# Evidence Supporting Measure of Similarity for Reducing the Complexity in Information Fusion

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**Abstract**

This paper proposes a new solution for reducing the number of sources of evidence to be combined in order to diminish the complexity of the fusion process required in some applications where the real-time constraint and strong computing resource limitation are of prime importance. The basic idea consists in selecting, among the whole set of sources of evidence, only the biggest subset of sources which are not too contradicting based on a criterion of Evidence Supporting Measure of Similarity (ESMS) in order to process solely the coherent information received. The ESMS criterion serves actually as a generic tool for outlier source identification and rejection. Since the ESMS between several belief functions can be defined using several distance measures, we browse the most common ones in this paper and we describe in detail the principle of our Generalized Fusion Machine (GFM). The last part of the paper shows the improvement of the performances of this new approach with respect to the classical one in a real-data based and real-time experiment for robot perception using sonar sensors.

*Key words:*
Information fusion; Belief function; Complexity reduction; Robot perception; DSmT; Measure of similarity; Distance; Lattice.

## 1. Introduction

Information fusion (IF) has gained more and more interest in the scientific community since the end of nineties because of the development of sophisticated multisensor and hybrid (involving human feedbacks in the loop) systems in many fields of applications (robotics, defense, security, medicine, etc.). IF appears through many scientific international conferences and workshops [11]. The main theories useful for information fusion are the Probability theory [16, 25] (and more recently the Imprecise Probability Theory [38]), the Possibility Theory [8] (based on Fuzzy Sets theory [42]), Neutrosophic Set Theory [15] and belief function theories, mainly Dempster-Shafer theory (DST) [29] and more recently Dezert-Smarandache theory (DSmT) [31, 32, 33].

In this work, we concentrate our attention on belief functions theories and specially on DSmT because of its ability to deal efficiently with uncertain, imprecise and conflicting quantitative and qualitative information. Basically, in DST, a basic belief assignment (bba) $m(.)$ is a mapping from the power set $2^{\Theta}$ (see section 2.2 for details) of the frame of discernment $\Theta$ into $[0, 1]$ such that

$$m(\emptyset) = 0 \quad \text{and} \quad \sum_{X \in 2^{\Theta}} m(X) = 1.$$

In DST, $\Theta$ represents the set of exclusive and exhaustive possibilities for the solution of the problem under consideration. In DSmT, $\Theta$ can be a set of possible non exclusive elements and the definition of bba is extended to the lattice structures of hyper-power set $D^{\Theta}$, and to super-power set $S^{\Theta}$ in UFT (Unification of Fusion Theories) [30, 32], Chap. 8 - see also section 2.3 for a brief presentation and [7, 10] and [33] for definitions, details and examples. In general $m(.)$ is not a measure of probability, except in the case when its focal elements (i.e. the elements which have a strictly positive mass of belief) are singletons; in such case, $m(.)$ is called a Bayesian bba [29] which can be considered as a subjective probability measure. In belief function theories, the main information fusion problem consists in finding an efficient way for combining several sources of evidence $s_1$, $s_2$, ..., $s_n$ characterized by their bba's $m_1(.)$, $m_2(.)$, ..., $m_n(.)$ assumed for simplicity here defined on the same fusion space, either $2^{\Theta}$, $D^{\Theta}$, or $S^{\Theta}$ depending on the underlying model associated with the nature of the frame $\Theta$. The difficulty in information fusion arises from the fact that the sources can be conflicting (i.e.

one source commits some belief in a proposition $A$ whereas another source commits some belief in a proposition $B$ but $A$ and $B$ are known to be truly exclusive ($A \cap B = \emptyset$)) and one needs a solution for dealing with conflicting information in the fusion process. In DST, Shafer proposes Dempster's rule of combination as the fusion operator for combining sources of evidence whereas in DSmT the recommended fusion operator is the PCR5 (Proportional Conflict redistribution rule # 5) rule of combination, see [29] and [33] for discussions and comparisons of these rules. PCR5 is more complex than Demspter's rule but it offers a better ability to deal with conflicting information.

Both rules however become intractable in some applications having only low computational capacities (as in some autonomous onboard systems by example) because their complexity increases drastically with the number $n$ of sources to combine and/or with the size of the frame $\Theta$, specially in the worst case (i.e. when a strict positive mass of belief is committed to all elements of the fusion space). To circumvent this problem, one has to play on both sides: 1) reducing the number of sources to combine and 2) reducing the size of the frame $\Theta$. In this paper, we propose a solution only for reducing the number of sources to combine because we are not concerned in our application of robot perception by the second aspect since in this application our frame $\Theta$ has only two elements representing the emptiness or occupancy states of the grid cells of the perceived map of the environment. To expect good performances of such limited-resource fusion scheme, it seems natural to search and combine altogether only the sources which are coherent (which are not too conflicting) according to a given measure of similarity.

Such idea has been already investigated by several authors who have proposed some distance measures between two evidential sources in different fields of applications. For example, Tessem [35] in 1993 proposed the distance $d_{ij} = max_{\theta_l \in \Theta}|BetP_i(\theta_l) - BetP_j(\theta_l)|$) according to the pignistic probability transform $BetP(.)$. In 1997, Bauer [1] introduced two other measures of error to take a decision based on pignistic probability distribution after approximation. In 1998, Zouhal and Denoeux [43] also introduced a distance based on mean square error between pignistic probability. In 1999, Petit-Renaud [26] has defined a measure directly on the power set of $\Theta$ and proposed an error criterion between two belief structures based on the generalized Hausdorff distance. In 2001, Jousselme et al. [14] proposed in DST framework a new distance measure $d_{ij} = 1 - \frac{1}{\sqrt{2}}\sqrt{m_1^2 + m_2^2 - 2\langle m_1, m_2\rangle}$ between two basic belief assignments (bba's) for measuring their similarity (closeness). In 2006, Ristic and Smets [27, 28] have defined in the TBM (Transferable Belief Model) framework a TBM-distance between bba's to solve the association of uncertain combat ID declarations. These authors recall also the Bhattacharya distance $d_{ij} = \sqrt{1 - \sum_{A\in\mathcal{F}_\rangle} \sum_{B\in\mathcal{F}_|} \sqrt{m_i(A)m_j(B)}}$ between two bba's. In 2006 also, Diaz et al. [6] proposed a new measure of similarity between bba's based on Tversky's similarity measure [37]. Note that in belief function theories, the direct use of classical measures used in Probability theory (say like Kullback Leibler (KL) distance [3]) cannot be applied directly because bba's are not probability measures in general.

In this paper, we develop an Evidence Support Measure of Similarity (ESMS) in a generalized fusion space according to different lattices [7, 10] for reducing the number of sources of evidence to combine and thus reducing the complexity of the computational burden. As shown in the next sections, we propose several possible measures of distance for ESMS and we compare their performances in our specific application of mobile robot perception. The purpose of this paper is not to select, nor to justify, the best measure of distance for ESMS but only to show the practical advantage of using the ESMS criteria as a generic tool for reducing the complexity of the fusion with keeping good performances for our application.

This paper is organized as follows. In section 2, we briefly recall the main paradigms for dealing with uncertain information. In section 3, we give a general mathematical definition of ESMS between two basic belief assignments and we establish some basic properties of ESMS. In section 4, we extend and present different possible ESMS functions (distance measures) fitting with the different mathematical paradigms listed in section 2. A comparison of the performances of five possible distances is made through a simple example in section 5. The simulation presented in section 6 shows in details how ESMS filter is used within GFM scheme. An application of ESMS filter in GFM for mobile robot perception with real-data (sonar sensors measurements) and in real-time is presented in section 7 to show the advantages of the approach proposed here. The conclusion is given in section 8.

## 2. The main paradigms for dealing with uncertainties

*2.1. Probability Theory and Bayes' rule.*

The (axiomatic) Probability Theory [16] is the most achieved theory for dealing with randomness. We will not present this theory in details since there exist dozens of very good classical books devoted to it, see for example [25]. We just recall that a random experiment is an experiment (action) whose result is uncertain before it is performed and a trial is a single performance of the random experiment. An outcome is the result of a trial and the sample space $\Theta$ is the set of all possible outcome of the random experiment. An event is the subset of the sample space $\Theta$ to which a probability measure can be assigned. Two events $A_i$ and $A_j$ are said exclusive (disjoint) if $A_i \cap A_j = \emptyset$, $\forall i \neq j$, where the empty set $\emptyset$ represents the impossible event. The sure event is the sample space $\Theta$. The probability theory is based on Set Theory and the measure theory on sets. The following axioms have been identified as necessary and sufficient for probability $P(.)$ as a measure: Axiom 1) (nonnegativity) $0 \leq P(A) \leq 1$, Axiom 2) (unity) $P(\Theta) = 1$, and Axiom 3) (finite additivity[1]), if $A_1$, $A_2$, $\ldots$, $A_n$ are disjoint events, then $P(A_1 \cup A_2 \cup \ldots \cup A_n) = \sum_{i=1}^{n} P(A_i)$. Events which are subsets of the sample space are put in one-to-one correspondence with propositions in belief fuction theory [29], pages 35-37 and that's why we use indifferently the terminology set, event or proposition in this paper. The probabilistic inference is (usually) carried out by Bayes' rule according to:

$$\forall B, P(B) > 0, \quad P(A_i|B) = \frac{P(A_i \cap B)}{P(B)} = \frac{P(B|A_i)P(A_i)}{\sum_{j=1}^{n} P(B|A_j)P(A_j)} \tag{1}$$

where the sample space $\Theta$ has been partitioned into exhaustive and exclusive events $A_1, A_2, \ldots, A_n$, i.e. such that $A_i \cap A_j = \emptyset, (i \neq j)$ and $A_1 \cup A_2 \cup \ldots \cup A_n = \Theta$; $P(.)$ is an a priori probability measure defined on $\Theta$ satisfying Kolmogorov's axioms. In Bayes formula, it is assumed that the denominator is strictly positive. A generalization of this rule has been proposed by Jeffrey [12, 13] for working in circumstances where the *parochialist* assumption is not a reasonable assumption, i.e. when $P(B|B) = 1$ is a fallacy, see [13, 22] for details and examples.

