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Cosine similarity measures of neutrosophic soft set

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Abstract. In this paper we have introduced the concept of cosine similarity measures for neutrosophic soft set and interval valued neutrosophic soft set.An application is given to show its practicality and effectiveness.

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1. Introduction

Neutrosophy is a branch of philosophy, which emphasizes the origin and nature of neutralities, along with their interaction with different conceptive domains. Fuzzy logic extends classical logic by assigning a membership function ranging in degree between 0 and 1 to the variables. As a generalization of fuzzy logic, neutrosophic logic introduces a new component called indeterminacy and carries more information than fuzzy logic. The application of neutrosophic logic would lead to better performance than fuzzy logic. Neutrosophic logic is so new that its use in many fields merit exploration.The words "neutrosophy" and "neutrosophic" were introduced by F. Smarandache in his 1998 book[23],. Etymologically, "neutro-sophy" (noun) [French *neutre* < Latin *neuter*, neutral, and Greek *sophia*, skill/wisdom] means knowledge of neutral thought. Neutrosophic set [24],[25] is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of]-0 , 1+[. A.A.Salama [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22] studied the various notions of neutrosophic sets.

In several application it is often needed to compare two sets and we are interested to know whether two patterns or images are identical or approximately identical of atleast to what degree they are identical. Several researchers like Hung

[3], Jun Ye [4, 5], P.Majumdar [6], C.Wang[26] and many authors have studied the problem of similarity measures between intuitionistic fuzzy sets, neutrosophic sets and vague soft sets. In this paper we have introduced some new cosine similarity measures for neutrosophic soft sets and derived some of their properties. A decision making method based on this similarity measure is constructed.[7], [8], [9]

2. Preliminaries

Definition 2.1 ([1])**.** A neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$,$x \in X$ where $T, I, F : X \to [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$, where T represents the truth value , I represents the indeterministic value and F represents the false value.

Definition 2.2. [1] Let X be a non empty set, and let $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ and $B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be neutrosophic sets. Then A is a subset of B if ∀ x ∈ X, $T_A(x) \leq T_B(x), I_A(x) \leq I_B(x)$ and $F_A(x) \geq F_B(x)$.

Definition 2.3 ([1])**.** Let X be a non empty set, and let $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ and $B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be neutrosophic sets. Then

$$A \cup B = < x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)), min(F_A(x), F_B(x)) >,$$
$$A \cap B = < x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)), max(F_A(x), F_B(x)) >.$$

Definition 2.4 ([1])**.** Let U be a the initial universal set and E be a set of parameters. Consider a non-empty set A, A ⊆ E. Let P(U) denote the set of all neutrosophic sets of U. The collection (F, A) is termed to be the neutrosophic soft set over U, where F is a mapping given by F:A⟶ P(U).(Neutrosophic soft set is denoted by NSS).

Definition 2.5 ([1])**.** A neutrosophic soft set (F,A) over the universe U is said to be empty neutrosophic soft set with respect to the parameter A if $T_{F(e)} = 0$, $I_{F(e)} = 0$,$F_{F(e)} = 1$, ∀ x ∈ U, ∀ e ∈ A. It is denoted by $\widetilde{0}$.

Definition 2.6 ([1])**.** A neutrosophic soft set (F,A) over the universe U is said to be universe neutrosophic soft set with respect to the parameter A if $T_{F(e)} = 1$, $I_{F(e)} = 1$, $F_{F(e)} = 0$, ∀ x ∈ U, ∀ e ∈ A. It is denoted by $\widetilde{1}$.

Definition 2.7 ([1])**.** A neutrosophic soft set (F,A) is said to be a subset of neutrosophic soft set (G,B) if A⊆ B and F(e)⊆ G(e) ∀ e ∈ E, u ∈ U. We denote it by (F,A)$\widetilde{\subseteq}$ (G,B).

Definition 2.8 ([1])**.** The complement of neutrosophic soft set (F,A) denoted by $(F, A)^c$ and is defined as $(F, A)^c = (F^c, \neg A)$ where $F^c : \neg A \longrightarrow P(U)$ is a mapping given by

$$F^c(\alpha) = < x, T_{F^c}(x) = F_F(x), I_{F^c}(x) = 1 - I_F(x), F_{F^c}(x) = T_F(x) >.$$

Definition 2.9. [1] The union of two neutrosophic soft sets (F,A)and (G,B) over (U,E) is neutrosophic soft set where C = A∪B, $\forall e \in C$

$$H(e) = \begin{cases} F(e) & \text{if e} \in A - B \\ G(e) & \text{if e} \in B - A \\ F(e) \cup G(e) & \text{if e} \in A \cap B \end{cases}$$

and is written as (F,A)$\widetilde{\cup}$(G,B)= (H,C).

