) - f^*(h_{\theta_k})(1 - i_k)|^\lambda \end{aligned}$$

and

$$\begin{aligned} f^*(h_{\theta_i})(1 - f_i) &\leq f^*(h_{\theta_j})(1 - f_j) \leq f^*(h_{\theta_k})(1 - f_k) \\ \Rightarrow |f^*(h_{\theta_i})(1 - f_i) - f^*(h_{\theta_j})(1 - f_j)|^\lambda &\leq |f^*(h_{\theta_i})(1 - f_i) - f^*(h_{\theta_k})(1 - f_k)|^\lambda. \end{aligned}$$

Then,

$$\begin{aligned} &\left(\frac{1}{3} \left(|f^*(h_{\theta_i})t_i - f^*(h_{\theta_j})t_j|^\lambda + |f^*(h_{\theta_i})(1 - i_i) - f^*(h_{\theta_j})(1 - i_j)|^\lambda + |f^*(h_{\theta_i})(1 - f_i) - f^*(h_{\theta_j})(1 - f_j)|^\lambda \right) \right)^{\frac{1}{\lambda}} \\ &\leq \left(\frac{1}{3} \left(|f^*(h_{\theta_i})t_i - f^*(h_{\theta_k})t_k|^\lambda + |f^*(h_{\theta_i})(1 - i_i) - f^*(h_{\theta_k})(1 - i_k)|^\lambda + |f^*(h_{\theta_i})(1 - f_i) - f^*(h_{\theta_k})(1 - f_k)|^\lambda \right) \right)^{\frac{1}{\lambda}}. \end{aligned}$$

Thus, $d(a_i, a_j) \leq d(a_i, a_k)$. The sentence $d(a_j, a_k) \leq d(a_i, a_k)$ can be proven in a similar way. The proof of Theorem 1 is thus completed.

Similarly, the Hausdorff distance measure can be applied to SNLNs. For two SNLNs a_i and a_j , the Hausdorff distance measure can be defined as follows:

$$d_{\text{haud}}(a_i, a_j) = \max \left(|f^*(h_{\theta_i})t_i - f^*(h_{\theta_j})t_j|, |f^*(h_{\theta_i})(1 - i_i) - f^*(h_{\theta_j})(1 - i_j)|, |f^*(h_{\theta_i})(1 - f_i) - f^*(h_{\theta_j})(1 - f_j)| \right). \tag{10}$$

In addition, several hybrid distance measures can be developed by combining the above distance measure.

1. The hybrid Hamming distance between a_i and a_j :

$$\begin{aligned} d_{\text{hhd}}(a_i, a_j) &= \frac{1}{2} \left(\frac{1}{3} \left(|f^*(h_{\theta_i})t_i - f^*(h_{\theta_j})t_j| + |f^*(h_{\theta_i})(1 - i_i) - f^*(h_{\theta_j})(1 - i_j)| + |f^*(h_{\theta_i})(1 - f_i) - f^*(h_{\theta_j})(1 - f_j)| \right) \right. \\ &\quad \left. + \max \left(|f^*(h_{\theta_i})t_i - f^*(h_{\theta_j})t_j|, |f^*(h_{\theta_i})(1 - i_i) - f^*(h_{\theta_j})(1 - i_j)|, |f^*(h_{\theta_i})(1 - f_i) - f^*(h_{\theta_j})(1 - f_j)| \right) \right). \end{aligned} \tag{11}$$

2. The hybrid Euclidean distance between a_i and a_j :

$$\begin{aligned} d_{\text{hed}}(a_i, a_j) &= \left(\frac{1}{2} \left(\frac{1}{3} \left(|f^*(h_{\theta_i})t_i - f^*(h_{\theta_j})t_j|^2 + |f^*(h_{\theta_i})(1 - i_i) - f^*(h_{\theta_j})(1 - i_j)|^2 + |f^*(h_{\theta_i})(1 - f_i) - f^*(h_{\theta_j})(1 - f_j)|^2 \right) \right. \right. \\ &\quad \left. \left. + \max \left(|f^*(h_{\theta_i})t_i - f^*(h_{\theta_j})t_j|^2, |f^*(h_{\theta_i})(1 - i_i) - f^*(h_{\theta_j})(1 - i_j)|^2, |f^*(h_{\theta_i})(1 - f_i) - f^*(h_{\theta_j})(1 - f_j)|^2 \right) \right) \right)^{\frac{1}{2}}. \end{aligned} \tag{12}$$

3. The generalized hybrid distance between a_i and a_j :

$$\begin{aligned} d_{\text{ghd}}(a_i, a_j) &= \left(\frac{1}{2} \left(\frac{1}{3} \left(|f^*(h_{\theta_i})t_i - f^*(h_{\theta_j})t_j|^\lambda + |f^*(h_{\theta_i})(1 - i_i) - f^*(h_{\theta_j})(1 - i_j)|^\lambda + |f^*(h_{\theta_i})(1 - f_i) - f^*(h_{\theta_j})(1 - f_j)|^\lambda \right) \right. \right. \\ &\quad \left. \left. + \max \left(|f^*(h_{\theta_i})t_i - f^*(h_{\theta_j})t_j|^\lambda, |f^*(h_{\theta_i})(1 - i_i) - f^*(h_{\theta_j})(1 - i_j)|^\lambda, |f^*(h_{\theta_i})(1 - f_i) - f^*(h_{\theta_j})(1 - f_j)|^\lambda \right) \right) \right)^{\frac{1}{\lambda}}. \end{aligned} \tag{13}$$

When $\lambda = 1, 2$, Eq. (13) is reduced to Eqs. (11) and (12), respectively.

