AN APPROXIMATE REASONING METHOD IN DEZERT-SMARANDACHE THEORY[1]

Li Xinde Wu Xuejian Sun Jiaming Meng Zhengda

(*School of Automation, Southeast University, Nanjing 210096, China*)

Abstract With the increment of focal elements number in discernment framework, the computation amount in Dezert-Smarandache Theory (DSmT) will exponentially go up. This has been the bottle-neck problem to block the wide application and development of DSmT. Aiming at this difficulty, in this paper, a kind of fast approximate reasoning method in hierarchical DSmT is proposed. Presently, this method is only fit for the case that there are only singletons with assignment in hyper-power set. These singletons in hyper-power set are forced to group through bintree or tri-tree technologies. At the same time, the assignments of singletons in those different groups corresponding to each source are added up respectively, in order to realize the mapping from the refined hyper-power set to the coarsened one. And then, two sources with the coarsened hyper-power set are combined together according to classical DSm Combination rule (DSmC) and Proportional Conflict Redistribution rule No. 5 (PCR5). The fused results in coarsened framework will be saved as the connecting weights between father and children nodes. And then, all assignments of singletons in different groups will be normalized respectively. Tree depth is set, in order to decide the iterative times in hierarchical system. Finally, by comparing new method with old one from different views, the superiority of new one over old one is testified well.

Key words Approximate reasoning; Information fusion; Hierarchical; Dezert-Smarandache Theory (DSmT)

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I. Introduction

With the rapid development of computer science, more and more imperfect information (*i.e.* uncertain, incomplete, inconsistent and imprecise information) is required to intelligently and efficiently disposed in information acquisition, fusion and management system. The requirement to information fusion theories and methods is greatly increased, because those conventional ones are hard to adapt this new situation. Dezert-Smarandache Theory (DSmT) proposed by Dr. Jean Dezert and Prof. Florentin Smarandache in 2003 is a new theory extended form Dempster-Shafer Theory (DST) and Probability Theory (PT)[1]. Presently, DSmT has been widely applied to many fields *i.e.* image processing, robot perception, Multiple Target Tracking (MTT), multiple target recognition, multiple object decision, radar target classification, geographical science, fault diagnosis, economy and finance. However, like DST, with the increment of focal elements number in discernment framework, the combinational computation exponentially goes up, and it has become a bottle-neck problem of DSmT's application and development.

In order to solve this difficulty, many experts and scholars have proposed many methods in the DST framework, *i.e.* Jean Gordon and Edward H. Shortliffe proposed an approximate reasoning method, when there were a great deal of singletons or disjoint subsets assigned basic belief assignment. This method was realized through three steps. However, because the method didnot assign belief to subsets that are not in the hierarchical tree, and the time complexity varied from exponential to linear[2], it was approximate and fast. When conflict was high, the effect of this method was not too good. Therefore, Shafer and Logan improved Jean Gordon and Edward H. Shortliffe's work. But Shafer and Logan's method couldn't deal with

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Supported by the National Natural Science Foundation of China (No. 60804063).
Communication author: Li Xinde, born in 1975, male, Ph.D.. School of Automation, Southeast University, Nanjing 210096, China.
Email: xindeli@seu.edu.cn.

evidence for $C_{A_i} \cup \{A_i^c\}$, where A_i was the set of elements in partitions. C_{A_i} represented the set of children A_i, A_i^c represented complementary set of A_i[3]. Shafer, Shenoy and Mellouli improved the method[3] and proposed a qualitative Markov trees algorithm. They also pointed out that this algorithm reduces the time complexity from being exponential in the size of the frame to being exponential in the size of the largest partition[4]. Ulla Bergsten and Johan Schubert proposed evidences in the complete directed acyclic graph (shown in Fig. 1). This algorithm made it feasible to calculate in advance support and plausibility symbolically for completely specified paths through a complete directed acyclic graph. However, its restriction was too strong[5]. B. Tessem tried to reduce the focal elements number in power-set by ignoring the influence of some focal elements with negligible belief assignments. This method would lead to information loss, moreover, the efficiency of approximate reasoning computation is also very limited[6]. Thierry Denoeux and Amel Ben Yaghlane coarsened the discernment framework in a hierarchical mapping way, and then yielded inner and outer approximations of the combined belief function by using the fast Möbius transform algorithm. This method could efficiently reduce the computation by coarsening the discernment framework and kept the real result in an interval. However, as we know, to deal with the imprecision induced by interval is also very disturbing. Moreover, the computation of inner and outer borders is also expensive[7]. Authors donot find other valuable references in these recent years. It seems as if the study about this theme stopped because of its difficulty.