Definition 2.10 ([1]). The intersection of two neutrosophic soft sets (F,A) and (G,B) over (U,E) is neutrosophic soft set where C = A∩B ,$\forall e \in C$ $H(e) = F(e) \cap G(e)$ and is written as (F,A)$\widetilde{\cap}$(G,B)= (H,C).

Definition 2.11 ([2]). An interval valued neutrosophic set (IVNS in short) on a universe X is an object of the form, where $T_A(x) = X \to Int([0,1])$, $I_A(x) = X \to Int([0,1])$ and $F_A(x) = X \to Int([0,1])$, Int([0,1]) stands for the set of all closed subinterval of [0,1] satisfies the condition : $\forall x \in X$,

$$supT_A(x) + supI_A(x) + supF_A(x) \leq 3.$$

Definition 2.12 ([2]). For an arbitrary set $A \subseteq [0,1]$, we define $\underline{A} = infA$ and $\overline{A} = supA$.

Definition 2.13 ([2]). Let U be an initial universe and E be a set of parameters. IVNS (U) denotes the set of all interval valued neutrosophic sets of U. Let A ⊆E. A pair (F,A) is an interval valued neutrosophic soft set over U, where F is a mapping given by F: A → IVNS(U).
Note: Interval valued neutrosophic soft set/sets is denoted by IVNSS/IVNSSs.

Definition 2.14 ([2]). Let U be an initial universe and E be a set of parameters. Suppose that A,B ⊆E, (F,A) and (G,B) be two IVNSSs, we say that (F,A) is an interval valued neutrosophic soft subset of (G,B) if and only if
(i) A ⊆ B,
(ii) e ∈A, F(e) is an interval valued neutrosophic soft subset of G(e), that is, for all x ∈U and e ∈ A,
$\underline{T}_{F(e)}(x) \leq \underline{T}_{G(e)}(x)$, $\overline{T}_{F(e)}(x) \leq \overline{T}_{G(e)}(x)$,
$\underline{I}_{F(e)}(x) \leq \underline{I}_{G(e)}(x)$, $\overline{I}_{F(e)}(x) \leq \overline{I}_{G(e)}(x)$,
$\underline{F}_{F(e)}(x) \geq \underline{F}_{G(e)}(x)$, $\overline{F}_{F(e)}(x) \geq \overline{F}_{G(e)}(x)$.
And it is denoted by (F,A) ⊆ (G,B).

Similarly (F,A) is said to be an interval valued neutrosophic soft super set of (G,B), if (G,B) is an interval valued neutrosophic soft subset of (F,A), we denote it by (F,A)⊇ (G,B).

Definition 2.15 ([2]). The complement of an IVNSS (F,A) is denoted by $(F,A)^c$ and is defined as $(F,A)^c = (F^c, \neg A)$ where $F^c : \neg A \to IVNSS(U)$ is a mapping given by

$$F^c(e) =< F_{F(\neg e)}(x), (I_{F(\neg e)}(x))^c, T_{F(\neg e)}(x) \text{ for all } x \in U \text{ and } e \in \neg A,$$

where $(I_{F(\neg e)}(x))^c = [1 - \underline{I}_{F(e)}(x), 1 - \overline{I}_{F(e)}(x)$.

