FLORENTIN SMARANDACHE

COLLECTED PAPERS

(Vol. II)

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FLORENTIN SMARANDACHE

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CONTENS

PARADOXIST MATHEMATICS

Abstract. The goal of this paper is to experiment new math concepts and theories, especially if they run counter to the classical ones. To prove that contradiction is not a catastrophe, and to learn to handle it in an (un)usual way. To transform the apparently unscientific ideas into scientific ones, and to develop their study (The Theory of Imperfections). And finally, to interconnect opposite (and not only) human fields of knowledge into as-heterogeneous-as-possible another fields.

The author welcomes any comments, notes, articles on this paper and/or the 120 open questions bothering him, which will be published in a collective monograph about the paradoxist mathematics.

Key words: non-mathematics, anti-mathematics, dadaist algebra, surrealist probability, cubist geometry, impressionist analysis, theory of non-choice, wild algorithms, infinite computability theory, symbolist mechanics, abstract physics, formalist chemistry, expressionist statistics, hermetic combinatorics, Sturm-und-Drang computer science, romanistics topology, letterist number theory, illuminist set theory, aesthetic differential/integral/functional equations, paradoxist logics, anti-leterature, experimental drama, nono-poems, MULTI-STRUCTURE, MULTI-SPACE, Euclidean spaces of non-integer or negative dimensione, non-system, anti-system, system with infinetely many independent axioms, unlimited theory, system of axioms based on a set with a single element, INCONSISTENT SYSTEMS OF AXIOMS, CONTRADICTORY THEORY, (unscientific, wrong, amalgam) geometry, (CHAOS or MESS) GEOMETRIES (PARADOXIST GEOMETRY, NON-GEOMETRY, COUNTER-PROJECTIVE GEOMETRY, ANTI-GEOMETRY), paradoxist model, critical area of a model, paradoxist axoims, counter-axioms, counter-model, counter-projective space, anti-axioms, anti-model, theory of distorted buildings of Tits, paradoxist trigonometry, DISCONTINUOUS MODELS, DISCONTINUOUS GEOMETRIES.

INTRODUCTION.

The "Paradoxist Mathematics" may be understood as Experimental Mathematics, Non-Mathematics, or even Anti-Mathematics: not in a nihilistic way, but in positive one. The truly innovative researchers will banish the old concepts in oder of check, by heuristic processes, some new ones: their opposites. Don't simply follow the crowd, and don't accept to be manipulated by any (political, economical, social, even scientific, or artistic, cultural, etc.) media! Learn to conradict everuthing and everybody!! "Duibito, ergo cogito; cogito, ergo sum", said Rene Descartes, "I doubt, therefore I think; I think, therefore I exist" (metaphysical doubt). See what happens if you deny the classics' theory?

Since my childhood I didn't like the term of 'exact' sciences... I hated it! I didn't like the 'truth' displayed and given to me on a plate - as food to be swallowed although not to my taste.

I considered the axioms as dogmas (not to think with your brain, but with others'!), and I refused to follow them. I wanted to be free in life - because at that time I was experiencing a political totalitarian system, without civil rights - hence I got the same feelings in science. That's why I didn't trust anybody, especially the 'official' peoples. (This is REVOLT against all petrified knowledge).

A system of axioms means to me a dictatorship model in science. It's not possible to perfectly formalize, i.e. without any intuition, but sometimes researchers like to trick themselves! Even Hilbert recognized that just in his 1898 book of <Foundations of Geometry> saying about the groups of axiomes that: "Each of these groups expresses, by itself, certain related fundamental facts of our intuition". And Kant in <Kritik der reinen Vernunft, Elementarlehre>, Part 2, Sec. 2: "All human knowledge begins with intuition, thence passes to concepts and ends with ideas". Therefore, axiomatization begins with intuition - is it a paradox? The "traditional concept of recognizing the axioms as obvious truths was replaced by the understanding that they are hypotheses for a theory" [<Encyclopedic Dictionary of Mathematics>, second edition, by the Mathematical Society of Japan, edited by Kiyosi Ito, translated in English, MIT Press, Cambridge, Massachusetts, London, 1993, 35A, p. 155].

The really avant-garde mind will entirely deny everything from the past. "No army can withstand the strenghth of an idea whose time has come" (Victor Hugo).

Questions 1-17 (one for each defined bellow section):

While, in a usual way, people apply mathematics to other human fields - what about inserting literary and art theory in mathematics?

How would we define the 'dadaist algebra', referring to the 1916-22 nihilistic mouvment in literature, painting, sculpture that rejected all accepted conventions and produced non-sens and un-readable creations? How can we introduce this style and similar <laws> in algebra??

But the 'surrealist probability'? (this syntagme makes a little sense, doesn't it?).

Or the 'cubist geometry', referring to the cubist paintings? (this may be exciting!).

The 'impressionist analysis'?

The 'theory of ... non-choice':

- from two possibilities, pick the third one! (Buridan's ass!)
- the best and unregrettable choice occurs when it's one and only possibility to choose from!

The 'wild algorithms', meaning algorithms with an infinite number of (non-linear) steps; And the 'infinite computability theory' = how much of mathematics can be decribed in such wild algorithms.

Same directions of study towards:

'symbolist mechanics',

'absract physics' (suppose, for example, as an axiom, that the speed of light is surpassed - [see Homer B. Tilton, "Light beyond belief", Echo Electronic Press, Tucson, 1995], but if the speed of a material body can be unbounded, even towards infinite? and see what you get by this anti-relativity theory: inventing new physics),

'formalist chemisttry',

'expressionist statistics',

'hermetic combinatorics',

'Sturm-und-Drang computer science' (!)

'romanticist topology' (wow, love is involving!)

'letterist number theory' (!)

'illuminist set theory',

'esthetic differential/integral/functional equations', etc.

Question 18:

The 'paradoxist logics', referring to the F.Smarandache's 1980 Paradoxist Literary Movment of avant-gardes, which may lead you to the anti-logic (which is logical!).

Features of the 'paradoxist logics':

The Basic Thesis of paradoxism:

everything has a meaning and a non-meaning in a harmony each other.

The Essence of the paradoxism:

a) the sense has a non-sense, and reciprocally

b) the non-sense has a sens.

The Motto of the paradoxism:

"All is possible, the impossible too!"

The Symbol of paradoxism:

(a spiral - optic illusion, or vicious circle)

The Delimitation from other avant-gardes:

- the paradoxism has a significance (in literature, art, science), while the dadaism, the lettrism, the absurd mouvment do not;
- the paradoxism especially reveals the contradictions, the anti-nomies, the anti-theses, the anti-phrases, the antagonism, the non-conformism, the paradoxes in other words of anything (in literature, art, science), while the futurism, cubism, surrealism, abstractism and all other avant-gardes do not focus on them.

The Directions of the paradoxism:

- to use scientific methods (especially algorithms) for generating (and also studing) contradictory literary and artistic works;
- to use artistic and literary methods for generating (and also studying) contradictory scientific works;
- to create contradictory literary and artistic works in scientific spaces (using scientific: symbols, meta-language, matrices, theorems, lemmas, definitions, etc.).

Question 19:

From Anti-Mathematics to Anti-Literature:

- I wrote a drama trilogy, called "MetaHistory", against the totalitarism of any kind: political, economical, social, cultural, artistic, even scientific (tendency of someones to monopolize the informational system, and to build not only political, economical, social dictatorships, but even distatorships in culture, art, and science ... promoting only their people and friends, and boycotting the others);

  one of them, called "A Upside-Down World", with the property that by combinations of its scenes (which are independrent modules) one gets 1, 000, 000, 000 of bilions of different dramas!

  another drama, called "The Country od the Animals", has no ... dialogue! (the characters' speech is showing on written placards).
- I wrote "Non-poems":

^ poems with no words!

^ universal poems: poem-grafitti, poem-drowing, etc.

^ poems in 3-dimensional spaces;

^ poems in Beltrami/Poincare/Hausdorff/etc. spaces;

^ poems poetical models of ... mathematics: poem-theorem, poem-lemma.

Try the reverse way: to apply math (and generally speaking science) in arts and literature. (There are famous people: as Lewis Carroll, Raymond Queneax, Ion Barbu, etc. mathematicians and writers simultaneously.)

Learn to deny (in a positive way) the masters and their work. Thus will progress our society. Thus will make revolutionary steps towards infinite... Look at some famous examples:
-Lobacevsky contradicted Euclid in 1826: "In geometry I find certain imperfections", he said in his <Theory of Parallels>.
-Riemann came to contradict both his predecessors in 1854.
-Einstein contradicted Newton in early years of the XX-th century, saying that if an object moves at velocity close to the speed of light, then time slows down, mass increases, and length in the direction of motion decreases, and so on...
Sometimes, people give new interpretations to old things... (and old interpretation to new things)!
[Don't talk about the humanistic field (art, literature, philosophy, sociology, etc.), where to reject other people's creation was and is being very common! And much easier, comparing with the scientific field.]
What would be happened if everybody had obeyed the predecessors? (a stagnation).

MULTI-STRUCTURE and MULTI-SPACE.

I consider that life and practice do not deal with 'pure' spaces, but with a group of many spaces, with a mixture of structures, a 'mongrel', a heterogeneity - the ardently preoccupation is to reunite them, to constitute a multi-structure.

I thought to a multi-space also: fragments (potsherds) of spaces put together, say as an example: Banach, Hausdorff, Tikhonov, compact, paracompact, Fock symmetric, Fock anti-symmetric, path-connected, simply connected , dsecrete metric, indescrete pseodo-metric, etc.

spaces that work together as a whole mechanism. The difficulty is to be the passage over 'frontiers' (borders between two dosjoint spaces); i.e. how can we organically tie a point P_1 from a space S_1 with a point P_2 from a structurally opposite space S_2? Does the problem become more complicated when the spaces' sets are not disjoint?

Question 20:

Can you define/construct Euclidean spaces of non-integer or negative dimension? [If so, are they connected in some way to Hausdorff's, or Kodaira's, Lebesgue's (of a normal space) algebraic/cohomological (of a topological space, a scheme, or an associative algebra)/homological/ (of a topological space, or a module) etc. dimension(s)?]

Question 21:

Let's have the case of Euclid + Lobachevsky + Reiman geometric spaces (with corresponding structures) into single space.What is the angles sum of a triangle with a vertex in each of these spaces equal to? and is it the same anytimes?

Especialy to find a model of the below geometry would be interesting, or properties and aplli-cations of it.

Paradoxically, the multi-, non-, or even anti- notions become after a while common notions. Their mystery, shock, novelty enter in the room of obvious things. This is the route of any invention and discovery.

Time is not uniform, but in a zigzag;

a today's truth will be the toomorrow's falsehood - and reciprocally, the opposite phenomena are complementary and may not survive independently.

The every-day reality is a sumum or multitude of rules, some of them opposite each other, accepted by ones and refused by others, on different surfaces of positive, negative, and null Gauss's curvatures in the same time (especially on non-constant curvature surfaces).

Question 22:

After all, what mathematical apparatus to use for subsequent improvement of this theory? [my defenition is elementary].

Logics without logics?

System without system? (will be a non-system or anti-system?)

Mathematics without mathematics!

World is an ordered disorder and disordered order! Homogeneity exists only in pure sciences without our imagination, but practice is quite different from theory.

There are systems with one axiom only [see Dr. Paul Welsh, "Primitivity in Mereology" (I and II), in <Notre Dame Journal of Formal Logic>, Vol. XIX, No. 1 and 3, January and July 1978, pp.25-62 and 355-85; or B.Sobocinski, "A note on an axiom system of atomistic mereology", in <Notre Dame Journal of Formal Logic>, Vol. XII, 1971, pp. 249-51.].

If one defines another system with a sole axiom, which is the negation of the previous axiom, one gets an opposite theory.

Question 23:

Try to construct a consistent system of axioms, with infinitely many independent axioms, in oder to define a Unlimited Theory. A theory to whom you may add at any time a new axiom to develop it in all directions you like.

Question 24;

Try to construct a consistent system of axioms based on a set with a single object (element). (But if the set is... empty?)

INCONSISTENT SYSTEMS OF AXIOMS and CONTRADICTORY THEORY.

5 Let $(a_1), (a_2), \ldots, (a_n), (b)$ be $n+1$ independent axioms, with $n \geq 1$; and let $b')$ be another

axiom contradictory to (b). We construct the system of $n+2$ axioms:

[I] $(a_1), (a_2), \ldots, (a_n), (b), (b')$

which is inconsistent. But this system may be shared into two consistent systems of independent axioms

[C] $(a_1), (a_2), \ldots, (a_n), (b)$,

and

[C'] $(a_1), (a_2), \ldots, (a_n), (b')$.

We also consider the partial system of independent axioms

[P] $(a_1), (a_2), \ldots, (a_n)$.

Developing [P], we find many propositions (theorems, lemmas) $(p_1), (p_2), \ldots, (p_m)$, by combinations of its axioms.

Developing [C], we find all propositions of [P] $(p_1), (p_2), \ldots, (p_m)$, resulted by combinations of $(a_1), (a_2), \ldots, (a_n)$, plus other propositions $(r_1), (r_2), \ldots, (r_t)$, results by combination of (b) with any of $(a_1), (a_2), \ldots, (a_n)$.

Similarly for [C′], we find the propositions of [P] $(p_1),(p_2),\ldots,(p_m)$, plus other propositions $(r_1'),(r_2'),\ldots,(r_t')$, resulted by combinations of (b') with any of $(a_1),(a_2),\ldots,(a_n)$, where (r_1') is an axiom contradictory to (r_1), and so on.

Now, developing [I], we'll findall the previous resulted propositions:

$$(p_1),(p_2),\ldots,(p_m),$$
$$(r_1),(r_2),\ldots,(r_t),$$
$$(r_1'),(r_2'),\ldots,(r_t').$$

Therefore, [I] is equivalent to [C] reunited to [C′]. From one pair of contradictory propositions $\{(b)\text{and}(b')\}$ in its begining, [I] adds t more such pairs, where $t \geq 1$, $\{(r_1)\text{and }(r_1'),\ldots,(r_t),\text{and }(r_t')\}$, after a complete step. The further we go, the more pairs of contradictory propositions are accumulating in [I].

Question 25:

Develop the study of an inconsistent system of axioms.

Question 26:

It is interesting to study the case when $n = 0$.

Why do people avoid thinking about the CONTRADICTORY THEORY ? As you know, nature is not perfect:

and opposite phenomena occur together,

and opposite ideas are simultaneously asserted and, ironically, proved that both of them are true! How is that possible ?...

A statement may be true in a referential system, but false in another one. The truth is subjective. The proof is relative. (In philosophy there is a theory: that "knowledge is relative to the mind, or things can be known only through their effects on the mind, and consequently there can be no knowlwdgw of reality as it is in itself", called "the Relativity of Knowledge"; <Webster's New World Dictionary of American English>, Third College Edition, Cleveland & New York, Simon & Schuster Inc., Editors: Victoria Neufeldt, David B. Guralnik, 1988, p. 1133.) You know? ... sometimes is good to be wrong!

Question 27:

Try to develop a particular contradictory theory.

I was attracted by Chaos Theory, deterministic behaviour which seems to be randomly: when initial conditions are verying little, the differential equation solutions are varying tremendously much. Originated by Poincare, and studied by Lorenz, a metereologist, in 1963, by computer help. These instabilities occuring in the numerical solutions of differential equations are thus connected to the phenomena of chaos. Look, I said, chaos in mathematics, like in life and world!

Somehow consequently are the following four concepts in the paradoxist mathematics, that may be altogether called, CHAOS (or MESS) GEOMETRIES!

PARADOXIST GEOMETRY

In 1969, intrigued by geometry, I simultaneously constracted a partially euclidean and partially nono-euclidean space by a strange replacement of the Euclid's fifth postulate (axiom of parallels) with the following five-statement proposition:

a) there are at least a strainght line and a point exterior to it in this space for which only one line passed through the point and does not intersect the initial line; [1 parallel]

b) there are at least a strainght line and point exterior to it in this space for which only a finite number of lines $l_1, \ldots, l_k$ $(k \geq 2)$ passe throught the point and do not intersect the initial line; [2 or more (in a finite number) parallels]

c) there are at least a strainght line and point exterior to it in this space for which any line that passes throught the point intersects the initial line; [0 parallels]

d) there are at least a strainght line and point exterior to it in this space for which an infinite number of lines that passes throught the point (but not all of them) do not intersect the initial line; [an infinite number of parallels, but not all lines passing throught]

e) there are at least a strainght line and a point exterior to it in this space for which any line that passes throught the point does not intersect the initial line; [an infinite number of parallels, all lines passing throught the point]

I have called it the PARADOXIST GEOMETRY. This geometry unites all together: Euclid, Lobachevsky/Bolyai, and Riemann geometries. And separates them as well!

Question 28:

Now, the problem is to find a nice model (on manifolds) for this Paradoxost Geometry, and study some of its characteristics.

NON-GEOMETRY

It's a lot easier to deny the Euclid's five postulates than Hilbert's twenty thorough axioms.

1. It is not always possible to draw a line from an arbitrary point to another arbitrary point.

   For example: this axiom can be denied only if the model's space has at least a discontinuity point; (in our bellow model MD, one takes an isolated point I in between f_1 and f_2, the only one which will not verify the axiom).

2. It is not always possible to extend by continuity a finite line to an infinite line.

   For example: consider the bellow Model, and the segment AB, the both A and B lie on f_1, A in between P and N, while B on the left side of N; one can not at all extend AB either beyond A or beyond B, because the resulted curve, noted say $A' - A - B - B'$, would not be a geodesic (i.e. line in our Model) anymore.

   If A and B lie in $\delta_1 - f_1$, both of them closer to f_1, A in the left side of P, while B in the right side of P, then the segment AB, which is in fact $A - P - B$, can be extended beyond A and also beyond B only up to f_1 (therefore one gets a finite line too, $A'_A - P - B - B'$), where A', B' are the intersections of PA, PB respectively with f_1).

   If A, B lie in $\delta_1 - f_1$, far enough from f_1 and P, such that AB is parallel to f_1, then AB verifies this postulate.

3. It is not always possible to draw a circle from an arbitrary point and of an arbitrary interval.

   For example: same as for the first axiom; the isolated point I, and a very small interval not reaching f_1 neither f_2, will deny this axiom.

4. Not all the right angles are congruent. (See example of the Anti-Geometry, explained bellow.)

5. If a line, cutting two other lines, forms the interior angles of the same side of it strictly less than two right angles, then not always the two lines extended towards infinite cut each other in the side where the angles are strictly less than two right angles.

   For example: let h_1, h_2, l be three lines in $\delta_1 - \delta_2$, where h_1 intersects f_1 in A, and h_2 intersects f_1 in B, with A, B, P different each other, such that h_1 and h_2 do not intersect, but l cuts h_1 and h_2 and forms the interior angles of one of its side (towards f_1) strictly less than two right angles;

the assumption of the fifth postulate is fulfilled, but the consequence does not hold, because h_1 and h_2 do not cut each other (they may not be extended beyond A and B respectively, because the lines would not be geodesics anymore).

Question 29

Find a more convincing midel for this non-geometry.

COUNTER-PROJECTIVE GEOMETRY

Let P, L be two sets, and r a relation included in $P \times L$. The elements of P are called points, and those of L lines. When (p, l) belongs to r, we say that the line l contains the point p. For these, one imposes the following COUNTER-AXIOMS:

(I) There exist: either at least two lines, or no line, that contains two given distinct points.

(II) Let p_1, p_2, p_3 be three non-collinear points, and q_1, q_2 two distinct points. Supoose that $\{p_1, q_1, p_3\}$ and $\{p_2, q_2, p_3\}$ are collinear triples. Then the line containing p_1, p_2, and the line containing q_1, q_2 do not intersect.

(III) Every line contains at most two distinct points.

Questions 30-31:

Find a model for the Counter-(General Projective) Geometry (the previous I and II counter-axioms hold), and a model for the Counter-Projective Geometry (the previous I, II, and III counter-axioms hold). [They are called COUNTER-MODELS for the general projective, and projective geometry, respectively.]

Questions 32-33:

Find geometric modls for each of the following two cases:

- There are points/lines that verify all the previous counter-axioms, and other points/lines in the same COUNTER-PROJECTIVE SPACE that do not verify any of them;
- Some of the counter-axioms I, II, III are verified, while the others are not (there are particular cases already known).

Question 34:

The study of these counter-models may be extended to Infinite-Dimensional Real (or Complex) Projective Spaces, denying the IV-th axioms, i.e.:

(IV) There exists no set of finite number of points for which any subspace that contains all of them contains P.

Question 35:

Does the Duality Principle hold in a counter-projective space?

What about Desargues's Theorem, Fundamental Theorem of Projective Geometry/Theorem of Pappus, and Staudt Algebra?

Or Pascal's Theorem, Brianchon's Theorem? (I think none of them will hold!)

Question 36:

The theory of Buildings of Tits, which contains the Projective Geometry as a particular case, can be 'distorted' in the same <paradoxist> way by deforming its axiom of a BN-pair (or Tits system) for the triple (G, B, N), where G is a group, and B, N its subgroups; [see J.Tits, "Buildings of spherical type and finite BN-pairs", Lecture notes in math. 386, Springer, 1974].

Notions as: simplex, complex, chamber, codimension, apartment, building will get contorted either...

Develop a Theory of Distorted Buldings of Tits!

ANTI-GEOMETRY

It is possible to de-formalize entirely Hilbert's groups of axioms of the Euclidean Geometry, and to construct a model such that none of his fixed axiom holds.

Let's consider the following things:

- a set of <points>: $A, B, C, \ldots$
- a set of <lines>: $h, k, l, \ldots$
- a set of <planes>: $\alpha, \beta, \gamma, \ldots$

and

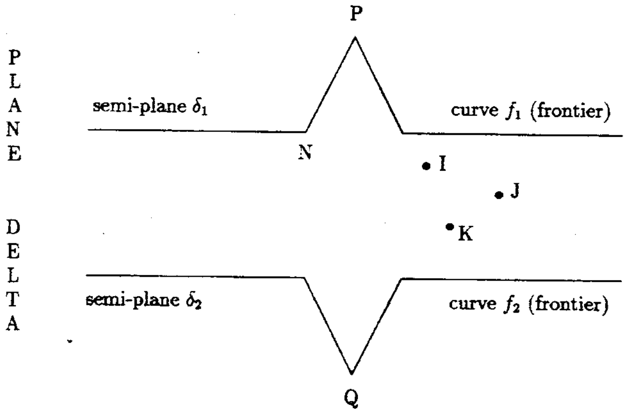
- a set of relationship among these elements: "are situated", "between", "parallel", "congruent", "continous", etc.

Then, we can deny all Hilbert's twenty axioms [see David Hilbert, "Foundation of Geometry", translated by E.J.Towsend, 1950; and Roberto Bonola, "Non-Euclidean Geometry", 1938]. There exist casses, whithin a geometric model, when the same axiom is verifyed by certain points/lines/planes and denied by others.

GROUP I. ANTI-AXIOMS OF CONNECTION

I.1. Two distinct points A and B do not always completely determine a line.

Let's consider the following model MD: get an ordinary plane δ, but with an infinite hole in of the following shape:



Plane delta is a reunion of two disjoint planar semi-planes; f_1 lies in MD, but f_2 does not; P, Q are two extreme points on f that belong to MD.

One defines a LINE l as a geodesic curve: if two points A, B that belong to MD lie in l, then the shortest curve lied in MD between A and B lies in l also. If a line passes two times throught the same point, then it is called double point ($KNOT$).

One defines a PLANE α as a surface such that for any two points A, B that lie in α and belong to MD there is a geodesic which passes trought A, B and lies in α also.

Now, let's have two strings of the same length: one ties P and Q with the first string s_1 such that the curve s_1 is folded in two or more different planes and s_1 is under the plane δ; next, do the same with string s_2, tie Q with P, but over the plane δ and such that s_2 has a different form from s_1; and a third string s_3, from P to Q, much longer than s_1. s_1, s_2, s_3 belongs to MD.

Let I, J, K be three isolated points - as some islands, i.e. not joined with any other poits of MD, exterior to the plane δ.

This model has measure, because the (pseudo-) line is the shortesr way (length) to go from a point to another (when possible).

Question 37:

Of course, the model is not perfect, and is far from the best. Readers are asked to improve it, or to make up a new one that is better.

(Let A, B be two distinct points in $\delta_1 - f_1$. P and Q are two points on s_1, but they do not completely determine a line, referring to the first axiom of Hilbert, because $A - P - s_1 - Q$ are different from $B - P - s_1 - Q$.)

I.2. There is at least a line l and at least two different points A and B of l, such that A and B do not completely determine the line l.

(Line $A - P - s_1 - Q$ are not completely determine by P and Q in the previous construction, because $B - P - s_1 - Q$ is another line passing through P and Q too.)

I.3. Three points A, B, C not situated in the same line do not always completely determine a plane α.

(Let A, B be two distinct points in $\delta_1 - f_1$, such that A, B, P are not co-linear. There are many planes containing these three points: δ_1 extended with any surface s containing s_1, but not cutting s_2 in between P and Q, for example.)

I.4. There is at least a plane, α, and at least three points A, B, C in it not lying in the same line, such that A, B, C do not completely determine the plane α.

(See the previous example.)

I.5. If two points A, B of line l lie in a plane α, it doesn't mean that every point of l lies in α.

(Let A be a point in $\delta_1 - f_1$, and B another point on s_1 in between P and Q. Let α be the following plane: δ_1 extended with a surface s containing s_1, but not cutting s_2 in between P and Q, and tangent to δ_2 on a line QC, where C is a point in $\delta_2 - f_2$. Let D be point in $\delta_2 - f_2$, not lying on the line QC. Now, A, B, D are lying on the same line $A - P - s_1 - Q - D, A, B$ are in the plane α, but D does not.)

I.6. If two planes α, β have a point A in common, it doesn't mean they have at least a second point in common.

(Construct the following plane α): a closed surface containing s_1 and s_2, and intersecting δ_1 in one point only, P. Then α and δ_1 have a single point in common.)

I.7. There exist lines where only one point lies, or planes where only two points lie, or space where only three points lie.

(Hilbert's I.7 axiom may be contradicted if the model has discontinuities. Let's consider the isolated points area.

The point I may be regarded as a line, because it's not possible to add any new point to I to form a line.

One constructs a surface that intersects the model only in the points I and J.)

GROUP II. ANTI-AXIOMS OF ORDER

II.1. If A, B, C are points of line and B lies between A and C, it doesn't mean that always B lies also between C and A.

[Let T lie in s_1, and V lie in s_2, both of them closer to Q, but different from it. Then:

P, T, V are points on the line $P - s_1 - Q - s_2 - P$ (i.e. the closed curve that starts from the point P) and lies in s_1 and passes through the point Q and lies back to s_2 and ends in P), and T lies between P and V

- because PT and TV are both geodesics, but T doesn't lie between V and P
- because from V the line goes to P and then to T, therefore P lies between V and T.]

[By defenition: a segment AB is a system of points lying upon a line between A and B (the extremes are included.)

Warning: AB may be different from BA; for example:]

the segment PQ formed by the system of points starting with P, ending with Q, and lying in s_1, is different from the segment PQ formed by the system of points starting with P, ending with Q, but belong to s_2.]

II.2. If A and C are two points of a line, then: there does not always exist a point B lying between A and C, or there does not always exist a point D such that C lies between A and D.

[For example:

let F be a point on f_1, F different from P, and G a point in δ_1, G doesn't belong to f_1; draw the line l which passes through G and F; then: there exists a point B

lying between G and F - because GF is an obvious segment, but there is no point D such that F lies between G and D - because GF is right bounded in F (GF may not be extended to the other side of F, because otherwise the line will not remain a geodesic anymore).]

II.3. There exist at least three points situated on a line such that:

one point lies between the other two, and another point lies also between the other two.

[For example:

let R, T be two distinct points, different from P and Q, situated on the line $P - s_1 - Q - s_2 - P$, such that the lenghts PR, RT, TP are all equal; then:

R lies between P and T, and T lies between R and P; also P lies between T and R].

II.4. Four points A, B, C, D of a line can not always be arranged: Such that B lies between A and C and also between A and D, and such that C lies between A and D and also between B and D.

[For example:

let R, T be two distinct points, different from P and Q, situated on the line $P - s_1 - Q - s_2 - P$ such that the lenghts PR, RQ, QT, TP are all equal, therefore R belongs to s_1, and T belongs to s_2; then P, Q, R, T are situated on the same line: such that R lies between P and Q, but not between P and T - because the geodesic PT does not pass through R, and such that Q does not lie between P and T, because the geodesic PT does not pass through Q, but lies between R and T; let A, B be two points in $\delta_2 - f_2$ such that A, Q, B are colinear, and C, D two points on s_1, s_2 respectively, all of the four points being different from P and Q; then A, B, C, D are points situated on the same line $A - Q - s_1 - P - s_2 - Q - B$, which qis the same with line $A - Q - s_2 - P - s_1 - Q - B$, therefore we may have two different orders of these four points in the same time: A, C, D, B and A, D, C, B.]

II.5. Let A, B, C be three points not lying in the same line, and l a line lying in the same plane ABC and not passsing through any of the points A, B, C. Then, if the line l passes through a point of the segment AB, it doesn't mean that always the line l will pass through either a point of the segment BC or a point of the segment AC.

[For example:

let AB be a segment passing through P in the semi-plane δ_1, and C point lying in δ_1 too on the left side of the line AB; thus A, B, C do not lie on the same line; now, consider the line $Q - s_2 - P - s_1 - Q - D$, where D is a point lying in the semi-plane δ_2 not on f_2: therefore this line passes through the point P of the segment AB, but does not pass through any point od the segment BC, nor through any point of the segment AC.]

GROUP III. ANTI-AXIOMS OF PARALLELS

In a plane α there can be drawn through a point A, lying outside of a line l, either no line, or only one line, or a finite number of lines which do not intersect the line l. (At least two of these situations should occur.) The line(s) is (are) called the parallel(s) to l through the given point A.

[For examples:

- let l_0 be the line $N - P - s_1 - Q - R$, where N is a point lying in δ_1 not on f_1, and R is a similar point lying in δ_2 not on f_2, and let A be a point lying on s_2, then: no parallel to l_0 can be drawn through A (because any line passing through A, hence through s_2, will intersect s_1, hence l_0, in P and Q);
-if the line l_1 lies in δ_1 such that l_1 does not intersect the frontier f_1, then: through any point lying on the left side of l_1 one and only one parellel will pass;
-let B be a point lying in f_1, different from P, and another point C lying in δ_1, not on f_1; let A be a point lying in δ_1 outside of BC; then: an infinite number of parallels to the line BC can be drawn through the point A.]

Theorem. *There are at least two lines l_1, l_2 of a plane, which do not meet a third line l_3 of the same plane, but they meet each other, (i.e. if l_1 is parallel to l_3, and l_2 is parallel to l_3, and all of them are in the same plane, it's not necessary that l_1 is parallel to l_2).*

[For example:

consider three points A, B, C lying in f_1, and different from P, and D a point in δ_1 not on f_1; draw the lines AD, BE and CE such that E is a point in δ_1 not on f_1 and both

BE and CE do not intersect AD; then: BE is parallel to AD, CE is also parallel to AD, but BE is not parallel to CE because the point E belong to both of them.]

GROUP IV. ANTI-AXIOMS OF CONGRUENCE

IV.1. If A, B are two points on a line l, and A' is a point upon the same or another line l', then: upon a given side of A' on the line l', we can not always find only one point B' so that the segment AB is congruent to the segment $A'B'$.

[For exemples:

- let AB be segment lying in δ_1 and having no point in common with f_1, and construct the line $C-P-s_1-Q-s_2-P$ (noted by l') which is the same with $C-P-s_2-Q-s_1-P$, where C is a point lying in δ_1 not on f_1 nor on AB; take the point A' on l', in between C and P, such that $A'P$ is smaller than AB; now, there exist two distinct points B_1' on s_1 and B_2' on s_2, such that $A'B_1'$ is congruent to AB and $A'B_2'$ is congruent to AB, with $A'B_1'$ different from $A'B_2'$;

- but if we consider a line l' lying in δ_1 and limited by the frontier f_1 on the right side (the limit point being noted by M), and take a point A' on l', close to M, such that $A'M$ is less than $A'B'$, then: there is no point B' on the right side of l' so that $A'B'$ is congruent to AB.]

A segment may not be congruent to itself!

[For example:

- let A be a point on s_1, closer to P, and B a point on s_2, closer to P also; A and B are lying on the same line $A-Q-B-P-A$ which is the same with line $A-P-B-Q-A$, but AB meseared on the first repersentation of the line is strictly greater than AB meseared on the second representation of their line.]

IV.2. If a segment AB is congruent to the segment $A'B'$ and also to the segment $A''B''$, then not always the segment $A'B'$ is congruent to the segment $A''B''$.

[For example:

- let AB be a segment lying in $\delta_1 - f_1$, and consider the line $C-P-s_1-Q-s_2-P-D$, where C, D are two distinct points in $\delta_1 - f_1$ such that C, P, D are colinear. Suppose tat the segment AB is congruent to the segment CD (i.e. $C-P-s_1-Q-s_2-P-D$). Get also an obvious segment $A'B'$ in $\delta_1 - f_1$, different from the preceding ones, but congruent to AB.

Then the segment $A'B'$ is not congruent to the segment CD (considered as $C-P-D$, i.e. not passing through Q.)

IV.3. If AB, BC are two segments of the same line l which have no points in common aside from the point B, and $A'B', B'C'$ are two segments of the same line or of another line l' having no point other than B' in common, such that AB is congruent to $A'B'$ and BC is congruent to $B'C'$, then not always the segment AC is congruent to $A'C'$.

[For example:

let l be a line lying in δ_1, not on f_1, and A, B, C three distinct points on l, such that AC is greater than s_1; let l' be the following line: $A' - P - s_1 - Q - s_2 - P$ where A' lies in δ_1, not on f_1, and get B' on s_1 such that $A'B'$ is congruent to AB, get C' on s_2 such that BC is congruent to $B'C'$ (the points A, B, C are thus chosen); then: the segment $A'C'$ which is first seen as $A' - P - B' - Q - C'$ is not congruent to AC, because $A'C'$ is the geodesic $A' - P - C'$ (the shortest way from A' to C' does not pass through B') which is strictly less than AC.]

Definitions. *Let h, k be two lines having a point O in common. Then the system (h, O, k) is called the angle of the lines h and k in the point O.*

(Because some of our lines are curves, we take the angle of the tangents to the curves in their common point.)

The angle formed by the lines h and k situated in the same plane, noted by $< h, k >$, is equal to the arithmetic mean of the angles formed by h and k in all their common points.

IV.4. Let an angle (h, k) be given in the plane α, and let a line h be given in the plane β. Suppose that in the plane β a definite side of the line h' is assigned, and a point O'. Then in the plane β there are one, or more , or even no half-line(s) k' emanating from the point O' such that the angle (h, k) is congruent to the angle (h', k'), and at the same time the interior points of the angle (h', k') lie upon one or both sides of h'.

[Examples:

- Let A be a point in $\delta_1 - f_1$, and B, C two distinct points in $\delta_2 - f_2$; let h be the line $A - P - s_1 - Q - B$, and k be the line $A - P - s_2 - Q - C$; because h and k intersect in an infinite number of points (the segment AP), where they normally coincide - i.e. in each such point their angle is congruent to zero, the angle (h, k) is congruent to zero.

Now, let A' be a point in $\delta_1 - f_1$, different from A, and B' a point in $\delta_2 - f_2$, different from B, and draw the line h' as $A' - P - s_1 - Q - B'$; there exist an infinite number of lines k', of the form $A' - P - s_2 - Q - C'$ (where C' is any point in $\delta_2 - f_2$, not on the line QB'), such that the angle (h, k) is congruent to (h', k'), because (h', k') is al- so congruent to zero, and the line $A' - P - s_2 - Q - C'$ is different from the line $A' - P - s_2 - Q - D'$ id D' is not on the line QC'

- If h, k and h' are three lines in $\delta_1 - P$, which intersect the frontier f_1 in at most one point, then there exists only one line k' on a given part of h' such that the angle (h, k) is congruent to the angle (h', k').

- *Is there any case when, with these hypotheses, no k' exists?

- Not every angle is congruent to itself; for example: $(< s_1, s_2 >)$ is not congruent to $(< s_1, s_2 >)$ [because one can construct two distinct lines: $P - s_1 - Q - A$ and $P - s_2 - Q - A$, where A is point in $\delta_2 - f_2$, for the first angle, which becomes equal to zero; and $P - s_1 - Q - A$ and $P - s_2 - Q - B$, where B is another point in $\delta_2 - f_2$, B different from A, for the second angle, which becomes strictly greater than zero!].

IV.5. If the angle (h, k) is congruent to the angle (h', k') and to the angle (h'', k''), then the angle (h', k') is not always congruent to the angle (h'', k'').
(A similar construction to the previous one.)

IV.6. Let ABC and $A'B'C'$ be two triangles such that AB is congruent to $A'B'$, AC is congruent to $A'C'$, $< BAC$ is congruent to $< B'A'C'$. Then not always $< ABC$ is congruent to $< A'B'C'$ and $< ACB$ is congruent to $< A'C'B'$.
[For example:
Let M, N be two distinct points in $\delta_2 - f_2$, thus obtaining the triangle PMN; now take three points R, M', N' in $\delta_1 - f_1$, such that RM' is congruent to PM, RN' is congruent to RN, and the angle (RM', RN') is congruent to the angle (PM, PN). $RM'N'$ is an obvious triangle. Of course, the two triangle are not congruent, because for example PM and PN cut each other twice - in p and Q - while RM' and RN' only once - in R. (These are geodesical triangles.)]

Definitions. *Two angles are called supplementary if they have the same vertex, one side in common, and the other sides not common form a line.*

A right angle is an angle congruent to its supplementary angle.

Two triangles are congruent if their angles are congruent two by two, and its sides are *congruent two by two.*

Propositions:

A right angle is not always congruent to another right angle.

For example:

Let $A-P-s_1-Q$ be a line, with A lying in δ_1-f_1, and $B-P-s_1-Q$ another line, with B lying in δ_1-f_1 and B not lying in the line AP; we consider the tangent t at s_1 in P, and B chosen in a way that $<(AP,t)$ is not congruent to $<(BP,t)$; let A', B' be other points lyng in δ_1-f_1 such that $<APA'$ is congruent to $<A'P-s_1-Q$, and $<BPB'$ is congruent to $<B'P-s_1-Q$. Then:

- the angle APA' is right, because it is congruent to its supplementary (by construction);

- the BPB' is also right, because it is congruent to its supplementary (by construction);

- but $<APA'$ is not congruent to $<BPB'$, because the first one is half of the angle $A-P-s_1-Q$, i.e. half of $<(AP,t)$, while the second one is half of the $B-P-s_1-Q$, i.e. half of $<(BP,t)$.

The theorems of congruence for triangles [side, side, and angle in between; angle, angle, and common side; side, side, side] may not hold either in the Critical Zone (s_1, s_2, f_1, f_2) of the Model.

Property:

The sum of the angles of a triangle can be:

- 180 degrees, if all its vertexes A, B, C are lying, for example, in δ_1-f_1;

-strictly less than 180 degrees [any value in the interval $(0,180)$], for example:

let R, T be two points in δ_2-f_2 such that Q does not lie in RT, and S another point on s_2; then the triangle SRT has $<(SR,ST)$ congruent to O because SR and ST have an infinite number of common points (the segment SQ), and $<QTR+<TRQ$ congruent to $180-<TQR$ [by construction we may very $<TQR$ in the interval $(0,180)$];

-even O degree!

let A be a point in δ_1-f_1, B a point in δ_2-f_2, and C a point on s_3, very close to P; then ABC is a non-degenerated triangle (because its vertexes are non-colinear), but $<(A-P-s_1-Q-B, A-P-s_3-C) =< (B-Q-s_1-P-A, B-Q-s_1-P-s_3-C) =$ $<(C-s_3-P-A, C-s_3-P-s_1-Q-B)=0$ (one considers the lenght $C-s_3-P-s_1-Q-B$ strictly less than $C-s_3-B$); the area of this triangle is also 0!

- more than 180 degrees, for example:

let A, B be two points in δ_1-f_1, such that $<PAB+<PBA+<(s_1,s_2;$ in $Q)$ is strictly greater than 180 degrees; then triangle ABQ, formed by the intersection of the lines $A - P - s_2 - Q, Q - s_1 - P - B, AB$ will have the sum of its angles strictly greater than 180 degrees.

Defenition. *A circle of center M is a totality of all points A for which the segments MA are congruent to one another.*

For example, if the center is Q, and the length of the segments MA is chosen greater than the length of s_1, then the circle is formed by the arc of circle centered in Q, of radius MA, and lying in δ_2, plus another arc of circle centered in P, of radius MA - length of s_1, lying in δ_1.

GROUP V. ANTI-AXIOMS OF CONTINUITY (ANTI-ARCHIMEDEAN AXIOM)

Let A, B be two points. Take the point $A_1, A_2, A_3, A_4, \ldots$ so that A_1 lies between A and A_2, A_2 lies between A_1 and A_3, A_3 lies between A_2 and A_4, etc. and the segments AA_1, A_1A_2, $A_2A_3, A_3A_4, \ldots$ are congruent to one another.

Then, among this series of points, not always there exists a certain point A_n such that B lies between A and A_n.

For example:

let A be a point in $\delta_1 - f_1$, and B a point on f_1, B different from P; on the line AB consider the points $A_1, A_2, A_3, A_4, \ldots$ in between A and B, such that $AA_1, A_1A_2, A_2A_3, A_3A_4$, etc. are congruent to one another; then we finde that there is no point behind B (considering the direction from A to B), because B is a limit point (the line AB ends in B).

The Bolzano's (intermediate value) theorem may not hold in the Critical Zone of the Model.

Question 38:

It's very intresting to find out if this system of axiom is complete and consistent (!) The apparent unsientific or wrong geometry, which looks more like an amalgam, is somehow supported by its attached model.

Question 39:

How will the differential equations look like in this field?

Question 40:

How will the (so called by us:) "PARADOXIST" TRIGONOMETRY look like in this field?

Question 41:

First, one can generalize this using more bridges (conections/strings between δ_1 and δ_2) of many lengths, and many gates (points like P and Q on f_1 and f_2, respectively) - from a finite to an infinite number of such bridges and gates.

If one put all bridges in the δ plane, one gates a dimension-2 model; otherwise, the dimension is ≥ 3.

Some bridges may be replaced with (round or not necessaryly) bodies, tangent (or not necessaryly) to the frontiers f_1 and f_2.

Question 42:

Should it be indicated to remove the discontinuities?

But what about DISCONTINUOUS MODELS (on spaces not everywhere continouus - like our MD)? generating in this way DISCONTINUOUS GEOMETRIES.

Question 43:

The model MD can also be generalized to n-dimensional space as a hypersurfece, considering the group of all projective transformations of an $(n+1)$-dimensional real projective space that leave MD invariant.

Questions 44-47:

Find geometric models for each of the following four cases:

- No point/line/plane in the model space verifies any of Hilbert's twenty axioms; (in our MD, some points/linse/planes did verify, and some others did not);

- The Hilbert's groups of axioms I, II, IV, V are denied for any point/line/plane in the model space, but the III-th one (axiom of parallels) is verified; this is an Opposite-(Lobachevski+Reimann) Geomatry:

neither hyperbolic, nor eliptic ... and yet Non-Euclidean!