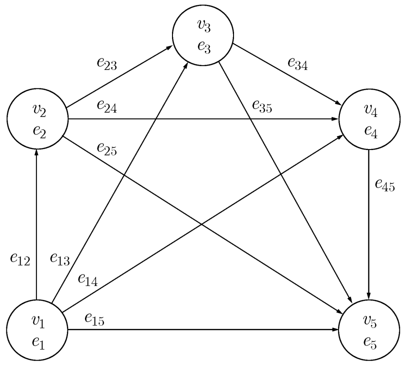

Fig. 1 Evidences in the complete directed acyclic graph

In this paper, aiming to the case that there is only a great deal of singletons with generalized basic assignment in DSmT framework, we proposed a new fast approximate reasoning method in hierarchical DSmT. These singletons in hyper power-set were forced to group by using bi-tree or tri-tree technologies, so that there is a mapping from the refined space to the different coarsened granular space. A recursive fusion way was chosen to fast obtain reliable approximate combination result.

II. Singletons' Grouping

For only singletons with generalized basic assignment in DSmT framework, supposed that there are two sources, *i.e.* S_1 and S_2, with the same discernment framework $\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\}$, and there is the most integrity constraint $\theta_i \cap \theta_j = \phi (i \neq j)$, that is, the Shafer model holds. These singletons in hyper power-set D^Θ is grouped, and mapped to new hyper power-set space $\Omega = \{\Theta_1', \Theta_2', \Theta_3', \cdots, \Theta_k'\}$, $k < n$. In other words, there is a mapping function $\rho(\cdot)$, $\rho(\Theta_k') = \{X_i, X_i \in D^\Theta\}$ between X_i and Θ_k'. According to the definition of Thierry Denoeux[7], there is a mapping function $\varphi(\cdot)$, $\varphi(m(\Theta_k')) = \sum m(\{X_i, X_i \in D^\Theta\})$. Here these singletons in hyper power-set are forced to group by using bi-tree or tri-tree technologies, their principles will be introduced in the following part. Because the case for non-singletons with belief assignments is very complex, we donot consider it here.

1. Singletons with non-zero assignments

Supposed that there are only all or part of singletons in hyper power-set with non-zero assignments, the set of these singletons is denoted $S_c \subseteq \Theta$.

(1) Grouping by bi-tree technology

If the number n of elements in $S_c = \{\theta_1, \theta_2, \cdots, \theta_n\}$ is even, the former $n/2$ elements in S_c are divided into a group, and the remaining ones are regarded as the other group. If n is odd, the former $[n/2]+1$ elements in S_c are divided into a group, *i.e.* $[x]$ represents the minimum integer close to x, similarly, the remaining ones as the other group. And then, we compute the sum of belief assignments involving the former and latter groups respectively. Of course, before grouping, the sum of belief assignments over the whole set $S_c = \{\theta_1, \theta_2, \cdots, \theta_n\}$ is 1. Therefore, we can get the coarsened focal elements and their corresponding belief assignments, *i.e.* $\Theta_1^{'}, \Theta_2^{'}$, and $m_1(\Theta_1^{'})$, $m_1(\Theta_2^{'})$, $m_2(\Theta_1^{'})$ $m_2(\Theta_2^{'})$. This grouping will continue, until there are at most 2 or 3 elements in the final group. The principle of grouping by bi-tree technology is shown in Fig. 2.

