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Analysis and Improvement for Proportional Conflict Redistribution Rules

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Abstract—**The Proportional Conflict Redistribution (PCR) rules based on Dezert-Smarandache theory (DSmT) is a useful method for dealing with uncertainty problems. It is more efficient in combining conflicting evidence. Therefore, it has been successfully applied in identity identification. However, there exist shortcomings in PCR rule. So in this paper we propose a new improved rule which is based on new Proportional Conflict Redistribution. The six PCR rules (PCR1-PCR6) and improved PCR rule are analyzed and compared through numerical examples, and the results show that the improved rule is effective.**

Keywords- Identity identification; DSm Theory; Proportional conflict redistribution

I. INTRODUCTION *(HEADING 1)*

Identity identification is an important content in information fusion which is full of vitality. Identity identification is not only the foundation of Situation and threat assessment but also providing supports for battlefield decisions. In the modern war, the targets are destroyed when they are detected. Therefore the war of high tech challenges the traditional method of identity identification. The category and behavior of targets become more and more complex especially in the environment of information confronting, which makes the traditional method based on single sensor difficult to obtain satisfying result.

Identity identification fused the multi-sensor information of targets' identity and obtained a more effective and precise estimation and judgment of the identity. D-S evidence theory(DST) is suitable for the fusion of prior information and at an advantage in denotation and combination of uncertain information. DST accords with decision process of human reasoning. But DST will obtain a result which is against instinct in the high conflicting condition[1]. It is an imminent problem to find an effective fusion of multi-sensor information which is high conflicting. Many experts say that the problem is caused by the combination rule and present some improvement[2] but the effect is not very satisfying. Dezert and Smarandache present DSm theory (DSmT) to solve this problem[3,4]. DSmT is an extension of classical DST, but DSmT is different from DST in nature.

Proportional conflict redistribution rules are presented by Dezert and Smarandache based on the DSmT [5,6]. Proportional conflict redistribution rules[7-10] are a series of effective method to deal with high conflicting evidence. This paper introduces the series of Proportional Conflict Redistribution rule, then analyses limitation of PCR rule and presents an improved PCR rule. The six PCR rules and improved PCR rule are analyzed and compared through numerical examples, and the results show that the improved rule is effective.

II. INTRODUTION OF DSMT

DST has essential limitation. When the conflict between sources of information becomes very high, the combination result is unreliable. It is difficult to judge the focal elements which are caused by the result. So the improvement of DST has inevitable limitations in dealing with high conflicting evidence. DSmT extends DST's basic belief assignment (bba) to generalized basic belief assignment (gbba). DSmT saves the conflicting focal elements as useful information to be fused which solves the problem of DST.

Let's $\Theta=\{\theta_1,\theta_2,\ldots,\theta_n\}$ be the frame of the fusion problem under consideration and two belief assignments m_1 , $m_2 : G^\Theta \to [0,1]$ such that $\sum_{Y\in G^\Theta} m_i(Y)=1$, $i=1,2$. The DSm's rule of combination is defined $\forall (X\neq\phi)\in G^\Theta$ by:

$$m(X)=\begin{cases}0 & X=\varnothing \\ \sum\limits_{\theta_i,\theta_j\in D^\Theta,\theta_i\cap\theta_j=X} m_1(\theta_i)m_2(\theta_j) & X\neq\varnothing\end{cases} \tag{1}$$

DSmT saves the conflicting focal elements instead of averagely distributing their basic belief assignment functions. So the DSm's rule of combination doesn't need to be normalized as DST. $m_\cap(A)$ is a new generalized basic belief assignment function of combination. In the case of multiple belief structures, evidences can be combined in a pair wise manner. But DSmT holds the conflicting focal elements which separates the value of gbba. So DSm's rule of combination is difficult for decision-making. To resolve this limitation of DST, Dezert and Smarandache present the proportional conflict redistribution rules to distribute the conflicting belief.

III. SERIES OF PROPORTIONAL CONFLICT REDISTRIBUTION RULE

The proportional conflict redistribution rules distribute conflicting belief in a certain proportion to the combination belief, which makes better use of the evidence. Proportional conflict redistribution rules are composed of PCR1 to PCR6 rule according to distributing proportion. PCR rules can be used in both DSmT and DST.

PCR rules have three advantages as follows: 1.the coherence of the combination result in all possible cases (i.e. for any number of sources, any values bba's and for any types of frames and models which can change or stay invariant over time); 2. the commutativity of the rule of combination;3. the neutral impact of the VBA into the fusion. Among all possible bba's or gbba's, the belief vacuous belief assignment (VBA), denoted $m_v(.)$ and defined by $m_v(\Theta)=1$ which characterizes a full ignorant source. The neutral impact is defined as $[m_1 \oplus \ldots \oplus m_s \oplus m_v](X)$ $=[m_1 \oplus \ldots \oplus m_s](X)$.