Like Eq. (5), it can be proven that Eqs. (10–13) satisfy the properties in Theorem 7.

Example 1 Assume $a_1 = \langle h_4, (0.7, 0.4, 0.6) \rangle$ and $a_2 = \langle h_5, (0.8, 0.3, 0.5) \rangle$ are two SNLNs, and let $\lambda = 2, t = 3$ and $f^* = f_1^*$. Then, the following results can be obtained.

$$d_{gd}(a_1, a_2) = 0.1625, d_{haud}(a_1, a_2) = 0.1833 \text{ and } d_{ghd}(a_1, a_2) = 0.1732.$$

4 A simplified neutrosophic linguistic MCDM approach based on MULTIMOORA

This section introduces an extended simplified neutrosophic linguistic MULTIMOORA approach based on the proposed SNLNBWM and SNLNBGBM operators as well as distance measures.

Consider a MCDM problem within the neutrosophic linguistic context. Let $A = \{a_1, a_2, \dots, a_m\}$ be a discrete set consisting of m alternatives, and let $C = \{c_1, c_2, \dots, c_n\}$ be a set consisting of n criteria. Assume that $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ is the weight vector of $c_j (j = 1, 2, \dots, n)$, and the weight information is unknown. The assessment information of $a_i (i = 1, 2, \dots, m)$ in terms of $c_j (j = 1, 2, \dots, n)$ is denoted by

$$B = [b_{ij}]_{m \times n} = [\langle h_{\theta_{ij}}, (t_{ij}, i_{ij}, f_{ij}) \rangle]_{m \times n},$$

where $b_{ij} = \langle h_{\theta_{ij}}, (t_{ij}, i_{ij}, f_{ij}) \rangle$ take the form of a SNLN.

Based on the above analysis, the simplified neutrosophic linguistic MCDM approach based on MULTIMOORA can be summarized as follows:

Step 1: Determine the criterion weights.

Motivated by the closeness coefficient of TOPSIS [62] and the variation coefficient method [63], a novel objective weight determination method can be developed based on the simplified neutrosophic distance measures.

- a. Determine the optimistic and pessimistic evaluation values for each criterion.

For each criterion, the optimistic and pessimistic evaluation values are denoted as follows:

$$\begin{aligned} \text{Optimistic values : } B^+ &= (b_1^+, b_1^+, \dots, b_n^+) \text{ and pessimistic values} \\ \text{: } B^- &= (b_1^-, b_1^-, \dots, b_n^-), \end{aligned} \tag{14}$$

For the sake of convenience, the maximal utopian reference point (MURP) can be chosen as a substitute for the maximal objective reference point. The MURPs can be defined referring to Ye [43], where

$$\begin{aligned} b_j^+ &= \begin{cases} \langle h_{2t+1}, (1, 0, 0) \rangle, j \in J_1 \\ \langle h_1, (0, 1, 1) \rangle, j \in J_2 \end{cases} \text{ and } b_j^- \\ &= \begin{cases} \langle h_1, (0, 1, 1) \rangle, j \in J_1 \\ \langle h_{2t+1}, (1, 0, 0) \rangle, j \in J_2 \end{cases}, \end{aligned} \tag{15}$$

in which J_1 and J_2 represent the maximizing and minimizing criteria, respectively.

- b. Calculate the overall distance measures between the criterion values and the optimistic/pessimistic values.

Utilize the distance measures described in Sect. 3.2 and calculate the overall distances between each criterion value and the optimistic/pessimistic values, respectively.

$$d_j^+ = \sum_{i=1}^m d(b_{ij}, b_j^+) \text{ and } d_j^- = \sum_{i=1}^m d(b_{ij}, b_j^-). \tag{16}$$

- c. Determine the dispersion measure of each criterion.

According to the distance-based method, the measure of dispersion of criterion $c_j (j = 1, 2, \dots, n)$ is expressed as follows:

$$\eta_j = \frac{d_j^+}{d_j^+ + d_j^-}. \tag{17}$$

The larger the value of $\eta_j (j = 1, 2, \dots, n)$ is, the larger of the dispersion measure and accordingly the more important of criterion c_j will be. This is consistent with the core principle of criterion weight determination.

- d. Obtain the normalized criterion weights.

