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Cosine Similarity Measure Of Rough Neutrosophic Sets And Its Application In Medical Diagnosis

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ABSTRACT
In this paper, we define a rough cosine similarity measure between two rough neutrosophic sets. The notions of rough neutrosophic sets (RNS) will be used as vector representations in 3D-vector space. The rating of all elements in RNS is expressed with the upper and lower approximation operator and the pair of neutrosophic sets which are characterized by truth-membership degree, indeterminacy-membership degree, and falsity-membership degree. A numerical example of the medical diagnosis is provided to show the effectiveness and flexibility of the proposed method.

General Terms
Rough neutrosophic set, Cosine similarity measure

Keywords
Rough cosine similarity measure, rough sets, neutrosophic sets, rough neutrosophic sets, indeterminacy-membership degree, 3D-vector space

1. INTRODUCTION
The concept of neutrosophic sets [1] was originated from the new branch of philosophy called 'neutrosophy'[1]. The neutrosophic set generalizes the classical set or crisp set proposed by Cantor, fuzzy set proposed by Zadeh [2], interval valued fuzzy set proposed independently by Zadeh [3], Grattan-Guiness [4], and Jahn [5], vague set proposed by Gau and Buehrer [6], grey set [7, 8], intuitionistic fuzzy set proposed by Atanassov [9], and interval valued intuitionistic fuzzy set proposed by Atanassov and Gargov [10]. Wang et al [11] introduced single valued neutrosophic set (SVNS) to deal realistic problem. It has been studied and applied in different fields such as medical diagnosis problem [12], decision making problems [13], [14], [15], [16], [17], social problems [18], [19], Educational problems [20], [21], conflict resolution [22] and so on.

The notion of rough set theory [23] was proposed by Pawlak. The concept of rough set theory [1] is an extension of the crisp set theory for the study of intelligent systems characterized by inexact, uncertain or insufficient information. It is a useful tool for dealing with uncertainty or imprecision information.

Literature survey reflects that the rough set theory has caught a great deal of attention and interest among the researchers. The concept of rough neutrosophic sets [24, 25] is recently proposed and very interesting. While the concept of neutrosophic sets is a powerful logic to handle indeterminate and inconsistent situation, the theory of rough neutrosophic sets is also a powerful mathematical logic to handle incompleteness. The rating of all alternatives is expressed with the upper and lower approximation operator and the pair of neutrosophic sets which are characterized by truth-membership degree, indeterminacy-membership degree, and falsity-membership degree.

GLOBAL JOURNAL OF ADVANCED RESEARCH
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To measure the degree of similarity between neutrosophic sets, many methods have been proposed in the literature. Broumi and Smarandache [26] studied the Hausdorff distance between neutrosophic sets and some similarity measures based on the distance, set theoretic approach, and matching function to calculate the similarity degree between neutrosophic sets. In 2013, Broumi and Smarandache [27] also proposed the correlation coefficient between interval neutrosphic sets. Majumdar and Smanta [28] studied several similarity measures of single valued neutrosophic sets based on distances, a matching function, membership grades, and **then proposed an entropy measure for a SVNS. Ye [29] proposed the distance-based similarity measure of SVNSs and applied it** to the group decision making problems with single valued neutrosophic information. Ye [30] also proposed three vector similarity measure for SNSs, an instance of SVNS and interval valued neutrosophic set, including the Jaccard, Dice, and cosine similarity and applied them to multi-criteria decision-making problems with simplified neutrosophic information. Recently, Biswas et al. [31] studied cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers. Broumi and Smarandache [32] proposed a cosine similarity measure of interval valued neutrosophic sets.

Literature review reflects that only one study related to rough neutrosophic decision making is done by Mondal and Pramanik [17]. There is no investigation on the similarity measure of rough neutrosophic sets. Because of more complexity and uncertainty nature of the problems, it is necessary to utilize more flexible method which can deal uncertain situation easily. In this situation, rough neutrosophic logic is very useful. In this paper, we propose rough neutrosophic similarity measure and establish some of its properties. To demonstrate the applicability and effectiveness of the proposed similarity measure, a numerical example of medical diagnosis is provided.