3. Cosine similarity measure neutrosophic soft set and interval valued neutrosophic soft set

Definition 3.1. Let U= $\{x_1, x_2, ...x_n\}$ be the universal set of elements and E = $\{e_1, e_2, ...e_m\}$ be the universal set of parameters. Let (F,A) and (G,B) be two neutrosophic soft sets over U. Then we define the cosine similarity measures between (F,A) and (G,B) as follows.

$$CS_1((F,A),(G,B)) = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} cos\left[\frac{\pi}{2}(|T_{F(e_i)}(x_j) - T_{G(e_i)}(x_j)|) \vee (|I_{F(e_i)}(x_j) - I_{G(e_i)}(x_j)|) \vee (|F_{F(e_i)}(x_j) - F_{G(e_i)}(x_j)|)\right],$$

$$CS_2((F,A),(G,B)) = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} cos\left[\frac{\pi}{6}(|T_{F(e_i)}(x_j) - T_{G(e_i)}(x_j)|) + (|I_{F(e_i)}(x_j) - I_{G(e_i)}(x_j)|) + (|F_{F(e_i)}(x_j) - F_{G(e_i)}(x_j)|)\right],$$

where the symbol $\vee$ is the maximum operation. The two similarity measures satisfy the axiomatic requirements of similarity measures.

Proposition 3.2. *Let U= $\{x_1, x_2, ...x_n\}$ be the universal set of elements and E = $\{e_1, e_2, ...e_m\}$ be the universal set of parameters. Then for two neutrosophic soft sets (F,A) and (G,B) over U the cosine similarity measure $CS_k((F,A),(G,B))$ (k=1,2)should satisfy the following properties (1)–(4).*

(1) $0 \leq CS_k((F,A),(G,B)) \leq 1$.
(2) $CS_k((F,A),(G,B)) = 1$ *if and only if (F,A)=(G,B).*
(3) $CS_k((F,A),(G,B)) = CS_k((G,B)(F,A))$.
(4) *If (H,C) is a neutrosophic soft set over U and (F,A) ⊆(G,B)⊆(H,C), then*

$$CS_k((F,A),(H,C)) \leq CS_k((F,A)(G,B))$$

and

$$CS_k((F,A),(H,C)) \leq CS_k((G,B),(H,C)).$$

Proof. (1) Since the truth membership degree , indeterminacy- membership degreee and falsity- membership degree in neutrosophic set and the value of cosine function are within [0,1], the similarity measure based on cosine function is also within [0,1]. Thus $0 \leq CS_k((F,A),(G,B)) \leq 1$ for k = 1, 2.

(2) For any two neutrosophic soft sets, (F,A) = (G,B) implies

$$T_{F(e_i)}(x_j) = T_{G(e_i)}(x_j),\ I_{F(e_i)}(x_j) = I_{G(e_i)}(x_j),\ F_{F(e_i)}(x_j) = F_{G(e_i)}(x_j)$$

for all i =1,2,...m and j=1,2,...n. Thus
$|T_{F(e_i)}(x_j) - T_{G(e_i)}(x_j)| = |I_{F(e_i)}(x_j) - I_{G(e_i)}(x_j)| = |F_{F(e_i)}(x_j) - F_{G(e_i)}(x_j)| = 0$.
So $CS_k((F,A),(G,B)) = 1$ for k =1, 2.

Conversely if $CS_k((F,A),(G,B)) = 1$ for k =1, 2, this implies

$$|T_{F(e_i)}(x_j)-T_{G(e_i)}(x_j)| = 0, |I_{F(e_i)}(x_j)-I_{G(e_i)}(x_j)| = 0, |F_{F(e_i)}(x_j)-F_{G(e_i)}(x_j)| = 0$$

since cos(0)=1 . Then these equalities indicate

$$T_{F(e_i)}(x_j) = T_{G(e_i)}(x_j),\ I_{F(e_i)}(x_j) = I_{G(e_i)}(x_j), F_{F(e_i)}(x_j) = F_{G(e_i)}(x_j)$$

for all i =1,2,...m and j=1,2,...n. Hence (F,A)= (G,B).

(3) Clearly $CS_k((F,A),(G,B)) = CS_k((G,B)(F,A))$.

(4) If (F,A)⊆(G,B)⊆(H,C), then

$$T_{F(e_i)}(x_j) \leq T_{G(e_i)}(x_j) \leq T_{H(e_i)}(x_j),$$

$$I_{F(e_i)}(x_j) \leq I_{G(e_i)}(x_j) \leq I_{G(e_i)}(x_j),$$

$$F_{F(e_i)}(x_j) \geq F_{G(e_i)}(x_j) \geq F_{H(e_i)}(x_j)$$

for all i =1,2,...m and j=1,2,...n.
Then we have the following inequalites :

$$|T_{F(e_i)}(x_j) - T_{G(e_i)}(x_j)| \leq |T_{F(e_i)}(x_j) - T_{H(e_i)}(x_j)|,$$

