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Contents

Regular and Totally Regular Interval Valued Neutrosophic Hypergraphs

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Abstract

In this paper, we define the regular and the totally regular interval valued neutrosophic hypergraphs, and discuss the order and size along with properties of the regular and the totally regular single valued neutrosophic hypergraphs. We extend work to completeness of interval valued neutrosophic hypergraphs.

Keywords

interval valued neutrosophic hypergraphs, regular interval valued neutrosophic hypergraphs, totally regular interval valued neutrosophic hypergraphs.

1 Introduction

Smarandache [8] introduced the notion of neutrosophic sets (NSs) as a generalization of the fuzzy sets [14], intuitionistic fuzzy sets [12], interval valued fuzzy set [11] and interval-valued intuitionistic fuzzy sets [13] theories.

The neutrosophic sets are characterized by a truth-membership function (t), an indeterminacy-membership function (i) and a falsity membership function (f) independently, which are within the real standard or non-standard unit interval $]^{-}0, 1^{+}[$.

In order to conveniently use NS in real life applications, Smarandache [8] and Wang et al. [9] introduced the concept of the single-valued neutrosophic set (SVNS), a subclass of the neutrosophic sets.

The same authors [10] introduced the concept of the interval valued neutrosophic set (IVNS), which is more precise and flexible than the single valued neutrosophic set.

The IVNS is a generalization of the single valued neutrosophic set, in which the three membership functions are independent and their value belong to the unit interval [0, 1]. More works on single valued neutrosophic sets, interval valued neutrosophic sets and their applications can be found on *http://fs.gallup.unm.edu/NSS/*.

Hypergraph is a graph in which an edge can connect more than two vertices, and can be applied to analyse architecture structures and to represent system partitions. J. Mordesen and P. S. Nasir gave the definitions for fuzzy hypergraphs. R. Parvathy and M. G. Karunambigai's paper introduced the concept of intuitionistic fuzzy hypergraphs and analysed its components. The regular intuitionistic fuzzy hypergraphs and the totally regular intuitionistic fuzzy hypergraphs were introduced by I. Pradeepa and S. Vimala [38].

In this paper, we extend the regularity and the totally regularity on interval valued neutrosophic hypergraphs.

2 Preliminaries

Definition 2.1.

Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth membership function $T_A(x)$, indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. For each point $x \in X$; $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$.

Definition 2.2.

Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set A (IVNS A) is characterized by truth membership function $T_A(x)$, indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. For each point $x \in X$; $T_A(x) = [TL_A(x), TU_A(x)]$, $I_A(x) = [IL_A(x), IU_A(x)]$ and $F_A(x) = [FL_A(x), FU_A(x)]$ are contained in $[0, 1]$.

Definition 2.3.

A hypergraph is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \ldots, x_n\}$ a finite set of vertices.

(2) $E = \{E_1, E_2, \ldots, E_m\}$ a family of subsets of X.

(3) E_j for $j = 1,2,3,\ldots,m$ and $\cup_j(E_j) = X$.

The set X is called set of vertices and E is the set of edges (or hyperedges).

Definition 2.4.

An interval valued neutrosophic hypergraph is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \ldots, x_n\}$ a finite set of vertices.

(2) $E = \{ E_1, E_2, ..., E_m \}$ a family of IVNSs of X.

(3) $E_j \neq O = ([0,0], [0,0], [0,0])$ for $j= 1,2,3,...,m$ and $\bigcup_j Supp(E_j)= X$.

The set X is called set of vertices and E is the set of IVN-edges (or IVN-hyperedges).

Example 2.5.

Consider an interval valued neutrosophic hypergraphs $H = (X , E)$, where $X = \{ a, b, c, d \}$ and $E = \{ P, Q, R \}$, defined by:

$P = \{(a, [0.8, 0.9], [0.4, 0.7], [0.2, 0.7]), (b, [0.7, 0.9], [0.5, 0.8], [0.3, 0.9])\}$,

$Q = \{(b, [0.9, 1.0], [0.4, 0.5], [0.8, 1.0]), (c, [0.8, 0.9], [0.4, 0.5], [0.2, 0.7])\}$,

$R = \{(c, [0.1, 0.9], [0.5, 0.7], [0.4, 1.0]), (d, [0.1, 1.0], [0.9, 1.0], [0.5, 0.9])\}$.

Proposition 2.6.

The Interval Valued Neutrosophic Hypergraph (IVNHG) is the generalization of fuzzy hypergraph, intuitionistic fuzzy hypergraphs, interval valued fuzzy hypergraph, interval valued intuitionistic fuzzy hypergraph and single valued neutrosophic hypergraph.

3 Regular and Totally Regular IVNHGs

Definition 3.1.

The open neighbourhood of a vertex x in the interval valued neutrosophic hypergraphs (IVNHGs) is the set of adjacent vertices of x, excluding that vertex, and it is denoted by $N(x)$.

Definition 3.2.

The closed neighbourhood of a vertex x in the interval valued neutrosophic hypergraphs (IVNHGs) is the set of adjacent vertices of x, including that vertex, and it is denoted by $N[x]$.

Example 3.3.

Consider the interval valued neutrosophic hypergraphs $H = (X , E)$, where $X = \{ a, b, c, d, e \}$ and $E=\{ P, Q, R, S \}$, defined by:

$P = \{(a, [0.1, 0.4], [0.2, 0.8], [0.3, 0.9]), (b, [0.4, 0.5], [0.5, 0.6], [0.6, 0.8])\}$,

$Q = \{(c, [0.1, 0.7], [0.2, 0.8], [0.3, 0.9]), (d, [0.4, 0.8], [0.5, 0.9], [0.6, 0.7]),$

(e, [0.7, 0.9], [0.8, 0.9], [0.9, 1.0])},

R = {(b, [0.1, 0.4], [0.2, 0.8], [0.3, 0.9]), (c, [0.4, 0.8], [0.5, 0.9], [0.6, 0.7])},

S = {(a, [0.4, 0.8], [0.5, 0.9], [0.6, 0.7]), (d, [0.1, 0.4], [0.2, 0.8], [0.3, 0.9])}.

Then, the open neighbourhood of a vertex *a* is *b* and *d*.

The closed neigh-bourhood of a vertex *b* is *b*, *a* and *c*.

Definition 3.4.

Let *H = (X, E)* be an IVNHG; the open neighbourhood degree of a vertex *x* is denoted and defined by:

$$deg(x) = ([deg_{TL}(\mathrm{x}), deg_{TU}(\mathrm{x})], [deg_{IL}(\mathrm{x}), deg_{IU}(\mathrm{x})], [deg_{FL}(\mathrm{x}), deg_{FU}(\mathrm{x})]), \quad (1)$$

where:

$$deg_{TL}(\mathrm{x}) = \sum_{x \in N(x)} TL_E(x), \quad (2)$$

$$deg_{IL}(\mathrm{x}) = \sum_{x \in N(x)} IL_E(x), \quad (3)$$

$$deg_{FL}(\mathrm{x}) = \sum_{x \in N(x)} FL_E(x), \quad (4)$$

$$deg_{TU}(\mathrm{x}) = \sum_{x \in N(x)} TU_E(x), \quad (5)$$

$$deg_{IU}(\mathrm{x}) = \sum_{x \in N(x)} IU_E(x), \quad (6)$$

$$deg_{FU}(\mathrm{x}) = \sum_{x \in N(x)} FU_E(x). \quad (7)$$

Example 3.5.

Consider the interval valued neutrosophic hypergraphs *H = (X, E),* where *X = {a, b, c, d, e }* and *E = { P, Q, R, S }*, defined by:

P = {(a, [0.1, 0.2], [0.2, 0.3] [0.3, 0.4]), (b, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7])},

Q = {(c, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (d, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7]), (e, [0.7, 0.8], [0.8, 0.9], [0.9, 1.0])},

R = {(b, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (c, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7]},

S = {(a, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (d, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7]}.

Then, the open neighbourhood of a vertex *a* is *b* and *d*.

Therefore, the open neighbourhood degree of a vertex *a* is *([0.8, 1.0], [1.0, 1.2], [1.2, 1.4]).*

Definition 3.6.

Let $H = (X, E)$ be an IVNHG; the closed neighbourhood degree of a vertex x is denoted and defined by:

$$deg[x] = ([deg_{TL}[x], deg_{TU}[x]], [deg_{IL}[x], deg_{IU}[x]], [deg_{FL}[x], deg_{FU}[x]]), \quad (8)$$

where:

$$deg_{TL}[x] = deg_{TL}(x) + TL_E(x), \quad (9)$$

$$deg_{IL}[x] = deg_{IL}(x) + IL_E(x), \quad (10)$$

$$deg_{FL}[x] = deg_{FL}(x) + FL_E(x), \quad (11)$$

$$deg_{TU}[x] = deg_{TU}(x) + TU_E(x), \quad (12)$$

$$deg_{IU}[x] = deg_{IU}(x) + IU_E(x), \quad (13)$$

$$deg_{FU}[x] = deg_{FU}(x) + FU_E(x). \quad (14)$$

Example 3.7.

Consider the interval valued neutrosophic hypergraphs $H = (X, E)$, where $X = \{ a, b, c, d, e \}$ and $E = \{ P, Q, R, S \}$, defined by:

$P = \{(a, [0.1, 0.2], [0.2, 0.3] [0.3, 0.4]), (b, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7])\}$,

$Q = \{(c, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (d, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7]), (e, [0.7, 0.8], [0.8, 0.9], [0.9, 1.0])\}$,

$R = \{(b, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (c, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7]\}$,

$S = \{(a, [0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), (d, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7]\}$.

The closed neighbourhood of a vertex a is a, b and d.

Hence the closed neighbourhood degree of a vertex $\underline{a}$ is $([0.9, 1.2], [1.2, 1.5], [1.5, 1.8])$.

Definition 3.8.

Let $H = (X , E)$ be an IVNHG; then H is said to be a n-regular IVNHG if all the vertices have the same open neighbourhood degree,

$$n = ([n_1 , n_2], [n_3 , n_4], [n_5 , n_6]). \quad (15)$$

Definition 3.9.

Let $H = (X, E)$ be an IVNHG; then H is said to be a m-totally regular IVNHG if all the vertices have the same closed neighbourhood degree,

$$m = ([m_1 , m_2], [m_3 , m_4], [m_5 , m_6]). \quad (16)$$

Proposition 3.10.

A regular IVNHG is the generalization of regular fuzzy hypergraphs, regular intuitionistic fuzzy hypergraphs, regular interval valued fuzzy hypergraphs and regular interval valued intuitionistic fuzzy hypergraphs.

Proposition 3.11.

A totally regular IVNHG is the generalization of the totally regular fuzzy hypergraphs, totally regular intuitionistic fuzzy hypergraphs, totally regular interval valued fuzzy hypergraphs and totally regular interval valued intuitionistic fuzzy hypergraphs.

Example 3.12.

Consider the interval valued neutrosophic hypergraphs $H = (X, E)$, where $X = \{ a, b, c, d \}$ and $E = \{ P, Q, R, S \}$, defined by:

$P = \{(a, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (b, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4])\}$,

$Q = \{(b, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (c, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4])\}$,

$R = \{(c, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4])\}$,

$S = \{(d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (a, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4])\}$.

Here, the open neighbourhood degree of every vertex is $([1.6, 1.8], [0.4, 0.6], [0.6, 0.8])$, hence H is regular IVNHG and the closed neighbourhood degree of every vertex is $([2.4, 2.7], [0.6, 0.9], [0.9, 1.2])$. Hence H is both a regular and a totally regular IVNHG.

Theorem 3.13.

Let $H = (X, E)$ be an IVNHG which is both a regular and a totally regular IVNHG; then E is constant.

Proof.

Suppose H is a n-regular and a m-totally regular IVNHG. Then,

$$deg(x) = n = ([n_1, n_2], [n_3, n_4], [n_5, n_6]), \tag{17}$$

$$deg[x] = m = ([m_1, m_2], [m_3, m_4], [m_5, m_6]), \tag{18}$$

for all $x \in E_i$.

Consider

$$deg[x] = m, \tag{19}$$

hence, by definition,

$$deg(x) + E_i(\mathrm{x}) = m; \tag{20}$$

this implies that

$$E_i(\mathrm{x}) = m - n, \tag{21}$$

for all $x \in E_i$.

Hence E is constant.

Remark 3.14.

The converse of above theorem need not to be true in general.

Example 3.15.

Consider the interval valued neutrosophic hypergraphs $H = (X, E)$, where $X = \{a, b, c, d\}$ and $E = \{P, Q, R, S\}$, defined by:

$P = \{ (a, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (b, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \}$,

$Q = \{ (b, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \}$,

$R = \{ (c, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \}$,

$S = \{ (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) \}$.

Here E is constant, but $deg(a) = ([1.6, 1.8], [0.4, 0.6], [0.6, 0.8])$ and $deg(d) = ([2.4, 2.7], [0.6, 0.9], [0.9, 1.2])$, i.e $deg(a)$ and $deg(d)$ are not equals, hence H is a not regular IVNHG. Next, $deg[a] = ([2.4, 2.7], [0.6, 0.9], [0.9, 1.2])$ and $deg[d]= ([3.2, 3.6], [0.8, 1.2], [1.2, 1.6])$, hence $deg[a]$ and $deg[d]$ are not equals, hence H is not a totally regular IVNHG.

We conclude that H is neither a regular and nor a totally regular IVNHG.

Theorem 3.16.

Let $H = (X, E)$ be an IVNHG; then E is constant on X if and only if the following are equivalent:

(1) H is a regular IVNHG;

(2) H is a totally regular IVNHG.

Proof.

Suppose $H = (X, E)$ is an IVNHG and E is constant in H, i.e.:

$$E_i(x) = c = ([c_1 , c_2], [c_3 , c_4], [c_5 , c_6]), \tag{22}$$

for all $\mathrm{x} \in E_i$.

Suppose H is a n-regular IVNHG; then

$$deg(x) = n = ([n_1 , n_2], [n_3 , n_4], [n_5 , n_6]), \tag{23}$$

for all x ∈ E_i .

Consider

$$deg[x] = deg(x) +E_i(x) = n + c, \tag{24}$$

for all x ∈ E_i.

Hence, H is a totally regular IVNHG.

Next, suppose that H is a m-totally regular IVNHG; then:

$$deg[x] = m = ([m_1 , m_2], [m_3 , m_4], [m_5 , m_6]), \tag{25}$$

for all x ∈ E_i. i.e.:

$$deg(x) + E_i(x) = m, \tag{26}$$

for all x ∈ E_i.

This implies that

$$deg(x) = m - c, \tag{27}$$

for all x ∈ E_i.

Thus, H is a regular IVNHG, and consequently *(1)* and *(2)* are equivalent.

Conversely.

Assume that *(1)* and *(2)* are equivalent, i.e. H is a regular IVNHG if and only if H is a totally regular IVNHG.

Suppose by contrary that E is not constant, that is E_i(x) and E_i(y) not equals for some x and y in X. Let $H = (X , E)$ be a n-regular IVNHG; then

$$deg(x) = n = ([n_1 , n_2], [n_3 , n_4], [n_5 , n_6]), \tag{28}$$

for all x ∈ E_i.

Consider:

$$deg[x] = deg(x) + E_i(x) = n + E_i(x), \tag{29}$$

$$deg[y] = deg(y) + E_i((y) = n + E_i(y), \tag{30}$$

since E_i(x) and E_i(y) are not equals for some x and y in X, hence *deg[x]* and *deg[y]* are not equals, thus H is not a totally regular IVNHG, which is a contradiction to our assumption.

Next, let H be a totally regular IVNHG, then

$$deg[x] = deg[y]. \tag{31}$$

That is

$$deg(x) + E_i(x) = deg(y) + E_i(y), \tag{32}$$

$$deg(x) - deg(y) = E_i(y) - E_i(x), \tag{33}$$

since RHS of above equation is nonzero, hence LHS of above equation is also nonzero, thus *deg(x)* and *deg(y)* are not equals, so *H* is not a regular IVNHG, which is again a contradiction to our assumption, thus our supposition was wrong, hence *E* must be constant, and this completes the proof.

Definition 3.17.

Let $H = (X, E)$ be a regular IVNHG; then the order of an IVNHG H is denoted and defined by:

$$O(H) = ([p, q], [r, s], [t, u]), \tag{34}$$

where

$$p = \sum_{x \in X} TL_{E_i}(x),\ q = \sum_{x \in X} TU_{E_i}(x),\ r = \sum_{x \in X} IL_{E_i}(x), \tag{35}$$

$$s = \sum_{x \in X} IU_{E_i}(x),\ t = \sum_{x \in X} FL_{E_i}(x),\ u = \sum_{x \in X} FU_{E_i}(x), \tag{36}$$

for every $x \in X$, and the size of a regular IVNHG is denoted and defined by:

$$S(H) = \sum_{i=1}^{n}(S_{E_i}), \tag{37}$$

where

$$S(E_i) = ([a, b], [c, d], [e, f]) \tag{38}$$

and

$$a = \sum_{x \in E_i} TL_{E_i}(x),\ b = \sum_{x \in E_i} TU_{E_i}(x),\ c = \sum_{x \in E_i} IL_{E_i}(x) \tag{39}$$

$$d = \sum_{x \in E_i} IU_{E_i}(x),\ e = \sum_{x \in E_i} FL_{E_i}(x),\ f = \sum_{x \in E_i} FU_{E_i}(x). \tag{40}$$

Example 3.18.

Consider the interval valued neutrosophic hypergraphs $H = (X, E)$, where $X = \{a, b, c, d\}$ and $E = \{P, Q, R, S\}$, defined by:

P = { (a, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (b, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) },

Q = { (b, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (c, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) },

R = { (c, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) },

S = { (d, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]), (a, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4]) }.

Here, the order and the size of *H* are given, *([3.2, 3.6], [.8, 1.2], [1.2, 1.6])*, and *([6.4, 7.2], [1.6,2.4], [2.4,3.2])* respectively.

Proposition 3.19.

The size of a *n*-regular IVNHG $H = (H , E)$ is $\frac{nk}{2}$ where $|X|= k$.

Proposition 3.20.

If $H = (X, E)$ is a m-totally regular IVNHG, then $2S(H) + O(H) = mk$, where $|X| = k$.

Corollary 3.21.

Let $H = (X, E)$ be a n-regular and a m-totally regular IVNHG; then $O(H) = k(m - n)$, where $|X|=k$.

Proposition 3.22.

The dual of a n-regular and a m-totally regular IVNHG $H = (X, E)$ is again a n-regular and a m-totally regular IVNHG.

Definition 3.23.

The interval valued neutrosophic hypergraph (IVNHG) is said to be a complete IVNHG if for every x in X, $N(x) = \{ x : x \text{ in } X\text{-}\{x\} \}$; that is $N(x)$ contains all remaining vertices of X except x.

Example 3.24.

Consider the interval valued neutrosophic hypergraphs $H = (X, E)$, where $X = \{ a, b, c, d \}$ and $E = \{ P, Q, R \}$, defined by:

$P = \{(a, [0.4, 0.5], [0.6, 0.7], [0.3, 0.4]), (c, [0.8, 0.9], [0.2, 0.3], [0.3, 0.4])\}$

$Q = \{(a, [0.8, 1.0], [0.7, 0.9], [0.3, 0.7]), (b, [0.8, 0.9], [0.2, 0.3], [0.1, 0.9]), (d, [0.8, 0.9], [0.2, 0.5], [0.1, 0.9])\}$

$R = \{(c, [0.4, 0.6], [0.9, 1.0], [0.9, 1.0]), (d, [0.7, 0.9], [0.2, 0.7], [0.1, 0.7]), (b, [0.4, 0.6], [0.2, 0.7], [0.1, 0.8])\}$

Here, $N(a) = \{b, c, d\}$, $N(b) = \{a, c, d\}$, $N(c) = \{a, b, d\}$, $N(d) = \{a, b, c\}$. Hence H is a complete IVNHG.

Remark 3.25.

In a complete IVNHG $H = (X, E)$, the cardinality of $N(x)$ is the same for every vertex.

Theorem 3.26.

Every complete IVNHG $H = (X , E)$ is both a regular and a totally regular if E is constant in H.

Proof.

Let $H = (X, E)$ be a complete IVNHG; suppose E is constant in H.

Consequently:

$$E_i(x) = c = ([c_1, c_2], [c_3, c_4], [c_5, c_6]), \quad (41)$$

for all x ∈ E_i; since IVNHG is complete, then by definition for every vertex *x* in *X*, *N(x) = { x : x in X-{x} }*, and the open neighbourhood degree of every vertex is same, that is:

$$deg(x) = n = ([n_1, n_2], [n_3, n_4], [n_5, n_6]), \quad (42)$$

for all x ∈ E_i.
Hence, a complete IVNHG is a regular IVNHG.
Also,

$$deg[x] = deg(x) + E_i(x) = n + c \quad (43)$$

for all x ∈ E_i.
Hence *H* is a totally regular IVNHG.

Remark 3.27.

Every complete IVNHG is totally regular even if *E* is not constant.

Definition 3.28.

An IVNHG is said to be *k*-uniform if all the hyper-edges have the same cardinality.

Example 3.29.

Consider an interval valued neutrosophic hypergraphs *H = (X , E),* where *X = { a, b, c, d }* and *E = { P, Q, R },* defined by:

P = {(a, [0.8, 0.9], [0.4,0.7], [0.2, 0.7]), (b, [0.7, 0.9], [0.5, 0.8], [0.3, 0.9])},

Q = {(b, [0.9, 1.0], [0.4, 0.5], [0.8, 1.0]), (c, [0.8, 0.9], [0.4, 0.5], [0.2, 0.7])},

R = {(c, [0.1, 0.9], [0.5, 0.7], [0.4, 1.0]), (d, [0.1, 1.0], [0.9, 1.0], [0.5, 0.9])}.

4 Conclusion

The theoretical concepts of graphs and hypergraphs are highly used in computer science applications. The interval valued neutrosophic hypergraphs are more flexible than the fuzzy hypergraphs and the intuitionistic fuzzy hypergraphs, the interval valued fuzzy hypergraphs and the interval valued intuitionistic fuzzy hypergraphs. The concept of interval valued neutrosophic hypergraphs can be applied in various areas of engineering and computer science.

In this paper, we defined the regular and the totally regular interval valued neutrosophic hypergraphs.

We plan to extend our research work to the irregular interval valued neutrosophic hypergraphs.

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Isomorphism of Single Valued Neutrosophic Hypergraphs

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Abstract

In this paper, we introduce the homomorphism, weak isomorphism, co-weak isomorphism, and isomorphism of single valued neutrosophic hypergraphs. The properties of order, size and degree of vertices, along with isomorphism, are included. The isomorphism of single valued neutrosophic hypergraphs equivalence relation and of weak isomorphism of single valued neutrosophic hypergraphs partial order relation is also verified.

Keywords

homomorphism, weak-isomorphism, co-weak-isomorphism, isomorphism of single valued neutrosophic hypergraphs.

1 Introduction

The neutrosophic set (NS) was proposed by Smarandache [8] as a generalization of the fuzzy sets [14], intuitionistic fuzzy sets [12], interval valued fuzzy set [11] and interval-valued intuitionistic fuzzy sets [13] theories, and it is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information in the real world. The neutrosophic sets are characterized by a truth-membership function (t), an indeterminacy-membership function (i) and a falsity membership function (f) independently, which are within the real standard or non-standard unit interval $]^{-}0, 1^{+}[$. To conveniently use NS in the real-life applications, Wang et al. [9] introduced the single-valued neutrosophic set (SVNS), as a subclass of the neutrosophic sets. The same authors [10] introduced the interval valued neutrosophic set (IVNS), which is even more precise and flexible than the single valued neutrosophic set. The IVNS is a generalization of the single valued neutrosophic set, in which the three membership functions are independent, and their values belong to the unit interval [0, 1]. The hypergraph is a graph in which an edge can connect more than two vertices. Hypergraphs can be applied to analyse architecture structures and to represent system partitions. In this paper, we extend the concept into isomorphism of single valued neutrosophic hypergraphs, and some of their properties are introduced.

2 Preliminaries

Definition 2.1

A hypergraph is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \dots, x_n\}$ a finite set of vertices;

(2) $E = \{E_1, E_2, \dots, E_m\}$ a family of subsets of X;

(3) E_j are not-empty for $j = 1,2,3, \dots, m$ and $\cup_j(E_j) = X$.

The set X is called set of vertices and E is the set of edges (or hyper-edges).

Definition 2.2

A fuzzy hypergraph H = (X, E) is a pair, where X is a finite set and E is a finite family of non-trivial fuzzy subsets of X, such that $X = \cup_j Supp(E_j)$, $j = 1, 2, 3, \dots, m$.

Remark 2.3

The collection $E = \{E_1, E_2, E_3, \dots, E_m\}$ is the collection of edge sets of H.

Definition 2.4

A fuzzy hypergraph with underlying set X is of the form $H = (X, E, R)$, where $E = \{E_1, E_2, E_3, \dots, E_m\}$ is the collection of fuzzy subsets of X, that is $E_j : X \rightarrow [0, 1]$, $j = 1, 2, 3, \dots, m$ and $R : E \rightarrow [0, 1]$ is a fuzzy relation on fuzzy subsets E_j, such that:

$$R(x_1, x_2, \dots, x_r) \leq \min(E_j(x_1), \dots, E_j(x_r)), \quad (1)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Definition 2.5

Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth membership function $T_A(x)$, indeterminacy membership function $I_A(x)$, and a falsity membership function $F_A(x)$. For each point x ∈X; $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$.

Definition 2.6

A single valued neutrosophic hypergraph (SVNHG) is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \dots, x_n\}$ a finite set of vertices.

(2) $E = \{E_1, E_2, \dots, E_m\}$ a family of SVNSs of X.

(3) $E_j \neq O = (0, 0, 0)$ for $j = 1, 2, 3, \dots, m$ and $\cup_j Supp(E_j) = X$.

The set X is called set of vertices and E is the set of SVN-edges (or SVN-hyper-edges).

Proposition 2.7

The SVNHG is the generalization of the fuzzy hypergraphs and of the intuitionistic fuzzy hypergraphs.

Let be given a SVNHG $H = (X, E, R)$, with underlying set X, where $E = \{E_1, E_2, \dots, E_m\}$ is the collection of non-empty family of SVN subsets of X, and R being SVN's relation on SVN subsets E_j such that:

$$R_T(x_1, x_2, \dots, x_r) \leq \min([T_{E_j}(x_1)], \dots, [T_{E_j}(x_r)]), \tag{2}$$

$$R_I(x_1, x_2, \dots, x_r) \geq \max([I_{E_j}(x_1)], \dots, [I_{E_j}(x_r)]), \tag{3}$$

$$R_F(x_1, x_2, \dots, x_r) \geq \max([F_{E_j}(x_1)], \dots, [F_{E_j}(x_r)]), \tag{4}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Example 2.8

Consider the SVNHG $H = (X, E, R)$ with underlying set $X = \{a, b, c\}$, where $E = \{A, B\}$ and R, which is defined in the *Tables* given below.

H	A	B
a	(0.2,0.3,0.9)	(0.5,0.2,0.7)
b	(0.5,0.5,0.5)	(0.1,0.6,0.4)
c	(0.8,0.8,0.3)	(0.5,0.9,0.8)

R	R_T	R_I	R_F
A	0.2	0.8	0.9
B	0.1	0.9	0.8

By routine calculations, $H = (X, E, R)$ is a SVNHG.

3 Isomorphism of SVNHGs

Definition 3.1

A homomorphism $f: H \rightarrow K$ between two SVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ is a mapping $f: X \rightarrow Y$, which satisfies:

$$\min[T_{E_j}(x)] \leq \min[T_{F_j}(f(x))], \quad (5)$$

$$\max[I_{E_j}(x)] \geq \max[I_{F_j}(f(x))], \quad (6)$$

$$\max[F_{E_j}(x)] \geq \max[F_{F_j}(f(x))], \quad (7)$$

for all $x \in X$, and

$$R_T(x_1, x_2, \dots, x_r) \leq S_T(f(x_1), f(x_2), \dots, f(x_r)), \quad (8)$$

$$R_I(x_1, x_2, \dots, x_r) \geq S_I(f(x_1), f(x_2), \dots, f(x_r)), \quad (9)$$

$$R_F(x_1, x_2, \dots, x_r) \geq S_F(f(x_1), f(x_2), \dots, f(x_r)), \quad (10)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Example 3.2

Consider the two SVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ with underlying sets $X = \{a, b, c\}$ and $Y = \{x, y, z\}$, where $E = \{A, B\}$, $F = \{C, D\}$, R and S, which are defined in the *Tables* given below, and $f: X \rightarrow Y$ defined by $f(a)=x$, $f(b)=y$ and $f(c)=z$.

H	A	B
a	(0.2,0.3,0.9)	(0.5,0.2,0.7)
b	(0.5,0.5,0.5)	(0.1,0.6,0.4)
c	(0.8,0.8,0.3)	(0.5,0.9,0.8)

K	C	D
x	(0.3,0.2,0.2)	(0.2,0.1,0.3)
y	(0.2,0.4,0.2)	(0.3,0.2,0.1)
z	(0.5,0.8,0.2)	(0.9, 0.7, 0.1)

R	R_T	R_I	R_F
A	0.2	0.8	0.9
B	0.1	0.9	0.8

S	S_T	S_I	S_F
C	0.2	0.8	0.3
D	0.1	0.7	0.3

By routine calculations, *f: H → K* is a homomorphism between *H* and *K*.

Definition 3.3

A weak isomorphism *f: H → K* between two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a bijective mapping *f : X → Y*, which satisfies *f* is homomorphism, such that:

$$\min[T_{E_j}(x)] = \min[T_{F_j}\big(f(x)\big)], \tag{11}$$

$$\max[I_{E_j}(x)] = \max[I_{F_j}\big(f(x)\big)], \tag{12}$$

$$\max[F_{E_j}(x)] = \max[F_{F_j}\big(f(x)\big)], \tag{13}$$

for all $x \in X$.

Note

The weak isomorphism between two SVNHGs preserves the weights of vertices.

Example 3.4

Consider the two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y = {x, y, z}*, where *E = {A, B}* , *F = {C, D}*, *R* and *S*, which are defined in the *Tables* given below, and *f: X* → *Y* defined by *f(a)=x*, *f(b)=y* and *f(c)=z*.