A. PCR1-PCR5 rule

PCR1 rule is the simplest and the easiest version of proportional conflict redistribution rule for combination. The basic idea of PCR1 rule is only to compute the total conflicting mass k_{12} .The total conflict mass is then distributed to all non-empty sets proportionally with respect to their corresponding non-empty column sum of the associated mass matrix. In PCR2 rule, the total conflicting mass k_{12} is distributed only to the non-empty sets involved in the conflict and taken the canonical form of the conflict proportionally with respect to their corresponding non-empty column sum. PCR3 rule transfers partial conflicting masses instead of the total conflicting mass to non-empty sets involved in partial conflict. The PCR3 rule works if at least one set between A and B is non-empty and its column sun is non-zero. PCR4 rule redistributes the partial conflicting mass to the elements involved in the partial conflict on considering the combination belief of partial conflict. PCR5 rule redistributes the partial conflicting mass to the elements involved in the partial conflict, considering the combination belief of partial conflict. PCR5 rule is a mathematically exact redistribution of conflicting mass to non-empty sets following the logic of the combination belief.

In a word, the PCR1 rule and PCR2 rule distribute the total conflicting mass, when the PCR3-PCR5 rules distribute the partial conflicting mass. On considering the proportion of redistribution, PCR1-PCR3 distribute conflicting mass according as the sum of elements' gbba, when PCR4 rule and PCR5 rule according as the bba of combination. The five PCR rules keep the neutral impact of the VBA into the fusion except for PCR1 rule. The accuracy of conflict distribution increases from PCR1 to PCR6 rule.

B. PCR6 rule

PCR6 rule was developed by A.Martin and C.Osswald in 2006 and it is an alternative of PCR5 rule for general case when the number of sources to combine becomes greater than two. The idea of PCR6 rule is to redistribute the masses of the focal elements giving a partial conflict proportionally to the initial masses on these elements. PCR6 rule is defined as follows: the number of sources is M , $m_{\text{PCR6}}(\phi)=0$ and for all $X \in G^{\Theta}, X \neq \phi$:

$$m_{\text{PCR6}}(X) = m_{\cap}(X) + (\sum_{i=1}^{M} m_i(X)^2) \cdot \sum_{\substack{\bigcap_{k=1}^{M-1} Y_{\sigma_i(k)} \cap X \equiv \phi \\ (Y_{\sigma_i(1)},\ldots,Y_{\sigma_i(M-1)}) \in (G^{\Theta})^{M-1}}} \left(\frac{\prod_{j=1}^{M-1} m_{\sigma_i(j)}(Y_{\sigma_i(j)})}{m_i(X) + \sum_{j=1}^{M-1} m_{\sigma_i(j)}(Y_{\sigma_i(j)})} \right) \quad (2)$$

Where $m_{\cap}(.)$ is the combination belief, $Y_j \in G^{\Theta}$ is the response of the source j , $m_j(Y_j)$ the associated belief function and σ_i counts from 1 to M avoiding i :

$$\begin{cases} \sigma_i(j) = j & if \quad j < i \\ \sigma_i(j) = j+1 & if \quad j \geq i \end{cases} \quad (3)$$

PCR6 rule does not follow back on the track of combination belief as PCR5 rule does, but it gets better intuitive results. For $M=2$ sources, PCR5 rule and PCR6 rule coincide. For $M \geq 3$ sources, one calculates the total conflict, which is a sum of products; if each product is formed by factors of masses of distinct hypothesis, then PCR6 rule coincide with PCR5 rule; if at least a product is formed by at least two factors masses of same hypotheses, then PCR6 rule is different form PCR5 rule. PCR6 rule computes the distributing proportion following the logic of the sum of evidence's gbba instead of combination belief used in PCR5 rule. So the calculation is less than PCR5 rule when dealing with large amount of evidence. PCR6 rule is suitable for the system which is sensitive to the calculation. There have been some applied fusion systems which adopt PCR6 rule[11,12].

IV. A IMPROVED PCR RULE

PCR rule can obtain a comparatively satisfying result. But traditional PCR rule only use originality evidence. Though the result is correct, the distribution method is conservative, the result is beyond perfect. So it is necessary to improve traditional PCR rule, particularly when the gbba between elements of proportion is disparity. The method of improved PCR rule is to improve the distribution proportion to consider more human organon. A resolution is to augment the proportion of different elements. Base on this idea, this paper presents a improved PCR rule, this method improve the traditional PCR6 rule, called PCR6f, the rule of combination is defined by $\forall (X \neq \phi) \in G^{\Theta}$

$$m_{\text{PCR6f}}(X) = m_{\cap}(X) + \sum_{i=1}^{M} m_i(X) f(m_i(X)) \sum_{\substack{\bigcap_{k=1}^{M-1} Y_{\sigma_i(k)} \cap X \equiv \phi \\ (Y_{\sigma_i(1)},\ldots,Y_{\sigma_i(M-1)}) \in (G^{\Theta})^{M-1}}} \left(\frac{\prod_{j=1}^{M-1} m_{\sigma_i(j)}(Y_{\sigma_i(j)})}{f(m_i(X)) + \sum_{j=1}^{M-1} f(m_{\sigma_i(j)}(Y_{\sigma_i(j)}))} \right) \quad (4)$$