This paper is organized as follow: In section 2, some basic definitions of neutrosophic set, single valued neutrosophic set, cosine similarity are presented briefly. In section 3, rough neutrosophic cosine similarity measure of rough neutrosophic sets and their basic properties are introduced. In section 4, the proposed similarity measure is applied to deal with the problem related to medical diagnosis. Section 5 presents the concluding remarks.

2. MATHEMATICAL PRELIMINARIES

2.1 Definitions on neutrosophic set [1]

Definition 2.1.1: Let H be a space of points (objects) with generic element in H denoted by x. Then a neutrosophic set $N1$ in H is characterized by a truth membership function T_{N1}, an indeterminacy membership function I_{N1} and a falsity membership function F_{N1}. The functions T_{N1}, I_{N1} ,and F_{N1} are real standard or non-standard subsets of $\left]^{-}0,1^{+}\right[$ that is T_{N1}: $H \to \left]^{-}0,1^{+}\right[$; I_{N1}: $H \to \left]^{-}0,1^{+}\right[$; F_{N1}: $H \to \left]^{-}0,1^{+}\right[$.

It should be noted that there is no restriction on the sum of $T_{N1}(x), I_{N1}(x), F_{N1}(x)$ i.e. $^{-}0 \le \sup T_{N1}(x) + \sup I_{N1}(x) + \sup F_{N1}(x) \le 3^{+}$

Definition 2.1.2: (complement) The complement of a neutrosophic set $N1$ is denoted by $N1^c$ and is defined by

$$T_{N1^c}(x) = \{1^+\} - T_{N1}(x);\ I_{N1^c}(x) = \{1^+\} - I_{N1}(x)$$

$$F_{N1^c}(x) = \{1^+\} - F_{N1}(x)$$

Definition 2.1.3: (Containment) A neutrosophic set $N1$ is contained in the other neutrosophic set $N2$, $N1 \subseteq N2$ if and only if the following results hold.

$$\inf T_{N1}(x) \le \inf T_{N2}(x),\ \sup T_{N1}(x) \le \sup T_{N2}(x)$$

$$\inf I_{N1}(x) \ge \inf I_{N2}(x),\ \sup I_{N1}(x) \ge \sup I_{N2}(x)$$

$$\inf F_{N1}(x) \ge \inf F_{N2}(x),\ \sup F_{N1}(x) \ge \sup F_{N2}(x)$$

for all x in H.

Definition 2.1.4: (Single-valued neutrosophic set). Let H be a universal space of points (objects) with a generic element of H denoted by x.

A single valued neutrosophic set [11] S is characterized by a truth membership function $T_N(x)$, a falsity membership function $F_N(x)$ and indeterminacy function $I_N(x)$ with $T_N(y)$, $F_N(x)$, $I_N(x) \in [0,1]$ for all x in H.

GLOBAL JOURNAL OF ADVANCED RESEARCH
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When H is continuous, a SNVS S can be written as follows:

$$S = \int_x \langle T_S(x), F_S(x), I_S(x) \rangle / x, \forall x \in H$$

and when H is discrete, a SVNS S can be written as follows:

$$S = \sum \langle T_S(x), F_S(x), I_S(x) \rangle / x, \forall x \in H$$

It should be observed that for a SVNS S,

$$0 \leq \sup T_S(x) + \sup F_S(x) + \sup I_S(x) \leq 3, \ \forall x \in H$$

Definition2.1.5: The complement of a single valued neutrosophic set S is denoted by S^c and is defined by

$$T_S{}^c(x) = F_S(x); \ I_S{}^c(x) = 1 - I_S x; \ F_S{}^c(x) = T_S(x)$$

Definition 2.1.6: A SVNS S_{N1} is contained in the other SVNS S_{N2}, denoted as $S_{N1} \subseteq S_{N2}$ iff, $T_{S_{N1}}(x) \leq T_{S_{N2}}(x)$; $I_{S_{N1}}(x) \geq I_{S_{N2}}(x)$; $F_{S_{N1}}(x) \geq F_{S_{N2}}(x)$, $\forall x \in H$.