- The groups og anti-axioms I, II, IV, V are all verified, but the III-th one (anti-axiom of parallels) is denied;

- Some of the groups of anti-axioms I, II, III, IV, V are verified, while the others atre not - except the previous case; (there are particular cases already known).

Question 48:

What connections may be found among this Paradoxist Model, and the Cayley, Klein, Poincare, Beltrami (differential geometric) models?

Questions 49-120: (combining by twos, each new geometry - out of 4 - with an old geometry - out of 18 - all mentionned below):

What connections among these Paradoxist Geometry, Non-Geometry, Counter-Projective Geometry, Anti-Geometry and the other ones: Conformal (Mobius) Geometry, Pseudo-Conformal Geometry, Laguerre Geometry, Spectral Geometry, Spherical Geometry, Hiper-Sphere Geometry, wave Geometry (Y. Mimura), Non-Holonomic Geometry (G. Vranceanu), Cartan's Geometry of Connection, Integral Geometry (W. Blaschke), Continuous Geometry (von Neumann), Affine Geometry, Generalized Geometries (of H. Weyl, O. Veblen, J.A. Schoutten), etc.

CONCLUSION

The above 120 OPEN QUESTIONS are not impossible at all. "The world is moving so fast nowadays that the person, who says <it can't be done>, is often interrupted by someone doing it"! [<Leadirship> journal, Editor Arthur F. Lenehan, October 24, 1995, p. 16, Fairfield, NJ].

The author encourages readers to send not only comments, but also new (solved or unsolved) questions arising from them.

Specials thanks to professors JoAnne Growney, Zahira S. Khan, and Paul Hartung of Bloomsburg University, Pennsylvania, for giving me the opportunity to write this article and to lecture it on November 13th, 1995, in their Department of Mathematics and Computer Sciences.

LOGICA SAU LOGICA MATEMATICĂ

Câte propoziţii sânt adevărate şi care anume dintre următoarele:

1. Există o propoziţie falsă printre cele n propoziţii.

2. Există două propoziţii false printre cele n propoziţii.

....

-. Există i propoziţii false printre cele n propoziţii.

....

n. Există n propoziţii false printre cele n propoziţii.

(O generalizare a unei proble propuse de prof. Francisco Bellot, revista NUMEROS, nr. 9/1984, p. 69, Insulele Canare, Spania).

Comentarii. Notăm cu P_i propoziţia $i, 1 \leq i \leq n$. Dacă n este par atunci propoziţiile $1, 2, \ldots, (n/2)$ sânt adevărate iar celelalte false. Se începe raţionamentul de la sfârşit; P_n nu poate să fie advărată, deci P_1 este adevărată; apoi P_{n-1} nu poate fi adevărată, deci P_2 este adevărată, etc.)

Remarcă. Dacă n este impar se obţine un **paradox**, deoarece urmând aceeaşi metodă de rezolvare găsim P_n falsă, implică P_1 adevărată; P_{n-1} falsă implică P_2 adevărată, ... $P_{\frac{n+1}{2}}$ falsă implică $P_{n+1-\frac{n+1}{2}}$ adevărată, adică $P_{\frac{n+1}{2}}$ falsă implică $P_{\frac{n+1}{2}}$ adevărată, absurd.

Dacă $n = 1$, se obţine o variantă a Paradoxului minciunosului ("Nu mint" este adevărat sau fals?)

1. Există o propoziţie falsă în acest dreptunghi.

Care este desigur un paradox.

["Gamma", Braşov, Anul IX, Nr. 1, noiembrie 1986.]

MATHEMATICS AND ALCOHOL AND GOD

Imaginary

God

Transcendentality

What's the probability

for God to be

in Number Theory ?

Artifica Mathematica -

in oder to kill the zero, the nothing

I don't like poeple who are happy, who had a great career (because it pushed on others and depersonalized them) - but those living unhappily, poorly, sickly and dying young.

If there exist irrational elements in man, then God exists. When we know what God is, we shall be gods ourselves (G.B.Show).

If there exist wars, genocides, then God does not exist.

If there exist sentences which cannot be proved or disproved within the system (K.Godel), then God exists.

"What's your religious beliefs?", and Lagrange answered: "I don't know"!

Mathematics and alcohol for God's sake, and instead of Him and to replace Him. Mystic Mathematics. Are science researchers guilty in front of God because of stealing Nature's secrets ?

God's Revolt

Mathematics' Revolt

Alcohol's Revolt

My revolt: I desire to create my own mathematics, new (and maybe strange, paradoxist) axioms - and put everything in it ! Develop an entire theory on this system of

anti-axioms. I'm writing the PARADOXIST MATHEMATICS

the PARADOXIST PHILOSOPHY

"Sommetimes great new ideas are born outside, not inside, the schools" (Dirk J. Struik).

God

Holy Mathematics with its fascinating infinite

Holy Alcohol with its degrees

Holy Philosophy

Music is sick in front of God

Picture is sick in front

Mathematics is sick

Creation is a drug I can't do without (Cecil B. Demille). Therefore: pray to God and drink because in this way you get closer to Him. Drink alcohol and solve diophantine equations !

CALCULATE - DRINK -PRAY

(This is not a paradox !)

SUBJECTIVE QUESTIONS AND ANWERS FOR A MATH INSTRUCTOR OF HIGHER EDUCATION

1) What are the instructor's general responsabilities ?

- participation in committee work and planning
- research and inovation
- in service training
- meetings
- to order necessary textbooks, audio-visual, and other instructional equipment for assigned courses
- to submit requests for supplies, equipment, and budgetary items in good order and on time
- to keep abreast of developments in subject field content and methods of instruction
- to assess and evaluate individual student progress; to maintain student records, and refer students to other appropriate college staff as necessary
- to participate on college-wide registration and advising
- effective and full use of the designated class meeting time
- adeqvated preparation for course instruction, course and curriculum planning
- teaching, advising students
- to be able to make decisions
- knowledge and use of material
- positive relationship
- knowlwdge of content
- to plan and implement these plans (or abandon them if they don't work) - short and long - term plannings
- to be a facilitator, motivator, model, assesor and evaluator of learning, counselor, classroom manager (i.e. to manage the behaviour of students, the environement, the curriculum)
- knowledge of teenage growth and development
- to continously develop instructional skills.

The most importand personal and academic characteristics of a teacher of higher education are: to be very good professionally in his/her field, to improve permanently his/her skills, to be dedicated to his/her work, to understand the stuidents' psychology, to be a good educator, to do attractive and intresting lessons, to make students learn to think (to solve not only mathematical problems, but also life ones), to try approching mathematics with what students are good at (telling'em, for example, that mathematics are applied anywhere in the nature), to conduct students in their scientific research, to advise them, to be involved in all scholar activeties and committee servicesl; to enjoy teaching.

The first day of school can be more mathematically recreative. Ask the students: What do you like in mathematics, and what don't you like ?

Tell them math jokes, games, proofs with mistakes (to be found!), stories about mathematicians lifes, connections between math and ... opposite fields, such as: arts, music, literature, poetry, foreing languages, etc.

2) What is the students evaluation of you as an instructor (negative opinions)?

- don't be too nice in the classroom (because some students take advantage of that matter and waste their and class time)
- to be more strict and respond firmly
- don't say: "this is easy, you should know this" beacouse one discourags students to ask questions
- attendence policy to be clear
- grammer skills, and listening skills
- patience with the students
- allow students to help each other when they don't understande me
- clear English
- sometimes there isn't enough time to cover all material
- to self-study the material and solve a lot of unassigned problems
- to talk louder to the class; to be more oriented towards the students and not the board/self
- to understand what the students ask me
- to take off points if the home work problems are wrong, instead of just giving points for trying
- to challenge students in learning
- to give examples of harder problems on the board
- to enjoy teaching (smile, joke ?)
- your methods should help students learning

3) What is the college's and university's mission and role in the society ?

- to assure that all students served by the College learn the skills, knowledge, behaviors, and attitudes necessary for productive living in a changing, democratic, multicultural society.

4) How do you see the future math teaching (new thechiques) ?

- teaching online
- telecourses (with videotapes and tapes)
- teaching using internet
- teaching by regular mail
- more electronic device tools in teaching (especially computers)
- interdisciplinarity teaching
- self-teaching (helping students to teach themselves)
- more mathimatics tought in connections with the social life (mathematical modeling)
- video conference style of teaching
- laboratory experiments

5) What about <Creative Solutions> ?

- the focus of the program is on developing student understanding of concepts and skills rather than <apparent understanding>
- students should be actively involved in problem - solving in new situations (creative solvers)
- non-routine problems should occur regularly in the student homework
- textbooks shall facilitate active involvement of students in the dicovery of mathematical ideas
- students should make conjectures and guesses, experiment and formulate hypothese and seek meaning
- the instructor should not let teaching of mathematics degenerate into mechanical manipulation without thought
- to teach students how to think, how to investigate a problem, how to do research in their own, how to solve a problem for which no method of solution has been provided

- homework assignments should draw the students' attention to underlying concepts
- to do a cognitive guided instruction
- to solve non-routine problems, multi-step problems
- to use a step-by-step procedure for problem solving
- to integrate ttradition with modern style teaching
- to emplasize the universality of mathematics
- to express mathematical ideas in a variety of ways
- to show students how to write mathematics, and how to reads mathematics
- interpretations of solutions
- using $MINITAB$ graphics to teach statistics (on the computer)
- tutorial programs on the computer
- developing manageable assessment procedures
- experemental teaching methods
- to mativate students to work and learn
- to stimulate mathematical reasoning
- to incorporate "real life" scenarios in teacher training programs
- homo faber + homo sapiens are inseparable (Antonio Gramsci, italian philosopher)
- to improve the critical thinking and reasoning skills of the students
- to teach students how to extend a concept
- to move from easy to medium and hard problems (gradually)
- math is learned by doing, not by watching
- the students should dedicate to the school
- to become familiar with symbols, rules, algorithms, key words and definitions
- to visualize math notions
- to use computer - generated patterns
- to use various problem-solving strategies such as:
    - perseverence
    - achievement motivation
    - role model
    - confidence
    - flexible thinking
    - fresh ideas
    - different approaches

· different data

- to use experimental teaching methods
- function ploters or computer algebra systems
- computer-based learning
- software development
- grant proposal writing
- inovative pedagogy
- to use multi-represantional strategies
- to try experimental tools
- to develop descussion groups
- symbol manipulation rules
- to solve template problems
- to do laboratory - based courses
- to think analytically
- to picture ourselves as teacher, or as students
- to use computer - generated patterns and new software tools
- to give the students educational and psychological tests to determine if any of them need special education (for handicaped or gifted students). - American Association on Mental Deficiency measures it.

6) How to diminish the computer anxiety ?

In oder to diminish the computer anxiety, a teacher needs to develop to the students:

- positive attitudes towards appropriate computer usege
- feeling of confidence in use of computers
- feeling of comfort with computers
- acceptance of computers as a problem-solving tool
- willingness to use a computer for tasks
- attitude of responsability for ethical use of computer
- attitude that computers are not responsible for "errors"
- free of fear and intimidation of computers (the students anxienty towards computer is diminished as their knowledge about computers increased)
- only after an algorithm is completely understood it is appropriate to rely on the computer to perform it

- computers help to remove the tedium of time - consuming calculations;
  - enable the students to consolidate the learning of the concepts and algorithms in math; the computer session is held at the end of the course when all the lectures and tutorials have been completed
  - to simulate real world phenomena
  - all students should learn to use calculaters
  - math is easier if a calculator is used to solve problems
  - calculator use is permissible on homework
  - using calculators makes students better problem solvers
  - calculators make mathematics fun
  - using calculator will make students try harder
  - the students should be able to
    - · assemble and start a computer
    - · understand the major parts of a computer
    - · use avariety of educational software
    - · distinguish the major instructional methodologies
    - · use word processor, datebase and spreadsheet programs
    - · attach and use a printer, peripherals, and lab probes
    - · use telecommunications networking
    - · use hypermedia tehnology
  - an instructor helps students to help themselves (it's interesting to study the epistemology of experience)

In the future the technology's role will increase due to the new kind of teaching: distance learning (internet, audio-visuals, etc.).

The technology is benefical becuase the students do not waste time graphing function anymore, but focusing on their interpretations.

7) Describe your experience teaching developmental mathematics including course names, semester taught and methods and techniques used.

In my teaching career of more than ten years experience I taught a variety of developmental mathematics courses, such as:

- Introductory Mathematics: Falls'88, '89, Springs'89, '90. Methods: problem solving participation in the class, small group work, guest speakers, discussion, student planning of assignments (to compose themselves problems of different styles and solve them by many methods), editing math problem solutions (there are students who know how to solve a problem, but they are not able to write correctly and completely their proof mathematically), math applied to real world problems (project), research work (how math is used in a job), recreational math approaches (logical games, jokes), etc.

- Prealgebra: Spring'82; - Algebra, Elem. Geomtry: '81, '89.

8) Briefly describe your **philosophy of theaching mathematics.** Decribe the aplication of this philosophy to a particular concept in a developmental mathematics course you have taught.

- My teaching philosophy is "concept centered" as well as "problem solving directed".

Makarenko: Everything can be taught to enybody if it's done at his/her level of knowledge. This focuses on promoting a student friendly environment where I not only lecture to provide the student a knowledge base by centering on concepts, but I also encourage peer mentoring with groups work to facilitate problem solving. It is my firm convinction that a student's perception, reasoning, and cognition can be strengthed with the application of both traditional and Alternative Learning Techniques and Student Interactive Activities.

- In my Introductory Mathematics course I taught about linear equations:
  - first I had to introduce the concept of variable, and then define the concept of equation; afterwards, tell the students why the equation is called linear; how the linear equation is used in the real world, its importance in the every day's life;
  - seconde I gave students an example of solving a linear equation on the board, showing them differnt methods; I classified them into consistent and inconsistent.

9) Describe **how you keep current with trends in mathematics instruction** and give one example of how you have integrated such a trend into the classroom.

- I keep current with trends in math instruction reading journals such as: "Journal for Reaserch in Mathematical Education", "Mathematics Teacher" (published be the National Council of Teachers of Mathematics, Reston, VA), "Journal of Computers in Mathematics and Science Teaching", "For the learning of mathematics", "Mathematics Teaching" (U.K.), "International Journal of Mathematical Education in Science and Techonology"; and participating with papers to the educational congresses, as: The

Fifth Conference on Teaching of Mathematics (Cambridge, June 21-22, 1996), etc.

Exemple: Intersubjectivity in Math: teaching to everybody at his/her level of understanding.

10) Describe your experience intergrating technology into teaching mathematics. Provide specific exemples of ways you have used technology in the mathematics classroom.

- I use graphic calculators (TI -85) in teaching Intermediate Algebra; for example: programming it to solve a quadratic equation (in all 3 cases, when D is $>, =, or < 0$).
- I used various software pachages of mathematics on IBM-PC or compatibles, such as: MPP, MAPLE, UA, etc. to give the students different approaches; for example in teaching Differential Equations I used MPP for solving a differential equation by Euler's method, changing many times the initial conditions, and graphing the solutions.

11) Describe your knowledge and/or experience as related to your ability to prepare clasroom materials.

Classroom materials that I use: handouts, different color markers, geometric instruments, take-home projects, cours notes, group projects, teaching outline, calculators, graphic calcultors, PC, projectors, books, journals, etc.

12) Describe the essential characteristics of an effective mathematics curriculum.

- To develop courses and programs that support the Coolege's vision of an educated person and a commitment to education as a lifelong process;
- To provide educational experiences designed to facilitate the individual's progress towards personal, academic, and work-based goals;
- To encourage the development of individual ideas and insights and aquisition of knowledge and skills that together result in an appreciation of cultural diversity and a quest for futher discovery;
- To respond to the changing educational, social, and technological needs of current and prospective students and community employers;

13) Provide specific examples of how you have and/or how you would develop and evaluate matematics curriculum.

In oder to develop a mathematics curriculum:

I identify unmet student need, faculty interest in a new area, request from employers, recommendation of advisory committee, result of program review, university curriculum development.

Criteria for evaluation of a mathematics curriculum:

- course/program is educationally sound and positively affects course/program offerings within district; course does not unnecessarily duplicate existing course or course content in other disciplines offered throughout the districr;
- development or modification of course/program does not adversely impact existing courses/programs offered throughout the district by competing for students and resources;
- course/program is compatible with the mission of the college.

14) Decribe your experience, education and training that has provided you with the knowledge of and ability to assess student achivement in methematics.

Courses I studed: History of Education, Introduction to Education, Philisophy, Child and Adolescent Psihology, Educational Psihology, General Psyhology, Methods of Teaching Mathematics, Analysis of Teaching and Research, Instructional Design and Evaluation, Learning Skills Theory, Historical / Philosophical / Social Education, Teaching Practice.

I taught mathematics in many countries, for many years, using various student assessments.

15) Provide specific examples of ways you have and/or ways you would assess student achivement in mathematics.

I assess students by: tests in the Testing Center, quizes in the classroom, homeworks, class participation (either solving problems on the board, or giving good answers for my questions), extra-work (voluntarily), take-home exams, research projects, frequency. Normally a test contains 10 problems, total being 100 points. For each homework I give 5 points, same for each extra-work, for each class participation. For more than 3 absences I sucstract points (one point for each absence), and later I withdraw the student.

Take-home exams, quizis, and research project have the worth of a test.

Finally I compute the average (my students know to assess themselves according to these rules, explained in the class and written in the syllabus).

16) This question is about motivating a typical community college class of students, which is very diverse.

a) What kinds of students are you likely to have in such a class?

Students of different races, genders, religions, ages, cultures, national origins, levels of preparendness, with or without phisical or mental handicaps.

b) How would you teach them ?

Catching their common interest, tutoing on a one-to-one basis students after class (according to each individual level of preparendness, knowledge), working differentially with categories of students on groups, being a resource to all students, using multirepresentational strategies, motivating and making them dedicate to the study, finding common factors of the class. Varying teaching stiels to respond to various student learning styles.

17) Given the fact that the community college philosophy encourages faculty members to contribute to the campus, the college, and the community, provide examples of how you have and /or would contribute to the campus, the college, and the community.

I have contributed to the college by:

- being an Associate Editor of the college (East Campus) "Math Power" journal;

- donating books, journals to the college (East Campus) Library;

- volunteering to help organizing the AMATYC math competitions (I have such experience from Romania and Morocco);

- representing the college at National/International Conferences on Mathematical and Educational Topics (as, for example, at Bloomsburg University, PA, Nov. 14, 1995);

- publishing papers, and therefore making free publicity for the college;

I would contribute to the college by:

- organizing a Math Club for interested students;

- cooperating with my fellow colleagues on educational projects sponsored by various foundations: National Science Foundation, Fullbright ... Guggemteim?

- socializing with my fellow coleagues to diverse activities needed to the college.

- being a liaison between the College and University in oder to frequenly update the University math software and documentation (public property, reach done will a grant from NSF).

18) Describe your experience within the last three years in teaching calculus for science and

engineering majors and/or survey calculus at a post secondary level.

I have taught Calculus I, II, III in many countries. I have insisted on solving most creatively problems in calculus, because most of them are open-ended (they have more than one correct answer or approach); sometimes, solving a problem relies on common sense ideas that are not stated in the problem. The fundamental basis od the Calculus class is what grphs symbolize, not how to draw them.

Using calclators or computers the students got reasonable approximation of a solution, which was usually just useful as an exact one.

19) Reform calculus a significant issue in math education today. Describe your thoughts on the strenghs and weaknesses of reform versus traditional calculus and indicate which form of calculculus you would prefer to teach.

Of course, I prefer to teach the Harvard Calculus, because it gives the students the skills to read graphs and think grphically, to read tables and think numerically, and to apply these skills along with their algebraic skills to modeling the real world (The Rule of Three); and Harvard Calculus also states that formal mathematical theory evolves from investigations of practical problems (The Way of Archimedes).

Wealnesses: the students mighn rely too much on calculators or computers ("the machines will think for us !"), forgetting to graph, solve, compute.

20) Describe your experience in curriculum development including course development, textbook or lab manual development, and development of alternative or innovative instructional methods.

I have developed a course of Calculus I, wrote and published a textbook of Calculus I for students, associated with various problems and solutions on the topic.

Concerning the alternative instructional methods, I'm studyng and developing The Intersujectivety Method of Teaching in Mathematics (inspired by some articles from "Journal for Research in Mathematical Education" and "International Journal of Mathamatical Education in Science and Technology").

21) Describe your education and/or experiences that would demonstrate your ability to proactively interact with and effectively teach students from each of the following: different races, cultures, ages, genders, and levels of preparednees. Provide examples of your interaction with and teaching of students from each of these groups.

I have taught mathematics in many countries: Romania (Europe), Morocco (Africa), Turkey (Asia), and USA. Therefore, I am accustomed to work with a diverse student population. More, each country had its educational rules, methods, styles, curriculum missions - including courses, programs, textbooks, math student competitions, etc. that I have acquired a very large experience. I like to work in a multi-cultural environement teaching in many languages, styles (according to the students' characteristics), being in touch with various professors arround the world, knowing many cultural habits.

Describe your professional development activities that help you stay in the field of mathematics. Give your best example of how you have integrated one thing into the classroom that came out of your professional development activities.

I subscribe to math journals, such as: "College Mathematics Journal", as a memeber of the Mathematical Association of America, and often go to the University Libreries, Science Section, to consult various publications.

I keep in touch with mathematicians and educators from all over, exchanging math papers and ideas, or meeting them at Conferences or Congresses of math or education. Studying about "intersurjectivity" in teaching, I got the idea of working differentially with my students, distributing them in groups of low level, medium level, high level according to their knowledge, and therefore assigning them appropriate special projects.

23)

a) What are the most important personal and academic characteristics of a teacher ?

b) At the end of your first year of district employment how will you determine whether or not you have been successfull ?

c) What are the greatest challenges in public education today ?

d)What do you want your students to learn ?

a) To be very good professional in his/her field, improving his/her skills permanently. To be dedicated to his/her work. To love the students and understand their psihology. To be a very good educator. To prepare every day the lesson (its objectives). To do attractive and interesting lessons.

b) Regarding the level of the class (the knowledge in math), the students grades, even their hobby for math (or at least their iterest).

c) To give the students a necessary luggage of knowledge and enough education such that they are able to fend for themselves in our society (they are prepared very well for the future).

d) To think. Brainstorm.

To solve not only mathematical problems, but also life problems.

24) What do you want to accomplish as a teacher ?

To get well prepared students with good behaviours.

25) How will (d0) you go about finding out students' attitudes and feelings about your class ?

I'll try to talk with every student to find out their opinions, dificulties, attitudes towards the teacher. Then, I'll try to adapt myself to the class level of knowledge and to be agreeable to the students. Besides that , I'll try to approach them in exracurriculum activities: soccer, tenis, chess, creative art and literature using mathematical algorithms/methods, improving my Spanish language.

26) An experienced teacher offers you following advice: "When you are teaching, be sure to command the respect of your students immediately and all will go well". How do you feel about this ?

I agree that in a good lesson the students should respect their teacher, and reciprocally. But the respect should not be "commanded", but earned. The teacher should not hurt the students by his/her words.

27) How do you go about deciding what it is that should be taught in your class ?

I follow the school plan, the mathematics text book, the school governing board directions. I talk with other mathematics teacher asking their opinions.

28) A parent comes to you and complains that what you are teaching his child is irrelevant to the child needs. How will you respond? I try to find out what he wants, what his needs are like. Then, maybe I have to change my teaching style. I respond that irrelevant subjects of today will be relevant subjects of tomorrow.

29) What do you think will (does) provide you the greatest pleasure in teaching ?

When students understand what I'm teaching about and they know how to use that in their life.

30) When you have some free time, what do you enjoy doing the most ?

Improving my mathematical skills (subscription to mathematical and education journals). Teaching mathematics beacame a hobby for me!

31) How do you go about finding what satudents are good at ?

I try to approach mathematics with what students are good at. For example: I tell'em : that mathematics are applied anywhere in the nuture and society, therefore in arts, in music, in literature, etc. Therfore, we can find a tangential joint between two apperent distinct (opposite) interests.

32) Would you rather try a lot of way-out teaching strategies or would you rather try to perfect the approaches which work best for you ? Explain your position.

Both: the way-out teaching strategies combined with approaches to students.

In each case the teacher should use the method/strategy that works better.

33) Do you like to teach with an overall plan in mind for the year, or would you rather just teach some interesting things and let the process determine the results ? Explain your position.

Normally I like to teach with an overall plan in mind, but some times - according with the class level and feelings - I may use the second strategy.

34) A student is doing poorly in your class. You talk to him/her, and he/she teills you that he/she considers you to be the poorest teacher he/she has every met. What would you do ?

I try to find out the opinions of other students about my teaching and to get a general opinion of the entire class. I give students a test with questions about my character, skills, style, teaching methods etc. in oder to find out my negative features and to correct/improve'em by working hard.

35) If there were absolutly no restrictions placed upon you, what would you most want to do in life ?

To set up a school (of mathematics espacially) for gifted and talented students with a math club for preparing students for school competitions.

36) How do you test what you teach ?

By written test, final exams, homeworks, class participation, special projects, extra homeworks, quizes, take-home exams.

37) Do you have and follow a course outline ? When would a variation from the outline be appropriate ?

- Yes, I follow a course outline.

- When I find out the students have gaphs in their knowledge and, therefore they are not able to understand the next topic to be taught. Or new topics are needed (due to scientific research or related to other disciplines).

38) Is student attendance important for your course ? Why or why not ? What are the student responsibilities necessary for success in your class ?

- Yes.

- If they miss many courses they will have difficulties to understand the others, because mathematics in like a chain.

- To work in the clasroom, to pay attention and ask questions, to do independent study at home too.

39) Describe your turnaround time for returning graded test and assignments.

I normally grade the tests over the weekends. Same for all other assignments.

40) Are you satisfied with the present textbooks ? Why or why not ?

- Yes.

- Because they gives the students the main ideas necessary in the technical world.

41) Describe some of the supplemental materials you minght use for this course.

- Personal computer with DERIVE sogtware pachage.

- TI-92 and an overhead projector.

- Tables of Laplace Transforms.

- Various handouts.

42) Describe your method of student recordkeeping.

- I keep track of: absences, homeworks, tests'grades, final exam's grade, class participations.

43) Describe how you assist or refer students who need remediation.

- I advise them to go to the College Tutoring Center.

- I encourage them to ask questions in the classroom, to work in groups with better students, to contact me before or after class.

44) What is your procedure for giving students feedback on their learning progress ?

- By the tests grades.

- By the work they are doing in the classroom.

45) How do you monitor your evaluation methods so that they are both fair and constructive ?

- My students are motivated to work and improve their grades by doing extra- (home)work.

- I compare my evaluation methods withe other instructors'.

- I also feel when a student masters or not a subject.

46) Describe your relationship with your colleagues.

- I share information, journals, books, samples of tests etc. with them.

- Good communication.

47) What procedures do you use to motivate students ?

- Giving'em a chance to improve their grades.

- Telling'em that if they don't learn a subject in mathematics, they would not understand the others (because mathematics is cyclic and linear).

48) Are you acquainted with district and campus policies and procedures ? Do you have any problems with any of the policies and procedures ?

I allways try to ajust myself to each campus's policy.

49) What mathematical education topic are you working in ?

- I'm studying the radical constructivism (jean liaget) and social constructivism (Vygotsky: to place communication and social life at the center of meaning - making), the intersubjectivity in mathematics, the meta-knowledge, the assessment standards.

Learning and teaching are processes of acculturation.

O GEOMETRIE PARADOXISTĂ[1]

În 1969, fascinat de geometrie, am construit un spaţiu parţial eucliadian şi parţial neeiclidian în acelaşi timp, înlocuind postulatul V al lui Euclid (axioma paralelelor) prin următoarea propoziţie stranie conţinând cinci aserţiuni:

a) există cel puţin o dreaptă şi un punct exterior ei în acest spaţiu astfel încât prin acel punct trece o singură dreaptă care nu intersectează dreapta iniţială;

[1 paralelă]

b) există cel puţin o dreaptă şi un punct exterior ei în acest spaţiu astfel încât un număr finit de drepte $l_1, \ldots, l_k$ $(k \geq 2)$ care trec prin acel punct nu intersectează dreapta iniţială;

[2 sau mai multe (dar în număr finit) paralele]

c) există cel puţin o dreaptă şi un punct exterior ei în acest spaţiu, astfel încât orice dreaptă trecând prin acel punct intersectează dreapta iniţială;

[0 (zero) paralele]

d) există cel puţin o dreaptă şi un punct exterior ei în acest spaţiu astfel încât un număr infinit de drepte care trec prin acest punct (dar nu toate) nu intersectează dreapta iniţială;

[un număr infinit de paralele, dar nu toate dreptele care trec prin acel punct]

e) există cel puţin o dreaptă şi un punct exterior ei în acest spaţiu astfel încât orice deaptă care trece prin acel punct nu intersectează dreapta iniţială;

[un număr infinit de paralele, toate dreptele trecând prin acel punct]

pe care am numit-o **geometrie paradoxistă**.

Aceasta reuneşte geometriile lui Euclid, Lobacrvski/Bolyai şi Riemann.

Important este găsirea unui model pentru această geometrie, şi studierea caracteristicilor ei.

[1] Prezentată la Bloomsburg, Pensylvania, pe 13 noiembrei 1995. Publicată în <Abracadabra>, Salinas, CA, ianuarie 1996, #39, p.22.

GEOMETRIC CONJECTURE

a) Let M be an interior point in an $A_1A_2 \ldots A_n$ convex polygon and P_i the projection of M on

$$A_iA_{i+1} \text{ i=1, 2, 3, \ldots, n.}$$

Then,

$$\sum_{i=1}^{n} \overline{MA_i} \geq c \sum_{i=1}^{n} \overline{MP_i}$$

where c is a constant to be found.

For $n = 3$, it was conjectured by Erdős in 1935 and solved by Mordell in 1937 and Kazarinoff in 1945. In this case $c = 2$ and the result is called the Erdős-Mordell Theorem.

Question: What happens in 3-space when the polygon is replaced by a polyhedron ?

b) More generally: If the projections P_i are considered under a given oriented angle $\alpha \neq 90$ degrees, what happens with the Erdős-Mordell Theorem and the various generalizations ?

c) In 3-space, we make the same generalization for a convex polyhedron

$$\sum_{i=1}^{n} \overline{MA_i} \geq c_1 \sum_{j=1}^{m} \overline{MP_j}$$

where $P_j, 1 \leq j \leq m$, are projections of M on all the faces of the polyhedron.

Futhermore,

$$\sum_{i=1}^{n} \overline{MA_i} \geq c_2 \sum_{k=1}^{r} \overline{MT_k}$$

where $T_k, 1 \leq k \leq r$, are projections of M on all sides of the polyhedron and c_1 and c_2 are constants to be determined.

[Kazarinoff conjectured that for the tetrahedron

$$\sum_{i=1}^{4} \overline{MA_i} \geq 2\sqrt{2} \sum_{i=1}^{4} \overline{MP_i}$$

and this is the best possible.

References

[1] P.Erdős, Letter to T.Yau, August, 1995.

[2] Alain Bouvier et Michel George, <Dictionnaire des Mathématiques>, Press Universitaires de France, Paris, p. 484.

A FUNCTION IN THE NUMBER THEORY

Summary

In this paper I shall construct a function η having the following properties:

$$\forall \eta \in Z \quad n \neq 0 \quad (\eta(n))! = M \cdot n, \tag{1}$$

$$\eta(n) \quad \text{is the smallest natural number with the property(1).} \tag{2}$$

We consider: $N = \{0, 1, 2, 3, \ldots\}$ and $N^* = \{1, 2, 3, \ldots\}$.

Lema 1. *$\forall k, p \in N^*, p \neq 1, k$ is uniquely written under the shape: $k = t_1 a_{n_1}^{(p)} + \ldots + t_l a_{n_l}^{(p)}$ where $a_{n_i}^{(p)} = \dfrac{p^{n_i - 1}}{p-1}$, $i = \overline{1,l}$, $n_1 > n_2 > \ldots > n_l > 0$ and $1 \leq t_j \leq p-1$, $j = \overline{1, l-1}$, $1 \leq t_l \leq p$, $n_j, t_j \in N$, $i = \overline{1,l}$ $l \in N^*$.*

Proof. The string $(a_n^{(p)})_{n \in N^*}$ consists of strictly increasing infinite natural numbers and $a_{n+1}^{(p)} - 1 = p \cdot a_n^{(p)}$, $;\forall n \in N^*, p$ is fixed,

$$a_1^{(p)} = 1, a_2^{(p)} = 1 + p, a_3^{(p)} = 1 + p + p^2, \ldots \Rightarrow N^* = \bigcup_{n \in N^*} ([a_n^{(p)}, a_{n+1}^{(p)}) \cap N^*)$$

where $[a_n^{(p)}, a_{n+1}^{(p)}) \cap [a_{n+1}^{(p)}, a_{n+2}^{(p)}) = \emptyset$ because $a_n^{(p)} < a_{n+1}^{(p)} < a_{n+2}^{(p)}$.

Let $k \in N^*$, $N^* = \bigcup_{n \in N^*} ([a_n^{(p)}, a_{n+1}^{(p)}) \cap N^*) \Rightarrow \exists! n_1 \in N^* : k \in ([a_{n_1}^{(p)}, a_{n_1+1}^{(p)}) \Rightarrow k)$ is uniquely written under the shape $k = \left[\dfrac{k}{a_{n_1}^{(p)}}\right] a_{n_1}^{(p)} + r_1$ (integer division theorem). We note $k = \left[\dfrac{k}{a_{n_1}^{(p)}}\right] = t_1 \Rightarrow k = t_1 a_{n_1}^{(p)} + r_1$, $r_1 < a_{n_1}^{(p)}$.

If $r_1 = 0$, as $a_{n_1}^{(p)} \leq k \leq a_{n_1+1}^{(p)} - 1 \Rightarrow 1 \leq t_1 \leq p$ and Lemma 1 is proved.

If $r_1 \neq 0 \Rightarrow \exists! n_2 \in N^* : r_1 \in [a_{n_2}^{(p)}, a_{n_2+1}^{(p)})$; $a_{n_1}^{(p)} > r_1 \Rightarrow n_1 > n_2$, $r_1 \neq 0$ and $a_{n_1}^{(p)} \leq k \leq$ $\leq a_{n_1+1}^{(p)} - 1 \Rightarrow 1 \leq t_1 \leq p-1$ because we have $t_1 \leq (a_{n_1+1}^{(p)} - 1 - r_1) : a_n^{(p)} < p_1$.

The procedure continues similarly. After a finite number of steps l, we achieve $r_l = 0$, as k =finite, $k \in N^*$ and $k > r_1 > r_2 \ldots > r_l = 0$ and between 0 and k there is only a finite number of distinct natural numbers.

Thus:

k is uniquely written: $k = t_1 a_{n_1}^{(p)} + r_1$, $1 \leq t_1 \leq p-1$, r is uniquely written: $r_1 = t_2 a_{n_2}^{(p)} + r_2$, $n_2 < n_1$,

$$1 \leq t_2 \leq p-1,$$

r_{l-1} is uniquely written: $r_{l-1} = t_l a_{n_l}^{(p)} + r_l$ and $r_l = 0$,

$$n_l < n_{l-1},\ 1 \leq t_l \leq p,$$

$\Rightarrow k$ is uniquely written under the shape $k = t_1 a_{n_1}^{(p)} + \ldots + t_l a_{n_l}^{(p)}$ with $n_1 > n_2 > \ldots > n_l;\ n_l > 0$ becuase $n_l \in N^*,\ 1 \leq t_j \leq p-1,\ j = \overline{1, l-1},\ 1 \leq t_l \leq p,\ l \geq 1$.

Let $k \in N^*$, $k = t_1 a_{n_1}^{(p)} + \ldots + t_l a_{n_l}^{(p)}$, with $a_{n_i}^{(p)} = \dfrac{p^{n_i} - 1}{p-1}$, $i = \overline{1, l}$, $l \geq 1$, $n_i, t_i \in N^*$, $i = \overline{1, l}$, $n_1 > n_2 > \ldots > n_l > 0$, $1 \leq t_j \leq p-1$, $j = \overline{1, l-1}$, $1 \leq t_l \leq p$.

I construct the function η_p, $p = \text{prime} > 0$, $\eta_p : N^* \to N^*$ thus:

$$\forall n \in N^* \ \ \eta_p(a_n^{(p)}) = p^n,$$

$$\eta_p(t_1 a_{n_1}^{(p)} + \ldots + t_l a_{n_l}^{(p)}) = t_1 \eta_p(a_{n_1}^{(p)}) + \ldots + t_l \eta_p(a_{n_l}^{(p)}).$$

Note 1. The function η_p is well defined for each natural number.

Proof.

Lema 2. $\forall k \in N^* \Rightarrow k$ *is uniquely written as* $k = t_1 a_n^{(p)} + \ldots + t_l a_{n_l}^{(p)}$ *with the conditions from Lemma 1* $\Rightarrow \exists!\ t_1 p^{n_1} + \ldots + t_l p^{n_l} = \eta_p(t_1 a_{n_1}^{(p)} + \ldots + t_l a_{n_l}^{(p)})$ *and* $t_{1p}^{n_1} + t_{lp}^{n_l} \in N^*$.

Lema 3. $\forall k \in N^*$, $\forall p \in N, p = prime \Rightarrow k = t_1 a_{n_1}^{(p)} + \ldots t_l a_{n_l}^{(p)}$ *with the conditions from Lemma 2* $\Rightarrow \eta_p(k) = t_1 p^{n_1} + \ldots + t_l p^{n_l}$.

It is known that $\left[\dfrac{a_1 + \ldots + a_n}{b}\right] \geq \left[\dfrac{a_1}{b}\right] + \ldots + \left[\dfrac{a_n}{b}\right]$ $\forall a_i, b \in N^*$ where through $[\alpha]$ we have written the integer side of number α. I shall prove that p's powers sum from the natural numbers make up the result factors $(t_1 p^{n_1} + \ldots + t_l p^{n_l})!$ is $\geq k$;

$$\left[\frac{t_1 p^{n_1} + \ldots + t_l p^{n_l}}{p}\right] \geq \left[\frac{t_1 p^{n_1}}{p}\right] + \ldots + \left[\frac{t_l p^{n_l}}{p}\right] = t_1 p^{n_1 - 1} + \ldots + t_l p^{n_l - 1}$$

$$\vdots$$

$$\left[\frac{t_1 p^{n_1} + \ldots + t_l p^{n_l}}{p^{n}}\right] \geq \left[\frac{t_1 p^{n_1}}{p^{n_l}}\right] + \ldots + \left[\frac{t_l p^{n_l}}{p^{n_l}}\right] = t_1 p^{n_1 - n_l} + \ldots + t_l p^0$$

$$\vdots$$

$$\left[\frac{t_1 p^{n_1} + \ldots + t_l p^{n_1}}{p^{n_1}}\right] \geq \left[\frac{t_1 p^{n_1}}{p^{n_1}}\right] + \ldots + \left[\frac{t_l p^{n_l}}{p^{n_l}}\right] = t_1 p^0 + \ldots + \left[\frac{t_l p^{n_l}}{p^{n_l}}\right].$$

Adding $\Rightarrow p$'s powers sum is $\geq t_1(p^{n_1 - 1} + \ldots + p^0) + \ldots + t_l(p^{n_l - 1} + \ldots + p^0) = t_1 a_{n_1}^{(p)} + \ldots t_l a_{n_l}^{(p)} = k$.

Theorem 1. *The function n_p, p =prime, defined previously, has the following properties:*

(1) $\forall k \in N^*$, $(n_p(k))! = Mp^k$.

(2) $\eta_p(k)$ *is the smallest number with the property (1).*

Proof.

(1) results from Lemma 3.

(2) $\forall k \in N^*$, $p \geq 2 \Rightarrow k = t_1 a_{n_1}^{(p)} + \ldots + t_l a_{n_l}^{(p)}$ (by Lemma 2) is uniquely written, where:

$n_i, t_i \in N^*$, $n_1 > n_2 > \ldots > n_l > 0$, $a_{n_i}^{(p)} = \dfrac{p^{n_i} - 1}{p - 1} \in N^*$, $i = \overline{1,l}$, $1 \leq t_j \leq p-1$, $j = \overline{1, l-1}$, $1 < t_l < p$.

$\Rightarrow \eta_p(k) = t_1 p^{n_1} + \ldots + t_l p^{n_l}$. I note: $z = t_1 p^{n_1} + \ldots t_l p^{n_l}$.

Let us prove the z is the smallest natural number with the property (1). I suppose by the method of reduction ad absurdum that $\exists \gamma \in N$, $\gamma < z$:

$\gamma! = Mp^k$;

$\gamma < z \Rightarrow \gamma \leq z - 1 \Rightarrow (z-1)! = Mp^k$.

$z - 1 = t_1 p^{n_1} + \ldots + t_l p_l^{n_l} - 1$; $n_1 > n_2 > \ldots > n_l \geq 0$ and $n_j \in N$, $j = \overline{1,l}$;

$\left[\dfrac{z-1}{p}\right] = t_1 p^{n_1 - 1} + \ldots + t_{l-1} p^{n_{l-1}-1} + t_l p^{n_l - 1} - 1$ as $\left[\dfrac{-1}{p}\right] = -1$ because $p \geq 2$,

$\left[\dfrac{z-1}{p^{n_l}}\right] = t_1 p^{n_1 - n_l} + \ldots + t_{l-1} p^{n_{l-1} - n_l} + t_l p^0 - 1$ as $\left[\dfrac{-1}{p^{n_l}}\right] = -1$ as $p \geq 2$, $n_l \geq 1$,

$\left[\dfrac{z-1}{p^{n_l+1}}\right] = t_1 p^{n_1 - n_l - 1} + \ldots + t_{l-1} p^{n_{l-1} - n_l - 1} + \left[\dfrac{t_l p^{n_l} - 1}{p^{n_l+1}}\right] = t_1 p^{n_1 - n_l - 1} + \ldots + t_{l-1} p^{n_{l-1} - n_l - 1}$

because $0 < t_l p^{n_l} - 1 \leq p \cdot p^{n_l} - 1 < p^{n_l + 1}$ as $t_l < p$;

$\left[\dfrac{z-1}{p^{n_{l-1}}}\right] = t_1 p^{n_1 n_{l-1}} + \ldots + t_{l-1} p^0 + \left[\dfrac{t_l p^{n_l} - 1}{p^{n_{l-1}}}\right] = t_1 p^{n_1 - n_{l-1}} + \ldots + t_{l-1} p^0$ as $n_{l-1} > n_l$,

$\left[\dfrac{z-1}{p^{n_1}}\right] = t_1 p^0 + \left[\dfrac{t_2 p^{n_2} + \ldots + t_l p^{n_l} - 1}{p^{n_1}}\right] = t_1 p^0$.

Because $0 < t_2 p^{n_2} + \ldots + t_l p^{n_l} - 1 \leq (p-1) p^{n_2} + \ldots + (p-1) p^{n_{l-1}} + p \cdot p^{n_l} - 1 \leq (p-1) \times$
$\times \sum\limits_{i=n_{l-1}}^{n_2} p^i + p^{n_l+1} - 1 \leq (p-1) \dfrac{p^{n_2+1}}{p-1} = p^{n_2+1} - 1 < p^{n_1} - 1 < p^{n_1} \Rightarrow \left[\dfrac{t_2 p^{n_2} + \ldots t_l p^{n_l} - 1}{p^{n_1}}\right] = 0$

$\left[\dfrac{z-1}{p^{n_1+1}}\right] = \left[\dfrac{t_1 p^{n_1} + \ldots t_l p^{n_l} - 1}{p^{n_1+1}}\right] = 0$

because: $0 < t_l p^{n_l} + \ldots + t_l p^{n_l} - 1 < p^{n_1+1} - 1 < p^{n_1+1}$ according to a reasoning similar to the previous one.