H	A	B
a	(0.2,0.3,0.9)	(0.5,0.2,0.7)
b	(0.5,0.5,0.5)	(0.1,0.6,0.4)
c	(0.8,0.8,0.3)	(0.5,0.9,0.8)

K	C	D
x	(0.2,0.3,0.2)	(0.2,0.1,0.8)
y	(0.2,0.4,0.2)	(0.1,0.6,0.5)
z	(0.5,0.8,0.9)	(0.9,0.9,0.1)

R	R_T	R_I	R_F
A	0.2	0.8	0.9
B	0.1	0.9	0.9

S	S_T	S_I	S_F
C	0.2	0.8	0.9
D	0.1	0.9	0.8

By routine calculations, *f: H* → *K* is a weak isomorphism between *H* and *K*.

Definition 3.5

A co-weak isomorphism *f: H* → *K* between two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a bijective mapping *f: X* → *Y* which satisfies *f* is homomorphism, i.e.:

$$R_T(x_1, x_2, \dots, x_r) = S_T(f(x_1), f(x_2), \dots, f(x_r)), \tag{14}$$

$$R_I(x_1, x_2, \dots, x_r) = S_I(f(x_1), f(x_2), \dots, f(x_r)), \tag{15}$$

$$R_F(x_1, x_2, \dots, x_r) = S_F(f(x_1), f(x_2), \dots, f(x_r)), \tag{16}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Note

The co-weak isomorphism between two SVNHGs preserves the weights of edges.

Example 3.6

Consider the two SVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ with underlying sets $X = \{a, b, c\}$ and $Y = \{x, y, z\}$, where $E = \{A, B\}$,$F = \{C, D\}$, R and S are defined in the *Tables* given below, and $f: X \rightarrow Y$ defined by $f(a)=x$, $f(b)=y$ and $f(c)=z$.

H	A	B
a	(0.2,0.3,0.9)	(0.5,0.2,0.7)
b	(0.5,0.5,0.5)	(0.1,0.6,0.4)
c	(0.8,0.8,0.3)	(0.5,0.9,0.8)

K	C	D
x	(0.3,0.2,0.2)	(0.2,0.1,0.3)
y	(0.2,0.4,0.2)	(0.3,0.2,0.1)
z	(0.5,0.8,0.2)	(0.9, 0.7, 0.1)

R	R_T	R_I	R_F
A	0.2	0.8	0.9
B	0.1	0.9	0.8

S	S_T	S_I	S_F
C	0.2	0.8	0.9
D	0.1	0.9	0.8

By routine calculations, $f: H \rightarrow K$ is a co-weak isomorphism between H and K.

Definition 3.7

An isomorphism *f: H → K* between two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a bijective mapping *f: X → Y*, which satisfies:

$$\min[T_{E_j}(x)] = \min[T_{F_j}(f(x))], \tag{17}$$

$$\max[I_{E_j}(x)] = \max[I_{F_j}(f(x))], \tag{18}$$

$$\max[F_{E_j}(x)] = \max[F_{F_j}(f(x))], \tag{19}$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) = S_T(f(x_1), f(x_2), \dots, f(x_r)), \tag{20}$$

$$R_I(x_1, x_2, \dots, x_r) = S_I(f(x_1), f(x_2), \dots, f(x_r)), \tag{21}$$

$$R_F(x_1, x_2, \dots, x_r) = S_F(f(x_1), f(x_2), \dots, f(x_r)), \tag{22}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Note

The isomorphism between two SVNHGs preserves both the weights of vertices and the weights of edges.

Example 3.8

Consider the two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y = {x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S,* which are defined in the *Tables* given below, and *f: X → Y* defined by, *f(a)=x*, *f(b)=y* and *f(c)=z*.

H	A	B
a	(0.2,0.3,0.7)	(0.5,0.2,0.7)
b	(0.5,0.5,0.5)	(0.1,0.6,0.4)
c	(0.8,0.8,0.3)	(0.5,0.9,0.8)

K	C	D
x	(0.2,0.3,0.2)	(0.2,0.1,0.8)
y	(0.2,0.4,0.2)	(0.1,0.6,0.5)
z	(0.5,0.8,0.7)	(0.9, 0.9, 0.1)

R	R_T	R_I	R_F
A	0.2	0.8	0.9
B	0.0	0.9	0.8

S	S_T	S_I	S_F
C	0.2	0.8	0.9
D	0.0	0.9	0.8

By routine calculations, $f: H \rightarrow K$ is an isomorphism between H and K.

Definition 3.9

Let $H = (X, E, R)$ be a SVNHG; then, the *order* of H is denoted and defined by:

$$O(H) = (\sum \min T_{E_j}(x), \sum \max I_{E_j}(x), \tag{23}$$

and the *size* of H is denoted and defined by:

$$S(H) = (\sum R_T(E_j), \sum R_I(E_j), \sum R_F(E_j)). \tag{24}$$

Theorem 3.10

Let $H = (X, E, R)$ and $K = (Y, F, S)$ be two SVNHGs, such that H is isomorphic to K.
Then:

(1) $O(H) = O(K)$;

(2) $S(H) = S(K)$.

Proof.

Let $f: H \rightarrow K$ be an isomorphism between H and K with underlying sets X and Y respectively.
Then, by definition, we have:

$$\min[T_{E_j}(x)] = \min[T_{F_j}(f(x))], \tag{25}$$

$$\max[I_{E_j}(x)] = \max[I_{F_j}(f(x))], \tag{26}$$

$$\max[F_{E_j}(x)] = \max[F_{F_j}(f(x))], \tag{27}$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) = S_T(f(x_1), f(x_2), \dots, f(x_r)), \quad (28)$$

$$R_I(x_1, x_2, \dots, x_r) = S_I(f(x_1), f(x_2), \dots, f(x_r)), \quad (29)$$

$$R_F(x_1, x_2, \dots, x_r) = S_F(f(x_1), f(x_2), \dots, f(x_r)), \quad (30)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Consider:

$$O_T(H) = \sum \min T_{E_j}(x) = \sum \min T_{F_j}(f(x)) = O_T(K) \quad (31)$$

Similarly, $O_I(H) = O_I(K)$ and $O_F(H) = O_F(K)$, hence *O(H) = O(K).*

Next,

$$S_T(H) = \sum R_T(x_1, x_2, \dots, x_r) = \sum S_T(f(x_1), f(x_2), \dots, f(x_r)) = S_T(K) \quad (32)$$

Similarly, $S_I(H) = S_I(K)$, $S_F(H) = S_F(K)$, hence $S(H) = S(K)$.

Remark 3.11

The converse of the above theorem need not to be true in general.

Example 3.12

Consider the two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S,* which are defined in the *Tables* given below, where *f* is defined by *f(a)=w, f(b)=x, f(c)=y, f(d)=z*.

H	A	B
a	(0.2, 0.5, 0.33)	(0.16,0.5,0.33)
b	(0.0,0.0,0.0)	(0.2,0.5,0.33)
c	(0.33,0.5,0.33)	(0.2,0.5,0.33)
d	(0.5,0.5,0.33)	(0.0,0.0,0.0)

K	C	D
w	(0.2,0.5,0.33)	(0.2,0.5,0.33)
x	(0.16,0.5,0.33)	(0.33,0.5,0.33)
y	(0.33,0.5,0.33)	(0.2,0.5,0.33)
z	(0.5,0.5,0.33)	(0.0,0.0,0.0)

R	R_T	R_I	R_F
A	0.2	0.5	0.33
B	0.16	0.5	0.33

S	S_T	S_I	S_F
C	0.16	0.5	0.33
D	0.2	0.5	0.33

Here, $O(H) = (1.06, 2.0, 1.32) = O(K)$ and $S(H) = (0.36, 1.0, 0.66) = S(K)$, but, by routine calculations, H is not isomorphism to K.

Corollary 3.13

The weak isomorphism between any two SVNHGs preserves the orders.

Remark 3.14

The converse of above corollary need not to be true in general.

Example 3.15

Consider the two SVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ with underlying sets $X = \{a, b, c, d\}$ and $Y = \{w, x, y, z\}$, where $E = \{A, B\}$, $F = \{C, D\}$, R and S, which are defined in the *Tables* given below, where f is defined by $f(a)=w, f(b)=x, f(c)=y, f(d)=z$.

H	A	B
a	(0.2,0.5,0.3)	(0.14,0.5,0.3)
b	(0.0,0.0,0.0)	(0.2,0.5,0.3)
c	(0.33,0.5,0.3)	(0.16,0.5,0.3)
d	(0.5,0.5,0.3)	(0.0,0.0,0.0)

K	C	D
w	(0.14,0.5,0.3)	(0.16,0.5,0.3)
x	(0.0,0.0,0.0)	(0.16,0.5,0.3)
y	(0.25,0.5,0.3)	(0.2,0.5,0.3)
z	(0.5,0.5,0.3)	(0.0,0.0,0.0)

Here, *O(H)= (1.0, 2.0, 1.2) = O(K),* but, by routine calculations, *H* is not weak isomorphism to *K*.

Corollary 3.16

The co-weak isomorphism between any two SVNHGs preserves sizes.

Remark 3.17

The converse of above corollary need not to be true in general.

Example 3.18

Consider the two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z}*, where *E = {A, B}, F = {C, D},R* and *S* are defined in the *Tables* given below, where *f* is defined by *f(a)=w, f(b)=x, f(c)=y, f(d)=z.*

H	A	B
a	(0.2,0.5,0.3)	(0.14,0.5,0.3)
b	(0.0,0.0,0.0)	(0.16,0.5,0.3)
c	(0.3,0.5,0.3)	(0.2,0.5,0.3)
d	(0.5,0.5,0.3)	(0.0,0.0,0.0)

K	C	D
w	(0.0,0.0,0.0)	(0.2,0.5,0.3)
x	(0.14,0.5,0.3)	(0.25,0.5,0.3)
y	(0.5,0.5,0.3)	(0.2,0.5,0.3)
z	(0.3,0.5,0.3)	(0.0,0.0,0.0)

R	R_T	R_I	R_F
A	0.2	0.5	0.3
B	0.14	0.5	0.3

S	S_T	S_I	S_F
C	0.14	0.5	0.3
D	0.2	0.5	0.3

Here, *S(H) = (0.34,1.0,0.6) = S(K)*, but, by routine calculations, *H* is not co-weak isomorphism to *K*.

Definition 3.19

Let *H = (X, E, R)* be a SVNHG; then the degree of vertex x_i is denoted and defined by:

$$\deg(x_i) = (deg_T(x_i), deg_I(x_i), deg_F(x_i)), \quad (33)$$

where

$$deg_T(x_i) = \sum R_T(x_1, x_2, \dots, x_r), \quad (34)$$

$$deg_I(x_i) = \sum R_I(x_1, x_2, \dots, x_r), \quad (35)$$

$$deg_F(x_i) = \sum R_F(x_1, x_2, \dots, x_r), \quad (36)$$

for $x_i \neq x_r$.

Theorem 3.20

If *H* and *K* are two isomorphic SVNHGs, then the degree of their vertices is preserved.

Proof.

Let *f: H → K* be an isomorphism between *H* and *K* with underlying sets *X* and *Y* respectively; then, by definition, we have

$$\min[T_{E_j}(x)] = \min[T_{F_j}(f(x))], \quad (37)$$

$$\max[I_{E_j}(x)] = \max[I_{F_j}(f(x))], \quad (38)$$

$$\max[F_{E_j}(x)] = \max[F_{F_j}(f(x))], \quad (39)$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) \quad = \ S_T(f(x_1)\,, f(x_2), \dots, f(x_r)\,), \tag{40}$$

$$R_I(x_1, x_2, \dots, x_r) \quad = S_I(f(x_1)\,, f(x_2), \dots, f(x_r)\,), \tag{41}$$

$$R_F(x_1, x_2, \dots, x_r) \quad = \ S_F(f(x_1)\,, f(x_2), \dots, f(x_r)\,), \tag{42}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Consider:

$$deg_T(x_i) = \sum R_T(x_1, x_2, \dots, x_r) = \sum S_T(f(x_1), f(x_2), \dots, f(x_r)) = deg_T(f(x_i)). \tag{43}$$

Similarly:

$$deg_I(x_i) = deg_I(f(x_i)), deg_F(x_i) = deg_F(f(x_i)) \tag{44}$$

Hence:

$$deg(x_i) = deg(f(x_i)). \tag{45}$$

Remark 3.21

The converse of the above theorem may not be true in general.

Example 3.22

Consider the two SVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b}* and *Y = {x, y}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S* are defined in the *Tables* given below, where *f* is defined by, *f(a)=x, f(b)=y,* here *deg(a) = (0.8, 1.0, 0.6) = deg(x)* and *deg(b) = (0.45, 1.0, 0.6) = deg(y).*

H	A	B
a	(0.5,0.5,0.3)	(0.3,0.5,0.3)
b	(0.25,0.5,0.3)	(0.2,0.5,0.3)

K	C	D
x	(0.3,0.5,0.3)	(0.5,0.5,0.3)
y	(0.2,0.5,0.3)	(0.25,0.5,0.3)

S	S_T	S_I	S_F
C	0.2	0.5	0.3
D	0.25	0.5	0.3

R	R_T	R_I	R_F
A	0.25	0.5	0.3
B	0.2	0.5	0.3

But H is not isomorphic to K, i.e. H is neither weak isomorphic nor co-weak isomorphic to K.

Theorem 3.23

The isomorphism between SVNHGs is an equivalence relation.

Proof.

Let *H = (X, E, R)*, *K = (Y, F, S)* and *M = (Z, G, W)* be SVNHGs with underlying sets X, Y and Z, respectively:

- Reflexive.

Consider the map (identity map) *f: X → X* defined as follows: *f(x) = x for all x*∈ *X,* since identity map is always bijective and satisfies the conditions:

$$\min[T_{E_j}(x)] = \min[T_{E_j}(f(x))], \tag{46}$$

$$\max[I_{E_j}(x)] = \max[I_{E_j}(f(x))], \tag{47}$$

$$\max[F_{E_j}(x)] = \max[F_{E_j}(f(x))], \tag{48}$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) = R_T(f(x_1), f(x_2), \dots, f(x_r)), \tag{49}$$

$$R_I(x_1, x_2, \dots, x_r) = R_I(f(x_1), f(x_2), \dots, f(x_r)), \tag{50}$$

$$R_F(x_1, x_2, \dots, x_r) = R_F(f(x_1), f(x_2), \dots, f(x_r)), \tag{51}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Hence *f* is an isomorphism of SVNHG *H* to itself.

- Symmetric.

Let *f: X → Y* be an isomorphism of *H* and *K*, then *f* is bijective mapping, defined as *f(x) = y* for all *x* ∈ *X.*

Then, by definition:

$$\min[T_{E_j}(x)] = \min[T_{F_j}(f(x))], \tag{52}$$

$$\max[I_{E_j}(x)] = \max[I_{F_j}(f(x))], \tag{53}$$

$$\max[F_{E_j}(x)] = \max[F_{F_j}(f(x))], \tag{54}$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) = S_T(f(x_1), f(x_2), \dots, f(x_r)), \tag{55}$$

$$R_I(x_1, x_2, \dots, x_r) = S_I(f(x_1), f(x_2), \dots, f(x_r)), \tag{56}$$

$$R_F(x_1, x_2, \dots, x_r) = S_F(f(x_1), f(x_2), \dots, f(x_r)), \tag{57}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Since f is bijective, then we have $f^{-1}(y) = x$ for all $y \in Y$.

Thus, we get:

$$\min[T_{E_j}\left(f^{-1}(y)\right)] = \min[T_{F_j}(y)], \tag{58}$$

$$\max[I_{E_j}\left(f^{-1}(y)\right)] = \max[I_{F_j}(y)], \tag{59}$$

$$\max[F_{E_j}\left(f^{-1}(y)\right)] = \max[F_{F_j}(y)], \tag{60}$$

for all $x \in X$, and:

$$R_T\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_T(y_1, y_2, \dots, y_r), \tag{61}$$

$$R_I\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_I(y_1, y_2, \dots, y_r), \tag{62}$$

$$R_F\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_F(y_1, y_2, \dots, y_r), \tag{63}$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of Y.

Hence, we have a bijective map $f^{-1}: Y \to X$, which is an isomorphism from K to H.

- Transitive.

Let $f: X \to Y$ and $g: Y \to Z$ be two isomorphism of SVNHGs of H onto K and K onto M, respectively. Then gof is a bijective mapping from X to Z, where gof is defined as $(gof)(x) = g(f(x))$ for all $x \in X$.

Since f is an isomorphism, then, by definition, $f(x) = y$ for all $x \in X$, which satisfies:

$$\min[T_{E_j}(x)] = \min[T_{F_j}(f(x))], \tag{64}$$

$$\max[I_{E_j}(x)] = \max[I_{F_j}(f(x))], \tag{65}$$

$$\max[F_{E_j}(x)] = \max[F_{F_j}(f(x))], \tag{66}$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) = S_T(f(x_1), f(x_2), \dots, f(x_r)), \tag{67}$$

$$R_I(x_1, x_2, \dots, x_r) = S_I(f(x_1), f(x_2), \dots, f(x_r)), \tag{68}$$

$$R_F(x_1, x_2, \dots, x_r) = S_F(f(x_1), f(x_2), \dots, f(x_r)), \tag{69}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Since $g : Y \to Z$is an isomorphism, then, by definition, $g(y) = z$ for all $y \in Y$, satisfying the conditions:

$$\min[T_{F_j}(y)] = \min\left[T_{G_j}(g(y))\right], \tag{70}$$

$$\max[I_{F_j}(y)] = \max[I_{G_j}(g(y))], \tag{71}$$

$$\max[F_{F_j}(y)] = \max[F_G(g(y))], \tag{72}$$

for all $x \in X$, and:

$$S_T(y_1, y_2, \dots, y_r) = W_T(g(y_1), g(y_2), \dots, g(y_r)), \tag{73}$$

$$S_I(y_1, y_2, \dots, y_r) = W_I(g(y_1), g(y_2), \dots, g(y_r)), \tag{74}$$

$$S_F(y_1, y_2, \dots, y_r) = W_F(g(y_1), g(y_2), \dots, g(y_r)), \tag{75}$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of *Y.*

Thus, from above equations, we conclude that:

$$\min[T_{E_j}(x)] = \min[T_{G_j}(g(f(x)))], \tag{76}$$

$$\max[I_{E_j}(x)] = \max[I_{G_j}(g(f(x)))], \tag{77}$$

$$\max[F_{E_j}(x)] = \max[F_{G_j}(g(f(x)))], \tag{78}$$

for all $x \in X$, and:

$$R_T(x_1, \dots, x_r) = W_T(g(f(x_1)), \dots, g(f(x_r))), \tag{79}$$

$$R_I(x_1, \dots, x_r) = W_I(g(f(x_1)), \dots, g(f(x_r))), \tag{80}$$

$$R_F(x_1, \dots, x_r) = W_F(g(f(x_1)), \dots, g(f(x_r))), \tag{81}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Therefore, gof is an isomorphism between H and M. Hence, the isomorphism between SVNHGs is an equivalence relation.

Theorem 3.24

The weak isomorphism between SVNHGs satisfies the partial order relation.

Proof.

Let *H = (X, E, R)*, *K = (Y, F, S)* and *M = (Z, G, W)* be SVNHGs with underlying sets X, Y and *Z*, respectively.

- Reflexive.

Consider the map (identity map) $f: X \to X$, defined as follows $f(x)=x$ for all $x \in X$, since the identity map is always bijective and satisfies the conditions:

$$\min[T_{E_j}(x)] = \min[T_{E_j}(f(x))], \tag{82}$$

$$\max[I_{E_j}(x)] = \max[I_{E_j}(f(x))], \tag{83}$$

$$\max[F_{E_j}(x)] = \max[F_{E_j}(f(x))], \tag{84}$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) \leq R_T(f(x_1), f(x_2), \dots, f(x_r)), \tag{85}$$

$$R_I(x_1, x_2, \dots, x_r) \geq R_I(f(x_1), f(x_2), \dots, f(x_r)), \tag{86}$$

$$R_F(x_1, x_2, \dots, x_r) \geq R_F(f(x_1), f(x_2), \dots, f(x_r)), \tag{87}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Hence f is a weak isomorphism of SVNHG H to itself.

- Anti-symmetric.

Let f be a weak isomorphism between H onto K, and g be a weak isomorphic between K and H, that is $f: X \to Y$ is a bijective map defined by $f(x) = y$ for all $x \in X$, satisfying the conditions:

$$\min[T_{E_j}(x)] = \min[T_{F_j}(f(x))], \tag{88}$$

$$\max[I_{E_j}(x)] = \max[I_{F_j}(f(x))], \tag{89}$$

$$\max[F_{E_j}(x)] = \max[F_{F_j}(f(x))], \tag{90}$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) \leq S_T(f(x_1), f(x_2), \dots, f(x_r)), \tag{91}$$

$$R_I(x_1, x_2, \dots, x_r) \geq S_I(f(x_1), f(x_2), \dots, f(x_r)), \tag{92}$$

$$R_F(x_1, x_2, \dots, x_r) \geq S_F(f(x_1), f(x_2), \dots, f(x_r)), \tag{93}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Since g is also a bijective map $g(y) = x$ for all $y \in Y$ satisfying the conditions:

$$\min[T_{F_j}(y)] = \min[T_{E_j}(g(y))], \tag{95}$$

$$\max[I_{F_j}(y)] = \max[I_{E_j}(g(y))], \tag{96}$$

$$\max[F_{F_j}(y)] = \max[F_{E_j}(g(y))], \tag{97}$$

for all $y \in Y$, and:

$$R_T(y, y_2, \dots, y_r) \ \leq \ S_T(g(y_1), g(y_2), \dots, g(y_r)), \tag{98}$$

$$R_I(y_1, y_2, \dots, y_r) \ \geq S_I(f(y_1), f(y_2), \dots, f(y_r)), \tag{99}$$

$$R_F(y_1, y_2, \dots, y_r) \ \geq \ S_F(f(y_1), f(y_2), \dots, f(y_r)), \tag{100}$$

for all $\{ y_1, y_2, \dots, y_r\}$ subsets of Y.

The above inequalities hold for finite sets X and Y only when H and K SVNHGs have same number of edges and the corresponding edge have same weight, hence H is identical to K.

- Transitive.

Let $f: X \to Y$ and $g: Y \to Z$ be two weak isomorphism of SVNHGs of H onto K and K onto M, respectively. Then gof is a bijective mapping from X to Z, where gof is defined as $(gof)(x) = g\big(f(x)\big)$ for all $x \in X$.

Since f is a weak isomorphism, then, by definition, $f(x) = y$ for all $x \in X$, which satisfies the conditions:

$$\min[T_{E_j}(x)] = \min[T_{F_j}\big(f(x)\big)], \tag{101}$$

$$\max[I_{E_j}(x)] = \max[I_{F_j}\big(f(x)\big)], \tag{102}$$

$$\max[F_{E_j}(x)] = \max[F_{F_j}\big(f(x)\big)], \tag{103}$$

for all $x \in X$, and:

$$R_T(x_1, x_2, \dots, x_r) \ \leq \ S_T(f(x_1), f(x_2), \dots, f(x_r)), \tag{104}$$

$$R_I(x_1, x_2, \dots, x_r) \ \geq S_I(f(x_1), f(x_2), \dots, f(x_r)), \tag{105}$$

$$R_F(x_1, x_2, \dots, x_r) \ \geq \ S_F(f(x_1), f(x_2), \dots, f(x_r)), \tag{106}$$

for all $\{ x_1, x_2, \dots, x_r\}$ subsets of X.

Since $g: Y \to Z$ is a weak isomorphism, then, by definition, $g(y) = z$ for all $y \in Y$ satisfying the conditions:

$$\min[T_{F_j}(y)] = \min[T_{G_j}\big(g(y)\big)], \tag{107}$$

$$\max[I_{F_j}(y)] = \max[I_{G_j}\big(g(y)\big)], \tag{108}$$

$$\max[F_{F_j}(y)] = \max[F_G\big(g(y)\big)], \tag{109}$$

for all $x \in X$, and:

$$S_T(y_1, y_2, \dots, y_r) \leq W_T(g(y_1), g(y_2), \dots, g(y_r)), \tag{110}$$

$$S_I(y_1, y_2, \dots, y_r) \ \geq W_I(g(y_1), g(y_2), \dots, g(y_r)), \tag{111}$$

$$S_F(y_1, y_2, \dots, y_r) \ \geq \ W_F(g(y_1), g(y_2), \dots, g(y_r)), \tag{112}$$

for all $\{ y_1, y_2, \dots, y_r\}$ subsets of Y.

Thus, from above equations, we conclude that:

$$\min[T_{E_j}(x)] = \min[T_{G_j}(g(f(x)))]\,, \tag{113}$$

$$\max[I_{E_j}(x)] = \max[I_{G_j}(g(f(x)))], \tag{114}$$

$$\max[F_{E_j}(x)] = \max[F_{G_j}(g(f(x)))], \tag{115}$$

for all $x \in X$, *and:*

$$R_T(x_1, \dots, x_r) \leq W_T(g\big(f(x_2)\big), \dots, g\big(f(x_r)\big)), \tag{116}$$

$$R_I(x_1, \dots, x_r) \geq W_I(g\big(f(x_2)\big), \dots, g\big(f(x_r)\big)), \tag{117}$$

$$R_F(x_1, \dots, x_r) \geq W_F(g\big(f(x_2)\big), \dots, g\big(f(x_r)\big)) \tag{118}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Therefore gof is a weak isomorphism between H and M.

Hence, a weak isomorphism between SVNHGs is a partial order relation.

4 Conclusion

Theoretical concepts of graphs and hypergraphs are highly used by computer science applications. Single valued neutrosophic hypergraphs are more flexible than fuzzy hypergraphs and intuitionistic fuzzy hypergraphs. The concepts of single valued neutrosophic hypergraphs can be applied in various areas of engineering and computer science.

In this paper, the isomorphism between SVNHGs is proved to be an equivalence relation and the weak isomorphism to be a partial order relation. Similarly, it can be proved that a co-weak isomorphism in SVNHGs is a partial order relation.

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Isomorphism of Interval Valued Neutrosophic Hypergraphs

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Abstract

In this paper, we introduce the homomorphism, weak isomorphism, co-weak isomorphism and isomorphism of interval valued neutrosophic hypergraphs. The properties of order, size and degree of vertices, along with isomorphism, are included. The isomorphism of interval valued neutrosophic hypergraphs equivalence relation and weak isomorphism of interval valued neutrosophic hypergraphs partial order relation are also verified.

Keywords

homomorphism, weak-isomorphism, co-weak-isomorphism, isomorphism of interval valued neutrosophic hypergraphs.

1 Introduction

The neutrosophic sets are characterized by a truth-membership function (t), an indeterminacy-membership function (i) and a falsity membership function (f) independently, which are within the real standard or non-standard unit interval $]^{-}0, 1^{+}[$.

Smarandache [8] proposed the notion of neutrosophic set (NS) as a generalization of the fuzzy set [14], intuitionistic fuzzy set [12], interval valued fuzzy set [11] and interval-valued intuitionistic fuzzy set [13] theories.

For convenient use of NS in real life applications, Wang et al. [9] introduced the concept of the single-valued neutrosophic set (SVNS), a subclass of the neutrosophic sets. The same authors [10] introduced the concept of the interval valued neutrosophic set (IVNS), which is more precise and flexible than the single valued neutrosophic set. The IVNS is a generalization of the single valued neutrosophic set, in which the three membership functions are independent and their value belong to the unit interval [0, 1].

More works on single valued neutrosophic sets, interval valued neutrosophic sets and their applications can be found on *http://fs.gallup.unm.edu/NSS/*.

Hypergraph is a graph in which an edge can connect more than two vertices. Hypergraphs can be applied to analyze architecture structures and to represent system partitions. Mordesen and Nasir gave the definitions for fuzzy hyper graphs. Parvathy R. and M. G. Karunambigai's paper introduced the concepts of intuitionistic fuzzy hypergraphs and analyze its components. Radhamani and Radhika introduced the concept of Isomorphism on Fuzzy Hypergraphs.

In this paper, we extend the concept to isomorphism of interval valued neutrosophic hypergraphs, and some of their important properties are introduced.

2 Preliminaries

Definition 2.1

A hypergraph is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \dots, x_n\}$ is a finite set of vertices.

(2) $E = \{E_1, E_2, \dots, E_m\}$ is a family of subsets of X.

(3) E_j are not-empty for $j = 1, 2, 3, \dots, m$ and $\cup_j(E_j) = X$.

The set X is called set of vertices and E is the set of edges (or hyper-edges).

Definition 2.2

A fuzzy hypergraph $H = (X, E)$ is a pair, where X is a finite set and E is a finite family of non-trivial fuzzy subsets of X, such that $X = \cup_j Supp(E_j)$, $j = 1, 2, 3, \dots, m$.

Remark 2.3

$E = \{E_1, E_2, E_3, \dots, E_m\}$ is the collection of edge set of H.

Definition 2.4

A fuzzy hypergraph with underlying set X is of the form $H = (X, E, R)$, where $E = \{E_1, E_2, E_3, \dots, E_m\}$ is the collection of fuzzy subsets of X, i.e. $E_j : X \to [0, 1]$, $j = 1, 2, 3, \dots, m$ and $R : E \to [0, 1]$ is a fuzzy relation on fuzzy subsets E_j, such that:

$$R(x_1, x_2, \dots, x_r) \leq \min(E_j(x_1), \dots, E_j(x_r)), \quad (1)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Definition 2.5

Let X be a space of points (objects) with generic elements in X, which is denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth membership function $T_A(x)$, indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. For each point x ∈X; $T_A(x)$, $I_A(x)$,$F_A(x)$ ∈ [0 , 1].

Definition 2.6

A single valued neutrosophic hypergraph is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \dots, x_n\}$ is a finite set of vertices.

(2) $E = \{E_1, E_2, \dots, E_m\}$ is a family of SVNSs of X.

(3)$E_j \neq O = (0, 0, 0)$ for $j = 1, 2, 3, \dots, m$ and $\bigcup_j Supp(E_j) = X$.

The set X is called set of vertices and E is the set of SVN-edges (or SVN-hyper-edges).

Proposition 2.7

The single valued neutrosophic hypergraph is the generalization of fuzzy hypergraphs and intuitionistic fuzzy hypergraphs.