Definition 2.1.7: Two single valued neutrosophic sets S_{N1} and S_{N2} are equal, i.e. $S_{N1} = S_{N2}$, iff, $S_{N1} \subseteq S_{N2}$ and $S_{N1} \supseteq S_{N2}$

Definition 2.1.8: (Union) The union of two SVNSs S_{N1} and S_{N2} is a SVNS S_{N3} , written as $S_{N3} = S_{N1} \cup S_{N2}$.

Its truth membership, indeterminacy-membership and falsity membership functions are related to S_{N1} and S_{N2} by the following equation

$$T_{S_{N3}}(x) = \max\left(T_{S_{N1}}(x), T_{S_{N2}}(x)\right);$$

$$I_{S_{N3}}(x) = \max\left(I_{S_{N1}}(x), I_{S_{N2}}(x)\right);$$

$$F_{S_{N3}}(x) = \min\left(F_{S_{N1}}(x), F_{S_{N2}}(x)\right) \text{ for all } x \text{ in } H.$$

Definition 2.1.9: (Intersection) The intersection of two SVNSs $N1$ and $N2$ is a SVNS $N3$, written as $N3 = N1 \cap N2$. Its truth membership, indeterminacy membership and falsity membership functions are related to $N1$ an $N2$ by the following equation

$$T_{S_{N3}}(x) = \min\left(T_{S_{N1}}(x), T_{S_{N2}}(x)\right);$$

$$I_{S_{N3}}(x) = \max\left(I_{S_{N1}}(x), I_{S_{N2}}(x)\right);$$

$$F_{S_{N3}}(x) = \max\left(F_{S_{N1}}(x), F_{S_{N2}}(x)\right), \forall x \in H$$

Distance between two neutrosophic sets.

The general SVNS can be presented in the follow form

$$S = \{(x/(T_S(x), I_S(x), F_S(x))) : x \in H\}$$

Finite SVNSs can be represented as follows:

$$S = \{(x_1/(T_S(x_1), I_S(x_1), F_S(x_1))), \cdots, (x_m/(T_S(x_m), I_S(x_m), F_S(x_m)))\}, \forall x \in H \tag{1}$$

Definition 2.1.10: Let

$$S_{N1} = \{(x_1/(T_{S_{N1}}(x_1), I_{S_{N1}}(x_1), F_{S_{N1}}(x_1))), \cdots, (x_n/(T_{S_{N1}}(x_n), I_{S_{N1}}(x_n), F_{S_{N1}}(x_n)))\} \tag{2}$$

$$S_{N2} = \{(x_1/(T_{S_{N2}}(x_1), I_{S_{N2}}(x_1), F_{S_{N2}}(x_1))), \cdots, (x_n/(T_{S_{N2}}(x_n), I_{S_{N2}}(x_n), F_{S_{N2}}(x_n)))\} \tag{3}$$

be two single-valued neutrosophic sets, then the Hamming distance [28]between two SNVS $N1$and $N2$ is defined as follows:

$$d_S(S_{N1}, S_{N2}) = \sum_{i=1}^{n} \left\langle \left|T_{S_{N1}}(x) - T_{S_{N2}}(x)\right| + \left|I_{S_{N1}}(x) - I_{S_{N2}}(x)\right| + \left|F_{S_{N1}}(x) - F_{S_{N2}}(x)\right| \right\rangle \tag{4}$$

and normalized Hamming distance between two SNVSs SN1 and SN2 is defined as follows:

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$$^{N}d_S(S_{N1}, S_{N2}) = \frac{1}{3n}\sum_{i=1}^{n}\left\langle \left|T_{S_{N1}}(x) - T_{S_{N2}}(x)\right| + \left|I_{S_{N1}}(x) - I_{S_{N2}}(x)\right| + \left|F_{S_{N1}}(x) - F_{S_{N2}}(x)\right| \right\rangle \quad (5)$$

with the following properties

1. $0 \le d_S(S_{N1}, S_{N2}) \le 3n$ (6)
2. $0 \le {}^{N}d_S(S_{N1}, S_{N2}) \le 1$ (7)

2.2 Definitions on rough neutrosophic set

Rough set theory consists of two basic components namely, crisp set and equivalence relation, which are the mathematical basis of RSs. The basic idea of rough set is based on the approximation of sets by a couple of sets known as the lower approximation and the upper approximation of a set. Here, the lower and upper approximation operators are based on equivalence relation. Rough neutrosophic sets [25, 26] are the generalization of rough fuzzy sets [33], [34], [35] and rough intuitionistic fuzzy sets [36].