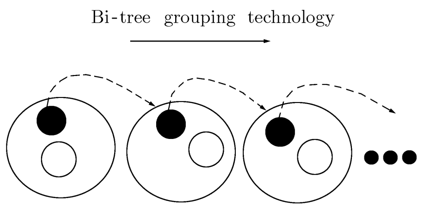

Fig. 2 The principle of grouping by bi-tree technology

Example 1 Supposed that $S_c = \{a, b, c, d, e, f\}$, $S_c^{\Omega} = \{\Theta_1^{'}, \Theta_1^{'}\}$ is the coarsened framework mapped to S_c, *i.e.* $\rho(\Theta_1^{'}) = \{a, b, c\}$, $\rho(\Theta_2^{'}) = \{d, e, f\}$. For two sources S_1 and S_2, they respectively supply with generalized basic belief assignments over $S_c = \{a, b, c, d, e, f\}$ as follows:

S_1: $m_1(a) = 0.3$, $m_1(b) = 0.1$, $m_1(c) = 0.1$, $m_1(d) = 0.15$, $m_1(e) = 0.05$, $m_1(f) = 0.3$

S_2: $m_2(a) = 0.2$, $m_2(b) = 0.2$, $m_2(c) = 0.3$, $m_2(d) = 0.1$, $m_2(e) = 0.05$, $m_2(f) = 0.15$

After mapping to the coarsened space, the new assignments are:

S_1:$m_1(\Theta_1^{'}) = 0.3 + 0.1 + 0.1 = 0.5$, $m_1(\Theta_2^{'}) = 0.15 + 0.05 + 0.3 = 0.5$

S_2:$m_2(\Theta_1^{'}) = 0.2 + 0.2 + 0.3 = 0.7$, $m_2(\Theta_2^{'}) = 0.1 + 0.05 + 0.15 = 0.3$

(2) Grouping by tri-tree technology

If n is divided exactly by 3, then all elements in $S_c = \{\theta_1, \theta_2, \cdots, \theta_n\}$ will be uniformly divided into 3 groups. Otherwise, at first, the former $[n/3]+1$ elements as 1st group, then, we check whether $n-1-[n/3]$ is even or not. If it is even, we regard the former half of $n-1-[n/3]$ as the 2nd group. The remaining ones are regarded as 3rd group. If $n-1-[n/3]$ is odd, we regard the former $[(n-1-[n/3]/2)]+1$ as the 2nd group. The remaining ones are regarded as 3rd group. Then, we can get the coarsened focal elements and their corresponding belief assignments, *i.e.* $\Theta_1^{'}, \Theta_2^{'}, \Theta_3^{'}$, and $m_1(\Theta_1^{'})$, $m_1(\Theta_2^{'})$, $m_1(\Theta_3^{'})$, $m_2(\Theta_1^{'})$, $m_2(\Theta_2^{'})$, $m_2(\Theta_3^{'})$. This grouping will continue, until there are at most 2 or 3 elements in the final group. The principle of grouping by tri-tree technology is shown in Fig. 3.

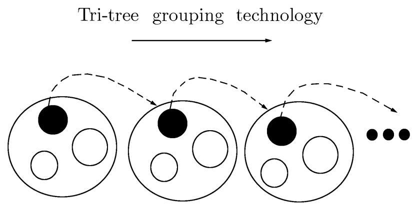

Fig. 3 The principle of grouping by tri-tree technology

Example 2 Supposed that $S_c = \{a, b, c, d, e, f\}$, $S_c^{\Omega} = \{\Theta_1^{'}, \Theta_2^{'}, \Theta_3^{'}\}$ is the coarsened framework mapped to S_c, *i.e.* $\rho(\Theta_1^{'}) = \{a, b\}$, $\rho(\Theta_2^{'}) = \{c, d\}$ and $\rho(\Theta_3^{'}) = \{e, f\}$. According to the original assignments over $S_c = \{a, b, c, d, e, f\}$ in example 1, after mapping to the coarsened space by tri-tree technology, the new assignments are:

S_1: $m_1(\Theta_1^{'}) = 0.3 + 0.1 = 0.4$, $m_1(\Theta_2^{'}) = 0.1 + 0.15 = 0.25$, $m_1(\Theta_3^{'}) = 0.05 + 0.3 = 0.35$

S_2: $m_2(\Theta_1^{'}) = 0.2 + 0.2 = 0.4$, $m_2(\Theta_2^{'}) = 0.3 + 0.1 = 0.4$, $m_2(\Theta_3^{'}) = 0.15 + 0.05 = 0.2$

(3) Singletons with zero assignment

There are at least two elements in S_c assigned 0 by one of two sources S_1 and S_2, in other words, if two sources both supply with assignment 0 to one element in S_c, then this element should be deleted from S_c.

