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两个近似伪Smarandache函数的均值计算

李育英[1],付瑞琴[1],李学功[2]

(1.西安石油大学 理学院,陕西 西安 710065;2.铜川职业技术学院,陕西 铜川 727031)

摘要:利用初等方法研究了近似伪Smarandache函数与简单数相关的渐近性质的值性质,并给出了两个有趣的渐近公式.

关键词:近似伪Smarandache函数;简单数;渐近公式;均值
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1 引言及结论

在文献[1]中,David Gorski将伪Smarandache函数$Z(n)$定义为最小的正整数t,使得$Z(n)=\min\{t: t\in N, n\mid(1+2+3+\cdots+t)\}$.在文献[2]中,A.W. Vyawahare将$Z(n)$做了如下变换:添加了一个最小自然数$k$,从而定义了一个新的函数$K(n)$,即:对任意的正整数$n$,$K(n)=m$,这里$m=\sum_{i=1}^{n} i+k$,$k$是使$n$能整除$m$的最小正整数.称这个函数$K(n)$为近似伪Smarandache函数.在文献[3]第23个问题中,如果一个正整数n的真因子的乘积不超过n,就称n为简单数.令A表示所有简单数的集合,即有$A=\{2,3,4,5,6,7,8,9,10,11,13,14,15,17,19,21,\cdots\}$.容易看出,$n$有4种情形,即$n=p$,或$n=p^2$,或$n=p^3$,或$n=pq$,这里$p$,$q$是不同的素数.本文利用初等数论方法研究了近似伪Smarandache函数与简单数相关的渐近性质,并给出了2个渐近公式,即证明了下面定理:

定理1　任意实数$x\geqslant 1$,令A表示所有简单数的集合.则有

$$\sum_{\substack{n\in A\\ n\leqslant x}} K(n)=\frac{x^2\ln\ln x}{3\ln x}+B\frac{x^2}{\ln x}+\frac{2x^2\ln\ln x}{9\ln^2 x}+O\left(\frac{x^2}{\ln^2 x}\right).$$

其中B是一个可计算的常数.

定理2　对任意实数$x\geqslant 1$,令A表示所有简单数的集合.则有

$$\sum_{\substack{n\in A\\ n\leqslant x}}\frac{1}{K(n)}=\frac{2}{3}(\ln\ln x)^2+D\ln\ln x+E+O\left(\frac{\ln\ln x}{\ln x}\right).$$

其中D和E都是可计算的常数.

2 几个引理

为了完成定理的证明,需要引入下面几个引理:

引理1　设n为任意正整数,则有

收稿日期:2010-07-19
作者简介:李育英(1960-),女,讲师,主要从事应用数学方面的研究.E-mail:lizhang@xsyu.edu.cn

$$K(n)=\begin{cases}\dfrac{n(n+3)}{2}, & 当\ n是奇数时,\\ \dfrac{n(n+2)}{2}, & 当\ n是偶数时.\end{cases}$$

证明　参阅文献[2].

引理 2　设 p 是一个素数,则有

$$\sum_{p\le x} p^2=\frac{x^3}{3\ln x}+\frac{x^3}{9\ln^2 x}+O\left(\frac{x^3}{\ln^3 x}\right);\tag{1}$$

$$\sum_{p\le x} p=\frac{x^2}{2\ln x}+\frac{x^2}{4\ln^2 x}+O\left(\frac{x^2}{\ln^3 x}\right);\tag{2}$$

$$\sum_{p^3\le x} p^6=O\left(\frac{x^3}{\ln^2 x}\right);\tag{3}$$

$$\sum_{p^3\le x} p^3=O\left(\frac{x^2}{\ln^2 x}\right).\tag{4}$$

证明　首先证明第一个式子. 注意到 $\pi(x)=\frac{x}{\ln x}+\frac{x}{\ln^2 x}+O\left(\frac{x}{\ln^3 x}\right)$. 由 Abel 恒等式[4]可得