Adding $\Rightarrow$ p's powers sum in the natural numbers which make up the product factors $(z-1)!$ is:

$t_1(p^{n_1-1}+\ldots+p^0)+\ldots+t_{l-1}(p^{n_{l-1}-1}+\ldots+p^0)+t_l(p^{n_l-1}+\ldots+p^0)-1\cdot n_l = k-n_l < k-1 < k$ because $n_l > 1 \Rightarrow (z-1)! \neq Mp^k$, this contradicts the supposition made.

$\Rightarrow \eta_p(k)$ is tha smallest natural number with the property $(\eta_p(k))! = Mp^k$.

I construct a new function $\eta : Z \setminus \{0\} \to N$ as follows:

$$\begin{cases} \eta(\pm 1) = 0, \\ \forall n = \epsilon p_1^{\alpha_1} \ldots p_s^{\alpha_s} \text{ with } \epsilon = \pm 1,\ p_i = \text{prime}, \\ p_i \neq p_j \text{ for } i \neq j,\ \alpha_i \geq 1,\ i = \overline{1,s},\ \eta(n) = \max\limits_{i=\overline{1,s}}\{\eta_{p_i}(\alpha_i)\}. \end{cases}$$

Note 2. η is well defined and defined overall.

Proof.

(a) $\forall n \in Z, n \neq 0, n \neq \pm 1, n$ is uniquely written, independent of the order of the factors, under the shape of $n = \epsilon p_1^{\alpha_1} \ldots p_s^{\alpha_s}$ with $\epsilon = \pm 1$ where p_i =prime, $p_i \neq p_j, \alpha_i \geq 1$ (decompose into prime factors in Z =factorial ring).

$\Rightarrow \exists! \eta(n) = \max\limits_{i=\overline{1,s}}\{\eta_{p_i}(\alpha_i)\}$ as s =finite and $\eta_{p_i}(\alpha_i) \in N^*$ and $\exists \max\limits_{i=\overline{1,s}}\{\eta_{p_i}(\alpha_i)\}$

(b) $n = \pm 1 \Rightarrow \exists! \eta(n) = 0$.

Theorem 2. *The function η previously defined has the following properties:*

(1) $(\eta(n))! = Mn,\ \forall n \in Z \setminus \{0\}$;

(2) $\eta(n)$ *is the smallest natural number with this property.*

Proof.

(a) $\eta(n) = \max\limits_{i=\overline{1,s}}\{\eta_{p_i}(\alpha_i)\},\ n = \epsilon \cdot p_1^{\alpha_1} \ldots p_s^{\alpha_s},\ (n \neq \pm 1);\ (\eta_{p_1}(\alpha_1))! = Mp_1^{\alpha_1}, \ldots (n_{p_s}(\alpha_s))! =$ $= Mp_s^{\alpha_s}$.

Supposing $\max\limits_{i=\overline{1,s}}\{\eta_{p_i}(\alpha_1)\} = \eta_{p_{i_0}}(\alpha_{i_0}) \Rightarrow (\eta_{p_{i_0}}(\alpha_{i0}))! = Mp_{i_0}^{\alpha_{i_0}},\ \eta_{p_{i_0}}(\alpha_{i_0}) \in N^*$ and because $(p_i, p_j) = 1,\ i \neq j$,

$\Rightarrow (\eta_{p_{i_0}}(\alpha_{i_0}))! = Mp_j^{\alpha_j},\ j = \overline{1,s}$.

$\Rightarrow (\eta_{p_{i_0}}(\alpha_{i_0}))! = Mp_1^{\alpha_1} \ldots p_s^{\alpha_s}$.

$$(b)\ \ n = \pm 1 \Rightarrow \eta(n) = 0;\ 0! = 1, 1 = M\epsilon \cdot 1 = Mn.$$

$$(2)\ (a)\ n \neq \pm 1 \Rightarrow n = \epsilon p_1^{\alpha_1} \dots p_s^{\alpha_s} \Rightarrow \eta(n) = \max_{i=\overline{1,s}} \eta_{p_i}.$$

Let $= \max_{i=\overline{1,s}}\{\eta_{p_i}(\alpha_i)\} = \eta_{p_{i_0}}(\alpha_{i_0}),\ 1 \leq i \leq s;$

$\eta_{p_{i_0}}(\alpha_{i_0})$ is the smallest natural number with the property:

$$(\eta_{p_{i_0}}(\alpha_{i_0}))! = Mp_{i_0}^{\alpha_{i_0}} \Rightarrow \forall \gamma \in N,\ \gamma < \eta_{p_{i_0}}(\alpha_{i_0}) \Rightarrow \gamma! \neq Mp_{i_0}^{\alpha_{i_0}} \Rightarrow$$
$$\Rightarrow \gamma! \neq M\epsilon \cdot p_1^{\alpha_i} \dots p_{i_0}^{\alpha_{i_0}} \dots p_s^{\alpha_s} = Mn.$$

$\eta_{p_{i_0}}(\alpha_{i_0})$ is the smallest natural number with the property.

(b) $n = \pm 1 \Rightarrow \eta(n) = 0$ and it is the smallest natural number $\Rightarrow 0$ is the smallest natural number with the property $0! = M(\pm 1)$.

Note 3. The functions η_p are increasing, not injective, on $N^* \to \{p^k | k = 1, 2, \dots\}$ they are surjective.

The function η is increasing, not injective, it is surjective on $Z \setminus \{0\} \to N \setminus \{1\}$.

CONSEQUENCE. Let $n \in N^*$, $n > 4$. Then n =prime $\Leftrightarrow \eta(n) = n$.

Proof.

" $\Rightarrow$ " n =prime and $n \geq 5 \Rightarrow \eta(n) = \eta_n(1) = n$.

" $\Leftarrow$ " Let $\eta(n) = n$ and suppose by absurd that $n \neq$prime $\Rightarrow$

(a) or $n = p_1^{\alpha_1} \dots p_s^{\alpha_s}$ with $s \geq 2,\ \alpha_i \in N^*,\ i = \overline{1,s}$,

$$\eta(n) = \max_{i=\overline{1,s}}\{\eta_{p_i}(\alpha_i)\} = \eta_{p_{i_0}}(\alpha_{i_0}) < \alpha_{i_0} p_{i_0} < n$$

contradicts the assumtion; or

(b) $n = p_1^{\alpha_1}$ with $\alpha_1 \geq 2 \Rightarrow \eta(n) = \eta_{p_1}(\alpha_1) \leq p_1 \cdot \alpha_1 < p_1^{\alpha_1} = n$

because $\alpha_1 \geq 2$ and $n > 4$ and it contradicts the hypothesis.

Application

1. Find the smallest natural number with the property: $n! = M(\pm 2^{31} \cdot 3^{27} \cdot 7^{13})$.

Solution

$\eta(\pm 2^{31} \cdot 3^{27} \cdot 7^{13}) = \max\{\eta_2(31), \eta_3(27), \eta_7(13)\}$.

Let us calculate $\eta_2(31)$; we make the string $(a_n^{(2)})_{n \in N^*} = 1, 3, 7, 15, 31, 63, \dots$

$31 = 1 \cdot 31 \Rightarrow \eta_2(31) = \eta_2(1 \cdot 31) = 1 \cdot 2^5 = 32$.

Let's calculate $\eta_3(27)$ making the string $(a_n^{(3)})_{n \in N^*} = 1, 4, 13, 40, \ldots$; $27 = 2 \cdot 13 + 1 \Rightarrow \eta_3^{(27)} = \eta_3(2 \cdot 13 + 1 \cdot 1) = 2 \cdot \eta_3(13) + 1 \cdot \eta_3(1) = 2 \cdot 3^3 + 1 \cdot 3^1 = 54 + 3 = 57$.

Let's calculate $\eta_7(13)$; making the string $(a_n^{(7)})_{n \in N^*} = 1, 8, 57, \ldots$; $13 = 1 \cdot 8 + 5 \cdot 1 \Rightarrow \eta_7(13) = 1 \cdot \eta_7(8) + 5 \cdot \eta_7(1) = 1 \cdot 7^2 + 5 \cdot 7^1 = 49 + 35 = 84 \Rightarrow \eta(\pm 2^{31} \cdot 3^{27} \cdot 7^{13}) = \max\{32, 57, 84\} = 84 \Rightarrow 84! = M(\pm 2^{31} \cdot 3^{27} \cdot 7^{13})$ and 84 is the smallest number with this property.

2. Which are the numbers with the factorial ending in 1000 zeros ?

Solution

$n = 10^{1000}$, $(\eta(n))! = M10^{1000}$ and it is the smallest number with this property.

$\eta(10^{1000}) = \eta(2^{1000} \cdot 5^{1000}) = \max\{\eta_2(1000), \eta_5(1000)\} = \eta_5(1 \cdot 781 + 1 \cdot 156 + 2 \cdot 31 + 1) = 1 \cdot 5^5 + 1 \cdot 5^4 + 2 \cdot 5^3 + 1 \cdot 5^7 = 4005$, 4005 is the smallest number with this property. 4006, 4007, 4008, 4009 verify the property but 4010 does not because $4010! = 4009!4010$ has 1001 zeros.

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AN INFINITY OF UNSOLVED PROBLEMS CONCERNING A FUNCTION IN THE NUMBER THEORY

§1. Abstract.

W.Sierpinski has asserted to an international conference that if mankind lasted for ever and numbered the unsolved problems, then in the long run all these unsolved problems would be solved.

The purpose of our paper is that making an infinite number of unsolved problems to prove his supposition is not true. Moreover, the author considers the unsolved problems proposed in this paper can never be all solved!

Every period of time has its unsolved problems which were not previously recommended until recent progress. Number of new unsolved problems are exponentially incresing in comparison with ancient unsolved ones which are solved at present. Research into one unsolved problem may produce many new interesting problems. The reader is invited to exhibit his works about them.

§2. Introduction

We have constructed (*) a function η which associates to each non-null integer n the smallest positive integer m such that $m!$ is multiple of n. Thus, if n has the standerd form: $n = \epsilon p_1^{a_1} \dots p_r^{a_r}$, with all p_i distinct primes, all $a_i \in N^*$, and $\epsilon = \pm 1$, then $\eta(n) = \max_{1 \le i \le r}\{\eta_{p_i}(a_i)\}$, and $\eta(\pm 1) = 0$.

Now, we define the η_p functions: let p be a prime and $a \in N^*$; then $\eta_p(a)$ is a smallest positive integer b such that $b!$ is a multiple of p^a. Constructing the sequence:

$$\alpha_k^{(p)} = \frac{p^k - 1}{p - 1},\ k = 1, 2, \dots$$

we have $\eta_p(\alpha_k^{(p)}) = p^k$, for all prime p, an all $k = 1, 2, \dots$. Because any $a \in N^*$ is uniquely written in the form:

$$a = t_1 \alpha_{n_1}^{(p)} + \dots + t_l \alpha_{n_l}^{(p)}, \text{ where } n_1 > n_2 > \dots > n_l > 0,$$

and $1 \le t_j \le p - 1$ for $j = 0, 1, \dots, e - 1$ and $1 \le t_e \le p$, with all n_i, t_i from N, the author proved that

$$\eta_p(a) = \sum_{i=1}^{e} \eta_p(\alpha_{n_i}^{(p)}) = \sum_{i=1}^{e} t_i p^{n_i}.$$

§3. Some Properties of the Function η

Clearly, the function η is even: $\eta(-n) = \eta(n)$, $n \in Z^*$. If $n \in N^*$ we have:

$$\frac{-1}{(n-1)!} \leq \frac{\eta(n)}{n} \leq 1, \tag{1}$$

and: $\dfrac{\eta(n)}{n}$ is maximum if and only if n is prime or $n = 4$; $\dfrac{\eta(n)}{n}$ is minimum if and only if $n = k!$

Clearly η is periodical function. For p prime, the functions η_p are increasing, not injective but on $N^* \to \{p^k | k = 1, 2, \ldots\}$ they are surjective. From (1) we find that $\eta = o(n^{1+\epsilon}$, $\epsilon > 0$, and $\eta = O(n)$.

The function η is **generally increasing** on N^*, that is : $(\forall)n \in N^*$, $(\exists)m_0 \in N^*$, $m_0 = m_0(n)$, such that for all $m \geq m_0$ we have $\eta(m) \geq \eta(n)$ (and generally descreasing on Z_-^*; it is not injective, but it is surjective on $Z \setminus \{0\} \to N \setminus \{1\}$.

The number n is called a **barrier** for a number-theoretic function $f(m)$ if, for all $m < n$, $m + f(m) \leq n$ (P.Erdös and J.Selfridge). Does $\epsilon\eta(m)$ have infinitely many barriers, with $0 < \epsilon \leq 1$? [No, becuase there is a $m_0 \in N$ such that for all $n - 1 \geq m_0$ we have $\eta(n-1) \geq \dfrac{2}{\epsilon}$ (η is generally increasing), whence $n - 1 + \epsilon\eta(n - 1 \geq n + 1$.]

$\sum_{n \geq 2} 1/\eta(n)$ is divergent, because $1/\eta(n) \geq 1/n$.

$$\eta\left(\underbrace{2^{2^{\cdot^{\cdot^{\cdot^{2}}}}}}_{\text{k times}}\right) = 2 + \underbrace{2^{\cdot^{\cdot^{\cdot^{2^{n}}}}}}_{\text{k times}}$$

Proof: Let $a_m^{(2)} = 2^m - 1$, where $m = \underbrace{2^{\cdot^{\cdot^{\cdot^{2^{n}}}}}}_{\text{k times}}$; then $\eta(2^{2^m}) = \eta_2(2^m) = \eta_2(1 + a_m^{(2)}) = \eta_2(1) + \eta_2(a_m^{(2)} = 2 + 2^m$.

§4. Glossary of Simbols and Notes

A-sequence:	an integer sequence $1 \leq a_1 < a_2 < \ldots$ so that no a_i is the sum of distinct members of the sequence other than a_i (R. K. Guy);
Average Order :	if $f(n)$ is an arithmetical function and $g(n)$ is any simple function of n such that $f(1)+\ldots+f(n)-g(1)+\ldots+g(n)$ we say that $f(n)$ is of the average order of $g(n)$;
$d(x)$:	number of pozitive divisors of x;
d_x:	difference between two consecutive primes: $p_{x+1}-p_x$;
Dirichlet Series:	a series of the form $F(s)=\sum_{n=1}^{\infty}\frac{a_n}{n^s}$, s may be real or complex;
Generating Function:	any function $F(s)=\sum_{n=1}^{\infty}\alpha_n u_n(s)$ is considered as a generating function of α_n; the most usual form of $u_n(s)$ is: $u_n(s)=e^{-\lambda_n \cdot s}$, where λ_n is a sequence of positive numbers which increases steadily to infinity;
$Logx$:	Napierian logarithm of x, to base e;
Normal Order:	$f(n)$ has the normal order $F(n)$ if $f(n)$ is approximately $F(n)$ for almost all values of n, i.e. (2), $(\forall)\epsilon > 0, (1-\epsilon)\cdot F(n) < f(n) <$ $< (1+\epsilon).F(n)$ for almost all values of n; "almost all" n means that the numbers less than n which do not possess the property (2) is $o(x)$;
Lipshitz-Condition:	a function f verifies the Lipshitz-condition of order $\alpha \in (0,1]$ if $(\exists)k > 0 : \|f(x)-f(y)\| \leq k\|x-y\|^{\alpha}$; if $\alpha = 1, f$ is called a k Lipshitz-function; if $k < 1, f$ is called a contractant function;
Multiplicative Function:	a function $f : N^* \to C$ for which $f(1)=2$, and $f(m \cdot n) = f(m) \cdot f(n)$ when $(m,n)=1$;
$p(x)$:	largest prime factor of x;
Uniformly Distributed:	a set of pionts in (a,b) is uniformly distributed if every sub-interval of (a,b) contains its proper quota of points;
Incongruent Roots:	two integers x, y which satisfy the congruence $f(x) \equiv f(y) \equiv 0 \pmod{m}$ and so that $x \not\equiv y \pmod{m}$;

s-additive secuence:	a sequence of the form: $a_1 = \ldots = a_s = 1$ and $a_{n+s+1} = a_{n+1} + \ldots + a_{n+s}$, $n \in N^*$ (R.Queneau);
$s(n)$:	sum of aliquot parts (divisors of n other than n) of n; $\sigma(n) - n$;
$s^k(n)$:	k^{th} iterate of $s(n)$;
$s^*(n)$:	sum of unitary aliquot parts of n;
$r_k(n)$:	least number of numbers not exceeding n, which must contain a k-term arithmetic progression;
$\pi(x)$:	number of primes not exceeding x;
$\pi(x; a, b)$:	number of primes not excedding x and congruent to a modulo b;
$\sigma(n)$:	sum of divisors of n; $\sigma_1(n)$;
$\sigma_k(n)$:	sum of k-th powers of divisors of n;
$\sigma^k(n)$:	k-th iterate of $\sigma(n)$;
$\sigma^*(n)$:	sum of unitary divisors of n;
$\varphi(n)$:	Euler's totient function; number of numbers not exceeding n and prime to n;
$\varphi^k(n)$:	k-th iterate of $\varphi(n)$;
$\overline{\phi}(n)$:	$= n\prod(1 - p^{-1})$, where the product is taken over the distinct prime divisors of n;
$\Omega(n)$:	number of prime factors of n, counting repetitions;
$w(n)$:	number of distinct prime factors of n, counting repetitions;
$\lfloor a \rfloor$:	floor of a; greatest integer not great than a;
(m, n):	g.c.d. (greatest common divisor) of m and n;
$[m, n]$:	l.c.d. (least common multiple) of m and n;
$\|f\|$:	modulus or absolute value of f;
$f(x) \rightarrow g(x)$:	$f(x)/g(x) \rightarrow 1$ as $x \rightarrow \infty$; f is asymptotic to g;

$f(x) = o(g(x))$: $f(x)/g(x) \to 0$ as $x \to \infty$;

$\left.\begin{array}{l} f(x) = O(g(x)) \\ f(x) << g(x) \end{array}\right\}$: there is a constant c such that $|f(x)| < c \cdot g(x)$ to any x;

$\Gamma(x)$: Euler's function of first case (γ function); $\Gamma : R^* \to R$, $\Gamma(x) = = \int_0^\infty e^{-t}t^{x-1}dt$. We have $\Gamma(x) = (x-1)!$

$\beta(x)$: Euler's function of second degree (beta function); $\beta : R_+^* \times R_+^* \to R$, $\beta(u,v) = \Gamma(u)\Gamma(v)/\Gamma(u+v) = \int_0^1 t^{u-1} \cdot (1-t)^{v-1}dt$;

$\mu(x)$: Mõbius' function; $\mu : N \to N$, $\mu(1) = 1$; $\mu(n) = (-1)^k$ if n is the product of $k > 1$*distinct primes*; $\mu(n) = 0$ in all others cases;

$\theta(x)$: Tchebycheff θ-function; $\theta : R_+ \to R$, $\theta(x) = \sum \log p$, where the summation is taken over all primes p not exceeding x;

$\Psi(x)$: Tchebycheff's Ψ-function; $\Psi(x) = \sum_{n \leq x} \Lambda(n)$, with

$$\Lambda(n) = \begin{cases} \log p, \text{if } n \text{ is an integer power of the prime } p \\ 0, \text{ in all other cases.} \end{cases}$$

This glossary can be continued with OTHER (ARITHMETICAL) FUNCTIONS.

§5. General Unsolved Problems Concerning the Function η

(1) Is there a closed expression for $\eta(n)$?

(2) Is there a good asymptotic expression for $\eta(n)$? (If yes, find it.)

(3) For a fixed non-null integer m, does $\eta(n)$ divide $n - m$? (Particularly when $m = 1$.) Of course, for $m = 0$ it is trivial: we find $n = k!$, or n is squarefree, etc.

(4) Is η an algebraic finction? (If no, is there the *max Card* $\{n \in Z^* | (\exists) p \in R[x, y], p$ non-null polynomial, with $p(n, \eta(n)) = 0$ for all these $n\}$?) More generally we introduce the notion: g is a f-**function** if $f(x, g(x)) = 0$ for all x, and $f \in R[x, y], f$ non-null. Is η a f-function? (If no, is there the *max Card* $\{n \in Z^* | (\exists) f \in R[x, y], f$ non-null, $f(n, \eta(n)) = 0$ for all these $n\}$?)

(5) Let A be a set of consecutive integers from N^*. Find *max Card A* for which η is monotonous. For example, *Card* $A \geq 5$, because for $A = \{1, 2, 3, 4, 5\}$ η is $0, 2, 3, 4, 5$, respectively.

(6) A nimber is called an η-**algebraic number of degree** $n \in N^*$ if it is a root of the polynomial

$$(p) \qquad p_\eta(x) = \eta(n)x^n + \eta(n-1)x^{n-1} + \ldots + \eta(1)x^1 = 0.$$

An η-**algebraic field** M is the aggregate of all numbers

$$R_\eta(\nu) = \frac{A(\nu)}{B(\nu)},$$

where ν is a given η-algebraic number, and $A(\nu), B(\nu)$ are polynomials in ν of the form (p) with $B(\nu) \neq 0$. Study M.

(7) Are the points $p_n = \eta(n)/n$ uniformly distributed in the interval $(0,1)$?

(8) Is $0.0234537465114\ldots$, where the sequence of digits is $\eta(n), n \geq 1$, an irrational number?

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Is it possible to repersent all integer n under the form:

(9) $n = \pm\eta(a_1)^{a_2} \pm \eta(a_2)^{a_3} \pm \ldots \pm \eta(a_k)^{a_1}$, where the integrs $k, a_1, \ldots, a_k$, and the signs are conveniently chosen?

(10) But as $n = \pm a_1^{\eta(a_1)} \pm \ldots \pm a_k^{\eta(a_k)}$?

(11) But as $n = \pm a_1^{\eta(a_2)} \pm a_2^{\eta(a_3)} \pm \ldots \pm a_k^{\eta(a_1)}$?

Find the smallest k for which: $(\forall)n \in N^*$ at least one of the numbers $\eta(n), \eta(n+1), \ldots, \eta(n+k-1)$ is:

(12) A perfect square.

(13) A divisor of k^n.

(14) A multiple of fixed nonzero integer p.

(15) A factorial of a positive integer.

*

(16) Find a general from of the continued fraction expansion of $\eta(n)/n$, for all $n \geq 2$.

(17) Are there integers m, n, p, q, with $m \neq n$ or $p \neq q$, for which: $\eta(m) + \eta(m+1) + \ldots + \eta(m+p) = \eta(n) + \eta(n+1) + \ldots + \eta(n+q)$?

(18) Are there integers m, np, k with $m \neq n$ and $p > 0$, such that:

$$\frac{\eta(m)^2 + \eta(m+1)^2 + \ldots + \eta(m+p)^2}{\eta(n)^2 + \eta(n+1)^2 + \ldots + \eta(n+p)^2} = k?$$

(19) How many primes have the form:

$$\overline{\eta(n)\,\eta(n+1)\ldots\eta(n+k)},$$

for a fixed integer k? For example: $\overline{\eta(2)\,\eta(3)} = 23$, $\overline{\eta(5)\,\eta(6)} = 53$ are primes.

(20) Prove that $\eta(x^n) + \eta(y^n) = n(z^n)$ has an infinity of integer solutions, for any $n \geq 1$. Look, for example, at the solution $(5, 7, 2048)$ when $n = 3$. (On Fermat's last theorem.) More generally: the diophantine equation $\sum_{i=1}^{k} \eta(x_i^s) = \sum_{j=1}^{m} \eta(y_j^t)$ has an infinite number of solutions.

(21) Are there m, n, k non-null positive integers, $m \neq 1 \neq n$, for which $\eta(m \cdot n) = m^k \cdot \eta(n)$? Clearly, η is not homogenous to degree k.

(22) Is it possilble to find two distinct numbers k, n for which $\log_{\eta(k^n)} \eta(n^k)$ be an integer? (The base is $\eta(k^n)$.)

(23) Let the congruens be: $h_\eta(x) = c_n x^{\eta(n)} + \ldots + c_1 x^{\eta(1)} \equiv 0 (mod\ m)$. How many incongruent roots has h_η, for some given constant integers $n, c_1, \ldots, c_n$?

(24) We know that $e^x = \sum_{n=0}^{\infty} x^n/n!$. Calcilate $\sum_{n=1}^{\infty} x^{\eta(n)}/n!$, $\sum_{n=1}^{\infty} x^n/\eta(n)!$ and eventually some of their properties.

(25) Find the average order of $\eta(n)$.

(26) Find some $u_n(s)$ for which $F(s)$ is a generating function of $\eta(n)$, and $F(s)$ have at all a simple form. Particularly, calculate Dirichlet series $F(s) = \sum_{n=1}^{\infty} \eta(n)/n^s$, with $s \in R$ (or $s \in C$).

(27) Does $\eta(n)$ have a normal order?

(28) We know that Euler's constant is

$$\nu = \lim_{n\to\infty} (1 + \frac{1}{2} + \ldots + \frac{1}{n} - \log n).$$

Is $\lim_{n\to\infty}[1 + \sum_{k=2}^{n} 1/\eta(k) - \log \eta(n)]$ a constant? If yes, find it.

(29) Is there an m for which $\eta^{-1}(m) = \{a_1, a_2, \ldots, a_{pq}\}$ such that the numbers $a_1, a_2, \ldots, a_{pq}$ can constitute a matrix of p rows and q columns with the sum of elements on each row and each column constant? Particularly when the matrix is square.

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(30) Let $\{x_n^{(s)}\}_{n\geq 1}$ be a s-additive sequence. Is it possible to have $\eta(x_n^{(s)}) = x_m^{(s)}$, $n \neq m$? But $x_{\eta(n)}^{(s)} = \eta(x_n^{(s)})$?

(31) Does η verify a Lipschitz Condition?

(32) Is η a k-Lipschitz Condition?

(33) Is η a contractant function?

(34) Is it possible to construct an A-sequence $a_1, \ldots, a_n$ such that $\eta(a_1), \ldots, \eta(a_n)$ is an A-sequence, too? Yes, for example $2, 3, 7, 31, \ldots$ Find such an infinite sequence.

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Find the greatest n such that: if $a_1, \ldots, a_n$ constitute a p-sequence then $\eta(a_1), \ldots, \eta(a_n)$ constitute a p-sequence too; where a p-sequence means:

(35) Arithmetical progression.

(36) Geometrical progression.

(37) A complete system of modulo n residues.

Remark: let p be a prime, and $p, p^2, \ldots, p^p$ a geometrical progression, then $\eta(p^i) = ip, i \in \{1, 2, \ldots, p\}$, constitute an arithmetical progression of length p. In this case $n \to \infty$.

(38) Let's use the sequence $a_n = \eta(n), n \geq 1$. Is there a recurring relation of the form $a_n = f(a_{n-1}, a_{n-2}, \ldots,)$ for any n?

(39) Are there blocks of consecutive composite numbers $m+1, \ldots, m+n$ such that $\eta(m+1), \ldots, \eta(m+n)$ are composite numbers, too? Find the greatest n.

(40) Find the number of partitions of n as sum od $\eta(m)$, $2 < m \leq n$.

MORE UNSOLVED GENERAL PROBLEMS CONCERNING THE FUNCTION η

§6. Unsolved Problems Concerning the Function η and Using the Number Sequences

41-2065) Are there non-null and non-prime integers $a_1, a_2, \ldots, a_n$ in the relation P, so that $\eta(a_1), \eta(a_2), \ldots, \eta(a_n)$ are in the relation R? Find the greatest n with this property. (Of course, all a_i are distinct). Where each P, R can represent one of the following number sequences:

(1) Abundant numbers; $a \in N$ is abundant id $\sigma(a) > 2a$.

(2) Almost perfect numbers; $a \in N, \sigma(a) = 2a - 1$.

(3) Amicable numbers; in this case we take $n = 2; a, b$ are called amicable if $a \neq b$ and $\sigma(a) = \sigma(b) = a + b$.

(4) Augmented amicable numbers; in this case $n = 2$; a, b are called augmented amicable if $\sigma(a) = \sigma(b) = a + b - 1$ (Walter E. Beck and Rudolph M. Najar).

(5) Bell numbers: $b_n = \sum_{k=1}^{n} S(n, k)$, where $S(n, k)$ are Stirling numbers of second case.

(6) Bernulli numbers (Jacques 1st): B_n, the coefficients of the development in integer sequence of

$$\frac{1}{e^t-1}=1-\frac{t}{2}+\frac{B_1}{2!}t^2-\frac{B_2}{4!}t^4+\ldots+(-1)^{n-1}\frac{B_n}{(2n)!}t^{2n}+\ldots,$$

for $0<|t|<2\pi$; (here we always take $\lfloor 1/B_n \rfloor$).

(7) Catalan numbers; $C_1=1, C_n=\frac{1}{n}\binom{2n-1}{n-1}$ for $n\geq 2$.

(8) Carmichael numbers; an odd composite number a, which is a pseudoprime to base b for every b relatively prime to a , is called a Charmicael number.

(9) Congruent numbers; let $n=3$, and the numbers a,b,c; we mist have $a=b(modc)$.

(10) Cullen numbers: $c_n=n*2^n+1, n\geq 0$.

(11) C_1-sequence of integers; the author introduced a sequens $a_1,a_2,\ldots$ so that:

$$(\forall)i\in N^*,\ (\exists)j,k\in N^*,\ j\neq i\neq k\neq j,:a_i\equiv a_j(mod\ a_k).$$

(12) C_2-sequence of integers; the author defined other sequence $a_1,a_2,\ldots$ so that:

$$(\forall)i\in N^*,\ (\exists)j,k\in N^*,\ i\neq j\neq k\neq i,:a_j\equiv a_k(mod\ a_j).$$

(13) Deficient numbers; $a\in N^*, \sigma(a)<2a$.

(14) Euler numbers: the coefficients E_n in the expansion of $secx=\sum_{n\geq 0}E_nx^n/n!$; here we will take $|E_n|$.

(15) Fermat numbers: $F_n=2^{2^n}+1, n\geq 0$.

(16) Fibonacci numbers: $f_1=f_2 1,\ f_n=f_{n-1}+f_{n-2},\ n\geq 3$.

(17) Genocchi numbers: $G_n=2(2^{2n}-1)B_n$, where B_n are Bernulli numbers; always $G_n\in Z$.

(18) Harmonic mean; in this case every member of the sequence is the harmonic mean of the preceding members.

(19) Harmonic numbers; a number n is called harmonic if the harmonic mean of all divisors of n is an integer (C. Pomerance).

(20) Heteromeous numbers: $h_n=n(n+1, n\in N^*)$.

(21) K-hyperperfect numbers; a is k-hyperperfect if $a=1+\sum d_i$, where the numeration is taken over all proper divisors, $1<d_i<a$, or $k\sigma(a)=(k+1)a+k-1$ (Daniel Minoli and Robert Bear).

(22) Kurepa numbers: $!n=0!+1!+2!+\ldots+(n-1)!$

(23) Lucas numbers: $L_1=1, L_2=3, L_n=L_{n-1}+L_{n-2}, n\geq 3$.

(24) Lucky numbers: from the natural numbers strike out all even numbers, leaving the odd numbers; apart from 1, the first remaining number is 3; strike out every third member in

the new sequence; the next member remaining is 7; strike out every seventh member in this sequence; next 9 remains; etc. (V.Gardiner, R.Lazarus, N.Metropolis, S.Ulam).

(25) Mersenne numbers: $M_p = 2^p - 1$.

(26) m-perfect numbers; a is m-perfect if $\sigma^m(a) = 2a$ (D.Bode).

(27) Multiply perfect (or k-fold perfect) numbers; a is k-fold perfect if $\sigma(a) = ka$.

(28) Perfect numbers; a is perfect if $\sigma(a) = 2a$.

(29) Polygonal numbers (reperesented on the perimeter of a polygon): $p_n^k = k(n-1)$.

(30) Polygonal numbers (represented on the closed surface of a polygon):
$p_n^k = \dfrac{(k-2)n^2 - (k-4)n}{2}$.

(31) Primitive abundant numbers; a is a primitive abundant if it is abundant, but none of its proper divisors are.

(32) Primitive pseudoperfect numbers; a is primitive pseudoperfect if it is pseudoperfect, but none of its proper divisors are.

(33) Pseudoperfect numbers; a is pseudoperfect if it is equal to the sum of some of its proper divisors (W.Sierpinski).

(34) Pseudoprime numbers to base b; a is pseudoprime to base b if a is an odd composite number for which $b^{a-1} \equiv 1 \pmod{a}$ (C.Pomerance, J.L. Selfridge, S.Wagstaff).

(35) Pyramidal numbers: $\pi_n = \dfrac{1}{6}n(n+1)(n+2), n \in N^*$.

(36)Pythagorian numbers; let $n = 3$ and a, b, c be integers; then one must have the relation: $a^2 = b^2 + c^2$.

(37) Quadratic residues of a fixed prime p: the nomzero number r for which the congruence $r \equiv x^2 \pmod{p}$ has solutions.

(38) Quasi perfect numbers; a is quasi perfect if $\sigma(a) = 2a + 1$.

(39) Reduced amicable numbers; we take $n = 2$; two integers a, b for which $\sigma(a) = \sigma(b) = a + b + 1$ are called reduced amicable numbers (Walter E. Beck and Rudolph M. Najar).

(40) Stirling numbers of first case: $s(0,0) = 1$, and $s(n,k)$ is the coefficient of x^k from the development $x(x-1)\ldots(x-n+1)$.

(41) Stirling numbers of second case: $S(0,0) = 1$, and $S(n,k)$ is the coefficient of the polynom $x^{(k)} = x(x-1)\ldots(x-k+1)$, $1 \le k \le n$, from the development (which is uniquely writen):

$$x^n = \sum_{k=1}^{n} S(n,k)x^{(k)}.$$

(42) Superperfect numbers; a is superperfect if $\sigma^2(a) = 2a$ (D.Surynarayana).

(43) Untouchable numbers; a is untouchable if $s(x) = 1$ has no solution (Jack Alanen).

(44) U-numbers: starting from arbitrary u_1 and u_2 continue with those numbers which can be expressed in just one way as the sum of two distinct earlier members of the sequence (S.M.Ulam).

(45) Weird numbers: a is called weird if it is abundant but not pseudoperfect (S.J.Benkoski).

MORE NUMBER SEQUENCES

The unsolved problem No. 41 is obtained by taking $P = (1)$ and $R = (1)$.

The unsolved problem No. 42 is obtained by taking $P = (1), R = (2)$.

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The unsolved problem No. 2065 is obtained by taking $p = (45), R = (45)$.

OTHER UNSOLVED PROBLEMS COMCERNING THE FUNCTION η AND USING NUMBER SEQUENCES

§7. Unsolved Diophantine Equations Concerning the Function η

2066) Let $0 < k \leq 1$ be a rational number. Does the diophantine equation $\eta(n)/n = k$ always have solutions? Find all k so that this equation has an infinite number of solutions. (For example, if $k = 1/r, r \in N^*$, then $n = rp_{a+h}, h = 1, 2, \ldots$, all p_{a+h} are primes, and a is a chosen index such that $p_{a+1} > r$.)

2067) Let $\{a_n\}_{n\geq 0}$ be a sequence, $a_0 = 1, a_1 = 2$, and $a_{n+1} = a_{\eta(n)} + \eta(a_n)$. Are there infinitely many pairs $(m, n), m \neq n$, for which $a_m = a_n$? (For example: $a_9 = a_{13} = 16$.)

2068) Conjecture: the equation $\eta(x) = \eta(x + 1)$ has no solution.

Let m, n be fixed integers. Solve the diophantine equations:

2069) $\eta(mx + n) = x$.

2070) $\eta(mx + n) = m + nx$.

2071) $\eta(mx + n) = x!$

2072) $\eta(x^m) = x^n$.

2073) $\eta(x)^m = \eta(x^n)$.

2074) $\eta(mx + n) = \eta(x)^y$.

2075) $\eta(x) + y = x + \eta(y), x$ and y are not primes.

2076) $\eta(x) + \eta(y) = \eta(x + y), x$ and y are not twin primes. (Generally, η is not additive.)

2077) $\eta(x + y) = \eta(x) \cdot \eta(y)$. (Generallly, η is not an exponential function.)

2078) $\eta(xy) = \eta(x)\eta(y)$. (Generally, η is not a multiplicative function.)

2079) $\eta(mx + n) = x^y$.

2080) $\eta(x)y = x\eta(y), x$ and y are not primes.

2081) $\eta(x)/y = x/\eta(y), x$ and y are not primes. (Particularly when $y = 2^k, k \in N$, i.e., $\eta(x)/2^k$ is a dyadic rational numbers.)

2082) $\eta(x)^y = x^{\eta(y)}, x$ and y are not primes.

2083) $\eta(x)^{\eta(y)} = \eta(x^y)$.

2084) $\eta(x^y) - \eta(z^w) = 1$, with $y \neq 1 \neq w$. (On Catalan's problem.)

2085) $\eta(x^y) = m, y \geq 2$.

2086) $\eta(x^x) = y^y$. (A trivial solution: $x = y = 2$).

2087) $\eta(x^y) = y^x$. (A trivial solution: $x = y = 2$).

2088) $\eta(x) = y!$ (An example: $x = 9, y = 3$.)

2089) $\eta(mx) = m\eta(x), m \geq 2$.

2090) $m^{\eta(x)} + \eta(x)^n = m^n$.

2091) $\eta(x^2)/m \pm \eta(y^2)/n = 1$.

2092) $\eta(x_1^{y_1} + \ldots + x_r^{y_r}) = \eta(x_1)^{y_1} + \ldots + \eta(x_r)^{y_r}$.

2093) $\eta(x_1! + \ldots + x_r!) = \eta(x_1)! + \ldots + \eta(x_r)!$

2094) $(x, y) = (\eta(x), \eta(y)), x$ and y are not primes.

2095) $[x, y] = [\eta(x), \eta(y)], x$ and y are not primes.

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OTHER UNSOLVED DIOPHANTINE EQUATIONS CONCERNING THE FUNCTION η ONLY

§8. Unsolved Diphantine Equations Concerning the Function η in Correlation with Other Functions

Let m, n be fixed integers. Solve the diophantine equations:

2096-2102) $\eta(x) = d(mx + n)$

$\eta(x)^m = d(x^n)$

$\eta(x) + y = x + d(y)$

$\eta(x) \cdot y = x \cdot d(y)$

$\eta(x)/y = d(y)/x$

$\eta(x)^y = x^{d(y)}$

$\eta(x)^y = d(y)^x$

2103-2221) Same equations as befor, but we substitute the function $d(x)$ with d_x, $p(x)$, $s(x)$, $s^k(x)$, $s^*(x), r_k(x), \pi(x), \pi(x; m, n)$, $\sigma_k(x), \sigma^k(x), \sigma^*(x)$, $\varphi(x), \varphi^k(x), \overline{\phi}(x), \Omega(x), \omega(x)$ respectively.

2222) $\eta(s(x, y)) = s(\eta^{(x)}, \eta(y))$.

2223) $\eta(S(x, y)) = S(\eta(x), \eta(y))$.

2224) $\eta(\lfloor x \rfloor) = \lfloor \Gamma(x) \rfloor$.

2225) $\eta(\lfloor x - y \rfloor) = \lfloor \beta(x, y) \rfloor$.

2226) $\beta(\eta(\lfloor x \rfloor), y) = \beta(x, \eta(\lfloor y \rfloor))$.

2227) $\eta(\lfloor \beta(x, y) \rfloor) = \lfloor \beta(\eta(\lfloor x \rfloor), \eta(\lfloor y \rfloor)) \rfloor$.

2228) $\mu(\eta(x)) = \mu(\varphi(x))$.

2229) $\eta(x) = \lfloor \Theta(x) \rfloor$.

2230) $\eta(x) = \lfloor \Psi(x) \rfloor$.

2231) $\eta(mx + n) = A_x^n = x(x-2)\ldots(x-n+1)$.

2232) $\eta(mx + n) = Ax^m$.

2233) $\eta(mx + n) = \begin{pmatrix} x \\ n \end{pmatrix} = \frac{x!}{n!(x-n)!}$.

2234) $\eta(mx + n) = \begin{pmatrix} x \\ m \end{pmatrix}$.

2235) $\eta(mx + n) = p_x$ =the x-th prime.

2236) $\eta(mx + n) = \lfloor 1/B_x \rfloor$.

2237) $\eta(mx + n) = G_x$.

2238) $\eta(mx + n) = k_x = \begin{pmatrix} x + n - 1 \\ n \end{pmatrix}$.

2239) $\eta(mx + n) = k_x^m$.

2240) $\eta(mx + n) = s(m, x)$.

2241) $\eta(mx + n) = s(x, n)$.

2242) $\eta(mx + n) = S(m, x)$.

2243) $\eta(mx+n)=S(x,n)$.

2244) $\eta(mx+n)=\pi_x$.

2245) $\eta(mx+n)=b_x$.

2246) $\eta(mx+n)=|E_x|$.

2247) $\eta(mx+n)=!x$.

2248) $\eta(x)\equiv\eta(y) \pmod{m}$.

2249) $\eta(xy)\equiv x \pmod{y}$.

2250) $\eta(x)(x+m)+\eta(y)(y+m)=\eta(z)(z+m)$.

2251) $\eta(mx+n)=f_x$.

2252) $\eta(mx+n)=F_x$.

2253) $\eta(mx+n)=M_x$.

2254) $\eta(mx+n)=c_x$.

2255) $\eta(mx+n)=C_x$.

2256) $\eta(mx+n)=h_x$.

2257) $\eta(mx+n)=L_x$.

More unsolved diophantine equations concerning the function η in correlation with other functions.

§9. Unsolved Diophantine Equations Concerning The Function η in Composition with Other Functions

2258) $\eta(d(x))=d(\eta(x)), x$ is not prime.

2259-2275) Same equations as this, but we substitute the function $d(x)$ with d_x, $p(x)$, $\ldots$, $\omega(x)$ respectively.

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More unsolved diophantine equations concerning the function η in composition with other functions. (For example: $\eta(\pi(4(x)))=\varphi(\eta(\pi(x)))$, etc.)

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§10. Unsolved Diophantine Inequations Concerning the Function η

Let m,n be fixed integers. Solve the following diophantine inequalities:

2276) $\eta(x)\geq\eta(y)$.

2277) is $0 < \{x/\eta(x)\} < \{\eta(x)/x\}$ infinitely often?

where $\{a\}$ is the factorial part of a.

2278) $\eta(mx+n) < d(x)$.

2279-2300) Same (or similar) inequalities as this, but we substitute the function $d(x)$ with d_x, $p(x)$, ..., $\omega(x)$, $\Gamma(x)$, $\beta(x,x)$, $\mu(x)$, $\Theta(x)$, $\Psi(x)$, respectively.

....

More unsolved diophantine inequations concerning the function η in correlation (or composition, etc.) with other function. (For example: $\Theta(\eta(\lfloor x \rfloor)) < \eta(\lfloor \Theta(x) \rfloor)$, etc.)

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§11. Arithmetic Functions Constructed by Means of the Function η

UNSOLVED PROBLEMS CONCERNING THESE NEW FUNCTIONS

I. The function $S_\eta : N^* \to N, S_\eta(x) = \sum\limits_{0<n\leq x} \eta(n)$.

2301) Is $\sum\limits_{x\geq 2} S_\eta(x)^{-1}$ a convergent series?