$$|T_{G(e_i)}(x_j) - T_{H(e_i)}(x_j)| \leq |T_{F(e_i)}(x_j) - T_{H(e_i)}(x_j)|,$$

$$|I_{F(e_i)}(x_j) - I_{G(e_i)}(x_j)| \leq |I_{F(e_i)}(x_j) - I_{H(e_i)}(x_j)|,$$

$$|I_{G(e_i)}(x_j) - I_{H(e_i)}(x_j)| \leq |I_{F(e_i)}(x_j) - I_{H(e_i)}(x_j)|,$$

$$|F_{F(e_i)}(x_j) - F_{G(e_i)}(x_j)| \leq |F_{F(e_i)}(x_j) - F_{H(e_i)}(x_j)|,$$

$$|F_{G(e_i)}(x_j) - F_{H(e_i)}(x_j)| \leq |F_{F(e_i)}(x_j) - F_{H(e_i)}(x_j)|.$$

Thus

$$CS_k((F,A),(H,C))) \leq CS_k((F,A),(G,B)))$$

and

$$CS_k((F,A),(H,C))) \leq CS_k((G,B),(H,C)))$$

for k = 1,2, since cosine function is a decreasing function within the interval $\left[0, \frac{\pi}{2}\right]$. □

Definition 3.3. Let U= $\{x_1, x_2, ...x_n\}$ be the universal set of elements and E = $\{e_1, e_2, ...e_m\}$ be the universal set of parameters and (F,A) and (G,B) be two neutrosophic soft sets over U. Now we consider the weights w_j of x_j(j=1,2,...n) then the weighted cosine similarity measures between (F,A) and (G,B) is defined as follows. $WCS_1((F,A),(G,B))$

$$= \frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{n} w_j cos\left[\frac{\pi}{2}(|T_{F(e_i)}(x_j) - T_{G(e_i)}(x_j)|) \vee (|I_{F(e_i)}(x_j) - I_{G(e_i)}(x_j)|) \vee (|F_{F(e_i)}(x_j) - F_{G(e_i)}(x_j)|)\right],$$

$WCS_2((F,A),(G,B))$

$$= \frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{n} w_j cos\left[\frac{\pi}{6}(|T_{F(e_i)}(x_j) - T_{G(e_i)}(x_j)|) + (|I_{F(e_i)}(x_j) - I_{G(e_i)}(x_j)|) + (|F_{F(e_i)}(x_j) - F_{G(e_i)}(x_j)|)\right]$$

where $w_j \in [0,1]$, j=1,2,...n and $\sum_{j=1}^{n} w_j = 1$.

In particular if we take $w_j = \frac{1}{n}$, j=1,2,3....n, then

$$WCS_k((F,A),(G,B)) = CS_k((F,A),(G,B)),$$

k=1, 2.

Definition 3.4. Let U= $\{x_1, x_2, ...x_n\}$ be the universal set of elements and E = $\{e_1, e_2, ...e_m\}$ be the universal set of parameters,and (F,A) and (G,B) be two interval valued neutrosophic soft sets over U. Then the cosine similarity measures between (F,A) and (G.B) is defined as

$CS_3((F,A),(G,B))$
$= \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} cos\Big[\frac{\pi}{4}((|\underline{T}_{F(e_i)}(x_j) - \underline{T}_{G(e_i)}(x_j)|) \vee (|\underline{I}_{F(e_i)}(x_j) - \underline{I}_{G(e_i)}(x_j)|) \vee$
$(|\underline{F}_{F(e_i)}(x_j) - \underline{F}_{G(e_i)}(x_j)|) + (|\overline{T}_{F(e_i)}(x_j) - \overline{T}_{G(e_i)}(x_j)|) \vee$
$(|\overline{I}_{F(e_i)}(x_j) - \overline{I}_{G(e_i)}(x_j)|) \vee (|\overline{F}_{F(e_i)}(x_j) - \overline{F}_{G(e_i)}(x_j)|))\Big].$
$CS_4((F,A),(G,B))$
$= \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} cos\Big[\frac{\pi}{12}((|\underline{T}_{F(e_i)}(x_j) - \underline{T}_{G(e_i)}(x_j)|) + (|\underline{I}_{F(e_i)}(x_j) - \underline{I}_{G(e_i)}(x_j)|) +$
$(|\underline{F}_{F(e_i)}(x_j) - \underline{F}_{G(e_i)}(x_j)|) + (|\overline{T}_{F(e_i)}(x_j) - \overline{T}_{G(e_i)}(x_j)|) +$
$(|\overline{I}_{F(e_i)}(x_j) - \overline{I}_{G(e_i)}(x_j)|) + (|\overline{F}_{F(e_i)}(x_j) - \overline{F}_{G(e_i)}(x_j)|))\Big]$
where the symbol $\vee$ is the maximum operation. The cosine similarity measures $CS_k((F,A),(G,B))$, k = 3, 4 satisfy the following properties.