The normalized weight of $c_j (j = 1, 2, \dots, n)$ can be obtained based on the dispersion measure.

$$\omega_j = \frac{\eta_j}{\sum_j^n \eta_j}. \tag{18}$$

Step 2: Construct the ratio system.

Since all the evaluation values are SNLNs, the ratio system can be constructed by adding and subtracting evaluation values for the maximizing and minimizing criteria, respectively, according to Eq. (6).

$$\begin{aligned} RS_i &= \left(\bigoplus_{\substack{i_1, j_1=1 \\ i_1 \neq j_1 \\ i_1, j_1 \in J_1}}^n \frac{\omega_{i_1} \omega_{j_1}}{1 - \omega_{i_1}} (b_{i_1}^p \otimes b_{j_1}^q) \right)^{\frac{1}{p+q}} \\ &\quad - \left(\bigoplus_{\substack{i_2, j_2=1 \\ i_2 \neq j_2 \\ i_2, j_2 \in J_2}}^n \frac{\omega_{i_2} \omega_{j_2}}{1 - \omega_{i_2}} (b_{i_2}^p \otimes b_{j_2}^q) \right)^{\frac{1}{p+q}}, \end{aligned} \tag{19}$$

where the summarizing index $RS_i(i = 1, 2, \dots, m)$ denotes the overall utility of alternative a_i . The alternatives are then ranked in descending order according to the scores of RS_i referring to Definition 6 and Definition 7.

Step 3: Derive the reference point.

According to Eq. (15), the simplified neutrosophic linguistic distance measures defined in Sect. 3.2 can be employed to identify the maximal deviation from the MURP for each alternative.

$$RP_i = \max_j \left(\omega_j d(b_{ij}, b_j^+) \right). \tag{20}$$

Then, the alternatives are ranked in ascending order based on the maximal deviances in Eq. (20).

Step 4: Construct the full multiplicative form.

The full multiplicative form can be established by calculating the overall utility $U_i (i = 1, 2, \dots, m)$ of alternative a_i as follows:

$$U_i = \frac{S(A_i)}{S(B_i)}, \tag{21}$$

where

$$A_i = \frac{1}{p+q} \otimes_{\substack{i_1, j_1=1 \\ i_1 \neq j_1 \\ i_1, j_1 \in J_1}}^n (pb_{i_1} \oplus qb_{j_1})^{\frac{\omega_{i_1} \omega_{j_1}}{1-\omega_{i_1}}}$$

and

$$B_i = \frac{1}{p+q} \otimes_{\substack{i_2, j_2=1 \\ i_2 \neq j_2 \\ i_2, j_2 \in J_2}}^n (pb_{i_2} \oplus qb_{j_2})^{\frac{\omega_{i_2} \omega_{j_2}}{1-\omega_{i_2}}}$$

according to Eq. (8). $S(A_i)$ and $S(B_i)$ are the scores of the aggregated evaluation values. The alternatives are then ranked based on descending U_i according to Definitions 6 and 7.

Step 5: Rank all the alternatives and select the best one(s).

The dominance theory is employed to summarize the three ranks provided by the respective parts of MULTIMOORA.

5 Illustrative example

The following example applies the proposed simplified neutrosophic linguistic MULTIMOORA approach to solve the MCDM problem of choosing an appropriate enterprise resource planning (ERP) system. The example demonstrates the application, validity, and effectiveness.

China’s economy has improved dramatically following economic reforms and the opening of markets. The use of a large amount of machinery has facilitated infrastructure

improvements, which help promote the overall investment environment. Consequently, an increasing number of enterprises both at home and abroad actively participate in the Chinese economy, creating fierce market competition. Numerous companies plan to implement ERP projects to improve operational efficiency and lower costs. Feasibility study involving in choosing ERP systems may involve more than one episode of MCDM, because various factors must be considered in order to guarantee the proper functioning of different systems.

ABC Machinery Manufacturing Co., Ltd. is a famous enterprise whose main business is developing and producing construction machinery transporters for use in China. The top managers of ABC plan to carry out an ERP project in order to enhance enterprise competitiveness. After thorough investigation, four ERP systems, denoted by $\{a_1, a_2, a_3, a_4\}$, are taken into account. Many criteria reflect the performance of an ERP system, and four criteria are chosen based on the experience of the chief information officer (CIO). These criteria include c_1 , function and technology; c_2 , strategic fitness; c_3 , vendor’s ability; and c_4 , vendor’s reputation.

The DMs, including the CIO and executive managers from different departments, were gathered to determine the evaluation information. The linguistic term set

$$H = \{s_1 = \text{very poor}, s_2 = \text{poor}, s_3 = \text{medium poor}, s_4 = \text{fair}, s_5 = \text{medium good}, s_6 = \text{good}, s_7 = \text{very good}\}$$

is used here to categorize the evaluation information in the form of SNLNs (Table 1).

5.1 Illustration of the proposed approach

The main procedures for obtaining the optimal ranking of alternatives are as follows (Let $f^* = f_1^* = (i - 1)/6, \lambda = 2, p = 1$ and $q = 2$):

Step 1: Determine the criterion weights.

