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RESEARCH ARTICLE

FUZZY NEUTROSOPHIC SUBGROUPS

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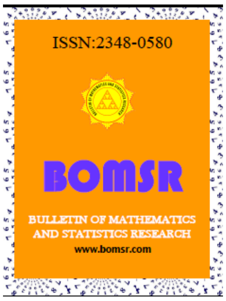

ABSTRACT
In this paper we introduce the notion of fuzzy neutrosophic subgroups. Also we obtain the fuzzy neutrosophic subgroups generated by fuzzy neutrosophic set and investigate some of their properties.

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1.INTRODUCTION

Smarandache [13] initiated the concept of neutrosophic set which overcomes the inherent difficulties that existed in fuzzy sets[14] and intuitionistic fuzzy sets [5,6].Following this, the neutrosophic sets are explored to different heights in all fields of science and engineering.I.Arockiarani et al. defined the notion of fuzzy neutrosophic sets [1].In 1989, R.Biswas [8] introduced the concept of intuitionistic fuzzy subgroups and studied some of their properties . In this paper we define fuzzy neutrosophic subgroups and discuss their properties.

2.PRELIMINARIES:

Definition 2.1:[1] A Fuzzy neutrosophic set A on the universe of discourse X is defined as A= $\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F: X \to [0,1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$

Definition 2.2: [1] Let X be a non empty set, and

$$A = \langle x, T_A(x), I_A(x), F_A(x)\rangle, B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$$

(i) $A \subseteq B$ for all x if $T_A(x) \le T_B(x)$, $I_A(x) \le I_B(x)$, $F_A(x) \ge F_B(x)$

(ii) $A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x))\rangle$

(iii) $A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x))\rangle$

(iv) A\B (x)= $\langle x, \min(T_A(x), F_B(x)), \min(I_A(x), 1 - I_B(x)), \max(F_A(x), T_B(x))\rangle$

Definition 2.3:[1] A Fuzzy neutrosophic set A over the universe X is said to be null or empty Fuzzy neutrosophic set if $T_A(x) = 0$,$I_A(x) = 0$, $F_A(x) = 1$ for all $x \in X$. It is denoted by 0_N

Definition 2.4:[1] A Fuzzy neutrosophic set A over the universe X is said to be absolute fuzzy neutrosophic set if $T_A(x) = 1$, $I_A(x) = 1$, $F_A(x) = 0$ for all $x \in X$. It is denoted by 1_N

Definition 2.5: [1]The complement of a Fuzzy neutrosophic set A is denoted by A^c and is defined as $A^c = \langle x, T_{A^c}(x), I_{A^c}(x), F_{A^c}(x)\rangle$ where $T_{A^c}(x) = F_A(x)$, $I_{A^c}(x) = 1 - I_A(x)$, $F_{A^c}(x) = T_A(x)$

The complement of a Fuzzy neutrosophic set A can also be defined as $A^c = 1_N - A$.

Definition 2.6:[2]Let X and Y be two non empty sets and $f : X \to Y$ be a function .

(i) If $B = \{\langle y, T_B(y), I_B(y), F_B(y)\rangle : y \in Y\}$ is a fuzzy neutrosophic set in Y then the pre image of B under f ,denoted by $f^{-1}(B)$, is the fuzzy neutrosophic set in X defined by $f^{-1}(B) = \{\langle x, f^{-1}(T_B(x)), f^{-1}(I_B(x)), f^{-1}(F_B(x))\rangle : x \in X\}$