$$\sum_{p\le x} p^2=\pi(x)x^2-\int_1^x\pi(t)2t\,dt=\frac{x^3}{\ln x}+\frac{x^3}{\ln^2 x}+O\left(\frac{x^3}{\ln^3 x}\right)-2\int_2^x\frac{t^2}{\ln t}dt-2\int_2^x\frac{t^2}{\ln^2 t}dt+O\left(\int_2^x\frac{t^2}{\ln^3 t}dt\right)=\frac{x^3}{\ln x}+\frac{x^3}{\ln^2 x}+$$
$$O\left(\frac{x^3}{\ln^3 x}\right)-\frac{2}{3}\frac{x^3}{\ln x}-\frac{8}{9}\frac{x^3}{\ln^2 x}+O\left(\int_2^x\frac{t^2}{\ln^3 t}dt\right)=\frac{x^3}{3\ln x}+\frac{x^3}{9\ln^2 x}+O\left(\frac{x^3}{\ln^3 x}\right).$$

利用同样的方法可以推出其他 3个式子:

$$\sum_{p\le x} p=\frac{x^2}{2\ln x}+\frac{x^2}{4\ln^2 x}+O\left(\frac{x^2}{\ln^3 x}\right).$$

$$\sum_{p^3\le x} p^6=\sum_{p\le\sqrt[3]{x}} p^6=\pi(\sqrt[3]{x})x^2-6\int_1^{\sqrt[3]{x}}\pi(t)t^5\,dt=O\left(\frac{x^3}{\ln^2 x}\right).$$

$$\sum_{p^3\le x} p^3=\sum_{p\le\sqrt[3]{x}} p^3=\pi(\sqrt[3]{x})x-3\int_1^{\sqrt[3]{x}}\pi(t)t^2\,dt=O\left(\frac{x^2}{\ln^2 x}\right).$$

于是完成了引理 2的证明.

引理 3　设 p 和 q 都是素数,则有

$$\sum_{pq\le x} p^2q^2=\frac{2x^3}{3\ln x}\ln\ln x+B_1\frac{x^3}{3\ln x}+\frac{2x^3\ln\ln x}{9\ln^2 x}+O\left(\frac{x^3}{\ln^2 x}\right);\tag{5}$$

$$\sum_{pq\le x} pq=\frac{x^2}{\ln x}\ln\ln x+B_2\frac{x^2}{2\ln x}+\frac{x^2\ln\ln x}{2\ln^2 x}+O\left(\frac{x^2}{\ln^2 x}\right).\tag{6}$$

证明　只需证明式(5)即可,类似于式(5)的推导即可得式(6)的证明. 注意到当 $x<1$ 时有 $\frac{1}{1-x}=1+x+x^2+x^3+\cdots+x^m+\cdots$, 于是可得

$$\sum_{p\le\sqrt{x}} p^2\sum_{q\le\frac{x}{p}} q^2=\sum_{p\le\sqrt{x}} p^2\left[\frac{1}{3}\frac{(x/p)^3}{\ln x-\ln p}+\frac{1}{3^2}\frac{(x/p)^3}{(\ln x-\ln p)^2}+O\left(\frac{(x/p)^3}{(\ln x-\ln p)^3}\right)\right]=\sum_{p\le\sqrt{x}}\left[\frac{x^3}{3\ln x}\frac{1}{p\left(1-\frac{\ln p}{\ln x}\right)}+\right.$$
$$\left.\frac{x^3}{9\ln^2 x}\frac{1}{p\left(1-\frac{\ln p}{\ln x}\right)^2}\right]+O\left(\sum_{p\le\sqrt{x}}\frac{x^3}{p\ln^3 x}\right)=\frac{x^3}{3\ln x}\sum_{p\le\sqrt{x}}\frac{1}{p}\left(1+\frac{\ln p}{\ln x}+\frac{\ln^2 p}{\ln^2 x}+\cdots+\frac{\ln^m p}{\ln^m x}+\cdots\right)+$$
$$\frac{x^3}{9\ln^2 x}\sum_{p\le\sqrt{x}}\frac{1}{p}\left(1+2\frac{\ln p}{\ln x}+\cdots+m\frac{\ln^{m-1} p}{\ln^{m-1} x}+\cdots\right)+O\left(\frac{x^3}{p\ln^3 x}\sum_{p\le\sqrt{x}}\frac{1}{p}\right).\tag{7}$$