Example 3 Supposed $S_c = \{a, b, c, d, e, f\}$. For two sources S_1 and S_2, they respectively supply with generalized basic belief assignments over $S_c = \{a, b, c, d, e, f\}$ as follows:

S_1: $m_1(a)= 0$, $m_1(b)= 0$, $m_1(c)= 0.1$, $m_1(d)= 0.15$, $m_1(e) = 0.45$, $m_1(f) = 0.3$

S_2: $m_2(a) = 0.2$, $m_2(b) = 0.2$, $m_2(c) = 0.3$, $m_2(d) = 0.1$, $m_2(e) = 0$, $m_2(f) = 0.2$

where the focal elements a, b, e are regarded as one group, denoted S_g, this is because $m_1(a) = 0$, $m_1(b) = 0$, $m_2(e) = 0$. Then, every assignment (non-zero) corresponding to singleton in S_g is divided by 2, and the average assignments $m(\theta_i), \theta_i \in S_g$ are obtained, as in Example 3, $m(a) = 0.1$, $m(b) = 0.1$,

$m(e)=0.225$. The sum of all assignments over $\theta_i \in S_g$ is $\sum m(\theta_i)$, then, the remaining focal elements involving $S_c \setminus S_g$ are forced to group with bintree or tritree technology. In this example, because the remaining focal elements involving $S_c \setminus S_g$ are c, d, f, the subdivision is not requisite. As we know, the assignments over c, d, f are not normal, before combination (see Section III), the normalization step (See section IV) is necessary. The combinational result by DSmT and PCR5 needs to multiply the factor $1-\sum m(\theta_i)$. So, in Example 3, $m(c)=0.575\times 0.3468$, $m(d)=0.575\times 0.1703$, $m(f)=0.575\times 0.4830$.

III. Evidence Combination

Dr. Jean Dezert and Prof. Florentin Smarandache[1] proposed the classical DSm combination rule and Proportional Conflict Redistribution rule (No. 5), *i.e.* PCR5. Here we simply introduce them as follows:

When dealing with information fusion in classical DSm model, $\mathrm{Bel}_1(\cdot)$ and $\mathrm{Bel}_2(\cdot)$ are the belief function of two independent sources S_1 and S_2 with the same discernment framework. $m_1(\cdot)$ and $m_2(\cdot)$ are the corresponding generalized basic belief assignments supplied by two sources. The free combination rule (two sources) is:

$$\forall C \in D^\Theta / \phi,\ m^f_{M(\Theta)}(C) \equiv m(C) = \sum_{\substack{A,B\in D^\Theta \\ A\cap B=C}} m_1(A)m_2(B) \tag{1}$$

Because hyper power-set D^Θ is closed under $\cup$ and $\cap$ operators, the assignment $m(\cdot)$ after combination in Eq. (1) is exact a generalized basic belief assignment, *i.e.* $m(\cdot): D^\Theta \mapsto [0,1]$, $m^f_{M(\Theta)}(\phi) \equiv 0$.