Where $f^{-1}(T_B(x)) = T_B(f(x))$

(ii) If $A = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle : x \in X\}$ is a fuzzy neutrosophic set in X then the image of A under f ,denoted by $f(A)$,is the fuzzy neutrosophic set in Y defined by $f(A) = \{\langle y, f(T_A(y)), f(I_A(y)), f_{\sim}(F_A(y))\rangle : y \in Y\}$

$$f(T_A(y)) = \begin{cases} \underset{x \in f^{-1}(y)}{Sup}\ T_A(x) & if\ f^{-1}(y) \neq 0_N \\ 0 & otherwise \end{cases}$$

where $$f(I_A(y)) = \begin{cases} \underset{x \in f^{-1}(y)}{Sup}\ I_A(x) & if\ f^{-1}(y) \neq 0_N \\ 0 & otherwise \end{cases}$$

$$f_{\sim}(F_A(y)) = \begin{cases} \inf_{x \in f^{-1}(y)} F_A(x) & if\ f^{-1}(y) \neq 0_N \\ 1 & otherwise \end{cases}$$

And $f_{\sim}(F_A(y)) = (1 - f(1 - F_A))y$

Definition 2.7:[3] Let $(X, .)$ be a group and let A be fuzzy neutrosophic set in X . Then A is called a fuzzy neutrosophic group (in short, FNG) in X if it satisfies the following conditions: (i) $T_A(xy) \geq T_A(x) \wedge T_A(y), I_A(xy) \geq I_A(x) \wedge I_A(y)$ and $F_A(xy) \leq F_A(x) \vee F_A(y)$ (ii) $T_A(x^{-1}) \geq T_A(x), I_A(x^{-1}) \geq I_A(x), F_A(x^{-1}) \leq F_A(x)$

Definition 2.8:[3]Let $(X, .)$ be a groupoid and let A and B be two fuzzy neutrosophic sets in X .Then the fuzzy neutrosophic product of A and B , $A \circ B$,is defined as follows: for any $x \in X$,

$$T_{A \circ B}(x) = \begin{cases} \vee_{yz=x} [T_A(y) \wedge T_B(z)] & for\ each\ (y,z) \in X \times X\ with\ yz = x, \\ 0 & otherwise \end{cases}$$

$$I_{A \circ B}(x) = \begin{cases} \vee_{yz=x} [I_A(y) \wedge I_B(z)] & for\ each\ (y,z) \in X \times X\ with\ yz = x, \\ 0 & otherwise \end{cases}$$

$$F_{A \circ B}(x) = \begin{cases} \wedge_{yz=x} [F_A(y) \vee F_B(z)] & for\ each\ (y,z) \in X \times X\ with\ yz = x, \\ 1 & otherwise \end{cases}$$

Definiion 2.9:[4] Let G be a groupoid and let $A \in FNS(G)$.Then A is called a :

(1) fuzzy neutrosophic left ideal (in short $FNLI$) of G if for any $x, y \in G$, $A(xy) \geq A(y)$.(i.e.,) $T_A(xy) \geq T_A(y), I_A(xy) \geq I_A(y)$ and $F_A(xy) \leq F_A(y)$

(2) fuzzy neutrosophic right ideal (in short $FNRI$) of G if for any $x, y \in G$, $A(xy) \geq A(x)$.(i.e.,) $T_A(xy) \geq T_A(x), I_A(xy) \geq I_A(x)$ and $F_A(xy) \leq F_A(x)$

(3) fuzzy neutrosophic ideal (in short FNI) of G if it is both a $FNLI$ and $FNRI$

It is clear that A is a FNI of G if and only if for any $x, y \in G$, $T_A(xy) \geq T_A(x) \vee T_A(y), I_A(xy) \geq I_A(x) \vee I_A(y)$ and $F_A(xy) \leq F_A(x) \wedge F_A(y)$.Moreover ,a FNI (respectively $FNLI$, $FNRI$) is a $FNSGP$ of G.Note that for any $FNSGP$ A of G we have $T_A(x^n) \geq T_A(x), I_A(x^n) \geq I_A(x)$ and $F_A(x^n) \leq F_A(x)$ for each $x \in G$, where x^n is any composite of x's.

We will denote the set of all $FNSGP$ s of G as $FNSGP(G)$.

Definition 2.10:[4]Let $(G,.)$ be a groupoid and let $0_N \neq A \in FNS(G)$.Then A is called a fuzzy neutrosophic subgroupoid in G (in short , $FNSGP$ in G) if $A \circ A \subset A$.