[1] Another axiom related to the countable additivity can be also considered as the fourth axiom of the probability theory.

Using the classical terminology adopted in belief function theories (DST and/or DSmT) and considering for example $\Theta = \{A, B\}$, a discrete probability measure $P(\cdot)$ can be interpreted as a specific Bayesian belief mass $m(.)$ such that

$$m(A) + m(B) = 1 \tag{2}$$

*2.2. Dempster-Shafer Theory (DST)*

In DST [29], the frame of discernment $\Theta$ of the fusion problem under consideration consists in a discrete finite set of $n$ exhaustive and exclusive elementary hypotheses $\theta_i$, i.e. $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$. This is called *Shafer's model* of the problem. Such model assumes that an ultimate refinement of the problem is possible, exists and is achievable, so that elements $\theta_i$, $i = 1, 2, \ldots, n$ are well precisely defined and identified in such a way that we are sure that they are truly exclusive and exhaustive (closed-world assumption). The set of all subsets of $\Theta$ is called the power set of $\Theta$ and is denoted $2^\Theta$. Its cardinality is $2^{|\Theta|}$. Since $2^\Theta$ is closed under $\cup$ and all $\theta_i$, $i = 1, 2, \ldots, n$ are exclusive, it defines a Boolean algebra. All composite propositions built from elements of $\Theta$ with $\cup$ operator such that:

1) $\emptyset, \theta_1, \ldots, \theta_n \in 2^\Theta$;
2) If $A, B \in 2^\Theta$, then $A \cup B \in 2^\Theta$;
3) No other elements belong to $2^\Theta$, except those obtained by using rules 1) or 2).

Shafer defines a basic belief assignment (bba), also called mass function, as a mapping $m(.) : 2^\Theta \to [0, 1]$ satisfying $m(\emptyset) = 0$ and the normalization condition. Typically, when $\Theta = \{A, B\}$ and Shafer's model holds, in DST one works with $m(.)$ such that

$$m(A) + m(B) + m(A \cup B) = 1 \tag{3}$$

$m(A \cup B)$ allows us to commit some belief on the disjunction $A \cup B$ which represents the ignorance in choosing between $A$ and $B$. From this very simple example, one sees clearly the ability of DST to offer a better modeling for a total ignorant/vacuous source of information by setting $m(A \cup B) = 1$, whereas in Probability Theory one would be forced to adopt the principle of insufficient reason (as known also as the principle of indifference) to justify taking $m(A) = m(B) = 1/2$ as default belief mass for representing a total ignorant body of evidence.

In DST framework, the combination of two belief assignments $m_1(.)$ and $m_2(.)$ is done using Dempster's rule of combination which can be seen as the normalized version of the conjunctive rule in order to remove the total conflicting mass and to get a proper normalized belief mass after the combination [29]. Dempster's rule is mathematically defined by $m(\emptyset) = 0$ and for $X \neq \emptyset$ by

$$m(X) = \frac{\sum\limits_{\substack{X_1, X_2 \in 2^\Theta \\ X_1 \cap X_2 = X}} m_1(X_1) m_2(X_2)}{1 - \sum\limits_{\substack{X_1, X_2 \in 2^\Theta \\ X_1 \cap X_2 = \emptyset}} m_1(X_1) m_2(X_2)} \tag{4}$$

Dempster's formula is defined if and only if the two sources of evidence are not fully conflicting; that is when $\sum_{\substack{X_1, X_2 \in 2^\Theta \\ X_1 \cap X_2 = \emptyset}} m_1(X_1) m_2(X_2) \neq 1$.

*2.3. Dezert-Smarandache Theory (DSmT)*

In DSmT framework [31, 32, 33], the frame $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$ is a finite set of $n$ exhaustive elements which are not necessary exclusive. The principle of the third excluded middle and Shafer's model are refuted in DSmT (but can be introduced if needed depending on the model of the frame one wants to deal with), since for a wide class of fusion problems, the nature of hypotheses can be only vague and imprecise or crude approximation of the reality and none ultimate refinement is achievable. As a simple example, if we consider two suspects Peter ($P$) and Mary ($M$) in some criminal investigations, it may be possible that Peter has committed the crime alone, as well as Mary, or maybe Peter and Mary have committed the crime together. In that case, one has to consider the possibility for $P \cap M \neq \emptyset$ but there is no way to refine the original frame $\Theta = \{P, M\}$ into a finer one with exclusive finer elements say as $\Theta' = \{P \setminus (P \cap M), P \cap M, M \setminus (P \cap M)\}$ because there is no physical meaning and no possible occurrence of the atomic granules $P \setminus (P \cap M)$ and $M \setminus (P \cap M)$. In other words, the finer exclusive elements of the refined frame satisfying Shafer's model cannot always be well identified and precisely separated and they may have no sense at all. This is the main reason why DSmT allows as foundation the possibility to deal with non exclusive, partially overlapped or vague elements and refute Shafer's model and third excluded middle assumptions. DSmT proposes to work on a fusion space defined by Dedekind's lattice also called *hyper-power set* $D^\Theta$ in DSmT.

The hyper-power set is defined as the set of all composite propositions built from elements of $\Theta$ with $\cap$ and $\cup$ operators such that [5]:

1) $\emptyset, \theta_1, \dots, \theta_n \in D^\Theta$;
2) If $A, B \in D^\Theta$, then $A \cup B \in D^\Theta$ and $A \cap B \in D^\Theta$;
3) No other elements belong to $D^\Theta$, except those obtained by using rules 1) or 2).

Following Shafer's idea, Dezert and Smarandache define a (generalized) basic belief assignment (or mass) as a mapping $m(.) : D^\Theta \to [0, 1]$ such that:

$$m(\emptyset) = 0 \quad \text{and} \qquad \sum_{X \in D^\Theta} m(X) = 1.$$

Typically, when $\Theta = \{A, B\}$ and Shafer's model doesn't hold, in DSmT one works with $m(.)$ such that

$$m(A) + m(B) + m(A \cup B) + m(A \cap B) = 1 \tag{5}$$

which appears actually as a direct and natural mathematical extension of (2) and (3).

Actually DSmT offers also the advantage to work with Shafer's model or with any hybrid model if some integrity constraints between elements of the frame are known to be true and must be taken into account in the fusion process. DSmT allows to solve static and/or dynamic[2] fusion problems in the same general mathematical framework. For notation convenience, one denotes by $G^\Theta$ the generalized fusion space or generalized power set including integrity constraints (i.e. exclusivity as well as possible non-existence restrictions between some elements of $\Theta$), so that $G^\Theta = D^\Theta$ when no constraint enters in the model, or $G^\Theta = 2^\Theta$ when one wants to work with Shafer's model (see [31] for details and examples), or $G^\Theta = \Theta$ when working with probability model. If one wants to work with the space closed under union $\cup$, intersection $\cap$, and complementarity $\mathcal{C}$ operators, then $G^\Theta = S^\Theta$, i.e. the super-power set (see next section). A more general introduction of DSmT

[2] i.e. when the frame and/or its model change with time.

can be found in Chapter 1 of [33].

In DSmT, the fusion of two sources of evidences characterized by $m_1(.)$ and $m_2(.)$ is defined by $m_{PCR5}(\emptyset) = 0$ and $\forall X \in G^\Theta \setminus \{\emptyset\}$

$$m_{PCR5}(X) = m_{12}(X) + \sum_{\substack{Y \in G^\Theta \\ X \cap Y = \emptyset}} [\frac{m_1(X)^2 m_2(Y)}{m_1(X) + m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_2(X) + m_1(Y)}] \quad (6)$$

where all sets involved in formulas are in canonical form; $m_{12}(X) \equiv m_\cap(X) = \sum_{\substack{X_1, X_2 \in G^\Theta \\ X_1 \cap X_2 = X}} m_1(X_1) m_2(X_2)$ corresponds to the conjunctive consensus on $X$ between the $n = 2$ sources and where all denominators are different from zero. If a denominator is zero, that fraction is discarded. A general formula of PCR5 for the fusion of $n > 2$ sources has been proposed in [32].