Definition 2.2.1: Let *Y* be a non-null set and *R* be an equivalence relation on *Y*. Let P be neutrosophic set in *Y* with the membership function T_P, indeterminacy function I_P and non-membership function F_P. The lower and the upper approximations of *P* in the approximation (*Y, R*) denoted by $\underline{N}(P)$ and $\overline{N}(P)$ are respectively defined as follows:

$$\underline{N}(P) = \left\langle < x, T_{\underline{N}(P)}(x), I_{\underline{N}(P)}(x), F_{\underline{N}(P)}(x) > / Y \in [x]_R, x \in Y \right\rangle, \quad (8)$$

$$\overline{N}(P) = \left\langle < x, T_{\overline{N}(P)}(x), I_{\overline{N}(P)}(x), F_{\overline{N}(P)}(x) > / Y \in [x]_R, x \in Y \right\rangle \quad (9)$$

Where, $T_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, T_P(Y)$,

$I_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, I_P(Y)$, $F_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, F_P(Y)$,

$T_{\overline{N}(P)}(x) = \vee_Y \in [x]_R\, T_P(Y)$, $I_{\overline{N}(P)}(x) = \vee_Y \in [x]_R\, T_P(Y)$,

$F_{\overline{N}(P)}(x) = \vee_Y \in [x]_R\, I_P(Y)$

So, $0 \le T_{\underline{N}(P)}(x) + I_{\underline{N}(P)}(x) + F_{\underline{N}(P)}(x) \le 3$

$0 \le T_{\overline{N}(P)}(x) + I_{\overline{N}(P)}(x) + F_{\overline{N}(P)}(x) \le 3$

Where $\vee$ and $\wedge$ mean “max” and “min’’ operators respectively, $T_P(Y)$, $I_P(Y)$ and $F_P(Y)$ are the membership , indeterminacy and non-membership of Y with respect to *P*. It is easy to see that $\underline{N}(P)$ and $\overline{N}(P)$ is two neutrosophic sets in *Y*.

Thus NS mapping $\underline{N}$, $\overline{N}$: *N*(*Y*) $\longrightarrow$ *N*(*Y*) are, respectively, referred to as the lower and upper rough NS approximation operators, and the pair $(\underline{N}(P), \overline{N}(P))$ is called the rough neutrosophic set in (*Y, R*).

From the above definition, it is seen that $\underline{N}(P)$ and $\overline{N}(P)$ have constant membership on the equivalence classes of *R* if $\underline{N}(P) = \overline{N}(P)$; i.e.

$T_{\underline{N}(P)}(x) = T_{\overline{N}(P)}(x)$, $I_{\underline{N}(P)}(x) = I_{\overline{N}(P)}(x)$,

$F_{\underline{N}(P)}(x) = F_{\overline{N}(P)}(x)$

for any x belongs to *Y*.

P is said to be a definable neutrosophic set in the approximation (*Y, R*). It can be easily proved that Zero neutrosophic set (0_N) and unit neutrosophic sets (1_N) are definable neutrosophic sets.

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Definition 2.2.2 If $N(P) = (\underline{N}(P), \overline{N}(P))$ is a rough neutrosophic set in (Y, R), the rough complement of $N(P)$ is the rough neutrosophic set denoted $\sim N(P) = (\underline{N}(P)^c, \overline{N}(P)^c)$,where $\underline{N}(P)^c, \overline{N}(P)^c$ are the complements of neutrosophic sets of $\underline{N}(P), \overline{N}(P)$ respectively.