Note that a given a SVNHG$H = (X, E, R)$ with underlying set X, where $E = \{E_1, E_2, \dots, E_m\}$ is the collection of non-empty family of SVN subsets of X, and R is SVN relation on SVN subsets E_j, such that:

$$R_T(x_1, x_2, \dots, x_r) \leq \min([T_{E_j}(x_1)], \dots, [T_{E_j}(x_r)]), \quad (2)$$

$$R_I(x_1, x_2, \dots, x_r) \geq \max([I_{E_j}(x_1)], \dots, [I_{E_j}(x_r)]), \quad (3)$$

$$R_F(x_1, x_2, \dots, x_r) \geq \max([F_{E_j}(x_1)], \dots, [F_{E_j}(x_r)]), \quad (4)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Definition 2.8

Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set A (IVNS A) is characterized by lower truth membership function $TL_A(x)$, lower indeterminacy membership function $IL_A(x)$, lower falsity membership function $FL_A(x)$, upper truth membership function $TU_A(x)$, upper indeterminacy membership function $IU_A(x)$, upper falsity membership function $FU_A(x)$, for each point x ∈ X; $[TL_A(x), TU_A]$, $[IL_A(x), IU_A(x)]$, $[FL_A(x), FU_A(x)]$ subsets of [0, 1].

Definition 2.9

An interval valued neutrosophic hypergraph is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \dots, x_n\}$ be a finite set of vertices.

(2) $E = \{E_1, E_2, \dots, E_m\}$ be a family of IVNSs of X.

(3)$E_j \neq O = ([0, 0], [0, 0], [0, 0])$ for $j = 1, 2, 3, \dots, m$ and $\bigcup_j Supp(E_j) = X$.

The set X is called set of vertices and E is the set of IVN-edges (or IVN-hyper-edges).

Note that a given IVNHG$H = (X, E, R)$ with underlying set X, where $E = \{E_1, E_2, \dots, E_m\}$ is the collection of non-empty family of IVN subsets of X, and R is IVN relation on IVN subsets E_j such that:

$$R_{TL}(x_1, x_2, \dots, x_r) \leq \min([TL_{E_j}(x_1)], \dots, [TL_{E_j}(x_r)]), \quad (5)$$

$$R_{IL}(x_1, x_2, \dots, x_r) \geq \max([IL_{E_j}(x_1)], \dots, [IL_{E_j}(x_r)]), \quad (6)$$

$$R_{FL}(x_1, x_2, \dots, x_r) \geq \max([FL_{E_j}(x_1)], \dots, [FL_{E_j}(x_r)]), \quad (7)$$

$$R_{TU}(x_1, x_2, \dots, x_r) \leq \min([TU_{E_j}(x_1)], \dots, [TU_{E_j}(x_r)]), \quad (8)$$

$$R_{IU}(x_1, x_2, \dots, x_r) \geq \max([IU_{E_j}(x_1)], \dots, [IU_{E_j}(x_r)]), \quad (9)$$

$$R_{FU}(x_1, x_2, \dots, x_r) \geq \max([FU_{E_j}(x_1)], \dots, [FU_{E_j}(x_r)]), \quad (10)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Proposition 2.10

The interval valued neutrosophic hypergraph is the generalization of fuzzy hypergraphs, intuitionistic fuzzy hypergraphs, interval valued fuzzy hypergraphs and interval valued intuitionistic fuzzy hypergraphs.

Example 2.11

Consider the IVNHG *H = (X, E, R)* with underlying set *X = {a, b, c}*, where *E = {A, B}* and *R*, which are defined in the *Tables* given below:

H	A	B
a	([0.5,0.7], [0.2, 0.9], [0.5,0.8])	([0.3,0.5],[0.5,0.6], [0.0,0.1])
b	([0.0,0.0], [0.0,0.0], [0.0,0.0])	([0.1,0.4],[0.3,0.9],[0.9,1.0])
c	([0.2,0.3], [0.1,0.5], [0.4,0.7])	([0.5,0.9],[0.2,0.3],[0.5,0.8])

R	R_T	R_I	R_F
A	[0.1, 0.2]	[0.6, 1.0]	[0.5, 0.9]
B	[0.1, 0.3]	[0.9, 0.9]	[0.9, 1.0]

By routine calculations, *H = (X, E, R)* is IVNHG.

2 Isomorphism of SVNHGs

Definition 3.1

A homomorphism *f: H → K* between two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a mapping *f: X → Y* which satisfies the conditions:

$$\min[TL_{E_j}(x)] \leq \min[TL_{F_j}(f(x))], \quad (11)$$

$$\max[IL_{E_j}(x)] \geq \max[IL_{F_j}(f(x))], \quad (12)$$

$$\max[FL_{E_j}(x)] \geq \max[FL_{F_j}(f(x))], \quad (13)$$

$$\min[TU_{E_j}(x)] \leq \min[TU_{F_j}(f(x))], \quad (14)$$

$$\max[IU_{E_j}(x)] \geq \max[IU_{F_j}(f(x))], \quad (15)$$

$$\max[FU_{E_j}(x)] \geq \max[FU_{F_j}(f(x))], \text{ for all } x \in X. \quad (16)$$

$$R_{TL}(x_1, x_2, \dots, x_r) \leq S_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (17)$$

$$R_{IL}(x_1, x_2, \dots, x_r) \geq S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (18)$$

$$R_{FL}(x_1, x_2, \dots, x_r) \geq S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (19)$$

$$R_{TU}(x_1, x_2, \dots, x_r) \leq S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (20)$$

$$R_{IU}(x_1, x_2, \dots, x_r) \geq S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (21)$$

$$R_{FU}(x_1, x_2, \dots, x_r) \geq S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (22)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Example 3.2

Consider the two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y = {x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S*, which are defined in the *Tables* given below:

H	A	B
a	([0.2,0.3], [0.3,0.4], [0.9,1.0])	([0.5,0.6], [0.2,0.3], [0.7,0.8])
b	([0.5,0.6], [0.5,0.6], [0.5,0.6])	([0.1,0.2], [0.6,0.7], [0.4,0.5])
c	([0.8,0.9], [0.8,0.9], [0.3, 0.4])	([0.5,0.6], [0.9,1.0], [0.8,0.9])
K	C	D
x	([0.3,0.4], [0.2,0.3], [0.2,0.3])	([0.2,0.3], [0.1,0.2], [0.3,0.4])
y	([0.2,0.4], [0.4,0.5], [0.2,0.3])	([0.3,0.4], [0.2,0.3], [0.1,0.2])
z	([0.5,0.6], [0.8,0.9], [0.2, 0.3])	([0.9,0.1], [0.7,0.8], [0.1,0.2])

R	R_T	R_I	R_F
A	[0.2,0.3]	[0.8,0.9]	[0.9,1.0]
B	[0.1,0.2]	[0.9,1.0]	[0.8,0.9]
S	S_T	S_I	S_F
C	[0.2,0.3]	[0.8,0.9]	[0.3,0.4]
D	[0.1,0.2]	[0.7,0.8]	[0.3,0.4]

and *f: X → Y* defined by, *f(a)=x* , *f(b)=y* and *f(c)=z*. Then, by routine calculations, *f: H → K* is a homomorphism between *H* and *K*.

Definition 3.3

A weak isomorphism *f: H → K* between two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a bijective mapping $f : X \to Y$ which satisfies the condition f is homomorphism, such that:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \tag{23}$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \tag{24}$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \tag{25}$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \tag{26}$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \tag{27}$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \tag{28}$$

for all $x \in X$.

Note

The weak isomorphism between two IVNHGs preserves the weights of vertices.

Example 3.4

Consider the two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y={x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S*, which are defined in the *Tables* given below:

H	A	B
a	([0.2,0.3], [0.3,0.4], [0.9,1.0])	([0.5,0.6], [0.2,0.3], [0.7,0.8])
b	([0.5,0.6], [0.5,0.6], [0.5,0.6])	([0.1,0.2], [0.6,0.7], [0.4,0.5])
c	([0.8,0.9], [0.8,0.9], [0.3, 0.4])	([0.5,0.6], [0.9,1.0], [0.8,0.9])
K	C	D
x	([0.2,0.3], [0.3,0.4], [0.2,0.3])	([0.2,0.3], [0.1,0.2], [0.8,0.9])
y	([0.2,0.3], [0.4,0.5], [0.2,0.3])	([0.1,0.2], [0.6,0.7], [0.5,0.6])
z	([0.5,0.6], [0.8,0.9], [0.9, 1.0])	([0.9,1.0], [0.9,1.0], [0.1,0.2])

R	R_T	R_I	R_F
A	[0.2,0.3]	[0.8,0.9]	[0.9,1.0]
B	[0.1,0.2]	[0.9,1.0]	[0.9,1.0]
S	S_T	S_I	S_F
C	[0.2,0.3]	[0.8,0.9]	[0.9,1.0]
D	[0.1,0.2]	[0.9,1.0]	[0.8,0.9]

and *f : X → Y* defined by, *f(a)=x*, *f(b)=y* and *f(c)=z*. Then, by routine calculations, *f: H → K* is a weak isomorphism between *H* and *K*.

Definition 3.5

A co-weak isomorphism *f: H → K* between two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a bijective mapping *f : X → Y* which satisfies the condition *f* is homomorphism, such that:

$$R_{TL}(x_1, x_2, \dots, x_r) = S_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (29)$$

$$R_{IL}(x_1, x_2, \dots, x_r) = S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (30)$$

$$R_{FL}(x_1, x_2, \dots, x_r) = S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (31)$$

$$R_{TU}(x_1, x_2, \dots, x_r) = S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (32)$$

$$R_{IU}(x_1, x_2, \dots, x_r) = S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (33)$$

$$R_{FU}(x_1, x_2, \ldots, x_r) = S_{FU}(f(x_1), f(x_2), \ldots, f(x_r)), \qquad (34)$$

for all $\{x_1, x_2, \ldots, x_r\}$ subsets of *X.*

Note

The co-weak isomorphism between two IVNHGs preserves the weights of edges.

Example 3.6

Consider the two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y = {x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S,* which are defined in the *Tables* given below:

H	A	B
a	([0.2,0.3], [0.3,0.4], [0.9,1.0])	([0.5,0.6], [0.2,0.3], [0.7,0.8])
b	([0.5,0.6], [0.5,0.6], [0.5,0.6])	([0.1,0.2], [0.6,0.7], [0.4,0.5])
c	([0.8,0.9], [0.8,0.9], [0.3, 0.4])	([0.5,0.6], [0.9,1.0], [0.8,0.9])
K	C	D
x	([0.3,0.4], [0.2,0.3], [0.2,0.3])	([0.2,0.3], [0.1,0.2], [0.3,0.4])
y	([0.2,0.3], [0.4,0.5], [0.2,0.3])	([0.3,0.4], [0.2,0.3], [0.1,0.2])
z	([0.5,0.6], [0.8,0.9], [0.2, 0.3])	([0.9,1.0], [0.7,0.8], [0.1,0.2])

R	R_T	R_I	R_F
A	[0.2,0.3]	[0.8,0.9]	[0.9,1.0]
B	[0.1,0.2]	[0.9,1.0]	[0.8,0.9]
S	S_T	S_I	S_F
C	[0.2,0.3]	[0.8,0.9]	[0.9,1.0]
D	[0.1,0.2]	[0.9,1.0]	[0.8,0.9]

and $f: X \rightarrow Y$ defined by, $f(a)=x$, $f(b)=y$ and $f(c)=z$. Then, by routine calculations, $f: H \rightarrow K$ is a co-weak isomorphism between *H* and *K.*

Definition 3.7

An isomorphism $f: H \rightarrow K$ between two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a bijective mapping $f: X \rightarrow Y$ which satisfies the conditions:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \qquad (35)$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \qquad (36)$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \qquad (37)$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \quad (38)$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \quad (39)$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \quad (40)$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) = S_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (41)$$

$$R_{IL}(x_1, x_2, \dots, x_r) = S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (42)$$

$$R_{FL}(x_1, x_2, \dots, x_r) = S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (43)$$

$$R_{TU}(x_1, x_2, \dots, x_r) = S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (44)$$

$$R_{IU}(x_1, x_2, \dots, x_r) = S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (45)$$

$$R_{FU}(x_1, x_2, \dots, x_r) = S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (46)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Note

The isomorphism between two IVNHGs preserves the both weights of vertices and weights of edges.

Example 3.8

Consider the two IVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ with underlying sets $X = \{a, b, c\}$ and $Y = \{x, y, z\}$, where $E = \{A, B\}$, $F = \{C, D\}$, R and S, which are defined in the *Tables* given below,

H	A	B
a	([0.2,0.3], [0.3,0.4], [0.7,0.8])	([0.5,0.6], [0.2,0.3], [0.7,0.8])
b	([0.5,0.6], [0.5,0.6], [0.5,0.6])	([0.1,0.2], [0.6,0.7], [0.4,0.5])
c	([0.8,0.9], [0.8,0.9], [0.3, 0.4])	([0.5,0.6], [0.9,1.0], [0.8,0.9])
K	C	D
x	([0.2,0.3], [0.3,0.4], [0.2,0.3])	([0.2,0.3], [0.1,0.2], [0.8,0.9])
y	([0.2,0.3], [0.4,0.5], [0.2,0.3])	([0.1,0.2], [0.6,0.7], [0.5,0.6])
z	([0.5,0.6], [0.8,0.9], [0.7, 0.8])	([0.9,1.0], [0.9,1.0], [0.1,0.2])

R	R_T	R_I	R_F
A	[0.2,0.3]	[0.8,0.9]	[0.9,1.0]
B	[0.0,0.1]	[0.9,1.0]	[0.8,0.9]
S	S_T	S_I	S_F
C	[0.2,0.3]	[0.8,0.9]	[0.9,1.0]
D	[0.0,0.1]	[0.9,1.0]	[0.8,0.9]

and $f: X \rightarrow Y$ defined by, $f(a)=x$, $f(b)=y$ and $f(c)=z$. Then, by routine calculations, $f: H \rightarrow K$ is a isomorphism between H and K.

Definition 3.9

Let $H = (X, E, R)$ be a IVNHG; then, the order of H, which is denoted and defined by:

$$O(H)= \\ ([\sum \min TL_{E_j}(x) , \sum \min TU_{E_j}(x)] ,\ [\sum \max IL_{E_j}(x) , \sum \max IU_{E_j}(x)], \\ [\sum \max FL_{E_j}(x) , \sum \max FU_{E_j}(x)]) \quad (47)$$

and the size of H, which is denoted and defined by:

$$S(H) = ([\sum R_{TL}(E_j), \sum R_{TU}(E_j)] , [\sum R_{IL}(E_j), \sum R_{IL}(E_j)], \\ [\sum R_{FL}(E_j), \sum R_{FU}(E_j)]\ ) \quad (48)$$

Theorem 3.10

Let $H = (X, E, R)$ and $K = (Y, F, S)$ be two IVNHGs such that H is isomorphic to K; then:

(1) $O(H) = O(K)$,

(2) $S(H) = S(K)$.

Proof.

Let $f : H \rightarrow K$ be an isomorphism between two IVNHGs H and K with underlying sets X and Y respectively; then, by definition, we have that:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \quad (49)$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \quad (50)$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \quad (51)$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \quad (52)$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \quad (53)$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \quad (54)$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) = S_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (55)$$

$$R_{IL}(x_1, x_2, \dots, x_r) = S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (56)$$

$$R_{FL}(x_1, x_2, \dots, x_r) = S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (57)$$

$$R_{TU}(x_1, x_2, \dots, x_r) = S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{58}$$

$$R_{IU}(x_1, x_2, \dots, x_r) = S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{59}$$

$$R_{FU}(x_1, x_2, \dots, x_r) = S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{60}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X*.

Consider:

$$O_{TL}(H) = \sum \min TL_{E_j}(x) = \sum \min TL_{F_j}(f(x)) = O_{TL}(K) \tag{61}$$

$$O_{TU}(H) = \sum \min TU_{E_j}(x) = \sum \min TU_{F_j}(f(x)) = O_{TU}(K) \tag{62}$$

Similarly:

$$O_{IL}(H) = O_{IL}(K) and O_{FL}(H) = O_{FL}(K), \tag{63}$$

$$O_{IU}(H) = O_{IU}(K) and O_{FU}(H) = O_{FU}(K). \tag{64}$$

Hence, *O(H) = O(K)*.

Next,

$$S_{TL}(H) = \sum R_{TL}(x_1, x_2, \dots, x_r)$$

$$= \sum S_{TL}(f(x_1), f(x_2), \dots, f(x_r)) = S_{TL}(K), \tag{65}$$

and similarly:

$$S_{TU}(H) = \sum R_{TU}(x_1, x_2, \dots, x_r)$$

$$= \sum S_{TU}(f(x_1), f(x_2), \dots, f(x_r)) = S_{TU}(K) \tag{66}$$

Similarly,

$$S_{IL}(H) = S_{IL}(K), S_{FL}(H) = S_{FL}(K), \tag{67}$$

$$S_{IU}(H) = S_{IU}(K), S_{FU}(H) = S_{FU}(K), \tag{68}$$

hence $S(H) = S(K)$.

Remark 3.11

The converse of the above theorem needs not to be true in general.

Example 3.12

Consider the two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S,* which are defined in the *Tables* given below:

H	A	B
a	([0.2, 0.3],[0.5,0.6],[0.33,0.43])	([0.16,0.26],[0.5,0.6],[0.33,0.43])
b	([0.0,0.0], [0.0,0.0], [0.0,0.0])	([0.2,0.3],[0.5,0.6],[0.33,0.43])
c	([0.33,0.43],[0.5,0.6],[0.33,0.43])	([0.2,0.3], [0.5,0.6],[0.33,0.43])
d	([0.5,0.6],[0.5,0.6],[0.33,0.43])	([0.0,0.0], [0.0,0.0], [0.0,0.0])
K	C	D
w	([0.2,0.3],[0.5,0.6],[0.33,0.43])	([0.16,0.26],[0.5,0.6],[0.33,0.43])
x	([0.0,0.0], [0.0,0.0], [0.0,0.0])	([0.2,0.3],[0.5,0.6],[0.33,0.43])
y	([0.33,0.43],[0.5,0.6],[0.33,0.43])	([0.2,0.3], [0.5,0.6],[0.33,0.43])
z	([0.5,0.6],[0.5,0.6],[0.33,0.43])	([0.0,0.0], [0.0,0.0], [0.0,0.0])

R	R_T	R_I	R_F
A	[0.2,0.3]	[0.5,0.6]	[0.33,0.43]
B	[0.16,0.26]	[0.5,0.6]	[0.33,0.43]
S	S_T	S_I	S_F
C	[0.16,0.26]	[0.5,0.6]	[0.33,0.43]
D	[0.2,0.3]	[0.5,0.6]	[0.33,0.43]

where *f* is defined by *f(a)=w, f(b)=x, f(c)=y, f(d)=z*.

Here, *O(H) = ([1.06, 1.46], [2.0, 2.4], [1.32, 1.72]) = O(K)* and *S(H) = ([0.36, 0.56], [1.0, 1.2], [0.66, 0.86]) = S(K)*.

By routine calculations, *H* is not isomorphism to *K*.

Corollary 3.13

The weak isomorphism between any two IVNHGs *H* and *K* preserves the orders.

Remark 3.14

The converse of the above corollary need not to be true in general.

Example 3.15

Consider the two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S,* which are defined in the *Tables* given below, where *f* is defined by *f(a)=w, f(b)=x, f(c)=y, f(d)=z.*

Here *O(H)= ([1.0,1.4], [2.0,2.4], [1.2,1.6]) = O(K).*

By routine calculations, *H* is not weak isomorphism to *K*.

H	A	B
a	([0.2,0.3],[0.5,0.6],[0.3,0.4])	([0.14,0.24],[0.5,0.6],[0.3,0.4])
b	([0.0,0.0], [0.0,0.0], [0.0,0.0])	([0.2,0.3],[0.5,0.6],[0.3,0.4])
c	([0.33,0.43],[0.5,0.6],[0.3,0.4])	([0.16,0.26], [0.5,0.6], [0.3,0.4])
d	([0.5,0.6], [0.5,0.6], [0.3,0.4])	([0.0,0.0], [0.0,0.0], [0.0,0.0])
K	C	D
w	([0.14,0.24],[0.5,0.6],[0.3,0.4])	([0.16,0.26],[0.5,0.6],[0.3,0.4])
x	([0.0,0.0], [0.0,0.0], [0.0,0.0])	([0.16,0.26],[0.5,0.6],[0.3,0.4])
y	([0.33,0.43],[0.5,0.6],[0.33,0.43])	([0.2,0.3], [0.5,0.6],[0.3,0.4])
z	([0.5,0.6],[0.5,0.6],[0.3,0.4])	([0.0,0.0], [0.0,0.0], [0.0,0.0])

Corollary 3.16

The co-weak isomorphism between any two IVNHGs *H* and *K* preserves the sizes.

Remark 3.17

The converse of the above corollary need not to be true in general.

Example 3.18

Consider the two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S*, which are defined in the *Tables* given below, where *f* is defined by, *f(a)=w, f(b)=x, f(c)=y, f(d)=z.* Here *S(H)= ([0.34,0.54], [1.0,1.2], [0.6,0.8]) = S(K)*, but, by routine calculations, *H* is not co-weak isomorphism to *K*.

H	A	B
a	([0.2,0.3],[0.5,0.6],[0.3,0.4])	([0.14,0.24],[0.5,0.6],[0.3,0.4])
b	([0.0,0.0], [0.0,0.0], [0.0,0.0])	([0.16,0.26],[0.5,0.6],[0.3,0.4])
c	([0.3,0.4],[0.5,0.6],[0.3,0.4])	([0.2,0.3], [0.5,0.6], [0.3,0.4])
d	([0.5,0.6], [0.5,0.6], [0.3,0.4])	([0.0,0.0], [0.0,0.0], [0.0,0.0])
K	C	D
w	([0.0,0.0],[0.0,0.0],[0.0,0.0])	([0.2,0.3],[0.5,0.6],[0.3,0.4])
x	([0.14,0.24], [0.5,0.6], [0.3,0.4])	([0.25,0.35],[0.5,0.6],[0.3,0.4])
y	([0.5,0.6], [0.5,0.6], [0.3,0.4])	([0.2,0.3], [0.5,0.6],[0.3,0.4])
z	([0.3,0.4], [0.5,0.6], [0.3,0.4])	([0.0,0.0], [0.0,0.0], [0.0,0.0])

R	R_T	R_I	R_F
A	[0.2,0.3]	[0.5,0.6]	[0.3,0.4]
B	[0.14,0.24]	[0.5,0.6]	[0.3,0.4]
S	S_T	S_I	S_F
C	[0.14,0.24]	[0.5,0.6]	[0.3,0.4]
D	[0.2,0.3]	[0.5,0.6]	[0.3,0.4]

Definition 3.19

Let $H = (X, E, R)$ be a IVNHG; then, the degree of vertex x_i is denoted and defined by:

$$\deg(x_i) = ([deg_{TL}(x_i), deg_{TU}(x_i)], [deg_{IL}(x_i), deg_{IU}(x_i)], [deg_{FL}(x_i), deg_{FU}(x_i)]), \tag{69}$$

where

$$deg_{TL}(x_i) = \sum R_{TL}(x_1, x_2, \dots, x_r), \tag{70}$$

$$deg_{IL}(x_i) = \sum R_{IL}(x_1, x_2, \dots, x_r), \tag{71}$$

$$deg_{FL}(x_i) = \sum R_{FL}(x_1, x_2, \dots, x_r), \tag{72}$$

$$deg_{TU}(x_i) = \sum R_{TU}(x_1, x_2, \dots, x_r), \tag{73}$$

$$deg_{IU}(x_i) = \sum R_{IU}(x_1, x_2, \dots, x_r), \tag{74}$$

$$deg_{FU}(x_i) = \sum R_{FU}(x_1, x_2, \dots, x_r), \tag{75}$$

for $x_i \neq x_r$.

Theorem 3.20

If H and K are two isomorphic IVNHGs, then the degree of their vertices are preserved.

Proof.

Let $f: H \rightarrow K$ be an isomorphism between two IVNHGs H and K with underlying sets X and Y, respectively. Then, by definition, we have:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \tag{75}$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \tag{77}$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \tag{78}$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \tag{79}$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \tag{80}$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \tag{81}$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) = S_{TL}(f(x_1), f(x_1), \dots, f(x_r)), \tag{82}$$

$$R_{IL}(x_1, x_2, \dots, x_r) = S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{83}$$

$$R_{FL}(x_1, x_2, \dots, x_r) = S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{84}$$

$$R_{TU}(x_1, x_2, \dots, x_r) = S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{85}$$

$$R_{IU}(x_1, x_2, \dots, x_r) = S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{86}$$

$$R_{FU}(x_1, x_2, \dots, x_r) = S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{87}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Consider,

$$deg_{TL}(x_i) = \sum R_{TL}(x_1, x_2, \dots, x_r) = \sum S_{TL}(f(x_1), f(x_2), \dots, f(x_r)) = deg_{TL}(f(x_i)) \tag{88}$$

and similarly:

$$deg_{TU}(x_i) = deg_{TU}(f(x_i)), \tag{89}$$

$$deg_{IL}(x_i) = deg_{IL}(f(x_i)), deg_{FL}(x_i) = deg_{FL}(f(x_i)), \tag{90}$$

$$deg_{IU}(x_i) = deg_{IU}(f(x_i)), deg_{FU}(x_i) = deg_{FU}(f(x_i)). \tag{91}$$

Hence,

$$deg(x_i) = deg(f(x_i)). \tag{92}$$

Remark 3.21

The converse of the above theorem may not be true in general.

Example 3.22

Consider the two IVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ with underlying sets $X = \{a, b\}$ and $Y = \{x, y\}$, where $E = \{A, B\}$, $F = \{C, D\}$, R and S, which are defined in the *Tables* given below, where f is defined by $f(a)=x$, $f(b)=y$, where $deg(a) = ([0.8,1.0], [1.0,1.2], [0.6,0.8]) = deg(x)$ and $deg(b) = ([0.45,0.65], [1.0,1.2], [0.6,0.8]) = deg(y)$. But H is not isomorphic to K, i.e. H is neither weak isomorphic nor co-weak isomorphic K.

H	A	B
a	([0.5,0.6], [0.5,0.6], [0.3,0.4])	([0.3,0.4],[0.5,0.6],[0.3,0.4])
b	([0.25,0.35], [0.5,0.6], [0.3,0.4])	([0.2,0.3], [0.5,0.6], [0.3, 0.4])

K	C	D
x	([0.3,0.4], [0.5,0.6], [0.3,0.4])	([0.5,0.6],[0.5,0.6],[0.3,0.4])
y	([0.2,0.3], [0.5,0.6], [0.3,0.4])	([0.25,0.34], [0.5,0.6], [0.3,0.4])

R	R_T	R_I	R_F
A	[0.25,0.35]	[0.5,0.6]	[0.3,0.4]
B	[0.2,0.3]	[0.5,0.6]	[0.3,0.4]

S	S_T	S_I	S_F
C	[0.2,0.3]	[0.5,0.6]	[0.3,0.4]
D	[0.25,0.35]	[0.5,0.6]	[0.3,0.4]

Theorem 3.23

The isomorphism between IVNHGs is an equivalence relation.

Proof.

Let $H = (X, E, R)$, $K = (Y, F, S)$ and $M = (Z, G, W)$ be IVNHGs with underlying sets X, Y and Z, respectively:

Reflexive.

Consider the map (identity map) $f: X \to X$, defined as follows: $f(x) = x$ for all $x \in X$, since the identity map is always bijective and satisfies the conditions:

$$\min[TL_{E_j}(x)] = \min[TL_{E_j}(f(x))], \tag{93}$$

$$\max[IL_{E_j}(x)] = \max[IL_{E_j}(f(x))], \tag{94}$$

$$\max[FL_{E_j}(x)] = \max[FL_{E_j}(f(x))], \tag{95}$$

$$\min[TU_{E_j}(x)] = \min[TU_{E_j}(f(x))], \tag{96}$$

$$\max[IU_{E_j}(x)] = \max[IU_{E_j}(f(x))], \tag{97}$$

$$\max[FU_{E_j}(x)] = \max[FU_{E_j}(f(x))], \tag{98}$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) = R_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{99}$$

$$R_{IL}(x_1, x_2, \dots, x_r) = R_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{100}$$

$$R_{FL}(x_1, x_2, \dots, x_r) = R_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{101}$$

$$R_{TU}(x_1, x_2, \dots, x_r) = R_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{102}$$

$$R_{IU}(x_1, x_2, \dots, x_r) = R_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{103}$$

$$R_{FU}(x_1, x_2, \ldots, x_r) = R_{FU}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{104}$$

for all $\{x_1, x_2, \ldots, x_r\}$ subsets of X.

Hence f is an isomorphism of IVNHG H to itself.

Symmetric.

Let $f: X \to Y$ be an isomorphism of H and K, then f is bijective mapping defined as: $f(x) = y$ for all $x \in X$. Then, by definition:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \tag{105}$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \tag{106}$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \tag{107}$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \tag{108}$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \tag{109}$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \tag{110}$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \ldots, x_r) = S_{TL}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{111}$$

$$R_{IL}(x_1, x_2, \ldots, x_r) = S_{IL}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{112}$$

$$R_{FL}(x_1, x_2, \ldots, x_r) = S_{FL}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{113}$$

$$R_{TU}(x_1, x_2, \ldots, x_r) = S_{TU}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{114}$$

$$R_{IU}(x_1, x_2, \ldots, x_r) = S_{IU}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{115}$$

$$R_{FU}(x_1, x_2, \ldots, x_r) = S_{FU}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{116}$$

for all $\{x_1, x_2, \ldots, x_r\}$ subsets of X. Since f is bijective, then we have $f^{-1}(y) = x$ for all $y \in Y$. Thus, we get:

$$\min[TL_{E_j}\left(f^{-1}(y)\right)] = \min[TL_{F_j}(y)], \tag{117}$$

$$\max[IL_{E_j}\left(f^{-1}(y)\right)] = \max[IL_{F_j}(y)], \tag{118}$$

$$\max[FL_{E_j}\left(f^{-1}(y)\right)] = \max[FL_{F_j}(y)], \tag{119}$$

$$\min[TU_{E_j}\left(f^{-1}(y)\right)] = \min[TU_{F_j}(y)], \tag{120}$$

$$\max[IU_{E_j}\left(f^{-1}(y)\right)] = \max[IU_{F_j}(y)], \tag{121}$$

$$\max[FU_{E_j}\left(f^{-1}(y)\right)] = \max[FU_{F_j}(y)], \tag{122}$$

for all $x \in X$.

$$R_{TL}\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_{TL}(y_1, y_2, \dots, y_r), \quad (123)$$

$$R_{IL}\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_{IL}(y_1, y_2, \dots, y_r), \quad (124)$$

$$R_{FL}\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_{FL}(y_1, y_2, \dots, y_r), \quad (125)$$

$$R_{TU}\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_{TU}(y_1, y_2, \dots, y_r), \quad (126)$$

$$R_{IU}\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_{IU}(y_1, y_2, \dots, y_r), \quad (127)$$

$$R_{FU}\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_{FU}(y_1, y_2, \dots, y_r), \quad (128)$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of Y.