Definition 2.11:[4]Let $(G,.)$ be a groupoid and let $A \in FNS(X)$.Then A is called a fuzzy neutrosophic subgroupoid in G (in short , $FNSGP$ in G) if for any $x, y \in G$, $T_A(xy) \geq T_A(x) \wedge T_A(y), I_A(xy) \geq I_A(x) \wedge I_A(y)$ and $F_A(xy) \leq F_A(x) \vee F_A(y)$.It is clear that 0_N and 1_N are both $FNSGP$ s of G.

Definition 2.12:[4]Let $A \in FNS(G)$.Then A is said to have the sup property if for any $T \in P(G)$,there exists a $t_0 \in T$ such that $A(t_0) = \bigcup_{t \in T} A(t)$.i.e.,

$T_A(t_0) = \vee_{t \in T} T_A(t), I_A(t_0) = \vee_{t \in T} I_A(t), F_A(t_0) = \wedge_{t \in T} F_A(t)$,where $P(G)$ denotes the power set of G.

Definition 2.13:[4]Let A be a fuzzy neutrosophic set in X and let $\lambda, \mu, \nu \in I$ with $\lambda + \mu + \nu \leq 3$ .Then the set $X_A^{(\lambda,\mu,\nu)} = \{x \in X : A(x) \geq C_{(\lambda,\mu,\nu)}(x)\} = \{x \in X : T_A(x) \geq \lambda, I_A \geq \mu, F_A \leq \nu\}$ is called a $(\lambda, \mu, \nu)-$ level subset of A.

3. FUZZY NEUTROSOPHIC SUBGROUPS

Definition 3.1:

Let G be a group and let $A \in FNSGP(G)$.Then A is called a fuzzy neutrosophic subgroup (in short , $FNSG$) of G if $A(x^{-1}) \geq A(x)$.(i.e.,) $T_A(x^{-1}) \geq T_A(x), I_A(x^{-1}) \geq I_A(x)$ and $F_A(x^{-1}) \leq F_A(x)$ for each $x \in G$.

Proposition 3.2:

Let $\{A_\alpha\}_{\alpha \in \beta} \subset FNSG(G)$. Then $\bigcap_{\alpha \in \beta} A_\alpha \in FNSG(G).$

Proposition 3.3:

Let A and B be any two $FNSG$ s of a group G.Then the following conditions are equivalent:

(1) $A \circ B \in FNSG(G).$

(2) $A \circ B = B \circ A$

Proof: Proof is immediate.

Proposition 3.4: Let $A \in FNSG(G)$.Then $A(x^{-1}) = A(x)$,

(i.e.,) $T_A(x^{-1}) = T_A(x), I_A(x^{-1}) = I_A(x), F_A(x^{-1}) = F_A(x)$ and $A(x) \leq A(e)$, (i.e.,) $T_A(x) \leq T_A(e)$, $I_A(x) \leq I_A(e)$, $F_A(x) \geq F_A(e)$ for each $x \in G$, where e is the identity element of G.

Proof:

Let $x \in G$. Then $T_A(x) = T_A((x^{-1})^{-1}) \geq T_A(x^{-1})$, for each $x \in G$.

$I_A(x) = I_A((x^{-1})^{-1}) \geq I_A(x^{-1})$, for each $x \in G$.

$F_A(x) = F_A((x^{-1})^{-1}) \leq F_A(x^{-1})$, for each $x \in G$.

Since $A \in FNSG(G)$, $T_A(x^{-1}) \geq T_A(x), I_A(x^{-1}) \geq I_A(x)$ and $F_A(x^{-1}) \leq F_A(x)$ for each $x \in G$.

Hence $T_A(x^{-1}) = T_A(x), I_A(x^{-1}) = I_A(x), F_A(x^{-1}) = F_A(x)$. (i.e.,) $A(x^{-1}) = A(x)$.