当 $m>2$ 时,注意到 $\pi(x)=\frac{x}{\ln x}+\frac{x}{\ln^2 x}+O\left(\frac{x}{\ln^3 x}\right)$, 于是有

$$\sum_{p\le\sqrt{x}}\frac{\ln^m p}{p}=\int_2^{\sqrt{x}}\frac{\ln^m y}{y}\,d\pi(y)=\frac{\ln^m\sqrt{x}}{\sqrt{x}}\pi(\sqrt{x})+O(1)-\int_2^{\sqrt{x}}\pi(y)\frac{m\ln^{m-1}y-\ln^m y}{y^2}\,dy=\frac{\ln^{m-1}x}{2^{m-1}}+O\left(\frac{\ln^{m-2}x}{2^{m-2}}\right)+\frac{1}{m2^m}\ln^m x+\frac{\ln^{m-1}x}{2^{m-1}}+O\left(\frac{1-m}{(m-2)2^{m-2}}\ln^{m-2}x\right)=\frac{1}{m2^m}\ln^m x+O\left(\frac{1}{2^{m-2}(2-m)}\ln^{m-2}x\right).\tag{8}$$

注意到$\sum_{m=1}^{\infty}\frac{1}{m2^m}$是收敛的并根据式（8）有

$$\sum_{p\le\sqrt{x}}\frac{1}{p}\left(1+\frac{\ln p}{\ln x}+\frac{\ln^2 p}{\ln^2 x}+\cdots+\frac{\ln^m p}{\ln^m x}+\cdots\right)=\ln\ln x+(C-\ln 2)+O\left(\frac{1}{\ln x}\right)+\frac{1}{\ln x}\left(\frac{1}{2}\ln x+O(1)\right)+\cdots+\frac{1}{\ln^m x}\left[\frac{1}{m2^m}\ln^m x+O\left(\frac{1}{2^{m-2}(2-m)}\ln^{m-2}x\right)\right]+\cdots=\ln\ln x+C_1+\left(\frac{1}{2}+\cdots+\frac{1}{m2^m}+\cdots\right)+O\left(\frac{1}{\ln x}\right)=\ln\ln x+C_2+O\left(\frac{1}{\ln x}\right).\tag{9}$$

利用同样的方法可得

$$\sum_{p\le\sqrt{x}}\frac{1}{p}\left(1+2\frac{\ln p}{\ln x}+3\frac{\ln^2 p}{\ln^2 x}+\cdots+(m+1)\frac{\ln^m p}{\ln^m x}+\cdots\right)=\ln\ln x+C_3+O\left(\frac{1}{\ln x}\right).\tag{10}$$

其中使用了估计式$\sum_{p\le\sqrt{x}}\frac{1}{p}=\ln\ln x+C+O\left(\frac{1}{\ln x}\right)$和$\sum_{p\le\sqrt{x}}\frac{\ln p}{p}=\frac{1}{2}\ln x+O(1)$.

于是由式（7）、式（9）和式（10）有

$$\sum_{p\le\sqrt{x}}p^2\sum_{q\le\frac{x}{p}}q^2=\frac{x^3}{3\ln x}\left[\ln\ln x+C_2+O\left(\frac{1}{\ln x}\right)\right]+\frac{x^3}{9\ln^2 x}\left[\ln\ln x+C_3+O\left(\frac{1}{\ln x}\right)\right]+O\left(\frac{x^3}{p\ln^3 x}\sum_{p\le\sqrt{x}}\frac{1}{p}\right)=\frac{x^3}{3\ln x}\ln\ln x+B_1\frac{x^3}{3\ln x}+\frac{x^3\ln\ln x}{9\ln^2 x}+O\left(\frac{x^3}{p\ln^3 x}\sum_{p\le\sqrt{x}}\frac{1}{p}\right).\tag{11}$$

同时可得　$$\sum_{p\le\sqrt{x}}p^2\sum_{q\le\sqrt{x}}q^2=\left[\frac{x^{\frac{3}{2}}}{3\ln\sqrt{x}}+O\left(\frac{x^{\frac{3}{2}}}{\ln^2\sqrt{x}}\right)\right]^2=O\left(\frac{x^3}{\ln^2 x}\right),\tag{12}$$

其中使用了估计式$\sum_{p\le\sqrt{x}}p^2=\frac{2x^{\frac{3}{2}}}{3\ln x}+O\left(\frac{x^{\frac{3}{2}}}{\ln^2\sqrt{x}}\right)$.