*2.4. Unification of Fusion Theory (UFT)*

Recently Smarandache has proposed in [30, 32] an extension of DSmT by considering a super-power set $S^\Theta$ as the Boolean algebra on $\Theta$, i.e. $S^\Theta = (\Theta, \cap, \cup, c(.))$. In other words, $S^\Theta$ is assumed to be closed under union $\cup$, intersection $\cap$, and complement $c(.)$ of sets respectively. With respect to the partial ordering relation, the inclusion $\subseteq$, the minimum element is the empty set $\emptyset$, and the maximal element is the total ignorance $I = \bigcup_{i=1}^n \theta_i$. Since it extends the power set space through the closed operation of $\cap$, $\cup$ and $c(.)$ operators, that is, UFT not only considers the non-exclusive situation among the elements, but also consider the exclusive, exhaustive, non-exhaustive situations, and even open and closed world. Typically, when $\Theta = \{A, B\}$ , in UFT one works with $m(.)$ such that

$$m(A) + m(B) + m(A \cap B) + m(A \cup B) \\ + m(c(A)) + m(c(B)) + m(c(A) \cup c(B)) = 1 \quad (7)$$

## 3. Evidence Support Measure of Similarity (ESMS)

**Definition 3.1.** *Let's consider a discrete and finite frame $\Theta$ and the fusion space $G^\Theta$ including integrity constraints of the model associated with $\Theta$. The infinite set of basic belief assignments defined on $G^\Theta$ is denoted by $\mathfrak{m}_{G^\Theta}$. An Evidence Support Measure of Similarity (ESMS) of two (generalized) basic belief assignments $m_1(.)$ and $m_2(.)$ in $\mathfrak{m}_{G^\Theta}$ is the functionSim$(.,.) : \mathfrak{m}_{G^\Theta} \times \mathfrak{m}_{G^\Theta} \to [0, 1]$ satisfying the following conditions:*

*1) Symmetry:* $\forall m_1(.), m_2(.) \in \mathfrak{m}_{G^\Theta}, Sim(m_1, m_2) = Sim(m_2, m_1)$*;*
*2) Consistency:* $\forall m(.) \in \mathfrak{m}_{G^\Theta}, Sim(m, m) = 1$*;*
*3) Non-negativity:* $\forall m_1(.), m_2(.) \in \mathfrak{m}_{G^\Theta}, Sim(m_1, m_2) \geq 0$

We will say that $m_2(.)$ is more similar to $m_1(.)$ than $m_3(.)$ if and only if $Sim(m_1, m_2) \geq Sim(m_1, m_3)$. The maximum degree of similarity is naturally obtained when both bba's $m_1(.)$ and $m_2(.)$ coincide, which is expressed by consistency condition 2). The equality $Sim(m_1, m_2) = 0$ must be obtained when bba's have no focal elements in common, in particular whenever $m_1(.)$ is focused on $X \in G^\Theta$, which is denoted $m_1^X(.)$ and corresponds to $m_1(X) = 1$, and $m_2(.)$ is focused on $Y \in G^\Theta$, i.e. $m_2(.) = m_2^Y(.)$ such that $m_2(Y) = 1$, with $X \cap Y = \emptyset$.

**Theorem 3.1.** *For any bba $m_1(.) \in \mathfrak{m}_{G^\Theta}$ (which is a $|G^\Theta|$-dimensional vector) and any small positive real number $\epsilon$, there exists at least one bba $m_2(.) \in \mathfrak{m}_{G^\Theta}$ for a given distance measure[3] $d(.,.)$ such that $d(m_1, m_2) \leq \epsilon$.*

**Proof:** Let's take $m_2(.) = m_1(.)$, then $d(m_1, m_2) = d(m_1, m_1) = d(m_2, m_2) = 0 < \epsilon$ which completes the proof.

**Definition 3.2.** *(Agreement of evidence) : If there exist two basic belief assignments $m_1(.)$ and $m_2(.)$ in $\mathfrak{m}_{G^\Theta}$ such that for some distance measure $d(.,.)$, one has $d(m_1, m_2) \leq \epsilon$ with $\epsilon > 0$, then $\epsilon$ is called the agreement of evidence supporting measure between $m_1(.)$ and $m_2(.)$ with respect to the chosen distance $d(.,.)$. $m_1(.)$ and $m_2(.)$ are said $\epsilon$-consistent with respect to the distance $d(.,.)$.*

**Theorem 3.2.** *The smaller $\epsilon > 0$ is, the closer the distance $d(m_1, m_2)$ between $m_1(.)$ and $m_2(.)$ is, that is, the more similar or consistent $m_1(.)$ and $m_2(.)$ are.*

**Proof:** According to the Definition 3.2, if the evidence measure between $m_1(.)$ and $m_2(.)$ is $\epsilon$-consistent, then $d(m_1, m_2) \leq \epsilon$. Let's take $\epsilon = 1 - Sim(m_1, m_2)$; when $\epsilon$ becomes smaller and smaller, $Sim(m_1, m_2)$ becomes greater and greater, according to the definition of ESMS and thus more similar or consistent $m_1(.)$ and $m_2(.)$ become. Finally, if $\epsilon = 1 - Sim(m_1, m_2) = 0$,

[3] Here we don't specify the distance measure and keep it only as a generic distance. Actually $d(.,.)$ can be any distance measure. In practice, the Euclidean distance is frequently used.

then $m_1(.)$ and $m_2(.)$ are totally consistent.

From the previous definitions and theorems, the ESMS appears as an interesting measure for evaluating the degree of similarity between two sources. We propose to use ESMS in a pre-processing/thresholding technique in order to reduce the complexity of the combination of sources of evidence by keeping in the fusion process only the sources which are $\epsilon$-consistent. $\epsilon$ is actually a threshold parameter which has to be tuned by the system designer and which depends on the application and computational resources.

## 4. Several possible ESMS

In this section we propose several possibilities for choosing an ESMS function $Sim(.,.)$ satisfying theorem 3.1.

### *4.1. Euclidean ESMS function $Sim_E(m_1, m_2)$*

**Definition 4.1.** *Let $\Theta = \{\theta_1, \dots, \theta_n\}$ $(n > 1)$, $m_1(.)$ and $m_2(.)$ in $\mathfrak{m}_{G^\Theta}$, $X_i$ the i-th (generic) element of $G^\Theta$ and $|G^\Theta|$ the cardinality of $G^\Theta$. The following simple Euclidean ESMS function can be extended from [14]:*

$$Sim_E(m_1, m_2) = 1 - \frac{1}{\sqrt{2}} \sqrt{\sum_{i=1}^{|G^\Theta|} (m_1(X_i) - m_2(X_i))^2} \tag{8}$$

The following theorem establishes that $Sim_E(m_1, m_2)$ is an ESMS function.

**Theorem 4.1.** *$Sim_E(m_1, m_2)$ defined in (8) is an ESMS function.*

**Proof:**

1) Let's prove that $Sim_E(m_1, m_2) \in [0, 1]$. If $Sim_E(m_1, m_2) > 1$, from (8) one would get $\frac{1}{\sqrt{2}}\sqrt{\sum_{i=1}^{|G^\Theta|} (m_1(X_i) - m_2(X_i))^2} < 0$ which is impossible, so that $Sim_E(m_1, m_2) \leq 1$. Let's prove $Sim_E(m_1, m_2) \geq 0$ or equivalently from (8), $\sum_{i=1}^{|G^\Theta|} (m_1(X_i) - m_2(X_i))^2 \leq 2$. This inequality is equivalent to $\sum_{i=1}^{|G^\Theta|} m_1(X_i)^2 + \sum_{i=1}^{|G^\Theta|} m_2(X_i)^2 \leq 2 + 2\sum_{i=1}^{|G^\Theta|} m_1(X_i)$ $m_2(X_i)$. We denote it (i) for short. (i) always holds because one has $(\sum_{i=1}^{|G^\Theta|} m_1(X_i)^2 + \sum_{i=1}^{|G^\Theta|} m_2(X_i)^2) \leq ([\sum_{i=1}^{|G^\Theta|} m_1(X_i)]^2 + [\sum_{i=1}^{|G^\Theta|} m_2(X_i)]^2)$ and thus

$(\sum_{i=1}^{|G^\Theta|} m_1(X_i)^2 + \sum_{i=1}^{|G^\Theta|} m_2(X_i)^2) \leq 2$ because $[\sum_{i=1}^{|G^\Theta|} m_s(X_i)]^2 = 1$ for $s = 1, 2$ ($m_s(.)$ being normalized bba). Therefore inequality (i) holds and thus $Sim_E(m_1, m_2) \geq 0$.

2) It is easy to check that $Sim_E(m_1, m_2)$ satisfies the first condition of Definition 3.1.
3) If $m_1(.) = m_2(.)$, then $Sim_E(m_1, m_2) = 1$ because

$$\sum_{i=1}^{|G^\Theta|} (m_1(X_i) - m_2(X_i))^2 = 0.$$

Thus the second condition of Definition 3.1 is also satisfied.

4) Non-negativity has been proven above in the first part. Herein we use a particular case to show that Sim(m1, m2)=0, i.e. there exist $m_1^X$ and $m_2^Y$ for some $X, Y \in G^\Theta \setminus \{\emptyset\}$ such that $X \neq Y$, then according to (8), one gets $\sum_{i=1}^{|G^\Theta|} (m_1(X_i) - m_2(X_i))^2 = [m_1^X(X)]^2 + [m_2^Y(Y)]^2 = 2$ and thus one has $Sim_E(m_1^X, m_2^Y) = 1 - (\sqrt{2}/\sqrt{2}) = 0$, so that $Sim_E(.,.)$ verifies the third condition of Definition 3.1.