Hence we have a bijective map $f^{-1} : Y \to X$, which is an isomorphism from K to H.

Transitive.

Let $f : X \to Y$ and $g : Y \to Z$ be two isomorphism of IVNHGs of H onto K and K onto M respectively. Then gof is bijective mapping from X to Z, where gof is defined as $(gof)(x) = g\big(f(x)\big)$ for all $x \in X$.

Since f is isomorphism, then, by definition, $f(x) = y$ for all $x \in X$, which satisfies the conditions:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}\big(f(x)\big)], \quad (129)$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}\big(f(x)\big)], \quad (130)$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}\big(f(x)\big)], \quad (131)$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}\big(f(x)\big)], \quad (132)$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}\big(f(x)\big)], \quad (133)$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}\big(f(x)\big)], \quad (134)$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) = S_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (135)$$

$$R_{IL}(x_1, x_2, \dots, x_r) = S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (136)$$

$$R_{FL}(x_1, x_2, \dots, x_r) = S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (137)$$

$$R_{TU}(x_1, x_2, \dots, x_r) = S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (138)$$

$$R_{IU}(x_1, x_2, \dots, x_r) = S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (139)$$

$$R_{FU}(x_1, x_2, \dots, x_r) = S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (140)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X. Since $g : Y \to Z$ is isomorphism, then by definition $g(y) = z$ for all $y \in Y$ satisfy the conditions:

$$\min[TL_{F_j}(y)] = \min\left[TL_{G_j}\big(g(y)\big)\right], \tag{141}$$

$$\max[IL_{F_j}(y)] = \max[IL_{G_j}\big(g(y)\big)], \tag{142}$$

$$\max[FL_{F_j}(y)] = \max[FL_{G_j}\big(g(y)\big)], \tag{143}$$

$$\min[TU_{F_j}(y)] = \min[TU_{G_j}\big(g(y)\big)], \tag{144}$$

$$\max[IU_{F_j}(y)] = \max[IU_{G_j}\big(g(y)\big)], \tag{145}$$

$$\max[FU_{F_j}(y)] = \max[FU_{G_j}\big(g(y)\big)], \tag{146}$$

for all $x \in X$.

$$S_{TL}(y_1, y_2, \dots, y_r) = W_{TL}(g(y_1), g(y_2), \dots, g(y_r)), \tag{147}$$

$$S_{IL}(y_1, y_2, \dots, y_r) = W_{IL}(g(y_1), g(y_2), \dots, g(y_r)), \tag{148}$$

$$S_{FL}(y_1, y_2, \dots, y_r) = W_{FL}(g(y_1), g(y_2), \dots, g(y_r)), \tag{149}$$

$$S_{TU}(y_1, y_2, \dots, y_r) = W_{TU}(g(y_1), g(y_2), \dots, g(y_r)), \tag{150}$$

$$S_{IU}(y_1, y_2, \dots, y_r) = W_{IU}(g(y_1), g(y_2), \dots, g(y_r)), \tag{151}$$

$$S_{FU}(y_1, y_2, \dots, y_r) = W_{FU}(g(y_1), g(y_2), \dots, g(y_r)), \tag{152}$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of Y. Thus, from the above equations, we conclude that:

$$\min[TL_{E_j}(x)] = \min\left[TL_{G_j}\left(g\big(f(x)\big)\right)\right], \tag{153}$$

$$\max[IL_{E_j}(x)] = \max[IL_{G_j}(g(f(x)))], \tag{154}$$

$$\max[FL_{E_j}(x)] = \max[FL_{G_j}(g(f(x)))], \tag{155}$$

$$\min[TU_{E_j}(x)] = \min\left[TU_{G_j}\left(g\big(f(x)\big)\right)\right], \tag{156}$$

$$\max[IU_{E_j}(x)] = \max[IU_{G_j}(g(f(x)))], \tag{157}$$

$$\max[FU_{E_j}(x)] = \max[FU_{G_j}(g(f(x)))], \tag{158}$$

for all $x \in X$.

$$R_{TL}(x_1, \dots, x_r) = W_{TL}(g\big(f(x_1)\big), \dots, g\big(f(x_r)\big)), \tag{159}$$

$$R_{IL}(x_1, \dots, x_r) = W_{IL}(g\big(f(x_1)\big), \dots, g\big(f(x_r)\big)), \tag{160}$$

$$R_{FL}(x_1, \dots, x_r) = W_{FL}(g\big(f(x_1)\big), \dots, g\big(f(x_r)\big)), \tag{161}$$

$$R_{TU}(x_1, \dots, x_r) = W_{TU}(g\big(f(x_1)\big), \dots, g\big(f(x_r)\big)), \tag{162}$$

$$R_{IU}(x_1, \dots, x_r) = W_{IU}(g(f(x_1)), \dots, g(f(x_r))), \quad (163)$$

$$R_{FU}(x_1, \dots, x_r) = W_{FU}(g(f(x_1)), \dots, g(f(x_r))), \quad (164)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Therefore, gof is an isomorphism between *H* and *M*. Hence, the isomorphism between IVNHGs is an equivalence relation.

Theorem 3.24

The weak isomorphism between IVNHGs satisfies the partial order relation.

Proof.

Let *H = (X, E, R), K = (Y, F, S)* and *M = (Z, G, W)* be IVNHGs with underlying sets X, Y and *Z* respectively,

Reflexive.

Consider the map (identity map) *f: X → X,* defined as follows: *f(x)=x* for all *x* ∈ *X*, since identity map is always bijective and satisfies the conditions:

$$\min[TL_{E_j}(x)] = \min[TL_{E_j}(f(x))], \quad (165)$$

$$\max[IL_{E_j}(x)] = \max[IL_{E_j}(f(x))], \quad (166)$$

$$\max[FL_{E_j}(x)] = \max[FL_{E_j}(f(x))], \quad (167)$$

$$\min[TU_{E_j}(x)] = \min[TU_{E_j}(f(x))], \quad (168)$$

$$\max[IU_{E_j}(x)] = \max[IU_{E_j}(f(x))], \quad (169)$$

$$\max[FU_{E_j}(x)] = \max[FU_{E_j}(f(x))], \quad (170)$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) \le R_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (171)$$

$$R_{IL}(x_1, x_2, \dots, x_r) \ge R_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (172)$$

$$R_{FL}(x_1, x_2, \dots, x_r) \ge R_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (173)$$

$$R_{TU}(x_1, x_2, \dots, x_r) \le R_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (174)$$

$$R_{IU}(x_1, x_2, \dots, x_r) \ge R_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (175)$$

$$R_{FU}(x_1, x_2, \dots, x_r) \ge R_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (176)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Hence *f* is a weak isomorphism of IVNHG *H* to itself.

Anti-symmetric.

Let f be a weak isomorphism between H onto K, and g be weak isomorphic between K and H, i.e. $f: X \to Y$ is a bijective map defined by: $f(x) = y$ for all $x \in X$ satisfying the conditions:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \tag{177}$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \tag{178}$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \tag{179}$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \tag{180}$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \tag{181}$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \tag{182}$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) \leq S_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{183}$$

$$R_{IL}(x_1, x_2, \dots, x_r) \geq S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{184}$$

$$R_{FL}(x_1, x_2, \dots, x_r) \geq S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{185}$$

$$R_{TU}(x_1, x_2, \dots, x_r) \leq S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{186}$$

$$R_{IU}(x_1, x_2, \dots, x_r) \geq S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{187}$$

$$R_{FU}(x_1, x_2, \dots, x_r) \geq S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{188}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.
Since g is also bijective map $g(y) = x$ for all $y \in Y$ satisfying the conditions:

$$\min[TL_{F_j}(y)] = \min[TL_{E_j}(g(y))], \tag{189}$$

$$\max[IL_{F_j}(y)] = \max[IL_{E_j}(g(y))], \tag{190}$$

$$\max[FL_{F_j}(y)] = \max[FL_{E_j}(g(y))], \tag{191}$$

$$\min[TU_{F_j}(y)] = \min[TU_{E_j}(g(y))], \tag{192}$$

$$\max[IU_{F_j}(y)] = \max[IU_{E_j}(g(y))], \tag{193}$$

$$\max[FU_{F_j}(y)] = \max[FU_{E_j}(g(y))], \tag{194}$$

for all $y \in Y$.

$$R_{TL}(y_1, y_2, \dots, y_r) \leq S_{TL}(g(y_1), g(y_2), \dots, g(y_r)), \tag{195}$$

$$R_{IL}(y_1, y_2, \dots, y_r) \geq S_{IL}(g(y_1), g(y_2), \dots, g(y_r)), \tag{196}$$

$$R_{FL}(y_1, y_2, \dots, y_r) \geq S_{FL}(g(y_1), g(y_2), \dots, g(y_r)), \tag{197}$$

$$R_{TU}(y,y_2,\dots,y_r) \le S_{TU}(g(y_1),g(y_2),\dots,g(y_r)), \tag{198}$$

$$R_{IU}(y_1,y_2,\dots,y_r) \ge S_{IU}(g(y_1),g(y_2),\dots,g(y_r)), \tag{199}$$

$$R_{FU}(y_1,y_2,\dots,y_r) \ge S_{FU}(g(y_1),g(y_2),\dots,g(y_r)), \tag{200}$$

for all $\{y_1,y_2,\dots,y_r\}$ subsets of Y.

The above inequalities hold for finite sets X and Y only whenever H and K have the same number of edges, and the corresponding edge have same weights, hence H is identical to K.

Transitive.

Let $f{:}\,X \to Y$ and $g{:}\,Y \to Z$ be two weak isomorphism of IVNHGs of H onto K and K onto M, respectively. Then gof is bijective mapping from X to Z, where gof is defined as $(gof)(x) = g(f(x))$ for all $x \in X$.

Since f is a weak isomorphism, then by definition $f(x) = y$ for all $x \in X$ which satisfies the conditions:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \tag{201}$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \tag{202}$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \tag{203}$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \tag{204}$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \tag{205}$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \tag{206}$$

for all $x \in X$.

$$R_{TL}(x_1,x_2,\dots,x_r) \le S_{TL}(f(x_1),f(x_2),\dots,f(x_r)), \tag{207}$$

$$R_{IL}(x_1,x_2,\dots,x_r) \ge S_{IL}(f(x_1),f(x_2),\dots,f(x_r)), \tag{208}$$

$$R_{FL}(x_1,x_2,\dots,x_r) \ge S_{FL}(f(x_1),f(x_2),\dots,f(x_r)), \tag{209}$$

$$R_{TU}(x_1,x_2,\dots,x_r) \le S_{TU}(f(x_1),f(x_2),\dots,f(x_r)), \tag{210}$$

$$R_{IU}(x_1,x_2,\dots,x_r) \ge S_{IU}(f(x_1),f(x_2),\dots,f(x_r)), \tag{211}$$

$$R_{FU}(x_1,x_2,\dots,x_r) \ge S_{FU}(f(x_1),f(x_2),\dots,f(x_r)), \tag{212}$$

for all $\{x_1,x_2,\dots,x_r\}$ subsets of X.

Since $g{:}\,Y \to Z$ is a weak isomorphism, then by definition $g(y) = z$ for all $y \in Y$ which satisfies the conditions:

$$\min[TL_{F_j}(y)] = \min[TL_{G_j}(g(y))], \tag{213}$$

$$\max[IL_{F_j}(y)] = \max[IL_{G_j}(g(y))], \tag{214}$$

$$\max[FL_{F_j}(y)] = \max[FL_G(g(y))], \quad (215)$$

$$\min[TU_{F_j}(y)] = \min[TU_{G_j}(g(y))], \quad (216)$$

$$\max[IU_{F_j}(y)] = \max[IU_{G_j}(g(y))], \quad (217)$$

$$\max[FU_{F_j}(y)] = \max[FU_G(g(y))], \quad (218)$$

for all $x \in X$.

$$S_{TL}(y_1, y_2, \dots, y_r) \leq W_{TL}(g(y_1), g(y_2), \dots, g(y_r)), \quad (219)$$

$$S_{IL}(y_1, y_2, \dots, y_r) \geq W_{IL}(g(y_1), g(y_2), \dots, g(y_r)), \quad (210)$$

$$S_{FL}(y_1, y_2, \dots, y_r) \geq W_{FL}(g(y_1), g(y_2), \dots, g(y_r)), \quad (211)$$

$$S_{TU}(y_1, y_2, \dots, y_r) \leq W_{TU}(g(y_1), g(y_2), \dots, g(y_r)), \quad (212)$$

$$S_{IU}(y_1, y_2, \dots, y_r) \geq W_{IU}(g(y_1), g(y_2), \dots, g(y_r)), \quad (213)$$

$$S_{FU}(y_1, y_2, \dots, y_r) \geq W_{FU}(g(y_1), g(y_2), \dots, g(y_r)), \quad (214)$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of Y.

Thus, from the above equations, we conclude that,

$$\min[TL_{E_j}(x)] = \min[TL_{G_j}(g(f(x)))], \quad (215)$$

$$\max[IL_{E_j}(x)] = \max[IL_{G_j}(g(f(x)))], \quad (216)$$

$$\max[FL_{E_j}(x)] = \max[FL_{G_j}(g(f(x)))], \quad (217)$$

$$\min[TU_{E_j}(x)] = \min[TU_{G_j}(g(f(x)))], \quad (219)$$

$$\max[IU_{E_j}(x)] = \max[IU_{G_j}(g(f(x)))], \quad (220)$$

$$\max[FU_{E_j}(x)] = \max[FU_{G_j}(g(f(x)))], \quad (221)$$

for all $x \in X$.

$$R_{TL}(x_1, \dots, x_r) \leq W_{TL}(g(f(x_1)), \dots, g(f(x_r))), \quad (222)$$

$$R_{IL}(x_1, \dots, x_r) \geq W_{IL}(g(f(x_1)), \dots, g(f(x_r))), \quad (223)$$

$$R_{FL}(x_1, \dots, x_r) \geq W_{FL}(g(f(x_1)), \dots, g(f(x_r))), \quad (224)$$

$$R_{TU}(x_1, \dots, x_r) \leq W_{TU}(g(f(x_1)), \dots, g(f(x_r))), \quad (225)$$

$$R_{IU}(x_1, \dots, x_r) \geq W_{IU}(g(f(x_1)), \dots, g(f(x_r))), \quad (226)$$

$$R_{FU}(x_1, \dots, x_r) \geq W_{FU}(g(f(x_1)), \dots, g(f(x_r))), \quad (227)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Therefore, gof is a weak isomorphism between H and M. Hence, the weak isomorphism between IVNHGs is a partial order relation.

4 Conclusion

The concepts of interval valued neutrosophic hypergraphs can be applied in various areas of engineering and computer science. In this paper, the isomorphism between IVNHGs is proved to be an equivalence relation and the weak isomorphism is proved to be a partial order relation. Similarly, it can be proved that the co-weak isomorphism in IVNHGs is a partial order relation.

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An Isolated Interval Valued Neutrosophic Graph

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Abstract

The interval valued neutrosophic graphs are generalizations of the fuzzy graphs, interval fuzzy graphs, interval valued intuitionstic fuzzy graphs, and single valued neutrosophic graphs. Previously, several results have been proved on the isolated graphs and the complete graphs. In this paper, a necessary and sufficient condition for an interval valued neutrosophic graph to be an isolated interval valued neutrosophic graph is proved.

Keyword

interval valued neutrosophic graphs, complete interval valued neutrosophic graphs, isolated interval valued neutrosophic graphs.

1 Introduction

To express indeterminate and inconsistent information which exists in real world, Smarandache [9] originally proposed the concept of the neutrosophic set from a philosophical point of view. The concept of the neutrosophic set (NS) is a generalization of the theories of fuzzy sets [14], intuitionistic fuzzy sets [15], interval valued fuzzy set [12] and interval-valued intuitionistic fuzzy sets [14].

The neutrosophic sets are characterized by a truth-membership function (t), an indeterminacy-membership function (i) and a falsity-membership function (f) independently, which are within the real standard or nonstandard unit interval $]^{-}0, 1^{+}[$.

Further on, Wang et al. [10] introduced the concept of a single-valued neutrosophic sets (SVNS), a subclass of the neutrosophic sets. The same authors [11] introduced the interval valued neutrosophic sets (IVNS), as a generalization of the single valued neutrosophic sets, in which three membership functions are independent and their value belong to the unit interval [0, 1]. Some more work on single valued neutrosophic sets, interval valued neutrosophic sets, and their applications, may be found in [1, 5, 7,8, 29, 30, 31, 37, 38].

Graph theory has become a major branch of applied mathematics, and it is generally regarded as a branch of combinatorics. Graph is a widely-used tool for solving combinatorial problems in different areas, such as geometry, algebra, number theory, topology, optimization and computer science. Most important thing which is to be noted is that, when we have uncertainty regarding either the set of vertices, or edges, or both, the model becomes a fuzzy graph.

In the literature, many extensions of fuzzy graphs have been deeply studied by several researchers, such as intuitionistic fuzzy graphs, interval valued fuzzy graphs, interval valued intuitionistic fuzzy graphs [2, 3, 16, 17, 18, 19, 20, 21, 22, 34].

But, when the relations between nodes (or vertices) in problems are indeterminate and inconsistent, the fuzzy graphs and their extensions fail. To overcome this issue Smarandache [5, 6, 7, 37] have defined four main categories of neutrosophic graphs: two are based on literal indeterminacy (I), (the I-edge neutrosophic graph and the I-vertex neutrosophic graph, [6, 36]), and the two others graphs are based on (t, i, f) components (the (t, i, f)-edge neutrosophic graph and the (t, i, f)-vertex neutrosophic graph, not developed yet).

Later, Broumi et al. [23] presented the concept of single valued neutrosophic graphs by combining the single valued neutrosophic set theory and the graph theory, and defined different types of single valued neutrosophic graphs (SVNG) including the strong single valued neutrosophic graph, the constant single valued neutrosophic graph, the complete single valued neutrosophic graph, and investigated some of their properties with proofs and suitable illustrations.

Concepts like size, order, degree, total degree, neighborhood degree and closed neighborhood degree of vertex in a single valued neutrosophic graph are introduced, along with theoretical analysis and examples, by Broumi al. in [24]. In addition, Broumi et al. [25] introduced the concept of isolated single valued neutrosophic graphs. Using the concepts of bipolar neutrosophic sets, Broumi et al. [32] also introduced the concept of bipolar single neutrosophic graph, as the generalization of the bipolar fuzzy graphs, N-graphs, intuitionistic fuzzy graph, single valued neutrosophic graphs and bipolar intuitionistic fuzzy graphs. Same authors [33] proposed different types of bipolar single valued neutrosophic graphs, such as bipolar single valued neutrosophic graphs, complete bipolar single valued neutrosophic graphs, regular bipolar single valued neutrosophic graphs, studying some of their related properties. Moreover, in [26, 27, 28], the authors introduced the concept of interval valued neutrosophic graph as a generalization of fuzzy graph, intuitionistic fuzzy graph and single valued neutrosophic graph, and discussed some of their properties with examples.

The aim of this paper is to prove a necessary and sufficient condition for an interval valued neutrosophic graph to be an isolated interval valued neutrosophic graph.

2 Preliminaries

In this section, we mainly recall some notions related to neutrosophic sets, single valued neutrosophic sets, fuzzy graph, intuitionistic fuzzy graph, single valued neutrosophic graphs and interval valued neutrosophic graph, relevant to the present work. See especially [2, 9, 10, 22, 23, 26] for further details and background.

Definition 2.1 [9]

Let X be a space of points (objects) with generic elements in X denoted by x; then the neutrosophic set A (NS A) is an object having the form A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$>, x ∈ X}, where the functions T, I, F: X→]−0,1+[define respectively the a truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element x ∈ X to the set A with the condition:

$$^{-}0 \leq T_A(x)+ I_A(x)+ F_A(x)\leq 3^{+}. \qquad (1)$$

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^{-}0,1^{+}[$.

Since it is difficult to apply NSs to practical problems, Wang et al. [10] introduced the concept of a SVNS, which is an instance of a NS, and can be used in real scientific and engineering applications.

Definition 2.2 [10]

Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by the truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X, $T_A(x), I_A(x), F_A(x) \in [0, 1]$. A SVNS A can be written as

$$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in X\} \tag{2}$$

Definition 2.3 [2]

A fuzzy graph is a pair of functions G = (σ, μ) where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ, i.e. $\sigma : V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$, for all u, v ∈ V, where uv denotes the edge between u and v and $\sigma(u) \wedge \sigma(v)$ denotes the minimum of σ(u) and σ(v). σ is called the fuzzy vertex set of V and μ is called the fuzzy edge set of E.

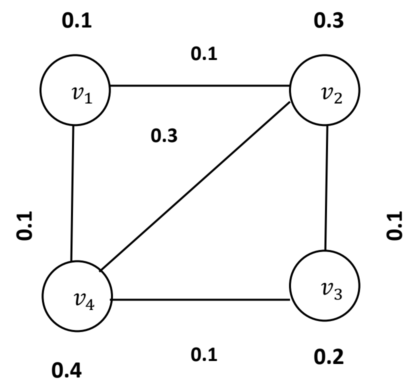

Figure 1. Fuzzy Graph.

Definition 2.4 [2]

The fuzzy subgraph H = (τ, ρ) is called a fuzzy subgraph of G = (σ, μ) if $\tau(u) \leq \sigma(u)$ for all u ∈ V and $\rho(u, v) \leq \mu(u, v)$ for all u, v ∈ V.

Definition 2.5 [22]

An intuitionistic fuzzy graph is of the form G = (V, E), where:

i. $V=\{v_1, v_2,\ldots, v_n\}$ such that $\mu_1: V\to [0,1]$ and $\gamma_1: V \to [0,1]$ denote the degree of membership and nonmembership of the element $v_i \in V$, respectively, and $0 \leq \mu_1(v_i) + \gamma_1(v_i)) \leq 1$ for every $v_i \in V$, (i = 1, 2, ..., n);

ii. $E \subseteq V \times V$ where $\mu_2: VxV\to[0,1]$ and $\gamma_2: VxV\to [0,1]$ are such that $\mu_2(v_i, v_j) \leq \min [\mu_1(v_i), \mu_1(v_j)]$ and $\gamma_2(v_i, v_j) \geq \max [\gamma_1(v_i), \gamma_1(v_j)]$ and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$, (i, j = 1, 2, ..., n).

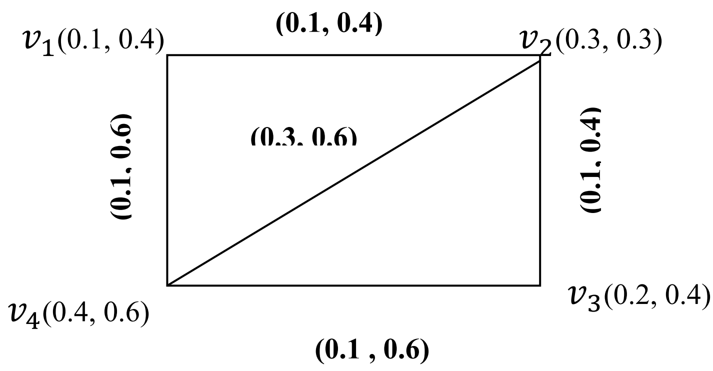

Figure 2. Intuitionistic Fuzzy Graph.

Definition 2.5 [23]

Let $A = (T_A, I_A, F_A)$ and $B = (T_B, I_B, F_B)$ be two single valued neutrosophic sets on a set X. If $A = (T_A, I_A, F_A)$ is a single valued neutrosophic relation on a set X, then $A = (T_A, I_A, F_A)$ is called a single valued neutrosophic relation on $B = (T_B, I_B, F_B)$ if

$$T_B(x, y) \leq \min(T_A(x), T_A(y)), \quad (3)$$

$$I_B(x, y) \geq \max(I_A(x), I_A(y)), \quad (4)$$

$$F_B(x, y) \geq \max(F_Ax), F_A(y)), \quad (5)$$

for all x, y ∈ X.

A single valued neutrosophic relation A on X is called symmetric if $T_A(x, y) = T_A(y, x)$, $I_A(x, y) = I_A(y, x)$, $F_A(x, y) = F_A(y, x)$ and $T_B(x, y) = T_B(y, x)$, $I_B(x, y) = I_B(y, x)$ and $F_B(x, y) = F_B(y, x)$, for all $x, y \in X$.

Definition 2.6 [23]

A single valued neutrosophic graph (SVN-graph) with underlying set V is defined to be a pair G = (A, B), where:

1. The functions T_A:V→[0, 1], I_A:V→[0, 1] and F_A:V→[0, 1] denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element $v_i \in$ V, respectively, and:

$$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3 \quad (6)$$

for all $v_i \in$ V (i = 1, 2, …,n).

2. The functions T_B: E ⊆ V x V →[0, 1], I_B:E ⊆ V x V →[0, 1] and F_B: E ⊆ V x V →[0, 1] are defined by:

$$T_B(\{v_i, v_j\}) \leq \min\,[T_A(v_i), T_A(v_j)], \quad (7)$$

$$I_B(\{v_i, v_j\}) \geq \max\,[I_A(v_i), I_A(v_j)], \tag{8}$$

$$F_B(\{v_i, v_j\}) \geq \max\,[F_A(v_i), F_A(v_j)], \tag{9}$$

denoting the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in$ E respectively, where:

$$0 \leq T_B(\{v_i, v_j\}) + I_B(\{v_i, v_j\}) + F_B(\{v_i, v_j\}) \leq 3 \quad \text{for all } \{v_i, v_j\} \in E \text{ (i, j = 1, 2,…, n)} \tag{10}$$

We have A - the single valued neutrosophic vertex set of V, and B - the single valued neutrosophic edge set of E, respectively. Note that B is a symmetric single valued neutrosophic relation on A. We use the notation (v_i, v_j) for an element of E. Thus, G = (A, B) is a single valued neutrosophic graph of G*= (V, E) if:

$$T_B(v_i, v_j) \leq \min\,[T_A(v_i), T_A(v_j)], \tag{11}$$

$$I_B(v_i, v_j) \geq \max\,[I_A(v_i), I_A(v_j)], \tag{12}$$

$$F_B(v_i, v_j) \geq \max\,[F_A(v_i), F_A(v_j)], \tag{13}$$

for all $(v_i, v_j) \in$ E.

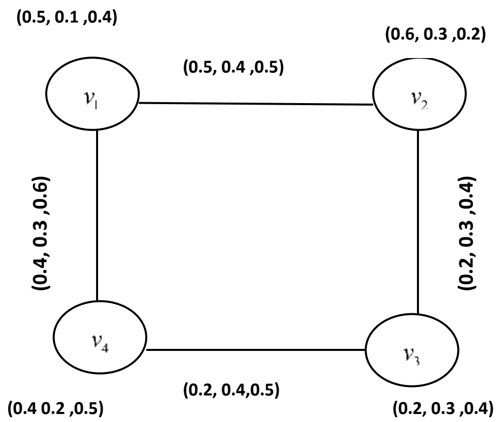

Figure 3. Single valued neutrosophic graph.

Definition 2.7 [23]

A single valued neutrosophic graph G= (A, B) is called complete if:

$$\mathrm{T_B(v_i, v_j) = \min\,[T_A(v_i),\ T_A(v_j)]} \tag{14}$$

$$\mathrm{I_B(v_i, v_j) = \max\,[I_A(v_i),\ I_A(v_j)]} \tag{15}$$

$$\mathrm{F_B(v_i, v_j) = \max\,[F_A(v_i), F_A(v_j)]} \tag{16}$$

for all $\mathrm{v_i, v_j} \in$ V.

Definition 2.8 [23]

The complement of a single valued neutrosophic graph G (A, B) on G^* is a single valued neutrosophic graph $\overline{G}$ on G^*, where:

$$1.\ \overline{A} = A. \tag{17}$$

$$2.\ \overline{T_A}(v_i) = T_A(v_i),\ \overline{I_A}(v_i) = I_A(v_i),\ \overline{F_A}(v_i) = F_A(v_i), \tag{18}$$

for all $v_j \in V$.

$$3.\ \overline{T_B}(v_i, v_j) = \min\left[T_A(v_i), T_A(v_j)\right] - T_B(v_i, v_j), \tag{19}$$

$$\overline{I_B}(v_i, v_j) = \max\left[I_A(v_i), I_A(v_j)\right] - I_B(v_i, v_j), \tag{20}$$

$$\overline{F_B}(v_i, v_j) = \max\left[F_A(v_i), F_A(v_j)\right] - F_B(v_i, v_j), \tag{21}$$

for all $(v_i, v_j) \in E$.

Definition 2.9 [26]

By an interval-valued neutrosophic graph of a graph $G^* = (V, E)$ we mean a pair $G = (A, B)$, where $A = < [T_{AL}, T_{AU}], [I_{AL}, I_{AU}], [F_{AL}, F_{AU}]>$ is an interval-valued neutrosophic set on V and $B =< [T_{BL}, T_{BU}], [I_{BL}, I_{BU}], [F_{BL}, F_{BU}]>$ is an interval valued neutrosophic relation on E, satisfying the following condition:

1. $V = \{v_1, v_2, \ldots, v_n\}$ such that $T_{AL}: V \to [0, 1]$, $T_{AU}: V \to [0, 1]$, $I_{AL}: V \to [0, 1]$, $I_{AU}: V \to [0, 1]$ and $F_{AL}: V \to [0, 1]$, $F_{AU}: V \to [0, 1]$, denoting the degree of truth-membership, the degree of indeterminacy-membership and falsity-membership of the element $y \in V$, respectively, and:

$$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3, \tag{22}$$

for all $v_i \in V$ (i=1, 2, …,n)

2. The functions $T_{BL}: V \times V \to [0, 1]$, $T_{BU}: V \times V \to [0, 1]$, $I_{BL}: V \times V \to [0, 1]$, $I_{BU}: V \times V \to [0, 1]$ and $F_{BL}: V \times V \to [0,1]$, $F_{BU}: V \times V \to [0, 1]$ are such that:

$$T_{BL}(\{v_i, v_j\}) \leq \min\left[T_{AL}(v_i), T_{AL}(v_j)\right], \tag{23}$$

$$T_{BU}(\{v_i, v_j\}) \leq \min\left[T_{AU}(v_i), T_{AU}(v_j)\right], \tag{24}$$

$$I_{BL}(\{v_i, v_j\}) \geq \max\left[I_{BL}(v_i), I_{BL}(v_j)\right], \tag{25}$$

$$I_{BU}(\{v_i, v_j\}) \geq \max\left[I_{BU}(v_i), I_{BU}(v_j)\right], \tag{26}$$

$$F_{BL}(\{v_i, v_j\}) \geq \max\left[F_{BL}(v_i), F_{BL}(v_j)\right], \tag{27}$$

$$F_{BU}(\{v_i, v_j\}) \geq \max\left[F_{BU}(v_i), F_{BU}(v_j)\right], \tag{28}$$

denoting the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in E$ respectively, where:

$$0 \le T_B(\{v_i, v_j\}) + I_B(\{v_i, v_j\}) + F_B(\{v_i, v_j\}) \le 3, \quad (29)$$

for all $\{v_i, v_j\} \in E$ (i, j = 1, 2, …, n).