2302) Find the smallest k for wich $\underbrace{(S_\eta \circ \ldots \circ S_\eta)}_{\text{k times}}(m) \geq n$, for m, n fixed integers.

2303-4602) Study S_η. The same (or similar) questions for S_η as for η.

II. The function $C_\eta : N^* \to Q, C_\eta(x) = \frac{1}{x}(\eta(1)+\eta(2)+\ldots+\eta(x))$ (sum of Cesaro concerning the function η).

4603) Is $\sum\limits_{x\geq 1} C_\eta(x)^{-1}$ a convergent series?

4604) Find the smallest k for which $\underbrace{(C_\eta \circ \ldots \circ C_\eta)}_{\text{k times}}(m) \geq n$, for m, n fixed integers.

4605-6904) Study C_η. The same (or similar) questions for C_η as for η.

III. The function $E_\eta : N^* \to N, E_\eta(x) = \sum\limits_{k=1}^{k_0} \eta^{(k)}(x)$, where $\eta^{(1)} = \eta$ and $\eta^{(k)} = \eta \circ \ldots \circ \eta$ of k times, and k_0 is the smallest integer k for which $\eta^{(k+1)}(x) = \eta^{(k)}(x)$.

6905) Is $\sum\limits_{x\geq 2} E_\eta(x)^{-1}$ aconvergent series?

6906) Find the smallest x for which $E_\eta(x) > m$, where m is a fixed integer.

6907-9206) Study E_η. The same (or similar) questions for S_η as for η.

IV. The function $F_\eta : N \setminus \{0,1\} \to N, F_\eta = \sum_{\substack{0<p\leq x \\ \text{p prime}}} \eta_p(x)$.

9207) Is $\sum_{x\geq 2} F_\eta(x)^{-1}$ aconvergent series?

9208-11507) Study the function F_η. The same (or similar) questions for F_η as for η .

V. The function $\alpha_\eta : N^* \to N, \alpha_\eta(x) = \sum_{n=1}^{x} \beta(n)$, where $\beta(n) = \begin{cases} 0, & \text{if } \eta(n) \text{ is even;} \\ 1, & \text{if } \eta(n) \text{ is odd.} \end{cases}$

11508) Let $n \in N^*$. Find the smallest k for which $\underbrace{(\alpha_\eta \circ \ldots \circ \alpha_\eta)}_{\text{k times}}(n) = 0$.

11509-13808) Study α_η. The same (or similar) questions for α_η as for η.

VI. The function $m_\eta : N^* \to N, m_\eta(j) = a_j, 1 \leq j \leq n$, fixed integers, and $m_\eta(n+1) = = min_i\{\eta(a_i + a_{n-i})\}, etc.$

13809) Is $\sum_{x\geq 1} m_\eta(x)^{-1}$ a convergent series?

13810-16109) Study m_η. The same (or similar) questions for m_η as for η.

VII. The function $M_\eta : N^* \to N$. A given finite positive integer sequence $a_1, \ldots, a_n$ is successively extended by:

$M_\eta(n+1) = \max_j\{\eta(a_i + a_{n-i})\}$, etc.

$M_\eta(j) = a_j, 1 \leq j \leq n$.

16110) Is $\sum_{x\geq 1} M_\eta(x)^{-1}$ a convergent series?

16111-18410) Study M_η. The same (or similar) questions for M_η as for η.

VIII. The function $\eta^{-1}_{\min} : N \setminus \{1\} \to N, \eta^{-1}_{\min}(x) = \min\{\eta^{-1}(x)\}$ where $\eta^{-1}(x) = = \{a \in N | \ \eta(a) = x\}$. For example $\eta^{-1}(x) = \{2^4, 2^4 \cdot 3, 2^4 \cdot 3^2, 3^2 \cdot 2, 3^2 \cdot 2^2, 3^2 \cdot 2^3\}$, whence $\eta^{-1}_{\min}(6) = 9$.

18411) Find the smallest k for which $\underbrace{(\eta^{-1}_{\min} \circ \ldots \circ \eta^{-1}_{\min})}_{\text{k times}}(m) \geq n$.

18412-20711) Study $\eta^{-1}_{\min}$. The same (or similar) questions for $\eta^{-1}_{\min}$ as for η.

IX. The function $\eta^{1}_{card} : N \to N, \eta^{-1}_{card}(x) = Card\{\eta^{-1}(x)\}$, where $CardA$ means the number of elements of the set A.

20712) Find the smallest k for which $\underbrace{(\eta^{-1}_{card} \circ \ldots \circ \eta^{-1}_{card})}_{\text{k times}}(m) \geq n$, for m, n fixed integers.

20713-23012) Study η^{-1}_{card}. The same (or similar) questions for η^{-1}_{card} as for η.

X. The function $d_\eta : N^* \to N, d_\eta(x) = |\eta(x+1) - \eta(x)|$. Let $d_\eta^{(k+1)} : N^* \to N, d_\eta(x) = |d_\eta^{(k)}(x+1) - d_\eta^{(k)}|$, for all $k \in N^*$, where $d_\eta^{(1)}(x) = d_\eta(x)$.

23013) Conjecture: $d_\eta^{(k)}(1) = 1$ or 0, for all $k \geq 2$. (This reminds us of Gillreath's conjecture on primes.) For example:

$\eta(1) = 0$	2	1	1	1	0	1	0	0	1	0	1	0	1	0	1	1	1	1	0
$\eta(2) = 2$	1	0	0	1	1	1	0	1	1	1	1	1	1	1	0	0	0	1	
$\eta(3) = 3$	1	0	1	0	0	1	1	2	2	0	0	0	2	1	0	0	1		
$\eta(4) = 4$	1	1	1	0	1	0	3	0	2	0	0	2	1	1	0	1			
$\eta(5) = 5$	2	2	1	1	1	3	3	2	2	0	2	1	0	1	1				
$\eta(6) = 3$	4	1	0	0	4	0	1	0	2	2	1	1	1	0					
$\eta(7) = 7$	3	1	0	4	4	1	1	2	0	1	0	0	1						
$\eta(8) = 4$	2	1	4	0	3	0	3	2	1	1	0	1							
$\eta(9) = 6$	1	5	4	3	3	3	1	1	0	1	1								
$\eta(10) = 5$	6	1	1	0	0	2	0	1	1	0									
$\eta(11) = 11$	7	2	1	0	2	2	1	0	1										
$\eta(12) = 4$	9	3	1	2	4	1	1	1											
$\eta(13) = 13$	6	4	3	6	5	2	2												
$\eta(14) = 7$	2	1	9	1	7	0													
$\eta(15) = 5$	1	10	10	8	7														
$\eta(16) = 6$	11	0	2	1															
$\eta(17) = 17$	11	2	1																
$\eta(18) = 6$	13	1																	
$\eta(19) = 19$	14																		
$\eta(20) = 5$																			

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23014-25313) Study $d_\eta^{(k)}$. The same (or similar) questions for $d_\eta^{(k)}$ as for η.

XI. The function $\omega_\eta : N^* \to N, \omega_\eta(x)$ is the number of m, with $0 < m \leq x$, so that $\eta(m)$ divides x. Hence, $\omega_\eta(x) \geq \omega(x)$, and we have equality if $x = 1$ or x is a prime.

25314) Find the smallest k for which $\underbrace{(\omega_\eta \circ \ldots \circ \omega_\eta)}_{\text{k times}}(x) = 0$, for a fixed integers x.

25315-27614) Study ω_η. The same (or similar) questions for ω_η as for η.

XII. The function $M_\eta : N^* \to N, M_\eta(x)$ is the number of m, with $0 < m \le x^k$, so that $\eta(m)$ is a multiple of x. For example $M_\eta(3) = Card(1,3,6,9,12,27) = 6$. If p is a prme $M_\eta(p) = Card\{1, a_2, \ldots, a_r\}$, then all $a_i, 2 \le i \le r$, are multiples of p.

27615) Let m, n be integer numbers. Find the smallest k for which $\underbrace{(M_\eta \circ \ldots \circ M_\eta)}_{\text{k times}}(m) \ge n$.

27616-29915) Study M_η. The same (or similar) questions for M_η as for η.

XIII. The function $\sigma_\eta : N^* \to N, \sigma_\eta(x) = \sum_{\substack{d|x \\ d>0}} \eta(d)$.

For example $\sigma_\eta(18) = \eta(1) + \eta(2) + \eta(3) + \eta(6) + \eta(9) + \eta(18) = 20, \sigma_\eta(9) = 9$.

29916) Are there an infinity of nonprimes n so that $\sigma_\eta(n) = n$?

29917-32216) Study σ_η. The same (or similar) questions for σ_η as for η.

XIV. The function $\pi_\eta : N \to N, \pi_\eta(x)$ is the number of numbers n so that $\eta(n) \le x$. If $p_1 < p_2 < \ldots < p_k \le n < p_{k+1}$ is the primes sequence, and for all $i = 1, 2, \ldots, k$ we have $p_i^{a_i}$ divides $n!$ but $p_i^{a_1+1}$ does not divide $n!$, then:

$$\pi_\eta(n) = (a_1 + 1) \ldots (a - k + 1).$$

32217-34516) Study π_η. The same (or similar) question for π_η as for η.

XV. The function $\varphi_\eta : N^* \to N; \varphi_\eta(x)$ is the number of m, with $0 < m \le x$, having the property $(\eta(m), x) = 1$.

34517) Is always true that $\varphi_\eta(x) < \varphi(x)$?

34518) Find x for which $\varphi(x) \ge \varphi(x)$.

34519) Find the smallest k so that $\underbrace{(\varphi_\eta \circ \ldots \circ \varphi_\eta)}_{\text{k times}}(x) = 1$, for a fixed integers x.

34520-36819) Study φ_η. The same (or similar) questions for φ_η as for η.

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More unsolved problems concerning these 15 functions.

....

More new (arithmetic) functions constructed by means of the function η, and new unsolved problems concerning them.

....

36820$\rightarrow \infty$. We can continue these recurring sequences of unsolved problems in number theory to infinity. Thus, we construct an infinity of more new functions: Using the functions $S_\eta, C_\eta, \ldots, \varphi_\eta$ construct the functions $f_{11}, f_{12}, \ldots, f_{1n_1}$ (by varied combinations between $S_\eta, C_\eta, \ldots, \varphi_\eta$; for example: $S_\eta^{(i+1)}(x) = \sum_{0<n\leq x} S_\eta^{(i)}$ far all $x \in N^*, S_\eta^{(i)} : N^* \rightarrow N$ for all $i = 0, 1, 2, \ldots$, where $S_\eta^{(0)} = S_\eta$. Or: $SC_\eta(x) = \frac{1}{x}\sum_{n=1}^{x} S_\eta(n), SC_\eta : N^* \rightarrow Q, SC_\eta$ being a combination between S_η and C_η; etc.); analogously by means of the functions $f_{11}, f_{12}, \ldots, f_{1n_1}$ we construct the functions $f_{21}, f_{22}, \ldots, f_{2n_2}$ etc. The method to obtain new functions continues to infinity. For each function we have at least 2300 unsolved problems, and we have an infinity-of thus functions. The method can be represented in the following way:

$$\eta \overset{\text{produces}}{\rightarrow} S_\eta, C_\eta, \ldots, \varphi \rightarrow f_{11}, f_{12}, \ldots, f_{1n_1}$$

$$f_{11}, f_{12}, \ldots, f_{1n_1} \rightarrow f_{21}, f_{22}, \ldots, f_{2n_2}$$

$$f_{21}, f_{22}, \ldots, f_{2n_1} \rightarrow f_{31}, f_{32}, \ldots, f_{3n_3}$$

....

$$f_{i1}, f_{i2}, \ldots, f_{in_i} \rightarrow f_{i+1,1}, f_{i+1,2}, \ldots, f_{i+1,n_{i+1}}$$

....

*

§12. Conclusion

With this paper the author wants to prove that we can construct infinitely many unsolved problems, especially in number theory: you "rock and roll" the numbers until you create interesting scenarios! Some problems in this paper could effect the subsequent development of mathematics.

The world is in a general crisis. Do the unsolved problems really constitute a mathematical crisis, or contrary to that, do their absence lead to an intellectual stagnation? Making will always have problems to solve, they even must again solve previously solved problems (!) For example, this paper shows that people will be more and more overwhelmed by (open) unsolved problems. [It is easier to ask than to answer.]

Here, there are proposed (un)solved problems which are enough for ever!! Suppose you solve an infinite number of problems, there will always be an infinity of problems remaining. Do not assume those proposals are trivial and non-important, rather, they are very substantial.

§13. References (books and papers which have inspired the author)

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Department of Mathematics, N. Bălcescu College, Craiova. [Presented at the 14th American Romanian Academy Annual Convention, held in Los-Angeles, California, hosted by the University of Southern California, from April 20 to April 22, 1989. An abstract was published by Prof. Constantin Corduneanu, Department of Mathematics, University of Texas at Arlington, in "Libertas Mathematica", tomus IX, p. 175, The Grid, Arlington, Texas. Another abstract had been published in the proceedings of the short communications, International Congress of Mathematicians, Berkeley, California, 1986.]

SOLVING PROBLEMS BY USING A FUNCTION IN THE NUMBER THEORY

Let $n \geq 1$, $h \geq 1$,and $a \geq 2$ be integers. For which values of a and n is $(n+h)!$ a multiple of a^n ? (A generalization of the problem $n^0 = 1270$, Mathematics Magazine, Vol. 60, No. 3, June 1987, p. 179, proposed by Roger B. Eggleton, The University of Newcastle, Australia.)

Solution (For $h = 1$ the problem $n^0 = 1270$ is obtained.)

§1. Introduction

We have constructed a function η (see [1]) having the fallowing properties:

(a) For each non-null integer n,$\eta(n)!$ is multiple of n;

(b) $\eta(n)$ is the smallest natural number with the property (a).

It is easy to prove:

Lemma 1. *$(\forall)k, p \in N^*$, $p \neq 1, k$ is uniquely written in the form:*

$$k = t_1 a_{n_1}^{(p)} + \ldots + t_l a_{n_l}^{(p)},$$

where $a_{n_i}^{(p)} = (p^{n_i} - 1)/(p-1)$,$i = 1, 2, \ldots, l$, $n_1 > n_2 > \ldots > n_l > 0$ and $1 \leq t_j \leq (p-1)$, $j = 1, 2, \ldots, l-1, 1 \leq t_l \leq p$, $n_j, t_j \in N$,$i = 1, 2, \ldots, l$, $l \in N^*$.

We have constructed the function η_p, p prime > 0, η_p:$N^* \to N^*$, thus :

$(\forall)n \in N^*$, $\eta_p(a_n^{(p)}) = p^n$, and $\eta_p(t_1 a_{n_1}^{(p)} + \ldots + t_l a_{n_l}^{(p)}) = t_1 \eta_p(a_{n_1}^{(p)}) + \ldots + t_l \eta_p(a_{n_l}^{(p)})$.

Of course:

Lemma 2. *(a) $(\forall)k \in N^*$, $\eta_p(k)! = Mp^k$.*

(b) $\eta_p(k)$ is the smallest number with the property (a). Now, we construct another function: $\eta : Z \setminus 0 \to N$ defined is follows:

$$\begin{cases} \eta(\pm 1) = 0, \\ (\forall)n = \epsilon p_1^{\alpha_1} \ldots p_s^{\alpha_s} \text{ with } \epsilon = \pm 1, p_i \text{ prime and } p_i \neq p_j \text{ for } i \neq j, \text{ all} \\ \alpha_i \in N^*, \eta(n) = \max\limits_{1 \leq i \leq s}\{\eta_p(\alpha_i)\} \end{cases}$$

It is not difficult to prove η has the demanded properties of §1.

§2. Now, let $a = p_1^{\alpha_1} \ldots p_s^{\alpha_s}$, with all $\alpha_i \in N^*$ and all p_i distinct primes. By the previous theory we have:

$\eta(a) = \max\limits_{1 \leq i \leq s}\{n_{p_i}(\alpha_i)\} = \eta_p(\alpha)$ (by notation).

Hance $\eta(a) = \eta(p^\alpha), \eta(p^\alpha)! = Mp^\alpha$.

We know:

$$(t_1p^{n_1}+\ldots+t_lp^{n_l})! = Mp^{t_1\frac{p^{n_1}-1}{p-1}+\ldots+t_l\frac{p^{n_l}-1}{p-1}}$$

We put:

$t_1p^{n_1}+\ldots+t_lp^{n_l} = n+h$ and $t_1\frac{p^{n_1}-1}{p-1}+\ldots+t_l\frac{p^{n_l}-1}{p-1} = \alpha n.$

Whence

$\frac{1}{\alpha}[\frac{p^{n_1}-1}{p-1}+\ldots+t_l\frac{p^{n_l}-1}{p-1}] \geq t_1p^{n_1}+\ldots+t_lp^{n_l}-h$ or

(1) $\alpha(p-1)h \geq (\alpha p-\alpha-1)[t_1p^{n_1}+\ldots+t_lp^{n_l}]+(t_1+\ldots+t_l)$.

On this condition we take $n_0 = t_1p^{n_1}+\ldots+t_lp^{n_l}-h$ (see Lemma 1), hence $n = \begin{cases} n_0, n_0>0; \\ 1, n_0\leq 0 \end{cases}$

Consider giving $a\neq 2$, we have a finite number of n. There is an infinite number of n if and only if $\alpha p-\alpha-1=0$ i.e., $\alpha=1$ and $p=2$, i.e.,$a=2$

§3 Particular Case

If $h=1$ and $a\neq 2$, bacause $t_1p^{n_1}+\ldots+t_lp^{n_l}\geq p^{n_l}>1$

and $t_1+\ldots+t_l\geq 1$, it follows from (1) that :

(1′) $(\alpha p-\alpha) > (\alpha p-\alpha-1)\cdot 1+1 = \alpha p-\alpha$,

which is impossible. If $h=1$ and $a=2$ then $\alpha=1, p=2$, or

(1″) $1\leq t_1+\ldots+t_l$,

hance $l=1, t_1=1$ whence $n = t_1p^{n_1}+\ldots+t_lp^{n_l}-h = 2^{n_1}-1, n_1\in N^*$ (the solution to problem 1270).

Example 1. *Let $h=16$ and $a=3^4\cdot 5^2$. Find all n such that*

$$(n+16)! = M2025^n.$$

Solution

$\eta(2025) = \max\{\eta_3(4),\eta_5(2)\} = \max\{9,10\} = 10 = \eta_5(2) = \eta(5^2)$. Whence $\alpha=2, p=5$. From (1) we have:

$$128\geq 7[t_15^{n_1}+\ldots t_l5^{n_l}]+t_1+\ldots+t_l.$$

Because $5^4>128$ and $7[t_15^{n_1}+\ldots t_l5^{n_l}]<128$ we find $l=1$,

$$128\geq 7t_15^{n_1}+t_1,$$

whence $n_1\leq 1$, i.e. $n_1=1$, and $t_1=1,2,3$. Then $n_0=t_15-16<0$, hence we take $n=1$.

Example 2. $(n+7)! = M3^n$ *when* $n = 1,2,3,4,5.$

$(n+7)! = M5^n$ *when* $n = 1.$

$(n+7)! = M7^n$ *when* $n = 1.$

But $(n+7)! \neq Mp^n$ *for* p *prime* $> 7, (\forall) n \in N^*.$

$(n+7)! \neq M2^n$ *when*

$$n_0 = t_1 2^{n_1} + \ldots + t_l 2^{n_l} - 7,$$

$$t_1, \ldots, t_{l-1} = 1,$$

$$1 \leq t_l \leq 2,\ t_1 + \ldots + t_l \leq 7$$

$$\text{and } n = \begin{cases} n_0, & n_0 > 0; \\ 1, & n_0 \leq 0. \end{cases} \quad etc.$$

Exercise for Readers

If $n \in N^*, a \in N^* \setminus \{1\}$, find all values of a and n such that:

$(n+7)!$ is a multiple of a^n.

Some Unsolved Problems (see [2])

Solve the diophantine equations:

(1) $\eta(x) \cdot \eta(y) = \eta(x+y)$.

(2) $\eta(x) = y!$ (A solution: $x = 9, y = 3$).

(3) Conjecture: the equation $\eta(x) = \eta(x+1)$ has no solution.

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[A comment about this generalization was published in "Mathematics Magazine"], Vol. 61, No. 3, June 1988, p. 202: "Smarandache considered the general problem of finding positive integers n, a and k, so that $(n+k)!$ should be a multiple of a^n. Also, for positive integers p and k, with p prime, he found a formula for determining the smallest integer $f(k)$ with the property that $(f(k))!$ is a multiple of p^k."]

SOME LINEAR EQUATIONS INVOLVING A FUNCTION IN THE NUMBER THEORY

We have constructed a function η which associates to each non-null integer m the smallest positive n such that $n!$ is a multiple of m.

(a) Solve the equation $\eta(x) = n$, where $n \in N$.

*(b) Solve the equation $\eta(mx) = x$, where $m \in Z$.

Discussion.

(c) Let $\eta^{(i)}$ denote $\eta \circ \eta \circ \ldots \circ \eta$ of i times. Prove that there is a k for which

$$\eta^{(k)}(m) = \eta^{(k+1)}(m) = n_m, \text{ for all } m \in Z^* \setminus \{1\}.$$

**Find n_m and the smallest k with this property.

Solution

(a) The cases $n = 0, 1$ are trivial.

We note the increasing sequence of primes less or equal than n by $P_1, P_2, \ldots, P_k$, and

$$\beta_t = \sum_{h \geq 1} [n/p_t^h], t = 1, 2, \ldots, k;$$

where $[y]$ is greatest integer less or equal than y.

Let $n = p_{i_1}^{\alpha_{i_1}} \ldots p_{i_s}^{\alpha_{i_s}}$, where all p_{i_j} are distinct primes and all α_{i_j} are from N.

Of course we have $n \leq x \leq n!$

Thus $x = p_1^{\sigma_1} \ldots p_k^{\sigma_k}$ where $0 \leq \sigma_t \leq \beta_t$ for all $t = 1, 2, \ldots, k$ and there exists at least a $j \in \{1, 2, \ldots, s\}$ for which

$$\sigma_{i_j} \in \beta_{ij}, \ \{\beta_{i_j}^{-1}, \ldots, \beta_{i_j} - \alpha_{i_j} + 1\}.$$

Clearly $n!$ is a multiple of x, and is the smallest one.

(b) See [1] too. We consider $m \in N^*$.

Lemma 1. *$\eta(m) \leq m$, and $\eta(m) = m$ if and only if $m = 4$ or m is a prime.*

Of course $m!$ is a multiple of m.

If $m \neq 4$ and m is not a prime, the Lemma is equivalent to there are m_1, m_2 such that $m = m_1 \cdot m_2$ with $1 < m_1 \leq m_2$ and $(2m_2 < m$ or $2m_1 < m)$. Whence $\eta(m) \leq 2m_2 < m$, respectively $\eta(m) \leq \max\{m_2, 2m\} < m$.

Lemma 2. *Let p be a prime ≤ 5. Then $= \eta(px) = x$ if and only if x is a prime $> p$, or $x = 2p$.*

Proof: $\eta(p) = p$. Hence $x > p$.

Analogously: x is not a prime and $x \neq 2p \Leftrightarrow x = x_1x_2, 1 < x_1 \leq x_2$ and $(2x_2 < x_1, x_2 \neq p_1,$ and $2x_1 < x) \Leftrightarrow \eta(px) \leq \max\{p, 2x_2\} < x$ respectively $\eta(px) \leq \max\{p, 2x_1, x_2\} < x$.

Observations

$$\eta(2x) = x \Leftarrow x = 4 \text{ or } x \text{ is an odd prime.}$$

$$\eta(3x) = x \Leftrightarrow x = 4, 6, 9 \text{ or } x \text{ is a prime} > 3.$$

Lemma 3. *If $(m, x) = 1$ then x is a prime $> \eta(m)$.*

Of course, $\eta(mx) = \max\{\eta(m), \eta(x)\} = \eta(x) = x$. And $x \neq \eta(m)$, because if $x = \eta(m)$ then $m \cdot \eta(m)$ divides $\eta(m)!$ that is m divides $(\eta(m) - 1)!$ whence $\eta(m) \leq \eta(m) - 1$.

Lemma 4. *If x is not a prime then $\eta(m) < x \leq 2\eta(m)$ and $x = 2\eta(m)$ if and only if $\eta(m)$ is a prime.*

Proof: If $x > 2\eta(m)$ there are x_1, x_2 with $1 < x_1 \leq x_2, x = x_1x_2$. For $x_1 < \eta(m)$ we have $(x-1)!$ is a multiple of mx. Same proof for other cases.

Let $x = 2\eta(m)$; if $\eta(m)$ is nopt a prime, then $x = 2ab, 1 < a \leq b$, but the product $(\eta(m)+1)(\eta(m)+2)\dots(2\eta(m)-1)$ is divided by x.

If $\eta(m)$ is a prime, $\eta(m)$ divides m, whence $m \cdot 2\eta(m)$ is divided by $\eta(m)^2$, it results in $\eta(m \cdot 2\eta(m)) \geq 2 \cdot \eta(m)$, but $(\eta(m)+1)(\eta(m)+2)\dots(2\eta(m))$ is a multiple of $2\eta(m)$, that is $\eta(m \cdot 2\eta(m)) = 2\eta(m)$.

Conclusion.

All x, prime number $> \eta(m)$, are solutions.

If $\eta(m)$ is prime, then $x = 2\eta(m)$ is a solution.

*If x is not a prime, $\eta(m) < x < 2\eta(m)$, and x does not divide $(x-1)!/m$ then x is a solution (semi-open question). If $m = 3$ it adds $x = 9$ too. (No other solution exists yet.)

(c)

Lemma 5. $\eta(ab) \leq \eta(a) + \eta(b)$.

Of course, $\eta(a) = a'$ and $\eta(b) = b'$ involves $(a'+b')! = b'!(b'+1\dots(b'+a')$. Let $a' \leq b'$. Then $\eta(ab) \leq a'+b'$, because the product of a' consecutive positive integers is a multiple of a'!

Clearly, if m is a prime then $k = 1$ and $n_m = m$.

If m is not a prime then $\eta(m) < m$, whence there is a k for which $\eta^{(k)}(m) = \eta^{(k+1)}(m)$.

If $m \neq 1$ then $2 \leq n_m \leq m$.

Lemma 6. *$n_m = 4$ or n_m is a prime.*

If $n_m = n_1 n_2, 1 < n_1 \leq n_2$, then $\eta(n_m) < n_m$. Absurd. $n_m \neq 4$.

(**) This question remains open.

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CONTRIBUŢII LA STUDIUL UNOR FUNCŢII ŞI CONJECTURI ÎN TEORIA NUMERELOR

Teoria Numerelor reprezintă pentru mine o pasiune. Rezultatele expuse mai departe constituie rodul câtorva ani buni de cercetări şi căutări.

Actualitatea temei aste evidentă, din moment ce la Universitatea din Craiova, Conf. dr. C. Dumitrescu & Conf. dr. V. Seleacu organizează <Prima Conferinţa Internaţională dedicată Noţiunilor de tip 'Smarandache' în Teoria Numerelor>, şi anume: funcţii (η şi extinderi ale sale, L, funcţii prime), secvenţe, operaţii speciale, criterii de divizibilitate, teoreme, etc. de tip 'Smarandache', în perioada 21-24 August 1997 [vezi şi anunţul din "Notices of the American Mathematical Societaty", University of Providence, RI, SUA, Vol. 42, No. 11, rubrica "Mathematics Calendar", p. 1366, Noiembrie 1995]. Conferinţa se va desfăfura sub egida UNESCO [240] [cf. Mircea Ichim, director, şi Lucreţia Băluţă, secretară, Filiala UNESCO din Bucureşti].

În felul acesta se deschid noi drumuri în Teoria Numerelor, formând un domeniu aparte, care a trezit interesul diverşilor specialişti.

Un grup de cercetare privind aceste noţiuni, în special concentrat asupra Funcţiei Smarandache, s-a format la Universitatea din Craiova, România, Catedra de Matematică, condus de către Prof. dr. A.Dincă (decan), Prof. dr. V.Boju, Conf. dr. V.Seleacu, Conf. dr. C.Dumitrescu, Conf. dr. I.Bălăcenoiu, Conf. dr. Şt.Zanfir, Conf. dr. N.Rădescu, Lect. E.Rădescu, Lect. dr. I.Cojocaru, Lect. dr. Paul Popescu, Asist. drd. Marcela Popescu, Asist. N.Vîrlan, Asist. drd. Carmen Rocşoreanu, prof. S.Cojocaru, prof. L.Tiţulescu, prof. E.Burton, prof. Panait Popescu, cercet. şt. M.Andrei, student Tomiţă Tiberiu Florin, şi alte cadre didactice împreună cu studenţi.

Membrii acestui grup se întâlnesc o dată pe săptămână, în timpul anului şcolar, şi expun ultimele cercetări asupra funcţiei η, precum şi încercări de generalizare.

În afara grupului de cercetare de la Craiova, destui matematicieni şi informaticieni străini s-au ocupat de studiul funcţiei η, cei mai activi fiind: Henry Ibstedt (Suedia), Pål Grønås (Norvegia), Jim Duncan, John C.MacCarthy, John R. Sutton (Anglia), Ken Tauscher (Australia), Th. Martin (SUA), Pedro Melendez (Brazilia), M.Costewitz (Franţa), J.Rodriguez (Mexic), etc. [Pentru o imagine mai detaliată, vezi cele 240 de "Referinţe" de la sfârşit.]

Despre însemnătatea "Funcţiei Smarandache", cum a fost botezată în revista londoneză

<Personal Computer World>, Iulie 1992, p. 420, şi-a dat pentru prima dată seamă scientistul englez Mike Mudge, editor al rubricii <Numbers Count> [10]. Iar valorile funcţiei, $\eta = 1, 2, 3, 4, 5, 3, 7, 4, 6, 5, 11, 4, 13, \ldots$ au fost etalate de N. J. A. Sloane & Simon Plouffe în <Encyclopedia of Integer Sequences>, Academic Press, [M0453], 1995, şi denumite "Numerele Smarandache" [140].

Articolele, notele, problemele (rezolvate sau deschise), conjecturile referitoare la această nouă funcţie în teoria numerelor sunt colectate într-o revistă specială numită "Smarandache Function Journal", publicată anual ori bianual, de Dr. R. Muller, Number Theory Publishing Co., Glendale, Arizona, SUA.

Mai mult, Ch. Ashbacher (SUA) i-a dedicat însăşi o monografie: "An introduction to the Smarandache function", Erhus Univ. Press, Vail, 1995 [194], iar Kenichiro Kashihara (Japonia) are în pregătire o altă carte despre η [235].

De asemenea, multe reviste şi chiar enciclopedii şi-au deschis paginile inserării de lucrări ce tratează, recenzează, sau citează funcţia η şi valorile ei [vezi "Personal Computer World" (Londra), "Humanistic Mathematics Network Journal" (Harvey Mudd College, Claremont, CA, Sua), "Libertas Mathematica" (Texas State University), "Octogon" (Braşov, România), "Encyclopedia of Integer Sequences" N. J. A. Sloane & Simon Plouffe (Academic Press; San Diego, New York, Boston, London, Sydney, Tokyo, Toronto; 1995), "Journal of Recreational Mathematics" (SUA), "Foaie Matematică" (Chişinău, Mildova), "The Mathematical Spectrum" (University of Sheffield, Anglia), "Elemente der Mathematik" Elveţia, "Zentralblatt für Matematik" (Berlin, Germania), "The Mathematical Reviews" (Ann Arbor, SUA), "The Fibonacci Quarterly" (SUA), etc.].

Iar la conferinţe naţionale şi internaţionale organizate, de exemplu la New Mexico State University of San Antonio (Texas), University of Arizona (Tucson), University of San Antonio (Texas), State University of New York at Farmingdale, University of Victoria (Canada), Congrès International < Henry Poincaré > (Université de Nancy, Franţa), <26th Annual Iranian Mathematics Conference> (Kerman, Iran), <The Second Asian Mathematics Conference> (Nakhon Ratchasima, Tilanda), <Programul manifestărilor organizate cu prilejul împlinirii a 100 ani de la apari'ia primului număr al revistei 'Gazeta Matematică' 1895-1995> (Alba-Iulia, România), etc. s-au prezentat articole ştiinţifice despre η în perioada 1991-5.

Arhivele care stochează cercetările asupra funcţiei η (cărţi, revicte, broşuri, manuscrise publicate ori inedite, articole note, comentarii, scrisori, - obişnuite ori electronice - de la diverşi matematicieni şi editori, probleme, aplicaţii, programe de conferinţe şi simpozioane, etc.), cât şi asupra altor noţiuni din teză, se gesesc la:

a) Arizona State University, Hayden Library, Colecţia Spercială (online) "The Florentine Smarandache papers", Tempe, AZ 85287, USA; phone:(602) 965-6515, e-mail:

ICCLM@ASUACAD.BITNET, responsabile:Carol Moore & Marilyn Wurzburger;

b) Archeves of American Mathematics, Center for American History SRH 2.109, University of Texas, Colecţia Specială "The Florentine Smarandache papers", Austin, TX 78713, USA; phone: (512) 495-4129, fax: (512) 495-4542, director Don Carleton;

c) Biblioteca University din Craiova, Str. Al. I. Cuza, Nr. 13, Secţia de Informare şi Documentare "Florentine Smarandache" din cadrul Seminarului Matematic <Gh. Ţiţeica>, director O. Lohon, bibliothecară Maria Buz, fax: (051) 411688, România;

d) Arhivele Statului, Filiala Vâlcea, Fondul Special "Floretin Smarandache", responsabil: Ion Soare, Str. General Praporgescu, Nr.32B, Rm. Vâlcea, Jud. Vâlcea, România; care sunt puse la dispoziţia publicului spre consultare.

Se defineşte, aşadar, o nouă funcţie:

$$n: Z^* \longrightarrow N,$$

$\eta(n)$ este cel mai mic întreg m astfel încât $m!$ este divizibil cu n.

Această funcţie este importantă deoarece caracterizează numerele prime - prin următoarea proprietate fundamentală:

Fie p un număr întreg > 4, atunci p este prim dacă şi numai dacă $\eta(p) = p$.

Deci, punctele fixe ale acestei funcţii sunt numere prime (la care se adaugă şi 4). Datorită acestei proprietăţi, funcţia η se foloseşte ca o sită pentru cernarea numerelor prime.

Studierea şi descoperirea unor relaţii despre funcţia η duce implicit la aprofundarea cunoştinţelor despre numerele prime, o preocupare în prezent fiind distribuirea lor. [F.Burton încearcă generalizarea funcţiei η în corpul numerelor complexe [169].]

Totodată, funcţia η intră în conexiune şi cu foarte cunoscuta Funcţie $\Pi(x)$, care reprezintă numărul de numere prime mai mici decât sau egale cu x, prin următoarea formulă:

$$\text{Pentru } x \geq 4, \ \Pi(x) = \sum_{k=2}^{x} \lfloor \eta(k)/k \rfloor - 1,$$

unde $\lfloor b \rfloor$ înseamnă partea întreagă a lui b.

[vezi L.Seagull [189]].

Alte proprietăţi:

Dacă $(a, b) = 1$, *atunci* $\eta(ab) = \max\{\eta(a), \eta(b)\}$.

Pentru orice numere pozitive nenule, $\eta(ab) \leq \eta(a) + \eta(b)$.

η *este o funcţie general crescătoare, adică:*

$$\forall a \in N\ \exists b \in N, b = b(a), \forall c \in N, c > b, \eta(c) > a.$$

Funcţia η face obiectul multor probleme deschise, care au ttrezit interesul matematicienilor.

De exemplu:

a) *Ecuaţia* $\eta(n) = \eta(n+1)$ *nu are nici o soluţie.*

Nu a fost încă demonstrată, deşi I.Prodănescu [29, 92] crezuse iniţial că i-a găsit soluţia. L.Ţuţescu [30] i-a dat o extindere acestei conjecturi.

b) A.Mullin [239], inspirat de problema anterioară, conjecturează că *ecuaţia* $\eta(n) = \eta(n+2)$ *are doar un număr finit de soluţii.*

c) T.Yau [63] a propus determinarea tuturor valorilor pentru care funcţia η *păstrează relaţia de recurenţă a lui Fibonacci, adică:*

$$\eta(n) + \eta(n+1) = \eta(n+2),$$

neştiindu-se dacă acestea sunt în număr finit sau infinit. El însuşi aflând pe $n = 9, 119$. Ch.Ashbacher [182, 207] a investigat relaţia de mai sus cu un program pe calculator până la $n = 1000000$, descoperind valori adiţionale pentru $n = 4900, 26243, 32110, 64008, 368138, 415662$, dar nedemonstrând cazul general. H.Ibstedt [224] presupune că există o infinitate de astfel de triplete.

d) Renumitul academician, P.Erdős [147], de la Academia Ungară de Ştiinţe, solicită cititorilor revistei engeleze <Mathematical Spectrum>, în care publică o scrisoare, să găsească o formulă asimptotică pentru:

$$\sum_{\substack{n < x \\ \eta(n) > P(n)}} \eta(n)^2,$$

unde $P(n)$ reprezintă cel mai mare factor prim al lui n.

Fiecare perioadă de timp are problemele ei deschise, cărora de obicei li se dă de cap mai târziu, odată cu progresul ştiinţei. Şi, totuşi, numărul noilor probleme nerezolvate, care apar datorită cercetărilor fireşte, creşte exponenţial, în comparaţie cu numărul vechilor probleme nerezolvate ce sunt în prezent soluţionate. Oare existenţa problemelor deschise constituie o criză matematică ori, dimpotrivă, absenţa lor ar însemna mai degrabă o stagnare intelectuală?

"Funcţia Smarandache" este pusă în combinaţii şi relaţii cu alte funcţii ori noţiuni din teoria numerelor şi analiză, precum: secvenţe-A, numărul de divizori, diferenţa dintre două numere prime consecutive, serii Dirichlet, funcţii generatoare, funcţia logaritm, ordin normal, condiţii Lipschitz, funcţii multiplicative ori aditive, cel mai mare factor, distribuţie uniformă, rădăcini necongruente, cardinal, triunghiul lui Pascal, secvenţă s-aditivă, suma părţilor alicuante, suma puterilor de ordin k ale părţilor alicuante, suma părţilor alicuante unitare, mediile aritmetică şi geometrică, şiruri recurente, ecuaţii şi inecuaţii diofantice, numărul de numere prime, numărul de numere prime congruente cu a *modulo* b, suma divizorilor, suma puterilor de ordin k ale divizorilor, suma divizorilor unitari, funcţia φ a lui Euler, funcţiile gamma şi beta, numărul de factori primi (cu repetiţie),numărul factorilor primi distincţi, partea întreagă, aproximaţii asimptoptice, câmpuri algebrice, funcţia Mobius, funcţiile Cebîşev Θ şi Ψ, etc.

Iar "Numerele Smarandache" sunt asociate şi întrepătrunse respectiv cu: numerele abundente, aproape perfecte, amicale, amicale mărite, numerele Bell, Bernoulli, Catalan, Carmichael, deficiente, Euler, Fermat, Fibonacci, Genocchi, numerele armonice, h-hiperperfecte, Kurepa, Mersenne, m-perfecte, numerele norocoase, k-îndoite perfecte, perfecte, poligonale, piramidale, poliedrale, primitive abundente, primitive pseodoperfecte, pseudoperfecte, pseudoprime, pitagoreice, reziduri pătratice, cvasiperfecte, Stirling de ordinul I şi II, superperfecte, intangibile, numerele sinistre, numerele Ulam, etc.

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presented as:

"SMARANDACHE NUMBERS": $S(n)$, for $n = 1, 2, 3, \ldots$, [MO543], (the values of the Smarandache Function),

and

"SMARANDACHE QUOTIENTS": for each integer $n > 0$, find the smallest k such that nk is a factorial, i.e. $S(n)/n$, for $n = 1, 2, 3 \ldots$;

and in the newest electronic version of the encyclopedia there are some other notions:

"SMARANDACHE DOUBLE FACTORIALS", "SMARANDACHE SQUARE BASE", "SMARANDACHE CUBIC BASE","SMARANDACHE PRIME BASE",

"SMARANDACHE SYMMETRIC SEQUENCE", "SMARANDACHE CONSECUTIVE SEQUENCE", "SMARANDACHE DESCONSTRUCTIVE SEQUENCE", "SMARANDACHE MIRROR SEQUENCE","SMARANDACHE PERMUTATION SEQUENCE", "SMARANDACHE REVERSE SEQUENCE", "SMARANDACHE CONSECUTIVE SIEVE";

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THE FUNCTION THAT YOU BEAR ITS NAME !

This quatation is from a Letter of September 24, 1993, by Constantin M. Popa, an essayist of the Paradoxist Literary Movement [a movement stating that <the sense has a non-sense and, reciprocally, the non-sense has a sense too>], referring to the <Smarandache> Function!

It's a comic sentence, somehow opposite to the Swiss-French mathematician Jacques Sturm (1803-55)'s lectures at l'Ecole Polytechnique, where, teaching students about the <Sturm> Theorem, Jacques said:

"Le théorème dont j'ai l'honneur de porter le nom" (the theorem that I am honored of to bear my name), i.e.:

let $p(x)$ be a real polynomial, $p_1 = p'$, and each $pi = -ri$, where ri are the succesive remainders computed by Euclidean Algorithm for the highest common factor of p and p' (this is called the Sturm Sequence);

if p is non-zero at the end points od an interval, then the number of roots in that interval, counting multiplicity, is the difference between the number of sign changes of the Sturm Sequence at the two end points.

Maybe it was accidentally that just this of my 40 math papers focused the attention of numbertheorists, a paper written when I was a high school student in 1970s, "A function in the number theory" :

$S(n)$ is defined as the smallest integer such that $S(n)!$ is divisible by n.

Some open problems and conjectures are related to it. For examples:

1. The equation $S(n) = S(n+1)$ has no solution.
2. The function verifies the Fibonacci relationship

$$S(n) + S(n+1) = S(n+2)$$

for infinitely many positive integers n.

{Some progress has been got, verifying by computer programs these previous assertions for n up to $100,000$; but it's seems to be still hard to find an analytic method for proving them.}

I attached some reference works published by various journals about "the function that I bear its name", and I'll be glad to here from you.

[For Professor Puaul Hartung and his students, the Number Theory Class, Department of Mathematics ans Computer Science, Bloomsburg University, PA; November 13th, 1995, time: 4:00-5:00 p.m.]

SMARANDACHE TYPE FUNCTION OBTAINED BY DUALITY[1]

Abstract. In this paper we extended the Smarandache function from the set N^* of positive integers to the set Q of rationals.

Using the inversion formula this function is also regarded as a generating function. We make in evidence a procedure to construct (numerical) function starting from a given function in two particular cases. Also conections between the Smarandache function and Euler's totient function as with Riemann's zeta function are etablished.

1. Introduction

The Smarandache function [13] is a numerical function $S : N^* \to N^*$ defined by $S(n) = \min\{m | m! \text{ is divisible by } n\}$.

From the definition it results that if

$$n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdots p_t^{\alpha_t} \tag{1}$$

is the decomposition of n into primes then

$$S(n) = \max S(p_i^{\alpha_i}) \tag{2}$$

and moreover, if $[n_1, n_2]$ is the smallest common multiple of n_1 and n_2 then

$$S([n_1, n_2]) = \max\{S(n_1), S(n_2)\} \tag{3}$$

The Smarandache function characterizes the prime in the sense that a positive integer $p \geq 4$ is prime if and only if it is a fixed point of S.