*4.2. Jousselme ESMS function $Sim_J(m_1, m_2)$*

**Definition 4.2.** *Let $m_1(.)$ and $m_2(.)$ be two basic belief assignments in $\mathfrak{m}_{G^\Theta}$ provided by the sources of evidence $S_1$ and $S_2$. Given a $|G^\Theta| \times |G^\Theta|$ assumed*[4] *positive definite matrix $\mathbf{D} = [D_{ij}]$, where $D_{ij} = |X_i \cap X_j|/|X_i \cup X_j|$, with $X_i, X_j \in G^\Theta$. Then, Jousselme ESMS function can be redefined from the Jousselme et al. measure [14]:*

$$Sim_J(m_1, m_2) = 1 - \frac{1}{\sqrt{2}} \sqrt{(m_1 - m_2)^T \mathbf{D} (m_1 - m_2)} \tag{9}$$

*or equivalently*

$$Sim_J(m_1, m_2) = 1 - \frac{1}{\sqrt{2}} \sqrt{m_1^2 + m_2^2 - 2\langle m_1, m_2 \rangle}$$

[4] Actually, Jousselme et al. in [14] did not prove that $\mathbf{D} = [D_{ij} = |X_i \cap X_j|/|X_i \cup X_j|]$ is truly a positive definite matrix. $\mathbf{D}$ is until now assumed to be positive definite. This is only a conjecture and proving it is not a trivial problem.

where $\langle m_1, m_2\rangle$ is the scalar product defined as

$$\langle m_1, m_2\rangle = \sum_{i=1}^{|G^\Theta|}\sum_{j=1}^{|G^\Theta|} D_{ij} m_1(X_i) m_2(X_j)$$

$X_i, X_j \in G^\Theta$, $i, j = 1, \ldots, s, |G^\Theta|$; $\|m\|^2$ represents the squared norm of the vector (bba) $m$, i.e. $\|m\|^2 = \langle m, m\rangle$.

**Theorem 4.2.** *$Sim_J(m_1, m_2)$ defined in formula (9) is an ESMS function.*

**Proof:**

1) Since the matrix $\mathbf{D}$ is conjectured to be a positive definite matrix, $Sim_J(m_1, m_2)$ satisfies the condition of symmetry.
2) If $m_1$ is equal to $m_2$, according to (9), one gets $Sim_J(m_1, m_2) = 1$. In other hand, if $Sim_J(m_1, m_2) = 1$, then the condition $m_1 = m_2$ holds. That is, the condition of consistency is satisfied.
3) According to (9), it can be drawn that $Sim_J(m_1, m_2) \leq Sim_E(m_1, m_2)$, and since the minimum value of $Sim_J(m_1, m_2)$ is zero, then $Sim_J(m_1, m_2)$ is non-negative.
4) According to the definition of $Sim_J(m_1, m_2)$, we can easily verify that $Sim_J(m_1, m_2)$ is a true distance measure between $m_1$ and $m_2$.

Actually $Sim_E(m_1, m_2)$ is nothing but a special case of $Sim_J(m_1, m_2)$ when taking $\mathbf{D}$ as the $|G^\Theta| \times |G^\Theta|$ identity matrix.

*4.3. Ordered ESMS function $Sim_O(m_1, m_2)$*

The definition of this (partial) ordered-based ESMS function is similar to $Sim_J(m_1, m_2)$ but instead of using Jousselme's matrix $\mathbf{D} = [D_{ij}]$, where $D_{ij} = |X_i \cap X_j|/|X_i \cup X_j|$, with $X_i, X_j \in G^\Theta$, we choose the DSm matrix $\mathbf{S} = [S_{ij}]$ where $S_{ij} = s(X_i \cap X_j)/s(X_i \cup X_j)$. Therefore, one has

$$Sim_O(m_1, m_2) = 1 - \frac{1}{\sqrt{2}}\sqrt{(m_1 - m_2)^T \mathbf{S}(m_1 - m_2)} \tag{10}$$

The function $s(X)$ corresponds to the intrinsic informational content of the proposition $X$ defined in details in [31] (Chap. 3) which is used for partially ordering the elements of $G^\Theta$. More precisely, $s(X)$ is the sum of the inverse of the length of the components of Smarandache's code[5] of $X$.

As a simple example, let's take $\Theta = \{\theta_1, \theta_2\}$ with free DSm model (i.e i.e. when all elements are non-exclusive two by two), then the partially[6] ordered hyper-power set $G^\Theta$ is given by $G^\Theta = \{\emptyset, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2\}$ because $s(\emptyset) = 0$, $s(\theta_1 \cap \theta_2) = 1/2$, $s(\theta_1) = 1 + 1/2$, $s(\theta_2) = 1 + 1/2$ and $s(\theta_1 \cup \theta_2) = 1 + 1 + 1/2$ since Smarandache's codes of $\emptyset$, $\theta_1$, $\theta_2$, $\theta_1 \cap \theta_2$ and $\theta_1 \cup \theta_2$ are respectively given by $\{< . >\}$ (empty code), $\{< 1 >, < 12 >\}$, $\{< 2 >, < 12 >\}$, $\{< 12 >\}$ and $\{< 1 >, < 12 >, < 2 >\}$. The matrix $\mathbf{S}$ is defined by[7]

$$\mathbf{S} = \begin{bmatrix} \frac{s(\theta_1 \cap \theta_2)}{s(\theta_1 \cap \theta_2)} & \frac{s(\theta_1 \cap \theta_2)}{s(\theta_1)} & \frac{s(\theta_1 \cap \theta_2)}{s(\theta_2)} & \frac{s(\theta_1 \cap \theta_2)}{s(\theta_1 \cup \theta_2)} \\ \frac{s(\theta_1 \cap \theta_2)}{s(\theta_1)} & \frac{s(\theta_1)}{s(\theta_1)} & \frac{s(\theta_1 \cap \theta_2)}{s(\theta_1 \cup \theta_2)} & \frac{s(\theta_1)}{s(\theta_1 \cup \theta_2)} \\ \frac{s(\theta_1 \cap \theta_2)}{s(\theta_2)} & \frac{s(\theta_1 \cap \theta_2)}{s(\theta_1 \cup \theta_2)} & \frac{s(\theta_2)}{s(\theta_2)} & \frac{s(\theta_2)}{s(\theta_1 \cup \theta_2)} \\ \frac{s(\theta_1 \cap \theta_2)}{s(\theta_1 \cup \theta_2)} & \frac{s(\theta_1)}{s(\theta_1 \cup \theta_2)} & \frac{s(\theta_2)}{s(\theta_1 \cup \theta_2)} & \frac{s(\theta_1 \cup X_2)}{s(\theta_1 \cup \theta_2)} \end{bmatrix} = \begin{bmatrix} 1 & 1/3 & 1/3 & 1/5 \\ 1/3 & 1 & 1/5 & 3/5 \\ 1/3 & 1/5 & 1 & 3/5 \\ 1/5 & 3/5 & 3/5 & 1 \end{bmatrix}$$

It is easy to verify the positiveness of the matrix $S$ by checking the positivity of all its eigenvalues which are $\lambda_1 = 0.800 > 0$, $\lambda_2 \approx 0.835 > 0$, $\lambda_3 \approx 0.205 > 0$ and $\lambda_4 \approx 2.160 > 0$. We have verified the positiveness of matrix $\mathbf{S}$ for $Card(\Theta) = n \leq 5$. Since a general proof of the positiveness of $\mathbf{D}$ and $\mathbf{S}$ seems difficult to obtain, we can only make a conjecture on the positiveness of $\mathbf{S}$ presently.

### *4.4. ESMS function $Sim_B(m_1, m_2)$*

Another ESMS function based on Bhattacharya's distance is defined as follows:

**Definition 4.3.** *Let $m_1(.)$, $m_2(.)$ be two basic belief assignments in $\mathfrak{m}_{G^\Theta}$, the ESMS function $Sim_B(m_1, m_2)$ is defined by:*

[5] Smarandache code is a representation of disjoint parts of the Venn diagram of the frame $\Theta$ under consideration. This code depends of the model for $\Theta$. For example, let's take $\Theta = \{\theta_1, \theta_2\}$. If $\theta_1 \cap \theta_2 = \emptyset$ (Shafer's model) is assumed, then the code of $\theta_1$ is $< 1 >$, whereas if $\theta_1 \cap \theta_2 \neq \emptyset$ (free DSm model) is assumed, then the code of $\theta_1$ will be $\{< 1 >, < 12 >\}$. The length of a component of a code is the number of characters between $<$ and $>$ in Smarandache's notation. For example, the length of component $< 12 >$ is 2. See [31], pp. 42–43 for details.

[6] This is a partial order since $s(\theta_1) = s(\theta_2)$.

[7] Actually, one works with $G^\Theta \setminus \{\emptyset\}$, and thus the column and row corresponding to the empty set do not enter in the definition of $\mathbf{S}$.

$$Sim_B(m_1, m_2) = 1 - \sqrt{1 - \sum_{X_i \in \mathcal{F}} \sqrt{m_1(X_i)m_2(X_i)}} \qquad (11)$$

*where $\mathcal{F}$ is the core of sources $S_1$ and $S_2$ , i.e. the set of elements of $G^\Theta$ having a positive belief mass: $\mathcal{F} = \{X \in G^\Theta | m_1(X) > 0 \text{ or } m_2(X) > 0\}$.*

**Theorem 4.3.** *$Sim_B(m_1, m_2)$ defined in formula (11) is an ESMS function.*

**Proof:**

1) Since $\sum_{X_i \in \mathcal{F}} \sqrt{m_1(X_i)m_2(X_i)} = \sum_{X_i \in \mathcal{F}} \sqrt{m_2(X_i)m_1(X_i)}$ then $Sim_B(m_1, m_2)$ satisfies the condition of symmetry.
2) if $m_1(.) = m_2(.)$, according to (11),

$$\sum_{X_i \in \mathcal{F}} \sqrt{m_1(X_i)m_2(X_i)} = \sum_{X_i \in \mathcal{F}} m_1(X_i) = 1$$

and therefore $Sim_B(m_1, m_1) = 1$. In other hand, if $Sim_B(m_1, m_2) = 1$, then the condition $m_1(.) = m_2(.)$ holds. That is, the condition of consistency is satisfied.
3) From the definition of bba, $\sum_{X_i \in \mathcal{F}} m_1(X_i) = 1$. Therefore,

$$\sum_{X_i \in \mathcal{F}} \sqrt{m_1(X_i)m_2(X_i)} \in [0, 1].$$

According to (11), it can be drawn that $Sim_B(m_1, m_2) \in [0, 1]$; that is, the minimum value of $Sim_B(m_1, m_2)$ is zero. Therefore, $Sim_B(m_1, m_2)$ is a non-negative measure.
4) According to the definition of $Sim_B(m_1, m_2)$, we can easily verify that $Sim_B(m_1, m_2)$ is a true distance measure between $m_1$ and $m_2$.