We have A - the interval valued neutrosophic vertex set of V, and B - the interval valued neutrosophic edge set of E, respectively. Note that B is a symmetric interval valued neutrosophic relation on A. We use the notation (v_i, v_j) for an element of E. Thus, G = (A, B) is an interval valued neutrosophic graph of G^*= (V, E), if:

$$T_{BL}(v_i, v_j) \le \min [T_{AL}(v_i), T_{AL}(v_j)], \quad (30)$$

$$T_{BU}(v_i, v_j) \le \min [T_{AU}(v_i), T_{AU}(v_j)], \quad (31)$$

$$I_{BL}(v_i, v_j) \ge \max [I_{BL}(v_i), I_{BL}(v_j)], \quad (32)$$

$$I_{BU}(v_i, v_j) \ge \max [I_{BU}(v_i), I_{BU}(v_j)], \quad (33)$$

$$F_{BL}(v_i, v_j) \ge \max [F_{BL}(v_i), F_{BL}(v_j)], \quad (34)$$

$$F_{BU}(v_i, v_j) \ge \max [F_{BU}(v_i), F_{BU}(v_j)], \quad (35)$$

for all $(v_i, v_j) \in E$.

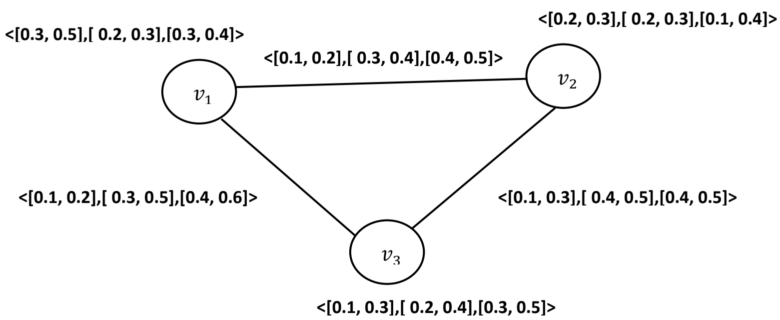

Figure 4. Interval valued neutrosophic graph.

Definition 2.10 [26]

The complement of a complete interval valued neutrosophic graph G = (A, B) of G^*= (V, E) is a complete interval valued neutrosophic graph $\overline{G}$= ($\overline{A}$, $\overline{B}$) = (A, $\overline{B}$) on G^*= (V, $\overline{E}$), where:

$$1.\ \overline{V} = V \quad (36)$$

$$2.\ \overline{T_{AL}}(v_i) = T_{AL}(v_i), \quad (37)$$

$$\overline{T_{AU}}(v_i) = T_{AU}(v_i), \quad (38)$$

$$\overline{I_{AL}}(v_i) = I_{AL}(v_i), \quad (39)$$

$$\overline{I_{AU}}(v_i) = I_{AU}(v_i), \quad (40)$$

$$\overline{F_{AL}}(v_i) = F_{AL}(v_i), \tag{41}$$

$$\overline{F_{AU}}(v_i) = F_{AU}(v_i), \tag{42}$$

for all $v_j \in V$.

$$3.\ \overline{T_{BL}}(v_i, v_j) = \min\left[T_{AL}(v_i), T_{AL}(v_j)\right] - T_{BL}(v_i, v_j), \tag{43}$$

$$\overline{T_{BU}}(v_i, v_j) = \min\left[T_{AU}(v_i), T_{AU}(v_j)\right] - T_{BU}(v_i, v_j), \tag{44}$$

$$\overline{I_{BL}}(v_i, v_j) = \max\left[I_{AL}(v_i), I_{AL}(v_j)\right] - I_{BL}(v_i, v_j), \tag{45}$$

$$\overline{I_{BU}}(v_i, v_j) = \max\left[I_{AU}(v_i), I_{AU}(v_j)\right] - I_{BU}(v_i, v_j), \tag{46}$$

$$\overline{F_{BL}}(v_i, v_j) = \max\left[F_{AL}(v_i), F_{AL}(v_j)\right] - F_{BL}(v_i, v_j), \tag{47}$$

$$\overline{F_{BU}}(v_i, v_j) = \max\left[F_{AU}(v_i), F_{AU}(v_j)\right] - F_{BU}(v_i, v_j), \tag{48}$$

for all $(v_i, v_j) \in E$.

Definition 2.11 [26]

An interval valued neutrosophic graph G= (A, B) is called complete, if:

$$T_{BL}(v_i, v_j) = \min(T_{AL}(v_i), T_{AL}(v_j)), \tag{49}$$

$$T_{BU}(v_i, v_j) = \min(T_{AU}(v_i), T_{AU}(v_j)), \tag{50}$$

$$I_{BL}(v_i, v_j) = \max\ (I_A(v_i), I_A(v_j)), \tag{51}$$

$$I_{BU}(v_i, v_j) = \max\ (I_{AU}(v_i), I_{AU}(v_j)), \tag{52}$$

$$F_{BL}(v_i, v_j) = \max\ (F_A(v_i), F_A(v_j)), \tag{53}$$

$$F_{BU}(v_i, v_j) = \max\ (F_{AU}(v_i), F_{AU}(v_j)), \tag{54}$$

for all $v_i, v_j \in V$.

3 Main Result

Theorem 3.1:

An interval valued neutrosophic graph G = (A, B) is an isolated interval valued neutrosophic graph if and only if its complement is a complete interval valued neutrosophic graph.

Proof

Let G= (A, B) be a complete interval valued neutrosophic graph.

Therefore:

$$T_{BL}(v_i, v_j) = \min(T_{AL}(v_i),\ T_{AL}(v_j)), \tag{55}$$

$$T_{BU}(v_i, v_j) = \min(T_{AU}(v_i), T_{AU}(v_j)), \tag{56}$$

$$I_{BL}(v_i, v_j) = \max\,(I_{AL}(v_i), I_{AL}(v_j)), \tag{57}$$

$$I_{BU}(v_i, v_j) = \max\,(I_{AU}(v_i), I_{AU}(v_j)), \tag{58}$$

$$F_{BL}(v_i, v_j) = \max\,(F_{AL}(v_i), F_{AL}(v_j)), \tag{59}$$

$$F_{BU}(v_i, v_j) = \max\,(F_{AU}(v_i), F_{AU}(v_j)), \tag{60}$$

for all $v_i, v_j \in V$.

Hence in $\overline{G}$,

$$\overline{T}_{BL}(v_i, v_j) = \min(T_{AL}(v_i), T_{AL}(v_j)) - T_{BL}(v_i, v_j) \tag{61}$$

for all i, j, ..., n.

$$= \min(T_{AL}(v_i), T_{AL}(v_j)) - \min(T_{AL}(v_i), T_{AL}(v_j)) \tag{62}$$

for all i, j, ..., n.

$$= 0 \tag{63}$$

for all i, j, ..., n.

$$\overline{T}_{BU}(v_i, v_j) = \min(T_{AU}(v_i), T_{AU}(v_j)) - T_{BU}(v_i, v_j) \tag{64}$$

for all i, j, ..., n.

$$= \min(T_{AU}(v_i), T_{AU}(v_j)) - \min(T_{AU}(v_i), T_{AU}(v_j)) \tag{65}$$

for all i, j, ..., n.

$$= 0 \tag{66}$$

for all i, j, ..., n.

And:

$$\overline{I}_{BL}(v_i, v_j) = \max(I_{AL}(v_i), I_{AL}(v_j)) - I_{BL}(v_i, v_j) \tag{67}$$

for all i, j, ..., n.

$$= \max(I_{AL}(v_i), I_{AL}(v_j)) - \max(I_{AL}(v_i), I_{AL}(v_j)) \tag{68}$$

for all i, j, ..., n.

$$= 0 \tag{69}$$

for all i, j, ..., n.

$$\overline{I}_{BU}(v_i, v_j) = \max(I_{AU}(v_i), I_{AU}(v_j)) - I_{BU}(v_i, v_j) \tag{70}$$

for all i, j, ..., n.

$$= \max(I_{AU}(v_i), I_{AU}(v_j)) - \max(I_{AU}(v_i), I_{AU}(v_j)) \tag{71}$$

for all i, j, ..., n.

$$= 0 \tag{72}$$

for all i, j, ..., n.

Also:

$$\bar{F}_{BL}(v_i, v_j) = \max(F_{AL}(v_i), F_{AL}(v_j)) - F_{BL}(v_i, v_j) \tag{73}$$

for all i, j, ..., n.

$$= \max(F_{AL}(v_i), F_{AL}(v_j)) - \max(F_{AL}(v_i), F_{AL}(v_j)) \tag{74}$$

for all i, j, ..., n.

$$= 0 \tag{75}$$

for all i, j, ..., n.

$$\bar{F}_{BU}(v_i, v_j) = \max(F_{AU}(v_i), F_{AU}(v_j)) - F_{BU}(v_i, v_j) \tag{76}$$

for all i, j, ..., n.

$$= \max(F_{AU}(v_i), F_{AU}(v_j)) - \max(F_{AU}(v_i), F_{AU}(v_j)) \tag{77}$$

for all i, j, ..., n.

$$= 0 \tag{78}$$

for all i, j, ..., n.

Thus,

$$([\bar{T}_{BL}(v_i, v_j), \bar{T}_{BU}(v_i, v_j)],\ [\bar{I}_{BL}(v_i, v_j), \bar{I}_{BU}(v_i, v_j)],\ [\bar{F}_{BL}(v_i, v_j), \bar{F}_{BU}(v_i, v_j)]) =$$

$$= ([0, 0], [0, 0], [0, 0]). \tag{79}$$

Hence, G = (A, B) is an isolated interval valued neutrosophic graph.

4 Conclusions

In this paper, we extended the concept of isolated single valued neutrosophic graph to an isolated interval valued neutrosophic graph. In future works, we plan to study the concept of isolated bipolar single valued neutrosophic graph.

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Isomorphism of Bipolar Single Valued Neutrosophic Hypergraphs

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Abstract

In this paper, we introduce the homomorphism, the weak isomorphism, the co-weak isomorphism, and the isomorphism of the bipolar single valued neutrosophic hypergraphs. The properties of order, size and degree of vertices are discussed. The equivalence relation of the isomorphism of the bipolar single valued neutrosophic hypergraphs and the weak isomorphism of bipolar single valued neutrosophic hypergraphs, together with their partial order relation, is also verified.

Keywords

homomorphism, weak-isomorphism, co-weak-isomorphism, isomorphism, bipolar single valued neutrosophic hypergraphs.

1 Introduction

The neutrosophic set - proposed by Smarandache [8] as a generalization of the fuzzy set [14], intuitionistic fuzzy set [12], interval valued fuzzy set [11] and interval-valued intuitionistic fuzzy set [13] theories - is a mathematical tool created to deal with incomplete, indeterminate and inconsistent information in the real world. The characteristics of the neutrosophic set are the truth-membership function (t), the indeterminacy-membership function (i), and the falsity membership function (f), which take values within the real standard or non-standard unit interval $]^{-}0 , 1^{+}[$.

A subclass of the neutrosophic set, the single-valued neutrosophic set (SVNS), was intoduced by Wang et al. [9]. The same authors [10] also introduced a generalization of the single valued neutrosophic set, namely the interval valued neutrosophic set (IVNS), in which the three membership functions are independent, and their values belong to the unit interval [0, 1]. The IVNS is more precise and flexible than the single valued neutrosophic set.

More works on single valued neutrosophic sets, interval valued neutrosophic sets and their applications can be found on *http://fs.gallup.unm.edu/NSS/*.

In this paper, we extend the isomorphism of the bipolar single valued neutrosophic hypergraphs, and introduce some of their relevant properties.

1 Preliminaries

Definition 2.1

A hypergraph is an ordered pair H = (X, E), where:

(1) $X = \{x_1, x_2, \ldots, x_n\}$ is a finite set of vertices.

(2) $E = \{E_1, E_2, \ldots, E_m\}$ is a family of subsets of X.

(3) E_j are non-void for j = 1, 2, 3, ..., m, and $\cup_j(E_j) = X$.

The set X is called 'set of vertices', and E is denominated as the 'set of edges' (or 'hyper-edges').

Definition 2.2

A fuzzy hypergraph H = (X, E) is a pair, where X is a finite set and E is a finite family of non-trivial fuzzy subsets of X, such that $X = \cup_j Supp(E_j)$, $j = 1, 2, 3, \ldots, m$.

Remark 2.3

The collection $E = \{E_1, E_2, E_3, \ldots., E_m\}$ is a collection of edge set of H.

Definition 2.4

A fuzzy hypergraph with underlying set X is of the form H = (X, E, R), where $E = \{E_1, E_2, E_3, \ldots, E_m\}$ is the collection of fuzzy subsets of X, that is $E_j : X \rightarrow [0, 1]$, j = 1, 2, 3, ..., m, and $R : E \rightarrow [0, 1]$ is the fuzzy relation of the fuzzy subsets E_j, such that:

$$R(x_1, x_2, \ldots, x_r) \leq \min(E_j(x_1), \ldots, E_j(x_r)), \quad (1)$$

for all $\{x_1, x_2, \ldots, x_r\}$ subsets of X.

Definition 2.5

Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by its truth membership function T_A(x), its indeterminacy membership function I_A(x), and its falsity membership function F_A(x). For each point, x ∈ X; T_A(x), I_A(x), F_A(x) ∈ [0, 1].

Definition 2.6

A single valued neutrosophic hypergraph is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \dots, x_n\}$ is a finite set of vertices.

(2) $E = \{E_1, E_2, \dots, E_m\}$ is a family of SVNSs of X.

(3)$E_j \neq O = (0, 0, 0)$ for $j= 1, 2, 3, \dots, m$, and $\bigcup_j Supp(E_j) = X$.

The set X is called set of vertices and E is the set of SVN-edges (or SVN-hyperedges).

Proposition 2.7

The single valued neutrosophic hypergraph is the generalization of fuzzy hypergraphs and intuitionistic fuzzy hypergraphs.

Note that a given SVNHG$H = (X, E, R)$, with underlying set X, where $E = \{E_1, E_2, \dots, E_m\}$, is the collection of the non-empty family of SVN subsets of X, and R is the SVN relation of the SVN subsets E_j, such that:

$$R_T(x_1, x_2, \dots, x_r) \leq \min([T_{E_j}(x_1)], \dots, [T_{E_j}(x_r)]), \quad (2)$$

$$R_I(x_1, x_2, \dots, x_r) \geq \max([I_{E_j}(x_1)], \dots, [I_{E_j}(x_r)]), \quad (3)$$

$$R_F(x_1, x_2, \dots, x_r) \geq \max([F_{E_j}(x_1)], \dots, [F_{E_j}(x_r)]), \quad (4)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Definition 2.8

Let X be a space of points (objects) with generic elements in X denoted by x.

A bipolar single valued neutrosophic set A (BSVNS A) is characterized by the positive truth membership function $PT_A(x)$, the positive indeterminacy membership function $PI_A(x)$, the positive falsity membership function $PF_A(x)$, the negative truth membership function $NT_A(x)$, the negative indeterminacy membership function $NI_A(x)$, and the negative falsity membership function $NF_A(x)$.

For each point x ∈X; $PT_A(x)$, $PI_A(x)$,$PF_A(x)$ ∈ [0, 1], and $NT_A(x)$, $NI_A(x)$, $NF_A(x)$ ∈ [-1, 0].

Definition 2.9

A bipolar single valued neutrosophic hypergraph is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \dots, x_n\}$ is a finite set of vertices.

(2) $E = \{E_1, E_2, \dots, E_m\}$ is a family of BSVNSs of X.

(3) $E_j \neq O = ([0, 0], [0, 0], [0, 0])$ for $j = 1, 2, 3, \dots, m$, and $\bigcup_j Supp(E_j) = X$.

The set X is called the 'set of vertices' and E is called the 'set of BSVN-edges' (or 'IVN-hyper-edges'). Note that a given BSVNHG$H = (X, E, R)$, with underlying set X, where $E = \{E_1, E_2, \dots, E_m\}$ is the collection of non-empty family of BSVN subsets of X, and R is the BSVN relation of BSVN subsets E_j such that:

$$R_{PT}(x_1, x_2, \dots, x_r) \leq \min([PT_{E_j}(x_1)], \dots, [PT_{E_j}(x_r)]), \quad (5)$$

$$R_{PI}(x_1, x_2, \dots, x_r) \geq \max([PI_{E_j}(x_1)], \dots, [PI_{E_j}(x_r)]), \quad (6)$$

$$R_{PF}(x_1, x_2, \dots, x_r) \geq \max([PF_{E_j}(x_1)], \dots, [PF_{E_j}(x_r)]), \quad (7)$$

$$R_{NT}(x_1, x_2, \dots, x_r) \geq \max([NT_{E_j}(x_1)], \dots, [NT_{E_j}(x_r)]), \quad (8)$$

$$R_{NI}(x_1, x_2, \dots, x_r) \leq \min([NI_{E_j}(x_1)], \dots, [NI_{E_j}(x_r)]), \quad (9)$$

$$R_{NF}(x_1, x_2, \dots, x_r) \leq \min([NF_{E_j}(x_1)], \dots, [NF_{E_j}(x_r)]), \quad (10)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Proposition 2.10

The bipolar single valued neutrosophic hypergraph is the generalization of the fuzzy hypergraph, intuitionistic fuzzy hypergraph, bipolar fuzzy hypergraph and intuitionistic fuzzy hypergraph.

Example 2.11

Consider the BSVNHG $H = (X, E, R)$, with underlying set $X = \{a, b, c\}$, where $E = \{A, B\}$, and R defined in *Tables* below:

H	A	B
a	(0.2, 0.3, 0.9, -0.2, -0.2, -0.3)	(0.5, 0.2, 0.7, -0.4, -0.2, -0.3)
b	(0.5, 0.5, 0.5, -0.4, -0.3, -0.3)	(0.1, 0.6, 0.4, -0.9, -0.3,-0.4)
c	(0.8, 0.8, 0.3, -0.9, -0.2, -0.3)	(0.5, 0.9, 0.8, -0.1, -0.2, -0.3)

R	R_{PT}	R_{PI}	R_{PF}	R_{NT}	R_{NI}	R_{NF}
A	0.2	0.8	0.9	-0.1	-0.4	-0.5
B	0.1	0.9	0.8	-0.1	-0.5	-0.6

By routine calculations, *H = (X, E, R)* is BSVNHG.

3 Isomorphism of BSVNHGs

Definition 3.1

A homomorphism *f: H →K* between two BSVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a mapping *f: X → Y* which satisfies the conditions:

$$\min[PT_{E_j}(x)] \leq \min[PT_{F_j}(f(x))], \tag{11}$$

$$\max[PI_{E_j}(x)] \geq \max[PI_{F_j}(f(x))], \tag{12}$$

$$\max[PF_{E_j}(x)] \geq \max[PF_{F_j}(f(x))], \tag{13}$$

$$\max[NT_{E_j}(x)] \geq \max[NT_{F_j}(f(x))], \tag{14}$$

$$\min[NI_{E_j}(x)] \leq \min[NI_{F_j}(f(x))], \tag{15}$$

$$\min[NF_{E_j}(x)] \leq \min[NF_{F_j}(f(x))], \tag{16}$$

for all $x \in X$.

$$R_{PT}(x_1, x_2, \dots, x_r) \leq S_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{17}$$

$$R_{PI}(x_1, x_2, \dots, x_r) \geq S_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{18}$$

$$R_{PF}(x_1, x_2, \dots, x_r) \geq S_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{19}$$

$$R_{NT}(x_1, x_2, \dots, x_r) \geq S_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{20}$$

$$R_{NI}(x_1, x_2, \dots, x_r) \leq S_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{21}$$

$$R_{NF}(x_1, x_2, \dots, x_r) \leq S_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{22}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X*.

Example 3.2

Consider the two BSVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y = {x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R*and*S,* which are defined in *Tables* given below:

H	A	B
a	(0.2, 0.3, 0.9,-0.2,-0.2,-0.3)	(0.5, 0.2, 0.7,-0.4,-0.2,-0.3)
b	(0.5, 0.5, 0.5,-0.4,-0.3,-0.3)	(0.1, 0.6, 0.4,-0.9,-0.3,-0.4)
c	(0.8, 0.8, 0.3,-0.9,-0.2,-0.3)	(0.5, 0.9, 0.8,-0.1,-0.2,-0.3)

K	C	D
x	(0.3, 0.2, 0.2,-0.9,-0.2,-0.3)	(0.2, 0.1, 0.3,-0.6,-0.1,-0.2)
y	(0.2, 0.4, 0.2,-0.4,-0.2,-0.3)	(0.3, 0.2, 0.1,-0.7,-0.2,-0.1)
z	(0.5, 0.8, 0.2,-0.2,-0.1,-0.3)	(0.9, 0.7, 0.1,-0.2,-0.1,-0.3)

R	R_{PT}	R_{PI}	R_{PF}	R_{NT}	R_{NI}	R_{NF}
A	0.2	0.8	0.9	-0.1	-0.4	-0.5
B	0.1	0.9	0.8	-0.1	-0.5	-0.6

S	S_{PT}	S_{PI}	S_{PF}	S_{NT}	S_{NI}	S_{NF}
C	0.2	0.8	0.3	-0.1	-0.2	-0.3
D	0.1	0.7	0.3	-0.1	-0.2	-0.3

and $f : X \rightarrow Y$ defined by: $f(a)=x$, $f(b)=y$ and $f(c)=z$. Then, by routine calculations, $f: H \rightarrow K$ is a homomorphism between H and K.

Definition 3.3

A weak isomorphism $f : H \rightarrow K$ between two BSVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ is a bijective mapping $f : X \rightarrow Y$ which satisfies f is homomorphism, such that:

$$\min[PT_{E_j}(x)] \leq \min[PT_{F_j}(f(x))], \tag{23}$$

$$\max[PI_{E_j}(x)] \geq \max[PI_{F_j}(f(x))], \tag{24}$$

$$\max[PF_{E_j}(x)] \geq \max[PF_{F_j}(f(x))], \tag{25}$$

$$\max[NT_{E_j}(x)] \geq \max[NT_{F_j}(f(x))], \tag{26}$$

$$\min[NI_{E_j}(x)] \leq \min[NI_{F_j}(f(x))], \tag{27}$$

$$\min[NF_{E_j}(x)] \leq \min[NF_{F_j}(f(x))], \tag{28}$$

for all $x \in X$.

Note

The weak isomorphism between two BSVNHGs preserves the weights of vertices.

Example 3.4

Consider the two BSVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ with underlying sets $X = \{a, b, c\}$ and $Y = \{x, y, z\}$, where $E = \{A, B\}$, $F = \{C, D\}$, R and S, which are defined by *Tables* given below, and $f: X \rightarrow Y$ defined by: $f(a)=x$, $f(b)=y$ and $f(c)=z$. Then, by routine calculations, $f: H \rightarrow K$ is a weak isomorphism between H and K.

H	A	B
a	(0.2, 0.3, 0.9,-0.2,-0.2,-0.3)	(0.5, 0.2, 0.7,-0.4,-0.2,-0.3)
b	(0.5, 0.5, 0.5,-0.4,-0.3,-0.3)	(0.1, 0.6, 0.4,-0.9,-0.3,-0.4)
c	(0.8, 0.8, 0.3,-0.9,-0.2,-0.3)	(0.5, 0.9, 0.8,-0.1,-0.2,-0.3)

K	C	D
x	(0.2, 0.3, 0.2,-0.9,-0.2,-0.3)	(0.2, 0.1, 0.8,-0.6,-0.1,-0.4)
y	(0.2, 0.4, 0.2,-0.4,-0.3,-0.3)	(0.1, 0.6, 0.5,-0.6,-0.2,-0.3)
z	(0.5, 0.8, 0.9,-0.2,-0.2,-0.3)	(0.9, 0.9, 0.1,-0.1,-0.3,-0.3)

R	R_{PT}	R_{PI}	R_{PF}	R_{NT}	R_{NI}	R_{NF}
A	0.2	0.8	0.9	-0.1	-0.4	-0.3
B	0.1	0.9	0.9	-0.1	-0.3	-0.5

S	S_{PT}	S_{PI}	S_{PF}	S_{NT}	S_{NI}	S_{NF}
C	0.2	0.8	0.9	-0.1	-0.3	-0.2
D	0.1	0.9	0.8	-0.1	-0.3	-0.4

Definition 3.5

A co-weak isomorphism *f: H → K* between two BSVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a bijective mapping *f: X → Y* which satisfies *f* is homomorphism, such that:

$$R_{PT}(x_1, x_2, \dots, x_r) = S_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{29}$$

$$R_{PI}(x_1, x_2, \dots, x_r) = S_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{30}$$

$$R_{PF}(x_1, x_2, \dots, x_r) = S_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{31}$$

$$R_{NT}(x_1, x_2, \dots, x_r) = S_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{32}$$

$$R_{NI}(x_1, x_2, \dots, x_r) = S_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{33}$$

$$R_{NF}(x_1, x_2, \dots, x_r) = S_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{34}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X*.

Note

The co-weak isomorphism between two BSVNHGs preserves the weights of edges.

Example 3.6

Consider the two BSVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y = {x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S,* which are defined in *Tables* given below, and *f : X → Y* defined by: *f(a)=x* , *f(b)=y* and *f(c)=z*. Then, by routine calculations, *f: H → K* is a co-weak isomorphism between *H* and *K*.

H	A	B
a	(0.2, 0.3, 0.9,-0.4,-0.2,-0.3)	(0.5, 0.2, 0.7,-0.1,-0.2,-0.3)
b	(0.5, 0.5, 0.5,-0.4,-0.2,-0.3)	(0.1, 0.6, 0.4,-0.4,-0.2,-0.3)
c	(0.8, 0.8, 0.3,-0.1,-0.2,-0.3)	(0.5, 0.9, 0.8,-0.4,-0.2,-0.3)

K	C	D
x	(0.3, 0.2, 0.2,-0.9,-0.2,-0.3)	(0.2, 0.1, 0.3,-0.4,-0.2,-0.3)
y	(0.2, 0.4, 0.2,-0.4,-0.2,-0.3)	(0.3, 0.2,0.1,-0.9,-0.2,-0.3)
z	(0.5, 0.8, 0.2,-0.1,-0.2,-0.3)	(0.9, 0.7, 0.1,-0.1,-0.2,-0.3)

R	R_{PT}	R_{PI}	R_{PF}	R_{NT}	R_{NI}	R_{NF}
A	0.2	0.8	0.9	-0.1	-0.2	-0.3
B	0.1	0.9	0.8	-0.1	-0.2	-0.3

S	S_{PT}	S_{PI}	S_{PF}	S_{NT}	S_{NI}	S_{NF}
C	0.2	0.8	0.9	-0.1	-0.2	-0.3
D	0.1	0.9	0.8	-0.1	-0.2	-0.3

Definition 3.7

An isomorphism $f : H \rightarrow K$ between two BSVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ is a bijective mapping $f : X \rightarrow Y$ which satisfies the conditions:

$$\min[PT_{E_j}(x)] = \min[PT_{F_j}(f(x))], \tag{35}$$

$$\max[PI_{E_j}(x)] = \max[PI_{F_j}(f(x))], \tag{36}$$

$$\max[PF_{E_j}(x)] = \max[PF_{F_j}(f(x))], \tag{37}$$

$$\max[NT_{E_j}(x)] = \max[NT_{F_j}(f(x))], \tag{38}$$

$$\min[NI_{E_j}(x)] = \min[NI_{F_j}(f(x))], \tag{39}$$

$$\min[NF_{E_j}(x)] = \min[NF_{F_j}(f(x))], \tag{40}$$

for all $x \in X$.

$$R_{PT}(x_1, x_2, \dots, x_r) = S_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{41}$$

$$R_{PI}(x_1, x_2, \dots, x_r) = S_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{42}$$

$$R_{PF}(x_1, x_2, \dots, x_r) = S_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{43}$$

$$R_{NT}(x_1, x_2, \dots, x_r) = S_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{44}$$

$$R_{NI}(x_1, x_2, \dots, x_r) = S_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{45}$$

$$R_{NF}(x_1, x_2, \dots, x_r) = S_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{46}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Note

The isomorphism between two BSVNHGs preserves the both weights of vertices and weights of edges.

Example 3.8

Consider the two BSVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y = {x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S,* which are defined by *Tables* given below:

H	A	B
a	(0.2, 0.3, 0.7,-0.2,-0.2,-0.3)	(0.5, 0.2, 0.7,-0.6,-0.6,-0.6)
b	(0.5, 0.5, 0.5,-0.4,-0.3,-0.3)	(0.1, 0.6, 0.4,-0.1,-0.2,-0.7)
c	(0.8, 0.8, 0.3,-0.9,-0.2,-0.4)	(0.5, 0.9, 0.8,-0.7,-0.2,-0.3)

K	C	D
x	(0.2, 0.3, 0.2,-0.2,-0.2,-0.4)	(0.2, 0.1, 0.8,-0.3,-0.2,-0.3)
y	(0.2, 0.4, 0.2,-0.6,-0.2,-0.3)	(0.1, 0.6, 0.5,-0.1,-0.2,-0.7)
z	(0.5, 0.8, 0.7,-0.4,-0.3,-0.3)	(0.9, 0.9, 0.1,-0.9,-0.6,-0.3)

R	R_{PT}	R_{PI}	R_{PF}	R_{NT}	R_{NI}	R_{NF}
A	0.2	0.8	0.9	-0.1	-0.3	-0.4
B	0.0	0.9	0.8	-0.1	-0.7	-0.8

S	S_{PT}	S_{PI}	S_{PF}	S_{NT}	S_{NI}	S_{NF}
C	0.2	0.8	0.9	-0.1	-0.3	-0.4
D	0.0	0.9	0.8	-0.1	-0.7	-0.8

and *f : X → Y* defined by: *f(a)=x , f(b)=y* and *f(c)=z*. Then, by routine calculations, *f: H → K* is an isomorphism between *H* and *K*.