From Legendre's formula:

$$m! = \prod_p p^{\sum\limits_{i \geq 1} [\frac{m}{p^i}]} \tag{4}$$

it results [2] that if $a_n(p) = \dfrac{(p^n - 1)}{(p-1)}$ and $b_n(p) = p^n$ then considering the standard numerical scale

$$[p] : b_0(p), b_1(p), \ldots, b_n(p), \ldots$$

[1]Together with C.Dumitrescu, N.Vîrlan, Şt. Zamfir, E.Rădescu and N.Rădescu

and the generalized scale

$$[p] : a_0(p), a_1(p), \ldots, a_n(p), \ldots$$

we have

$$S(p^k) = p(\alpha_{[p]})_{(p)} \tag{5}$$

that is $S(p^k)$ is calculated multiplying by p the number obtained writing the exponent α in the generalised scale $[p]$ and "reading" it in the standard scale (p).

Let us observe that the calculus in the generalised scale $[p]$ is essentilly different from the calculus in the usual scale (p), becuase the usual relationship $b_{n+1}(p) = pb_n(p)$ is modified in $a_{n+1}(p) = pa_n(p) + 1$ (for more detals see [2]).

In the following let us note $S_p(\alpha) = S(p^\alpha)$. In [3] it is proved that

$$S_p(\alpha) = (p-1)\alpha + \sigma_{[p]}(\alpha) \tag{6}$$

where $\sigma_{[p]}(\alpha)$ is the sum of the digits of α written in the scale $[p]$, and also that

$$S_p(\alpha) = \frac{(p-1)^2}{p}(E_p(\alpha) + \alpha) + \frac{p-1}{p}\sigma_{(p)}(\alpha) + \sigma_{[p](\alpha)} \tag{7}$$

where $\sigma_{(p)}(\alpha)$ is the sum of the digits of α written in the standard scale (p) and $E_p(\alpha)$ is the exponent of p in the decomposition into primes of α! From (4) it results that $E_p(\alpha) = \sum_{i\geq 1}\left[\frac{\alpha}{p^i}\right]$, where $[h]$ is the integral part of h. It is also said [11] that

$$E_p(\alpha) = \frac{\alpha - \sigma_{(p)}(\alpha)}{p-1} \tag{8}$$

We can observe that this equality may be writen as

$$E_p(\alpha) = \left(\left[\frac{\alpha}{p}\right]_{(p)}\right)_{[p]}$$

that is the exponent of p in the decomposition into primes of α! is obtained writing the integral part of α/p in the base (p) and reading in the scale $[p]$.

Finally we note that in [1] it is proved that

$$S_p(\alpha) = p\left(\alpha - \left[\frac{\alpha}{p}\right] + \left[\frac{\sigma_{[p]}(\alpha)}{p}\right]\right) \tag{9}$$

From the definition of S it results that $S_p(E_p(\alpha)) = p\left[\frac{\alpha}{p}\right] = \alpha - \alpha_p$ (α_p is the remainder of α with respect to the modulus m) and also that

$$E_p(S_p(\alpha)) \geq \alpha; \quad E_p(S_p(\alpha) - 1) < \alpha \tag{10}$$

so

$$\frac{S_P(\alpha) - \sigma_{(p)}(S_P(\alpha))}{p-1} \geq \alpha; \ \frac{S_p(\alpha) - 1 - \sigma_{(p)}(S_p(\alpha) - 1)}{p-1} < \alpha.$$

Using (6) we obtain that $S_p(\alpha)$ is the unique solution of the system

$$\sigma_{(p)}(x) \leq \sigma_{[p]}(\alpha) \leq \sigma_{(p)}(x-1) + 1 \tag{11}$$

2. Connections with classical numerical functions

It is said that Riemann's zeta function is

$$\zeta(s) = \sum_{n \geq 1} \frac{1}{n^s}.$$

We may establish a connection between the function S_p and Riemann's function as follows:

Proposition 2.1. *If* $n = \prod_{i=1}^{t_n} p_i^{\alpha_{i_n}}$ *is the dcomposition into primes of the pozitive integer* n *then*

$$\frac{\zeta(s-1)}{\zeta(s)} = \sum_{n \geq 1} \prod_{i=1}^{t_n} \frac{S_{p_i}(p_i^{\alpha_{i_n}-1}) - p_i}{p_i^{s\alpha_{i_n}}}$$

Proof. We firs establish a connection with Euler's totient function φ. Let us observe that, for $\alpha \geq 2, p^{\alpha-1} = (p-1)a_{\alpha-1}(p) + 1$, so $\sigma_{[p]}(p^{\alpha-1}) = p$. Then by means of (6) it results (for $\alpha \geq 2$) that

$$S_p(p^{\alpha-1}) = (p-1)p^{\alpha-1} + \sigma_{[p]}(p^{\alpha-1}) = \varphi(p^\alpha) + p$$

Using the well known relation between φ and ζ given by

$$\frac{\zeta(s-1)}{\zeta(s)} = \sum_{n \geq 1} \frac{\varphi(n)}{n^n}$$

and (12) it results the required relation.

Let us remark also that, if n is given by (1), then

$$\varphi(n) = \prod_{i=1}^{t} \varphi(p_i^{\alpha_i}) = \prod_{i=1}^{t} (S_{p_i}(p_i^{\alpha_i-1}) - p_i)$$

and

$$S(n) = \max(\varphi(p^{\alpha_i+1}) + p_i)$$

Now it is said that $1+\varphi(p_i)+\ldots+\varphi(p_i^{\alpha_i})=p_i^{\alpha_i}$ and then

$$\sum_{k=1}^{\alpha_i-1} Sp_i(p_i^k)-(\alpha_i-1)p_i=p_i^{\alpha_i}.$$

Consequently we may write

$$S(n)=\max(S\sum_{k=0}^{\alpha_i-1} Sp_i(p_i^k)-(\alpha_i-1)p_i)$$

To establish a connection with Mangolt's function let us note $\bigwedge=\min, \bigvee=\max$, $\bigwedge\limits_{d}$ =the greatest common divisor and $\overset{d}{\bigvee}$=the smallest common multiple.

We shall write also $n_1\bigwedge\limits_{d} n_2=(n_1,n_2)$ and $n_1\overset{d}{\bigvee} n_2=[n_1,n_2]$.

The Smarandache function S may be regarded as function from the lattice $\mathcal{L}_d=(N^*,\bigwedge\limits_{d},\overset{d}{\bigvee})$, into lattice $\mathcal{L}=(N^*,\bigwedge,\bigvee)$ so that

$$S(\bigvee_{i=\overline{1,k}} n_i)=\bigvee_{i=\overline{1,k}} S(n_i). \tag{12}$$

Of course S is also order preserving in the sense that $n_1\leq_d n_2\rightarrow S(n_1)<S(n_2)$.

It is said [10] that if $(V,\bigwedge,\bigvee)$ is a finite lattice, $V=\{x_1,x_2,\ldots,x_n\}$ with the induced order $\leq$, then for every function $f:V\rightarrow N$ the asociated generating function is defined by

$$F(x)=\sum_{y\leq x} f(y) \tag{13}$$

Magolt's function Λ is

$$\Lambda(n)=\begin{cases}\ln p \text{ if } n=p^i\\ 0 \text{ otherwise}\end{cases}$$

The generating function of Λ in the lattice $\mathcal{L}_d$ is

$$F^d(n)=\sum_{k\leq_d n}\Lambda(k)=ln\ n \tag{14}$$

The last equality follows from the fact that

$$k\leq_d n\Leftrightarrow k\bigwedge_d n=k\Leftrightarrow k\backslash n(k \text{ divides } n)$$

The generating function of Λ in the lattice $\mathcal{L}$ is the function Ψ

$$F(n)=\sum_{k\leq n}\Lambda(k)=\Psi_{(n)}=\ln[1,2,\ldots,n] \tag{15}$$

Then we have the diagram from below.

We observe that the definition of S is in a closed connection with the equalities (1.1) and (2.2) in this diagram. If we note the Mangolt's function by f then the relations

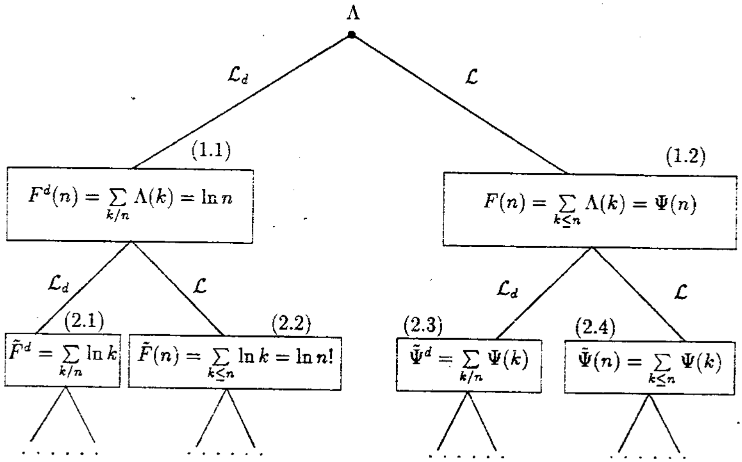
$$[1,2,\ldots,n] = e^{F(n)} = e^{f(1)}e^{f(2)}\ldots e^{f(n)} = e^{\Psi(n)}$$

$$n! = e^{\tilde{F}} = e^{F^d(1)}e^{F^d(2)}\ldots e^{F^d(n)}$$

together with the definition of S suggest us to consider numerical functions of the from:

$$\nu(n) = \min\{m/n \leq_d [1,2,\ldots,m]\} \tag{16}$$

where will be detailed in section 5.



3. The Smarandache function as generating function

Let V be a partitial order set. A function $f : V \to N$ may be obtained from its generating function F, defined as in (15), by the inversion formula

$$f(x) = \sum_{z \leq x} F(z)\mu(z,x) \tag{17}$$

where μ is Moebius function on V, that is $\mu : VXV \to N$ satisfies:

$$(\mu_1)\ \mu(x,y) = 0 \qquad \text{if } x \not\leq y$$

$$(\mu_2)\ \mu(x,x) = 1$$

$$(\mu_3)\ \sum_{x \leq y \leq z} \mu(x,y) = 0 \quad \text{if } x < z$$

It is said [10] that if $V = \{1, 2, \ldots, n\}$ then for $(V, \leq_d)$ we have $\mu(x,y) = \mu\left(\frac{y}{x}\right)$, where $\mu(k)$ is the numerical Meobius function $\mu(1) = 1, \mu(k) = (-1)^i$ if $k = p_1 p_2 \ldots p_k$ and $\mu(k) = 0$ if k is divisible by the square of an integer $d > 1$.

If f is the Smarandache function it results

$$F_S(n) = \sum_{d/n} S(n).$$

Until now it is not known a closed formula for F_S, but in[8] it is proved that

$(i) F_S(n) = n$ if and only if n is prime, $n = 9, n = 16$ or $n = 24$.

$(ii) F_S(n) > n$ if and only if $n \in \{8, 12, 18, 20\}$ or $n = 2p$ with p a prime (hence it results $F_S(n) \leq n + 4$ for every pozitive integer n) and in [2] it is showed that

$(iii) F(p_1 p_2 \ldots p_t) = \sum\limits_{i=1}^{t} 2^{i-1} p_i$.

In this section we shall regard the Smarandache function as a generating function that is using the inversion formula we shall construct the function s so that

$$s(n) = \sum_{d/n} \mu(d) S\left(\frac{n}{d}\right). \tag{18}$$

If n is given by (1) it results that

$$s(n) = \sum_{p_{i_1} p_{i_2} \cdots p_{i_r}} (-1)^r S\left(\frac{n}{p_{i_1} p_{i_2} \ldots p_{i_r}}\right).$$

Let us consider $S(n) = \max S(p_i^{\alpha_i}) = S(p_{i_0}^{\alpha_{i_0}})$. We distinguish the following cases:

(a_1) if $S(p_{i_0}^{\alpha_{i_0}}) \geq S(p_i^{\alpha_i})$ foe all $i \neq i_0$ then we observe that the divisors d for which $\mu(d) \neq 0$ are of the form $d = 1$ or $d = p_{i_1}p_{i_2}\dots p_{i_r}$. A divisor of the last form may contain p_{i_0} or not, so using (2) it results

$$s(n) = S(p_{i_0}^{\alpha_{i_0}})(1-C_{t-1}^1+C_{t-1}^2+\ldots+(-1)^{t-1}C_{t-1}^{t-1})+S(p_{i_0}^{\alpha_{i_0}-1})(-1+C_{t-1}^1-C_{t-1}^2+\ldots+(-1)^tC_{t-1}^{t-1})$$

that is $s(n) = 0$ if $t \geq 2$ or $S(p_{i_0}^{\alpha_{i_0}-1})$ and $s(n) = p_{i_0}$ otherwise.

(a_2) if there exists, j_0 so that $S(p_{i_0}^{\alpha_{i_0}-1}) < S(p_{j_0}^{\alpha_{j_0}})$ and

$$S(p_{j_0}^{\alpha_{j_0}-1}) \geq S(p_i^{\alpha_i}) \text{ for } i \neq i_0, j_0$$

we also suppose that $S(p_{j_0}^{\alpha_{j_0}}) = \max\{S(p_j^{\alpha_j})/S(p_{i_0}^{\alpha_{i_0}-1}) < S(p_j^{\alpha_j})\}$.

Then

$$\begin{aligned} s(n) &= S(p_{i_0}^{\alpha_{i_0}})(1 - C_{t-1}^1 + C_{t-1}^2 - \ldots + (-1)^{t-1}C_{t-1}^{t-1})+ \\ &+S(p_{j_0}^{\alpha_{j_0}})(-1 + C_{t-2}^1 - C_{t-2}^2 - \ldots + (-1)^{t-1}C_{t-2}^{t-2})+ \\ &+S(p_{j_0}^{\alpha_{j_0}-1})(1 - C_{t-2}^1 + C_{t-2}^2 - \ldots + (-1)^{t-2}C_{t-2}^{t-2}) \end{aligned}$$

so $s(n) = 0$ if $t \geq 3$ or $S(p_{j_0}^{\alpha_{j_0}-1}) = S(p_{j_0}^{\alpha_{j_0}})$ and $s(n) = -p_{j_0}$ otherwise.

Consequently, to obtain $s(n)$ we construct as above a maximal sequence $i_1, i_2, \ldots, i_k$, so that $S(n) = S(p_{i_1}^{\alpha_{i_1}}), S(p_{i_1}^{\alpha_{i_1}-1}) < S(p_{i_2}^{\alpha_{i_2}}),\ldots, S(p_{i_{k-1}}^{\alpha_{i_{k-1}}-1}) < S(p_{i_k}^{\alpha_{i_k}})$ and it results that $s(n) = 0$ if $t \geq k+1$ or $S(p_{i_k}^{\alpha_{i_k}}) = S(p_{i_k}^{\alpha_{i_k}-1})$ and $s(n) = (-1)^{k+1}$ otherwise.

Let us observe that

$$S(p^\alpha) = S(p^{\alpha-1}) \Leftrightarrow (p-1)\alpha+\sigma_{[p]}(\alpha) = (p-1)(\alpha-1)+\sigma_{[p]}(\alpha-1) \Leftrightarrow \sigma_{[p]}(\alpha-1)-\sigma_{[p]}(\alpha) = p-1.$$

Otherwise we have $\sigma_{[p]}(\alpha - 1) - \sigma_{[p]}(\alpha) = -1$. So we may write

$$s(n) = \begin{cases} o \text{ if } t \geq k+1 \text{ or } \sigma_{[p]}(\alpha_k - 1) - \sigma_{[p]}(\alpha_k) = p - 1 \\ (-1)^{k+1}p_k \text{ otherwise} \end{cases}$$

Application. It is said [10] that $(V, \wedge, \vee)$ is a finit lattice, with the indused order $\leq$ and for the function $f : V \to N$ we consider the generating function F defined as in (15) then if $g_{ij} = F(x_i \wedge x_j)$ it results $\det g_{ij} = f(x_1) \cdot f(x_2) \cdot \ldots \cdot f(x_n)$. In [10] it is shown also that this assertion may be generalized for partial ordered set by defining

$$g_{ij} = \sum_{\substack{x \leq x_i \\ x \leq x_j}} f(x).$$

Using these results, if we denote by (i,j) the greatest common divisor of i and j, and $\Delta(r) = \det(S((i,j)))$ for $i,j = \overline{1,r}$ then $\Delta(r) = s(1)\cdot s(2)\cdot\ldots\cdot s(r)$. That is for a suffisient large r we have $\Delta(r) = 0$ (in fact for $r \geq 8$). Moreover, for every n there exists a sufficient large r so that $\Delta(n,r) = \det(S(n+i,n+j)) = 0$, for $i,j = \overline{1,r}$ because $\Delta(n,r) = \prod_{i=1}^{n} S(n+1)$.

4. The extension of S to the rational numbers

To obtain this extension we shalll define first a dual function of the Smarandache function.

In [4] and [6] a duality principale is used to obtain, starting from a given lattice on the unit interval, other lattices on the same set. The results are used to propose a definition of bitopological spaces and to introduce a new point of view for studying the fuzzy sets. In [5] the method to obtain news lattices on the unit interval is generalised for an arbitrary lattice.

In the following we adopt a method from [5] to construct all the functions tied in a certain sense by duality to the Smarandache function.

Let us observe that if we note $\Re_d(n) = \{m/n \leq_d m!\}$, $\mathcal{L}_d(n) = \{m/m! \leq_d n\}, \Re(n) = \{m/n \leq m!\}, \mathcal{L}(n) = \{m/m! \leq n\}$ then we may say that the function S is defined by the triplet $(\bigwedge, \in, \Re_d)$, because $S(n) = \bigwedge\{m/m \in \Re_d(n)\}$. Now we may investigate all the functions defined by means of a triplet (a,b,c), where a is one of the symbols $\bigvee, \bigwedge, \overset{d}{\bigwedge}, \underset{d}{\bigvee}$, b is one of the symbols $\in$ and $\notin$, and c is one of the sets $\Re_d(n), \mathcal{L}_d(n), \Re(n), \mathcal{L}(n)$ defined above.

Not all of these functions are non-trivial. As we have already seen the triplet $(\bigwedge, \in, \Re_d)$ defined the function $S_1(n) = S(n)$, but the thriplet $(\bigwedge, \in, \mathcal{L}_d)$ defines the function $S_2(n) = \bigwedge\{m/m! \leq_d n\}$, wich is identically one.

Many of the functions obtained by this method are step functions. For instance let S_3 be the function defined by $(\bigwedge, \in, \Re)$. We have $S_3(n) = \bigwedge\{m/n \leq m!\}$ so $S_3(n) = m$ if only if $n \in [(m-1)!+1, m!]$. Let us focus the attention on the function defined by $(\bigwedge, \in, \mathcal{L}_d)$

$$S_4(4) = \bigvee\{m/m! \leq_d n\} \tag{19}$$

where there is, in a certain sense, the dual of Smarandache function.

Proposition 4.1. *The function S_4 satisfies*

$$S_4(n_1 \underset{d}{\bigvee} n_2) = S_4(n_1) \bigvee S_4(n_2) \tag{20}$$

so is a morphism from $(\mathbf{N}^*, \underset{d}{\bigvee})$ to $(\mathbf{N}^*, \bigvee)$

Proof. Let us denote by $p_1, p_2, \ldots, p_i, \ldots$ the sequence of the prime numbers and let

$$n_1 = \prod p_i^{\alpha_i},\ n_2 = \prod p_i^{\beta_i}.$$

The $n_1 \bigwedge_d n_2 = \prod p_i^{\min(\alpha_i, \beta_i)}$. If $S_4(n_1 \bigvee_d n_2) = m, S_4(n_i) = m_i$, for $i = 1, 2$ and we suppose $m_1 \le m_2$ then the right hand in (22) is $m_1 \bigwedge m_2 = m$. By the definition S_4 we have $E_{p_i}(m) \le \min(\alpha_i, \beta_i)$ for $i \ge 1$ and there exists j so that $E_{p_i}(m+1) > \min(\alpha_i, \beta_i)$. Then $\alpha_i > E_{p_i}(m)$ and $\beta_i \ge E_{p_i}(m)$ for all $i \ge 1$. We also wave $E_{p_i}(m_r) \le \alpha_i$ for $r = 1, 2$. In addition there exist h and k so that $E_{p_h}(m+1) > \alpha_h, e_{p_j}(m+1) > \alpha_k$.

Then $\min(\alpha_i, \beta_i) \ge \min(\varepsilon_{p_1}(m_1), \varepsilon_{p_i}(m_2)) = E_{p_i}(m_1)$, because $m_1 \le m_2$, so $m - 1 \le m$. If we assume $m_1 < m$ it results that $m! \le n_1$, so it exists h that $E_{p_h}(m) > \alpha_h$ and we have the contradiction $E_{p_h}(m) > \min\{\alpha_h, \beta_h\}$. Of course $S_4(2n+1) = 1$ and

$$S_4(n) > 1 \text{ if and only if } n \text{ is even.} \tag{21}$$

Proposition 4.2. *Let $p_1, p_2, \ldots, p_i, \ldots$ be the sequence of all consecutive primes and*

$$n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot \ldots \cdot p_k^{\alpha_k} \cdot q_1^{\beta_1} \cdot q_2\beta_2 \cdot \ldots \cdot q_r^{\beta_r}$$

the decomposition of $n \in N^*$ inte primes such that the first part of the decomposition contains the (eventualy) consecutive primes, and let

$$t_i = \begin{cases} S(p_i^{\alpha_i}) - 1 \text{ if } E_{p_i}(S(p_i^{\alpha_i})) > \alpha_i \\ S(p_i^{\alpha_i}) + p_i - 1 \text{ if } E_{p_i}(S(p_i^{\alpha_i})) = \alpha_i \end{cases} \tag{22}$$

then $S_n(n) = \min\{t_1, t_2, \ldots, t_k, p_{k+1} - 1\}$.

Proof. If $E_{p_i}(S(p_i^{\alpha_i})) > \alpha_i$, then from the definition of the function S results that $S(p_i^{\alpha_i}) - 1$ is the greatest positive integer m much than $E_{p_i}(m) \le \alpha_i$. Also if $E_{p_i}(S(p_i^{\alpha_i})) = \alpha_i$ then $S(p_i^{\alpha_i}) + p_i - 1$ is the greatest integer m with the property that $E_{p_i}(m) = \alpha_i$.

It results that $\min\{t_1, t_2, \ldots, t_k, p_{k+1} - 1\}$ is the greatest integer m much that $E_{p-i}(m!) \le \alpha_i$, for $i = 1, 2, \ldots, k$.

Proposition 4.3. *The function S_4 satisfies*

$$S_4((n_1 + n_2)) \bigwedge S_4([n_1, n_2]) = S_4(n_1) \bigwedge S_4(n_2)$$

for all positive integers n_1 and n_2.

Proof. The equality results using (22) from the fact that $(n_1+n_2,[n_1,n_2])=(n_1,n_2))$.

We point out now some morphism properties of the functions defined bu a triplet (a,b,c) as above.

Proposition 4.4. *(i) The functions* $S_5:\mathbf{N}^*\to\mathbf{N}^*, S_5(n)=\overset{d}{\bigvee}\{m/m!\leq_d n\}$ *satisfies*

$$S_5(n_1\bigwedge_d n_2)=S_5(n_1)\bigwedge_d S_5(n_2)=S_5(n_1)\bigwedge S_5(n_2) \tag{23}$$

(ii) The function $\mathbf{S}_6:\mathbf{N}^*\to\mathbf{N}^*$, $S_6(n)=\overset{d}{\bigvee}\{m/n\leq_d m!\}$ *satisfies*

$$S_6(n_1\overset{d}{\bigvee} n_2)=S_6(n_1)\overset{d}{\bigvee} S_6(n_2) \tag{24}$$

(iii) The function $S_7:\mathbf{N}^*\to\mathbf{N}^*, S_7(n)=\overset{d}{\bigvee}\{m/m!\leq n\}$ *satisfies*

$$S_7(n_1\bigwedge n_2)=S_7(n_1)\bigwedge S_7(n_2); S_7(n_1\bigvee n_2)=S_7(n_1)\bigvee S_7(n_2). \tag{25}$$

Proof. (i) Let $A=\{a_i/a_i!\leq_d n_1\}, B=\{b_j/b_j!\leq_d n_2\}$ and $C=\{c_k/c_k!\leq_d n_1\bigvee_d n_2\}$. Then we have $A\subset B$ or $B\subset A$. Indeed, let $A=\{a_1,a_2,\ldots,a_h\}, B=\{b_1,b_2,\ldots,b_r\}$ so that $a_i<a_{i+1}$and $b_j<b_{j+1}$. Then if $a_h\leq b_r$ it results that $a_i\leq b_r$ for $i=\overline{1,h}$ so $a_i!\leq_d b_r!\leq_d n_2$. That minds $A\subset B$. Analogously, if $b_r\leq a_h$ it results $B\subset A$. Of course we have $C=A\dot{\cup}B$ so if $A\subset B$ it results

$$S_5(n_1\bigwedge_d n_2)=\overset{d}{\bigvee} c_k=\overset{d}{\bigvee} a_i=S_5(n_1)=\min\{S_5(n_1),S_5(n_2)\}=S_5(n_1)\bigwedge_d S_5(n_2)$$

From (25) it results that S_5 is order preserving in $\mathcal{L}_d$ (but not in $\mathcal{L}$, becuase $m!<m!+1$ but $S_5(m!)=[1,2,\ldots,m]$ and $S_5(m!+1)=1$, because $m!+1$ is odd).

(ii) Let us observe that $S_6(n)=\overset{d}{\bigvee}\{m/\exists i\in\overline{1,t}$ so that $E_{p_i}(m)<\alpha_i\}$. If $a=\bigvee\{m/n\leq_d m!\}$ then $n\leq_d(a+1)!$ and $a+1=\bigwedge\{m/n\leq_d m!\}=S(n)$, so $S_6(n)=[1,2,\ldots,S(n)-1]$.

Then we have $S_6(n_1\overset{d}{\vee} n_2)=[1,2,\ldots,S(n_1\overset{d}{\vee} n_2)-1]=[1,,2\ldots,S(n_1)\vee S(n_2)-1]$ and $S_6(n_1)\overset{d}{\vee} S_6(n_2)=[[1,2,\ldots,S_6(n_1)-1],[1,2,\ldots,S_6(n_2)-1]]=[1,2,\ldots,S_6(n_1)\vee S_6(n_2)-1]$.

(iii) The relations (27) result from the fact that $S_7(n)=[1,2,\ldots,m]$ if and only if $n\in[m!,(m+1)!-1]$.

Now we may extend the Smarandache function to the rational numbers. Every positive rational number a possesses a unique prime decomposition of the form

$$a=\prod_p p^{\alpha_p} \tag{26}$$

with integer exponents α_p, of which only finitely many are nonzero. Multiplication of rational numbers is reduced to addition of their integer exponent systems. As a consequence of this reduction questions concerning divisibility of rational numbers are reduced to questions concerning ordering of the corresponding exponent systems. That is if $b = \prod_p P^{\beta_p}$ then b divides a if and only if $\beta_p \leq \alpha_p$ for all p. The greatest common divisors d and the least common multiple e are given by

$$d = (a, b, \ldots) = \prod_p p^{\min(\alpha_p, \beta_p, \ldots)}, \quad e = [a, b, \ldots] = \prod_p p^{\max(\alpha_p, \beta_p, \ldots)} \tag{27}$$

Futhermore, the least coomon multiple of nonzero numbers (multiplicatively bounded above) is reduced by the rule

$$[a, b, \ldots] = \frac{1}{(\frac{1}{a}, \frac{1}{b}, \ldots)} \tag{28}$$

to the greatest common divisor of their reciprocals (multiplicatively bounded below).

Of course we may write every positive rational a under the form $a = n/n_1$, with n and n_1 positive integers.

Definition 4.5. *The extencion $S : Q_+^* \to Q_+^*$ of the Smarandache function is defined by*

$$S\left(\frac{n}{n_1}\right) = \frac{S_1(n)}{S_4(n_1)}. \tag{29}$$

A consequence of this definition is that if n_1 and n_2 are positive integers then

$$S\left(\frac{1}{n_1} \overset{d}{\bigvee} \frac{1}{n_2}\right) = S\left(\frac{1}{n_1}\right) \bigvee S\left(\frac{1}{n_2}\right). \tag{30}$$

Indeed

$$S\left(\frac{1}{n_1} \overset{d}{\bigvee} \frac{1}{n_2}\right) = S\left(\frac{1}{n_1 \underset{d}{\bigwedge} n_2}\right) = \frac{1}{S_4(n_1 \underset{d}{\bigwedge} n_2)} = \frac{1}{S_4(n_1) \bigwedge S_4(n_2)} = \frac{1}{S_4(n_1)} \bigvee \frac{1}{S_4(n_2)} =$$

$$= S\left(\frac{1}{n}\right) \bigvee S\left(\frac{1}{n_2}\right)$$

and we can imediately deduce that

$$S\left(\frac{n}{n_1} \overset{d}{\bigvee} \frac{m}{m_1}\right) = (S(n) \bigvee S(m)) \cdot \left(S\left(\frac{1}{n_1}\right) \bigvee S\left(\frac{1}{m_1}\right)\right) \tag{31}$$

It results that function $\tilde{S}$ defined by $\tilde{S}(a) = \dfrac{1}{S(\frac{1}{a})}$ satisfies

$$\tilde{S}(n_1 \bigwedge_d n_2) = \tilde{S}(n_1) \bigwedge \tilde{S}(n_2) \text{ and}$$

$$\tilde{S}\left(\frac{1}{n_1} \bigwedge_d \frac{1}{n_2}\right) = \tilde{S}\left(\frac{1}{n_1}\right) \bigwedge \tilde{S}\left(\frac{1}{n_2}\right) \tag{32}$$

for every positive integers n_1 and n_2. Moreover, it results that

$$\tilde{S}\left(\frac{n_1}{m_1} \bigwedge_d \frac{n_2}{m_2}\right) = (\tilde{S}(n_1) \bigwedge \tilde{S}(n_2)) \cdot \left(\tilde{S}\left(\frac{1}{m_1}\right) \bigwedge \tilde{S}\left(\frac{1}{m_2}\right)\right)$$

and of course the restriction of $\tilde{S}$ to the positive integers is S_4. The extention of S to all the rationals is given by $S(-a) = S(a)$.

5. Numerical functions inspired from the definition of the Smarandache function

We shall use now the equality (21) and the relation (18) to consider numerical functions as the Smarandache function.

We may say that $m!$ is the product of all positive "smaller" than m in the lattice $\mathcal{L}$. Analogously the product p_m of all the divisors of m is the product of all the elements "smaller" than m in the lattice $\mathcal{L}$. So we may consider functions of the form

$$\Theta(n) = \bigwedge\{m | n \geq_d p(m)\}. \tag{33}$$

It is said that if $m = p_1^{x_1} \cdot p_2^{x_2} \cdot \ldots \cdot p_t^{x_t}$ then the product of all the divisors of m is $p(m) = \sqrt{m^{\tau(m)}}$ where $\tau(m) = (x_1+1)(x_2+1)\ldots(x_t+1)$ is the number of all the divisors of m.

If n is given as in (1) then $n \geq_d p(m)$ id and only if

$$\begin{aligned} g_1 &= x_1(x_1+1)(x_2+1)\ldots(x_t+1) - 2\alpha_1 \geq 0 \\ g_2 &= x_2(x_1+1)(x_2+1)\ldots(x_t+1) - 2\alpha_2 \geq 0 \\ g_t &= x_t(x_1+1)(x_2+1)\ldots(x_t+1) - 2\alpha_t \geq 0 \end{aligned} \tag{34}$$

so $\Theta(n)$ may be obtained solving the problem of non linear programming

$$(\min) f = p_1^{x_1} \cdot p_2^{x_2} \cdot \ldots \cdot p_t^{x_t} \tag{35}$$

under the restrictions (37).

The solutions of this problem may be obtained applying the algorithm SUMT (Sequencial Unconstrained Minimization Techniques) due to Fiacco and Mc Cormick [7].

Examples

1. For $n = 3^4 \cdot 5^{12}$, (37) and (38) become $(\min) f(x) = 3^{x_1} 5^{x_2}$ with $x_1(x_1+1)(x_2+1 \geq 8)$, $x_2(x_1+1)(x_2+1) \geq 24$. Considering the function $U(x,n) = f(x) - r \sum_{i=1}^{k} \ln g_1(x)$, and the system

$$\sigma U / \sigma x_1 = 0, \ \sigma U / \sigma x_2 = 0 \tag{36}$$

in [7] it is showed that if the solution $x_1(r), x_2(r)$ can't be explained from the system we can make $r \to 0$. Then the system becomes $x_1(x_1+1)(x_2+1) = 8$, $x_2(x_1+1)(x_2+1) = 24$ with the (real) solution $x_1 = 1, x_2 = 3$.

So we have $\min\{m/3^4 \cdot 5^{12} \leq \rho(m)\} = m_0 = 3 \cdot 5^3$.

Indeed $\rho(m_0) = m_0^{\tau(m_0)/2} = m_0^4 = 3^4 \cdot 5^{12} = n$.

2. For $n = 3^2 \cdot 567$, from the system (39) it results for x_2 the equation $2x_2^3 + 9x_2^2 + 7x_2 - 98 = 0$, with th real solution $x_2 \in (2,3)$. It results $x_1 \in (4/6, 5/7)$. Considering $x_1 = 1$, we observe that for $x_2 = 2$ the pair (x_1, x_2) is not an admissible solution of the problem, but $x_2 = 3$ give $\Theta(3^2 \cdot 5^7) = 3^4 \cdot 5^{12}$.

3. Generaly for $n = p_1^{\alpha_1} \cdot p_2^{\alpha_3}$, from the system (39) it results the equation

$$\alpha_1 x_2^3 + (\alpha_1 + \alpha_2) \cdot x_2^2 + \alpha_2 x_2 - 2\alpha_2^2 = 0$$

with solutions given by Cartan's formula.

Of course, using "the method of the triplets", as for the Smarandache function, many other functions may be associated to Θ.

For the function ν given by (18) it is also possible to generate a class of function by means of such triplets.

In the sequel we'll focus the attention on the analogous of the Smarandache function and on his dual in this case.

Proposition 5.1. *If n has the decomposition into primes given by (1) then*

(i) $\nu(n) = \max_{i=\overline{1,t}} p_i^{\alpha_i}$

(ii) $\nu(n_1 \overset{d}{\bigvee} n_2) = \nu(n_1) \bigvee \nu(n_2)$

Proof.

(i) Let $\max p_i^{\alpha_i} = p_u^{\alpha_u}$. Then $p_i^{\alpha_i} \leq p_u^{\alpha_u}$ for all $\overline{1,t}$, so $p_i^{\alpha_i} \leq_d [1, 2, \ldots p_u^{\alpha_u}]$. But $(p_i^{\alpha_i}, p_j^{\alpha_j}) = 1$ for $i \neq j$ and then $n \leq_d [1, 2, \ldots p_u^{\alpha_u}]$.

Now if for some $m < p_u^{\alpha_u}$ we have $n \leq_d [1, 2, \ldots, m]$, it results the contradiction $p_u^{\alpha_u} \leq_d [1, 2, \ldots, m]$.

(ii) If $n_1 = \prod p^{\alpha_p}$, $n_2 = \prod p^{\beta_p}$ then $n_1 \overset{d}{\vee} n_2 = \prod p^{\max(\alpha_p \beta_p)}$ so

$$\nu(n_1 \overset{d}{\bigvee} n_2) = \max p^{\max(\alpha_p, \beta_p)} = \max(\max p^{\alpha_p}, \max p^{\beta_p}).$$

The function $\nu_1 = \nu$ is defined by means of the triplet $(\bigvee, \in, \Re_{[d]})$ where $\mathbf{R}_{[d]} = \{m/n \leq_d [1, 2, \ldots, m]\}$. His dual, in the sense of above section, is the function defined by the triplet $(\bigvee, \in, \mathcal{L}_{[d]})$. Let us note ν_4 this function

$$\nu_4(n) = \bigvee\{m | [1, 2, \ldots, m] \leq_d n\}$$

That is $\nu_4(n)$ is the greatest natural number with the property that all $m \leq \nu_4(n)$ divide n.

Let us observe that necessary and sufficient condition to have $\nu_4(n) > 1$ is to exist $m > 1$ so that every prime $p \leq m$ devides n. From the definition of ν_4 it also results that $\nu_4(n) = m$ if and only if n is divisible by every $i \leq n$ and not by $m + 1$.

Proposition 5.2. *The function ν_4 satisfies*

$$\nu_4(n_1 \overset{d}{\bigvee} n_2) = \nu_4(n_1) \bigwedge \nu_4(n_2)$$

Proof. Let us note $n = n_1 \overset{d}{\wedge} n_2$, $\nu_4(n) = m$, $\nu_4(n_i) = m_i$ for $i = 1, 2$. If $m_1 = m_1 \wedge m_2$ than we prove that $m = m_1$. From the definition of ν_4 it results

$$\nu_4(n_i) = m_i \leftrightarrow [\forall i \leq m_i \rightarrow n \text{ is divisible by } i \text{ but not by } m + 1]$$

If $m < m_1$ then $m + 1 \leq m_1 \leq m$ so $m + 1$ divides n_1 and n_2. That is $m + 1$ divides n.

If $m > m_1$ then $m_1 + 1 \leq n$, so $m_1 + 1$ divides n. But n divides n_1, so $m_1 + 1$ divides n_1.

If $t_0 = \max\{i | j \leq i \Rightarrow n \text{ divides } n\}$ then $\nu_4(n)$ may be obtained solving the integer linear programming problem

$$\begin{aligned} &(\max)f = \sum_{i=1}^{t_0} x_i \ln p \\ &x_i \leq \alpha_i \; for \; i = \overline{1, t_0} \end{aligned} \tag{37}$$

$$\sum_{i=1}^{t_0} x_i \ln p_i \leq \ln p_{t_0+1}.$$

If f_0 is the maximal value of f for above problem, then $\nu_4(n) = e^{f_0}$.

For instance $\nu_4(2^3 \cdot 3^2 \cdot 5 \cdot 11) = 6$.

Of course, the function ν may be extinded to the rational numbers in the same way as Smarandache function.

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FUNCŢII ARITMETICE

Este bine cunoscută importanţa funcţiilor aritmetice în teoria nimerelor, importanţă datorată pe de-o parte bogăţiei rezultatelor ce se obţin cu ajutorul acestor funcţii, şi pe de altă parte frumuseţii acestor rezultate.

Este într-adevăr nu numai util, dar şi frumos să ştim că dacă $\Pi(x)$ este numărul numerelor prime mai mici sau egale cu x, atunci $\Pi(x)$ este asimptotic egal cu $x \ln x$ sau că dacă se cunoaşte functia sumatoare $F(n) = \sum f(d)$ pentru funcţia numerică f, atunci f se poate exprima cu ajutorul funcţiei F prin formula de inversiune

$$f(n) = \sum \mu(d) F(n/d).$$

În cele ce urmează vom prezenta o funcţie numerică definită recent [19] ale cărei proprietăţi sunt deci prea puţin cunoscute până acum.

Această funcţie $f : Z^* \to N$ este caracterizată de proprietăţile:

(i) $\forall n \in Z^* (\eta(n)! = M \cdot n)$ (multiplu de n);

(ii) $\eta(n)$ este cel mai mic număr natural cu proprietatea (i).

Lema 1 *Pentru orice $k, p \in N^*, p \neq 1$, numărul k se poate scrie în mod unic sub forma:*

$$k = t_1 a_{n_1}(p) + t_2 a_{n_2}(p) + \ldots + t_l a_{n_l}(p) \tag{1}$$

unde $a_{n_i}(p) = \dfrac{(p^{n_i} - 1)}{(p-1)}$, *pentru* $i = 1, \ldots, l, n_1 > n_2 > \ldots > n_l > 0$ *şi* $t_i \in [1, p-1] \cap N$ *pentru* $j = 1, \ldots l-1$, *iar* $t_l \in [1, p] \cap N$.

Demonstraţia este evidentă, fiind vorba be scrierea numărului k în baza generalizată:

$$[p] a_1(p), a_2(p), \ldots, a_n(p), \ldots$$

Pentru fiecare număr prim $p \in N^*$ putem defini acum o funcţie :

$$\eta_p : N^* \to N$$

având proprietăţile:

$$(\eta_1) \quad \eta_p(a-n(p)) = p^n;$$

$$(\eta_2) \quad \eta_p(t_1 a_{n_1}(p) + t_2 a_{n_2}(p) + \ldots + t e a_{n_e}) = t_1 \eta_p(a_{n_1}(p) + t_2 \eta_p(a_{n_2}(p)) + \ldots + t_e \eta_p(a_{n_e}(p)).$$

Într-adevăr, utilizând lema precedentă orice număr $k \in N^*$ poate fi scris sub forma (1) şi atunci putem defini:

$$\eta_p(k) = t_1 p^{n_1} + t_2 p^{n_2} + \ldots + t_e p^{n_e}.$$

Teorema 1 *Fiecare funcţie η_p, cu $p > 0$ număr prim, are proprietăţile:*

(iii) $\forall k \in N^*(\eta_p(k))! = M \cdot p^k$;

(iv) $\eta_p(k)$ *este cel mai mic număr natural având proprietatea (iii).*

Demonstraţie. Se ştie că exponentul $e_{p,n}$ la care apare p în descompunerea în factori a lui $n!$ este dat de formula lui Legendre:

$$e_{p,n} = \sum \left[\frac{n}{p^i}\right].$$

Prin urmare exponentul la care apare p în descompunerea în factori a lui $(\eta_p(k))!$ este:

$$e_{p,\eta(k)} = \sum \left[\frac{t_1p^{n_1} + t_2p^{n_2} + \ldots + t_ep^{n_e}}{p^i}\right] = \left[\frac{t_1p^{n_1} + t_2p^{n_2} + \ldots + t_ep^{n_e}}{p}\right] +$$

$$+ \left[\frac{t_1p^{n_1} + t_2p^{n_2} + \ldots + t_{ep}p^{n_e}}{p_2}\right] + \ldots \left[\frac{t_1p^{n_1} + t_2p^{n_2} + \ldots + t_ep^{n_e}}{p^{n_1}}\right] =$$

$$= (t_1p^{n_1} - 1 + t_2p^{n_2} - 1 + \ldots + t_ep^{n_e} - 1) + (t_1p^{n_1} - 2 + t_2p^{n_2} - 2 + \ldots) + \ldots + t_1 =$$

$$= t_1(p^{n_1} - 1 + p^{n_2} - 1 + \ldots + p^0) + t_2(p^{n_1} - 1 + p^{n_2} - 2 + \ldots + p^0) + \ldots + t_e(p^{n_e} - 1 + p^{n_e} - 2 + \ldots)$$

Deci:

$$e_{p,\eta(k)} = k;$$

şi teorema este demonstrată.