Proposition 3.5. *Let U=* $\{x_1, x_2, ...x_n\}$ *be the universal set of elements and E =* $\{e_1, e_2, ...e_m\}$ *be the universal set of parameters. Then for two interval valued neutrosophic soft sets (F,A) and (G,B) over U the cosine similarity measure* $CS_k((F,A),(G,B))$ *(k=3,4)should satisfy the following properties (1)–(4).*
(1) $0 \leq CS_k((F,A),(G,B)) \leq 1$.
(2) $CS_k((F,A),(G,B)) = 1$ *if and only if (F,A)=(G,B).*
(3) $CS_k((F,A),(G,B)) = CS_k((G,B)(F,A))$.
(4) *If (H,C) is a interval valued neutrosophic soft set over U and (F,A)⊆(G,B)⊆(H,C), then*

$$CS_k((F,A),(H,C)) \leq CS_k((F,A)(G,B))$$

and

$$CS_k((F,A),(H,C)) \leq CS_k((G,B),(H,C)).$$

Proof. (1) Since the truth membership degree, indeterminacy- membership degreee and falsity- membership degree in neutrosophic set and the value of cosine function are within [0,1], the similarity measure based on cosine function is also within [0,1]. Then $0 \leq CS_k((F,A),(G,B)) \leq 1$ for k = 3, 4.

(2) For any two interval valued neutrosophic soft sets, (F,A) = (G,B) implies

$$T_{F(e_i)}(x_j) = T_{G(e_i)}(x_j),\ I_{F(e_i)}(x_j) = I_{G(e_i)}(x_j), F_{F(e_i)}(x_j) = F_{G(e_i)}(x_j)$$

for all i =1,2,...m and j=1,2,...n. Then

$$|\underline{T}_{F(e_i)}(x_j) - \underline{T}_{G(e_i)}(x_j)| = |\underline{I}_{F(e_i)}(x_j) - \underline{I}_{G(e_i)}(x_j)| = |\underline{F}_{F(e_i)}(x_j) - \underline{F}_{G(e_i)}(x_j)| = 0$$

and

$$|\overline{T}_{F(e_i)}(x_j) - \overline{T}_{G(e_i)}(x_j)| = |\overline{I}_{F(e_i)}(x_j) - \overline{I}_{G(e_i)}(x_j)| = |\overline{F}_{F(e_i)}(x_j) - \overline{F}_{G(e_i)}(x_j)| = 0.$$

Thus $CS_k((F,A),(G,B)) = 1$ for k =3, 4.

Conversely if $CS_k((F,A),(G,B)) = 1$ for k =3, 4, then, since cos(0)=1,

$$|\underline{T}_{F(e_i)}(x_j) - \underline{T}_{G(e_i)}(x_j)| = 0,$$

$$|\underline{I}_{F(e_i)}(x_j) - \underline{I}_{G(e_i)}(x_j)| = 0,$$

$$|\underline{F}_{F(e_i)}(x_j) - \underline{F}_{G(e_i)}(x_j)| = 0,$$

$|\overline{T}_{F(e_i)}(x_j) - \overline{T}_{G(e_i)}(x_j)| = |\overline{I}_{F(e_i)}(x_j) - \overline{I}_{G(e_i)}(x_j)| = |\overline{F}_{F(e_i)}(x_j) - \overline{F}_{G(e_i)}(x_j)| = 0.$

Then these equalities indicate

$$T_{F(e_i)}(x_j) = T_{G(e_i)}(x_j),\ I_{F(e_i)}(x_j) = I_{G(e_i)}(x_j),\ F_{F(e_i)}(x_j) = F_{G(e_i)}(x_j)$$

for all i =1,2,...m and j=1,2,...n. Thus (F,A)= (G,B).