Definition 3.9

Let *H = (X, E, R)* be a BSVNHG, then the order of *H* is denoted and defined by as follows:

$$O(H) = (\sum min\left(PT_{E_j}(x)\right), \sum max\left(PI_{E_j}(x)\right), \sum max\left(PF_{E_j}(x)\right), \sum max\left(NT_{E_j}(x)\right), \sum min\left(NI_{E_j}(x)\right), \sum min\left(NF_{E_j}(x)\right)) \tag{47}$$

The size of *H* is denoted and defined by:

$$S(H) = (\sum R_{PT}(E_j), \sum R_{PI}(E_j), \sum R_{PF}(E_j), \sum R_{NT}(E_j), \sum R_{NI}(E_j), \sum R_{NF}(E_j)\ ) \tag{48}$$

Theorem 3.10

Let *H = (X, E, R)* and *K = (Y, F, S)* be two BSVNHGs such that *H* is isomorphic to *K*, then:

(1) O(H) = O(K),

(2) S(H) = S(K).

Proof

Let *f: H → K* be an isomorphism between two BSVNHGs *H* and *K* with underlying sets *X* and *Y* respectively; then, by definition:

$$\min[PT_{E_j}(x)] = \min[PT_{F_j}(f(x))], \tag{49}$$

$$\max[PI_{E_j}(x)] = \max[PI_{F_j}(f(x))], \tag{50}$$

$$\max[PF_{E_j}(x)] = \max[PF_{F_j}(f(x))], \tag{51}$$

$$\max[NT_{E_j}(x)] = \max[NT_{F_j}(f(x))], \tag{52}$$

$$\min[NI_{E_j}(x)] = \min[NI_{F_j}(f(x))], \tag{53}$$

$$\min[NF_{E_j}(x)] = \min[NF_{F_j}(f(x))], \tag{54}$$

for all $x \in X$.

$$R_{PT}(x_1, x_2, \dots, x_r) = S_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{55}$$

$$R_{PI}(x_1, x_2, \dots, x_r) = S_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{56}$$

$$R_{PF}(x_1, x_2, \dots, x_r) = S_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{57}$$

$$R_{NT}(x_1, x_2, \dots, x_r) = S_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{58}$$

$$R_{NI}(x_1, x_2, \dots, x_r) = S_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{59}$$

$$R_{NF}(x_1, x_2, \dots, x_r) = S_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{60}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X*.

Consider:

$$O_{PT}(H) = \sum \min PT_{E_j}(x) = \sum \min PT_{F_j}(f(x)) = O_{PT}(K) \tag{61}$$

$$O_{NT}(H) = \sum \max NT_{E_j}(x) = \sum \max NT_{F_j}(f(x)) = O_{NT}(K) \tag{62}$$

Similarly, $O_{PI}(H) = O_{PI}(K)$ and $O_{PF}(H) = O_{PF}(K)$, $O_{NI}(H) = O_{NI}(K)$ and $O_{NF}(H) = O_{NF}(K)$**,** hence *O(H) = O(K).*

Next:

$$S_{PT}(H) = \sum R_{PT}(x_1, x_2, \dots, x_r)$$

$$= \sum S_{PT}(f(x_1), f(x_2), \dots, f(x_r)) = S_{PT}(K). \tag{63}$$

Similarly,

$$S_{NT}(H) = \sum R_{NT}(x_1, x_2, \dots, x_r)$$

$$= \sum S_{NT}(f(x_1), f(x_2), \dots, f(x_r)) = S_{NT}(K). \tag{64}$$

and $S_{PI}(H) = S_{PI}(K)$, $S_{PF}(H) = S_{PF}(K)$, $S_{NI}(H) = S_{NI}(K)$, $S_{NF}(H) = S_{NF}(K)$, hence $S(H) = S(K)$.

Remark 3.11

The converse of the above theorem need not to be true in general.

Example 3.12

Consider the two BSVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S* are defined in *Tables* given below:

H	A	B
a	(0.2, 0.5, 0.3, -0.1, -0.2, -0.3)	(0.14, 0.5, 0.3, -0.1, -0.2, -0.3)
b	(0.0,0.0,0.0, 0.0, 0.0, 0.0)	(0.2, 0.5, 0.3, -0.4, -0.2, -0.3)
c	(0.33, 0.5, 0.3, -0.4, -0.2, -0.3)	(0.16, 0.5, 0.3, -0.1, -0.2, -0.3)
d	(0.5, 0.5, 0.3, -0.1, -0.2, -0.3)	(0.0,0.0,0.0, 0.0, 0.0, 0.0)

K	C	D
w	(0.14, 0.5, 0.3, -0.1, -0.2, -0.3)	(0.2, 0.5, 0.33, -0.4, -0.2, -0.3)
x	(0.16, 0.5, 0.3, -0.1, -0.2, -0.3)	(0.33,0.5, 0.33, -0.1, -0.2, -0.3)
y	(0.25, 0.5, 0.3, -0.1,-0.2, -0.3)	(0.2, 0.5, 0.33, -0.1, -0.2, -0.3)
z	(0.5, 0.5, 0.3, -0.4, -0.2, -0.3)	(0.0, 0.0, 0.0, 0.0, 0.0, 0.0)

R	R_{PT}	R_{PI}	R_{PF}	R_{NT}	R_{NI}	R_{NF}
A	0.2	0.5	0.3	-0.1	-0.2	-0.3
B	0.14	0.5	0.3	-0.1	-0.2	-0.3

S	S_{PT}	S_{PI}	S_{PF}	S_{NT}	S_{NI}	S_{NF}
C	0.14	0.5	0.3	-0.1	-0.2	-0.3
D	0.2	0.5	0.3	-0.1	-0.2	-0.3

where *f* is defined by: *f(a)=w, f(b)=x, f(c)=y, f(d)=z.*

Here, *O(H) = (1.0,2.0, 1.2,-0.7,-0.8,-1.2) = O(K)* and *S(H)=(0.34, 1.0, 0.9, -0.2, -0.4, -0.9)=S(K)*, but, by routine calculations, *H* is not an isomorphism to *K*.

Corollary 3.13

The weak isomorphism between any two BSVNHGs *H* and *K* preserves the orders.

Remark 3.14

The converse of the above corollary need not to be true in general.

Example 3.15

Consider the two BSVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S* are defined in *Tables* given below, where *f* is defined by: *f(a)=w, f(b)=x, f(c)=y, f(d)=z:*

H	A	B
a	(0.2, 0.5, 0.3,-0.1,-0.2,-0.3)	(0.14, 0.5, 0.3,-0.4,-0.2,-0.3)
b	(0.0,0.0,0.0,0.0,0.0,0.0)	(0.2, 0.5, 0.3,-0.1,-0.2,-0.3)
c	(0.33, 0.5, 0.3,-0.4,-0.2,-0.3)	(0.16, 0.5, 0.3,-0.1,-0.2,-0.3)
d	(0.5, 0.5, 0.3,-0.1,-0.2,-0.3)	(0.0,0.0,0.0,0.0,0.0,0.0)

K	C	D
w	(0.14, 0.5, 0.3,-0.1,-0.2,-0.3)	(0.16, 0.5, 0.3,-0.1,-0.2,-0.3)
x	(0.0, 0.0, 0.0,0.0,0.0,0.0)	(0.16, 0.5, 0.3,-0.1,-0.2,-0.3)
y	(0.25, 0.5, 0.3,-0.1,-0.2,-0.3)	(0.2, 0.5, 0.3,-0.4,-0.2,-0.3)
z	(0.5, 0.5, 0.3,-0.1,-0.2,-0.3)	(0.0, 0.0, 0.0,0.0,0.0,0.0)

Here, *O(H)= (1.0, 2.0, 1.2, -0.4, -0.8, -1.2) = O(K),* but, by routine calculations, *H* is not a weak isomorphism to *K*.

Corollary 3.16

The co-weak isomorphism between any two BSVNHGs *H* and *K* preserves sizes.

Remark 3.17

The converse of the above corollary need not to be true in general.

Example 3.18

Consider the two BSVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z}*, where *E = {A, B}*, *F = {C, D}*,*R* and *S* are defined in *Tables* given below,

H	A	B
a	(0.2, 0.5, 0.3,-0.1,-0.2,-0.3)	(0.14, 0.5, 0.3, -0.1,-0.2,-0.3)
b	(0.0,0.0,0.0,0.0,0.0,0.0)	(0.16, 0.5, 0.3, -0.1,-0.2,-0.3)
c	(0.3, 0.5, 0.3, -0.1,-0.2,-0.3)	(0.2, 0.5, 0.3,-0.4,-0.2,-0.3)
d	(0.5, 0.5, 0.3, -0.1,-0.2,-0.3)	(0.0,0.0,0.0,0.0,0.0,0.0)

K	C	D
w	(0.0, 0.0, 0.0, 0.0, 0.0, 0.0)	(0.2, 0.5, 0.3, -0.1,-0.2,-0.3)
x	(0.14,0.5,0.3, -0.1,-0.2,-0.3)	(0.25, 0.5, 0.3, -0.1,-0.2,-0.3)
y	(0.5, 0.5, 0.3, -0.1,-0.2,-0.3)	(0.2, 0.5, 0.3,-0.4,-0.2,-0.3)
z	(0.3, 0.5, 0.3, -0.1,-0.2,-0.3)	(0.0,0.0,0.0,0.0,0.0,0.0)

R	R_{PT}	R_{PI}	R_{PF}	R_{NT}	R_{NI}	R_{NF}
A	0.2	0.5	0.3	-0.1	-0.2	-0.3
B	0.14	0.5	0.3	-0.1	-0.2	-0.3

S	S_{PT}	S_{PI}	S_{PF}	S_{NT}	S_{NI}	S_{NF}
C	0.14	0.5	0.3	-0.1	-0.2	-0.3
D	0.2	0.5	0.3	-0.1	-0.2	-0.3

where *f* is defined by: *f(a)=w, f(b)=x, f(c)=y, f(d)=z.*

Here *S(H)= (0.34, 1.0, 0.6, -0.2, -0.4, -0.6) = S(K)*, but, by routine calculations, *H* is not a co-weak isomorphism to *K*.

Definition 3.19

Let *H = (X, E, R)* be a BSVNHG, then the degree of vertex x_i, which is denoted and defined by:

$$\deg(x_i) = \\ (deg_{PT}(x_i), deg_{PI}(x_i), deg_{PF}(x_i), \\ deg_{NT}(x_i), deg_{NI}(x_i),\ deg_{NF}(x_i) \quad (65)$$

where:

$$deg_{PT}(x_i) = \sum R_{PT}(x_1, x_2, \dots, x_r), \quad (66)$$
$$deg_{PI}(x_i) = \sum R_{PI}(x_1, x_2, \dots, x_r), \quad (67)$$
$$deg_{PF}(x_i) = \sum R_{PF}(x_1, x_2, \dots, x_r), \quad (68)$$
$$deg_{NT}(x_i) = \sum R_{NT}(x_1, x_2, \dots, x_r), \quad (69)$$
$$deg_{NI}(x_i) = \sum R_{NI}(x_1, x_2, \dots, x_r), \quad (70)$$
$$deg_{NF}(x_i) = \sum R_{NF}(x_1, x_2, \dots, x_r), \quad (71)$$

for $x_i \neq x_r$.

Theorem 3.20

If *H* and *K* be two isomorphic BSVNHGs, then the degree of their vertices are preserved.

Proof

Let *f: H → K* be an isomorphism between two BSVNHGs *H* and *K* with underlying sets *X* and *Y* respectively, then, by definition, we have:

$$\min[PT_{E_j}(x)] = \min[PT_{F_j}(f(x))], \quad (72)$$
$$\max[PI_{E_j}(x)] = \max[PI_{F_j}(f(x))], \quad (73)$$
$$\max[PF_{E_j}(x)] = \max[PF_{F_j}(f(x))], \quad (74)$$
$$\max[NT_{E_j}(x)] = \max[NT_{F_j}(f(x))], \quad (75)$$
$$\min[NI_{E_j}(x)] = \min[NI_{F_j}(f(x))], \quad (76)$$
$$\min[NF_{E_j}(x)] = \min[NF_{F_j}(f(x))], \quad (77)$$

for all $x \in X$.

$$R_{PT}(x_1, x_2, \dots, x_r) = S_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{78}$$

$$R_{PI}(x_1, x_2, \dots, x_r) = S_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{79}$$

$$R_{PF}(x_1, x_2, \dots, x_r) = S_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{80}$$

$$R_{NT}(x_1, x_2, \dots, x_r) = S_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{81}$$

$$R_{NI}(x_1, x_2, \dots, x_r) = S_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{82}$$

$$R_{NF}(x_1, x_2, \dots, x_r) = S_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{83}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Consider:

$$deg_{PT}(x_i) = \sum R_{PT}(x_1, x_2, \dots, x_r)$$
$$= \sum S_{PT}(f(x_1), f(x_2), \dots, f(x_r))$$
$$= deg_{PT}(f(x_i)), \tag{84}$$

and similarly:

$$deg_{NT}(x_i) = deg_{NT}(f(x_i)), \tag{85}$$

$$deg_{PI}(x_i) = deg_{PI}(f(x_i)), deg_{PF}(x_i) = deg_{PF}(f(x_i)) \tag{86}$$

$$deg_{NI}(x_i) = deg_{NI}(f(x_i)), deg_{NF}(x_i) = deg_{NF}(f(x_i)) \tag{87}$$

Hence:

$$deg(x_i) = deg(f(x_i)). \tag{88}$$

Remark 3.21

The converse of the above theorem may not be true in general.

Example 3.22

Consider the two BSVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b}* and *Y = {x, y}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S* are defined by *Tables* given below:

H	A	B
a	(0.5, 0.5, 0.3, -0.1, -0.2, -0.3)	(0.3, 0.5, 0.3, -0.1, -0.2, -0.3)
b	(0.25, 0.5, 0.3, -0.1, -0.2, -0.3)	(0.2, 0.5, 0.3, -0.1, -0.2, -0.3)

K	C	D
x	(0.3, 0.5, 0.3, -0.1, -0.2, -0.3)	(0.5,0.5,0.3, -0.1, -0.2, -0.3)
y	(0.2, 0.5, 0.3, -0.1, -0.2, -0.3)	(0.25, 0.5, 0.3, -0.1, -0.2, -0.3)

S	S_{PT}	S_{PI}	S_{PF}	S_{NT}	S_{NI}	S_{NF}
C	0.2	0.5	0.3	-0.1	-0.2	-0.3
D	0.25	0.5	0.3	-0.1	-0.2	-0.3

R	R_{PT}	R_{PI}	R_{PF}	R_{NT}	R_{NI}	R_{NF}
A	0.25	0.5	0.3	-0.1	-0.2	-0.3
B	0.2	0.5	0.3	-0.1	-0.2	-0.3

where *f* is defined by: *f(a)=x, f(b)=y,* here *deg(a) = (0.8, 1.0, 0.6, -0.2, -0.4, -0.6) = deg(x)* and *deg(b) = (0.45, 1.0, 0.6, -0.2, -0.4, -0.6) = deg(y).*

But *H* is not isomorphic to *K*, i.e. *H* is neither weak isomorphic, nor co-weak isomorphic to *K*.

Theorem 3.23

The isomorphism between BSVNHGs is an equivalence relation.

Proof

Let *H = (X, E, R), K = (Y, F, S)* and *M = (Z, G, W)* be BSVNHGs with underlying sets X, Y and *Z*, respectively:

Reflexive

Consider the map (identity map) *f : X → X* defined as follows: *f(x) = x* for all *x* ∈ *X,* since the identity map is always bijective and satisfies the conditions:

$$\min[PT_{E_j}(x)] = \min[PT_{E_j}(f(x))], \tag{89}$$

$$\max[PI_{E_j}(x)] = \max[PI_{E_j}(f(x))], \tag{90}$$

$$\max[PF_{E_j}(x)] = \max[PF_{E_j}(f(x))], \tag{91}$$

$$\max[NT_{E_j}(x)] = \max\left[NT_{E_j}(f(x))\right], \tag{92}$$

$$\min[NI_{E_j}(x)] = \min[NI_{E_j}(f(x))], \tag{93}$$

$$\min[NF_{E_j}(x)] = \min[NF_{E_j}(f(x))], \tag{94}$$

for all *x*∈ X.

$$R_{PT}(x_1, x_2, \dots, x_r) = R_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{95}$$

$$R_{PI}(x_1, x_2, \dots, x_r) = R_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{96}$$

$$R_{PF}(x_1, x_2, \dots, x_r) = R_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{97}$$

$$R_{NT}(x_1, x_2, \dots, x_r) = R_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{98}$$

$$R_{NI}(x_1, x_2, \dots, x_r) = R_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{99}$$

$$R_{NF}(x_1, x_2, \dots, x_r) = R_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{100}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Hence *f* is an isomorphism of BSVNHG *H* to itself.

Symmetric

Let *f: X → Y* be an isomorphism of *H* and *K*, then *f* is a bijective mapping defined as *f(x) = y* for all *x* ∈ *X*.

Then, by definition:

$$\min[PT_{E_j}(x)] = \min[PT_{F_j}(f(x))], \tag{101}$$

$$\max[PI_{E_j}(x)] = \max[PI_{F_j}(f(x))], \tag{102}$$

$$\max[PF_{E_j}(x)] = \max[PF_{F_j}(f(x))], \tag{103}$$

$$\max[NT_{E_j}(x)] = \max[NT_{F_j}(f(x))], \tag{104}$$

$$\min[NI_{E_j}(x)] = \min[NI_{F_j}(f(x))], \tag{105}$$

$$\min[NF_{E_j}(x)] = \min[NF_{F_j}(f(x))], \tag{106}$$

for all *x*∈ X.

$$R_{PT}(x_1, x_2, \ldots, x_r) = S_{PT}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{107}$$

$$R_{PI}(x_1, x_2, \ldots, x_r) = S_{PI}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{108}$$

$$R_{PF}(x_1, x_2, \ldots, x_r) = S_{PF}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{109}$$

$$R_{NT}(x_1, x_2, \ldots, x_r) = S_{NT}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{101}$$

$$R_{NI}(x_1, x_2, \ldots, x_r) = S_{NI}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{111}$$

$$R_{NF}(x_1, x_2, \ldots, x_r) = S_{NF}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{112}$$

for all $\{x_1, x_2, \ldots, x_r\}$ subsets of *X*.

Since *f* is bijective, then we have:

$$f^{-1}(y) = x \text{ for all } y \in Y.$$

Thus, we get:

$$\min[PT_{E_j}(f^{-1}(y))] = \min[PT_{F_j}(y)], \tag{113}$$

$$\max[PI_{E_j}(f^{-1}(y))] = \max[PI_{F_j}(y)], \tag{114}$$

$$\max[PF_{E_j}(f^{-1}(y))] = \max[PF_{F_j}(y)], \tag{115}$$

$$\max[NT_{E_j}(f^{-1}(y))] = \max[NT_{F_j}(y)], \tag{116}$$

$$\min[NI_{E_j}(f^{-1}(y))] = \min[NI_{F_j}(y)], \tag{117}$$

$$\min[NF_{E_j}(f^{-1}(y))] = \min[NF_{F_j}(y)], \tag{118}$$

for all *x*∈ X.

$$R_{PT}(f^{-1}(y_1), f^{-1}(y_2), \ldots, f^{-1}(y_r)) = S_{PT}(y_1, y_2, \ldots, y_r), \tag{119}$$

$$R_{PI}(f^{-1}(y_1), f^{-1}(y_2), \ldots, f^{-1}(y_r)) = S_{PI}(y_1, y_2, \ldots, y_r), \tag{120}$$

$$R_{PF}(f^{-1}(y_1), f^{-1}(y_2), \ldots, f^{-1}(y_r)) = S_{PF}(y_1, y_2, \ldots, y_r), \tag{121}$$

$$R_{NT}\big(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\big) = S_{NT}(y_1, y_2, \dots, y_r), \quad (122)$$

$$R_{NI}\big(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\big) = S_{NI}(y_1, y_2, \dots, y_r), \quad (123)$$

$$R_{NF}\big(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\big) = S_{NF}(y_1, y_2, \dots, y_r), \quad (124)$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of Y.

Hence, we have a bijective map $f^{-1} : Y \to X$ which is an isomorphism from K to H.

Transitive

Let $f : X \to Y$ and $g : Y \to Z$ be two isomorphism of BSVNHGs of H onto K and K onto M, respectively. Then $g \; o \; f$ is bijective mapping from X to Z, where $g \; o \; f$ is defined as $(g \; o \; f)(x) = g\big(f(x)\big)$ for all $x \in X$.

Since f is an isomorphism, then by definition $f(x) = y$ for all $x \in X$, which satisfies the conditions:

$$\min[PT_{E_j}(x)] = \min[PT_{F_j}\big(f(x)\big)], \quad (125)$$

$$\max[PI_{E_j}(x)] = \max[PI_{F_j}\big(f(x)\big)], \quad (126)$$

$$\max[PF_{E_j}(x)] = \max[PF_{F_j}\big(f(x)\big)], \quad (127)$$

$$\max[NT_{E_j}(x)] = \max[NT_{F_j}\big(f(x)\big)], \quad (128)$$

$$\min[NI_{E_j}(x)] = \min\Big[NI_{F_j}\big(f(x)\big)\Big], \quad (129)$$

$$\min[NF_{E_j}(x)] = \min[NF_{F_j}\big(f(x)\big)], \quad (130)$$

for all $x \in X$.

$$R_{PT}(x_1, x_2, \dots, x_r) = S_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \quad (131)$$

$$R_{PI}(x_1, x_2, \dots, x_r) = S_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \quad (132)$$

$$R_{PF}(x_1, x_2, \dots, x_r) = S_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \quad (133)$$

$$R_{NT}(x_1, x_2, \dots, x_r) = S_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \quad (134)$$

$$R_{NI}(x_1, x_2, \dots, x_r) = S_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \quad (135)$$

$$R_{NF}(x_1, x_2, \dots, x_r) = S_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \quad (136)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Since $g : Y \to Z$ is an isomorphism, then by definition $g(y) = z$ for all $y \in Y$ satisfying the conditions:

$$\min[PT_{F_j}(y)] = \min[PT_{G_j}\big(g(y)\big)], \quad (137)$$

$$\max[PI_{F_j}(y)] = \max[PI_{G_j}\big(g(y)\big)], \quad (138)$$

$$\max[PF_{F_j}(y)] = \max[PF_{G_j}(g(y))], \tag{139}$$

$$\max[NT_{F_j}(y)] = \max\left[NT_{G_j}(g(y))\right], \tag{140}$$

$$\min[NI_{F_j}(y)] = \min[NI_{G_j}(g(y))], \tag{141}$$

$$\min[NF_{F_j}(y)] = \min\left[NF_{G_j}(g(y))\right], \tag{142}$$

for all $x \in X$.

$$S_{PT}(y_1, y_2, \dots, y_r) = W_{PT}(g(y_1), g(y_2), \dots, g(y_r)), \tag{143}$$

$$S_{PI}(y_1, y_2, \dots, y_r) = W_{PI}(g(y_1), g(y_2), \dots, g(y_r)), \tag{144}$$

$$S_{PF}(y_1, y_2, \dots, y_r) = W_{PF}(g(y_1), g(y_2), \dots, g(y_r)), \tag{145}$$

$$S_{NT}(y_1, y_2, \dots, y_r) = W_{NT}(g(y_1), g(y_2), \dots, g(y_r)), \tag{146}$$

$$S_{NI}(y_1, y_2, \dots, y_r) = W_{NI}(g(y_1), g(y_2), \dots, g(y_r)), \tag{147}$$

$$S_{NF}(y_1, y_2, \dots, y_r) = W_{NF}(g(y_1), g(y_2), \dots, g(y_r)), \tag{148}$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of *Y*.

Thus, from above equations we conclude that:

$$\min[PT_{E_j}(x)] = \min[PT_{G_j}(g(f(x)))], \tag{149}$$

$$\max[PI_{E_j}(x)] = \max[PI_{G_j}(g(f(x)))], \tag{150}$$

$$\max[PF_{E_j}(x)] = \max[PF_{G_j}(g(f(x)))], \tag{151}$$

$$\max[NT_{E_j}(x)] = \max\left[NT_{G_j}\left(g(f(x))\right)\right], \tag{152}$$

$$\min[NI_{E_j}(x)] = \min[NI_{G_j}(g(f(x)))], \tag{153}$$

$$\min[NF_{E_j}(x)] = \min[NF_{G_j}(g(f(x)))], \tag{154}$$

for all $x \in X$.

$$R_{PT}(x_1, \dots, x_r) = W_{PT}(g(f(x_1)), \dots, g(f(x_r))), \tag{155}$$

$$R_{PI}(x_1, \dots, x_r) = W_{PI}(g(f(x_1)), \dots, g(f(x_r))), \tag{156}$$

$$R_{PF}(x_1, \dots, x_r) = W_{PF}(g(f(x_1)), \dots, g(f(x_r))), \tag{157}$$

$$R_{NT}(x_1, \dots, x_r) = W_{NT}(g(f(x_1)), \dots, g(f(x_r))), \tag{158}$$

$$R_{NI}(x_1, \dots, x_r) = W_{NI}(g(f(x_1)), \dots, g(f(x_r))), \tag{159}$$

$$R_{NF}(x_1, \dots, x_r) = W_{NF}(g(f(x_1)), \dots, g(f(x_r))), \tag{160}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X*.

Therefore $g \ o \ f$ is an isomorphism between *H* and *M*.

Hence, the isomorphism between BSVNHGs is an equivalence relation.

Theorem 3.24

The weak isomorphism between BSVNHGs satisfies the partial order relation.

Proof

Let $H = (X, E, R)$, $K = (Y, F, S)$ and $M = (Z, G, W)$ be BSVNHGs with underlying sets X, Y and Z, respectively:

Reflexive

Consider the map (identity map) $f : X \to X$ defined as follows: $f(x)=x$ for all $x \in X$, since the identity map is always bijective and satisfies the conditions:

$$\min[PT_{E_j}(x)] = \min[PT_{E_j}(f(x))], \tag{161}$$

$$\max[PI_{E_j}(x)] = \max[PI_{E_j}(f(x))], \tag{162}$$

$$\max[PF_{E_j}(x)] = \max[PF_{E_j}(f(x))], \tag{163}$$

$$\max[NT_{E_j}(x)] = \max[NT_{E_j}(f(x))], \tag{164}$$

$$\min[NI_{E_j}(x)] = \min[NI_{E_j}(f(x))], \tag{165}$$

$$\min[NF_{E_j}(x)] = \min[NF_{E_j}(f(x))], \tag{166}$$

for all $x \in X$.

$$R_{PT}(x_1, x_2, \dots, x_r) \leq R_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{167}$$

$$R_{PI}(x_1, x_2, \dots, x_r) \geq R_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{168}$$

$$R_{PF}(x_1, x_2, \dots, x_r) \geq R_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{169}$$

$$R_{NT}(x_1, x_2, \dots, x_r) \geq R_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{170}$$

$$R_{NI}(x_1, x_2, \dots, x_r) \leq R_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{171}$$

$$R_{NF}(x_1, x_2, \dots, x_r) \leq R_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{172}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.
Hence, f is a weak isomorphism of BSVNHG H to itself.

Anti-symmetric

Let f be a weak isomorphism between H onto K, and g be a weak isomorphic between K and H, that is $f: X \to Y$ is a bijective map defined by: $f(x) = y$ for all $x \in X$ satisfying the conditions:

$$\min[PT_{E_j}(x)] = \min[PT_{F_j}(f(x))], \tag{173}$$

$$\max[PI_{E_j}(x)] = \max[PI_{F_j}(f(x))], \tag{174}$$

$$\max[PF_{E_j}(x)] = \max[PF_{F_j}(f(x))], \tag{175}$$

$$\max[NT_{E_j}(x)] = \max[NT_{F_j}(f(x))], \tag{176}$$

$$\min[NI_{E_j}(x)] = \min[NI_{F_j}(f(x))], \tag{177}$$

$$\min[NF_{E_j}(x)] = \min[NF_{F_j}(f(x))], \tag{178}$$

for all $x \in X$.

$$R_{PT}(x_1, x_2, \dots, x_r) = S_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{179}$$

$$R_{PI}(x_1, x_2, \dots, x_r) = S_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{180}$$

$$R_{PF}(x_1, x_2, \dots, x_r) = S_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{181}$$

$$R_{NT}(x_1, x_2, \dots, x_r) = S_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{182}$$

$$R_{NI}(x_1, x_2, \dots, x_r) = S_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{183}$$

$$R_{NF}(x_1, x_2, \dots, x_r) = S_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{184}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Since g is also bijective map $g(y) = x$ for all $y \in Y$ satisfying the conditions:

$$\min[PT_{F_j}(y)] = \min[PT_{E_j}(g(y))], \tag{185}$$

$$\max[PI_{F_j}(y)] = \max[PI_{E_j}(g(y))], \tag{186}$$

$$\max[PF_{F_j}(y)] = \max[PF_{E_j}(g(y))], \tag{187}$$

$$\max[NT_{F_j}(y)] = \max[NT_{E_j}(g(y))], \tag{188}$$

$$\min[NI_{F_j}(y)] = \min[NI_{E_j}(g(y))], \tag{189}$$

$$\min[NF_{F_j}(y)] = \min[NF_{E_j}(g(y))], \tag{190}$$

for all $y \in Y$.

$$R_{PT}(y, y_2, \dots, y_r) \leq S_{PT}(g(y_1), g(y_2), \dots, g(y_r)), \tag{191}$$

$$R_{PI}(y_1, y_2, \dots, y_r) \geq S_{PI}(f(y_1), f(y_2), \dots, f(y_r)), \tag{192}$$

$$R_{PF}(y_1, y_2, \dots, y_r) \geq S_{PF}(f(y_1), f(y_2), \dots, f(y_r)), \tag{193}$$

$$R_{NT}(y, y_2, \dots, y_r) \geq S_{NT}(g(y_1), g(y_2), \dots, g(y_r)), \tag{194}$$

$$R_{NI}(y_1, y_2, \dots, y_r) \leq S_{NI}(f(y_1), f(y_2), \dots, f(y_r)), \tag{195}$$

$$R_{NF}(y_1, y_2, \dots, y_r) \leq S_{NF}(f(y_1), f(y_2), \dots, f(y_r)), \tag{196}$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of Y.