On the other hand ,

$T_A(e) = T_A(xx^{-1}) \geq T_A(x) \wedge T_A(x^{-1}) = T_A(x)$, $I_A(e) = I_A(xx^{-1}) \geq I_A(x) \wedge I_A(x^{-1}) = I_A(x)$,

$F_A(e) = F_A(xx^{-1}) \leq F_A(x) \vee F_A(x^{-1}) = F_A(x)$

Hence $T_A(x) \leq T_A(e)$, $I_A(x) \leq I_A(e)$, $F_A(x) \geq F_A(e)$ for each $x \in G$.(i.e.,) $A(x) \leq A(e)$.

This completes the proof.

Proposition 3.5: If $A \in FNSG(G)$, then

$G_A = \{x \in G : A(x) = A(e), i.e., T_A(x) = T_A(e), I_A(x) = I_A(e), F_A(x) = F_A(e)\}$ is a subgroup of G.

Proof:

Let $x, y \in G_A$. Then $T_A(x) = T_A(e), I_A(x) = I_A(e), F_A(x) = F_A(e)$ and

$T_A(y) = T_A(e), I_A(y) = I_A(e), F_A(y) = F_A(e)$.

Thus $T_A(xy^{-1}) \geq T_A(x) \wedge T_A(y^{-1})$

$= T_A(x) \wedge T_A(y)$ (by Proposition 3.4)

$= T_A(e) \wedge T_A(e) = T_A(e)$.

Similarly , $I_A(xy^{-1}) \geq I_A(e)$.

$F_A(xy^{-1}) \leq F_A(x) \vee F_A(y^{-1})$

$= F_A(x) \vee F_A(y)$ (by Proposition 3.4)

$= F_A(e) \vee F_A(e) = F_A(e)$.

On the other hand , by proposition 3.4 $T_A(xy^{-1}) \leq T_A(e)$, $I_A(xy^{-1}) \leq I_A(e)$, $F_A(xy^{-1}) \geq F_A(e)$ .So $T_A(xy^{-1}) = T_A(e)$, $I_A(xy^{-1}) = I_A(e)$, $F_A(xy^{-1}) = F_A(e)$.(i.e.,) $A(xy^{-1}) = A(e)$ .Thus $xy^{-1} \in G_A$.Hence G_A is a subgroup of G.

Proposition 3.6:

Let $A \in FNSG(G)$. If $A(xy^{-1}) = A(e)$. (i.e.,) $T_A(xy^{-1}) = T_A(e)$, $I_A(xy^{-1}) = I_A(e)$, $F_A(xy^{-1}) = F_A(e)$ for any $x, y \in G$,then $A(x) = A(y)$,(i.e.,) $T_A(x) = T_A(y), I_A(x) = I_A(y), F_A(x) = F_A(y)$.

Proof:

Let $x, y \in G$. Then $T_A(x) = T_A((xy^{-1})y) \geq T_A(xy^{-1}) \wedge T_A(y) = T_A(e) \wedge T_A(y) = T_A(y)$.

On the other hand , by Proposition 3.4 $T_A(x^{-1}) = T_A(x)$,then we have

$T_A(xy^{-1}) = T_A((yx^{-1})^{-1}) = T_A(yx^{-1})$ and thus

$T_A(y) = T_A((yx^{-1})x) \geq T_A(yx^{-1}) \wedge T_A(x) = T_A(xy^{-1}) \wedge T_A(x) = T_A(e) \wedge T_A(x) = T_A(x).$

So $T_A(x) = T_A(y)$ .By the similar arguments, we have $I_A(x) = I_A(y), F_A(x) = F_A(y).$

This completes the proof.

Corollary 3.7 :

Let $A \in FNSG(G)$ .If G_A is a normal subgroup of G , then A is constant on each coset of G_A .