Funcţia $\eta : Z \to N$ se poate construi cu ajutorul funcţiilor η_p în felul următor:

(a) $\eta(\pm 1) = 0$;

(b) pentru orice $n = \varepsilon p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot \ldots \cdot p_t^{\alpha_t}$, cu $\varepsilon = \pm 1$ şi p_i numere prime distincte, iar $\alpha_i \geq 1$ definim:

$$\eta(n) = \max \eta_p(\alpha_i).$$

Teorema 2 *Funcţia η definită prin condiţiile (a) şi (b) are proprietăţile (i) şi (ii).*

Demonstraţie. (i) este evident[, deoarece $(\eta(n)! = \max_i(\eta_p(\alpha_i)))$, deci $\eta(n)!$ este divizibil cu n. Proprietatea (ii) rezultă din (iv). Să observăm că funcţiile η_p sunt crescătoare, nu sunt injective, dar considernd $\eta_p : N^* \to \{p^k / k = 1, 2, \ldots\}$ se verifică surjectivitatea. Funcţia η nu este nici ea injectivă, dar $\eta : Z^* \to N \setminus \{1\}$ este surjectivă.

Consecinţă. Fie $n \geq 4$. Atunci n este număr prim dacă şi numai dacă $\eta(n) = n$.

Demonstraţie. Dacă $n = p$ este număr prim, cu $p \geq 5$, atunci $\eta(n) = \eta_p(1) = p$.

Fie acum $\eta(n) = n$. Dar $\eta(n) = \max \eta(p_i)$, deci $n = p$.

APLICAŢII

1. Care este cel mai mic număr natural n cu proprietatea: $n! = M(2^{31}.3^{27}.7^{13})$?

Soluţie.

Pentru a calcula $\eta_2(31)$ scriem numărul $\alpha_1 = 31$ în baza generalizată [2], unde:

[2]: 1, 3, 7, 15, 31, 63, ...

Pentru a calcula $\eta_3(27)$ considerăm baza generalizată

[3]: $1, 4, 13, 40, \ldots$ şi deducem $27 = 2*13+1 = 2a_3(3)+a_1(3)$, deci $\eta_3(27) = 2*3^3+1*3^1 = = 57$.

Analog obţinem $\eta_7(13) = 84$. Deci $\eta(\pm 2^{31} * 3^{27} * 7^{13}) = \max(32, 57, 84) = 84$. Prin urmare 84! este divizibil cu $\pm 2^{31} * 3^{27} * 7^{13}$ şi este cel mai mic număr natural cu această proprietate.

2. Care sunt numerele ale căror factoriale se termină în o mie de zerouri?

Soluţie: Dacă $n = 10^{1000}$ atunci $\eta(n)! = M10^{1000}$ şi este cel mai mic număr natural cu această proprietate.

Avem:

$\eta(10^{1000}) = \eta(2^{1000} * 5^{1000}) = \max(\eta_2(1000), \eta_5(1000)) = \eta_5(1000)$, iar cum:

$[5] = 1, 6, 31, 156, 781, \ldots$

deducem $1000 = 1*a_3(5)+1*a_4(5)+2a_3(5)+a_1(5)$, deci $\eta_5(1000) = 1*5^3+1*5^4+1*5 = 4005$.

Aşadar numărul 4005 este cel mai mic număr natural al cărui factorial se termină cu 1000 de zerouri. Factorialul numerelor 4006, 4007, 4008 şi 4009 se termină şi el cu o mie de zerouri, dar $4010! = 4009! \cdot 4010$ are 1001 zerouri.

În legătură cu funcţia η am alcătuit [20] o listă de probleme nerezolvate. Iată câteva dintre acestea:

(1) Să se găsească formule pentru exprimarea lui $\eta(n)$.

În [1] şi [2] se dau astfel de formule. În fond, din cele prezentate mai sus putem spune că dacă $n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot \ldots \cdot p_t^{\alpha_t}$, atunci $\eta(n) = \max_i \eta(p_i^{\alpha_i}) = \max_i \eta_{p_i}(\alpha_i)$, adică $\eta(n) = \max_{i=\overline{1,t}} \eta(p_i(\alpha_i)_{[p_i D_t]})$, deci $\eta(p_i^{\alpha_i})$ se obţine înmulţind numărul p_i cu numărul obţinut scriind exponentul α_i în baza generalizată $[p_i]$ şi "citind" rezultatul în baza standard $(p) : 1, p, p^2, \ldots, p^n, \ldots$.

(2) Există exprimări asimptotice pentru $\eta(n)$?

(3) Pentru un număr întreg fixat m, în ce condiţii $\eta(n)$ divide diferenţa $n-m$? (în particular pentru $m = 1$). Desigur, pentru $m = 0$ avem soluţiile $n = k!$ sau n este un număr liber de pătrate.

(4) Este η o funcţie algebrică? Mai general, să spunem că g este o f-funcţie, f nenulă, dacă $f(x, g(x)) = 0$ pentru orice x şi $f \in R[x, y]$. Este η o f-funcţie?

(5) Fie A o mulţime de numere naturale nenule consecutive. Să se determine max *card* A pentru care η este monotonă pe A. Se poate observa că avem max *card* $A \geq 5$ deaoarece pentru $A = \{1, 2, 3, 4, 5\}$ valorile lui η sunt respectiv $0, 2, 3, 4, 5$.

(6) Un număr se spune că este număr η algebric de grad n dacă el este rădăcina polinomului: $P_\eta(x) = \eta(n) \cdot x^n + \eta(n-1) \cdot x^{n-1} + \ldots + \eta(1) \cdot x^1$. Pentru ce fel de numere n există numere algebrice de ordinul n care să fie numere întregi?

(7) Sunt numerele $P_n = \eta(n)/n$ uniform distribuite în intervalul $(0, 1)$? Răspunsul este negativ şi a fost demonstrat de Gh.Ashbacher.

(8) Este numărul $0,0234537465114\ldots$, format prin concatenarea valorilor lui $\eta(n)$, un număr iraţional ? Răspunsul este afirmativ şi a fost demonstrat de Gh.Ashbacher.

(9) Se pot reprezenta numerele întregi n sub forma:

$$n = \pm\eta(a_1)^{a_2} \pm \eta(a_2)^{a_3} \pm \ldots \pm \eta(a_k)^{a_1},$$

unde întregii $k, a_1, a_2, \ldots a_k$ şi semnele sunt convenabil alese?

Dar sub forma:

$$n = \pm a_1^{\eta(a_1)} \pm \ldots \pm a_k^{\eta(a_k)}?$$

Sau sub forma:

$$n = \pm a_1^{\eta(a_2)} \pm a_2^{\eta(a_2)} \pm \ldots \pm a_k^{\eta(a_1)}?$$

(10) Găsiţi o formă generală a exprimării în fracţii continue a lui $\eta(n)/n$, pentru $n \geq 2$.

(11) Există întregii m, n, p, q cu $m \neq n$ sau $p \neq q$ pentru care:

$$\eta(m) + \eta(m+1) + \ldots + \eta(m+p) = \eta(n) + \eta(n+1) + \ldots + \eta(n+q)?$$

(12) Există întregii m, n, p, k sau $m \neq n$ şi $p > 0$ astfel încât:

$$\frac{\eta(m)^2 + \eta(m+1)^2 + \ldots + \eta(m+p)^2}{\eta(n)^2 + \eta(n+1)^2 + \ldots + \eta(n+p)^2} = k?$$

(13) Câte numere prime au forma $\eta(n) \cdot \eta(n+1) \cdot \ldots \eta(n+k)$ pentru o valoare fixată a lui k? Se observă că $\eta(2) \cdot \eta(3) = 23$ şi $\eta(5) \cdot \eta(6) = 53$ sunt prime.

(14) Există două numere distincte k şi n pentru care:

$\log_{\eta(k^n)} \eta(n^k)$ este număr întreg?

(15) Este numărul:

$\lim\limits_{n\to\infty} \left(1 + \sum\limits_{k=2}^{\infty} \frac{1}{\eta(k)} - \ln \eta(n)\right)$ număr finit?

Răspunsul este negativ [9].

(16) Verifică η o condiţie de tip Lipschitz?

Răspunsul este negativ şi aparţine tot lui Gh.Ashbacher.

(17) Există o formulă de recurenţă pentru şirul $a_n = \eta(n)$?

Un alt grup de probleme nerezolvate este următorul:

Există numere nenule nonprime $a_1, a_2, \ldots, a_n$ în relaţia P astfel încât $\eta(a_1), \eta(a_2), \ldots, \eta(a_n)$ să fie în relaţia R? Căsiţi cel mai mare n cu această proprietate (unde P şi R reprezintă una din următoarele categorii de numere):

(i) numere abundente: $a \in N$ este abundent dacă $\sigma(a) > 2a$.

(ii) numere aproape perfecte: a este aproape perfect dacă $\sigma(a) = 2a - 1$;

(iii) numere amicale: a şi b sunt amicale dacă $\sigma(a) = \sigma(b) = a + b$;

(iv) numere Bell: $S_n = \sum_{k=1}^{n} S(n,k)$, unde $S(n,k)$ sunt numerele Stirling de categoria a doua $S(0,0) = 1$, iar $S(n,k)$ se deduc din $x^n = \sum_{k=1}^{n} S(n,k) \cdot x^k, x^{(k)} = x(x-1)\ldots(x_k+1)$, pentru $1 \leq k \leq n$;

(v) numerele Cullen: $C_n = n * 2^n + 1, n \geq 0$;

(vi) numerele Fermat: $F_n = 2^{2^n} + 1$;

(vii) numerele Fibonacci: $f_1 = f_2 = 1, f_{n+2} = f_{n+1} + f_n$;

(viii) numerele armonice: a este armonic dacă media armonică a divizorilor lui a este număr întreg;

(ix) numerele Mersenne: $M_p = 2^p - 1$;

(x) numerele perfecte: $\sigma(a) = 2a$ şi desigur lista ar putea continua.

Desigur se pot formula probleme interesante conţinând funcţia η, probleme în legătură cu funcţii numerice sau categorii speciale de numere (printre care sunt şi cele enumărate mai sus). Rezolvarea acestor probleme va oferi legătura încă necunoscută, dintre funcţia η şi celelalte categorii de funcţii numerice.

Demersul spre această legăturaă poate fi făcut de exemplu şi cu ajutorul ecuaţiilor (inecuaţiilor) diofantice. Iată câteva dintre aceastea:

(i) $\eta(m * x + n) = A$, unde A poate fi: C_n^m,

- P_n (al n-lea număr prim),

- $[\Theta(x)]$ ($\Theta(x) = \sum_{p \leq x} \ln p$, este funcţia Θ a lui Cebâşev),

- $[\Psi(x)]$ ($\Psi(x) = \sum_{n \leq x} \Lambda(n)$, unde $\Lambda(n)$ are valoarea lui p dacă n este o putere întreagă a numărului prim p şi este zero în caz contrar),

- $S(n, x)$ sau $S(m, x)$,

- $\Pi(x)$ (numărul numerelor prime ce nu depăşesc pe x) şi desigur lista posibilităţilor pentru A poate continua.

(ii) $\eta(mx+n) < B$, unde B poate fi:

-$d(x)$ (numărul divizorilor pozitivi ai lui x),

- $\Gamma(x)$ (funcţia lui Euler de speţa întâi),

$$\Gamma(x) = \int_0^\infty \frac{t^{x-1}}{e^t} dt,$$

- $1/\beta(x,x)$ (funcţia lui Euler de speţa a doua, $\beta(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$),

- $\mu(x)$ (funcţia lui Mobius).

Există multe posibilităţi de a alege pe B. Rămâne de descoperit cele într-adevăr interesante, care dau legătura lui η cu noţiuni devenite clasice în teoria numerelor.

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FUNCŢII PRIME ŞI COPRIME

Vom construi următoarele funcţii (pe care le numim **prime**):

$$P_1 : \mathbf{N} \to \{0, 1\},$$

$$P_1(n) = \begin{cases} 0, & \text{dacă } n \text{ este prim;} \\ 1, & \text{în caz contrar.} \end{cases}$$

De exemplu

$$P_1(0) = P_1(1) = P_1(4) = P_1(6) = \ldots = 1, P_1(2) = P_1(3) = P_1(5) = \ldots = 0.$$

Analog:

$$P_2 : \mathbf{N}^2 \to \{0, 1\},$$

$$P_2(m, n) = \begin{cases} 0, & \text{dacă } m \text{ şi } n \text{ sunt amândouă prime;} \\ 1, & \text{în caz contrar.} \end{cases}$$

Şi în general:

$$P_k : \mathbf{N}^k \to \{0, 1\},$$

$$P_k(n_1, n_2, \ldots, n_k) = \begin{cases} 0, & \text{dacă } m \text{ şi } n \text{ sunt toate prime;} \\ 1, & \text{în caz contrar.} \end{cases}$$

Funcţiile **coprime** se definesc similar, doar că se impune o condiţie mai slabă: în acolada de mai sus $n_1, n_2, \ldots, n_k$ sunt prime între ele.

ASUPRA UNOR CONJECTURI ŞI PROBLEME NEREZOLVATE REFERITOARE LA O FUNCŢIE ÎN TEORIA NUMERELOR

1. Introducere

Am construit [19] o functie η care asociază fiecărui întreg nenul n cel mai mic întreg pozitiv m astfel încât $m!$ este multiplu de n.

De aici rezultă că dacă n are descompunerea în factori primi:

$$n = \varepsilon \cdot p_1^{a_1} \cdot p_2^{a_2} \dots p_t^{a_t};$$

cu p_i numere distincte, $a_i \in N^*$ şi $\varepsilon = \pm 1$ atunci:

$$\eta(n) = \max_{1 \leq i \leq t} \eta(p_i^{a_i});$$

şi $\eta(\pm 1) = 0$.

Pentru calculul lui $\eta(p_i^{a_i})$ observăm că dacă:

$$\alpha_k(p) = \frac{p^k - 1}{p - 1},\ k = 1, 2, \dots;$$

atunci din formula lui Legendre:

$$n! = \prod p_i^{\sum\limits_{k>1} \frac{n}{p^k}};$$

rezultă $\eta(p^{\alpha_k(p)}) = p^k$.

Mai general, considerând baza generalizată:

$$[p] : \alpha_1(p), \alpha_2(p), \dots;$$

şi scriind exponentul a în această bază:

$$a_{[p]} = t_1 \cdot a_{n_1}(p) + \dots + t_l \cdot a_{n_l}(p);$$

cu $n_1 > n_2 > \dots > n_e > 0$ şi $t_1 \in [1, p-1]$ pentru $j = 0, 1, \dots, l-1$ şi $t_l \in [1, p]$, în [19] am arătat că:

$$\eta(p^a) = \sum_{i=1}^{e} t_i p^{n_i}. \tag{1}$$

2. Proprietăţi ale funcţiei η

Din felul în care a fost definită rezultă imediat că funcţia η este pară: $\eta(-n) = \eta(n)$. De asemenea pentru orice $n \in N^*$ avem:

$$\frac{-1}{(n-1)!} \leq \frac{\eta(n)}{n} \leq 1;$$

Raportul $\frac{\eta(n)}{n}$ este maxim dacă şi numai dacă n este prim sau $n = 4$ şi are valoare minimă dacă şi numai dacă $n = k!$. Evident η nu este o funcţie periodică.

Pentru orice număr prim p funcţia $\eta_p : N^* \to N, \eta_p(a) = \eta(p^n)$ este crescătoare, noninjectivă, dar considerând $\eta_p : N^* \to \{p^k | k = 1, 2, \ldots\}$ este verificată surjectivitatea.

Funcţia η este *în general crescătoare* pe N^*, în sensul că:

$$\forall n \in N^* \, (\exists) m_0 \in N^* \, \forall m \geq m_0 \; \eta(m) \geq n.$$

Prin urmare funcţia este *în general descrescătoare* pe Z_-^* adică:

$$\forall n \in Z_-^* \, (\exists) m_0 \in Z_-^* \, \forall m \leq m_0 \; \eta(m) \leq n.$$

De asemenea nu este injectivă, dar considerând: $\eta : Z^* \to N \setminus \{1\}$ este verificată surjectivitatea.

Definiţia 1. *(P.Erdös şi J.L.Selfridge)*

Numărul n se numeşte barieră pentru funcţia numerică f dacă pentru orice $m < n$ avem $m + f(m) \leq n$.

Se observă că pentru orice $\varepsilon \in [0, 1]$ funcţia f definită prin $f(m) = \varepsilon \cdot \eta(m)$ nu are o infinitate de bariere deoarece există $m_0 \in N$ astfel încât pentru orice $n \geq m_0$ avem:

$$\eta(n) \geq \frac{2}{\varepsilon} \text{ dacă } n + \varepsilon \cdot \eta(n) \geq n.$$

Seria $\sum\limits_{n \geq 2} \frac{1}{\eta(n)}$ este divergentă deoarece $\frac{1}{\eta(n)} \geq \frac{1}{n}$.

Avem de asemenea:

$$\eta\Bigg(\underbrace{2^{2^{\cdot^{\cdot^{2^n}}}}}_{k \text{ ori}}\Bigg) = 2 + \underbrace{2^{2^{\cdot^{\cdot^{2^n}}}}}_{k-1 \text{ ori}}.$$

Într-adevăr, pentru $m = \underbrace{2^{2^{\cdot^{\cdot^{2^n}}}}}_{k-2 \text{ ori}}$ avem $\eta(2^{2^m}) = 2 + 2^m$.

3. Formule de calcul pentru $\eta(n)$

În [2] se arată c[formula (1) poate fi scrisă sub forma:

$$\eta(p^{a_{[p]}}) = p(a_{[p]})_{(p)} \tag{2}$$

adică pentru a calcula pe $\eta(p^a)$ scriem exponentul a în baza generalizată $[p]$ şi "îl citim" în baza standard (p):

$$(p): \ 1, p, p^2, \ldots, p^n, \ldots$$

Să observăm că "cititrea" în baza (p) presupune uneori calcule cu *cifra* p, care nu este cifră în această bază, dar poate apare ca cifră în baza $[p]$. Vom exemplifica utilizarea formulei (2) pentru calculul lui $\eta(3^{89})$. Parcurgem următoarele etape:

(i) scriem exponentul $a = 89$ în baza

$[3]: \ 1, 4, 13, 40, 121, \ldots$

obţinem $3_{[3]} = 2021$;

(ii) "citim" numărul 2021 în baza $(3): \ 1, 3, 9, 27, \ldots$. Avem $2021_{(3)} = 183_{(10)}$, deci $\eta(3^{89}) =$ $= 183$, ceea ce înseamnă că cel mai mic număr natural al cărui factorial este divizibil cu 3^{89} este 189.

Într-adevăr: $\sum\limits_{i \leq 1} \left[\frac{183}{3^i}\right] = 89$.

Facem observaţia că în baza generalizată $[p]$ tehnica de lucru este esenşţial diferită de tehnica de lucru din baza standard (p); aceasta datorită faptului că şirul $b_n(p) = p^n$, care determînă baza (p) satisface relaţia de recurenţă:

$$b_{n+1}(p) = p \cdot bn(p);$$

În timp ce şirul $an(p) = (p^n - 1)/(p - 1)$ cu ajutorul căruia se generează baza $[p]$ satisface relaţia de recurenţă:

$$a_{n+1}(p) = p \cdot a_n(p) + 1. \tag{3}$$

Datorită relaţiei (3) pentru a face adunarea în baza $[p]$ procedăm astfel: începem adunând cifrele de ordinul zecilor şi nu al unităţilor (cifrele corespunzătoare coloanei $a_2(p)$). Dacă adunând aceste cifre obţinem numărul $pa_2(p)$, vom utiliza o unitate din clasa unităţilor (coeficienţii lui $a_1(p)$) pentru a obţine $pa_2(p) + 1 = a_3(p)$.

Continuând adunarea pe coloana "zecilor" dacă obţinem din nou $pa_2(p)$, vom utiliza o nouă unitate din clasa unităţilor, etc. De exemplu pentru:

$$m_{[5]} = 441, n_{[5]} = 412 \text{ şi } r_{[5]} \text{ avem}$$

$$\begin{array}{r} m+n+r = 442+ \\ 412 \\ 44 \\ \hline dcba \end{array}$$

Începem adunarea cu coloana zecilor:

$$4 \cdot a_2(5) + a_2(5) + 4 \cdot a_2(5) = 5 \cdot a_2(5) + 4 \cdot a_2(5);$$

şi utilizând o unitate din coloana unităţilor obţinem:

$$a_3(5) + 4 \cdot a_2(5), \text{ deci } b = 4.$$

Continuând obţinem:

$$4 \cdot a_3(5) + 4 \cdot a_3(5) + a_3(5) = 5 \cdot a_3(5) + 4 \cdot a_3(5);$$

şi utilizând o nouă "unitate":

$$a_4(4) + 4 \cdot a_3(5), \text{ deci } c = 4 \text{ şi} d = 1.$$

În sfârşit, adunând unităţile rămase:

$$4 \cdot a_1(5) + 2 \cdot a_1(5) = 5 \cdot a_1(5) + a_1(5) = 5 \cdot a_1(5) + 1 = a_2(5),$$

rezultă că trebuie modificat şi $a = 0$. Deci $m + n + r = 1450_{[5]}$.

Aplicarea formulei (2) la calculul valorilor lui η pentru toate numerele între $N_1 = 31000000$ şi $N_2 = 31001000$, pe un PC 386 a dus la obţinerea unui timp de lucru de mai mult de 16 minute, din care cea mai mare parte a fost utilizată pentru descompunerea numerelor în factori primi.

Algoritmul a fost următorul:

1. Descompunerea numerelor n în factori primi $n = p_1^{d_1} p_2^{d_2} \dots p_t^{d_t}$;
2. Pentru n fixat, determinarea valorii $\max p_i \cdot d_i$;
3. $\eta_0 = \eta(p_i^{d_i})$, pentru i determinat la 2;
4. Deoarece $\eta(p^{d_i}) \leq p_j \cdot d_j$ ignorăm factorii pentru care $p_i \cdot a_1 \leq \eta_0$;
5. Calculăm $\eta(p_j^{d_j})$ pentru $p_j \cdot a_j > \eta_0$ şi determinăm cea mai mare dintre aceste valori, care va fi $\eta(n)$. Pentru punctele $2 - 5$ din program au trebuit mai puţin de 3 secunde.

Pentru a obţine alte formule de calcul pentru funcţia η (de fapt pentru $\eta(p^a)$) să considerăm exponentul a scris în cele două baze:

$$a_{(p)} = \sum_{i=0}^{n} c_i \cdot p^i \text{ şi } a_{[p]} = \sum_{j=1}^{v} k_j a_j(p) = \sum_{j=1}^{v} k_j \cdot \frac{p^j - 1}{p - 1}.$$

Obţinem:

$(p-1) \cdot a = \sum_{j=1}^{v} k_j p^j - \sum_{j=1}^{v} k_j$, deci notând:

$\sigma_{(a)} = \sum_{i=0}^{n} c_i$ - suma cifrelor lui a scris în baza (p);

$\sigma_{[p]}(a) = \sum_{j=1}^{v} k_j$ -suma cifrelor lui a scris în baza $[p]$; şi ţinând cont de faptul că $\sum_{j=1}^{v} k_j p^j = p(a_{[p]})_{(p)}$ obţinem:

$$\eta(p^a) = (p-1) \cdot a + \sigma_{[p]}(a). \tag{4}$$

Ţinând cont de exprimarea lui a în baza (p) obţinem:

$$p \cdot a_{(p)} = \sum_{i=0}^{n} c_i(p^{i+1} - 1) + \sum_{i=0}^{n} c_i \text{ sau:}$$

$$\frac{p}{p-1} \cdot a = \sum_{i=0}^{n} c_i \cdot a_{i+1}(p) + \frac{1}{p-1} \cdot \sigma_{(p)}(a), \tag{5}$$

prim urmare:

$$a = \frac{p-1}{p} \cdot (a_{[p]})_{[p]} + \frac{1}{p} \cdot \sigma_{(p)}(a) \tag{6}$$

Înlocuind această valoare a lui a în (4), se obţine:

$$\eta(p^a) = \frac{(p-1)^2}{p} \cdot (a_{(p)})_{[p]} + \frac{p-1}{p} \cdot \sigma_{(p)}(a) + \sigma_{[p]}(a). \tag{7}$$

Notând cu $E_{n,p}$ exponentul lui p în expresia lui $n!$,

$$E_{n,p} = \sum_{i \geq 1} \left[\frac{n}{p^i}\right];$$

se ştie [18] că $E_{n,p} = (n - \sigma_{(p)}(n))/(p-1)$, deci exprimând pe $\sigma_{(p)}(a)$ din (6), se deduce:

$$E_{n,p} = (a_{(p)})_{[p]} - a. \tag{8}$$

O altă formulă pentru $E_{n,p}$ se poate obţine astfel:

$$a = C_n \cdot p^n + C_{n-1} \cdot p^{n-1} + \ldots + C_1 \cdot p + C_0 \text{ deci:}$$

$$E_{a,p} = \frac{a}{p^n} + \frac{a}{p^{n-1}} + \ldots \frac{a}{p} = C_n + (C_n p + C_{n-1}) + \ldots + (C_n p^{n-1} + C_{n-1} p^{n-2} + \ldots + C_1) =$$
$$= C_n a_n(p) + C_{n-1} a_{n-1}(p) + \ldots + C_1 a_1.$$

Cu alte cuvinte dacă $a_{(p)} = \overline{C_n \cdot C_{n-1} \cdot \ldots \cdot C_1 \cdot C_0}$ atunci:

$$E_{a,p} = ((a - C_0)_{(p)})_{[p]} \equiv \left(\left(\left[\frac{a}{p}\right]\right)_{(p)}\right)_{[p]}.$$

Din (7) şi (8) se obţine:

$$\eta(p^a) = \frac{(p-1)^2}{p} \cdot (E_{a,p} + a) + \frac{p-1}{p} \cdot \sigma_{(p)(a)} + \sigma_{[p]}(a);$$

iar din (2) ţi (7) se deduce:

$$p \cdot \sigma_{[p]}(a) + (p-1) \cdot \sigma_{(p)}(a) = p^2 \cdot (a_{[p]})_{(p)} - (p-1)^2 \cdot (a_{(p)})_{[p]}.$$

4. FUNCŢIA SUMATOARE F_η

Se ştie că oricărei funcţii numerice f i se poate ataşa funcţia sumatoare F_f definită prin:

$$F_f(n) = \sum_{d/n} f(d);$$

şi că f se poate exprima cu ajutorul lui F_f prin formula de inversiune:

$$f(n) = \sum_{i,j=n} \mu(i) \cdot F(j), \tag{9}$$

unde μ este funcţia lui Mõbius ($\mu(1) = 1, \mu(k) = (-1)^q$) dacă numărul i este produsul a q numere prime diferite şi $\mu(i) = 0$ dacă i este divizibil cu un pătrat).

Pentru η avem:

$$F(n) = F_\eta(n) = \sum_{d/n} \eta(d) \text{ şi}$$

$$F(p^a = \eta(1) + \eta(p) + \ldots + \eta(p^a).$$

Din (4) deducem $\eta(p^j) = (p-1) \cdot j + \sigma_{[p]}(j)$ deci:

$$F(p^a) = \sum_{j=0}^{a} \eta(p^j) = (p-1) \cdot \sum_{j=1}^{a} j + \sum_{j=1}^{a} \sigma_{[p]}(j) = (p-1) \cdot \frac{a \cdot (a+1)}{2} + \sum_{j=1}^{a} \sigma_{[p]}(j).$$

În consecinţă:

$$F(p^a) = (p-1) \cdot \frac{a \cdot (a+1)}{2} + \sum_{j=1}^{a} \sigma_{[p]}(j) \tag{10}$$

Să considerăm acum:

$$n = p_t \cdot p_{t-1} \cdot \ldots \cdot p_1$$

cu $p_1 < p_2 < \ldots < p_t$ numere prime nu neapărat consecutive

Desigur $\eta(n) = p_t$ şi din:

$$F(1) = \eta(1) = 0;$$

$$F(p_1) = \eta(1) + \eta(p_1) = p_1;$$

$$F(p_1 \cdot p_2) = p_1 + 2p_2 = F(p_1) + 2p_2;$$

$$F(p_1 \cdot p_2 \cdot p_3) = p_1 + 2p_2 + 2^2 p_3 = F(p_1 \cdot p_2) + 2^2 p_3;$$

rezultă prin inducţie:

$$F(p_1 \cdot p_2 \cdot \ldots \cdot p_t) = F(p_1 \cdot p_2 \cdot \ldots \cdot p_{t-1}) + 2^{t-1} p^t;$$

adică:

$$F(p_1 \cdot p_2 \cdot \ldots \cdot p_t) = \sum_{i=1}^{t} 2^{i-1} p_i.$$

Egalitatea (9) devine:

$$p_t = \eta(n) = \sum_{u,v=n} \mu(u) \cdot F(v) = F(n) - \sum_i F\left(\frac{n}{p_i}\right) + \sum_{i,j} F\left(\frac{n}{p_i \cdot p_j}\right) + \ldots + (-1)^{t-1} \cdot \sum_{i=1}^{t} F(p_i);$$

şi deoarece $F(p_i) = p_i$, obţinem:

$$F\left(\frac{n}{p_i}\right) = F(p_1 \cdot p_2 \cdot \ldots \cdot p_{i-1} \cdot p_{i+1} \cdot \ldots \cdot p_k) = \sum_{j=1}^{i-1} 2^{j-1} \cdot p_j + \sum_{j=i+1}^{t} 2^{j-1} \cdot p_j =$$

$$= F(p_1 \cdot p_2 \cdot \ldots \cdot p_{i-1}) + 2^{i-1} \cdot F(p_{i+1} \cdot p_{i+2} \cdot \ldots \cdot p_t).$$

În mod analog avem

$$F\left(\frac{n}{p_i p_j}\right) = F(p_1 \cdot p_2 \cdots p_{i-1}) + 2^{i-1} \cdot F(p_{i+1} \cdot p_{i+2} \cdots p_{j-1}) + 2^{j-1} F(p_{j+1} \cdot p_{j+2} \cdots p_t).$$

Notând $N_{ij} = p_i \cdot \ldots \cdot p_j$, obţinem atunci:

$$\sum_{i=1}^{t-1} p_i = -F(n) + \sum_i F(N_{i-1} + 2^{i-1} \cdot F(_{i+1,t})) - \sum_{i<j} (F(N_{i-1}) + 2^{i-1} \cdot F(N_{i+1,j-1}) +$$

$$+ 2^{j-1} \cdot F(N_{j+1,t})) + \ldots$$

Generalizări ale funcşiei η

I.Bălăcenoiu [3] propune trei funcţii care generalizează funcţia η. În cele ce urmează vom prezenta aceste generalizări.

Fie X o mulţime nevidă, r o relaţie de echivalenţă pe X pentru care notăm cu X_r mulţimea cât şi $(I, \leq)$ o mulţime total ordonată. Dacă $g : X \to I$ este o funcţie injectivă oarecare, atunci funcţia $f : X \to I, f(x) = g(x)$ se spune că este o funcţie de standartizare. În acest caz despre mulţimea X se spune că este $(r, (I, \leq), f)$ stadartizată.

Dacă r_1 şi r_2 sunt două relaţii de echivalenţă pe X se ştie că relaţia $r = r_1 \bigwedge r_2$ unde:

$$xry \Leftrightarrow xr_1y \text{ şi } xr_2y$$

este o relaţie d echivalenţă.

Despre funcţiile $f_i : X \to I, i = \overline{1,s}$ se spune că au aceeaşi monoticitate dacă pentru orice $x, y \in X$ avem :

$$f_i(x) \leq f_i(y) / f_i(x) \leq f_i(y)$$

pentru orice $i, j = \overline{1,s}$.

În [3] se demonstrează următoarea teoremă:

Dacă funcţiile de standardizare $f_i : X \to I$ corespunzătoare relaţiilor de echivalenţă $r_i, i = \overline{1,s}$ sunt de aceeaţi monotonie atunci funcţia $f = \max f_i$ este funcţia de standardizare corespunzătoare relaţiei $r = \bigwedge\limits_i r_i$ şi are aceeaşi monotonie cu funcţiile f_i. Un alt element preliminar considerării celor trei generalizări ale funcţiei η prezentate în [3] este definiţia următoare.

Dacă $\top$şi $\perp$ sunt legi binare pe X respectiv I, spunem despre funcţia de standardizare $f : X \to I$ că este $\sum$ compozabilă dacă tripletul $(f(x), f(y), f(x \top y))$ satisface condiţia $\sum$. În acest caz se mai spune că funcţia f $\sum$ standardizează structura $(X, \top)$ pe structura $(I, \leq, \perp)$.

De exemplu funcţia η determină următoarele standardizări:

(a) funcţia η standardizează $\sum_1$ structura $(N^*, \cdot)$ pe structura $(N^*, \leq, +)$ prin:

$$\sum_1 : \eta(a \cdot b) \leq \eta(a) + \eta(b);$$

(b) funcţia η standardizează $\sum_2$ aceleaşi structuri, considernd:

$$\sum_2 : \max\{\eta(a), \eta(b)\} \leq \eta(a \cdot b) \leq \eta(a) \cdot \eta(b).$$

Funcţia Smarandache $\eta : N^* \to N$ a fost definită în [16] cu ajutorul următoarelor funcţii η_p:

Pentru orice număr prim p fie $\eta_p : N^* \to N^*$ astfel

(i) $(\eta_p(n))!$ este divizibil cu p^n;

(ii) $\eta_p(n)$ este cel mai mic întreg pozitiv cu proprietatea (i).

Pentru fiecare $n \in N^*$ să considerăm şi relaţiile $r_n \subset N^* X N^*$ definite prin condiţiile:

1. Dacă n este de forma $n = p^i$ cu $p = 1$ sau p număr prin şi $i \in N^*$ vom spune că a este în relaţia r_n cu b dacă şi numai dacă $\min\{k/k! = Mp^s\}$;

2. Dacă $n = p_1^{i_1}, p_2^{i_2}, \ldots, p_t^{i_t}$ atunci

$$r_n = r_{p_1}^{i_1} \bigwedge r_{p_2}^{i_2} \bigwedge \cdots \bigwedge r_{p_t}^{i_t}.$$

Definiţia 2. *Pentru orice $n \in N^*$ funcţia Smarandache de primul tip este funcţia $\eta_n : N^* \to N^*$ definită astfel:*

1. Dacă $n = p^i$, cu $p = 1$ sau p număr prim atunci $S_n(a) = k, k$ fiind cel mai mic întreg pozitiv pentru care $k! = M_{p^{ia}}$;

2. Dacă $n = p_1^{i_1}, p_2^{i_2}, \ldots, p_t^{i_t}$ atunci $\eta_k = \max\limits_{j=\overline{1,t}} \eta_{p_j^{i_j}}(a)$.

Se observă că:

a) Funcţiile η_n sunt funcţii de satndarduzare, corespunzătoare relaţiilor r_n şi pentru $n = 1$ avem $X_{r_1} = N^*$;

b) Dacă $n = p$ atunci η_n este funcţia η_p definită în [16];

c) Funcţiile η_n sunt crescătoare, deci sunt de aceeaşi monotonicitate, în sensul dat mai sus.

Theorem 1. *Funcţiile $\eta_n, \sum_1$ standardizează structura $(N^*, +)$ pe structura $(N^*, \leq, +)$ prin:*

$\sum_1 : \max\{\eta_n(a), \eta_n(b)\} \leq \eta_n(a+b) \leq \eta_n(a) + \eta_n(b)$ pentru orice $a, b \in N^*$ şi deasemenea $\sum_2$ standardizează $(N^*, +)$ pe $(N, \leq, .)$ prin

$\sum_2 : \max\{\eta_n(a), \eta_n(b)\} \leq \eta_n(a+b) \leq \eta_n(a) * \eta_n(b)$ pentru orice $a, b \in N^*$.

Demonstraţia este dată în [3].

Definiţia 3. *Funcţiile Smarandache de al doilea tip sunt funcţiile $\eta^k : N^* \to N^*$ definite prin $\eta^k(n) = \eta^n(k)$ pentru orice $k \in N^*$, unde η_n sunt funcţiile Smarandache de primul tip.*

Observăm că pentru $k = 1$ funcţia η^k este funcţia η definită în [17], cu modificarea $\eta(1) = 1$. Întradevăr, pentru $n > 1$ avem:

$$\eta^1(n) = \eta_n(1) = \max_j \eta_{p_j^{i_j}}(1) = \max_j \eta_{p_j}(i_j) = \eta(n).$$

Theorem 2. *Funcţiile Smarandache de al doilea tip $\sum_3$ standardizează structura $(N^*, *)$ pe structura $(N^*, \leq, t)$ prin*

$\sum_3 : \max\{\eta^k(a), \eta^k(b)\} \leq \eta^k(a * b) \leq \eta^k(a) + \eta^k(b)$ *pentru orice* $a, b \in N^*$ *şi* $\sum_4$ *standardizează* $(N^*, *)$ *pe* $(N^*, \leq, *)$ *prin*

$\sum_4 : \max\{\eta^k(a), \eta^k(b)\} \leq \eta^k(a * b) \leq \eta^k(a) * \eta^k(b)$ *pentru orice* $a, b \in N^*$.

Pentru a defini funcşiile Smarandache de al treilea tip să considerăm şirurile:

$$(a)\ \ 1 = a_1, a_2, \ldots, a_n, \ldots$$

$$(b)\ \ 1 = b_1, b_2, \ldots, b_n, \ldots$$

staisfăcând relaţii de recurenţă $a_{kn} = a_k * a_n$ şi respectiv $b_{kn} = b_k * b_n$.

Desigur există oricâte astfel de şiruri deoarece putem alege o valoare arbitrară pentru a_2 şi apoi să determinăm ceilalţi termeni cu ajutorul relaţiei de recurenţă. Cu ajutorul şirurilor (a) şi (b) definim funcţia $f_a^b : N^* \to N^*$ prin

$f_a^b(n) = \eta_{a_n}(b_n)$, unde η_{a_n} este funcţia Smarandache de primul tip. Se observă că:

(u) Dacă $a_n = 1$ şi $b_n = 1$ pentru orice $n \in N^*$ atunci $f_a^b = \eta_1$;

(v) Dacă $a_n = n$ şi $b_n = 1$ pentru orice $n \in N^*$ atunci $f_a^b = \eta^1$.

Definiţia 4. *Funcţiile Smarandache de al treilea tip sunt funcţiile $\eta_a^b = f_a^b$ în cazul în care şirurile (a) şi (b) sunt diferite de cele de la (u) şi (v).*

Theorem 3. *Funcţiile f_a^b realizează standardizarea $\sum_5$ între structurile $(N^*, *)$ şi $(N^*, \leq, +, *)$ prin*

$$\textstyle\sum_5 : \max\{f_a^b(k), f_a^b(n)\} \leq f_a^b(k * n) \leq b_n f_a^b(k) + b_k f_a^b(n).$$

Demonstraţia acestei teoreme este deasemenea dată în [3]. De aici rezultă că funcţiile Smarandache de al treilea tip satisfac:

$$\textstyle\sum_6 : \max\{\eta_a^b(k), \eta_a^b(n)\} \leq \eta_a^b(k \cdot n) \leq b_n \eta_a^b(k) + b_k \eta_a^b(n).$$

Exemplu: Considerând şirurile (a) şi (b) date prin $a_n = b_n = n$, pentru orice $n \in N^*$, funcţia Smarandache de al treilea tip corespunzătoare este $\eta_a^b : N^* \to N^*, \eta_a^b(n) = \eta_n(n)$ şi $\sum_6$ devine:

$$\max\{\eta_k(k), \eta_n(n) \leq \eta_{kn}(k \cdot n) \leq n\eta_k(k) + k\eta_n(n)$$

pentru orice $k, n \in N^*$.

Această relaţie este echivalentă cu relaţia următoare, scrisă cu ajutorul funcţiei η:

$$\max\{\eta(k^k), \eta(n^n) \leq \eta((kn)^{kn}) \leq n\eta(k^k) + k\eta(n^n).$$

5. Probleme rezolvate şi probleme nerezolvate referitoare la funcţia η

În [20] se dă o listă cuprinzătoare de probleme deschise referitoare la funcţia η. În [22] M.Mudge reia o parte din aceste probleme. Apari'ia articolului lui M.Mudge ca şi apariţia unei reviste dedicată studiului funcţiei η (Smarandache Function Journal în colaborare cu Facultatea de Matematică din Craiova şi Number Theory Publishing Co. din Glendale, Arizona) au determinat creşterea interesului pentru această funcţie. În cele ce urmează vom enumera câteva dintre problemele propuse în [8] şi reluate în [22] arătând stadiul rezolvării lor, dar vom aminti şi alte probleme interesante apărute ulterior articolului lui M.Mudge.

1. Să se investigheze şirurile $i, i+1, i+2, \ldots, i+x$ pentru care valorile lui η sunt crescătoare (descrescătoare). Răspunsul la această problemă au dat J.Duncan [7] şi Grønas [11]. Acesta din urmă arată că există şiruri crescătoare $u_1 < u_2 < \ldots < u_r$ de lungime oricât de mare pentru care valorile funcţie η sunt descrescătoare.

Referitor la următoarele trei probleme nu cunoaştem publicarea vreunui rezultat.

2. Găsiţi cel mai mic număr natural k astfel încât pentru orice n mai mic sau egal cu n_0 cel puţin unul dintre numerele: $\eta(n), \eta(n+1), \ldots, \eta(n+k-1)$ este:

(A) un pătrat perfec;

(B) un divizor al lui k^n.

Ce se întâmplă pentru k şi n_0 tinzând la infinit?

3. Construiţi numere prime având forma $\overline{\eta(n)\eta(n+1)\ldots\eta(n+k)}$ unde $\overline{ab}$ desemnează întregul obţinut prin concatenarea numerelor a şi b.

Un şir $1 \leq a_1 \leq \ldots$ de numere întregi se spune că este un A-şir dacă nici un a_i nu este suma a cel puţin doi termeni din şir.

4. Investigaţi posibilitatea construirii unui A-şir astfel încât şirul asociat $\eta(a_1), \eta(a_2), \ldots, \eta(a_n), \ldots$ este de asemenea un A-şir.

Notând $D_n(x) = |\eta(x+1) - \eta(x)|$ şi $D_n^{(k+1)}(x) = |D_n^{(k)}(x+1) - D_n^{(k)}(x)|$, pentru $k \in N^*$, unde $D_n^{(1)}(x) = D_n(x)$ articolul lui M.Mudge reia următoarea problemă.

5. Investigaţi conjectura $D_n^{(k)}(x)$ are valoarea unu sau zero pentru oricare $k \geq 2$.

J.Duncan [8] verifică conjectura pentru toate numerele naturale până la 32000. În acelaşi articol se arată că raportul între numărul de 1-uri şi numărul zerourilor este aproximativ egal cu 1 pentru valori mari ale lui k. De asemenea se arată că pentru $k > 100$ şi până la 32000 raportul $D_n^{(k)}(x)/D_n^{(k-1)}(x)$ este aproximativ egal cu -2.

T.Yau [24] pune următoarea problemă: pentru ce triplet de numere consecutive $n.n+1, n+2$ funcţia η verifică o egalitate de tip Fibonacci, adică $\eta(n) + \eta(n+1) = \eta(n+2)$. El observă că în primele 1200 de numere naturale există două soluţii şi anume $n = 11$ şi $n = 121$, dar nu găseşte o soluţie generală.

P.Gronas [10] dă răspuns următoarei întrebări: "Există o funcţie de numere n pentru care $\sigma_\eta(n) = n$?" unde $\sigma_\eta(n) = \sum\limits_{d/n:d>0} \eta(d)$. El arată că singurele soluţii ale acestei ecuaţii sunt $n \in \{8, 12, 18, 20, 2p\}$ unde p este număr prim.