The idea of the improved PCR rule is to augment the proportion of different elements. The method makes the distribution proportion of large gbba increase, when the distribution proportion of small gbba decreases. We need an increasing function which increases more quickly for large gbba than small gbba. The function is increasing function and the differential coefficient of the function is an increasing function. It is an easy method to choose $ax+b$ as the differential coefficient to construct the function.

Let's assume the differential coefficient of the function is $ax+b$, the primary function is $\frac{a^2}{2}x^2+bx+c$. bx is a liner function, c is a constant. So the function would not accord with the need, if the b is lager than a, so the same with c. Therefore it is reasonable to choose function as $\frac{a^2}{2}x^2$. The numerator and denominator both have the item $\frac{a^2}{2}$, so $\frac{a^2}{2}$ can be ignored. The function is chosen as $f(x)=x^2$. $f(x)=x^2$ is the simplest form of the function. The function $f(x)$ requires its differential coefficient which is an increasing function in the interval $[0,1]$. And we can define the standard form of $f(x)$ as $f(x)=\frac{k_n}{n!}x^n+\frac{k_{n-1}}{(n-1)!}x^{n-1}+\cdots+\frac{k_3}{6}x^3++\frac{k_2}{2}x^2$, which is $f(x)=\sum_{n=0}^{\infty}\frac{k_n}{n!}x^n \quad (n=2,3\cdots)$ for short. But the calculation increases when the degree of the function increases. So we choose $f(x)=x^2$ and $f(x)=x^3$, then put them into formula (4), we obtain:

$$m_{\mathrm{PCR6f}_{x^2}}(X)=m_{\cap}(X)+\sum_{i=1}^{M}m_i(X)m_i^2(X) \sum_{\substack{\bigcap_{k=1}^{M-1}Y_{\sigma_i(k)}\cap X\equiv\phi \\ (Y_{\sigma_i(1)},\ldots,Y_{\sigma_i(M-1)})\in(G^{\Theta})^{M-1}}}\left(\frac{\prod_{j=1}^{M-1}m_{\sigma_i(j)}(Y_{\sigma_i(j)})}{m_i^2(X)+\sum_{j=1}^{M-1}m_{\sigma_i(j)}^2(Y_{\sigma_i(j)})}\right) \tag{5}$$

$$m_{\mathrm{PCR6f}_{x^3}}(X)=m_{\cap}(X)+\sum_{i=1}^{M}m_i(X)m_i^3(X) \sum_{\substack{\bigcap_{k=1}^{M-1}Y_{\sigma_i(k)}\cap X\equiv\phi \\ (Y_{\sigma_i(1)},\ldots,Y_{\sigma_i(M-1)})\in(G^{\Theta})^{M-1}}}\left(\frac{\prod_{j=1}^{M-1}m_{\sigma_i(j)}(Y_{\sigma_i(j)})}{m_i^3(X)+\sum_{j=1}^{M-1}m_{\sigma_i(j)}^3(Y_{\sigma_i(j)})}\right) \tag{6}$$

In $\mathrm{PCR6f}_{x^3}$ rule $f(x)=x^3$, the differential coefficient is $f'(x)=3x^2$. In $\mathrm{PCR6f}_{x^2}$ rule $f(x)=x^2$, the differential coefficient is $f'(x)=2x$. The two differential coefficient are increasing function which are suitable for the need. The differential coefficient of $\mathrm{PCR6f}_{x^3}$ rule increases faster than that of $\mathrm{PCR6f}_{x^2}$ rule , so the result of $\mathrm{PCR6f}_{x^3}$ rule is better than the result of $\mathrm{PCR6f}_{x^2}$ rule. When the calculation of $\mathrm{PCR6f}_{x^3}$ rule is larger that of $\mathrm{PCR6f}_{x^2}$ rule. In the practice, we choose rule between different improved PCR rules in the need of the system.

V. ANALYSIS OF EXAMPLES

PCR1-PCR6 rule and improved PCR rules are analyzed and compared through two numerical examples. Example 1 is high conflict evidence; example 2 is low conflict evidence.