Since all criteria are of the maximizing type, the MURPs are $b_j^+ = \langle h_7, (1, 0, 0) \rangle$ and $b_j^- = \langle h_1, (1, 0, 0) \rangle$. The overall distance measures can be calculated based on Eq. (13), and the normalized weight vector can be obtained by using Eqs. (16–18).

$$d_1^+ = 2.0951, d_2^+ = 2.2728, d_3^+ = 2.3818 \text{ and } d_4^+ = 2.2795. \\ d_1^- = 2.3285, d_2^- = 2.0285, d_3^- = 1.8895 \text{ and } d_4^- = 1.9867. \\ \omega = (0.2262, 0.2523, 0.2663, 0.2552).$$

Step 2: Construct the ratio system.

Construct the ratio system by adding the evaluation values for the maximizing criteria according to Eq. (6). Then, rank all alternatives in descending order of the scores of $RS_i (i = 1, 2, 3, 4)$ according to Definitions 6 and 7.

Table 1 Evaluation information

	c_1	c_2	c_3	c_4
a_1	$\langle h_5, (0.6, 0.2, 0.3) \rangle$	$\langle h_6, (0.6, 0.2, 0.3) \rangle$	$\langle h_5, (0.6, 0.2, 0.3) \rangle$	$\langle h_4, (0.6, 0.3, 0.3) \rangle$
a_2	$\langle h_6, (0.7, 0.2, 0.3) \rangle$	$\langle h_4, (0.7, 0.2, 0.2) \rangle$	$\langle h_4, (0.6, 0.4, 0.2) \rangle$	$\langle h_5, (0.7, 0.2, 0.2) \rangle$
a_3	$\langle h_6, (0.5, 0.2, 0.1) \rangle$	$\langle h_5, (0.6, 0.2, 0.3) \rangle$	$\langle h_5, (0.5, 0.3, 0.2) \rangle$	$\langle h_5, (0.5, 0.2, 0.2) \rangle$
a_4	$\langle h_5, (0.5, 0.3, 0.2) \rangle$	$\langle h_5, (0.5, 0.2, 0.2) \rangle$	$\langle h_5, (0.7, 0.2, 0.2) \rangle$	$\langle h_6, (0.6, 0.4, 0.2) \rangle$

Table 2 Ranking results according to MULTIMOORA

	The ratio system	The reference point	The full multiplicative form	MULTIMOORA
a_1	3	3	3	3
a_2	4	2	4	4
a_3	1	4	1	1
a_4	2	1	2	2

Table 3 Ranking results with different f^* , λ , p , and q values

		$p = 1, q = 0$	$p = q = 1$	$p = q = 2$	$p = q = 5$	$p = 1, q = 2$	$p = 2, q = 5$
f_1^*	$\lambda = 1$	a_3, a_4, a_1, a_2	a_3, a_4, a_1, a_2	a_3, a_4, a_1, a_2	a_3, a_4, a_1, a_2	a_3, a_4, a_1, a_2	a_3, a_4, a_2, a_1
	$\lambda = 2$	a_3, a_4, a_1, a_2	a_3, a_4, a_1, a_2	a_3, a_4, a_1, a_2	a_3, a_4, a_1, a_2	a_3, a_4, a_1, a_2	a_3, a_4, a_1, a_2
	$\lambda = 5$	a_4, a_3, a_1, a_2	a_4, a_3, a_1, a_2	a_4, a_3, a_1, a_2	a_4, a_3, a_1, a_2	a_4, a_3, a_1, a_2	a_4, a_3, a_1, a_2
f_2^*	$\lambda = 1$	a_3, a_4, a_2, a_1	a_3, a_4, a_2, a_1	a_3, a_4, a_2, a_1	a_3, a_4, a_2, a_1	a_3, a_4, a_2, a_1	a_3, a_4, a_2, a_1
	$\lambda = 2$	a_3, a_4, a_2, a_1	a_3, a_4, a_2, a_1	a_3, a_4, a_2, a_1	a_3, a_4, a_2, a_1	a_3, a_4, a_2, a_1	a_3, a_4, a_2, a_1
	$\lambda = 5$	a_4, a_3, a_1, a_2	a_4, a_3, a_1, a_2	a_4, a_3, a_1, a_2	a_4, a_3, a_1, a_2	a_4, a_3, a_2, a_1	a_4, a_3, a_1, a_2
f_3^*	$\lambda = 1$	a_3, a_4, a_1, a_2	a_3, a_4, a_1, a_2	a_3, a_4, a_1, a_2	a_3, a_4, a_2, a_1	a_3, a_4, a_1, a_2	a_3, a_4, a_2, a_1
	$\lambda = 2$	a_3, a_4, a_1, a_2	a_3, a_4, a_1, a_2	a_3, a_4, a_1, a_2	a_3, a_4, a_2, a_1	a_3, a_4, a_1, a_2	a_3, a_4, a_2, a_1
	$\lambda = 5$	a_4, a_3, a_1, a_2	a_4, a_3, a_1, a_2	a_4, a_3, a_1, a_2	a_4, a_3, a_1, a_2	a_4, a_3, a_1, a_2	a_4, a_3, a_1, a_2