M.Costewitz [15] abordează pentru prima oară problema găsirii cardinalului mulţimii $M_n = \{x/\eta(x) = x\}$. În [25] se arată că dacă descompunerea lai n în factori primi este $n_0 = p_1^{a_1} \cdot p_2^{a_2} \cdot \ldots \cdot p_t^{a_t}$ cu $p_1 < p_2 < \ldots < p_t$ şi notăm $e_i = \sum[n/p_i^j]$ iar $n_0 = p_1^{e_1} \cdot p_2^{e_2} \cdot \ldots \cdot p_t^{e_t}$ şi $p_1^{e_1-a_1} \cdot p_2^{e_2-a_2} \cdot \ldots \cdot p_t^{e_t-a_t}$, atunci *card* $M_n = (\sigma(n_0) - \sigma(n_0)\sigma(Q))$ unde $\sigma(n)$ este suma divizorilor lui n, iar $Q = \prod\limits_k q_k^{f_k}$, numerele $q_1, q_2, \ldots, q_r$ fiind toate numerele prime mai mici decât n şi care nu sunt divizori ai lui n. Exponentul f_k este $f_k = \sum\limits_j \left[\frac{n}{q_k^j}\right]$.

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K-Divisibility and K-Strong Divisibility Sequences

A sequence of rational integers g is called a **divisibility sequence** if and only if

$$n|m \Rightarrow g(n)|g(m)$$

for all positive integers n, m. [See [3] and [4]]

Also, g is called a **strong divisibility sequence** if ans only if

$$(g(n), g(m)) = g((n, m))$$

for all positive integers n, m. [See [1], [2], [3], [4] and [5]]

Of course, it is easy to show that the results of the Smarndache function $S(n)$ is niether a divibility or a strong divisibility sequence because $4|20$ but $S(4) = 4$ does not divide $5 = S(20)$, and $(S(4), S(20)) = (4,5) = 1 \neq 4 = S(4) = S((4,20))$.

a) However, is there an infinite subsequence of integers $M = \{m_1, m_2, \ldots\}$ such that S is a divisibility sequence on M?

b) If $P\{p_1, p_2, \ldots\}$ is the set of prime numbers, the S is not a strong divisibility sequence on P, because for $i \neq j$ we have

$$(S(p_i), S(p_j)) = (p_i, p_j) = 1 \neq 0 = S(1) = S((p_i, p_j)).$$

And the same question can be asked about P as was asked in part (a).

We introduce the following two notions, which are generalizations of a "divisibility sequence" and "sttrong divisibility sequrnce" respectively.

1) A k-divisibility sequence, where $l \geq 1$ is an integer, is defined in the following way:

If $n|m \Rightarrow g(n)|g(m) \Rightarrow g(g(n))|g(g(m)) \Rightarrow \ldots \Rightarrow \underbrace{g(\ldots(g(n))\ldots)}_{k \text{ times}} | \underbrace{g(\ldots(g(m))\ldots)}_{k \text{ times}}$ for all positive integers n, m.

For example, $g(n) = n!$ is a k-divisibility sequence.

Also: any constant sequence is a k-divisibility sequence.

2) A k-strong divisibility sequence, where $k \geq 1$ is an integer, is defined in the following way:

If $(g(n_1), g(n_2), \ldots, g(n_k)) = g((n_1, n_2, \ldots, n_k))$ for all positive integers $n_1, n_2, \ldots, n_k$.

For example, $g(n) = 2n$ is a k-strong divisibility sequence, because $(2n_1, 2n_2, \ldots, 2n_k) = 2 * (n_1, n_2, \ldots, n_k) = g((n_1, n_2, \ldots, n_k))$.

Remarks: If g is a divisibility sequence and we apply its definition k-times, we get that g is a k-divisibility sequence for any $k \geq 1$. The converse is also true. If g is k-strong divisibility sequence, $k \geq 2$, then g is a strong divisibility sequence. This can be seen by taking the definition of a k-strong divisibility sequence and replacing n by n_1 and all $n_2, \ldots, n_k$ by m to obtain $(g(n), g(m), \ldots, g(m)) = g((n, m, \ldots, m))$ or $(g(n), g(m)) = g((n, m))$.

The converse is also true, as

$$(n_1, n_2, \ldots, n_k) = ((\ldots((n_1, n_2), n_3), \ldots), n_k).$$

Therefore, we found that:

a) The divisibility sequence notion is equivalent to a k-divisibility sequence, or a generalization of a notion id equivalent to itself.

Is there any paradox or dilemma?

b) The strong divisibility sequence is equivalent to the k-strong divisibility sequence notion. As before, a generalization of a notion is equivalent to itself.

Again, is there any paradox or dilemma?

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Conjecture (General Fermat Numbers)

Let a, b be integers ≥ 2 and k an integer such that $(a, c) = 1$.

One construct the function $P(k) = a^{b^k} + c$, where $k \in \{0, 1, 2, \ldots\}$.

Then:

a) For any geven triplett (a, b, c) there is at least a k_0 such that $P(k_0)$ is prime.

b) There are no (a, b, c) triplett such that $P(k)$ is prime for all $k \geq 0$.

c) Is it possible to find a triplett (a, b, c) such that $P(k)$ is prime for ifinetely many k's?

ASUPRA UNEI METODE A LUI W.SIERPINSKI DE REZOLVARE ÎN NUMERE INTREGI A ECUAŢIILOR LINIARE

În nota următoare se fac câteva remarci privind metoda expusă de Sierpinski în [1], remarci ce au ca scop simplificarea şi extinderea acestei metode (vezi [2]).

Fie o ecuaţie liniară $a_1x_1 + \ldots + a_nx_n = b$ având coeficienţii numere întregi.

a) În cazul în care un coeficient a_i este negativ W.S. înlocuieşte necunoscuta x_i cu $-x_i$ pentru ca toţi coeficienţii să fie pozitivi.

Considerăm că această înlocuire nu este necesară, deoarece în rezolvare nu întâmpinăm dificultăţi cauzate de coeficienţii negativi, şi apoi se măreşte inutil numărul variabilelor - fie ele şi auxiliare; (chiar în [1], în momentul când se compară coeficienţii ar putea fi consideraţi în valoare absolută).

b) Dacă doi din ceficienţii $a_1, \ldots, a_n$ ar fi egali, de exemplu $a_1 = a_2$, W.S. punea $x_1 + x_2 = x$, în care ideea de a micşora numărul necunoscutelor; considerăm că aceast pas poate fi extins, şi anume dacă $a_1 = \pm a_2 = \ldots \pm a_k$ putem lua $x_1 \pm x_2 \pm \ldots \pm x_k = x$ semnele fiind corespunzătoare coeficienţilor, (substituţie care nu lasă să se întrezăreascăîn [1] p. 94); putem extinde chiar mai mult; dacă spre exemplu coeficienţii $a_1, a_2, \ldots, a_r$ au un divizor pozitiv comun $d \neq 1$, deci $a_i = da'_i, i = 1, \ldots, r$, atunci se notează $a'_1x_1 + \ldots + a'_rx_r = x$, şi reducerea numărului de necunoscute este mai masivă; de fiecare dată ecuaţia nou obţinută are mai puţine necunoscute, şi este echivalentă cu prima; justificarea rămâne aceeaşi ca în [1].

c) Apoi W.S. alege cel mai mare coeficient (toţi presupuţi de el fiind naturali), a_1 de exepmlu, şi prin împărţirea întregă la un altul, a_2 să zicem se obţine $a_1 = a_2 \cdot p + a'_2, p \in N$, înlocuindu-se $x'_1 = px_1 + x_2, x'_2 = x_1, a'_1 = a_2$ deducând astfel la reducerea coeficientului cal mai mare; considerăm că nu este în mod forfetar să se efectuieze această operaţie având drept coeficient pe cel mai mare (în modul), ci să se aleagă acei coeficienţi a_i şi a_j pentru care împărţirea întreagă să aibă forma $a_i = pa_j \pm r$ cu $r = 1$ sau, dacă nu e posibil, în aşa fel ca restul să fie cât mai mic în modul, nenul (vezi [2], capitolul "Another whole number algorithm to solve linear equations (using congruency)" p. 16-21) deoarece se caută să se obţină printr-un număr cât mai mic de paşi coeficientul ± 1 pentru cel puţin una din necunoscute (este posibil să se obţină acest coeficient în cazul în care ecuaţia admite soluţii întregi - vezi [2], p. 19, Lemma 5); iar în alte cazuri se alege chiar cel mai mic (!) coeficient în modul (din aceleaşi considerente - vezi [2], capitolul "A whole number algorithm to solve linear equations" p. 11-15), alteori un coeficient intermediar între aceste extreme; (vezi [2] p. 14, Note); această operaţie este mai importantă, decât a) şi b) şi ar fi deci indicat să se execute prima-aplicarea ei făcând apoi inutilă folosirea celorlalte.

Ca **exemplu** vom prelua aceeaşi ecuaţie din [1] p. 95, pe care o vom rezolva în conformitate cu cele expuse aici $6x+10y-7z=11$. **Soluţia I.** $-7=6(-1)-1$ şi $6(x-z)-z+10y=11$, deci am obţinut din primul pas coeficientul -1. Notând $x-z=t\in Z$, atunci $z=6t+10y-11$ de unde $x=t+z=7t+10y-11$, iar y este arbitrar în Z.**Soluţia II.** $6(x+2y-z)-2y-z=11$ şi tot din primul pas am obţinut coeficientul -1. Punând $x+2y-z=u\in Z$ obţinem $6u-2y-z=11$ şi astfel $z=6u-2y-11$. Rezultă $x=u-2y+z=7u-4y-11$ cu $y\in Z$ arbitrar. Observăm că cele două soluţii sunt diferite ca expresie între ele ţi diferite de cea dată de W.Sierpinski în [1], p. 95, dar toate trei sunt echivalente ca soluţii generale pentru ecuaţia dată (vezi [3], sau [2] p. 4-10).

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ÎN LEGĂTURĂ CU O PROBLEMĂ DE LA CONCURSUL DE MATEMATICĂ, FAZA LOCALĂ, RÂMNICUL VÂLCEA

Se prezintă în această notă o extindere a unei probleme dată la Olimpiada de matematică, faza locală, la Râmnicul Vâlcea, clasa a VI-a, 1980.

Fie $a_1, \ldots, a_{2n+1}$ numere întregi şi $b_1, \ldots, b_{2n+1}$ aceleaşi numere în altă ordine. Să se arate că expresia: $E = (a_1 \pm b_1) \cdot (a_2 \pm b_2) \cdot \ldots \cdot (a_{2n+1} \pm b_{2n+1})$, unde semnele + sau - sînt luate arbitrar în fiecare paranteză, este un număr par.

Soluţie:

Presupunem că expresia E este un număr impar. Atunci rezultă că fiecare paranteză este un număr impar, deci în fiecare paranteză avem un număr par şi unul impar.

Avem astfel $2n + 1$ numere pare. (1)

Dacă într-o paranteză există, să zicem, un a_{i_0} număr par, atunci există o altă paranteză în care un $b_{j_0} = a_{i_0}$ şi deci b_{j_0} este număr par.

Astfel pentru fiecare a_i =număr par dintr-o paranteză, există un b_j număr par şi ar trebui să avem în total, în expresia E, un număr par de numere pare. Dar aceasta contrazice (1), contradicţie care demonstrează problema.

Observaţia 1. Demonstraţia ar fi decurs într-un mod analog dacă ne-am fi referit la numărul de numere impare din expresie. O propunem cititorului.

Observaţia 2. Pentru $n = 3$ se obţine problema dată la olimpiadă, problemă de care am amintit în partea anterioară a notei.

["Caiet 32/matematică", Craiova, Anul IV, Nr. 4, pp. 44-5, Reprografia Universităţii din Craiova]

NUMEROLOGY (I)

or

Properties of the Numbers

1) Reverse sequence:

1, 21, 321, 4321, 54321,654321,7654321, 87654321,987654321,10987654321, 1110987654321, 121110987654321, ...

2) Multiplicative sequence:

2, 3 ,6, 12, 18, 24, 36, 48, 54, ...

General definition: if m_1, m_2, are the first two terms of the sequence, then m_k, for $k \geq 3$, is the smallest number equal to the product of two previous distinct terms.

All terms of rank ≥ 3 are divisible by m_1 and m_2.

In our case the first two terms are 2, respectively 3.

3) Wrong numbers:

(A number $n = \overline{a_1 a_2 \ldots a_k}$, of at least two digits, with the following property:

the sequence $a_1, a_2, \ldots a_k, b_{k+1}, b_{k+2}, \ldots$ (where b_{k+i} is the product of the previous k terms, for any $i \geq 1$) contains n as its term.)

The author conjectures that there is no wrong number (!)

Therefore, this sequance is empty.

4) Impotent numbers:

2, 3, 4, 5, 7, 9, 11, 13, 17, 19, 23, 25, 29, 31, 41, 43, 47, 49, 53, 59, 61, ...

(A number n those proper divisors product is less than n.)

Remark: this sequence is $\{p, p^2;$ where p is a positive prime$\}$.

5) Random sieve:

1, 5, 6, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, 37, 41, 43, 47, 53, 59, ...

General definition:

- choose a positive number u_1 at random;
- delete all multiples of all its divisors, exept this number;
- chose another number u_2 greater than u_1 among those remaining;
- delete all multiples of all its divisors, ecxept this second number;

... so on.

The remaining numbers are all coprime two by two.

The sequence obtained $u_k, k \geq 1$, is less dense than the prime number sequence, but it tends to the prime number sequence as k tends to infinite. That's why this sequence may be important.

In our case, $u_1 = 6, u_2 = 19, u_3 = 35, \ldots$.

6) Cubic base:

0, 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 30, 31, 32, 100, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 117, 120, 121, 122, 123, ...

(Each number n written in the cubic base.)

(One defines over the set of natural numbers the following infinte base: for $k \geq 1$ $s_k = k\hat{}3$.)

We prove that every positive integer A may be uniquely written in the cubic base as:

$A = (\overline{a_n \ldots a_2 a_1})_{(C3)} \stackrel{def}{=} \sum_{i=1}^{n} a_i c_i$, with $0 \leq a_1 \leq 7, 0 \leq a_2 \leq 3, 0 \leq a_3 \leq 2$ and $0 \leq a_i \leq 1$ for $i \geq 4$, and of course $a_n = 1$, in the following way:

- if $c_n \leq A < c_{n+1}$ then $A = c_n + r_1$;

- if $c_m \leq r_1 < c_{m+1}$ then $r_1 = c_m + r_2, m < n$;

and so on untill one obtains a rest $r_j = 0$.

Therefore, any number may be written as a sum of cubes (1 not counted as cube - being obvious)$+e$, where $e = 0, 1, \ldots$, or 7.

If we denote by $c(A)$ the superior square part of A (i.e. the largest cube less than or equal to A), then A is written in the cube base as:

$$A = c(A) + c(A - c(A)) + c(A - c(A) - c(A - c(A))) + \ldots.$$

This base may be important for partitions with cubes.

7) Anti-symmetric sequence:

11, 1212, 123123, 12341234, 1234512345, 123456123456, 12345671234567, 1234567812345678, 123456789123456789, 1234567891012345678910, 12345678910111234567891011, 123456789101112123456789101112, ...

8-16) Recurence type sequences:

A. $1, 2, 5, 26, 29, 677, 680, 701, 842, 845, 866, 1517, 458330, 458333, 458354, \ldots$

($ss2(n)$ is the smallest number, strictly greater than the previous one, which is the squares sum of two previous distinct terms of the sequence;
in our particular case the first two terms are 1 and 2.)

Recurrence definition: 1) The number $a \leq b$ belong to SS2;

2) If b, c belong to SS2, then $b^2 + c^2$ belong to SS2 too;

3) Only numbers, obtained by rules 1) and/or 2) applied a finite number of times, belongs to SS2.

The sequence (set) SS2 is increasingly ordered.

[Rule 1) may be changed by: the given numbers $a_1, a_2, \ldots, a_k$, where $k \geq 2$, belongs to SS2.]

B. 1, 1, 2, 4, 5, 6, 16, 17, 18, 20, 21, 22, 25, 26, 27, 29, 30, 31, 36, 37, 38, 40, 41, 42, 43, 45, 46, ...

($ss1(n)$ is the smallest number, strictly greater than the previous one (for $n \geq 3$), which is the squares sum of one or more previous distinct terms of the sequence;

in our particular case the first term is 1.)

Recurrence definition:

1) The number a belongs to SS1;

2) If $b_1, b_2, \ldots, b_k$ belongs to SS1, where $k \geq 1$, then $b_1\hat{}2 + b_2\hat{}2 + \ldots + b_k\hat{}2$ belongs to SS1 too;

3) Only numbers, obtained by reles 1) and/or 2) applied a finite number of times, belong to SS1.

The sequence (set) SS1 is increasingly ordered.

[Rule 1) may be changed by: the given numbers $a_1, a_2, \ldots, a_k$, where $k \geq 1$, belong to SS1.]

C. 1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 14, 15, 16, 18, 19, 21, ...

($nss2(n)$ is the smallest number, strictly greater than the previous one, which is NOT the squares sum of two previous distinct terms of the sequence;

in our particular case the first two terms are 1 and 2.)

Recurrence definition:

1) The numbers $a \leq b$ belong to NSS2;

2) If b, c belomg to NSS2, then $b\hat{}2 + c\hat{}2$ DOES NOT belong to NSS2; any other numbers belong to NSS2;

3) Only numbers, obtained by rules 1) and/or 2) applied a finite number of times, belong to NSS2.

The sequence (set) NSS2 is increasingly ordered.

[Rule 1) may be changed by: the given numbers $a_1, a_2, \ldots, a_k$, where $k \geq 2$, belong to NSS2.]

D. 1, 2, 3, 6, 7, 8, 11, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 47, ...

($nss1(n)$ is the smallest number, strictly greater than the previous one, which is NOT the squares sum of the one or more previous distinct terms of the sequence;

in our particular case the first term is 1.)

Recurrence definition:

1) The number a belongs to NSS1;

2) If $b_1, b_2, \ldots, b_k$ belongs to NSS1, where $k \geq 1$, then $b_1\hat{}2 + b_2\hat{}2 + \ldots + b_k\hat{}2$ DO NOT belong to NSS1; any other numbers belong to NSS1;

3) Only numbers, obtained by rules 1) and/or 2) applied a finite number of times, belong to NSS1.

The sequence (set) NSS1 is increasingly ordered.

[Rule 1) may change by: the given numbers $a_1, a_2, \ldots, a_k$, where $k \geq 1$, belong to NSS1.]

E. 1, 2, 9, 730, 737, 389017001, 389017008, 389017729, ...

($cs2(n)$ is the smallest number, strictly greater than the previous one, which is the cubes sum of two previous distinct terms of the sequence;

in our particular case the first two terms are 1 and 2.)

Recurrence definition:

1) The numbers $a \leq b$ belong to CS2;

2) If c, d belong to CS2, then $c\hat{}3 + d\hat{}3$ belongs to CS2 too;

3) Only numbers, obtained by rules 1) and/or 2) applied a finite number of times, belong to CS2.

The sequence (set) CS2 is increasingly ordered.

[Rule 1) may be changed by: the given numbers $a_1, a_2, \ldots a_k$, where $k \geq 2$, belong to CS2.]

F. 1, 1, 2, 8, 9, 10, 512, 513, 514, 520, 521, 522, 729, 730, 731, 737, 738, 739, 1241, ...

($cs1(n)$ is the smallest number, strictly greater than the previous one (for $n \geq 3$), which is the cubes sum of one or more previous distinct terms of the sequence;

in our particular case the first term is 1.)

Recurrence definition:

1) The numbers $a \leq b$ belong to CS1;

2) If $b_1, b_2, \ldots, b_k$ belongs to CS1, where $k \geq 1$, then $b_1\hat{}3 + b_2\hat{}3 + \ldots + b_k\hat{}3$ belong to CS1 too;

3) Only numbers, obtained by rules 1) and/or 2) applied a finite number of times, belong to CS1.

The sequence (set) CS1 is increasingly ordered.

[Rule 1) may be changed by: the given numbers $a_1, a_2, \ldots a_k$, where $k \geq 2$, belong to CS1.]

G. 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 36, 37, 38, ...

($ncs2(n)$ is the smallest number, strictly greater than the previous one, which is NOT the cubes sum of two previous distinct terms of the sequence; in our particular case the first term is 1 and 2.)

Recurrence definition:

1) The numbers $a \leq b$ belong to NCS2;

2) If c, d belong to NCS2, then $c\hat{}3+d\hat{}3$ DOES NOT belong to NCS2; any other numbers do belong to NCS2;

3) Only numbers, obtained by rules 1) and/or 2) applied a finite number of times, belong to NCS2.

The sequence (set) NCS2 is increasingly ordered.

[Rule 1) may be changed by: the given numbers $a_1, a_2, \ldots a_k$, where $k \geq 2$, belong to NCS2.]

H. 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 34, 37, 38, 39, ...

($ncs1(n)$ is the smallest number, strictly greater than the previous one, which is NOT the cubes sum of the one or more previous distinct terms of the sequence;
in our particular case the first term is 1.)

Recurrence definition:

1) The number a belongs to NCS1;

2) If $b_1, b_2, \ldots, b_k$ belongs to NCS1, where $k \geq 1$, then $b_1\hat{}2 + b_2\hat{}2 + \ldots + b_k\hat{}2$ DO NOT belong to NCS1; any other numbers belong to NCS1;

3) Only numbers, obtained by rules 1) and/or 2) applied a finite number of times, belong to NCS1.

The sequence (set) NCS1 is increasingly ordered.

[Rule 1) may change by: the given numbers $a_1, a_2, \ldots, a_k$, where $k \geq 1$, belong to NCS1.]

I. General-recurrence type sequence:

General recurrence definition:

Let $k \geq j$ be natural numbers, $a_1, a_2, \ldots, a_k$ given elements, and R a j-relationship (relation among j elements).

Then:

1) The elements $a_1, a_2, \ldots, a_k$ belong to SGR.

2) If $m_1, m_2, \ldots, m_j$ belong to SGR, then $R(m_1, m_2, \ldots, m_j)$ belongs to SGR too.

3) only elements, obtained by rules 1) and/or 2) applied a finite number of times, belong to SGR.

The sequence (set) SGR is increasingly ordered.

Method of consttruction of the general recurrence sequence:

- level 1: the given elements $a_1, a_2, \ldots, a_k$ belong to SGR;
- level 2: apply the relationship R for all combinations of j elements among $a_1, a_2, \ldots, a_k$; the results belong to SGR too;

order all elements of level 1 and 2 together;

....

- level $i+1$:

if $b_1, b_2, \ldots, b_m$ are all elements of levels $1, 2, \ldots, i-1$, and $c_1, c_2, \ldots, c_n$ are all elements of level i, then apply the relationship R for all combinations of j elements among $b_1, b_2, \ldots, b_m, c_1, c_2, \ldots, c_n$ such that at least an element is from the level i;

the results belong to SGR too;

order all elements of levels i and $i+1$ together;

and so on...

17)-19) Partition type sequences:

A. 1, 1, 1, 2, 2, 2, 2, 3, 4, 4, ...

(How many times is n written as sum of non-nul squares, desregarding the terms order; for example:

$9 = 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2$

$= 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 2^2$

$= 1^2 + 2^2 + 2^2$

$= 3^2$,

therefore $ns(9) = 4$.)

B. 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, ...

(How many times is n written as a sum of non-null cubes, desregarding the terms order; for example:

$9 = 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3$

$= 1^3 + 2^3$,

therefore $nc(9) = 2$.)

C. General-partition type sequence:

Let f be an arithmetic function, and R a relation among numbers.

{ How many times can n be written under the form:

$$n = R(f(n_1), f(n_2), \ldots, f(n_k))$$

for some k and $n_1, n_2, \ldots, n_k$ such that

$$n_1 + n_2 + \ldots + n_k = n?\}$$

20) Concatenate sequence:

1, 22, 333, 4444, 55555, 666666, 7777777, 88888888, 999999999, 10101010101010101010, 1111111111111111111111, 121212121212121212121212, 13131313131313131313131313, 1414141414141414141414141414, 151515151515151515151515151515,...

21) Triangular base:

1, 2, 10, 11, 12, 100, 101, 102, 110, 1000, 1001, 1002, 1010, 1011, 10000, 10001, 10002, 10010, 10011, 10012, 100000, 100001, 100002, 100010, 100011, 100012, 100100, 1000000, 1000001, 1000002, 1000010, 1000011, 1000012, 1000100,...

(Numbers written in the triangular base, defined as follows: $t(n) = n(n+1)/2$, for $n \geq 1$.)

22) Double factorial base:

1, 10, 100, 101, 110, 200, 201, 1000, 1001, 1010, 1100, 1101, 1110, 1200, 10000, 1001, 10010, 10100, 10101, 10110, 10200, 10201, 11000, 11001, 11010, 11100, 11101, 11110, 11200, 11201, 12000, ...

(Numbers written in the double factorial base, defined as follows: $df(n) = n!!$)

23) Non-multiplicative sequence:

General definition: let $m_1, m_2, \ldots, m_k$ be the first k given terms of the sequence, where $k \geq 2$;

then m_i, for $i \geq k+1$, is the smallest number not equal to the product of k previous distinct terms.

24) Non-arithmetic progression:

1, 2, 4, 5, 10, 11, 13, 14, 28, 29, 31, 32, 37, 38, 40, 41, 64, ...

General definition: if m_1, m_2, are the first two terms of the sequence, then m_k, for $k \geq 3$, is the smallest number such that no 3-term arithmetic progression is in the sequence.

in our case the first two terms are 1, respectively 2.

Generalization: same initial conditions, but no i-term arithmetic progression in the sequence (for a given $i \geq 3$).

25) Prime product secuence:

2, 7, 31, 211, 2311, 30031, 510511, 9699691, 223092871, 6469693231, 200560490131, 7420738134811, 304250263527211, ...

$P_n = 1 + p_1 p_2 \dots p_n$, where p_k is the k-th prime.

Question: How many of them are prime?

26) Square product sequence:

2, 5, 37, 577, 14401, 518401, 25401601, 1625702401, 131681894401, 13168189440001, 1593350922240001, ...

$S_n = 1 + s_1 s_2 \dots s_n$, where s_k is the k-th square number.

Question: How many of them are prime?

27) Cubic product sequence:

2, 9, 217, 13825, 1728001, 373248001, 128024064001, 65548320768001, ...

$C_n = 1 + c_1 c_2 \dots c_n$, where c_k is the k-th cubic number.

Question: How many of them are prime?

28) Factorial product sequence:

2, 3, 13, 289, 34561, 24883201, 125411328001, 5056584744960001, ...

$F_n = 1 + f_1 f_2 \dots f_n$, where f_k is the k-th factorial number.

Question: How many of them are prime?

29) U-product sequence {generalization}:

Let $u_n, n \geq 1$, be a positive integer sequence. Then we define a U-sequence as follows:

$U_n = 1 + u_1 u_2 \dots u_n$.

30) Non-geometric progression:

1, 2, 3, 5, 6, 7, 8, 10, 11, 13, 14, 15, 16, 17, 19, 21, 22, 23, 24, 26, 27, 29, 30, 31, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 50, 51, 53, ...

General definition: if m_1, m_2, are the first two terms of the sequence, then m_k, for $k \geq 3$, is the smallest number such that no 3-term geometric progression is in the sequence.

In our case the first two terms are 1, respectively 2.

31) Unary sequence:

11, 111, 11111, 1111111, 11111111111, 1111111111111, 11111111111111111, 1111111111111111111, 11111111111111111111111, 11111111111111111111111111111, 1111111111111111111111111111111, ...

$u(n) = \overline{11\ldots1}$, p_n digits of "1", where p_n is the n-th prime.

The old quenstion: are there an infinite number of primes belomging to the sequence?

32) No prime digits sequence:

1, 4, 6, 8, 9, 10, 11, 1, 1, 14, 1, 16, 1, 18, 19, 0, 1, 4, 6, 8, 9, 0, 1, 4, 6, 8, 9, 40, 41, 42, 4, 44, 4, 46, 4, 48, 49, 0, ...

(Take out all prime digits of n.)

33) No square digits sequence:

2, 3, 5, 6, 7, 8, 2, 3, 5, 6, 7, 8, 2, 2, 22, 23, 2, 25, 26, 27, 28, 2, 3, 3, 32, 33, 3, 35, 36, 37, 38, 3, 2, 3, 5, 6, 7, 8, 5, 5, 52, 52, 5, 55, 56, 57, 58, 5, 6, 6, 62, ...

(Take out all square degits of n.)

34) Concatenated prime sequence:

2, 23, 235, 2357, 235711, 23571113, 2357111317, 235711131719, 23571113171923, ...

Conjecture: there are infinetely many primes among these numbers!

35) Concatenated odd sequence:

1, 13, 135, 1357, 13579, 1357911, 135791113, 13579111315, 1357911131517, ...

Conjecture: there are infinetely many primes among these numbers!

36) Concatenated even sequence:

2, 24, 246, 2468, 246810, 24681012, 2468101214, 246810121416, ...

Conjecture: none of them is a perfect power!

37) Concatenated S-sequence {generalization}:

Let $s_1, s_2, s_3, s_4, \ldots, s_n, \ldots$ be an infinite sequence (noted by S.)

Then:

$s_1, \overline{s_1, s_2}, \overline{s_1 s_2 s_3}, \overline{s_1 s_2 s_3 s_4}, \overline{s_1 s_2 s_3 s_4 \ldots s_n}, \ldots$ is called the Concatenated S-sequence.

Question:

a) How many terms of the Concatenated S-sequence belong to the initial S-sequence?

b) Or, how many terms of the Concatenated S-sequence verify the realtion of other given sequences?

The first three cases are particular.

Look now at some other examples, when S is the sequence of squares, cubes, Fibonacci respectively (and one can go so on):

Concatenated Square sequence:

1, 14, 149, 14916, 1491625, 149162536, 14916253649, 1491625364964, ...

How many of them are perfect squares?

Concatenated Cubic sequence:

1, 18, 1827, 182764, 182764125, 182764125216, 1827631252166343, ...

How many of them are perfect cubes?

Concatenated Fibonacci sequence:

1, 11, 112, 1123, 11235, 112358, 11235813, 1123581321, 112358132134, ...

Does any of these numbers is a Fibonacci number?

References

[1] F.Smarandache, "Properties of Numbers", University of Craiova Archives, 1975; [see also Arzona State University Special Collections, Tempe, Arizona, USA].

38) Teh Smallest Power Function:

$SP(n)$ is the smallest number m such that $m\hat{}m$ is divisible by n.

The following sequence $SP(n)$ is generated:

1, 2, 3, 2, 5, 6, 7, 4, 3, 10, 11, 6, 13, 14, 15, 4, 17, 6, 19, 10, 21, 22, 23, 6, 5, 26, 3, 14, 29, 30, 31, 4, 33, 34, 35, 6, 37, 38, 39, 20, 41, 42, ...

Remark:

If p is prime, then $SP(n) = p$.

If r is square free, then $SP(r) = r$.

If $n = (p_1\hat{}s_1) \cdot \ldots \cdot (p_k\hat{}s_k)$ and all $s_i \leq p_i$, then $SP(n) = n$.

If $n = p\hat{}s$, where p is prime, then:

$$SP(n) = \begin{cases} p, \text{ if } 1 \leq s \leq p; \\ p\hat{}2, \text{ if } p+1 \leq s \leq 2p\hat{}2; \\ p\hat{}3, \text{ if } 2p\hat{}2+1 \leq s \leq 3p\hat{}3; \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ p\hat{}t, \text{ if } (t-1)p\hat{}(t-1)+1 \leq s \leq tp\hat{}t. \end{cases}$$

Generally, if $n = (p_1\hat{}s_1) \cdot \ldots \cdot (p_k\hat{}s_k)$, with all p_i prime, then:

$SP(n) = (p_1\hat{}t_1) \cdot \ldots \cdot (p_k\hat{}t_k)$, where $t_i = u_i$ if $(u_i - 1)p\hat{}(u_i - 1)+ \leq s_i \leq u_i p_i\hat{}u_i$ for $1 \leq i \leq k$.

39) A $3n$-digital subsequence:

13, 26, 39, 412, 515, 618, 721, 824, 927, 1030, 1133, 1236, ...

(numbers that can be partitioned into two groups such that the second is three times biger than the first)

40) A $4n$-digital subsequence:

14, 28, 312, 416, 520, 624, 728, 832, 936, 1040, 1144, 1248, ...

(numbers that can be partitioned into two grpoups such that the second is four times biger than the first)

41) A $5n$-digital subsequence:

15, 210, 315, 420, 525, 630, 735, 840, 945, 1050, 1155, 1260, ... (numbers that can be partitioned into two groups such that the second is five times biger than the first)

42) A second function (numbers):

1, 2, 3, 2, 5, 6, 7, 4, 3, 10, 11, 6, 13, 14, 15, 4, 17, 6, 19, 10, 21, 22, 23, 12, 5, 26, 9, 14, 29, 30, 31, 8, 33, ...

($S2(n)$ is tha smallest integer m such that m^2 is divisible by n)

43) A third function (numbers):

1, 2, 3, 2, 5, 6, 7, 8, 3, 10, 11, 6, 13, 14, 15, 4, 17, 6, 19, 10, 21, 22, 23, 6, 5, 26, 3, 14, 29, 30, 31, 4, 33, ...

($S3(n)$ is the smallest integer m such that m^3 is divisible by n)

NUMERALOGY (II)

or

Properties of Numbers

1) Factorial base:

0, 1, 10, 11, 20, 21, 100, 101, 110, 111, 120, 121, 200, 201, 210, 211, 220, 221, 300, 301, 310, 311, 320, 321, 1000, 1001, 1010, 1011, 1020, 1021, 1100, 1101, 1110, 1111, 1120, 1121, 1200, ...

(Each number n written in the factorial base.)

(We define over the set of natural numbers the following infinite base: for $k \geq 1$ $f_k = k!$)

It is proved that every positive integer A may be uniquely written in the factorial base as:

$A = (\overline{a_n \dots a_2 a_1})_{(F)} \stackrel{def}{=} \sum_{i=1}^{n} a_i f_i$, with all $a_i = 0, 1, \dots i$ for $i \geq 1$.

in the following way:

- if $f_n \leq A < f_{n+1}$ then $A = f_n + r_1$;

- if $f_m \leq r_1 < f_{m+1}$ then $r_1 = f_m + r_2, m < n$;

and so on untill one obtains a rest $r_j = 0$.

What's very interesting: $a_1 = 0$ or 1; $a_2 = 0, 1$, or 2; $a_3 = 0, 1, 2$ or 3, and so on...

If we note by $f(A)$ the superior factorial part of A (i.e. the largest factorial less than or equal to A), then A is written in the factorial base as:

$$A = f(A) + f(A - f(A)) + f(A - f(A) - f(A - f(A))) + \dots.$$

Rules of addition and substraction in factorial base:

for each digit a_i we add and substract in base $i + 1$, for $i \geq 1$.

For examplu, addition:

base	5	4	3	2	
		2	1	0	+
		2	2	1	
	1	1	0	1	

because: $0 + 1 = 1$ (in base 2);

$1 + 2 = 10$ (in base 3); therefore we write 0 and keep 1;

$2 + 2 + 1 = 11$ (in base 4).

Now substraction:

base	5	4	3	2	
	1	0	0	1	-
		3	2	0	
			1	1	

becuase: $1 - 0 = 1$ (in base 2);

$0 - 2 = ?$ it's not possible (in base 3), go to the next left unit, which is 0 again (in base 4), go again to the next left unit, which is 1 (in base 5), therefore $1001 \rightarrow 0401 \rightarrow 0331$ and then $0331 - 320 = 11$.

Find some rules for multiplication and division.

In a general case:

if we want to design a base such that any number

$A = (\overline{a_n \dots a_2 a_1})_{(B)} \stackrel{def}{=} \sum_{i=1}^{n} a_i b_i$, with all $a_i = 0, 1, \dots t_i$ for $i \geq 1$, where all $t_i \geq 1$, then:

this base should be

$$b_1 = 1, b_{i+1} = (t_i + 1) * b_i \text{ for } i \geq 1.$$

2) More general-sequence sieve:

For $i = 1, 2, 3, \dots$, let $u_i > 1$, be a strictly increasing positive integer sequence, and $v_i < u_i$ another positive integer sequence. Then:

From the natural numbers set:

- keep the v_1-th number among $1, 2, 3, \dots, u_1 - 1$, and delete every u_1-th numbers;

- keep the v_2-th number among the next $u_2 - 1$ remaining numbers, and delete every u_2-th numbers;

... and so on, for step $k (k \geq 1)$:

-keep the v_k-th number among the next $u_k - 1$ remaining numbers, and delete every u_k-th numbers;

... .

Problem: study the relationship between sequences $u_i, v_i, i = 1, 2, 3, \dots$, and the remaining sequence resulted from the more general sieve.

u_i and v_i previously defined, are called sieve generators.

3) Mobile periodicals (I):

...000000000000000000000000000000010000000000000000000000000000000000...
...000000000000000000000000000000111000000000000000000000000000000000...
...000000000000000000000000000001101100000000000000000000000000000000...
...000000000000000000000000000000111000000000000000000000000000000000...
...000000000000000000000000000000010000000000000000000000000000000000...
...000000000000000000000000000000111000000000000000000000000000000000...
...000000000000000000000000000001101100000000000000000000000000000000...
...000000000000000000000000000011000110000000000000000000000000000000...
...000000000000000000000000000001101100000000000000000000000000000000...
...000000000000000000000000000000111000000000000000000000000000000000...
...000000000000000000000000000000010000000000000000000000000000000000...
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...000000000000000000000000000000010000000000000000000000000000000000...
...000000000000000000000000000000111000000000000000000000000000000000...
...000000000000000000000000000001101100000000000000000000000000000000...
...000000000000000000000000000011000110000000000000000000000000000000...
...000000000000000000000000000110000011000000000000000000000000000000...
...000000000000000000000000001100000001100000000000000000000000000000...
...000000000000000000000000011000000000110000000000000000000000000000...
...000000000000000000000000110000000000011000000000000000000000000000...

....

This sequence has the form

$$\underbrace{1,111,11011,111,}_{5}\underbrace{1,111,11011,1100011,11011,111,}_{7}\underbrace{1,111,11011,1100011,110000011,\ldots}_{9}$$

4) Mobile periodicals (II):

...000000000000000000000000000010000000000000000000000000000000...
...000000000000000000000000000111000000000000000000000000000000...
...000000000000000000000000001121100000000000000000000000000000...
...000000000000000000000000000111000000000000000000000000000000...
...000000000000000000000000000010000000000000000000000000000000...
...000000000000000000000000000111000000000000000000000000000000...
...000000000000000000000000001121100000000000000000000000000000...
...000000000000000000000000011232110000000000000000000000000000...
...000000000000000000000000001121100000000000000000000000000000...
...000000000000000000000000000111000000000000000000000000000000...
...000000000000000000000000000010000000000000000000000000000000...
...000000000000000000000000000111000000000000000000000000000000...
...000000000000000000000000001121100000000000000000000000000000...
...000000000000000000000000011232110000000000000000000000000000...
...000000000000000000000000112343211000000000000000000000000000...
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...000000000000000000000000001121100000000000000000000000000000...
...000000000000000000000000011232110000000000000000000000000000...
...000000000000000000000000112343211000000000000000000000000000...
...000000000000000000000001123454321100000000000000000000000000...
...000000000000000000000011234565432110000000000000000000000000...
...000000000000000000000112345676543211000000000000000000000000...

....

This sequence has the form

$$\underbrace{1,111,11211,111,}_{5}\underbrace{1,111,11211,1123211,11211,111,}_{7}\underbrace{1,111,11211,1123211,112343211,\ldots}_{9}$$

5) Infinite numbers (I):

...11111111111111111111111111111101111111111111111111111111111111111...
...11111111111111111111111111111000111111111111111111111111111111111...
...11111111111111111111111111110010011111111111111111111111111111111...
...11111111111111111111111111111000111111111111111111111111111111111...
...11111111111111111111111111111101111111111111111111111111111111111...
...11111111111111111111111111111000111111111111111111111111111111111...
...11111111111111111111111111110010011111111111111111111111111111111...
...11111111111111111111111111100111001111111111111111111111111111111...
...11111111111111111111111111110010011111111111111111111111111111111...
...11111111111111111111111111111000111111111111111111111111111111111...
...11111111111111111111111111111101111111111111111111111111111111111...
...11111111111111111111111111111000111111111111111111111111111111111...
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...11111111111111111111111111100111001111111111111111111111111111111...
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...11111111111111111111111111111000111111111111111111111111111111111...
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...11111111111111111111111111001111100111111111111111111111111111111...
...11111111111111111111111110011111110011111111111111111111111111111...
...11111111111111111111111100111111111001111111111111111111111111111...
...11111111111111111111111001111111111100111111111111111111111111111...

....

6) Infinite numbers (II):

...1111111111111111111111111111111111111211111111111111111111111111111111111111111111111111...
...11111111111111111111111111111111111122211111111111111111111111111111111111111111111111...
...1111111111111111111111111111111111122322111111111111111111111111111111111111111111...
...11111111111111111111111111111111111122211111111111111111111111111111111111111111111111...
...1111111111111111111111111111111111111211111111111111111111111111111111111111111111111111...
...11111111111111111111111111111111111122211111111111111111111111111111111111111111111111...
...1111111111111111111111111111111111122322111111111111111111111111111111111111111111...
...1111111111111111111111111111111122343221111111111111111111111111111111111111111111...
...1111111111111111111111111111111111122322111111111111111111111111111111111111111111...
...11111111111111111111111111111111111122211111111111111111111111111111111111111111111111...
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...1111111111111111111111111111111111122322111111111111111111111111111111111111111111...
...1111111111111111111111111111111122343221111111111111111111111111111111111111111111...
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...1111111111111111111111111111111122343221111111111111111111111111111111111111111111...
...1111111111111111111111111111111111122322111111111111111111111111111111111111111111...
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...1111111111111111111111111111111122343221111111111111111111111111111111111111111111...
...1111111111111111111111111111111112234543221111111111111111111111111111111111111111...
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...11111111111111111111111111111111111122211111111111111111111111111111111111111111111111...
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...1111111111111111111111111111111122343221111111111111111111111111111111111111111111...
...1111111111111111111111111111111112234543221111111111111111111111111111111111111111...
...1111111111111111111111111111111122345654322111111111111111111111111111111111111111...
...1111111111111111111111111111111223456765432211111111111111111111111111111111111111...
...111111111111111111111111111111223456787654322111111111111111111111111111111111111...

....

7) Car:

...0000000000000000000000000000000000000000000000000000000000000000...
...0000000000000000011111111111111111111111110000000000000000000000...
...0000000000000000111111111111111111111111111000000000000000000000...
...0000000000000001100000000000000000000000001100000000000000000000...
...0000000000000001100000000000000000000000001100000000000000000000...
...0000001111111111100000000000000000000000001111111111000000000000...
...0000001111111111100000000000000000000000001111111111000000000000...
...0000001100000000000000000000000000000000000000000011000000000000...
...0000001100000000000000000000000000000000000000000011000000000000...
...0000001100000444000000000000000000000000000044400011000000000000...
...0000001111144444111111111111111111111111111144444111000000000000...
...0000001111444444411111111111111111111111111444444411000000000000...
...0000000000044444000000000000000000000000000044444000000000000000...
...0000000000004440000000000000000000000000000004440000000000000000...
...0000000000000000000000000000000000000000000000000000000000000000...
....