Proof:

Let $a \in G$ and let $x \in aG_A$ .Then there exists $x' \in G_A$ such that $x = ax'$ .Since G_A is normal and $x' \in G_A$, $xa^{-1} = ax'a^{-1} \in G_A$ .Thus $T_A(xa^{-1}) = T_A(e)$, $I_A(xa^{-1}) = I_A(e)$ and $F_A(xa^{-1}) = F_A(e)$ .By Proposition 3.6 , $T_A(x) = T_A(a)$, $I_A(x) = I_A(a)$ and $F_A(x) = F_A(a)$ .So A is constant on aG_A for each $a \in G$. By the similar arguments , we can see that A is constant on $G_A a$ for each $a \in G$. Hence A is constant on each coset of G_A .

Note:

Let H be a subgroup of G .Then the number of right [respectively left] cosets of H in G is called index of H in G and denoted by $[G:H]$. If G is a finite group ,then there can be only a finite number of distinct right [respectively left] cosets of H .Hence the index $[G:H]$ is finite .If G is an infinite group ,then $[G:H]$ may be either finite or infinite.

Corollary 3.8 :

Let $A \in FNSG(G)$ and let G_A be normal .If G_A has a finite index, then A has the sup -property.

Proof:

Let $T \subset G$ .Since G_A has finite index ,let the index $[G:G_A] = n$,say $\mathcal{A} = \{a_1 G_A, ...a_n G_A\}$,where $a_i \in G (i = 1,....n)$ and $a_i G_A \cap a_j G_A = \phi$ for any $i \neq j$ .Let $t \in T$ .Since $G = \cup \mathcal{A} = \bigcup_{i=1}^{n} a_i G_i$,there exists $i \in \{1,2,...n\}$ such that $t \in a_i G_A$ .Since G_A is normal ,by corollary 3.7, $T_A(t) = T_A(a_i), I_A(t) = I_A(a_i), F_A(t) = F_A(a_i)$ on $a_i G_A$,say $T_A(t) = \alpha_i, I_A(t) = \beta_i, F_A(t) = \gamma_i$,where $\alpha_i, \beta_i, \gamma_i \in I$ and $\alpha_i + \beta_i + \gamma_i \leq 3$ .Thus there exists a $t_0 \in T$ such that

$T_A(t_0) = \vee_{t \in T} T_A(t) = \bigvee_{i=1}^{n} \alpha_i, I_A(t_0) = \vee_{t \in T} I_A(t) = \bigvee_{i=1}^{n} \beta_i, F_A(t_0) = \wedge_{t \in T} F_A(t) = \bigwedge_{i=1}^{n} \gamma_i$. Hence A has the sup- property.

Proposition 3.9 :

$A \in FNSG(G)$ if and only if $T_A(xy^{-1}) \geq T_A(x) \wedge T_A(y)$, $I_A(xy^{-1}) \geq I_A(x) \wedge I_A(y)$, $F_A(xy^{-1}) \leq F_A(x) \vee F_A(y)$ for any $x, y \in G$.

Proof:

Proof follows from Definition 3.1 and Proposition 3.4.

Proposition 3.10 :

A group G cannot be the union of two proper $FNSGs$.

Proof:

Let A and B be proper $FNSG$ s of a group G such that $A \cup B = 1_N$, $A \neq 1_N$ and $B \neq 1_N$.

$A \cup B = 1_N \Rightarrow T_A \vee T_B = 1, I_A \vee I_B = 1, F_A \wedge F_B = 0$ .Then $T_A = 1$ or $T_B = 1, I_A = 1$ or $I_B = 1$, $F_A = 0$ or $F_B = 0$ .Since $A \neq 1_N$ and $B \neq 1_N$, $T_A \neq 1 or\, I_A \neq 1 or\, F_A \neq 0$ and $T_B \neq 1 or\, I_B \neq 1 or\, F_B \neq 0$ .In either cases,this is a contradiction.This completes the proof.

Proposition 3.11:

If A is a $FNSGP$ of a group G ,then A is a $FNSG$ of G .