PCR5 does a better redistribution of the conflicting mass than Dempster's rule since PCR5 goes backwards on the tracks of the conjunctive rule and redistributes the partial conflicting masses only to the sets involved in the conflict and proportionally to their masses put in the conflict, considering the conjunctive normal form of the partial conflict. PCR5 is quasi-associative and preserves the neutral impact of the vacuous belief assignment. we just remind PCR5 rule for only two sources: $m_{\mathrm{PCR5}}(\phi)=0$, and $\forall X \in G\setminus\{\phi\}$

$$m_{\mathrm{PCR5}}(X) = m_{12} + \sum_{\substack{Y\in G^\Theta\setminus\{X\} \\ X\cap Y=\phi}} \left[\frac{m_1(X)^2 m_2(Y)}{m_1(X)+m_2(Y)} + \frac{m_1(Y)^2 m_2(X)}{m_2(X)+m_1(Y)}\right] \tag{2}$$

where G^Θ corresponds either to classical power-set or hyper-power set (depending on the underlying model chosen), all sets that involved in the formula are in canonical form and where $m_{12}(X)=\sum_{\substack{X_1,X_2\in G^\Theta \\ X_1\cap X_2=X}} m_1(X_1)m_2(X_2)$ corresponds to the con junctive consensus on X between the two sources and where all denominators are different from zero. If a denominator is zero, that fraction is discarded.

Example 4 According to the grouping results in Example 1, belief masses are resigned to the coarsened focal elements as follows:

S_1: $m_1(\Theta_1')=0.3+0.1+0.1=0.5$, $m_1(\Theta_2')=0.15+0.05+0.3=0.5$

S_2: $m_2(\Theta_1')=0.2+0.2+0.3=0.7$, $m_2(\Theta_2')=0.1+0.05+0.15=0.3$

At first, by applying classical DSm combination rule in Eq. (1), one obtains, $m_c(\Theta_1')=0.5\times 0.7=0.35$, $m_c(\Theta_1')=0.5\times 0.3=0.15$, $m_c(\Theta_1'\cap\Theta_2')=0.5\times 0.3+0.5\times 0.7=0.50$. And then, according to the integrity constraint, by applying PCR5 in Eq. (2). $m_c(\Theta_1'\cap\Theta_2')$ is redistributed to $m_c(\Theta_1')$ and $m_c(\Theta_2')$ one obtain,

$$m_c(\Theta_1') = 0.35 + \frac{0.5^2\times 0.3}{0.8} + \frac{0.7^2\times 0.5}{1.2} = 0.648$$

$$m_c(\Theta_2') = 0.15 + \frac{0.3^2\times 0.5}{0.8} + \frac{0.5^2\times 0.7}{1.2} = 0.352$$

IV. Normalization Step

Because the belief assignments are normalized before grouping, after grouping by bi-tree or tri-tree technology, generally speaking, these assignments in subgroup are not normal. Before applying DSmC and PCR5, the normalization step is necessary. For example, in Example 1, $\rho(\Theta_1')=\{a,b,c\}$, $\rho(\Theta_2')=\{d,e,f\}$ the original belief assignments over $S_c=\{a,b,c,d,e,f\}$ are:

S_1: $m_1(a)=0.3$, $m_1(b)=0.1$, $m_1(c)=0.1$, $m_1(d)=0.15$, $m_1(e)=0.05$, $m_1(f)=0.3$

S_2: $m_2(a)=0.2$, $m_2(b)=0.2$, $m_2(c)=0.3$, $m_2(d)=0.1$, $m_2(e)=0.05$, $m_2(f)=0.15$

After grouping, one obtains the normalization results for two subgroups in different sources, respectively,

Subgroup 1

$$S_1: m_1(a)=\frac{0.3}{0.5},\ m_1(b)=\frac{0.1}{0.5},\ m_1(c)=\frac{0.1}{0.5}$$

$$S_2: m_2(a)=\frac{0.2}{0.7},\ m_2(b)=\frac{0.2}{0.7},\ m_2(c)=\frac{0.3}{0.7}$$

Subgroup 2

$$S_1: m_1(d)=\frac{0.15}{0.5},\ m_1(e)=\frac{0.05}{0.5},\ m_1(f)=\frac{0.3}{0.5}$$

$$S_2: m_2(d)=\frac{0.1}{0.3},\ m_2(e)=\frac{0.05}{0.3},\ m_2(f)=\frac{0.15}{0.3}$$

V. Program Realization

The procedure about approximate reasoning fusion in hierarchical DSmT is shown in Fig. 4. Its main steps are introduced as follows:

Step 1 At first, check whether the number n of singletons in S_c is greater than 3. If yes, then next step. Otherwise, go to Step 4.