## 5. Comparison of ESMS functions

In this section we analyze the performances of the five ESMS functions aforementioned through a very simple example, where $\Theta = \{\theta_1, \theta_2, \theta_3\}$. We assume the free DSm model for $\Theta$. In such case, $G^\Theta = D^\Theta$ has eighteen non-empty elements $a_i$, $i = 1, 2, \ldots, s, \ldots, 18$. $G^\Theta$ is closed under $\cap$ and $\cup$

operators according to Dedekind's lattice. Of course, we also may choose other theoretical frameworks in a similar way according to the appropriate model of $G^\Theta$.

Let's assume that $\theta_2$ is the true identity of the object under consideration. Its optimal belief assignment is denoted $m_2(.) \triangleq \{m_2(\theta_2) = 1, m_2(X) = 0 \text{ for } X \in G^\Theta \setminus \{\theta_2\}\}$. We perform a comparison of the four ESMS functions in order to show the evolution of the measure of similarity between $m_1(.)$ and $m_2(.)$ when $m_1(.)$ is varying from an uniform distributed bba to $m_2(.)$. More precisely, we start our simulation by choosing $m_1(.)$ with all elements in $D^\Theta$ uniformly distributed, i.e. $m_1(a_i) = 1/18$, for $i = 1, 2, \ldots, s, \ldots, 18$. Then, step by step we increase the mass of belief of $\theta_2$ by a constant increment $\Delta = 0.01$ until reaching $m_1(\theta_2) = 1$. In the meantime the mass $m_1(X)$ of belief of all elements $X \neq \theta_2$ of $G^\Theta$ take value $[1 - m_1(\theta_2)]/17$ in order to work with a normalized bba $m_1()$. The basic belief mass committed to empty set is always zero, i.e. $m_1(\emptyset) = m_2(\emptyset) = 0$.

The degree of similarity of the four ESMS functions are plotted in Figure 1. The speed of convergence[8] of a similarity measure is characterized by the angle $\alpha$ of the slope of the curve at origin, or by its tangent. Based on this speed of convergence criterion, the analysis of the figure 1 yields the following remarks:

1) According to Figure 1., $\tan(\alpha_B) \approx 0.86$, $\tan(\alpha_E) \approx 0.68$, $\tan(\alpha_O) \approx 0.6$ and $\tan(\alpha_J) \approx 0.57$. $Sim_J(m_1, m_2)$ has the slowest convergence speed, then $Sim_O(m_1, m_2)$ takes second place.
2) $Sim_E(m_1, m_2)$ has a faster speed of convergence than $Sim_O(m_1, m_2)$ and $Sim_J(m_1, m_2)$ because it doesn't consider the intrinsic complexity of the elements in $G^\Theta$.
3) The speed of convergence of $Sim_B(m_1, m_2)$ is the fastest. When $m_1(.)$ and $m_2(.)$ become very similar, $Sim_B(m_1, m_2)$ becomes very quickly close to 1. But if a small dissimilarity between $m_1()$ and $m_2()$ occurs, then $Sim_B(m_1, m_2)$ becomes very small which actually makes it very sensitive to small dissimilarity perturbations.

[8] Here the convergence speed refers to how much the global agreement degree (similarity) is between $m_1(.)$ and $m_2(.)$ with the continuous decrease of $m_1(\theta_2)$.

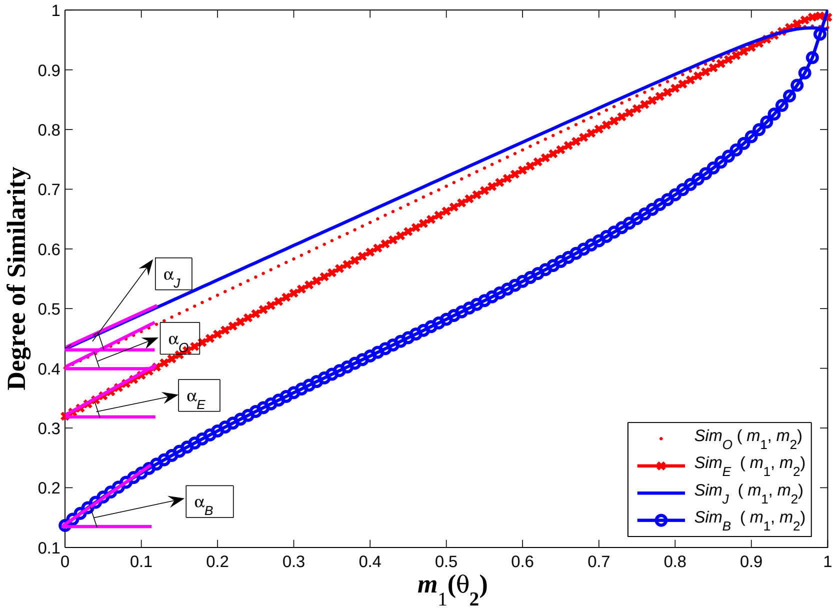


Figure 1: Comparison of performance among ESMS functions.

4) In summary, one sees that no definitive conclusion about the best choice among these four ESMS functions can be drawn in general, but if one considers as important the speed of convergence criterion of the dissimilarity/ difference between two evidential sources, $Sim_B(m_1, m_2)$ is the best choice, because it is very sensitive to such difference, whereas $Sim_J(m_1, m_2)$ is the worst choice with respect to such criterion.

## 6. Simulation results

We present a simulation result to show how the ESMS filter performs in generalized fusion machine (GFM), and its advantage. Let's take a 2D frame of discernment $\Theta = \{\theta_1, \theta_2\}$ and consider twenty equireliable sources of evidence according to the Table 1. We consider the free DSm model and the fusion space is the hyper-power set $D^\Theta = \{\theta_1, \theta_2, \theta_1 \cap \theta_2, \theta_1 \cup \theta_2\}$. $\mathbf{S}_{c1}$ denotes the barycentre of the front 10 belief masses, while $\mathbf{S}_{c2}$ denotes the barycentre[9] of all belief masses . The measure of similarity based on Euclidean ESMS function defined in equation (8) has been used here, but any other measures of similarity could be used instead. In this example if we take 0.75 for the threshold value we see from the Table 1 and for the 10 front sources of evidences, that the measures of similarity of $S_5$ and $S_{10}$ with respect to $\mathbf{S}_{c1}$ are lower than 0.75. Therefore, the sources $S_5$ and $S_{10}$ will be discarded/filtered of the fusion process. If the threshold value is set to 0.8, then the sources $S_5$, $S_{10}$, $S_4$ and $S_8$ will be discarded. That is, the higher the given threshold is, the less the number of information sources through the filter is.

| $S$ | $m(\theta_1)$ | $m(\theta_2)$ | $m(\theta_1 \cap \theta_2)$ | $m(\theta_1 \cup \theta_2)$ | $Sim_E$ |
|---|---|---|---|---|---|
| $S_1$ | 0.3 | 0.4 | 0.2 | 0.1 | 0.8735 |
| $S_2$ | 0.3 | 0.2 | 0.4 | 0.1 | 0.800 |
| $S_3$ | 0.4 | 0.1 | 0.2 | 0.3 | 0.8268 |
| $S_4$ | 0.7 | 0.1 | 0.1 | 0.1 | 0.7592 |
| $S_5$ | 0.1 | 0.8 | 0.1 | 0.0 | 0.5550 |
| $S_6$ | 0.5 | 0.2 | 0.2 | 0.1 | 0.9106 |
| $S_7$ | 0.4 | 0.3 | 0.1 | 0.2 | 0.9368 |
| $S_8$ | 0.3 | 0.1 | 0.2 | 0.4 | 0.7592 |
| $S_9$ | 0.4 | 0.5 | 0.1 | 0.0 | 0.8103 |
| $S_{10}$ | 0.8 | 0.1 | 0.0 | 0.1 | 0.6806 |
| $\mathbf{S}_{c1}$ | 0.42 | 0.28 | 0.16 | 0.14 | 1.0000 |
| $S_{11}$ | 0.5 | 0.0 | 0.2 | 0.3 | 0.7569 |
| $S_{12}$ | 0.2 | 0.6 | 0.1 | 0.1 | 0.7205 |
| $S_{13}$ | 0.4 | 0.3 | 0.2 | 0.1 | 0.9360 |
| $S_{14}$ | 0.9 | 0.1 | 0.0 | 0.0 | 0.6230 |
| $S_{15}$ | 0.5 | 0.2 | 0.1 | 0.2 | 0.9100 |
| $S_{16}$ | 0.5 | 0.3 | 0.0 | 0.2 | 0.8900 |
| $S_{17}$ | 0.5 | 0.0 | 0.1 | 0.4 | 0.7205 |
| $S_{18}$ | 0.7 | 0.2 | 0.1 | 0.0 | 0.7807 |
| $S_{19}$ | 0.1 | 0.7 | 0.1 | 0.1 | 0.6217 |
| $S_{20}$ | 0.3 | 0.6 | 0.1 | 0.0 | 0.7390 |
| $\mathbf{S}_{c2}$ | 0.44 | 0.29 | 0.13 | 0.14 | 1.0000 |

Table 1: A list of given sources of evidences.