Proof:

Let $x \in G$ .Since G is finite , x has the finite order say n .Then $x^n = e$,where e is the identity of G . Thus $x^{-1} = x^{n-1}$ .Since A is a $FNSGP$ of a group G , $T_A(x^{-1}) = T_A(x^{n-1}) = T_A(x^{n-2}x) \geq T_A(x)$

$I_A(x^{-1}) = I_A(x^{n-1}) = I_A(x^{n-2}x) \geq I_A(x)$

$F_A(x^{-1}) = F_A(x^{n-1}) = F_A(x^{n-2}x) \leq F_A(x)$

Hence A is a $FNSG$ of G .

Proposition 3.12:

Let A be a $FNSG$ of a group G and let $x \in G$ .Then $A(xy) = A(y)$,(i.e.,) $T_A(xy) = T_A(x), I_A(xy) = I_A(x), F_A(xy) = F_A(x)$ for each $y \in G$ if and only if $A(x) = A(e)$.(i.e.,) $T_A(x) = T_A(e), I_A(x) = I_A(e), F_A(x) = F_A(e)$,where e is the identity of G .

Proof:

Suppose $A(xy) = A(y)$ for each $y \in G$ .Then clearly $A(x) = A(e)$.

Conversely, suppose $A(x) = A(e)$ .Then by Proposition 3.4,

$T_A(y) \leq T_A(x), I_A(y) \leq I_A(x), F_A(y) \leq F_A(x)$ for each $y \in G$ .Since A is a $FNSG$ of G ,then $T_A(xy) \geq T_A(x) \wedge T_A(y), I_A(xy) \geq I_A(x) \wedge I_A(y), F_A(xy) \leq F_A(x) \vee F_A(y)$ .Thus

$T_A(xy) \geq T_A(y), I_A(xy) \geq I_A(y), F_A(xy) \leq F_A(y)$ for each $y \in G$.

On the other hand ,by Proposition 3.4,

$T_A(y) = T_A(x^{-1}xy) \geq T_A(x) \wedge T_A(xy), I_A(y) \geq I_A(x) \wedge I_A(xy), F_A(y) \leq F_A(x) \vee F_A(xy)$.

Since $T_A(x) \geq T_A(y), I_A(x) \geq I_A(y), F_A(x) \leq F_A(y)$ for each $y \in G$.

$T_A(x) \wedge T_A(xy) = T_A(xy), I_A(x) \wedge I_A(xy) = I_A(xy), F_A(x) \vee F_A(xy) = F_A(xy)$ .So

$T_A(y) \geq T_A(xy), I_A(y) \geq I_A(xy), F_A(y) \leq F_A(xy)$ for each $y \in G$ .Hence

$T_A(xy) = T_A(y), I_A(xy) = I_A(y), F_A(xy) = F_A(y)$ for each $y \in G$.

Proposition 3.13:

Let $f : G \to G'$ be a group homomorphism, let $A \in FNSG(G)$ and let $B \in FNSG(G')$ .Then the following hold:

(i) If A has the sup- property , then $f(A) \in FNG(G')$.

(ii) $f^{-1}(B) \in FNSG(G)$.

Proof:

(i) By Proposition 5.4 in [4],(i.e.,)Let $f : G \to G''$ be a groupoid homomorphism and let $A \in FNS(G)$ have the sup property .

(1) If $A \in FNSGP(G)$, then $f(A) \in FNSGP(G'')$.

(2) If A is a $FNI(FNLI, FNRI)$ of G , then $f(A)$ is a $FNI(FNLI, FNRI)$ of G'' .

Since $f(A)\in FNSGP(G)$, it is enough to show that

$T_{f(A)}(y^{-1})\geq T_{f(A)}(y), I_{f(A)}(y^{-1})\geq I_{f(A)}(y), F_{f(A)}(y^{-1})\leq F_{f(A)}(y)$ for each $y\in f(G)$.

Let $y\in f(G)$.Then $\phi\neq f^{-1}(y)\subset G.$ Since A has the sup- property , there exists $x_0\in f^{-1}(y)$ such that $T_A(x_0)=\underset{t\in f^{-1}(y)}{\vee}T_A(t), I_A(x_0)=\underset{t\in f^{-1}(y)}{\vee}I_A(t), F_A(x_0)=\underset{t\in f^{-1}(y)}{\wedge}F_A(t)$.