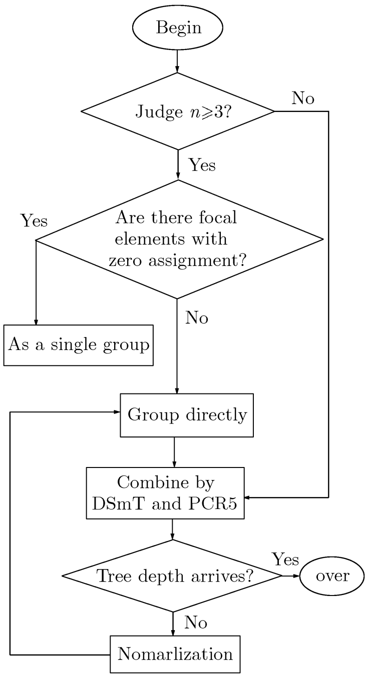

Fig. 4 The procedure about approximate reasoning fusion in hierarchical DSmT

Step 2 Judge whether there are more than 2 singletons with zero assignment. If yes, then these singletons with zero assignment are regarded as a single group, which can be further disposed according to the method (Singletons with zero assignment) aforementioned in Section II. Otherwise, go to next step.

Step 3 Divide these focal elements with non-zero assignments by using bi-tree or tri-tree technology, and compute the sum of belief assignments involving the former and latter groups respectively. Therefore, we regard the sum as the belief assignment over the coarsened focal element mapped to the refined ones, *i.e.* $m(\Theta_k^{'})=\sum m(\{X_i, X_i\in D^{\Theta}\})$ and then, go to next step.

Step 4 Combine the two new evidence sources in the coarsened space by applying DSmC and PCR5. The combinational result is regarded as the connected weight between father and children node. Go to next step.

Step 5 Judge whether the tree depth arrives. If yes, compute $m(\theta_i)$ over every focal elements in S_c, and stop it. For example, for bi-tree, shown in Fig. 5 $m(\theta_1)=m_{11}m_{211}m_{311}m_{411}$. For tri-tree, shown in Fig. 6, $m(\theta_1)=m_{11}m_{211}m_{311}$. Otherwise, go to next step.

Step 6 Carry out the normalization step for two subgroups in different sources, respectively. Go to Step 1.

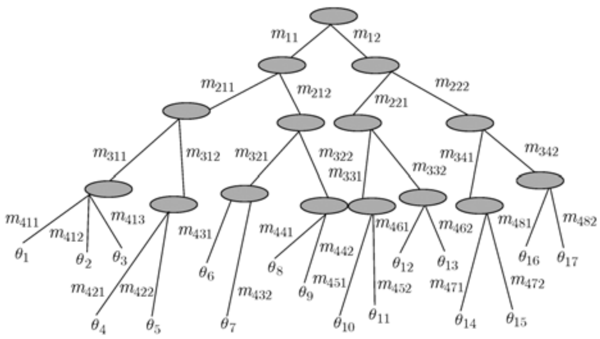

Fig. 5 Fusion result by bi-tree

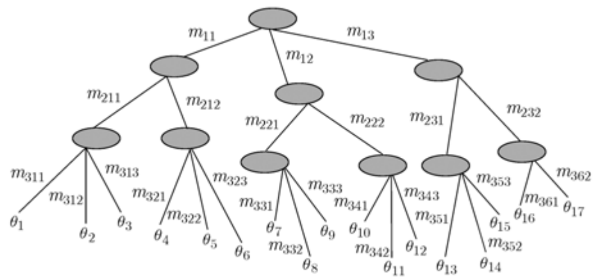

Fig. 6 Fusion result by tri-tree

VI. Comparison and Analysis of Fusion Results

In order to show the advantage of new method, we compare and analyze it from 3 views, *i.e.* similarity, efficiency, and robustness respectively.