所以由式（11）和式（12）可得

$$\sum_{pq\le x}p^2q^2=2\sum_{p\le\sqrt{x}}p^2\sum_{q\le\frac{x}{p}}q^2-\left(\sum_{p\le\sqrt{x}}p^2\right)\left(\sum_{q\le\sqrt{x}}q^2\right)=\frac{2x^3}{3\ln x}\ln\ln x+B_1\frac{x^3}{3\ln x}+\frac{2x^3\ln\ln x}{9\ln^2 x}+O\left(\frac{x^3}{\ln^2 x}\right).\tag{13}$$

于是完成了引理 3的证明.

引理 4　对任意的正数 $x>1$，有

$$\sum_{pq\le x}\frac{1}{pq}=(\ln\ln x)^2+D_1\ln\ln x+D_2+O\left(\frac{\ln\ln x}{\ln x}\right).$$

证明　注意到$\sum_{p\le x}\frac{1}{p}=\ln\ln x+C+O\left(\frac{1}{\ln x}\right)$，以及$\sum_{p\le\sqrt{x}}\frac{1}{p}\ln\ln x=(\ln\ln x)^2+D_1\ln\ln x+D_2+O\left(\frac{\ln\ln x}{\ln x}\right)$（参阅文献[5]），有

$$\sum_{pq\le x}\frac{1}{pq}=2\sum_{p\le\sqrt{x}}\frac{1}{p}\sum_{q\le\frac{x}{p}}\frac{1}{q}-\left(\sum_{p\le\sqrt{x}}\frac{1}{p}\right)\left(\sum_{q\le\sqrt{x}}\frac{1}{q}\right)=2\sum_{p\le\sqrt{x}}\frac{1}{p}\left[\ln\ln\frac{x}{p}+C+O\left(\frac{1}{\ln x}\right)\right]-\left[\ln\ln\sqrt{x}+C+O\left(\frac{1}{\ln x}\right)\right]^2=2\sum_{p\le\sqrt{x}}\frac{1}{p}\ln\ln x+2C\sum_{p\le\sqrt{x}}\frac{1}{p}+O\left(\frac{1}{\ln x}\sum_{p\le\sqrt{x}}\frac{1}{p}\right)-\left[(\ln\ln x)^2+C_4\ln\ln x+C_5+O\left(\frac{\ln\ln x}{\ln x}\right)\right]=(\ln\ln x)^2+D_1\ln\ln x+D_2+O\left(\frac{\ln\ln x}{\ln x}\right).\tag{14}$$

于是完成了引理 4的证明.

3 定理的证明

本节来完成 2个定理的证明. 由引理 1、引理 2和引理 3有

$$\sum_{n\in A,n\leqslant x}K(n)=\sum_{p\leqslant x}K(p)+\sum_{p^2\leqslant x}K(p^3)+\sum_{p^3\leqslant x}K(p^3)+\sum_{pq\leqslant x,p\neq q}K(pq)=\sum_{p\leqslant x}\frac{p(p+3)}{2}+\sum_{p^2\leqslant x}\frac{p^3(p^3+3)}{2}+\sum_{p^3\leqslant x}\frac{p^3(p^3+3)}{2}+\sum_{pq\leqslant x,p\neq q}\frac{pq(pq+3)}{2}=\frac{1}{2}\left(\sum_{p\leqslant x}p^2+\sum_{p^3\leqslant x}p^6+\sum_{pq\leqslant x}p^2q^2\right)+\frac{3}{2}\left(\sum_{p\leqslant x}p+\sum_{p^3\leqslant x}p^3+\sum_{pq\leqslant x}pq\right)=\frac{x^3\ln\ln x}{3\ln x}+B\frac{x^3}{\ln x}+\frac{2x^3\ln\ln x}{9\ln^2 x}+O\left(\frac{x^3}{\ln^2 x}\right).$$