[9] Let's denote $k = |G^\Theta|$ the cardinality of $G^\Theta$ and consider $S$ independent sources of evidence. If all sources are equireliable, the barycentre of belief masses of the $S$ sources is given by: $\forall j = 1, \ldots, k$, $\bar{m}(X_j) = \frac{1}{S}\sum_{s=1}^{S} m_s(X_j)$, see [18] for details.

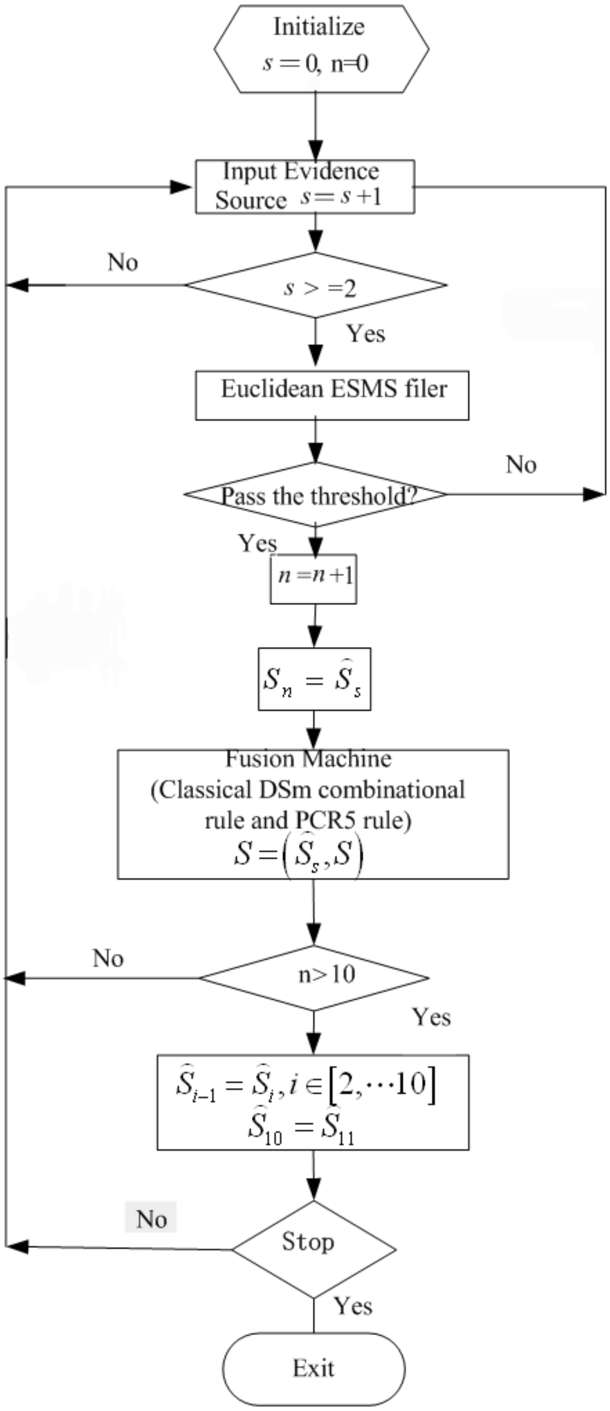


Figure 2: The procedure flow chart of DSmT-based generalized fusion machine.

In order to embed ESMS function to GFM [19], we give its working principle summarized in Figure 2, whose main steps of the algorithm for implementing the GFM are enumerated as follows:

1) Initialization of the parameters: the number of sources of evidence is set to zero, (i.e. one has initially no source, $s = 0$), so that the number of sources in the filter window is $n = 0$.
2) Include[10] a source of evidence $\hat{S}_s$ and then test if the number of sources $s$ is less than 2. If $s \geq 2$, then go to next step, otherwise include/take into account another source of evidence $\hat{S}_2$.
3) Based on the barycentre of gbba of the front $n \leq 10$ evidence sources, the degrees of similarity are computed according to the formula (8), and compared with a prior tuned threshold. If it is larger than the threshold, then let $n = n + 1$. Otherwise, introduce a new source of evidence $\hat{S}_{s+1}$.
4) If $n = 1$, the current source, say $S$, is not involved in the fusion process. If $n = 2$, then the fusion step must apply between $S$ and $\hat{S}_2$ with classical DSm rule [31], i.e. the conjunctive consensus. We then use PCR5 rule [32] to redistribute the remaining partial conflicts only to the sets involved in the corresponding partial conflicts. We get a new combined source affected with same index $S$. If $2 < n \leq 10$, after the current evidence source $\hat{S}_s$ is combined with the final source of evidence produced last time, a new source of evidence is obtained and assigned to $S$ again. Whenever $n \leq 10$, go back to step 2), otherwise, the current source of evidence $\hat{S}_s$ under test has been accepted by the ESMS filter, $\hat{S}_i$ is assigned to $\hat{S}_{i-1}, i \in [2, s, 10]$, and $\hat{S}_s$ is assigned to $\hat{S}_{10}$. Then, $\hat{S}_{10}$ is combined with the last source $S$, the combined result is reassigned[11] to $S$, and then, go back to step 2).
5) Test whether to stop or not[12]: if no, then introduce a new source of evidence $\hat{S}_{s+1}$, otherwise stop and exit.

We show two simulation results in Figures 3 and 4 following the working principle of GFM, when we use the sources of evidences listed in Table 1.

[10] We assume the free DSm model and consider that the general basic belief assignments are given.

[11] In this work, we use also an ESMS filter window in a sliding mode.

[12] In our experiment, judge whether the mobile robot stops receiving sonar's data.

The comparison of Figures 3 and 4 yields the following remarks:

1) On the Figure 3, we don't see the real advantage of ESMS filter since the convergence to $\theta_1$ without ESMS filter (green curve) is better than with ESMS filter in terms of improving fusion precision. This is because some useful information sources are filtered and thrown away with the increase of the threshold, the number of information sources to enter into the final fusion will become fewer and fewer. Generally speaking, this yields a slower speed of convergence to $\theta_1$. For example, for one source $S_3$ in the Table 1, if $S_3$ is combined with itself once according to (6), the combinational result is $S = [m_N(\theta_1), m_N(\theta_2), m_N(\theta_1 \cap \theta_2), m_N(\theta_1 \cup \theta_2)]$[13] $= [0.5707, 0.0993, 0.1680, 0.1620]$ . If twice, the result is $S = [0.6765, 0.0852, 0.1429, 0.0954]$. If thrice, then the result is $S = [0.7453, 0.00693, 0.1216, 0.0638]$. More the combinational times is, nearer by 1 $m_N(\theta_1)$ is and nearer by 0 $m_N(\theta_2)$ is .Therefore, ESMS filter might also result in losing some useful information, while it filter some bad information.
2) On the Figure 4, one sees the role played by the ESMS filter. When there are highly conflicting sources, the result of the fusion process will not converge if the ESMS filter is not used. With the fine tuning of the ESMS threshold, the convergence becomes better and better because ESMS filter processes the fused information, and withdraws the sources which might cause the results to be incorrect or imprecise, so that it improves the fusion precision and correctness.
3) The reduction of computing burden is obtained. Even if we have introduced an ESMS preprocessing step, it turns out that finally a drastic reduction of computing burden is obtained because we can significantly reduce the number of sources to combine with ESMS criterion.
4) We increase the applicability of the classical rules of combination. Since we reduce the number of conflicting sources of evidence thanks to the ESMS preprocessing, the degree of conflict between sources to combine is kept low. Therefore, classical rules, like Dempster's rule, which do not perform well in high conflicting situations can be used also in applications like the one studied here. Without ESMS preprocessing step, the classical fusion rules cannot work very well [23, 29]. Therefore, we extend their domain of applicability when using ESMS filtering step.

[13] $m_N(\cdot)$ refers to the new generalized basic belief assignment

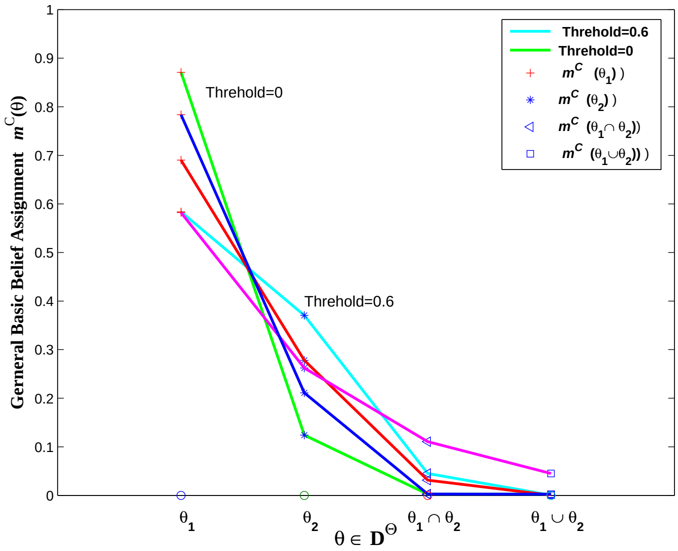


Figure 3: Fusion result of the front 10 sources using different thresholds ($0 \sim 0.9$).