Thus $T_{f(A)}(y^{-1})=f(T_A)(y^{-1})=\underset{t\in f^{-1}(y^{-1})}{\vee}T_A(t)\geq T_A(x_0^{-1})\geq T_A(x_0)=T_{f(A)}(y)$.

$$I_{f(A)}(y^{-1})=f(I_A)(y^{-1})=\underset{t\in f^{-1}(y^{-1})}{\vee}I_A(t)\geq I_A(x_0^{-1})\geq I_A(x_0)=I_{f(A)}(y).$$

$$F_{f(A)}(y^{-1})=f(F_A)(y^{-1})=\underset{t\in f^{-1}(y^{-1})}{\wedge}F_A(t)\leq F_A(x_0^{-1})\leq F_A(x_0)=F_{f(A)}(y).$$

Hence $f(A)\in FNSG(G)$.

(ii) By Proposition 5.1 in [ ],(i.e.,) Let $f:G\to G''$ be a groupoid homomorphism and let $B\in FNS(G'')$

(1) If $B\in FNSGP(G'')$, then $f^{-1}(B)\in FNSGP(G)$.

(2) If B is a $FNI(FNLI, FNRI)$ of G'' then $f^{-1}(B)$ is a $FNI(FNLI, FNRI)$ of G. Since $f^{-1}(B)\in FNSGP(G)$, it is enough to show that $f^{-1}(B)(x^{-1})\geq f^{-1}(B)(x)$ for each $x\in G$.

Let $x\in G$.Then

$T_{f^{-1}(B)}(x^{-1})=f^{-1}(T_B)(x^{-1})=T_B(f(x^{-1}))=T_B(((f(x))^{-1})\geq T_B(f(x))=T_{f^{-1}(B)}(x)$,

$I_{f^{-1}(B)}(x^{-1})=f^{-1}(I_B)(x^{-1})=I_B(f(x^{-1}))=I_B(((f(x))^{-1})\geq I_B(f(x))=I_{f^{-1}(B)}(x)$,and

$F_{f^{-1}(B)}(x^{-1})=f^{-1}(F_B)(x^{-1})=F_B(f(x^{-1}))=F_B(((f(x))^{-1})\leq F_B(f(x))=F_{f^{-1}(B)}(x)$.

Hence $f^{-1}(B)\in FNSG(G)$.

Proposition 3.14:

Let G_p be the cyclic group of prime order p.Then $A\in FNSG(G_p)$ if and only if $A(x)=A(1)\leq A(0)$, (i.e.,) $T_A(x)=T_A(1)\leq T_A(0)$, $I_A(x)=I_A(1)\leq I_A(0)$, $F_A(x)=F_A(1)\geq F_A(0)$ for each $0\neq x\in G_p$.

Proof:

Suppose $A\in FNSG(G_p)$ and let $0\neq x\in G_p$.Then

$T_A(xy)\geq T_A(x)\wedge T_A(xy), I_A(xy)\geq I_A(x)\wedge I_A(xy), F_A(xy)\leq F_A(x)\vee F_A(xy)$ for any $x,y\in G_p$ .Since G_p is the cyclic group of prime order p, $G_p=\{0,1,2,....p-1\}$.Since x is the sum of $1's$ and 1 is the sum of $x's$, $T_A(x)\geq T_A(1)\geq T_A(x), I_A(x)\geq I_A(1)\geq I_A(x)$ and $F_A(x)\leq I_A(1)\leq I_A(x)$ .Thus $T_A(x)=T_A(1), I_A(x)=I_A(1), F_A(x)=F_A(1).$ Since 0 is the identity element of G_p,

$T_A(x)\leq T_A(0), I_A(x)\leq I_A(0), F_A(x)\geq F_A(0)$.Hence the necessary conditions hold.