(3) Clearly $CS_k((F,A),(G,B)) = CS_k((G,B)(F,A))$.

(4) If (F,A)⊆(G,B)⊆(H,C), then

$$\underline{T}_{F(e_i)}(x_j) \le \underline{T}_{G(e_i)}(x_j) \le \underline{T}_{H(e_i)}(x_j),$$
$$\overline{T}_{F(e_i)}(x_j) \le \overline{T}_{G(e_i)}(x_j) \le \overline{T}_{H(e_i)}(x_j),$$
$$\underline{I}_{F(e_i)}(x_j) \le \underline{I}_{G(e_i)}(x_j) \le \underline{I}_{H(e_i)}(x_j),$$
$$\overline{I}_{F(e_i)}(x_j) \le \overline{I}_{G(e_i)}(x_j) \le \overline{I}_{H(e_i)}(x_j),$$
$$\underline{F}_{F(e_i)}(x_j) \ge \underline{F}_{G(e_i)}(x_j) \ge \underline{F}_{H(e_i)}(x_j),$$
$$\overline{F}_{F(e_i)}(x_j) \ge \overline{F}_{G(e_i)}(x_j) \ge \overline{F}_{H(e_i)}(x_j),$$

for all i =1,2,...m and j=1,2,...n. Thus we have the following inequalites :

$$|\underline{T}_{F(e_i)}(x_j) - \underline{T}_{G(e_i)}(x_j)| \le |\underline{T}_{F(e_i)}(x_j) - \underline{T}_{H(e_i)}(x_j)|,$$
$$|\underline{T}_{G(e_i)}(x_j) - \underline{T}_{H(e_i)}(x_j)| \le |\underline{T}_{F(e_i)}(x_j) - \underline{T}_{H(e_i)}(x_j)|,$$
$$|\overline{T}_{F(e_i)}(x_j) - \overline{T}_{G(e_i)}(x_j)| \le |\overline{T}_{F(e_i)}(x_j) - \overline{T}_{H(e_i)}(x_j)|,$$
$$|\overline{T}_{G(e_i)}(x_j) - \overline{T}_{H(e_i)}(x_j)| \le |\overline{T}_{F(e_i)}(x_j) - \overline{T}_{H(e_i)}(x_j)|,$$
$$|\underline{I}_{F(e_i)}(x_j) - \underline{I}_{G(e_i)}(x_j)| \le |\underline{I}_{F(e_i)}(x_j) - \underline{I}_{H(e_i)}(x_j)|,$$
$$|\underline{I}_{G(e_i)}(x_j) - \underline{I}_{H(e_i)}(x_j)| \le |\underline{I}_{F(e_i)}(x_j) - \underline{I}_{H(e_i)}(x_j)|,$$
$$|\overline{I}_{F(e_i)}(x_j) - \overline{I}_{G(e_i)}(x_j)| \le |\overline{I}_{F(e_i)}(x_j) - \overline{I}_{H(e_i)}(x_j)|,$$
$$|\overline{I}_{G(e_i)}(x_j) - \overline{I}_{H(e_i)}(x_j)| \le |\overline{I}_{F(e_i)}(x_j) - \overline{I}_{H(e_i)}(x_j)|,$$
$$|\underline{F}_{F(e_i)}(x_j) - \underline{F}_{G(e_i)}(x_j)| \le |\underline{F}_{F(e_i)}(x_j) - \underline{F}_{H(e_i)}(x_j)|,$$
$$|\underline{F}_{G(e_i)}(x_j) - \underline{F}_{H(e_i)}(x_j)| \le |\underline{F}_{F(e_i)}(x_j) - \underline{F}_{H(e_i)}(x_j)|,$$
$$|\overline{F}_{F(e_i)}(x_j) - \overline{F}_{G(e_i)}(x_j)| \le |\overline{F}_{F(e_i)}(x_j) - \overline{F}_{H(e_i)}(x_j)|,$$
$$|\overline{F}_{G(e_i)}(x_j) - \overline{F}_{H(e_i)}(x_j)| \le |\overline{F}_{F(e_i)}(x_j) - \overline{F}_{H(e_i)}(x_j)|.$$