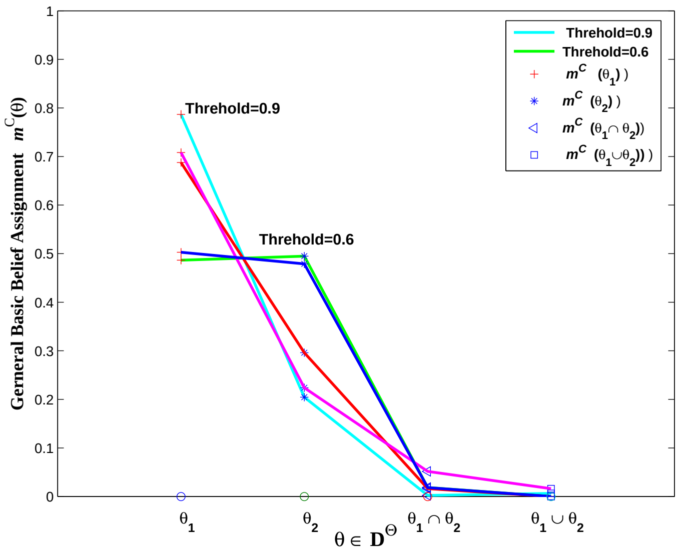


Figure 4: Fusion result of the total 20 sources using different thresholds ($0 \sim 0.9$).

## 7. An application in mobile robot perception

The information acquired in building grid map using sonar sensors on a mobile robot is usually uncertain, imprecise and even highly conflicting. Such application in autonomous robot perception and navigation provides a good platform to verify experimentally the capability of the ESMS filter in GFM. Although there exist many methods of building map based either on Probability theory [36], FST (Fuzzy System Theory) [24], DST [34], GST (Grey System Theory) [39, 40, 41], or DSmT [20], we just compare the performances of the map building using a classical fusion machine without ESMS filter (i.e. CFMW) with respect to the classical fusion machine with ESMS filter (called GFM) in the DSmT framework only. A detailed comparison between our current ESMS-based approach with other methods is given in a companion paper in [21] where we show that ESMS-based approach outperforms other approaches using almost the same experimental conditions and inputs. In order to further reduce the measurement noises, we improve our past belief assignment model of sonar sensors in DSmT framework[14] as follows:

$$m(\theta_1) = \begin{cases} (1-\rho/(R-2\epsilon)) \times (1-\lambda/2) & \text{if} \quad \begin{cases} R_{min} \leq \rho \leq R-\epsilon \\ 0 \leq \varphi \leq \omega/2 \end{cases} \\ 0 \quad \text{otherwise} \end{cases} \tag{12}$$

$$m(\theta_2) = \begin{cases} \exp(-3(\rho-R)^2) \times \lambda & \text{if} \quad \begin{cases} R_{min} \leq \rho \leq R+\epsilon \\ 0 \leq \varphi \leq \omega/2 \end{cases} \\ 0 \quad \text{otherwise} \end{cases} \tag{13}$$

[14]We assume that there are only two focal elements $\theta_1$ and $\theta_2$ in the frame of discernment. Elements of hyper-power set are $\theta_1$,$\theta_2$, $\theta_1 \cap \theta_2$ and $\theta_1 \cup \theta_2$. $\theta_1$ represents the emptiness of a given grid cell, $\theta_2$ represents the occupancy for a given grid cell, $\theta_1 \cap \theta_2$ means that there is some conflict between two sonar measurements for the same grid cell and $\theta_1 \cup \theta_2$ represents the ignorance for a grid cell because of the possible lack of measurement acquisition.

$$m(\theta_1 \cap \theta_2) = \begin{cases} 1 - (2(\rho - (R-\epsilon)/R)^2) & \text{if} \quad \begin{cases} R_{min} \le \rho \le R + \epsilon \\ 0 \le \varphi \le \omega/2 \end{cases} \\ 0 \quad \text{otherwise} \end{cases} \tag{14}$$

$$m(\theta_1 \cup \theta_2) = \begin{cases} \tan(2(\rho - R)) \times (1 - \lambda) & \text{if} \quad R \le \rho \le R + \epsilon \\ 0 \quad \text{otherwise} \end{cases} \tag{15}$$

Where, $\lambda$ is given by (see [9] for justification)

$$\lambda = \begin{cases} 1 - (2\varphi/\omega)^2 & \text{if} \quad 0 \le |\varphi| \le \omega/2 \\ 0 \quad \text{otherwise} \end{cases} \tag{16}$$

The parameters $R$, $\rho$, $\epsilon$, $R_{min}$, $\omega$, and $\varphi$ in formulas (12)-(16) were defined and used in [19, 20, 21]. $R$ is the range measurement. $\rho$ is the distance between the grid cell and sonar's emitting point. $\epsilon$ is the range measurement error. $R_{min}$ is the minimal range distance of sonar sensors. $\omega$ is the scattering angle of sonar. $\varphi$ is the angle between the line (from the grid cell to sonar emitting point) and the sonar's emitting direction. The following functions $C_1$ and $C_2$ play an important role in reducing noises in the process of map building. $C_1$ function, proposed by Wang in [39, 40, 41], is a constrict function[15] for sonar measurements defined by:

$$C_1 = \begin{cases} 0 & \text{if} \quad \rho > \rho_{l_2} \\ \frac{\rho_{l_2} - \rho}{\rho_{l_2} - \rho_{l_1}} & \text{if} \quad \rho_{l_1} \le \rho \le \rho_{l_2} \\ 1 & \text{if} \quad \rho < \rho_{l_2} \end{cases} \tag{17}$$

Where, $\rho_{l_1}$ and $\rho_{l_2}$ represents the upper and lower limits of valid measurements. $C_2$ is the constraint function[16] for sonar's uncertainty defined as follows:

$$C_2 = \begin{cases} (\frac{\rho - R + 0.5\epsilon}{0.5\epsilon})^2 & \text{if} \quad \rho - R > -0.5\epsilon \\ (\frac{\rho - R - 0.5\epsilon}{0.5\epsilon})^2 & \text{if} \quad \rho - R < 0.5\epsilon \\ 0 & \text{if} \quad \mid \rho - R \mid > 0.5\epsilon \end{cases} \tag{18}$$

[15] The main idea is that the sonar readings must be discounted according to sonar characteristics.

[16] The main idea consists in committing high belief assignments to sonar readings close to the sonar sensor.

Where, the product of $C_1$ and $C_2$ is multiplied by the belief assignment function, i.e. $m(\theta_2), m(\theta_1 \cap \theta_2), m(\theta_1 \cup \theta_2)$ respectively.

The experiment is performed by running Pioneer II mobile robot with 16 sonar detectors in the indoor laboratory environment as shown in Figure 5. The environment's size is $4550mm \times 3750mm$. The environment is divided into $91 \times 75$ rectangular cells having the same size according to the grid map method. The robot starts to move from the location (1 m, 0.6 m), which faces towards 0 degree. We take the corner of left bottom as the global coordinate origin of the map. Objects/obstacles in rectangular grid map are sketched in Figure 6. The processing steps of our intelligent perception and fusion system have been implemented with our software Toolbox developed under C++ 6.0 and with OpenGL server as a client end. When the robot moves in the environment, the server end collects much information (i.e. the location of robot, sensors measurements, velocity, etc.) from the mobile robot and its sensors onboard. Through the protocol of TCP/IP, the client end can get any information from the server end and fuse them.

Since our environment is small,the robot moves less time or a short distance. Then one only considers the self-localization method based on $\delta$-NFAM[17] method [17, 19] with the search from $\theta - \delta_\theta$ to $\theta + \delta_\theta$. In order to reduce the computation burden, the restricted spreading arithmetic has been used. The flow chart of this procedure for this experiment is given in Figure 7. Its main steps are the following ones:

1) Initialize the parameters of the robot (location, velocity, etc.).
2) Acquire 16 sonar measurements, and robot's location from odometer, when the robot is running in the environment. We can calibrate the robot's pose with our $\delta$-NFAM method [17, 19]. Here we set the first timer, of which interval is 100 ms.
3) Compute gbba of the fan-form area detected by each sonar sensor according to the formulas in [20].
4) Apply the DSmT-based GFM, that is, adopt Euclidean information filter to choose basic consistent sources of evidence according to the formula (8). Then combine the consistent sources with the DSm conjunctive rule [4, 5, 31] and compute gbbas after combination. Then,

[17]One has taken $\delta_\theta = 5^o$ in our experiment.

redistribute partial conflicting masses to the gbbas of sets involved in the partial conflict only with the PCR5 rule [32].

5) Compute the belief of occupancy $Bel(\theta_2)$ of some grid cells according to [31]. Save them into the map matrix and then go to step 6).
6) Update the map of the environment (here we set the second timer, of whose interval is 100 ms). Generally speaking, more the times of scanning map are, more accurate the final map reconstructed is. At the same time, also test whether the robot stops receiving the detecting data: if yes, then stop fusion and exit, otherwise, go back to step 2).

Figure 5: The real experimental environment.