$\underline{N}(P)^c = \left\langle < x, T_{\underline{N}(P)}(x), 1 - I_{\underline{N}(P)}(x), F_{\underline{N}(P)}(x) > /, x \in Y \right\rangle$, and

$$\overline{N}(P)^c = \left\langle < x, T_{\underline{N}(P)}(x), 1 - I_{\overline{N}(P)}(x), F_{\overline{N}(P)}(x) > /, x \in Y \right\rangle \quad (10)$$

Definition 2.2.3. If $N(P_1)$ *and* $N(P_2)$ are two rough neutrosophic sets of the neutrosophic sets respectively in Y, then the following definitions holds [25, 26]:

$N(P_1) = N(P_2) \Leftrightarrow \underline{N}(P_1) = \underline{N}(P_2) \wedge \overline{N}(P_1) = \overline{N}(P_2)$

$N(P_1) \subseteq N(P_2) \Leftrightarrow \underline{N}(P_1) \subseteq \underline{N}(P_2) \wedge \overline{N}(P_1) \subseteq \overline{N}(P_2)$

$N(P_1) \bigcup N(P_2) = < \underline{N}(P_1) \bigcup \underline{N}(P_2), \overline{N}(P_1) \bigcup \overline{N}(P_2) >$

$N(P_1) \bigcap N(P_2) = < \underline{N}(P_1) \bigcap \underline{N}(P_2), \overline{N}(P_1) \bigcap \overline{N}(P_2) >$

$N(P_1) + N(P_2) = < \underline{N}(P_1) + \underline{N}(P_2), \overline{N}(P_1) + \overline{N}(P_2) >$

$N(P_1) . N(P_2) = < \underline{N}(P_1) . \underline{N}(P_2), \overline{N}(P_1) . \overline{N}(P_2) >$

If A, B, C are rough neutrosophic sets in (Y, R), then the following proposition are stated from definitions

Proposition 1:

1. $\sim A(\sim A) = A$
2. $A \bigcup B = B \bigcup A, A \bigcup B = B \bigcup A$
3. $(A \bigcup B) \bigcup C = A \bigcup (B \bigcup C)$,
   $(A \bigcap B) \bigcap C = A \bigcap (B \bigcap C)$
4. $(A \bigcup B) \bigcap C = (A \bigcup B) \bigcap (A \bigcup C)$,
   $(A \bigcap B) \bigcup C = (A \bigcap B) \bigcup (A \bigcap C)$

Proposition 2:

De Morgan‘s Laws are satisfied for rough neutrosophic sets

1. $\sim (N(P_1) \bigcup N(P_2)) = (\sim N(P_1)) \bigcap (\sim N(P_2))$
2. $\sim (N(P_1) \bigcap N(P_2)) = (\sim N(P_1)) \bigcup (\sim N(P_2))$

Proposition 3:

If P_1 and P_2 are two rough neutrosophic sets in U such that $P_1 \subseteq P_2$ *, then* $N(P_1) \subseteq N(P_2)$

1. $N(P_1 \bigcap P_2) \subseteq N(P_2) \bigcap N(P_2)$
2. $N(P_1 \bigcup P_2) \supseteq N(P_2) \bigcup N(P_2)$

Proposition 4:

1. $\underline{N}(P) = \sim \overline{N}(\sim P)$
2. $\overline{N}(P) = \sim \underline{N}(\sim P)$

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3. $\underline{N}(P) \subseteq \overline{N}(P)$

2.3 Cosine similarity function

Definition 2.3.1

A fundamental angle-based similarity measure between two vectors of n dimensions using the cosine of the angle between them is known as cosine similarity measure. It calculates the similarity between two vectors based on the direction, neglecting the impact of the distance between them. Given two attribute vectors $X = (x_1, x_2, \ldots, x_n)$ and $Y = (y_1, y_2, \ldots, y_n)$, the cosine similarity, *cosθ,* is presented as follows:

$$\cos\theta = \frac{\sum_{i=1}^{n} x_i\, y_i}{\sqrt{\sum_{i=1}^{n} x_i^2}\sqrt{\sum_{i=1}^{n} y_i^2}}$$

A cosine similarity measure based on Bhattacharya's distance [37] between two fuzzy set $\mu_A(x_i)$ and $\mu_B(x_i)$ are defined as follows:

$$C_F(A,B) = \frac{\sum_{i=1}^{n} \mu_A(x_i)\,\mu_B(x_i)}{\sqrt{\sum_{i=1}^{n}(\mu_A(x_i))^2}\sqrt{\sum_{i=1}^{n}(\mu_B(x_i))^2}}$$