于是完成了定理 1的证明. 下面来证明定理 2. 由引理 2和引理 4有

$$\sum_{n\in A,n\leqslant x}\frac{1}{K(n)}=\sum_{p\leqslant x}\frac{1}{K(p)}+\sum_{p^2\leqslant x}\frac{1}{K(p^3)}+\sum_{p^3\leqslant x}\frac{1}{K(p^3)}+\sum_{pq\leqslant x,p\neq q}\frac{1}{K(pq)}=\sum_{p\leqslant x}\frac{2}{p(p+3)}+\sum_{p^2\leqslant x}\frac{2}{p^3(p^3+3)}+\sum_{p^3\leqslant x}\frac{2}{p^3(p^3+3)}+\sum_{pq\leqslant x,p\neq q}\frac{2}{pq(pq+3)}=2\left(\sum_{p\leqslant x}\frac{1}{p(p+3)}+\sum_{p^3\leqslant x}\frac{2}{p^3(p^3+3)}+\sum_{pq\leqslant x}\frac{1}{pq(pq+3)}\right).$$

注意到$\sum\limits_{p}\frac{1}{p(p+3)}$和$\sum\limits_{p^3}\frac{1}{p^3(p^3+3)}$都是收敛的, 则容易推出

$$\sum_{n\in A,n\leqslant x}\frac{1}{K(n)}=\frac{2}{3}(\ln\ln x)^2+D\ln\ln x+E+O\left(\frac{\ln\ln x}{\ln x}\right).$$其中 D和 E都是可计算的常数.

于是完成了定理 2的证明.

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model is established by taking leakage rate reliability economical performance and the convenient degree of installation and maintenance as 4 factors for comprehensive evaluation. Taking some refining chemical factory as an example the network implementation of the comprehensive evaluation is presented by using ASP technique. The study provides a effective way for the comprehensive evaluation of mechanical seal.

Key words: mechanical seal; comprehensive evaluation; fuzzy mathematics; ASP

LI Ya-jun[1], ZHANG Xin[2], LV Rui-dian[2], YOU Wen-qing (1. Petrochemical Engineering Branch, Tarim Oilfield Company, Korla 841100, Xinjiang, China; 2. School of Graduate, Southwest Petroleum University, Chengdu 610500, Sichuan, China) JXSYU 2010 V.25 N.5 P.85-88

Necessity and trend of developing electric vehicle in China

Abstract: The development history of electric vehicle is reviewed. The necessity of developing electric vehicle is described in our country at present. The key technologies of the electric vehicle are discussed. The factors of restricting the development of the electric vehicle at present are pointed, and the development trend of the electric vehicle is presented in the future.

Key words: electric vehicle; key technology; development trend; restrictive factors

TIAN Mei-e (Periodical Center, Xi'an Shiyou University, Xi'an 710065, Shaanxi, China) JXSYU 2010 V.25 N.5 P.89-91

An image magnification method based on nonlinear PDE

Abstract: An image magnification method based on nonlinear PDE (partial differential equation) is proposed. The magnification results of grey image and color image show that the method has very good magnification effect, but there is grid effect when amplification factor is great.

Key words: image magnification; nonlinear partial differential equation; curvature and edge

ZHANG Hui-yu[1], ZHU Xuan[2] (1. School of Management, Xi'an University of Finance and Economics, Xi'an 710061, Shaanxi, China; 2. School of Information and Science, Northwest University, Xi'an 710069, Shaanxi, China) JXSYU 2010 V.25 N.5 P.92-95

Some further periodic properties of the element in BCI-algebras

Abstract: Some new properties of element period in BCI-algebras are presented, and associative, p-semisimple, quasi-associative and k-associative BCI-algebras are characterized by maximum and minimun of all non-element periods.

Key words: BCI-algebras; period; associative BCI-algebra; p-semisimple BCI-algebra; quasi-associative BCI-algebra; k-associative BCI-algebra

YANG Wen-qi (Department of Mathematics, Baoji University of Arts and Sciences, Baoji 721013, Shaanxi, China) JXSYU 2010 V.25 N.5 P.96-98

Calculation of the mean value of two approximate pseudo-Smarandache functions

Abstract: The asymptotic properties of two approximate pseudo-Smarandache functions related to simple numbers are studied using elementary mathematics method, and two interesting asymptotic formulae are presented.

Key words: approximate pseudo-Smarandache function; simple numbers; asymptotic formula; mean value

LI Yu-ying[1], FU Rui-qin[1], LI Xue-gong[2] (1. College of Sciences, Xi'an Shiyou University, Xi'an 710065, Shaanxi, China; 2. Deparment of Mathematics, Tongchuan Polytechnic College, Tongchuan 727031, Shaanxi, China) JXSYU 2010 V.25 N.5 P.99-102

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