Thus

$$CS_k((F,A),(H,C)) \le CS_k((F,A),(G,B))$$

and

$$CS_k((F,A),(H,C)) \le CS_k((G,B),(H,C))$$

for k = 3,4, since cosine function is a decreasing function within the interval $\left[0, \frac{\pi}{2}\right]$.
□

Definition 3.6. Let U= $\{x_1, x_2, ...x_n\}$ be the universal set of elements and E = $\{e_1, e_2, ...e_m\}$ be the universal set of parameters,and (F,A) and (G,B) be two interval valued neutrosophic soft sets over U. Then the weighted cosine similarity measures between (F,A) and (G.B) is defined as

$$CS_3((F, A), (G, B))$$
$$= \frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{n} w_j cos\Big[\frac{\pi}{4}((|\underline{T}_{F(e_i)}(x_j) - \underline{T}_{G(e_i)}(x_j)|) \vee (|\underline{I}_{F(e_i)}(x_j) - \underline{I}_{G(e_i)}(x_j)|) \vee$$
$$(|\underline{F}_{F(e_i)}(x_j) - \underline{F}_{G(e_i)}(x_j)|) + (|\overline{T}_{F(e_i)}(x_j) - \overline{T}_{G(e_i)}(x_j)|) \vee$$
$$(|\overline{I}_{F(e_i)}(x_j) - \overline{I}_{G(e_i)}(x_j)|) \vee (|\overline{F}_{F(e_i)}(x_j) - \overline{F}_{G(e_i)}(x_j)|))\Big],$$
$$CS_4((F, A), (G, B))$$
$$= \frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{n} w_j cos\Big[\frac{\pi}{12}((|\underline{T}_{F(e_i)}(x_j) - \underline{T}_{G(e_i)}(x_j)|) + (|\underline{I}_{F(e_i)}(x_j) - \underline{I}_{G(e_i)}(x_j)|) +$$
$$(|\underline{F}_{F(e_i)}(x_j) - \underline{F}_{G(e_i)}(x_j)|) + (|\overline{T}_{F(e_i)}(x_j) - \overline{T}_{G(e_i)}(x_j)|) +$$
$$(|\overline{I}_{F(e_i)}(x_j) - \overline{I}_{G(e_i)}(x_j)|) + (|\overline{F}_{F(e_i)}(x_j) - \overline{F}_{G(e_i)}(x_j)|))\Big]$$

where $w_j \in [0, 1]$, j=1,2,...n and $\sum_{j=1}^{n} w_j = 1$.

In particular if we take $w_j = \frac{1}{n}$, j=1,2,3....n, then

$$WCS_k((F, A), (G, B)) = CS_k((F, A), (G, B)), \text{ k=3,4.}$$

4. Application of cosine similarity measure of neutrisophic soft set

Example 4.1. Consider areas of a state are affected by flood. A team of three members $U = \{m_1, m_2, m_3\}$ from Rehabilitation department inspect the flood affected areas and the relief measures recommended by them are described by the parameter set $E = \{e_1, e_2, e_3, e_4\}$ where e_1 = relief for crop loss, e_2 = relief for ecological damage, e_3 = relief for livestock and e_4 = provision for alternate livelihood.Based on these recommendations the government has to allocate funds according to the level of damage. Algorithm:

Step 1: An interval valued neutrosophic soft set (F,A)over U is constructed based on previous records of relief measures in similar situation.

Step 2: An interval valued neutrosophic soft set (G,B) over U based on the recommendations of the team visiting Area I is constructed.

Step 3: Cosine similarity measures between (F,A)and (G,B) is calculated.

Step 4: An interval valued neutrosophic soft set (H,C) over U based on the recommendations of the team visiting Area II is constructed.

Step 5: Cosine similarity measures between (F,A)and (H,C) is calculated.

Step 6: Estimate result by using the similarity value. An interval valued (F,A) over U with A = E based on the previous records of relief measures in similar situation is given by.