In 2D vector space, a cosine similarity measure between two intuitionistic fuzzy sets proposed by Ye [38] is as follows:

$$C_{IFS}(A,B) = \frac{\sum_{i=1}^{n}\left[\mu_A(x_i)\,\mu_B(x_i) + \nu_A(x_i)\,\nu_B(x_i)\right]}{\sqrt{\sum_{i=1}^{n}(\mu_A(x_i))^2 + (\nu_A(x_i))^2}\sqrt{\sum_{i=1}^{n}(\mu_B(x_i))^2 + (\nu_B(x_i))^2}}$$

Where all vectors lies between 0 and 1.

3. Cosine similarity measure of rough neutrosophic sets

The cosine similarity measure is calculated as the inner product of two vectors divided by the product of their lengths. It is the cosine of the angle between the vector representations of two rough neutrosophic sets. The cosine similarity measure is a fundamental measure used in information technology. Existing cosine similarity measures does not deal with rough neutrosophic sets till now. Therefore, a new cosine similarity measure between rough neutrosophic sets is proposed in 3-D vector space.

Definition 3.1: Assume that there are two rough neutrosophic sets $A = \left\langle \left(\underline{T}_A(x_i), \underline{I}_A(x_i), \underline{F}_A(x_i)\right), \left(\overline{T}_A(x_i), \overline{I}_A(x_i), \overline{F}_A(x_i)\right) \right\rangle$ and $B = \left\langle \left(\underline{T}_B(x_i), \underline{I}_B(x_i), \underline{F}_B(x_i)\right), \left(\overline{T}_B(x_i), \overline{I}_B(x_i), \overline{F}_B(x_i)\right) \right\rangle$ in $X = \{x_1, x_2, \ldots, x_n\}$. A cosine similarity measure between rough neutrosophic sets A and B is proposed as follows:

$$C_{RNS}(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{\delta T_A(x_i)\,\delta T_B(x_i) + \delta I_A(x_i)\,\delta I_B(x_i) + \delta F_A(x_i)\,\delta F_B(x_i)}{\sqrt{(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2}\sqrt{(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2}}$$

Where, $\delta T_A(x_i) = \left(\dfrac{\underline{T}_A(x_i) + \overline{T}_A(x_i)}{2}\right)$, $\delta T_B(x_i) = \left(\dfrac{\underline{T}_B(x_i) + \overline{T}_B(x_i)}{2}\right)$, $\delta I_A(x_i) = \left(\dfrac{\underline{I}_A(x_i) + \overline{I}_A(x_i)}{2}\right)$,

$\delta I_B(x_i) = \left(\dfrac{\underline{I}_B(x_i) + \overline{I}_B(x_i)}{2}\right)$, $\delta F_A(x_i) = \left(\dfrac{\underline{F}_A(x_i) + \overline{F}_A(x_i)}{2}\right)$, $\delta F_B(x_i) = \left(\dfrac{\underline{F}_B(x_i) + \overline{F}_B(x_i)}{2}\right)$.

Proposition 5

Let A and B be rough neutrosophic sets then

1. $0 \le C_{RNS}(A,B) \le 1$
2. $C_{RNS}(A,B) = C_{RNS}(B,A)$
3. $C_{RNS}(A, B) = 1$, iff $A = B$
4. If C is a RNS in Y and $A \subset B \subset C$ then, $C_{RNS}(A, C) \le C_{RNS}(A, B)$, and $C_{RNS}(A, C) \le C_{RNS}(B, C)$

Proofs :

1. It is obvious because all positive values of cosine function are within 0 and 1.
2. It is obvious that the proposition is true.
3. When $A = B$, then obviously $C_{RNS}(A, B) = 1$. On the other hand if $C_{RNS}(A, B) =1$ then,

$\delta T_A(x_i) = \delta T_B(x_i)$, $\delta I_A(x_i) = \delta I_B(x_i)$, $\delta F_A(x_i) = \delta F_B(x_i)$ *ie,*

$\underline{T}_A(x_i) = \underline{T}_B(x_i)$, $\overline{T}_A(x_i) = \overline{T}_B(x_i)$, $\underline{I}_A(x_i) = \underline{I}_B(x_i)$, $\overline{I}_A(x_i) = \overline{I}_B(x_i)$, $\underline{F}_A(x_i) = \underline{F}_B(x_i)$, $\overline{F}_A(x_i) = \overline{F}_B(x_i)$

This implies that A = B.