$$RS_1 = \langle h_{5.0138}, (0.6, 0.2171, 0.3) \rangle, RS_2 = \langle h_{4.7681}, (0.6813, 0.2369, 0.2346) \rangle,$$

$$RS_3 = \langle h_{5.2543}, (0.5236, 0.2229, 0.1923) \rangle \text{ and } RS_4 = \langle h_{5.2543}, (0.5781, 0.2828, 0.2) \rangle.$$

$$S(RS_1) = 0.4645, S(RS_2) = 0.4626, S(RS_3) = 0.4983 \text{ and } S(RS_4) = 0.4952.$$

$$A_1 = \langle h_{4.9743}, (0.6, 0.2214, 0.3) \rangle,$$

$$A_2 = \langle h_{4.7129}, (0.6775, 0.2459, 0.2302) \rangle,$$

$$A_3 = \langle h_{5.2412}, (0.5235, 0.2246, 0.1981) \rangle \text{ and } A_4 = \langle h_{5.2412}, (0.5736, 0.2813, 0.2) \rangle.$$

$$U_1 = S(A_1) = 0.4589, U_2 = S(A_2) = 0.4541, U_3 = S(A_3) = 0.4950 \text{ and } U_4 = S(A_4) = 0.4930.$$

Step 3: Derive the reference point.

As described in Eq. (15), identify the maximal deviation RP_i ($i = 1, 2, 3, 4$) from the MURPs for each alternative using Eq. (13). Then rank all alternatives in ascending order of the maximal deviances.

$$RP_1 = 0.5711, RP_2 = 0.5585, RP_3 = 0.5746 \text{ and } RP_4 = 0.5530.$$

Step 4: Construct the full multiplicative form.

Calculate the overall evaluation values using Eq. (8), and obtain the scores $S(A_i)$ ($i = 1, 2, 3, 4$) for all maximizing criteria. Then, rank all alternatives in descending order of U_i ($i = 1, 2, 3, 4$) according to Definitions 6 and 7.

Step 5: Rank all the alternatives and select the best one(s).

Utilize the dominance theory to summarize the three ranks determined above. The results are shown in Table 2. The last column in Table 2 presents the final rankings.

According to the multi-criteria evaluation, the ERP system a_3 should be selected as the optimal system.

In order to investigate the influence of semantics, distance measure parameters, and BM operator on the ranking results, different values for f^* , λ , p and q can be taken into account. The results are shown in Table 3. Due to limited space, “ a_i, a_j, a_k, a_l ” denotes “ $a_i \succ a_j \succ a_k \succ a_l$ ”.

Table 4 Ranking results of different methods

Methods	Ranking results
Ye's method [43]	$a_2 \succ a_4 \succ a_1 \succ a_3$
Tian et al.'s method [45]	
Method with f_1^* and $p = 1, q = 2$	$a_3 \succ a_4 \succ a_1 \succ a_2$
Method with f_2^* and $p = 1, q = 2$	$a_3 \succ a_4 \succ a_2 \succ a_1$
Method with f_3^* and $p = 1, q = 2$	$a_3 \succ a_4 \succ a_1 \succ a_2$
The proposed approach with $f_1^*, \lambda = 2$ and $p = 1, q = 2$	$a_3 \succ a_4 \succ a_1 \succ a_2$

Table 3 shows that the ranking results vary as the distance parameter λ changes under different semantic situations. The best alternative is a_3 when $\lambda = 1, 2$. However, a_4 is a better alternative when $\lambda = 5$. The rankings of alternatives may change slightly with the same λ, p , and q values but different semantics. Furthermore, the parameters p and q in the SNLNWBM and SNLNWGBM operators play a key role in affecting the ranking of alternatives.

The above analysis suggests that the semantics, distance measure parameters, and simplified neutrosophic linguistic BM aggregation operators indeed play a vital role in solving the MCDM problem. Since selecting an optimal ERP system involves experts from different fields' assessing a group of alternatives with multiple criteria, it is reasonable and necessary to take these factors into consideration.

5.2 Comparative analysis and discussion

This subsection describes a comparative study conducted to validate the flexibility and feasibility of the proposed approach. Different methods are employed to solve the same MCDM problem with SNLNs.

1. Ye's method [43] contains two major phases. First, the weighted evaluation information is derived based on the operations of SNLNs, and then, an extended TOPSIS method is used to rank all the alternatives. The criterion weight information is obtained using the proposed method with $f^* = f_1^*$ and $\lambda = 2$.
2. In Tian et al.'s method [45], a SNLNWBM operator based on the linguistic scale function is employed to aggregate the evaluation information, after which the ranking of alternatives is determined using the score function. The criterion weight information is obtained using the proposed method with $\lambda = 2$. The ranking results obtained by the different methods are summarized in Table 4.