The above inequalities hold for finite sets X and Y only whenever H and K have same number of edges and corresponding edge have same weights, hence H is identical to K.

Transitive

Let $f: X \to Y$ and $g: Y \to Z$ be two weak isomorphism of BSVNHGs of H onto K and K onto M, respectively. Then $g\ o\ f$ is bijective mapping from X to Z, where $g\ o\ f$ is defined as $(g\ o\ f)(x) = g(f(x))$ for all $x \in X$.

Since f is a weak isomorphism, then by definition $f(x) = y$ for all $x \in X$ which satisfies the conditions:

$$\min[PT_{E_j}(x)] = \min[PT_{F_j}(f(x))], \tag{197}$$

$$\max[PI_{E_j}(x)] = \max[PI_{F_j}(f(x))], \tag{198}$$

$$\max[PF_{E_j}(x)] = \max[PF_{F_j}(f(x))], \tag{199}$$

$$\max[NT_{E_j}(x)] = \max[NT_{F_j}(f(x))], \tag{200}$$

$$\min[NI_{E_j}(x)] = \min[NI_{F_j}(f(x))], \tag{201}$$

$$\min[NF_{E_j}(x)] = \min[NF_{F_j}(f(x))], \tag{202}$$

for all $x \in X$.

$$R_{PT}(x_1, x_2, \dots, x_r) \leq S_{PT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{203}$$

$$R_{PI}(x_1, x_2, \dots, x_r) \geq S_{PI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{204}$$

$$R_{PF}(x_1, x_2, \dots, x_r) \geq S_{PF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{205}$$

$$R_{NT}(x_1, x_2, \dots, x_r) \geq S_{NT}(f(x_1), f(x_2), \dots, f(x_r)), \tag{206}$$

$$R_{NI}(x_1, x_2, \dots, x_r) \leq S_{NI}(f(x_1), f(x_2), \dots, f(x_r)), \tag{207}$$

$$R_{NF}(x_1, x_2, \dots, x_r) \leq S_{NF}(f(x_1), f(x_2), \dots, f(x_r)), \tag{208}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Since $g : Y \to Z$ is a weak isomorphism, then by definition $g(y) = z$ for all $y \in Y$, satisfying the conditions:

$$\min[PT_{F_j}(y)] = \min[PT_{G_j}(g(y))], \tag{209}$$

$$\max[PI_{F_j}(y)] = \max[PI_{G_j}(g(y))], \tag{210}$$

$$\max[PF_{F_j}(y)] = \max[PF_{G_j}(g(y))], \tag{211}$$

$$\max[NT_{F_j}(y)] = \max[NT_{G_j}(g(y))], \tag{212}$$

$$\min[NI_{F_j}(y)] = \min[NI_{G_j}(g(y))], \tag{213}$$

$$\min[NF_{F_j}(y)] = \min[NF_{G_j}(g(y))], \tag{214}$$

for all $x \in X$.

$$S_{PT}(y_1, y_2, \dots, y_r) \leq W_{PT}(g(y_1), g(y_2), \dots, g(y_r)), \tag{215}$$

$$S_{PI}(y_1, y_2, \dots, y_r) \geq W_{PI}(g(y_1), g(y_2), \dots, g(y_r)), \tag{216}$$

$$S_{PF}(y_1, y_2, \dots, y_r) \geq W_{PF}(g(y_1), g(y_2), \dots, g(y_r)), \tag{217}$$

$$S_{NT}(y_1, y_2, \dots, y_r) \geq W_{NT}(g(y_1), g(y_2), \dots, g(y_r)), \tag{218}$$

$$S_{NI}(y_1, y_2, \dots, y_r) \leq W_{NI}(g(y_1), g(y_2), \dots, g(y_r)), \tag{219}$$

$$S_{NF}(y_1, y_2, \dots, y_r) \leq W_{NF}(g(y_1), g(y_2), \dots, g(y_r)), \tag{220}$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of Y.

Thus, from above equations, we conclude that:

$$\min[PT_{E_j}(x)] = \min[PT_{G_j}(g(f(x)))], \quad (221)$$

$$\max[PI_{E_j}(x)] = \max[PI_{G_j}(g(f(x)))], \quad (222)$$

$$\max[PF_{E_j}(x)] = \max[PF_{G_j}(g(f(x)))], \quad (223)$$

$$\max[NT_{E_j}(x)] = \max[NT_{G_j}(g(f(x)))], \quad (224)$$

$$\min[NI_{E_j}(x)] = \min[NI_{G_j}(g(f(x)))], \quad (225)$$

$$\min[NF_{E_j}(x)] = \min[NF_{G_j}(g(f(x)))], \quad (226)$$

for all $x \in X$.

$$R_{PT}(x_1, \dots, x_r) \leq W_{PT}(g(f(x_1)), \dots, g(f(x_r))), \quad (227)$$

$$R_{PI}(x_1, \dots, x_r) \geq W_{PI}(g(f(x_1)), \dots, g(f(x_r))), \quad (228)$$

$$R_{PF}(x_1, \dots, x_r) \geq W_{PF}(g(f(x_1)), \dots, g(f(x_r))), \quad (229)$$

$$R_{NT}(x_1, \dots, x_r) \geq W_{NT}(g(f(x_1)), \dots, g(f(x_r))), \quad (230)$$

$$R_{NI}(x_1, \dots, x_r) \leq W_{NI}(g(f(x_1)), \dots, g(f(x_r))), \quad (231)$$

$$R_{NF}(x_1, \dots, x_r) \leq W_{NF}(g(f(x_1)), \dots, g(f(x_r))), \quad (232)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of X.

Therefore $g\ o\ f$ is a weak isomorphism between H and M.

Hence, the weak isomorphism between BSVNHGs is a partial order relation.

4 Conclusion

The bipolar single valued neutrosophic hypergraph can be applied in various areas of engineering and computer science. In this paper, the isomorphism between BSVNHGs is proved to be an equivalence relation and the weak isomorphism is proved to be a partial order relation. Similarly, it can be proved that co-weak isomorphism in BSVNHGs is a partial order relation.

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Subtraction and Division of Neutrosophic Numbers

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Abstract

In this paper, we define the subtraction and the division of neutrosophic single-valued numbers. The restrictions for these operations are presented for neutrosophic single-valued numbers and neutrosophic single-valued overnumbers / undernumbers / offnumbers. Afterwards, several numeral examples are presented.

Keywords

neutrosophic calculus, neutrosophic numbers, neutrosophic summation, neutrosophic multiplication, neutrosophic scalar multiplication, neutrosophic power, neutrosophic subtraction, neutrosophic division.

1 Introduction

Let $A = (t_1, i_1, f_1)$ and $B = (t_2, i_2, f_2)$ be two single-valued neutrosophic numbers, where $t_1, i_1, f_1, t_2, i_2, f_2 \in [0, 1]$, and $0 \leq t_1, i_1, f_1 \leq 3$ and $0 \leq t_2, i_2, f_2 \leq 3$.

The following operational relations have been defined and mostly used in the neutrosophic scientific literature:

1.1 Neutrosophic Summation

$$A \oplus B = (t_1 + t_2 - t_1 t_2, i_1 i_2, f_1 f_2) \tag{1}$$

1.2 Neutrosophic Multiplication

$$\mathrm{A} \otimes B = (t_1 t_2, i_1 + i_2 - i_1 i_2, f_1 + f_2 - f_1 f_2) \tag{2}$$

1.3 Neutrosophic Scalar Multiplication

$$\lambda A = (1 - (1 - t_1)^{\lambda}, i_1^{\lambda}, f_1^{\lambda}), \tag{3}$$

where $\lambda \in \mathbb{R}$, and $\lambda > 0$.

1.4 Neutrosophic Power

$$A^{\curlywedge} = (t_1^{\curlywedge}, 1-(1-i_1)^{\curlywedge}, 1-(1-f_1)^{\curlywedge}), \tag{4}$$

where $\curlywedge \in \mathbb{R}$, and $\curlywedge > 0$.

2 Remarks

Actually, the neutrosophic scalar multiplication is an extension of neutrosophic summation; in the last, one has $\curlywedge = 2$.

Similarly, the neutrosophic power is an extension of neutrosophic multiplication; in the last, one has $\curlywedge = 2$.

Neutrosophic summation of numbers is equivalent to neutrosophic union of sets, and neutrosophic multiplication of numbers is equivalent to neutrosophic intersection of sets.

That's why, both the neutrosophic summation and neutrosophic multiplication (and implicitly their extensions neutrosophic scalar multiplication and neutrosophic power) can be defined in many ways, i.e. equivalently to their neutrosophic union operators and respectively neutrosophic intersection operators.

In general:

$$A \oplus B = (t_1 \vee t_2, i_1 \wedge i_2, f_1 \wedge f_2), \tag{5}$$

or

$$A \oplus B = (t_1 \vee t_2, i_1 \vee i_2, f_1 \vee f_2), \tag{6}$$

and analogously:

$$A \otimes B = (t_1 \wedge t_2, i_1 \vee i_2, f_1 \vee f_2) \tag{7}$$

or

$$A \otimes B = (t_1 \wedge t_2, i_1 \wedge i_2, f_1 \vee f_2), \tag{8}$$

where "$\vee$" is the fuzzy OR (fuzzy union) operator, defined, for $\alpha, \beta \in [0, 1]$, in three different ways, as:

$$\alpha \overset{1}{\vee} \beta = \alpha + \beta - \alpha\beta, \tag{9}$$

or

$$\alpha \overset{2}{\vee} \beta = max\{\alpha, \beta\}, \tag{10}$$

or

$$\alpha \overset{3}{\vee} \beta = min\{x + y, 1\}, \tag{11}$$

etc.

While "∧" is the fuzzy AND (fuzzy intersection) operator, defined, for $\alpha, \beta \in [0, 1]$, in three different ways, as:

$$\alpha \underset{1}{\wedge} \beta = \alpha\beta, \tag{12}$$

or

$$\alpha \underset{2}{\wedge} \beta = min\{\alpha, \beta\}, \tag{13}$$

or

$$\alpha \underset{3}{\wedge} \beta = max\{x + y - 1, 0\}, \tag{14}$$

etc.

Into the definitions of $A \oplus B$ and $A \otimes B$ it's better if one associates $\overset{1}{\vee}$ with $\underset{1}{\wedge}$, since $\overset{1}{\vee}$ is opposed to $\underset{1}{\wedge}$, and $\overset{2}{\vee}$ with $\underset{2}{\wedge}$, and $\overset{3}{\vee}$ with $\underset{3}{\wedge}$, for the same reason. But other associations can also be considered.

For examples:

$$A \oplus B = (t_1 + t_2 - t_1t_2, i_1 + i_2 - i_1i_2, f_1f_2), \tag{15}$$

or

$$A \oplus B = (max\{t_1, t_2\}, min\{i_1, i_2\}, min\{f_1, f_2\}), \tag{16}$$

or

$$A \oplus B = (max\{t_1, t_2\}, max\{i_1, i_2\}, min\{f_1, f_2\}), \tag{17}$$

or

$$A \oplus B = (min\{t_1 + t_2, 1\}, max\{i_1 + i_2 - 1, 0\}, max\{f_1 + f_2 - 1, 0\}). \tag{18}$$

where we have associated $\overset{1}{\vee}$ with $\underset{1}{\wedge}$, and $\overset{2}{\vee}$ with $\underset{2}{\wedge}$, and $\overset{3}{\vee}$ with $\underset{3}{\wedge}$.

Let's associate them in different ways:

$$A \oplus B = (t_1 + t_2 - t_1t_2, min\{i_1, i_2\}, min\{f_1, f_2\}), \tag{19}$$

where $\overset{1}{\vee}$ was associated with $\underset{2}{\wedge}$ and $\underset{3}{\wedge}$; or:

$$A \oplus B = (max\{t_1, t_2\}, i_1, i_2, max\{f_1 + f_2 - 1, 0\}), \tag{20}$$

where $\overset{2}{\vee}$ was associated with $\underset{1}{\wedge}$ and $\underset{3}{\wedge}$; and so on.

Similar examples can be constructed for $A \otimes B$.

3 Neutrosophic Subtraction

We define now, for the first time, the subtraction of neutrosophic number:

$$A \ominus B = (t_1, i_1, f_1) \ominus (t_2, i_2, f_2) = \left(\frac{t_1 - t_2}{1 - t_2}, \frac{i_1}{i_2}, \frac{f_1}{f_2}\right) = C, \tag{21}$$

for all $t_1, i_1, f_1, t_2, i_2, f_2 \in [0, 1]$, with the restrictions that: $t_2 \neq 1$, $i_2 \neq 0$, and $f_2 \neq 0$.

So, the neutrosophic subtraction only partially works, i.e. when $t_2 \neq 1$, $i_2 \neq 0$, and $f_2 \neq 0$.

The restriction that

$$\left(\frac{t_1 - t_2}{1 - t_2}, \frac{i_1}{i_2}, \frac{f_1}{f_2}\right) \in ([0, 1], [0, 1], [0, 1]) \tag{22}$$

is set when the classical case when the neutrosophic number components t, i, f are in the interval $[0, 1]$.

But, for the general case, when dealing with neutrosophic overset / underset /offset [1], or the neutrosophic number components are in the interval $[\Psi, \Omega]$, where Ψ is called *underlimit* and Ω is called *overlimit*, with $\Psi \leq 0 < 1 \leq \Omega$, i.e. one has *neutrosophic overnumbers / undernumbers / offnumbers*, then the restriction (22) becomes:

$$\left(\frac{t_1 - t_2}{1 - t_2}, \frac{i_1}{i_2}, \frac{f_1}{f_2}\right) \in ([\Psi, \Omega], [\Psi, \Omega], [\Psi, \Omega]). \tag{23}$$

3.1 Proof

The formula for the subtraction was obtained from the attempt to be consistent with the neutrosophic addition.

One considers the most used neutrosophic addition:

$$(a_1, b_1, c_1) \oplus (a_2, b_2, c_2) = (a_1 + a_2 - a_1 a_2, b_1 b_2, c_1 c_2), \tag{24}$$

We consider the $\ominus$ neutrosophic operation the opposite of the $\oplus$ neutrosophic operation, as in the set of real numbers the classical subtraction $-$ is the opposite of the classical addition $+$.

Therefore, let's consider:

$$(t_1, i_1, f_1) \ominus (t_2, i_2, f_2) = (x, y, z), \tag{25}$$

$$\oplus\ (t_2, i_2, f_2) \qquad\qquad \oplus\ (t_2, i_2, f_2)$$

where $x, y, z \in \mathbb{R}$.

We neutrosophically add $\oplus\ (t_2, i_2, f_2)$ on both sides of the equation. We get:

$$(t_1, i_1, f_1) = (x, y, z) \oplus (t_2, i_2, f_2) = (x + t_2 - x t_2, y i_2, z f_2). \tag{26}$$

Or,

$$\begin{cases} t_1 = x + t_2 - x t_2, \text{whence } x = \frac{t_1 - t_1}{1 - t_2}\ ; \\ i_1 = y i_2, \text{whence } y = \frac{i_1}{i_2}\ ; \\ f_1 = z f_2, \text{whence } z = \frac{f_1}{f_2}\ . \end{cases} \tag{27}$$

3.2 Checking the Subtraction

With $A = (t_1, i_1, f_1)$, $B = (t_2, i_2, f_2)$, and $C = \left(\frac{t_1-t_2}{1-t_2}, \frac{i_1}{i_2}, \frac{f_1}{f_2}\right)$,
where $t_1, i_1, f_1, t_2, i_2, f_2 \in [0,1]$, and $t_2 \neq 1$, $i_2 \neq 0$, and $f_2 \neq 0$, we have:

$$A \ominus B = C. \tag{28}$$

Then:

$$B \oplus C = (t_2, i_2, f_2) \oplus \left(\frac{t_1-t_2}{1-t_2}, \frac{i_1}{i_2}, \frac{f_1}{f_2}\right) = \left(t_2 + \frac{t_1-t_2}{1-t_2} - t_2 \cdot \frac{t_1-t_2}{1-t_2}, i_2 \cdot \frac{i_1}{i_2}, f_2 \cdot \frac{f_1}{f_2}\right) = \left(\frac{t_2-t_2^2+t_1-t_2-t_1t_2+t_2}{1-t_2}, i_1, f_1\right) = \left(\frac{t_1(1-t_2)}{1-t_2}, i_1, f_1\right) = (t_1, i_1, f_1). \tag{29}$$

$$A \ominus C = (t_1, i_1, f_1) \ominus \left(\frac{t_1-t_2}{1-t_2}, \frac{i_1}{i_2}, \frac{f_1}{f_2}\right) = \left(\frac{t_1 - \frac{t_1-t_2}{1-t_2}}{1-\frac{t_1-t_2}{1-t_2}}, \frac{i_1}{\frac{i_1}{i_2}}, \frac{f_1}{\frac{f_1}{f_2}}\right) = \left(\frac{\frac{t_1-t_1t_2-t_1+t_2}{1-t_2}}{\frac{1-t_2-t_1+t_2}{1-t_2}}, i_2, f_2\right) = \left(\frac{-t_1t_2+t_2}{1-t_2}, i_2, f_2\right) = \left(\frac{t_2(-t_1+1)}{1-t_2}, i_2, f_2\right) = (t_2, i_2, f_2). \tag{30}$$

4 Division of Neutrosophic Numbers

We define for the first time the division of neutrosophic numbers:

$$A \oslash B = (t_1, i_1, f_1) \oslash (t_2, i_2, f_2) = \left(\frac{t_1}{t_2}, \frac{i_1-i_2}{1-i_2}, \frac{f_1-f_2}{1-f_2}\right) = D, \tag{31}$$

where $t_1, i_1, f_1, t_2, i_2, f_2 \in [0,1]$, with the restriction that $t_2 \neq 0$, $i_2 \neq 1$, and $f_2 \neq 1$.
Similarly, the division of neutrosophic numbers only partially works, i.e. when $t_2 \neq 0$, $i_2 \neq 1$, and $f_2 \neq 1$.
In the same way, the restriction that

$$\left(\frac{t_1}{t_2}, \frac{i_1-i_2}{1-i_2}, \frac{f_1-f_2}{1-f_2}\right) \in ([0,1], [0,1], [0,1]) \tag{32}$$

is set when the traditional case occurs, when the neutrosophic number components t, i, f are in the interval [0, 1].
But, for the case when dealing with neutrosophic overset / underset /offset [1], when the neutrosophic number components are in the interval $[\Psi, \Omega]$, where Ψ is called *underlimit* and Ω is called *overlimit*, with $\Psi \leq 0 < 1 \leq \Omega$, i.e. one has *neutrosophic overnumbers / undernumbers / offnumbers*, then the restriction (31) becomes:

$$\left(\frac{t_1}{t_2}, \frac{i_1-i_2}{1-i_2}, \frac{f_1-f_2}{1-f_2}\right) \in ([\Psi,\Omega],[\Psi,\Omega],[\Psi,\Omega]). \tag{33}$$

4.1 Proof

In the same way, the formula for division $\oslash$ of neutrosophic numbers was obtained from the attempt to be consistent with the neutrosophic multiplication.

We consider the $\oslash$ neutrosophic operation the opposite of the $\otimes$ neutrosophic operation, as in the set of real numbers the classical division $\div$ is the opposite of the classical multiplication $\times$.

One considers the most used neutrosophic multiplication:

$$(a_1, b_1, c_1) \otimes (a_2, b_2, c_2)$$
$$= (a_1 a_2, b_1 + b_2 - b_1 b_2, c_1 + c_2 - c_1 c_2), \tag{34}$$

Thus, let's consider:

$$(t_1, i_1, f_1) \oslash (t_2, i_2, f_2) = (x, y, z), \tag{35}$$
$$\otimes(t_2, i_2, f_2) \qquad\qquad \otimes(t_2, i_2, f_2)$$

where $x, y, z \in \mathbb{R}$.

We neutrosophically multiply $\otimes$ both sides by (t_2, i_2, f_2). We get

$$(t_1, i_1, f_1) = (x, y, z) \otimes (t_2, i_2, f_2)$$
$$= (xt_2, y + i_2 - yi_2, z + f_2 - zf_2). \tag{36}$$

Or,

$$\begin{cases} t_1 = xt_2, \text{whence } x = \frac{t_1}{t_2};: \\ i_1 = y + i_2 - yi_2, \text{whence } y = \frac{i_1-i_2}{1-i_2}; \\ f_1 = z + f_2 - zf_2, \text{whence } z = \frac{f_1-f_2}{1-f_2}. \end{cases} \tag{37}$$

4.2 Checking the Division

With $A = (t_1, i_1, f_1)$, $B = (t_2, i_2, f_2)$, and $D = \left(\frac{t_1}{t_2}, \frac{i_1-i_2}{1-i_2}, \frac{f_1-f_2}{1-f_2}\right)$,

where $t_1, i_1, f_1, t_2, i_2, f_2 \in [0,1]$, and $t_2 \neq 0$, $i_2 \neq 1$, and $f_2 \neq 1$, one has:

$$A * B = D. \tag{38}$$

Then:

$$\frac{B}{D} = (t_2, i_2, f_2) \times \left(\frac{t_1}{t_2}, \frac{i_1-i_2}{1-i_2}, \frac{f_1-f_2}{1-f_2}\right) = \left(t_2 \cdot \frac{t_1}{t_2}, i_2 + \frac{i_1-i_2}{1-i_2} - i_2 \cdot \frac{i_1-i_2}{1-i_2}, f_2 + \frac{f_1-f_2}{1-f_2} - f_2 \cdot \frac{f_1-f_2}{1-f_2}\right) =$$

$$\left(t_1, \frac{i_2-i_2^2+i_1-i_2-i_1 i_2+i_2^2}{1-i_2}, \frac{f_2-f_2^2+f_1-f_2-f_1 f_2+f_2^2}{1-f_2}\right) =$$
$$\left(t_1, \frac{i_1(1-i_2)}{1-i_2}, \frac{f_1(1-f_2)}{1-f_2}\right) = (t_1, i_1, f_1) = A. \tag{39}$$

Also:

$$\frac{A}{D} = \frac{(t_1, i_1, f_1)}{\left(\frac{t_1}{t_2}, \frac{i_1-i_2}{1-i_2}, \frac{f_1-f_2}{1-f_2}\right)} = \left(\frac{t_1}{\frac{t_1}{t_2}}, \frac{i_1-\frac{i_1-i_2}{1-i_2}}{1-\frac{i_1-i_2}{1-i_2}}, \frac{f_1-\frac{f_1-f_2}{1-f_2}}{1-\frac{f_1-f_2}{1-f_2}}\right) =$$
$$\left(t_2, \frac{\frac{i_1-i_1 i_2-i_1+i_2}{1-i_2}}{\frac{1-i_2-i_1+i_2}{1-i_2}}, \frac{\frac{f_1-f_1 f_2-f_1+f_2}{1-f_2}}{\frac{1-f_2-f_1+f_2}{1-f_2}}\right) = \left(t_2, \frac{\frac{i_2(-i_1+1)}{1-i_2}}{\frac{1-i_1}{1-i_2}}, \frac{\frac{f_2(-f_1+1)}{1-f_2}}{\frac{1-f_1}{1-f_2}}\right) =$$
$$\left(t_2, \frac{i_2(1-i_1)}{1-i_1}, \frac{f_2(1-f_1)}{1-f_1}\right) = (t_2, i_2, f_2) = B. \tag{40}$$

5 Conclusion

We have obtained the formula for the subtraction of neutrosophic numbers $\ominus$ going backwords from the formula of addition of neutrosophic numbers $\oplus$.

Similarly, we have defined the formula for division of neutrosophic numbers $\oslash$ and we obtained it backwords from the neutrosophic multiplication $\otimes$.

We also have taken into account the case when one deals with classical neutrosophic numbers (i.e. the neutrosophic components t, i, f belong to [0, 1]) as well as the general case when t, i, f belong to $[\Psi, \Omega]$, where the underlimit $\Psi \leq 0$ and the overlimit $\Omega \geq 1$.

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Rough Neutrosophic Hyper-complex Set and its Application to Multi-attribute Decision Making

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Abstract

This paper presents multi-attribute decision making based on rough neutrosophic hyper-complex sets with rough neutrosophic hyper-complex attribute values. The concept of neutrosophic hyper-complex set is a powerful mathematical tool to deal with incomplete, indeterminate and inconsistent information. We extend the concept of neutrosophic hyper-complex set to rough neutrosophic hyper-complex environ-ment. The ratings of all alternatives are expressed in terms of the upper / lower approximations and pairs of neutrosophic hyper-complex sets, which are characterized by two hyper-complex functions and an indeterminacy component. We also define cosine function based on rough neutrosophic hyper-complex set to determine the degree of similarity between rough neutrosophic hyper-complex sets. We establish a new decision making approach based on rough neutrosphic hyper-complex set. Finally, a numerical example is provided to prove the applicability of the proposed approach.

Keywords

neutrosophic set, rough neutrosophic set, rough neutrosophic hyper-complex set, cosine function, decision making.

1 Introduction

The concept of rough neutrosophic set has been introduced by Broumi et al. [1, 2]. It has been derived as a combination of the concepts of rough set proposed by Z. Pawlak [3] and of neutrosophic set introduced by F. Smarandache [4, 5]. The rough sets and the neutrosophic sets and are both capable of dealing with partial information and uncertainty.

To deal with real world problems, Wang et al. [6] introduced the single valued netrosophic sets (SVNSs).

Recently, Mondal and Pramanik proposed a few decision making models in rough neutrosophic environment.

Mondal and Pramanik [7] applied the concept of grey relational analysis based rough neutrosophic set in multi-attribute decision making.

Mondal and Pramanik [8] also studied the cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis.

The same authors [9] proposed the multi-attribute decision making using rough accuracy and score function.

The same authors [10] also proposed the cotangent similarity measure under rough neutrosophic sets.

The same authors [11] further studied some similarity measures, namely Dice similarity measure [12] and Jaccard similarity measure [12] in rough neutrosophic sets.

The rough neutrosophic hyper-complex set is the generalization of rough neutrosophic set and of neutrosophic hyper-complex set [13].

S. Olariu [14] introduced the concept of hyper-complex number, and studied some of its properties.

Mandal and Basu [15] studied hyper-complex similarity measure for SVNS and its application in decision making.

Mondal and Pramanik [16] studied tri-complex rough neutrosophic similarity measure and presented an application in multi-attribute decision making.

In this paper, we have defined the rough neutrosophic hyper-complex set and the rough neutrosophic hyper-complex cosine function (RNHCF).

We have also proposed a multi-attribute decision making process in rough neutrosophic hyper-complex environment.

The paper is organized in the following way: *Section 2* presents preliminaries of neutrosophic sets and of single valued neutrosophic sets, and some basic ideas of hyper-complex sets. *Section 3* gives the definition of the rough neutro-sophic hyper-complex sets. *Section 4* gives the definition of the rough neutrosophic hyper-complex cosine function. *Section 5* introduces the multi-attribute decision-making method based on rough neutrosophic hyper-complex cosine function. *Section 6* offers a numerical example of the proposed approach. Finally, *Section 7* produces the concluding remarks and some aims of future research.

2 Neutrosophic Preliminaries

Neutrosophic set is derived from neutrosophy [4].

2.1 Neutrosophic Set

Definition 2.1 [4, 5]

Let U be a universe of discourse. Then a neutrosophic set A can be presented in the form:

$$A = \{< x:T_A(x), I_A(x), F_A(x)>, x \in U\}, \quad (1)$$

where the functions T, I, F: U→ $]^-0,1^+[$ represent respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in U$ to the set Asatisfying the following the condition.

$$^-0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+ \quad (2)$$

Wang et al. [6] mentioned that the neutrosophic set assumes the values from the real standard or non-standard subsets of $]^-0, 1^+[$ based on philosophical point of view. So instead of $]^-0, 1^+[$ Wang et al. [6] consider the interval [0, 1] for technical applications, because $]^-0, 1^+[$ is difficult to apply in the real applications such as scientific and engineering problems. For two netrosophic sets (NSs),

$$A_{NS} = \{<x: T_A(x), I_A(x), F_A(x)> \mid x \in X\} \quad (3)$$

and

$$B_{NS} = \{< x, T_B(x), I_B(x), F_B(x)> \mid x \in X\}, \quad (4)$$

the two relations are defined as follows:

(1) $A_{NS} \subseteq B_{NS}$ if and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$

(2) $A_{NS} = B_{NS}$ if and only if $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$.

2.2 Single valued neutrosophic sets (SVNS)

Definition 2.2 [6]

Assume that X is a space of points (objects) with generic elements in X denoted by x. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity membership function $F_A(x)$, and for each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. When X is continuous, a SVNS A can be written as:

$$A = \int_X \frac{<T_A(x), I_A(x), F_A(x)>}{x} : x \in X . \quad (5)$$

When X is discrete, a SVNS A can be written as:

$$A = \sum_{i=1}^{n} \frac{<T_A(x_i), I_A(x_i), F_A(x_i)>}{x_i} : x_i \in X . \quad (6)$$

For two SVNSs,

$$A_{SVNS} = \{<x: T_A(x), I_A(x), F_A(x)> \mid x \in X\} \quad (7)$$

and

$$B_{SVNS} = \{<x, T_B(x), I_B(x), F_B(x)> \mid x \in X\}, \quad (8)$$

the two relations are defined as follows:

(i) $A_{SVNS} \subseteq B_{SVNS}$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$;

(ii) $A_{SVNS} = B_{SVNS}$ if and only if $T_A(x) = T_Q(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$,

for any $x \in X$.

2.3 Basic concept of Hyper-complex number of dimension *n* [12]

The hyper-complex number of dimension *n* (or *n*-complex number) was defined by S. Olariu [13] as a number of the form:

$$u = x_0 + h_1x_1 + h_2x_2 + \dots + h_{n-1}x_{n-1}, \quad (9)$$

where $n \ge 2$, and the variables $x_0, x_1, x_2, \dots, x_{n-1}$ are real numbers, while $h_1, h_2, \dots, h_{n-1}$ are the complex units, $h_0 = 1$, and they are multiplied as follows:

$$h_jh_k = h_{j+k} \text{ if } 0 \le j+k \le n-1, \text{ and } h_jh_k = h_{j+k-n} \text{ if } n \le j+k \le 2n-2. \quad (10)$$

The above complex unit multiplication formulas can be written in a simpler form as:

$$h_jh_k = h_{j+k} \ (mod\ n) \quad (11)$$

where *mod n* means *modulo n*. For example, if n = 5, then:

$$h_3h_4 = h_{3+4}(\text{mod } 5) = h_7\ (mod\ 5) = h_2. \quad (12)$$

The formula above allows us to multiply many complex units at once, as follows:

$$h_{j1}h_{j2}\dots h_{jp} = h_{j1+j2+\dots+jp}\ (mod\ n), \quad (13)$$

for $p \ge 1$.