GLOBAL JOURNAL OF ADVANCED RESEARCH
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4. If $A \subset B \subset C$ then we can write $\underline{T}_A(x_i) \le \underline{T}_B(x_i) \le \underline{T}_C(x_i)$, $\overline{T}_A(x_i) \le \overline{T}_B(x_i) \le \overline{T}_C(x_i)$, $\underline{I}_A(x_i) \ge \underline{I}_B(x_i) \ge \underline{I}_C(x_i)$, $\overline{I}_A(x_i) \ge \overline{I}_B(x_i) \ge \overline{I}_B(x_i)$, $\underline{F}_A(x_i) \ge \underline{F}_B(x_i) \ge \underline{F}_C(x_i)$, $\overline{F}_A(x_i) \ge \overline{F}_B(x_i) \ge \overline{F}_C(x_i)$.

The cosine function is decreasing function within the interval $\left[0, \frac{\pi}{2}\right]$. Hence we can write $C_{RNS}(A, C) \le C_{RNS}(A, B)$, and $C_{RNS}(A, C) \le C_{RNS}(B, C)$.

If we consider the weights of each element x_i, a weighted rough cosine similarity measure between rough neutrosophic sets *A* and *B* can be defined as follows:

$$C_{WRNS}(A,B) = \frac{1}{n}\sum_{i=1}^{n} w_i \frac{\delta T_A(x_i)\,\delta T_B(x_i) + \delta I_A(x_i)\,\delta I_B(x_i) + \delta F_A(x_i)\,\delta F_B(x_i)}{\sqrt{(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2}\,\sqrt{(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2}}$$

$w_i \in [0,1]$, i = 1, 2,…, n and $\sum_{i=1}^{n} w_i = 1$. If we take $w_i = \frac{1}{n}$, i = 1, 2,…, n, then $C_{WRNS}(A, B) = C_{RNS}(A, B)$

The weighted rough cosine similarity measure (WRNS) between two rough neutrosophic sets *A* and *B* also satisfies the following properties:

1. $0 \le C_{WRNS}(A,B) \le 1$
2. $C_{WRNS}(A,B) = C_{WRNS}(B,A)$
3. $C_{WRNS}(A, B) = 1$, iff $A = B$
4. If C is a WRNS in Y and $A \subset B \subset C$ then, $C_{WRNS}(A, C) \le C_{WRNS}(A, B)$, and $C_{WRNS}(A, C) \le C_{WRNS}(B, C)$

Proof :

The proofs of above properties are similar proofs of the propositions (5).

4 EXAMPLES ON MEDICAL DIGNOSIS

We consider a medical diagnosis problem from practical point of view for illustration of the proposed approach. Medical diagnosis comprises of uncertainties and increased volume of information available to physicians from new medical technologies. The process of classifying different set of symptoms under a single name of a disease is very difficult task. In some practical situations, there exists possibility of each element within a lower and an upper approximation of neutrosophic sets. It can deal with the medical diagnosis involving more indeterminacy. Actually this approach is more flexible and easy to use. The proposed similarity measure among the patients versus symptoms and symptoms versus diseases will provide the proper medical diagnosis. The main feature of this proposed approach is that it considers truth membership, indeterminate and false membership of each element between two approximations of neutrosophic sets by taking one time inspection for diagnosis.

Now, an example of a medical diagnosis is presented. Let P = {P_1, P_2, P_3} be a set of patients, D = {Viral Fever, Malaria, Stomach problem, Chest problem} be a set of diseases and S = {Temperature, Headache, Stomach pain, cough, Chest pain.} be a set of symptoms. Our solution is to examine the patient and to determine the disease of the patient in rough neutrosophic environment.