Table 4 reveals differences between the ranking results obtained by the three methods. The optimal alternative is consistently a_3 , except in the results obtained by the method of Ye [43]. The reason for this inconsistency is that

Ye's method [43] conducts the transformation from qualitative information to quantitative data according to the labels of linguistic terms, and the processes of the linguistic parts and SNLN parts are separately carried out in the operations for SNLNs. These operations may create information distortion, and the defects and irrationalities involved are minutely discussed in [45]. Therefore, the result obtained by this method is unacceptable.

The results obtained by Tian et al.'s method [45] are consistent with those of the proposed approach. The two methods are based on the same linguistic scale functions and operations for SNLNs. Moreover, Tian et al.'s method [45] employs the SNLNWBM operator to aggregate the evaluation values, the ratio system in the proposed approach. However, the proposed simplified neutrosophic linguistic MULTIMOORA method consists of three parts, namely the ratio system, the reference point, and the full multiplicative form, which represent different approaches to data aggregation and ranking of alternatives.

According to the comparative analysis, the approach proposed in this paper has the following advantages over other methods.

1. Linguistic scale functions are used to conduct the transformation between qualitative and quantitative information. This approach maintains the fuzziness of original evaluation information and fully integrates it into the ranking of all alternatives. Moreover, the proposed approach can provide DMs more choices considering semantics, creating flexibility that is necessary in a complex MCDM problem.
2. This paper develops an objective distance-based method for criterion weight determination, which eliminates the subjectivity of the DMs in determining criterion weights. This is very useful in cases where DMs disagree on the values of weights. Therefore, the proposed approach is more reliable than the other two methods, in which criterion weights are subjectively provided by the DMs.
3. The proposed algorithm benefits from the ability of the simplified neutrosophic linguistic MULTIMOORA method to consider three different viewpoints when analyzing alternatives. This creates a more robust

system than the other single decision-making methods. Moreover, this paper employs the SNLNWBM and SNLNWGBM operators to capture the interrelationship of input arguments with parameters p and q . This effectively relieves the influence of unbalanced data on the aggregated results in the process of inputting interdependent information.

6 Conclusions

This paper develops an extended MULTIMOORA method that considers interdependent information based on SNLNs, providing a novel way to solve simplified neutrosophic linguistic MCDM problems. A family of distance measures for SNLNs is defined based on the linguistic scale functions, and a distance-based method is established to determine criterion weights. Then, the SNLNNWBM and SNLNWGBM operators as well as the simplified neutrosophic linguistic distance measure are incorporated into the core structure of MULTIMOORA. Finally, a ranking result is obtained by comparing the three parts of MULTIMOORA using dominance theory.

The proposed approach can yield robust results in solving complex MCDM problems involving interdependent data. Moreover, this approach enables DMs to obtain reliable and objective weight information under different semantic situations. Therefore, this study has great potential applications in solving simplified neutrosophic linguistic MCDM problems with completely unknown criterion weights. In future studies, we will focus on establishing a fuzzy stochastic MULTIMOORA decision-making methodology, as well as applying these methods to fields like fault diagnosis and project management.

Acknowledgments This work was supported by the National Natural Science Foundation of China (Nos. 71271218, 71571193 and 71431006).