8) Finite lattice:

...0000000000000000000000000000000000000000000000000000000000000000...
...0777000000000070000007777777700777777770077007777777700777777770...
...0777000000000777000007777777700777777770077007777777700777777770...
...0777000000007707700000007700000000770000077007770000000770000000...
...0777000000077000770000007700000000770000077007770000000777770000...
...0777000000777777777000007700000000770000077007770000000770000000...
...0777777000770000000770007700000000770000077007777777700777777770...
...0777777007700000000077007700000000770000077007777777700777777770...
...0000000000000000000000000000000000000000000000000000000000000000...
....

9) Infinite lattice:

...1111111111111111111111111111111111111111111111111111111111111111...
...1777111111111171111117777777711777777771177117777777711777777771...
...1777111111111777111117777777711777777771177117777777711777777771...
...1777111111117717711111117711111111771111177117771111111771111111...
...1777111111177111771111117711111111771111177117771111111777771111...
...1777111111777777777111117711111111771111177117771111111771111111...
...1777777111771111111771117711111111771111177117777777711777777771...
...1777777117711111111177117711111111771111177117777777711777777771...
...1111111111111111111111111111111111111111111111111111111111111111...
....

Remark: of course, it's interesting to "design" a large variety of numerical <object sequences> in the same way. Their numbers may be infinte if the picture's background is zeroed, or infinite if the picture's background is not zeroed - as for the previous examples.

10) Multiplication:

Another way to multiply two integer numbers, A and B:

- let k be an integer ≥ 2;
- write A and B on two different vertical columns: $c(A)$, respectively $c(B)$;
- multiply A by k and write the product A_1 on the column $c(A)$;

- divide B by k, and write the integer part of the quotient B_1 on the column $c(B)$;

... and so on with the new numbers A_1 and B_1, until we get a $B_i < k$ on the column $c(B)$;

Then:

- write another column $c(r)$, on the right side of $c(B)$, such that:

  for each number of column $c(B)$, which may be a multiple of k plus the rest r (where $r = 0, 1, 2, \ldots, k-1$), the corresponding number on $c(r)$ will be r;

- multiply each number of column A by its corresponding r of $c(r)$, and put the new products on another column $c(P)$ on the right side of $c(r)$;

- finally add all numbers of column $c(P)$.

$A \times B =$ the sum of all numbers of $c(P)$.

Remark that any multiplication of integer numbers can be done only by multiplication with $2, 3, \ldots, k$, division by k, and additions.

This is a generalization of Russian multiplication (where $k = 2$).

This multiplication is usefull when k is very small, the best values being for $k = 2$ (Russian multiplication - known since Egyptian time), or $k = 3$. If k is greater than or equal to $\min\{10, B\}$, tjis multiplication is trivial (the obvious multiplication).

Example 1. *(if we choose $k = 3$):*

$73 \times 97 = ?$

x_3	$/3$		
$c(A)$	$c(B)$	$c(r)$	$c(P)$
73	*97*	*1*	*73*
219	*32*	*2*	*438*
657	*10*	*1*	*657*
1971	*3*	*0*	*0*
5913	*1*	*1*	*5913*
			7081 total

therefore: $73 \times 97 = 7081$.

Remark that any multiplication of integer numbers can be done only by multiplication with $2, 3$, divisions by 3, and additions.

Example 2. *(if we choose $k = 4$):*

$73 \times 97 = ?$

x_4	$/4$		
$c(A)$	$c(B)$	$c(r)$	$c(P)$
73	*97*	*1*	*73*
292	*24*	*0*	*0*
1168	*6*	*2*	*2336*
4672	*1*	*1*	*4672*
			7081 total

therefore: $73 \times 97 = 7081$.

Remark that any multiplication of integer numbers can be done only by multiplication with $2, 3, 4$, divisions by 4, and additions.

Example 3. *(if we choose $k = 5$):*

$73 \times 97 = ?$

x_5	$/5$		
$c(A)$	$c(B)$	$c(r)$	$c(P)$
73	*97*	*2*	*146*
365	*19*	*4*	*1460*
1825	*3*	*3*	*5475*
			7081 total

therefore: $73 \times 97 = 7081$.

Remark that any multiplication of integer numbers can be done only by multiplication with $2, 3, 4, 5$, divisions by 5, and additions.

This multiplication becomes less usefull when k increases.

Look at another example (4), what happens when $k = 10$:

Example 4. $73 \times 97 = ?$

x_10	$/10$		
$c(A)$	$c(B)$	$c(r)$	$c(P)$
73	*97*	*7*	*511* $(= 73 \times 7)$
730	*9*	*9*	*6570* $(= 73 \times 9)$
			7081 total

therefore: $73 \times 97 = 7081$.

Remark that any multiplication of integer numbers can be done only by multiplication with $2, 3, \ldots, 10$, divisions by 10, and additions - hence we obtain just the obvious multiplication!

11) Division by $k\hat{}n$:

Another way to devide an integer numbers A by $k\hat{}n$, where k, n are integers ≥ 2:

- write A and $k\hat{}n$ on two different vertical columns: $c(A)$, respectively $c(k\hat{}n)$;
- devide A by k, and write the integer quotient A_1 on the column $c(A)$;
- devide $k\hat{}n$ by k, and write the quotient $q_1 = k\hat{}(n-1)$ on the column $c(k\hat{}n)$;

... and so on with the new numbers A_1 and q_1, untill we get $q_n = 1 (= k\hat{}0)$ on the column $c(k\hat{}n)$;

Then:

- write another column $c(r)$, on the left side of $c(A)$, such that:

  for each number of column $c(A)$, which may be multiple of k plus the rest r (where $r = 0, 1, 2, \ldots, k-1$), the corresponding number on $c(r)$ will be r;
- write another column $c(P)$, on the left side of $c(r)$, in the following way: the element on line i (except the last line which is 0) will be $k\hat{}(i-1)$;
- multiply each number of column $c(P)$ by its corresponding r of $c(r)$, and put the new products on another column $c(R)$ on the left side of $c(P)$;
- finally add all numbers of column $c(R)$ to get the final rest R, while the final quotient will be stated in front of $c(k\hat{}n)$'s 1.

Therefore:

$A/(k\hat{}n) = A_n$ and rest R_n.

Remark that any division of an integer number by $k\hat{}n$ can done only by divisions to k, calculations of powers of k, multiplications with $1, 2, \ldots, k-1$, additions.

This division is usefull when k is small, the best values being when k is an one-digit number, and n large. If k is very big and n very small, this division becomes useless.

Example 1. $1357/(2\hat{}7) = ?$

			/2	/2	
$c(R)$	$c(P)$	$c(r)$	$c(A)$	$c(2\hat{}7)$	
1	$2\hat{}0$	*1*	*1357*	$2\hat{}7$	$line_1$
0	$2\hat{}1$	*0*	*678*	$2\hat{}6$	$line_2$
4	$2\hat{}2$	*1*	*339*	$2\hat{}5$	$line_3$
8	$2\hat{}3$	*1*	*169*	$2\hat{}4$	$line_4$
0	$2\hat{}4$	*0*	*84*	$2\hat{}3$	$line_5$
0	$2\hat{}5$	*0*	*42*	$2\hat{}2$	$line_6$
64	$2\hat{}6$	*1*	*21*	$2\hat{}1$	$line_7$
			10	$2\hat{}0$	*last_line*
77					

Therefore: $1357/(2\hat{}7) = 10$ *and rest* 77.

Remark that the division of an integer number by any power of 2 can be done only by divisions to 2, calculations of power of 2, multiplications and additions.

Example 2. $19495/(3\hat{}8) = ?$

			/3	/3	
$c(R)$	$c(P)$	$c(r)$	$c(A)$	$c(3\hat{}8)$	
1	$3\hat{}0$	*1*	*19495*	$3\hat{}8$	$line_1$
0	$3\hat{}1$	*0*	*6498*	$3\hat{}7$	$line_2$
0	$3\hat{}2$	*0*	*2166*	$3\hat{}6$	$line_3$
54	$3\hat{}3$	*2*	*722*	$3\hat{}5$	$line_4$
0	$3\hat{}4$	*0*	*240*	$3\hat{}4$	$line_5$
486	$3\hat{}5$	*2*	*80*	$3\hat{}3$	$line_6$
1458	$3\hat{}6$	*2*	*26*	$3\hat{}2$	$line_7$
4374	$3\hat{}7$	*2*	*8*	$3\hat{}1$	$line_8$
			2	$3\hat{}0$	*last_line*
6373					

Therefore: $19495/(3\hat{}8) = 2$ *and rest* 6373.

Remark that the division of an integer number by any power of 3 can be done only by divisions to 3, calculations of power of 3, multiplications and additions.

References

[1] Alain Bouvier et Michel George, sous la deriction de Francois Le Lionnais, "Dictionnaire des Mathematiques", Presses Universitaires de France, Paris, 1979, p. 659;

"The Florentin Smarandache papers" special collection, Arizona State University, Tempe, AZ 85287.

12) Almost prime of first kind:

$a_1 \geq 2$, and for $n \geq 1$ a_{n+1} is the smallest number that is not divisible by any of the previous terms (of the sequence) $a_1, a_2, \ldots, a_n$.

Example for $a_1 = 10$:

10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 25, 27, 29, 31, 35, 37, 41, 43, 47, 49, 53, 57, 61, 67, 71, 73, ...

If one starts by $a_1 = 2$, it obtains the complete prime sequence and only it.

If one starts by $a_2 > 2$, it obtains after a rank r, where $a_r = p(a_1)^2$ with $p(x)$ the strictly superior prime part of x, i.e. the largest prime strictly less than x, the prime sequence:

- between a_1 and a_r, the sequence contains all prime numbers of this interval and some composite numbers;

- from a_{r+1} and up, the sequence contains all prime numbers greater than a_r and no composite numbers.

13) Almost primes of second kind:

$a_1 \geq 2$, and for $n \geq 1$ a_{n+1} is the smallest number that is coprime with all of the previous terms (of the sequence) $a_1, a_2, \ldots, a_n$.

This second kind sequence merges faster to prime numbers than the first kind sequence.

Example for $a_1 = 10$:

10, 11, 13, 17, 19, 21, 23, 29, 31, 37, 41, 43, 47, 53, 57, 61, 67, 71, 73, ...

If one starts by $a_1 = 2$, it obtains the complete prime sequence and only it.

If one starts by $a_2 > 2$, it obtains after a rank r, where $a_r = p_i p_j$ with p_i and p_j prime number strictly less than and not dividing a_1, the prime sequence:

- between a_1 and a_r, the sequence contains all prime numbers of this interval and some composite numbers;

- from a_{r+1} and up, the sequence contains all prime numbers greater than a_r and no composite numbers.

NUMEROLOGY (III)

or

Properties of Numbers

1) Odd Sequence:

1, 13, 135, 1357, 13579, 1357911, 135791113, 13579111315, 1357911131517, ...

How many of them are primes?

2) even Sequence:

1, 24, 246, 2468, 246810, 24681012, 2468101214, 246810121416, ...

Conjecture: No number in this sequence is an even power.

3) Prime Sequence:

2, 23, 235, 2357, 235711, 23571113, 2357111317, 235711131719, ...

How many of them are primes?

Conjecture: A finite number.

4)S-sequence:

General definition: Let $S = \{s_1, s_2, s_3, \ldots, s_n, \ldots\}$ be an infinite sequence of integers.

Then the corresponding S-sequence is $\{s_1, s_1s_2, \ldots, s_1s_2\cdots s_n, \ldots\}$ where the numbers are concatenated together.

Question 1: How many termsof the S-sequence are found in the original set S?

Question 2: How many terms of the S-sequence satisfy the properties of other given sequence?

For example, the odd sequence above is built from the set $S = \{1,3,5,7,9,\ldots\}$ and every element of the S-sequence is found in S. The even sequence is built from the set $S = \{2,4,6,8,10,\ldots\}$ and every element of the corresponding S-sequence is also in S. However, the question is much harder for the prime sequence.

Study the case when S is the Fibonacci numbers $\{1,1,2,3,5,8,13,21,\ldots\}$. The corresponding F-sequenceis then $\{1,11,112,1123,11235,112358,11235813,\ldots\}$. In particular, how many primes are in the F-sequence?

5) Uniform sequences:

General definition: Let $n \neq 0$ be an integer and $d_1, d_2, \ldots, d_r$ distinct digits in base $B > r$. Then, multiples of n, written using only the comlete set of digits $d_1, d_2, \ldots, d_r$ in base B, increasingly ordered, is called the uniform sequence.

Some particular examples involve one digit only.

a) Multiples of 7 written in base 10 using only the digit 1.

111111, 111111111111, 111111111111111111, 111111111111111111111111, ...

b) Multiples of 7 written in base 10 using only digit 2.

222222, 222222222222, 222222222222222222, 222222222222222222222222, ...

c) Multiples of 79365 written in base 10 using only the digit 5.

55555, 555555555555, 555555555555555555, 555555555555555555555555, ...

In many cases, the uniform sequence is empty.

d) It is possible to create multiples of 79365 in base 10 using only the digit 6.

Remark: If there exists at least one such multiple of n written with the digits $d_1, d_2, \ldots, d_r$ in base B, then there exists an infinite number of multiples of n. If m is the initial multiple, then they all have the form, $m, mm, mmm, \ldots$

With a computer program it is easy to select all multiples of a given number written with a set of digits, up to a maximum number of digits.

Exercise: Find the general term expression for multiples of 7 using only the digits $\{1, 3, 5\}$ in base 10.

6) Operation Sequence:

General definition: Let E be an ordered set of elements, $E = \{e_1, e_2, \ldots\}$ and Θ a set of binary operations well-defined on E. Then

$$a_1 \in \{e_1, e_2, \ldots\}$$

$$a_{n+1} = \min\{e_1\theta_1 e_2\theta_2 \ldots \theta_n e_{n+1}\} > a_n, \text{ for } n \geq 1.$$

where all θ_i are oprations belonging to Θ.

Some examples:

a) When E is the set of natural numbers and $\Theta = \{+, -, *, /\}$, the four standard arithmetic operations.

Then

$a_1 = 1$

$a_{n+1} = \min\{1\theta_1\theta_2 \ldots \theta_n(n+1)\} > a_n$, for $n \geq 1$.

where $\theta_i \in \Theta$.

Questions:

a) Given N as the set of numbers and $\Theta = \{+, -, *, /\}$ as the set of operations, is there a general formula for the sequence?

b) If the finite sequence is defined with the finite set of numbers $\{1, 2, 3, \ldots, 99\}$ and the set of operations the same as above, where

$a_1 = 1$

$$a_{n+1} = \min\{1\theta_1 2\theta_2 \ldots \theta_{98} 99\} > a_n, \text{ for } n \geq 1.$$

Same questions as in (a).

c) Let N be the set of numbers and $\Theta = \{+, -, *, /, **, (\sqrt{})\}$, where $x^{**}y$ is x to the power y and $x(\sqrt{})y$ is the x-th root of y. Define the sequence by

$a_1 = 1$

$$a_{n+1} = \min\{1\theta_1 2\theta_2 \ldots \theta_n (n+1)\} > a_n, \text{ for } n \geq 1.$$

The same questions can be asked, althought they are harder and perhaps more intresting.

d) Using the same set of operations, the algebraic operation finite sequence can be defined:

$a_1 = 1$

$a_{n+1} = \min\{1\theta_1 2\theta_2 \ldots \theta_{98} 99\} > a_n$, for $n \geq 1$.

And pose the same questions as in (b).

More generally, the binary operations can be replaced by k_i-ary operations, where all k_i are integers.

$a_1 \in \{e_1, e_2, \ldots\}$

$a_{n+1} = \min\{1\theta_1 2\theta_2 \ldots \theta_1 k_1 \theta_2 (k_1 + 1)\theta_2 \ldots \theta_2 (k_1 + k_2 - 1) \ldots (n + 2 - k_r \theta_r \ldots \theta_r (n+1)\} > a_n$

where $n \geq 1$.

Where each θ_i is a k_i-ary relation and $k_1 + (k_2 - 1) + \ldots + (k_r - 1) = n + 1$. Note that the last element of the k_i relation is the first element of the k_{i+1} realtion.

Remark: The questions are much easier when $\Theta = \{+, -\}$. Study the operation type sequences in this easier case.

e) Operators sequences at random:

Same definition and questions as the previous sequences, except that the minimum condition is removed.

$a_{n+1} = \{e_1\theta_1 e_2\theta_2 \dots \theta_n e_{n+1}\} > a_n$, for $n \geq 1$.

Therefore, a_{n+1} will be chosen at random, with the only restriction being that it be greater than a_n.

Study these sequences using a computer program with a random number generator to choose a_{n+1}.

References

[1] F.Smarandache, "Properties of the Numbers", University of Craiova Archives, 1975. [Also see the Arizona State University Special Collections, Tempe, Arizona, USA].

P-Q Relationships and Sequences

Let $A = \{a_n\}, n \geq 1$ be a sequence of numbers and q, p integers ≥ 1.

We say that the terms $a_{k+1}, a_{k+2}, \dots, a_{k+p}, a_{k+p+1}, a_{k+p+2}, \dots, a_{k+p+q}$ satisfy a $p-q$ relationship if

$$a_{k+1} \diamond a_{k+2} \diamond \dots \diamond a_{k+p} = a_{k+p+1} \diamond a_{k+p+2} \diamond \dots \diamond a_{k+p+q}$$

where $\diamond$ may be any arithmetic operation, although it is generally a binary relation on A. If this relationship is satisfied for any $k \geq 1$, then $\{a_n\}, n \geq 1$ is said to be a $p-q-\diamond$ sequence. For operations such as addition, where $\diamond = +$, the sequence is called a $p-q$-additive sequence.

As a specific case, we can easily see that the Fibonacci/Lucas sequence ($a_n + a_{n+1} = a_{n+2}$, for $n \geq 1$), is a $3-1$-additive sequence.

Definition. *Given any integer $n \geq 1$, the value of the Smarandache function $S(n)$ is the smallest integer m such that n divides $m!$.*

If we consider the sequence of numbers that are the values of the Smarandache function for the integers $n \geq 1$,

$1, 2, 3, 4, 5, 3, 7, 4, 6, 5, 11, 4, 13, 7, 5, 6, 17, \dots$

they can be incorporated into questions involving the $p-q-\diamond$ relationships.

a) How many ordered quadruples are there of the form $(S(n), S(n+1), S(n+2), S(n+3))$ such that $S(n+1) + S(n+2) = S(n+3) + S(n+4)$ which is a $2-2$-additive relationship?

The three quadruples

$S(6) + S(7) = S(8) + S(9),\ \ 3 + 7 = 4 + 6;$

$S(7) + S(8) = S(9) + S(10),\ \ 7 + 4 = 6 + 5;$

$S(28) + S(29) = S(30) + S(31),\ \ 7 + 29 = 5 + 31.$

are known. Are there any others? At this time, these are the only known solutions.

b) How many quadruples satisfy the $2-2$-subtrac relationship $S(n+1) - S(n+2) = S(n+3) - S(n+4)$?

The three quadruples

$S(1) - S(2) = S(3) - S(4),\ \ 1 - 2 = 3 - 4;$

$S(2) - S(3) = S(4) - S(5),\ \ 2 - 3 = 4 - 5;$

$S(49) - S(50) = S(51) - S(52),\ \ 14 - 10 = 17 - 13$

are known. Are there any others?

c) How many 6-tuples satisfy the $2-3$-additive relationship $S(n+1)+S(n+2)+S(n+3) = S(n+4) + S(n+5) + S(n+6)$?

The only known solution is

$$S(5) + S(6) + S(7) = S(8) + S(9) + S(10),\ 5 + 3 + 7 = 4 + 6 + 5.$$

Charles Ashbacher has a computer program that calculates the values of the Smarandache function. Therefore, he may be able to find additional solutions to theese problems.

More general, if f_p is a p-ary raltion and g_q a q-ary relation, both defined on the set $\{a_1, a_2, a_3, \ldots\}$, then $a_{i_1}, a_{i_2}, \ldots, a_{i_p}, a_{j_1}, a_{j_2}, \ldots, a_{j_q}$ satisfies a $f_p - g_q$ relationship if

$$f(a_{i_1}, a_{i_2}, \ldots, a_{i_p}) = g(a_{j_1}, a_{j_2}, \ldots, a_{j_q}).$$

If this relationship holds for all terms of the sequence, then $\{a_n\}, n \geq 1$ is called a $f_p - g_q$ sequence.

Study some $f_p - g_p$ relationship for welll-known sequences, such as the perfect numbers, Ulam numbers, abundant numbers, Catalan numbers and Cullen numbers. For example, a $2-2$-additive, subtractive or multiplicative relationship.

If f_p is a p-ary relationship on $\{a_1, a_2, a_3, \ldots\}$ and $f_p(a_{i_1}, a_{i_2}, \ldots, a_{i_p}) = f(a_{j_1}, a_{j_2}, \ldots, a_{j_q})$ for all a_{i_k}, a_{j_k} where $k = 1, 2, 3, \ldots, p$ and for all $p \geq 1$, the $\{a_n\}, n \geq 1$ is called a perfect f-sequence.

If not all p-plets $(a_{i_1}, a_{i_2}, \ldots, a_{i_p})$ and $(a_{j_1}, a_{j_2}, \ldots, a_{j_q})$ satisfy the f_p relation or the relation is not satisfied for all $p \geq 1$, then $\{a_n\}, n \geq 1$ is called a partial perfect f-sequence. For example, the sequence $1, 1, 0, 2, -1, 1, 1, 3, -2, 0, 0, 2, 1, 1, 3, 5, -4, -2, -1, 1, -1, 1, 3, 0, 2, \ldots$ is a partial perfect additive-sequence. This sequence has the property that $\sum_{i=1}^{p} a_i = \sum_{j=p+1}^{2p} a_j$, for all $p \geq 1$.

It is constructed in the following way:

$a_1 = a_2$,

$a_{2p+1} = a_{p+1} - 1$,

$a_{2p+2} = a_{p+1} + 1$

for all $p \geq 1$.

a) Can you, the reader, find a general expression of a_n (as a function of n)? Is it periodic, convergent or bounded?

b) Develop other perfect or partial perfect f-sequences. Think about multiplicative sequences of this type.

References

[1] Sloane N.J.A., Plouffe Simon, "The Encyclopedia of Integer Sequences", Academic Press, San Diego, New York, Boston, Sydnei, Tokyo, Toronto, 1995/M0453.

[2] Smarandache F., "Properties of the Numbers", 1975, University of Craiova, Archives; (See also Arizona State University Special Collections, Tempe, AZ, USA.)

Digital Subsequences

Let $\{a_n\}$ $n \geq 1$ be a sequence defined by a property (or a relationship involving its terms) P. We then screen this sequence, selecting only the terms whose digits also satisfy the property or relationship.

1) The new sequence is then called a P-digital subsequence.

Examples:

a) Sqare-digital subsequence:

Given the sequence of perfect squares $0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, \ldots$ only those terms whose digits are all perfect squares $\{0, 1, 4, 9\}$ are chosen. The first few terms are $0, 1, 4, 9, 49, 100, 144, 400, 441$.

Disregarding squares of the form $N00\ldots0$, where N is also a perfect square, how many numbers belong to this subsequence?

b) Given the sequence of perfect cubes, $0, 1, 8, 27, 64, 125, \ldots$ only those terms whose digits are all perfect cubes $\{0, 1, 8\}$ are chosen. The first few terms are $0, 1, 8, 1000, 8000$.

Disregarding cubes of the form $N00\ldots0$, where N is also a perfect cube, how many numbers belong to this subsequence?

c) Prime-digital subsequence:

Given the sequence of prime numbers, $2, 3, 5, 7, 11, 13, 17, 19, 23, \ldots$. Only those primes where all digits are prime numbers are chosen. The first few terms are $2, 3, 5, 7, 23, 29, \ldots$.

Conjecture: This subsequence is infinite.

In the same vein, elements of a sequence can be chosen if groups of digits, except the complete number, satisfy a property (or relationship) P. The subsequence is then called a P-partial-digital subsequence.

Examples:

a) Squares-partial-digital subsequence:

$49, 100, 144, 169, 361, 400, 441, \ldots$

In other words, perfect squares whose digits can be partioned into two or more groups that are perfect squares.

For example 169 can be partitioned into 16 and 9.

Disregarding square numbers of the form $N00\ldots0$, where N is also a perfect square, how many numbers belong to this sequence?

b) Cube-partial-digital subsequence:

$1000, 8000, 10648, 27000, \ldots$

i.e. all perfect cubes where the digits can be partioned into two or more groups that are perfect cubes. For example 10648 can be partitioned into 1, 0, 64 and 8.

Disregarding cube numbers of the form $\overline{N\underbrace{00\ldots0}_{2k \text{ times}}}$, where N is also a perfect cube, how many numbers belong to this sequence?

c) Prime-partial-digital subsequence:

23, 37, 53, 73, 113, 137, 173, 193, 197,...

i.e. all prime numbers where the digits can be partioned into two or more groups of digits that are prime numbers. For example, 113 can be partioned into 11 and 3.

Conjecture: This subset of the prime numbers is infinte.

d) Lucas-partial-digital subsequence:

Definition. *A number is a Lucas number of sequence* $L(0) = 2, L(1) = 1$ *and* $L(n+2) = L(n+1) + L(n)$ *for* $n \geq 1$.

The first few elements of this sequence are $2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, \ldots$

A number is an element of the Lucas-partial-digital subsequence if it is a Lucas number and the digits can be partioned into three groups such that the third group, moving left to right, is the sum of the first two groups. For example, 123 satisfies all these properties.

Is 123 the only Lucas number that satisfies the properties of this partition?

Study some P-partial-digital subsequences using the sequences of numbers.

i) Fibonacci numbers. A search was conducted looking for Fibonacci numbers that satisfy the properties of such a partition, but none were found. Are there any such numbers?

ii) Smith numbers, Eulerian numbers, Bernouli numbers, Mock theta numbers and Smarandache type sequences are other candidate sequences.

Remark: Some sequences may not be partitionable in this manner.

If a sequence $\{a_n\}, n \geq 1$ is defined by $a_n = f(n)$, a function of n, then an f-digital sequence is obtained by sceeening the sequence and selecting only those numbers that can be partioned into two groups of digits g_1 and g_2 such that $g_2 = f(g_1)$.

Examples:

a) If $a_n = 2n, n \geq 1$, then the even-digital subsequence is $12, 24, 36, 48, 510, 612, 716, 816, 918, 1020, \ldots$

where 714 can be partitioned into 7 and 14 in that order and

b) Lucky-digital subsequence:

Definition: *Given the set of natural numbers* $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, \ldots$. *First strike out every even numbers, leaving* $1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, \ldots$. *Then strike out* *every third in the remaining list, every fourth number in what remains after that, every fifth number remaining after that and so on. The set of numbers that remains after this infinite sequence is performed are the Lucky numbers.*

$1, 3, 7, 9, 13, 15, 21, 25, 31, 33, 37, 43, 49, 51, 63, \ldots$

A number is said to be a member of the lucky-digital subsequence if the digits can be partitioned into two number mn in that order such that $L_m = n$.

37 and 49 are both elements of this sequence. How many others are there?

Study this type of sequence for other well-known sequences.

References

[1] F.Smarandache, "Properties of the Numbers", University of Craiova Archives, 1975. [See also the Arizona State Special Collections, Tempe, AZ., USA].

Magic Squares

For $n \geq 2$, let A be set of n^2 elements and l an n-ary relation defined on A. As a generalization of the XVIth-XVIIth century magic squares, we present the magic square of order n. This is square array of elements of A arranged so that l applied to all rows and columns yields the same result.

If A is an arithmetic progression and l addition, then many such magic squares are known. The following appeared in Durer's 1514 engraving, "Melancholia"

$$\begin{array}{rrrr} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{array}$$

Questions:

1) Can you find magic square of order at least three or four where A is a set of prime numbers and l is addition?

2) Same question when A is a set of square, cube or other spacial numbers such as the Fibonacci, Lucas, triangular or Smarandache quotients. Given any m, the Smarandache Quotient $q(m)$ is the smallest number k such that mk is a factorial.

A similar definition for the magic cube of order n, where the elements of A are arranged in the form of a cube of length n.

3) Study questions similar to tose above for the cube. An interesting law may be

$$l(a_1, a_2, \ldots, a_n) = a_1 + a_2 - a_3 + a_4 - a_5 \ldots$$

References

[1] F.Smarandache, "Properties of the Numbers", University of Craiova Archives, 1975. [See also the Arizona State Special Collections, Tempe, AZ., USA].

Prime Conjecture

Any odd number can be expressed as a sum of two primes minus a third prime, not including the trivial solution $p = p + q - q$.

For example,

$1 = 3 + 5 - 7 = 5 + 7 - 11 = 7 + 11 - 17 = 11 + 13 - 23 = \ldots$

$3 = 5 + 11 - 13 = 7 + 19 - 23 = 17 + 23 - 37 = \ldots$

$5 = 3 + 13 - 11 = \ldots$

$7 = 11 + 13 - 17 = \ldots$

$9 = 5 + 7 - 3 = \ldots$

$11 = 7 + 17 - 13 = \ldots$

a) Is this conjecture equivalent to Coldbach's conjecture? The conjecture is that any odd prime ≥ 9 can be expressed as a sum of three primes. This was solved by Vinogradov in 1937 for any odd number greater then $3^{3^{15}}$.

b) The number of times each odd number can be expressed as a sum of two primes minus a third prime are called prime conjecture numbers. None of them is known!

c)Write a computer program to check this conjecture for as many positive numbers as possible.

There are infinitely many numbers that cannot be expressed as the absolute difference between a cube and a square. These are called bad numbers(!)

For example, F.Smarandache has conjuctured [1] that 5, 6, 7, 10, 13 and 14 are bad numbers. However, 1, 2, 3, 4, 8, 9, 11, 12, and 15 are not as

$$1 = |2^3 - 3^2|,\ 2 = |3^3 - 5^2|,\ 3 = |1^3 - 2^2|,\ 4 = |5^3 - 11^2|,\ 8 = |1^3 - 3^2|,$$

$$9 = |6^3 - 15^2|,\ 11 = |3^3 - 4^2|,\ 12 = |13^3 - 47^2|,\ 15 = |4^3 - 7^2|.$$

a) Write a computer program to determine as many bad numbers as possible. Find an ordered array of a's such that $a = |x^3 - y^2|$, for x and y integers ≥ 1.

References

[1] F.Smarandache, "Properties of the Numbers", University of Craiova Archives, 1975. [See also the Arizona State Special Collections, Tempe, AZ., USA].

SOME PERIODICAL SEQUENCES

1) Let N be a positive integer with not all digits the same, and N' its digital reverse.

Then, let N_1 =abs($N - N'$), and N_1' its digital reverse. Again, let N_2 =abs($N_1 - N_1'$), N_2' its digital reverse, and so on.

After a finite number of steps one finds an N_j which is equal to a previous N_i, therefore the sequence is periodical [because if N has, say, n digits, all other integers following it will have n digits or less, hence their number is limited, and one applies the Dirichlet's box principle].

Foe examples:

a) If one starts with $N = 27$, then $N' = 72$;

abs$(27 - 72) = 45$; its reverse is 54;

abs$(45 - 54) = 09, \ldots$

thus one gets: $27, 45, 09, 81, 63,\ 27, 45, \ldots$;

the Lentgh of the Period $LP = 5$ numbers $(27, 45, 09, 91, 63)$, and Length of the Sequence 'till the first repetition occurs $LS = 5$ numbers either.

b) If one starts with 52, then one gets:

$52, 27, 45, 09, 81, 63,\ 27, 45, \ldots$;

then $LP = 5$ numbers, while $LS = 6$.

c) If one starts with 42, then one gets:

$42, 18, 63, 27, 45, 09, 81,\ 63, 27, \ldots$;

then $LP = 5$ numbers, while $LS = 7$.

For the sequences of integers of two digits, it seems like: $LP = 5$ numbers $(27, 45, 09, 81, 63)$; or circular permutation of them), and $5 \leq LS \leq 7$.

Question 1: To find the Length of the Period (with its corresponding numbers), and the Length of the Sequence'till the firs repetition ocurrs for: the integers of three digits, and integers of four digits. (It's easier to write a computer programm in these cases to check the LP and LS.)

An example for three digits: $321, 198, 693, 297, 495, 099, 891,\ 693, \ldots$;

(similar to the previous period, just inserting 9 in the middle of each number).

Generalization for the sequences of numbers of n digits.

2) Let N be a positive integer, and N' its digital reverse. For a given positive integer C,

let N_1 =abs$(N' - C)$ and N_1' its digital reverse. Again, let N_2 =abs$(N_1 - C)$, N_2' its digital reverse, and so on.

After a finite number of steps one finds an an N_j wich is equal to a previous N_i, therefore the sequence is periodical [same proof].

For example:

If $N = 52$, and $c = 1$, than one gets:

$52, 24, 41, 13, 30, 02, 19, 90, 08, 79, 96, 68, 85, 57, 74, 46, 63, 35, \ 52, \ldots;$

thus $LP = 18, LS = 18$.

Question 2: To find the Length of the Period (with its corresponding numbers), and the Length of the Sequence'till the first repetition occurs (with a given non-null c) for: integers of two digits, and the integers of three digits.

(It's easier to write a computer program in these cases to check the LP and LS.)

Generalization for sequences of numbers of n digits.

3) Let N be a positive integer with n digits $a_1 a_2 \ldots a_n$, and c a given integer > 1.

Multiply each digit a_i of N by c, and replace a_i with the last digit of the product $a_i x c$, say it is b_i. Note $N_1 = b_1 b_2 \ldots b_n$, do the same procedure for N_1, and so on.

After a finite number of steps one finds an N_j which is equal to a previous N_i, therefore the sequence is periodical [same proof].

For exemple:

If $N = 68$ and $c = 7$:

$68, 26, 42, 84, \ 68, \ldots$

thus $LP = 4, LS = 4$.

Question 3: To find the Length of the Period (with its corresponding numbers), and the Length of the Sequence'till the first repetition occurs (with a given c) for: integers of two digits, and the integers of three digits.

(It's easier to write a computer program in these cases to check the LP and LS.)

Generalization for sequences of numbers of n digits.

4.1) Generalized periodical sequence:

Let N be a positive integer with n digits $a_1 a_2 \ldots a_n$. If f is a function defined on the set of integers with n digits or less, and the values of f are also in the same set, then: there exist two natural numbers $i < j$ such that

$$f(f(\dots f(s)\dots)) = f(f(f(\dots f(s)\dots))),$$

where f occurs i times in the left side, and j times in the right side of the previuos equality.

Particularizing f, one obtaines many periodical sequences.

Say: If N has two digits $a_1 a_2$, then: add'em (if the sum is greater than 10, add the resulted digits again), and substruct'em (take the absolute value) - they will be the first, and second digit respectively of N_1. And same procedure for N_1.

Example: $75, 32, 51, 64, 12, 31, 42, 62, 84, 34, 71, 86, 52, 73, 14, 53, 82, 16,\ 75, \dots$

4.2) More General:

Let S be a finite set, and $f : S \to S$ a function. Then: for any element s belonging to S, there exist two natural numbers $i < j$ such that

$$f(f(\dots f(s)\dots)) = f(f(f(\dots f(s)\dots))),$$

where f occurs i times in the left side, and j times in the right side of the previuos equality.

SEQUENCES OF SUB-SEQUENCES

For all of the following sequences:

a) Crescendo Sub-sequences:

$1, \ 1,2, \ 1,2,3, \ 1,2,3,4, \ 1,2,3,4,5, \ 1,2,3,4,5,6, \ 1,2,3,4,5,6,7, \ 1,2,3,4,5,6,7,8, \ \ldots$

b) Descrescendo Sub-sequences:

$1, \ 2,1, \ 3,2,1, \ 4,3,2,1, \ 5,4,3,2,1, \ 6,5,4,3,2,1, \ 7,6,5,4,3,2,1, \ 8,7,6,5,4,3,2,1, \ \ldots$

c) Crescenco Pyramidal Sub-sequences:

$1, \ 1,2,1, \ 1,2,3,2,1, \ 1,2,3,4,3,2,1, \ 1,2,3,4,5,4,3,2,1, \ 1,2,3,4,5,6,5,4,3,2,1, \ldots$

d) Descrescenco Pyramidal Sub-sequences:

$1, \ 2,1,2, \ 3,2,1,2,3, \ 4,3,2,1,2,3,4, \ 5,4,3,2,1,2,3,4,5, \ 6,5,4,3,2,1,2,3,4,5,6, \ldots$

e) Crescendo Symmetric Sub-sequences:

$1,1, \ 1,2,2,1, \ 1,2,3,3,2,1, \ 1,2,3,4,4,3,2,1, \ 1,2,3,4,5,5,4,3,2,1,$

$1,2,3,4,5,6,6,5,4,3,2,1, \ldots$

f) Descrescenco Symmetric Sub-sequences:

$1,1, \ 2,1,1,2, \ 3,2,1,1,2,3, \ 4,3,2,1,1,2,3,4, \ 5,4,3,2,1,1,2,3,4,5,$

$6,5,4,3,2,1,1,2,3,4,5,6, \ldots$

g) Permutation Sub-sequences:

$1,2, \ 1,3,4,2, \ 1,3,5,6,4,2, \ 1,3,5,7,8,6,4,2, \ 1,3,5,7,9,10,8,6,4,2, \ldots$

find a formula for the general term of the sequence.

Solutions:

For purposes of notatipn in all problems, let $a(n)$ denote the n-th term in the complete sequence and $b(n)$ the n-th subsequence. Therefore, $a(n)$ will be a number and $b(n)$ a sub-sequence.

a) Clearly, $b(n)$ contains n terms. Using a well-known summation formula, at the end of $b(n)$ there would be a total of $\dfrac{n(n+1)}{2}$ terms. Therefore, since the last number of $b(n)$ is n, $a((n(n+1))/2) = n$. Finally, since this would be the terminal number in the sub-sequence $b(n) = 1, 2, 3, \ldots, n$ the general formula is $a(((n(n+1)/2) - i) = n - i$ for $n \geq 1$ and $0 \leq i \leq n-i$.

b) With modifications for decreasing rather than increasing, the proof is essentialy the same. The final formula is $a(((n(n+1)/2) - i) = 1 + i$ for $n \geq 1$ and $0 \leq i \leq n-1$.

c) Clearly, $b(n)$ has $2n-1$ terms. Using the well-known formula of summation $1+3+5+$

$\ldots + (2n-1) = n^2$, the last term of $b(n)$ is n, so counting back $n-1$ positions, they increase in value by one each step until n is reached.

$$a(n^2 - i) = 1 + i, \text{ for } 0 \leq i \leq n-1.$$

After the maximum value at $n-1$ position back from n^2, the values descreases by one. So at the n-th position back, the value is $n-1$, at the $(n-1)$-st position back the value is $n-2$ and so forth.

$$a(n^2 - n - i) = n - i - 1 \text{ for } 0 \leq i \leq n-2.$$

d) Using similar reasoning $a(n^2) = n$ for $n \geq 1$ and

$$a(n_2^2 - i) = n - i, \text{ for } 0 \leq i \leq n-1$$

$$a(n^2 - n - i) = 2 + i, \text{ for } 0 \leq i \leq n-2.$$

e) Clearly, $b(n)$ contains $2n$ terms. Applying another well-known summation formula $2 + 4 + 6 + \ldots + 2n = n(n+1)$, for $n \geq 1$. Therefore, $a(n(n+1)) = 1$. Counting backwards $n-1$ positions, each term descreases by 1 up to a maximum of n.

$$a((n(n+1)) - i) = 1 + i, \text{ for } 0 \leq i \leq n-1.$$

The value n psitions down is also n and then the terms descrease by one back down to one.

$$a((n(n+1)) - n - i) = n - i, \text{ for } 0 \leq i \leq n-1.$$

f) The number of terms in $b(n)$ is the same as that for (e). The only difference is that now the direction of increase/decrease is reversed.

$$a((n(n+1)) - i) = n - i, \text{ for } 0 \leq i \leq n-1.$$

$$a((n(n+1)) - n - i) = 1 + i, \text{ for } 0 \leq i \leq n-1.$$

g) Given the following circular permutation on the first n integers.

$$\varphi_n = \begin{vmatrix} 1 & 2 & 3 & 4 & \ldots & n-2 & n-1 & n \\ 1 & 3 & 5 & 7 & \ldots & 6 & 4 & 2 \end{vmatrix}$$

Once again, $b(n)$ has $2n$ terms. Therefore, $a(n(n+1)) = 2$. Counting backwards $n - 1$ positions, each term is two larger than the successor

$$a((n(n+1)) - i) = 2 + 2i, \text{ for } 0 \leq i \leq n-1.$$

The next position down is one less than the previuos and after that, each term is again two less the successor.

$$a((n(n+1)) - n - i) = 2n - 1 - 2i, \text{ for } 0 \leq i \leq n-1.$$

As a single formula using the permutation

$$a((n(n+1)) - i) = \varphi_n(2n - i), \text{ for } 0 \leq i \leq 2n-1.$$

References

[1] F.Smarandache, "Numerical Sequences", University of Craiova, 1975; [See Arizona State University, Special Collection, Tempe, AZ, USA].

RECREATIONAL MATHEMATICS

ARITMOGRAF I

Aflaţi, de la A la B, denumirea unei ştiinţe fundamentale, iar pe orizontal noţiuni din această ştiinţă, înlocuind cifrele prin litere.

A

	7	4	8	5	1	4	3	8	9	
	10	9	7	8	2	11	2	3	2	
		1	9	12	3	5	1	4		
		13	9	1	4	8	2	12		
		4	12	4	1	4	13	3		
		4	6	9	2	3	5	4		
		5	7	14	3	4	15	2		
		7	14	12	5	16	14	13		
3	8	5	10	4	6	3	14	2	8	4
16	4	13	4	8	2	3	14	2	8	4

B

Soluţie:

A

	P	E	R	I	M	E	T	R	U	
	S	U	P	R	A	F	A	T	A	
		M	U	L	T	I	M	E		
		N	U	M	E	R	A	L		
		E	L	E	M	E	N	T		
		E	C	U	A	T	I	E		
		I	P	O	T	E	Z	A		
		P	O	L	I	G	O	N		
T	R	I	S	E	C	T	O	A	R	E
G	E	N	E	R	A	T	O	A	R	E

B

ARIFMOGRAF II

Înlocuind cifrele prin litere veţi obţine, de la A la B, denumirea unei ramuri matematice, iar orizontal noţiuni din această ramură.

A

			8	9	1	10	7			
			11	12	2	6	13			
				14	3	9				
		11	2	1	4	2	9	5		
	7	15	3	5	2	9	8	16	13	
		13	15	3	5	2	4	13		
11	2	4	7	17	6	2	13	15	5	13
		4	2	17	7	13	9	13		
		5	6	7	2	17	6	8		

B

Soluţie:

A

			U	N	G	H	I			
			S	F	E	R	A			
				C	O	N				
		S	E	G	M	E	N	T		
	I	P	O	T	E	N	U	Z	A	
		A	P	O	T	E	M	A		
S	E	M	I	D	R	E	A	P	T	A
		M	E	D	I	A	N	A		
		T	R	I	E	D	R	U		

B

The Lucky Mathematics!

If, by a wrong calculation (method, algorithm, operation, etc.) one arrives to the right answer, that is called a Lucky Calculation (Method, Algorithm, Operation, etc.)!

The wrong calculation (method, algorithm, operation, etc.) should by funny (somehow similar to a correct one, producing confusion and sympathy)!

Can somebody find a Lucky Integration or Differention?

FLORENTIN SMARANDACHE

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