(F,A)=

$\{F(e_1)(m_1) = ([0.6, 0.7], [0.3, 0.4], [0.2, 0.3]), F(e_2)(m_1) = ([0.4, 0.5], [0.3, 0.4], [0.2, 0.3]),$
$F(e_3)(m_1) = ([0.5, 0.6], [0.4, 0.5], [0.1, 0.2]), F(e_4)(m_1) = ([0.5, 0.6], [0.2, 0.3], [0.3, 0.4]),$

$F(e_1)(m_2) = ([0.5, 0.6], [0.3, 0.4], [0.2, 0.3]), F(e_2)(m_2) = ([0.4, 0.5], [0.2, 0.3], [0.5, 0.6]),$
$F(e_3)(m_2) = ([0.5, 0.6], [0.1, 0.2], [0.3, 0.4]), F(e_4)(m_2) = ([0.5, 0.6], [0.3, 0.5], [0.4, 0.5]),$
$F(e_1)(m_3) = ([0.7, 0.8], [0.4, 0.5], [0.2, 0.3]), F(e_2)(m_3) = ([0.5, 0.6], [0.3, 0.4], [0.2, 0.3]),$
$F(e_3)(m_3) = ([0.6, 0.7], [0.4, 0.5], [0.3, 0.4]), F(e_4)(m_3) = ([0.6, 0.7], [0.2, 0.3], [0.4, 0.5])\}.$
An interval valued (G,B) over U with B = E based on recommendations of the expert team visiting Area I.
(G,B)=
$\{G(e_1)(m_1) = ([0.6, 0.7], [0.5, 0.6], [0.4, 0.5]), G(e_2)(m_1) = ([0.3, 0.4], [0.5, 0.6], [0.4, 0.5]),$
$G(e_3)(m_1) = ([0.3, 0.4], [0.4, 0.5], [0.5, 0.6]), G(e_4)(m_1) = ([0.2, 0.3], [0.3, 0.4], [0.5, 0.6]),$
$G(e_1)(m_2) = ([0.8, 0.9], [0.4, 0.5], [0.2, 0.3]), G(e_2)(m_2) = ([0.5, 0.6], [0.1, 0.2], [0.3, 0.4]),$
$G(e_3)(m_2) = ([0.5, 0.6], [0.1, 0.2], [0.2, 0.3]), G(e_4)(m_2) = ([0.3, 0.4], [0.3, 0.4], [0.2, 0.3]),$
$G(e_1)(m_3) = ([0.5, 0.6], [0.4, 0.5], [0.3, 0.4]), G(e_2)(m_3) = ([0.4, 0.5], [0.2, 0.3], [0.5, 0.6]),$
$G(e_3)(m_3) = ([0.2, 0.3], [0.4, 0.5], [0.4, 0.5]), G(e_4)(m_3) = ([0.2, 0.3], [0.5, 0.6], [0.3, 0.4])\}.$
An interval valued (H,C) over U with C= E based on recommendations of the expert team visiting Area II.
(H,C)=
$\{H(e_1)(m_1) = ([0.4, 0.5], [0.2, 0.3], [0.1, 0.2]), H(e_2)(m_1) = ([0.3, 0.4], [0.2, 0.3], [0.1, 0.2]),$
$H(e_3)(m_1) = ([0.6, 0.7], [0.2, 0.3], [0.1, 0.2]), H(e_4)(m_1) = ([0.1, 0.2], [0.3, 0.4], [0.6, 0.7]),$
$H(e_1)(m_2) = ([0.4, 0.5], [0.2, 0.3], [0.5, 0.6]), H(e_2)(m_2) = ([0.2, 0.3], [0.3, 0.4], [0.4, 0.5]),$
$H(e_3)(m_2) = ([0.1, 0.2], [0.5, 0.6], [0.4, 0.5]), H(e_4)(m_2) = ([0.5, 0.6], [0.3, 0.4], [0.1, 0.2]),$
$H(e_1)(m_3) = ([0.2, 0.3], [0.4, 0.5], [0.3, 0.4]), H(e_2)(m_3) = ([0.1, 0.2], [0.3, 0.4], [0.4, 0.5]),$
$H(e_3)(m_3) = ([0.2, 0.3], [0.4, 0.5], [0.4, 0.5]), H(e_4)(m_3) = ([0.3, 0.4], [0.5, 0.6], [0.6, 0.7])\}.$
Using Definition 3.4, we get

$$CS_3((F, A), (G, B)) = 0.904$$

and

$$CS_3((F, A), (H, C)) = 0.871.$$

We have $CS_3((F, A), (G, B)) > CS_3((F, A), (H, C))$. Hence we conclude that Area I is severely affected by flood.

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