References

- Mardani A, Jusoh A, Zavadskas EK (2015) Fuzzy multiple criteria decision-making techniques and applications—two decades review from 1994 to 2014. *Expert Syst Appl* 42(8):4126–4148
- Zadeh LA (1975) The concept of a linguistic variable and its application to approximate reasoning—I. *Inf Sci* 8(3):199–249
- Herrera F, Herrera-Viedma E, Verdegay JL (1996) A model of consensus in group decision-making under linguistic assessments. *Fuzzy Sets Syst* 78(1):73–87
- Herrera F, Herrera-Viedma E (2000) Linguistic decision analysis: steps for solving decision problems under linguistic information. *Fuzzy Sets Syst* 115(1):67–82
- Merigó JM, Casanovas M, Palacios-Marqués D (2014) Linguistic group decision making with induced aggregation operators and probabilistic information. *Appl Soft Comput* 24:669–678
- Liu PD, Wang YM (2014) Multiple attribute group decision making methods based on intuitionistic linguistic power generalized aggregation operators. *Appl Soft Comput* 17:90–104
- Meng FY, Chen XH, Zhang Q (2014) Some interval-valued intuitionistic uncertain linguistic Choquet operators and their application to multi-attribute group decision making. *Appl Math Model* 38:2543–2557
- Mehrjerdi YZ (2014) Strategic system selection with linguistic preferences and grey information using MCDM. *Appl Soft Comput* 18:323–337
- Tian ZP, Wang J, Wang JQ, Chen XH (2015) Multi-criteria decision-making approach based on gray linguistic weighted Bonferroni mean operator. *Int Trans Oper Res*. doi:10.1111/itor.12220
- Meng FY, Chen XH, Zhang Q (2014) Multi-attribute decision analysis under a linguistic hesitant fuzzy environment. *Inf Sci* 267:287–305
- Zhou H, Wang J, Li XE, Wang JQ (2016) Intuitionistic hesitant linguistic sets and their application in multi-criteria decision-making problems. *Oper Res Int J* 16(1):131–160
- Rodríguez RM, Martínez L, Herrera F (2012) Hesitant fuzzy linguistic term sets for decision making. *IEEE Trans Fuzzy Syst* 20(1):109–119
- Wang J, Wang JQ, Zhang HY, Chen XH (2015) Multi-criteria decision-making based on hesitant fuzzy linguistic term sets: an outranking approach. *Knowl-Based Syst* 86:224–236
- Herrera F, Martínez L (2000) A 2-tuple fuzzy linguistic representation model for computing with words. *IEEE Trans Fuzzy Syst* 8(6):746–752
- Wang J, Wang JQ, Zhang HY, Chen XH (2016) Multi-criteria group decision making approach based on 2-tuple linguistic aggregation operators with multi-hesitant fuzzy linguistic information. *Int J Fuzzy Syst* 18(1):81–97
- Chen TY, Chang CH, Rachel Lu JF (2013) The extended QUALIFLEX method for multiple criteria decision analysis based on interval type-2 fuzzy sets and applications to medical decision making. *Eur J Oper Res* 226(3):615–625
- Wang JC, Tsao CY, Chen TY (2015) A likelihood-based QUALIFLEX method with interval type-2 fuzzy sets for multiple criteria decision analysis. *Soft Comput* 19(8):2225–2243
- Zhang HY, Ji P, Wang JQ, Chen XH (2016) A neutrosophic normal cloud and its application in decision-making. *Cognit Comput*. doi:10.1007/s12559-016-9394-8
- Smarandache F (1999) A unifying field in logics. *Neutrosophy: neutrosophic probability, set and logic*. American Research Press, Rehoboth
- Atanassov KT (1986) Intuitionistic fuzzy sets. *Fuzzy Sets Syst* 20(1):87–96
- Wang HB, Smarandache F, Zhang YQ, Sunderraman R (2010) Single valued neutrosophic sets. *Multispace Multistruct* 4:410–413
- Biswas P, Pramanik S, Giri BC (2015) TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment. *Neural Comput Appl*. doi:10.1007/s00521-015-1891-2
- Şahin R, Liu PD (2015) Maximizing deviation method for neutrosophic multiple attribute decision making with incomplete weight information. *Neural Comput Appl*. doi:10.1007/s00521-015-1995-8
- Ye J (2015) Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses. *Artif Intell Med* 63(3):171–179