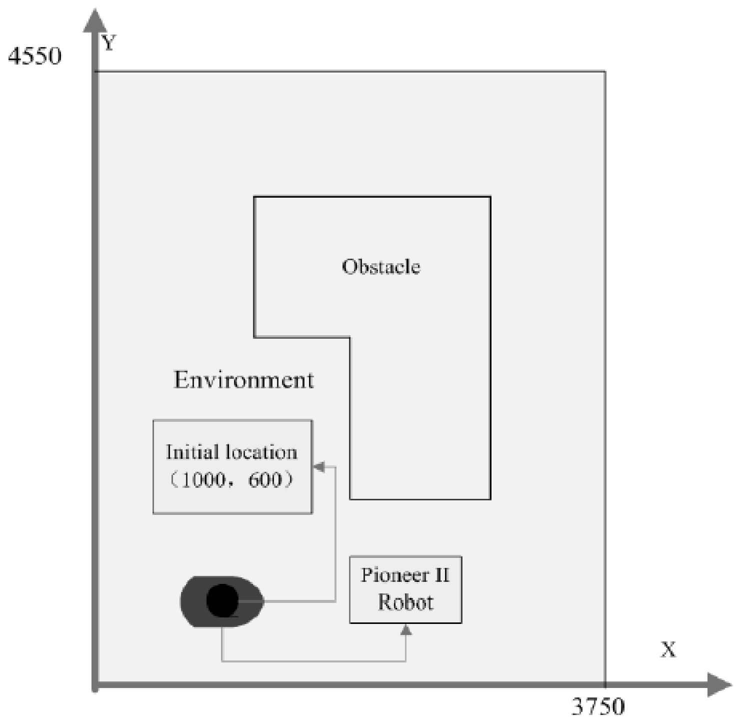


Figure 6: Global coordinate system for the experiment.

In this experiment, we obtain the maps built by the GFM before and after improving the sonar model as shown in Figure 8 and 9 respectively. In order to show the advantage of the ESMS filter in the GFM, we also compare our approach with the classical fusion machine which doesn't use the ESMS filter (called CFMW). The maps built by the CFMW before and after improving the sonar model are shown in Figures 10 and 11 respectively. Whenever the map is built before or after improving the sonar model, one sees that the GFM always outperforms the CFMW because one obtains clearer boundary outlines and less noises in the map reconstruction. In addition, the ESMS information filter coupled with PCR5 fusion rule, allows to reduce drastically the computational burden because ESMS filter can filter the outlier-sources. With the GFM approach, only the most consistent sources of evidence are combined and this allows to reduce the uncertainty in the fusion result and to improve the robot perception of the surrounded environment.

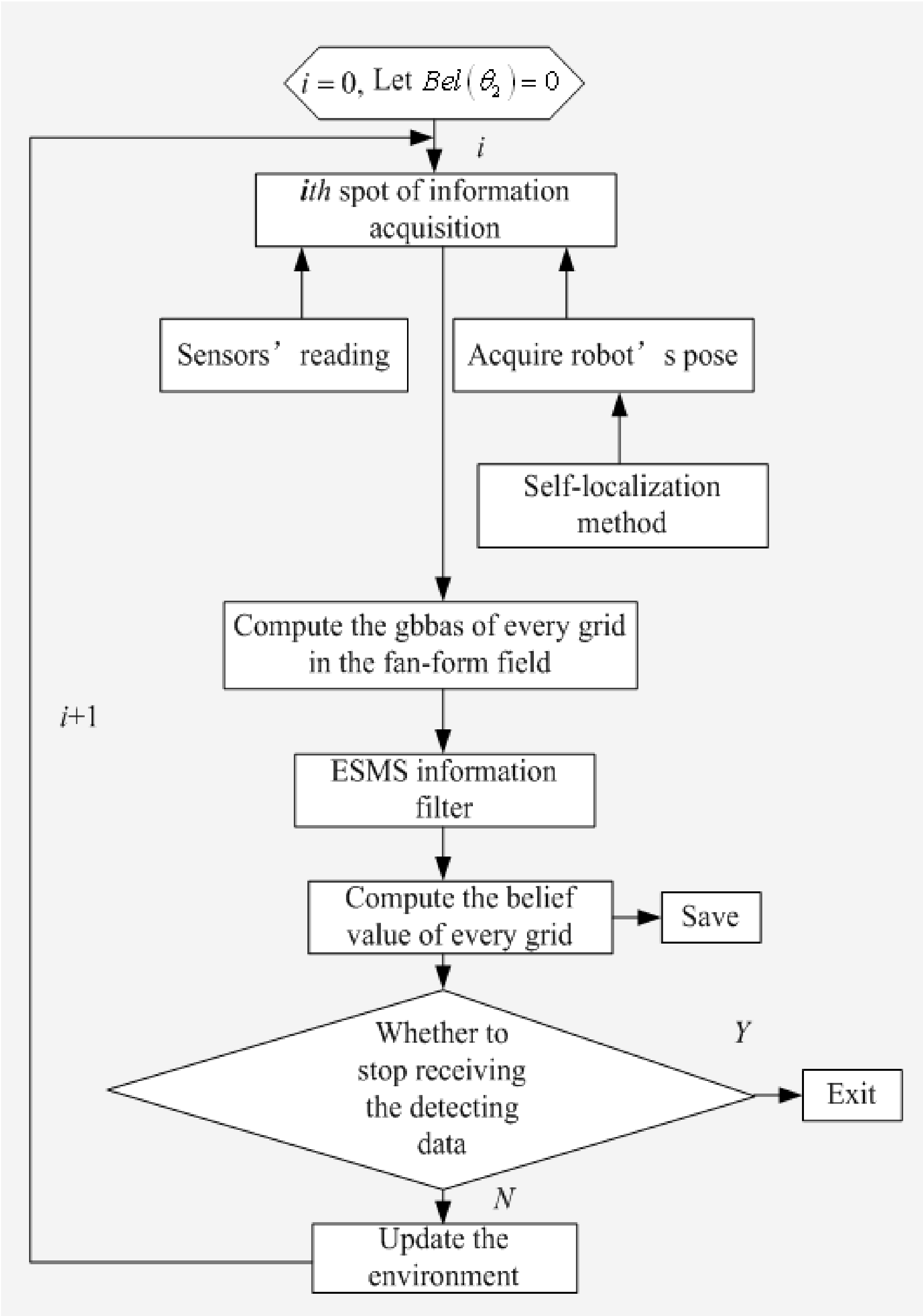


Figure 7: Flow chart of the map building with the GFM.

Figure 8: Map building based on the GFM before improving the sonar model.

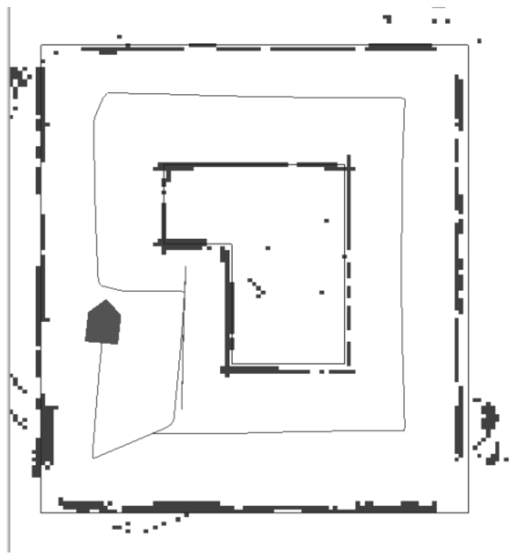

Figure 9: Map building based on the GFM after improving the sonar model.

Figure 10: Map building based on the CFMW before improving the sonar model.

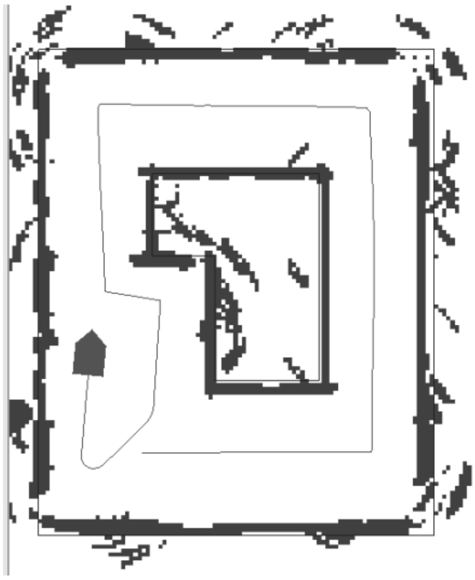

Figure 11: Map building based on the CFMW after improving the sonar model.

## 8. Conclusions

In this paper, a general Evidence Supporting Measure of Similarity (ESMS) between two basic belief assignments has been proposed. ESMS can be used on different fusion spaces (lattice structures) and with different distance measures. This approach allows to select the most coherent subset of sources of evidence available and to reject outlier-sources which are too conflicting with other sources. Hence, a drastic reduction of computational burden is possible with keeping good performances which is very attractive for real-time applications having limited computing resources. The hybrid of ESMS with the sophisticated and efficient PCR5 fusion rule of DSmT, called GFM (Generalized Fusion Machine), is specially useful and interesting in robotic applications involving real-time perception and navigation systems. The real application of GFM for mobile robot perception from sonar sensors presented in this work shows clearly a substantial improvement of the fusion result in map building/estimation of the surrounded environment. This work shows also the crucial role played by the most advanced fusion techniques for applications in robotics.

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