1. Similarity

Example 5 Supposed $S_c = \{a, b, c, d\}$, the assignments over these elements in S_c are supplied by two sources, S_1 and S_2 as follows:

S_1: $m_1(a) = x - \varepsilon$, $m_1(b) = \varepsilon$, $m_1(c) = 1 - x - \varepsilon$, $m_1(d) = \varepsilon$

S_2: $m_2(a) = \varepsilon$, $m_2(b) = y - \varepsilon$, $m_2(c) = \varepsilon$, $m_2(d) = 1 - y - \varepsilon$

where bi-tree technology is chosen to group, $\varepsilon = 0.01$. In order to assure the belief assignment of every focal element greater than zero, let $x, y \in [0.02, 0.98]$. The Euclidean evidence support measurement of similarity is chosen to estimate the similarity between new and old methods, *i.e.* $N_E(m_1, m_2)$[8],

$$N_E(m_1, m_2) = 1 - \frac{1}{\sqrt{2}} \sqrt{\sum_{i=1}^{D^\Theta} \left(m_1(X_i) - m_2(X_i)\right)^2} \quad (3)$$

When x, y vary in $[0.02, 0.98]$, the similarity between new and old methods is computed through Eq. (3). The minimum of similarity is 0.7110. Therefore, even if the conflict between two sources is very high, then the result by new method is also very similar to the old one.

2. High efficiency

Whether can the new method solve the bottle-neck problem of DSmT? On the basis of high similarity, high efficiency is very important. In Tab. 1, we can give the comparison result as times of addition, product and division operation and the time of operation, when the focal elements number varies from 10000 to 50000.

Seen from Tab. 1, new method has high computation efficiency, especially, bi-tree is more obvious. And even, we can reach a conclusion that more the number of branches is, greater the computation amount is at the same layer. Bi-tree is the best one to group in DSmT framework.

Tab. 1 Comparison of operation efficiency

The number of focal elements	Methods	Times of addition (+)	Times of product (×)	Times of division (/)	Operation time (ms)
10000	Old	399,953,796	399,963,796	199,976,898	3688
	Bi-tree	335,344	166,532	82,958	15
	Tri-tree	260,724	155,583	83,882	62
20000	Old	1,599,901,648	1,599,921,648	799,950,824	14,672
	Bi-tree	709,510	340,888	165,714	16
	Tri-tree	574,778	333,670	178,224	94
30000	Old	3,599,846,872	3,599,876,872	1,799,923,436	33,625
	Bi-tree	1,085,018	520,204	244,394	31
	Tri-tree	843,766	480,924	252,234	469
50000	Old	9,999,701,168	9,999,751,168	4,999,850,584	93,709
	Bi-tree	1,950,442	877,918	429,000	47
	Tri-tree	1,493,368	845,091	437,782	609

3. Robustness

In order to testify the robustness of new method, several bi-tree examples are given as follows:

(1) Consistent evidence sources

Example 6 Supposed $S_c = \{a, b, c, d\}$, the assignments over these elements in S_c are supplied by two sources, S_1 and S_2 as follows:

S_1: $m_1(a) = 0.3,\ m_1(b) = 0.2,\ m_1(c) = 0.4\ \ m_1(d) = 0.1$

S_2: $m_2(a) = 0.5,\ m_2(b) = 0.1,\ m_2(c) = 0.3,\ m_2(d) = 0.1$

The combinational results from the above two sources S_1 and S_2 by new and old methods is shown in Tab. 2.

Tab. 2 Combinational results in Example 6

Method	$m(a)$	$m(b)$	$m(c)$	$m(d)$
New	0.4618	0.1129	0.3703	0.0549
Old	0.4642	0.1064	0.3764	0.0530

Example 7 We exchange the assignments over the elements b, c in Example 6, as

S_1: $m_1(a)=0.3$, $m_1(b)=0.4$, $m_1(c)=0.2$, $m_1(d)=0.1$

S_2: $m_2(a)=0.5$, $m_2(b)=0.3$, $m_2(c)=0.1$, $m_2(d)=0.1$

The combinational results from the above two sources S_1 and S_2 by new and old methods is shown in Tab. 3.