25. Ye J (2015) Improved correlation coefficients of single valued neutrosophic sets and interval neutrosophic sets for multiple attribute decision making. *J Intell Fuzzy Syst* 27(5):2453–2462
26. Wu XH, Wang J, Peng JJ, Chen XH (2016) Cross-entropy and prioritized aggregation operator with simplified neutrosophic sets and their application in multi-criteria decision-making problems. *Int J Fuzzy Syst*. doi:10.1007/s40815-016-0180-2
27. Ye J (2014) A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. *J Intell Fuzzy Syst* 26(5):2459–2466
28. Peng JJ, Wang JQ, Wang J, Zhang HY, Chen XH (2016) Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. *Int J Syst Sci* 47(10):2342–2358
29. Wang HB, Smarandache F, Zhang YQ, Sunderraman R (2005) *Interval neutrosophic sets and logic: theory and applications in computing*. Hexis, Phoenix
30. Zhang HY, Ji P, Wang J, Chen XH (2015) An improved weighted correlation coefficient based on integrated weight for interval neutrosophic sets and its application in multi-criteria decision-making problems. *Int J Comput Intell Syst* 8(6):1027–1043
31. Tian ZP, Zhang HY, Wang J, Wang JQ, Chen XH (2015) Multi-criteria decision-making method based on a cross-entropy with interval neutrosophic sets. *Int J Syst Sci*. doi:10.1080/00207721.2015.1102359
32. Zhang HY, Wang J, Chen XH (2016) An outranking approach for multi-criteria decision-making problems with interval-valued neutrosophic sets. *Neural Comput Appl* 27(3):615–627
33. Peng JJ, Wang JQ, Wu XH, Wang J, Chen XH (2015) Multi-valued neutrosophic sets and power aggregation operators with their applications in multi-criteria group decision-making problems. *Int J Comput Intell Syst* 8(2):345–363
34. Biswas P, Pramanik S, Giri BC (2015) Cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers. *Neutrosophic Sets Syst* 8:47–57
35. Ye J (2015) Trapezoidal neutrosophic set and its application to multiple attribute decision-making. *Neural Comput Appl* 26(5):1157–1166
36. Liu PD, Shi LL (2015) The generalized hybrid weighted average operator based on interval neutrosophic hesitant set and its application to multiple attribute decision making. *Neural Comput Appl* 26(2):457–471
37. Liu PD, Teng F (2015) Multiple attribute decision making method based on normal neutrosophic generalized weighted power averaging operator. *Int J Mach Learn Cybernet*. doi:10.1007/s13042-015-0385-y
38. Ali M, Deli I, Smarandache F (2015) The theory of neutrosophic cubic sets and their applications in pattern recognition. *J Intell Fuzzy Syst*. doi:10.3233/IFS-151906
39. Deli I, Broumi S (2015) Neutrosophic soft relations and some properties. *Ann Fuzzy Math Inf* 9(1):169–182
40. Deli I (2015) Interval-valued neutrosophic soft sets and its decision making. *Int J Mach Learn Cybernet*. doi:10.1007/s13042-015-0461-3
41. Deli I (2015) NPN-soft sets theory and applications. *Ann Fuzzy Math Inf* 10(6):847–862
42. Deli I, Broumi S (2015) Neutrosophic soft matrices and NSM-decision making. *J Intell Fuzzy Syst* 28(5):2233–2241
43. Ye J (2015) An extended TOPSIS method for multiple attribute group decision making based on single valued neutrosophic linguistic numbers. *J Intell Fuzzy Syst* 28(1):247–255
44. Ye J (2014) Some aggregation operators of interval neutrosophic linguistic numbers for multiple attribute decision making. *J Intell Fuzzy Syst* 27(5):2231–2241
45. Tian ZP, Wang J, Zhang HY, Chen XH, Wang JQ (2015) Simplified neutrosophic linguistic normalized weighted Bonferroni mean operator and its application to multi-criteria decision-making problems. *FILOMAT*. doi:10.2298/FIL1508576F
46. Broumi S, Smarandache F (2014) Single valued neutrosophic trapezoid linguistic aggregation operators based on multi-attribute decision making. *Bull Pure Appl Sci Math Stat* 33:135–155
47. Broumi S, Ye J, Smarandache F (2015) An extended TOPSIS method for multiple attribute decision making based on interval neutrosophic uncertain linguistic variables. *Neutrosophic Sets Syst* 8:22–31
48. Ye J (2015) Multiple attribute group decision making based on interval neutrosophic uncertain linguistic variables. *Int J Mach Learn Cybernet*. doi:10.1007/s13042-015-0382-1
49. Ma YX, Wang JQ, Wang J, Wu XH (2016) An interval neutrosophic linguistic multi-criteria group decision-making method and its application in selecting medical treatment options. *Neural Comput Appl*. doi:10.1007/s00521-016-2203-1
50. Xu ZS (2008) Group decision making based on multiple types of linguistic preference relations. *Inf Sci* 178:452–467
51. Bao GY, Lian XL, He M, Wang LL (2010) Improved two-tuple linguistic representation model based on new linguistic evaluation scale. *Control Decis* 25(5):780–784
52. Kahneman D, Tversky A (1979) Prospect theory: an analysis of decision under risk. *Econometrica* 47(2):263–291
53. Brauers WKM, Zavadskas EK (2010) Project management by MULTIMOORA as an instrument for transition economies. *Technol Econ Dev Econ* 16(1):5–24
54. Brauers WKM, Zavadskas EK (2006) The MOORA method and its application to privatization in a transition economy. *Control Cybern* 35(2):445–469
55. Brauers WKM, Zavadskas EK (2012) Robustness of MULTIMOORA: a method for multi-objective optimization. *Informatica* 23(1):1–25
56. Datta S, Sahu N, Mahapatra S (2013) Robot selection based on grey-MULTIMOORA approach. *Grey Syst Theory Appl* 3(2):201–232
57. Farzannia E, Babolghani MB (2014) Group decision-making process for supplier selection using MULTIMOORA technique under fuzzy environment. *Kuwait Chapter Arab J Bus Manag Rev* 3(11a):203–218
58. Liu HC, You JX, Lu C, Shan MM (2014) Application of interval 2-tuple linguistic MULTIMOORA method for health-care waste treatment technology evaluation and selection. *Waste Manag* 34(11):2355–2364
59. Deliktas D, Ustun O (2015) Student selection and assignment methodology based on fuzzy MULTIMOORA and multichoice goal programming. *Int Trans Oper Res*. doi:10.1111/itor.12185
60. Brauers WKM, Zavadskas EK (2011) MULTIMOORA optimization used to decide on a bank loan to buy property. *Technol Econ Dev Econ* 17(1):174–188
61. Tian ZP, Wang J, Wang JQ, Zhang HY (2016) Simplified neutrosophic linguistic multi-criteria group decision-making approach to green product development. *Group Decis Negot*. doi:10.1007/s10726-016-9479-5
62. Chen SJ, Hwang CL (1992) *Fuzzy multiple attribute decision making: methods and applications*. Springer, Berlin
63. Yu L, Lai KK (2011) A distance-based group decision-making methodology for multi-person multi-criteria emergency decision support. *Decis Support Syst* 51(2):307–315