Table 1: (Relation-1) The relation between Patients and Symptoms

Relation-1	Temperature	Headache	Stomach pain	Cough	Chest pain
P_1	$\langle (0.6,0.4,0.3), (0.8,0.2,0.1) \rangle$	$\langle (0.4,0.4,0.4), (0.6,0.2,0.2) \rangle$	$\langle (0.5,0.3,0.2), (0.7,0.1,0.2) \rangle$	$\langle (0.6,0.2,0.4), (0.8,0.0,0.2) \rangle$	$\langle (0.4,0.4,0.4), (0.6,0.2,0.2) \rangle$
P_2	$\langle (0.5,0.3,0.4), (0.7,0.3,0.2) \rangle$	$\langle (0.5,0.5,0.3), (0.7,0.3,0.3) \rangle$	$\langle (0.5,0.3,0.4), (0.7,0.1,0.4) \rangle$	$\langle (0.5,0.3,0.3), (0.9,0.1,0.3) \rangle$	$\langle (0.5,0.3,0.3), (0.7,0.1,0.3) \rangle$
P_3	$\langle (0.6,0.4,0.4), (0.8,0.2,0.2) \rangle$	$\langle (0.5,0.2,0.3), (0.7,0.0,0.1) \rangle$	$\langle (0.4,0.3,0.4), (0.8,0.1,0.2) \rangle$	$\langle (0.6,0.1,0.4), (0.8,0.1,0.2) \rangle$	$\langle (0.5,0.3,0.3), (0.7,0.1,0.1) \rangle$

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Table 2: (Relation-2) The relation among Symptoms and Diseases

Relation-2	Viral Fever	Malaria	Stomach problem	Chest problem
Temperature	$\left\langle \begin{matrix}(0.6,0.5,0.4),\\(0.8,0.3,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.1,0.4,0.4),\\(0.5,0.2,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.3,0.4,0.4),\\(0.5,0.2,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.2,0.4,0.6),\\(0.4,0.4,0.4)\end{matrix}\right\rangle$
Headache	$\left\langle \begin{matrix}(0.5,0.3,0.4),\\(0.7,0.3,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.2,0.3,0.4),\\(0.6,0.3,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.2,0.3,0.3),\\(0.4,0.1,0.1)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.1,0.5,0.5),\\(0.5,0.3,0.3)\end{matrix}\right\rangle$
Stomach pain	$\left\langle \begin{matrix}(0.2,0.3,0.4),\\(0.4,0.3,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.1,0.4,0.4),\\(0.3,0.2,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.4,0.3,0.4),\\(0.6,0.1,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.1,0.4,0.6),\\(0.3,0.2,0.4)\end{matrix}\right\rangle$
Cough	$\left\langle \begin{matrix}(0.4,0.3,0.3),\\(0.6,0.1,0.1)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.3,0.3,0.3),\\(0.5,0.1,0.3)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.1,0.6,0.6),\\(0.3,0.4,0.4)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.5,0.3,0.4),\\(0.7,0.1,0.2)\end{matrix}\right\rangle$
Chest pain	$\left\langle \begin{matrix}(0.2,0.4,0.4),\\(0.4,0.2,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.1,0.3,0.3),\\(0.3,0.1,0.1)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.1,0.4,0.4),\\(0.3,0.2,0.2)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.4,0.4,0.4),\\(0.6,0.2,0.2)\end{matrix}\right\rangle$

Table 3: The Correlation Measure between Relation-1 and Relation-2

Rough cosine similarity measure	Viral Fever	Malaria	Stomach problem	Chest problem
P_1	**0.9595**	0.9114	0.8498	0.8743
P_2	**0.9624**	0.9320	0.8935	0.8307
P_3	**0.9405**	0.8873	0.8487	0.8372

The highest correlation measure (see the Table 3) reflects the proper medical diagnosis. Therefore, all three patients P_1, P_2, P_3 suffer from viral fever.

5. CONCLUSION

In this paper, we have proposed rough cosine similarity measure of rough neutrosophic sets and proved some of their basic properties. We have presented an application of rough cosine similarity measure of rough neutrosophic sets in medical diagnosis problems. The authors hope that the proposed concept can be applied in solving realistic multi-criteria decision making problems.

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