Tab. 3 Combinational results in Example 7

Method	$m(a)$	$m(b)$	$m(c)$	$m(d)$
New	0.4556	0.3879	0.0977	0.0589
Old	0.4642	0.3764	0.1064	0.0530

Example 8 We exchange the assignments over the elements b, d in Example 6, as

S_1: $m_1(a)=0.3$, $m_1(b)=0.1$, $m_1(c)=0.4$, $m_1(d)=0.2$

S_2: $m_2(a)=0.5$, $m_2(b)=0.1$, $m_2(c)=0.3$, $m_2(d)=0.1$

The combinational results from the above two sources S_1 and S_2 by new and old methods is shown in Tab. 4.

Tab. 4 Combinational results in Example 8

Method	$m(a)$	$m(b)$	$m(c)$	$m(d)$
New	0.4438	0.0562	0.3971	0.1029
Old	0.4642	0.0530	0.3764	0.1064

(2) Inconsistent evidence sources

Example 9 Supposed $S_c=\{a, b, c, d\}$, the assignments over these elements in S_c are supplied by two sources, S_1 and S_2 as follows:

S_1: $m_1(a)=0.3$, $m_1(b)=0.2$, $m_1(c)=0.4$, $m_1(d)=0.1$

S_2: $m_2(a)=0.01$, $m_2(b)=0.59$, $m_2(c)=0.3$, $m_2(d)=0.1$

The combinational results from the above two sources S_1 and S_2 by new and old methods is shown in Tab. 5.

Tab. 5 Combinational results in Example 9

Method	$m(a)$	$m(b)$	$m(c)$	$m(d)$
New	0.1344	0.4403	0.3703	0.0549
Old	0.1304	0.4657	0.3548	0.0491

Example 10 Supposed $S_c=\{a, b, c, d\}$, the assignments over these elements in S_c are supplied by two sources, S_1 and S_2 as follows:

S_1: $m_1(a)=0.49$, $m_1(b)=0.01$, $m_1(c)=0.4$, $m_1(d)=0.1$

S_2: $m_2(a)=0.01$, $m_2(b)=0.59$, $m_2(c)=0.1$, $m_2(d)=0.3$

The combinational results from the above two sources S_1 and S_2 by new and old methods is shown in Tab. 6.

Tab. 6 Combinational results in Example 10

Method	$m(a)$	$m(b)$	$m(c)$	$m(d)$
New	0.2859	0.2888	0.2286	0.1967
Old	0.2682	0.3552	0.2220	0.1546

Example 11 Supposed $S_c=\{a, b, c, d\}$, the assignments over these elements in S_c are supplied by two sources, S_1 and S_2 as follows:

S_1: $m_1(a)=0.49$, $m_1(b)=0.01$, $m_1(c)=0.4$, $m_1(d)=0.1$

S_2: $m_2(a)=0.01$, $m_2(b)=0.59$, $m_2(c)=0.3$, $m_2(d)=0.1$

The combinational results from the above two sources S_1 and S_2 by new and old methods is shown in Tab. 7.

Tab. 7 Combinational results in Example 11

Method	$m(a)$	$m(b)$	$m(c)$	$m(d)$
New	0.2859	0.2888	0.3703	0.0549
Old	0.2682	0.3552	0.3325	0.0442

Seen from the comparison of fusion results from new and old methods, no matter whether consistent sources or inconsistent sources are, the result from the new method is always very similar to the old one, which shows the robustness of new one adequately. At the same time, we also find a fact that when the conflict between two sources becomes greater, the new method might be influenced a bit, although no too big. Therefore, an intelligent hierarchical grouping is needed.

VII. Conclusion

With the wide application of DSmT in different fields, because of the increment of focal elements number in discernment framework, its computation amount will go up exponentially. Then, it is very valuable to solve this problem. In this paper, we propose a fast approximate reasoning method in hierarchical DSmT, in order to solve the bottle-neck problem of its computation. Presently, this method is only fit to adapt singletons with assignments. For non-singletons with assignments, those interesting readers can pay attention to our later reports before long.

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