A. Example 1——high conflicting evidence

Let's consider the frame of discernment $\Theta=\{A,B,C\}$, there are two sensors in the identification system. Table 2 shows the result of identity system. The gbba at a certain time as:

TABLE I. THE GBBA OF HIGH CONFLICTING EVIDENCE

gbba / target	m_1	m_2
A	0.7	0.1
B	0.1	0.7
C	0.2	0.2

TABLE II. IDENTITY RESULT OF HIGH CONFLICTING EVIDENCE

gbba / target	$m(A)$	$m(B)$	$m(C)$
PCR1	0.398	0.398	0.204
PCR2	0.398	0.398	0.204
PCR3	0.42667	0.42667	0.14667
PCR4	0.42182	0.42094	0.15636
PCR5	0.43556	0.43556	0.12889
PCR6	0.43556	0.43556	0.12889
$\mathrm{PCR6f}_{x^2}$	0.45343	0.45343	0.09314
$\mathrm{PCR6f}_{x^3}$	0.45903	0.45903	0.08194

The Table 2 shows that A, B, C all have relations to the conflict, so every focal element in the discernment frame is involved in the conflict. $\frac{c_{12}(X)}{d_{12}}$ is equal to $\frac{c_{12}(X)}{e_{12}}$, so PCR1 and PCR2 rule have a same result. The distributing proportion of PCR3 rule is $\frac{c_{12}(F)}{c_{12}(F)+c_{12}(H)}=\frac{0.7+0.1}{(0.7+0.1)+(0.7+0.1)}=0.5$, $\frac{c_{12}(F)}{c_{12}(F)+c_{12}(N)}=\frac{0.7+0.1}{(0.7+0.1)+(0.2+0.2)}=0.67$; the distributing proportion of PCR4 rule is $\frac{m_{\cap}(X)}{m_{\cap}(X)+m_{\cap}(Y)}=\frac{0.7*0.1}{0.7*0.1+0.7*0.1}=0.5$, $\frac{m_{\cap}(X)}{m_{\cap}(X)+m_{\cap}(Y)}=\frac{0.7*0.1}{0.7*0.1+0.2*0.2}=0.64$. So PCR3 rule and PCR4 rule have the similar result. The number of sources is two, result of PCR5 rule and PCR6 rule coincide. The result of the PCR rules is gradually precise. PCR6f_{x^3} rule and PCR6f_{x^2} rule have better result than any other PCR rule.

B. Example 2——low conflicting evidence

Let's consider the frame of discernment $\Theta = \{A, B, C\}$, there are two sensors in the identification system. Table 4 shows the result of identity system. The gbba at a certain time as:

TABLE III. THE GBBA OF LOW CONFLICTING EVIDENCE

gbba \ target	m_1	m_2
A	0.7	0.7
B	0.1	0.2
C	0.2	0.1

TABLE IV. IDENTITY RESULT OF LOW CONFLICTING EVIDENCE

gbba \ target	$m(A)$	$m(B)$	$m(C)$
PCR1	0.819	0.0905	0.0905
PCR2	0.819	0.0905	0.0905
PCR3	0.83588	0.082059	0.094118
PCR4	0.89353	0.052863	0.053235
PCR5	0.83028	0.084861	0.084861
PCR6	0.83028	0.084861	0.084861
PCR6f_{x^2}	0.88607	0.056966	0.056966
PCR6f_{x^3}	0.90321	0.048394	0.048394

The Table 4 shows that PCR4 rule, PCR6f_{x^3} rule and PCR6f_{x^2} rule have better results than other methods when dealing with low conflicting evidence. We can study that PCR rules can obtain a satisfying result in dealing with low conflicting evidence.

C. Analysis of two examples

We can study from the examples above that distribute conflicting belief in a certain proportion to the combination belief, which make better use of the evidence. Among the series of proportional conflict redistribution rule, PCR1 rule and PCR2 rule redistribute the total conflicting mass, PCR3 to PCR6 rule redistribute the partial conflicting mass. When considering distributing proportion, PCR1 rule, PCR2 rule and PCR3 rule use sum of belief of different sources, PCR4 rule and PCR5 rule use the combination belief of partial conflict, PCR6 rule uses focal elements involved in the conflict. The five PCR rules keep the neutral impact of the VBA into the fusion except for PCR1 rule. PCR rules can be used in both DSmT and DST. PCR6f_{x^3} rule and PCR6f_{x^2} rule have better results than other methods when dealing with low conflicting evidence and high conflicting evidence. PCR6f_{x^3} rule has more precise result and lager calculation than PCR6f_{x^2} rule. From PCR1 up to PCR6f_{x^3}, one increases the complexity of the rules and also the exactitude of the redistribution of conflicting masses.

VI. CONCLUSIONS

In identity identification, dealing with high conflicting evidence by PCR rules is effective and feasible. With the development of the rules, improved PCR rule PCR6f rule rule have better results than other methods when dealing with low conflicting evidence and high conflicting evidence. But the improved PCR rule has lager calculation, especially in dealing with large amount of evidence, which is the direction to research and improve.

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