The Neutrosophic hyper-complex number of dimension n [12] which is a number and it can be written of the form:

$$u + vI, \tag{14}$$

where u and v are n-complex numbers, and I is the indeterminacy.

3 Rough Neutrosophic Hyper-complex Set in Dimension n

Definition 3.1

Let Z be a non-null set and R be an equivalence relation on Z. Let A be a neutrosophic hyper-complex set of dimension n (or neutrosophic n-complex number), and its elements of the form u+vI, where u and v are n-complex numbers and I is the indeterminacy.

The lower and the upper approximations of A in the approximation space (Z, R) denoted by $\underline{N}(A)$ and $\overline{N}(A)$ are respectively defined as follows:

$$\underline{N}(A) = \left\langle < x, [u + vI]_{\underline{N}(A)}(x) > / z \in [x]_R, x \in Z \right\rangle \tag{15}$$

$$\overline{N}(A) = \left\langle < x, [u + vI]_{\overline{N}(A)}(x) > / z \in [x]_R, x \in Z \right\rangle, \tag{16}$$

where

$$[u + vI]_{\underline{N}(A)}(x) = \wedge_z \in [x]_R\, [u + vI]_A(z), \tag{17}$$

$$[u + vI]_{\overline{N}(A)}(x) = \vee_z \in [x]_R\, [u + vI]_A(z). \tag{18}$$

So, $[u + vI]_{\underline{N}(A)}(x)$ and $[u + vI]_{\overline{N}(A)}(x)$ are neutrosophic hyper-complex number of dimension n.

Here $\vee$ and $\wedge$ denote 'max' and 'min' operators respectively. $[u + vI]_A(z)$ and $[u + vI]_A(z)$ are the neutrosophic hyper-complex sets of dimension n of z with respect to A. $\underline{N}(A)$ and $\overline{N}(A)$ are two neutrosophic hyper-complex sets of dimension n in Z.

Thus, NS mappings $\underline{N}, \overline{N}$: $N(Z) \to N(Z)$ are respectively referred to as the lower and upper rough neutrosophic hyper-complex approximation operators, and the pair $(\underline{N}(A), \overline{N}(A))$ is called the rough neutrosophic hyper-complex set in (Z, R).

Based on the above mentioned definition, it is observed that $\underline{N}(A)$ and $\overline{N}(A)$ have constant membership on the equivalence clases of R, if $\underline{N}(A) = \overline{N}(A)$; i.e. $[u + vI]_{\underline{N}(A)}(x) = [u + vI]_{\overline{N}(A)}(x)$.

Definition 3.2

Let N(A) = ($\underline{N}(A), \overline{N}(A)$) is a rough neutrosophic hyper-complex set in (Z, R). The rough complement of N(A) is denoted by $\sim N(A) = (\underline{N}(A)^c, \overline{N}(A)^c)$, where $\underline{N}(A)^c$ and $\overline{N}(A)^c$ are the complements of neutrosophic hyper-complex set of $\underline{N}(A)$ and $\overline{N}(A)$ respectively.

$$\underline{N}(A)^c = \left\langle < x, [u + v(1 - I)]_{\underline{N}(A)}(x) > /, x \in Z \right\rangle, \tag{19}$$

and

$$\overline{N}(A)^c = \left\langle < x, [u + v(1 - I)]_{\overline{N}(A)}(x) > /, x \in Z \right\rangle \tag{20}$$

Definition 3.3

Let N(A) and N(B) be two rough neutrosophic hyper-complex sets, respectively in Z, then the following definitions holds:

$$N(A) = N(B) \Leftrightarrow \underline{N}(A) = \underline{N}(B) \wedge \overline{N}(A) = \overline{N}(B) \tag{21}$$

$$N(A) \subseteq N(B) \Leftrightarrow \underline{N}(A) \subseteq \underline{N}(B) \wedge \overline{N}(A) \subseteq \overline{N}(B) \tag{22}$$

$$N(A) \cup N(B) = < \underline{N}(A) \cup \underline{N}(B), \overline{N}(A) \cup \overline{N}(B) > \tag{23}$$

$$N(A) \cap N(B) = < \underline{N}(A) \cap \underline{N}(B), \overline{N}(A) \cap \overline{N}(B) > . \tag{24}$$

If A, B, C are the rough neutrosophic hyper-complex set in (Z, R), then the following propositions are stated from definitions:

Proposition 1

I. $\sim A(\sim A) = A$ (25)

II. $\underline{N}(A) \subseteq \overline{N}(B)$ (26)

III. $<\sim (\underline{N}(A) \cup \underline{N}(B)) = \sim (\underline{N}(A)) \cap \sim (\underline{N}(B))$ (27)

IV. $<\sim (\underline{N}(A) \cap \underline{N}(B)) = \sim (\underline{N}(A)) \cup \sim (\underline{N}(B))$ (28)

V. $<\sim (\overline{N}(A) \cup \overline{N}(B)) = \sim (\overline{N}(A)) \cap \sim (\overline{N}(B))$ (29)

VI. $<\sim (\overline{N}(A) \cap \overline{N}(B)) = \sim (\overline{N}(A)) \cup \sim (\overline{N}(B))$ (30)

Proof I

If N(A) = $[\underline{N}(A), \overline{N}(A)]$ is a rough neutrosophic hyper-complex set in (Z, R), the complement of N(A) is the rough neutrosophic hyper-complex set defined as follows:

$$\underline{N}(A)^c = \langle < x, [u + v(1 - I)]_{\underline{N}(A)}(x) > /, x \in Z \rangle, \tag{31}$$

and

$$\overline{N}(A)^c = \langle < x, [u + v(1 - I)]_{\overline{N}(A)}(x) > /, x \in Z \rangle \tag{32}$$

From this definition, we can write:

$$\sim A(\sim A) = A. \tag{33}$$

Proof II

The lower and the upper approximations of A in the approximation (Z, R) denoted by $\underline{N}(A)$ and $\overline{N}(A)$ are respectively defined as follows:

$$\underline{N}(A)^c = \langle < x, [u + v(1 - I)]_{\underline{N}(A)}(x) > /, x \in Z \rangle, \tag{34}$$

and

$$\overline{N}(A)^c = \langle < x, [u + v(1 - I)]_{\overline{N}(A)}(x) > /, x \in Z \rangle, \tag{35}$$

where

$$[u + vI]_{\underline{N}(A)}(x) = \wedge_z \in [x]_R \, [u + vI]_A(z), \tag{36}$$

$$[u + vI]_{\overline{N}(A)}(x) = \vee_z \in [x]_R \, [u + vI]_A(z). \tag{37}$$

So,

$$\underline{N}(A) \subseteq \overline{N}(A). \tag{38}$$

Proof III

Consider:

$$x \in \sim (\underline{N}(A) \cup \underline{N}(B))$$

$$\Rightarrow x \in \sim \underline{N}(A) \text{ and } x \in \sim \underline{N}(B)$$

$$\Rightarrow x \in \sim (\underline{N}(A)) \cap \sim (\underline{N}(B))$$

$$\Rightarrow x \in \sim (\underline{N}(A)) \cap \sim (\underline{N}(B))$$

$$\Rightarrow \sim (\underline{N}(A) \cup \underline{N}(B)) \subseteq \sim ((\underline{N}(A)) \cap \sim (\underline{N}(B))). \tag{39}$$

Again, consider:

$$y \in \sim ((\underline{N}(A)) \cap \sim (\underline{N}(B)))$$

$$\Rightarrow y \in \sim \underline{N}(A) \text{ or } y \in \sim \underline{N}(B)$$

$$\Rightarrow y \in \Rightarrow \sim (\underline{N}(A) \cup \underline{N}(B))$$

$$\Rightarrow \sim (\underline{N}(A) \cup \underline{N}(B)) \supseteq \sim ((\underline{N}(A)) \cap \sim (\underline{N}(B))). \tag{40}$$

Hence,

$$\sim\left(\underline{N}(A) \cup \underline{N}(B)\right) = \sim\left(\left(\underline{N}(A)\right) \cap \sim\left(\underline{N}(B)\right)\right). \tag{41}$$

Proof IV

Consider:

$$x \in \sim\left(\underline{N}(A) \cap \underline{N}(B)\right)$$

$$\Rightarrow x \in \sim \underline{N}(A) \text{ or } x \in \sim \underline{N}(B)$$

$$\Rightarrow x \in \sim\left(\underline{N}(A)\right) \cup \sim\left(\underline{N}(B)\right)$$

$$\Rightarrow x \in \sim\left(\underline{N}(A)\right) \cup \sim\left(\underline{N}(B)\right)$$

$$\Rightarrow \sim\left(\underline{N}(A) \cap \underline{N}(B)\right) \subseteq \sim\left(\left(\underline{N}(A)\right) \cup \sim\left(\underline{N}(B)\right)\right) \tag{42}$$

Again, consider:

$$y \in \sim\left(\left(\underline{N}(A)\right) \cup \sim\left(\underline{N}(B)\right)\right)$$

$$\Rightarrow y \in \sim \underline{N}(A) \text{ and } y \in \sim \underline{N}(B)$$

$$\Rightarrow y \in \sim\left(\underline{N}(A) \cap \underline{N}(B)\right)$$

$$\Rightarrow \sim\left(\underline{N}(A) \cap \underline{N}(B)\right) \supseteq \sim\left(\left(\underline{N}(A)\right) \cup \sim\left(\underline{N}(B)\right)\right). \tag{43}$$

Hence,

$$\sim\left(\underline{N}(A) \cap \underline{N}(B)\right) = \sim\left(\left(\underline{N}(A)\right) \cup \sim\left(\underline{N}(B)\right)\right). \tag{44}$$

Proof V

Consider:

$$x \in \sim\left(\overline{N}(A) \cup \overline{N}(B)\right)$$

$$\Rightarrow x \in \sim \overline{N}(A) \text{ and } x \in \sim \overline{N}(B)$$

$$\Rightarrow x \in \sim\left(\overline{N}(A)\right) \cap \sim\left(\overline{N}(B)\right)$$

$$\Rightarrow x \in \sim\left(\overline{N}(A)\right) \cap \sim\left(\overline{N}(B)\right)$$

$$\Rightarrow \sim\left(\overline{N}(A) \cup \overline{N}(B)\right) \subseteq \sim\left(\left(\overline{N}(A)\right) \cap \sim\left(\overline{N}(B)\right)\right). \tag{45}$$

Again, consider:

$$y \in \sim\left(\left(\overline{N}(A)\right) \cap \sim\left(\overline{N}(B)\right)\right)$$

$$\Rightarrow y \in \sim \overline{N}(A) \text{ or } y \in \sim \overline{N}(B)$$

$$\Rightarrow y \in \sim\left(\overline{N}(A) \cup \overline{N}(B)\right)$$

$$\Rightarrow \sim\left(\overline{N}(A) \cup \overline{N}(B)\right) \supseteq \sim\left(\left(\overline{N}(A)\right) \cap \sim\left(\overline{N}(B)\right)\right). \tag{46}$$

Hence,

$$\sim\left(\overline{N}(A)\cup\overline{N}(B)\right)=\sim\left(\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)\right). \quad (47)$$

Proof VI

Consider:

$$x \in \sim\left(\overline{N}(A)\cap\overline{N}(B)\right)$$

$$\Rightarrow x \in \sim\overline{N}(A) \text{ or } x \in \sim\overline{N}(B)$$

$$\Rightarrow x \in \sim\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)$$

$$\Rightarrow x \in \sim\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)$$

$$\Rightarrow \sim\left(\overline{N}(A)\cap\overline{N}(B)\right)\subseteq\sim\left(\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)\right). \quad (48)$$

Again, consider:

$$y \in \sim\left(\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)\right)$$

$$\Rightarrow y \in \sim\overline{N}(A) \text{ and } y \in \sim\overline{N}(B)$$

$$\Rightarrow y \in \sim\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)$$

$$\Rightarrow \sim\left(\overline{N}(A)\cap\overline{N}(B)\right)\supseteq\sim\left(\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)\right). \quad (49)$$

Hence,

$$\sim\left(\overline{N}(A)\cap\overline{N}(B)\right)=\sim\left(\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)\right). \quad (50)$$

Proposition 2

I. $\sim[N(A) \cup N(B)] = (\sim N(A)) \cap (\sim N(B))$; (51)

II. $\sim[N(A) \cap N(B)] = (\sim N(A)) \cup (\sim N(B))$. (52)

Proof I

$$\sim[N(A) \cup N(B)]$$

$$= \sim < \underline{N}(A)\cup\underline{N}(B), \overline{N}(A)\cup\overline{N}(B) >$$

$$= <\sim\left(\underline{N}(P)\cap\underline{N}(Q)\right), \sim\left(\overline{N}(P)\cap\overline{N}(Q)\right)>$$

$$= (\sim N(A)) \cap (\sim N(B)) \quad (53)$$

Proof II

$$\sim[N(A) \cap N(B)]$$

$$= \sim < \underline{N}(A)\cap\underline{N}(B), \overline{N}(A)\cap\overline{N}(B) >$$

$$= <\sim (\underline{N}(A) \cup \underline{N}(B)), \sim (\overline{N}(A) \cup \overline{N}(B))>$$

$$= (\sim N(A)) \cup (\sim N(B)) \tag{54}$$

4 Rough neutrosophic hyper-complex cosine function (RNHCF)

The cosine similarity measure is calculated as the inner product of two vectors divided by the product of their lengths. It is the cosine of the angle between the vector representations of two rough neutrosophic sets. The cosine similarity measure is a fundamental measure used in information technology. Now, a new cosine function between rough neutrosophic hyper-complex sets has been proposed as follows.

Definition 4.1

Assume that there are two rough neutrosophic hyper-complex sets

$$A = \left\langle [u+vI]_{\underline{N}(A)}(x), [u+vI]_{\overline{N}(A)}(x) \right\rangle, \tag{55}$$

and

$$B = \left\langle [u+vI]_{\underline{N}(B)}(x), [u+vI]_{\overline{N}(B)}(x) \right\rangle \tag{56}$$

in X = {$x_1, x_2, \ldots, x_n$).
Then rough neutrosophic hyper-complex cosine function between two sets A and B is proposed as follows:

$$C_{RNHCF}(A,B) =$$

$$\frac{1}{n}\sum_{i=1}^{n} \frac{\Delta u_A(x_i).\Delta u_B(x_i) + \Delta v_A(x_i).\Delta v_B(x_i) + \Delta I_A(x_i).\Delta I_B(x_i)}{\sqrt{(\Delta u_A(x_i))^2 + (\Delta v_A(x_i))^2 + (\Delta I_A(x_i))^2}\sqrt{(\Delta u_B(x_i))^2 + (\Delta v_B(x_i))^2 + (\Delta I_B(x_i))^2}} \tag{57}$$

where

$$\Delta u_A(x_i) = 0.5.\left| u_{\underline{N}(A)(x_i)} + u_{\overline{N}(A)(x_i)} \right|, \tag{58}$$

$$\Delta u_B(x_i) = 0.5.\left| u_{\underline{N}(B)(x_i)} + u_{\overline{N}(B)(x_i)} \right|, \tag{59}$$

$$\Delta v_A(x_i) = 0.5.\left| v_{\underline{N}(A)(x_i)} + v_{\overline{N}(A)(x_i)} \right|, \tag{60}$$

$$\Delta v_B(x_i) = 0.5.\left| v_{\underline{N}(B)(x_i)} + v_{\overline{N}(B)(x_i)} \right|, \tag{61}$$

$$\Delta I_A(x_i) = 0.5.\left| I_{\underline{N}(A)(x_i)} + I_{\overline{N}(A)(x_i)} \right|, \tag{62}$$

$$\Delta I_B(x_i) = 0.5.\left| I_{\underline{N}(B)(x_i)} + I_{\overline{N}(B)(x_i)} \right|. \tag{63}$$

Proposition 3

Let A and B be rough neutrosophic sets; then:

I. $0 \le C_{RNHCF}(A, B) \le 1$ (64)

II. $C_{RNHCF}(A, B) = C_{RNHCF}(B, A)$ (65)

III. $C_{RNHCF}(A, B) = 1$, if and only if A = B (66)

IV. If C is a RNHCF in Y and $A \subset B \subset C$ then, $C_{RNHCF}(A, C) \le C_{RNHCF}(A, B)$, and $C_{RNHCF}(A, C) \le C_{RNHCF}(B,C)$. (67)

Proofs

I. It is obvious because all positive values of cosine function are within 0 and 1

II. It is obvious that the proposition is true.

III. When A = B, then obviously $C_{RNHCF}(A, B) = 1$. On the other hand if $C_{RNHCF}(A, B) = 1$ then, $\Delta T_A(x_i) = \Delta T_B(x_i)$, $\Delta I_A(x_i) = \Delta I_B(x_i)$, $\Delta F_A(x_i) = \Delta F_B(x_i)$.

This implies that A = B.

IV. If $A \subset B \subset C$ then we can write:

$$u_{\underline{N}(A)}(x_i) \le u_{\underline{N}(B)}(x_i) \le u_{\underline{N}(C)}(x_i), \tag{68}$$

$$u_{\overline{N}(A)}(x_i) \le u_{\overline{N}(B)}(x_i) \le u_{\overline{N}(C)}(x_i), \tag{69}$$

$$v_{\underline{N}(A)}(x_i) \le v_{\underline{N}(B)}(x_i) \le v_{\underline{N}(C)}(x_i), \tag{70}$$

$$v_{\overline{N}(A)}(x_i) \le v_{\overline{N}(B)}(x_i) \le v_{\overline{N}(C)}(x_i), \tag{71}$$

$$I_{\underline{N}(A)}(x_i) \ge I_{\underline{N}(B)}(x_i) \ge I_{\underline{N}(C)}(x_i), \tag{72}$$

$$I_{\overline{N}(A)}(x_i) \ge I_{\overline{N}(B)}(x_i) \ge I_{\overline{N}(C)}(x_i) \tag{73}$$

The cosine function is decreasing function within the interval $\left[0, \frac{\pi}{2}\right]$. Hence we can write $C_{RNHCF}(A, C) \le C_{RNHCF}(A, B)$, and $C_{RNHCF}(A, C) \le C_{RNHCF}(B, C)$.

If we consider the weights of each element x_i, a weighted rough neutrosophic hyper-complex cosine function (WRNHCF) between two sets A and B can be defined as follows:

$$C_{WRNHCF}(A, B) =$$

$$\sum_{i=1}^{n} W_i \frac{\Delta u_A(x_i).\Delta u_B(x_i) + \Delta v_A(x_i).\Delta v_B(x_i) + \Delta I_A(x_i).\Delta I_B(x_i)}{\sqrt{(\Delta u_A(x_i))^2 + (\Delta v_A(x_i))^2 + (\Delta I_A(x_i))^2}\sqrt{(\Delta u_B(x_i))^2 + (\Delta v_B(x_i))^2 + (\Delta I_B(x_i))^2}} \tag{74}$$

where

$$\Delta u_A(x_i) = 0.5.\left|u_{\underline{N}(A)(x_i)} + u_{\overline{N}(A)(x_i)}\right|, \tag{75}$$

$$\Delta u_B(x_i) = 0.5.\left|u_{\underline{N}(B)(x_i)} + u_{\overline{N}(B)(x_i)}\right|, \tag{76}$$

$$\Delta v_A(x_i) = 0.5.\left|v_{\underline{N}(A)(x_i)} + v_{\overline{N}(A)(x_i)}\right|, \tag{77}$$

$$\Delta v_B(x_i) = 0.5.\left|v_{\underline{N}(B)(x_i)} + v_{\overline{N}(B)(x_i)}\right|, \tag{78}$$

$$\Delta I_A(x_i) = 0.5.\left|I_{\underline{N}(A)(x_i)} + I_{\overline{N}(A)(x_i)}\right|, \tag{79}$$

$$\Delta I_B(x_i) = 0.5.\left|I_{\underline{N}(B)(x_i)} + I_{\overline{N}(B)(x_i)}\right|. \tag{80}$$

$w_i \in [0,1]$, i = 1, 2,…, n and $\sum_{i=1}^{n} w_i = 1$. If we take $w_i = \frac{1}{n}$, i = 1, 2,…, n, then:

$$C_{WRNHCF}(A, B) = C_{RNHCF}(A, B) \tag{81}$$

The weighted rough neutrosophic hyper-complex cosine function (WRNHCF) between two rough sets A and B also satisfies the following properties:

I. $0 \le C_{WRNHCF}(A, B) \le 1$ (82)

II. $C_{WRNHCF}(A, B) = C_{WRNHCF}(B, A)$ (83)

III. $C_{WRNHCF}(A, B) = 1$, if and only if A = B (84)

IV. If C is a WRNHCF in Y and $A \subset B \subset C$ then, $C_{WRNHCF}(A, C) \le C_{WRNHCF}(A, B)$, and $C_{WRNHCF}(A, C) \le C_{WRNHCF}(B, C)$ (85)

5 Decision making procedure based on rough hyper-complex neutrosophic function

In this section, we apply rough neutrosophic hyper-complex cosine function between RNHSs to the multi-attribute decision making problem. Let $A_1, A_2, \ldots, A_m$ be a set of alternatives and $C_1, C_2, \ldots, C_n$ be a set of attributes. The proposed decision making method is described using the following steps.

Step1: Construction of the decision matrix with rough neutrosophic hyper-complex number

The decision maker considers a decision matrix with respect to *m* alternatives and *n* attributes in terms of rough neutrosophic hyper-complex numbers, as follows:

$$DM = \left\langle \underline{dm}_{ij}, \overline{dm}_{ij} \right\rangle_{m\times n} =$$

$$\begin{array}{c|cccc}
 & C_1 & C_2 & \cdots & C_n \\
\hline
A_1 & \langle \underline{dm}_{11}, \overline{dm}_{11} \rangle & \langle \underline{dm}_{12}, \overline{dm}_{12} \rangle & \cdots & \langle \underline{dm}_{1n}, \overline{dm}_{1n} \rangle \\
A_2 & \langle \underline{dm}_{21}, \overline{dm}_{21} \rangle & \langle \underline{dm}_{22}, \overline{dm}_{22} \rangle & \cdots & \langle \underline{dm}_{2n}, \overline{dm}_{2n} \rangle \\
. & \cdots & \cdots & \cdots & \cdots \\
. & \cdots & \cdots & \cdots & \cdots \\
A_m & \langle \underline{dm}_{m1}, \overline{dm}_{m1} \rangle & \langle \underline{dm}_{m2}, \overline{dm}_{m2} \rangle & \cdots & \langle \underline{dm}_{mn}, \overline{dm}_{mn} \rangle
\end{array} \tag{86}$$

Table 1. Rough neutrosophic hyper-complex decision matrix.

Here $\left\langle \underline{dm}_{ij}, \overline{dm}_{ij} \right\rangle$ is the rough neutrosophic hyper-complex number according to the *i*-th alternative and the *j*-th attribute.

Step2: Determination of the weights of attribute

Assume that the weight of the attributes C (j = 1, 2, … , n) considered by the decision-maker be w_j (j = 1, 2, … , n) such that $\forall w_j \in [0, 1]$ (j = 1, 2, …, n) and $\sum_{j=1}^{n} w_j = 1$.

Step 3: Determination of the benefit type attribute and cost type attribute

Generally, the evaluation attribute can be categorized into two types: benefit attribute and cost attribute. Let K be a set of benefit attribute and M be a set of cost attribute. In the proposed decision-making method, an ideal alternative can be identified by using a maximum operator for the benefit attribute and a minimum operator for the cost attribute to determine the best value of each criterion among all alternatives. Therefore, we define an ideal alternative as follows:

$$A^* = \{C_1^*, C_2^*, \dots, C_m^*\}, \tag{87}$$

Benefit attribute:

$$C_j^* = \left[\max_i u_{C_j}{}^{(A_i)}, \max_i v_{C_j}{}^{(A_i)}, \min_i I_{C_j}{}^{(A_i)} \right]. \tag{88}$$

Cost attribute:

$$C_j^* = \left[\min_i T_{C_j}{}^{(A_i)}, \min_i I_{C_j}{}^{(A_i)}, \max_i F_{C_j}{}^{(A_i)} \right] \tag{89}$$

where

$$u_{C_j}{}^{(A_i)} = 0.5.\left| \left(u_{C_j}\right)_{\underline{N}(A_i)} + \left(u_{C_j}\right)_{\overline{N}(A_i)} \right|, \tag{90}$$

$$v_{C_j}{}^{(A_i)} = 0.5.\left| \left(v_{C_j}\right)_{\underline{N}(A_i)} + \left(v_{C_j}\right)_{\overline{N}(A_i)} \right|, \tag{91}$$

and

$$I_{C_j}{}^{(A_i)} = 0.5.\left| \left(I_{C_j}\right)_{\underline{N}(A_i)} + \left(I_{C_j}\right)_{\overline{N}(A_i)} \right|. \tag{92}$$

Step4: Determination of the over all weighted rough hyper-complex neutrosophic cosine function (WRNHCF)of the alternatives

Weighted rough neutrosophic hyper-complex cosine function is given as follows:

$$C_{WRNHCF}(A, B) = \sum_{j=1}^{n} w_j C_{WRNHCF}(A, B) \tag{93}$$

Step5: Ranking the alternatives

Using the weighted rough hyper-complex neutrosophic cosine function between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be easily selected with the highest similarity value.

Step 6: End

6 Numerical Example

Assume that a decision maker (an adult man/woman who is eligible to marry) intends to select the most suitable life partner for marriage from the three initially chosen candidates (S_1, S_2, S_3) by considering five attributes, namely: physical and mental health C_1, education and job C_2, management power C_3, family background C_4, risk factor C_5.

Based on the proposed approach discussed in section 5, the considered problem has been solved using the following steps:

Step1: Construction of the decision matrix with rough neutrosophic hyper-complex numbers

The decision maker considers a decision matrix with respect to three alternatives and five attributes in terms of rough neutrosophic hyper-complex numbers shown in the Table 2.

$$DM = \left\langle \underline{dm}_{ij}, \overline{dm}_{ij} \right\rangle_{3\times 5} =$$

$$\begin{array}{c|ccccc}
 & C_1 & C_2 & C_3 & C_4 & C_5 \\
\hline
A_1 & \left\langle \begin{matrix} (i+0.6(1+i)), \\ (2i+0.4(2+i)) \end{matrix} \right\rangle & \left\langle \begin{matrix} ((1+i)+0.65(2i)), \\ ((1+2i)+0.55(3i)) \end{matrix} \right\rangle & \left\langle \begin{matrix} ((1+i)+0.4(2+i)), \\ ((1+2i)+0.2(2+3i)) \end{matrix} \right\rangle & \left\langle \begin{matrix} (4i+0.55(1+i)), \\ ((4+i)+0.45(2+i)) \end{matrix} \right\rangle & \left\langle \begin{matrix} (3i+0.78(2+3i)), \\ ((1+3i)+0.72(3+3i)) \end{matrix} \right\rangle \\
A_2 & \left\langle \begin{matrix} (i+0.6(1+2i)), \\ (3i+0.5(1+3i)) \end{matrix} \right\rangle & \left\langle \begin{matrix} ((1+i)+0.55(i)), \\ ((1+2i)+0.45(3i)) \end{matrix} \right\rangle & \left\langle \begin{matrix} (2i+0.3(2+i)), \\ ((2+i)+0.2(1+3i)) \end{matrix} \right\rangle & \left\langle \begin{matrix} (i+0.52(2+3i)), \\ (2i+0.48(4+3i)) \end{matrix} \right\rangle & \left\langle \begin{matrix} ((1+i)+0.82(2+i)), \\ (2i+0.78(4+3i)) \end{matrix} \right\rangle \\
A_3 & \left\langle \begin{matrix} (2i+0.5(1+i)), \\ (3i+0.4(1+3i)) \end{matrix} \right\rangle & \left\langle \begin{matrix} ((2+i)+0.69(5i)), \\ ((2+i)+0.51(6i)) \end{matrix} \right\rangle & \left\langle \begin{matrix} (i+0.6(1+i)), \\ (2i+0.4(3+2i)) \end{matrix} \right\rangle & \left\langle \begin{matrix} ((1+i)+0.48(3+4i)), \\ ((1+2i)+0.42(5+3i)) \end{matrix} \right\rangle & \left\langle \begin{matrix} ((1+i)+0.9(i)), \\ ((1+2i)+0.7(2+3i)) \end{matrix} \right\rangle
\end{array} \quad (94)$$

where, $i=\sqrt{-1}$.

Table 2. Decision matrix with rough neutrosophic hyper-complex number.

Step 2: Determination of the weights of the attributes

The weight vectors considered by the decision maker are 0.25, 0.20, 0.25, 0.10, and 0.20 respectively.

Step 3: Determination of the benefit attribute and cost attribute

Here four benefit types attributes C_1, C_2, C_3, C_4 and one cost type attribute C_5. Using equations (12) and (13) we calculate A^* as follows:

$$A^* = [(5.00, 2.69, 0.45), (4.47, 5.50, 0.50), (3.60, 2.83, 0.25), (6.40, 5.30, 0.45), (3.16, 2.24, 0.80)]$$

Step 4: Determination of the over all weighted rough hyper-complex neutrosophic similarity function (WRHNSF)of the alternatives

We calculate weighted rough neutrosophic hyper-complex similarity values as follows:

$$S_{WRHCF}(A_1, A^*) = 0.9622;$$

$$S_{WRHCF}(A_2, A^*) = 0.9404;$$

$$S_{WRHCF}(A_3, A^*) = 0.9942.$$

Step 5: Ranking the alternatives

Ranking the alternatives is prepared based on the descending order of similarity measures. Highest value reflects the best alternative.
Here,

$$S_{WRHCF}(A_3, A^*) \prec S_{WRHCF}(A_1, A^*) \prec S_{WRHCF}(A_2, A^*). \quad (95)$$

Hence, the decision maker must choose the candidate A_3 as the best alternative for marriage.

Step 6: End

7 Conclusion

In this paper, we have proposed the rough neutrosophic hyper-complex set and the rough neutrosophic hyper-complex cosine function, and proved some of their basic properties.

We have also proposed the rough neutrosophic hyper-complex similarity measure based multi-attribute decision making.

We have presented an application, namely selection of best candidate for marriage for Indian context.

The concept presented in this paper can be applied for other multiple attribute decision making problems in rough neutrosophic hyper-complex environments.

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