Conversely, suppose the necessary conditions hold, and let $x,y\in G_p$.Then we have four cases : (i) $x\neq 0, y\neq 0$ *and* $x=y$ (ii) $x\neq 0, y=0$ (iii) $x=0, y\neq 0$ (iv) $x\neq 0, y\neq 0$ *and* $x\neq y$.

Case (i) Suppose $x\neq 0, y\neq 0$ *and* $x=y$ .Then by the hypothesis, $T_A(x)=T_A(y)=T_A(1)\leq T_A(0)$, $I_A(x)=I_A(y)=I_A(1)\leq I_A(0)$ and $F_A(x)=F_A(y)=F_A(1)\geq F_A(0)$.So

$T_A(x-y)=T_A(0)\geq T_A(x)\wedge T_A(y), I_A(x-y)=I_A(0)\geq I_A(x)\wedge I_A(y)$,

$F_A(x-y)=F_A(0)\leq F_A(x)\vee F_A(y)$.

Case (ii) Suppose $x\neq 0$ *and* $y=0$. Since $x-y\neq 0$,by the hypothesis,

$T_A(x-y)=T_A(x)=T_A(1)\leq T_A(0)=T_A(y)$, $I_A(x-y)=I_A(x)=I_A(1)\leq I_A(0)=I_A(y)$ and

$F_A(x-y)=F_A(x)=F_A(1)\geq F_A(0)=F_A(y)$ .So

$T_A(x-y)\geq T_A(x)\wedge T_A(y), I_A(x-y)\geq I_A(x)\wedge I_A(y), F_A(x-y)\leq F_A(x)\vee F_A(y)$.

Case (iii) is the same as Case (ii).

Case (iv) Suppose $x\neq 0, y\neq 0$ *and* $x\neq y$ .Since $x-y\neq 0$, by the hypothesis,

$T_A(x-y)=T_A(x)=T_A(y)=T_A(1)\leq T_A(0)$, $I_A(x-y)=I_A(x)=I_A(y)=I_A(1)\leq I_A(0)$,

$F_A(x-y)=F_A(x)=F_A(y)=F_A(1)\geq F_A(0)$ .So

$T_A(x-y)\geq T_A(x)\wedge T_A(y), I_A(x-y)\geq I_A(x)\wedge I_A(y), F_A(x-y)\leq F_A(x)\vee F_A(y)$ .In all,

$T_A(x-y)\geq T_A(x)\wedge T_A(y), I_A(x-y)\geq I_A(x)\wedge I_A(y), F_A(x-y)\leq F_A(x)\vee F_A(y)$ .Hence by Proposition 3.5, $A\in FNSG(G_p)$.

Proposition 3.15:

The $FNI(FNLI, FNRI)$ in a group G are just the constant mappings.

Proof:

Suppose A is a constant mapping and let $x,y\in G$ .Then $T_A(xy)=T_A(x)=T_A(y)$, $I_A(xy)=I_A(x)=I_A(y)$, $F_A(xy)=F_A(x)=F_A(y)$ .So A is a FNI of G .

Now suppose A is a $FNLI$ of G . Then $T_A(xy)\geq T_A(y), I_A(xy)\geq I_A(y), F_A(xy)\leq F_A(y)$ for any $x,y\in G$ .In particular , $T_A(x)\geq T_A(e), I_A(x)\geq I_A(e), F_A(x)\leq F_A(e)$ for each $x\in G$.

Moreover , $T_A(e)=T_A(x^{-1}x)\geq T_A(x), I_A(e)=I_A(x^{-1}x)\geq I_A(x), F_A(e)=F_A(x^{-1}x)\leq F_A(x)$

For each $x\in G$ .So $T_A(x)=T_A(e), I_A(x)=I_A(e)$, $F_A(x)=F_A(e)$ for each $x\in G$. Hence A is a constant mapping.

Proposition 3.16:

Let A be a $FNSG$ of a group G .Then for each $(\lambda,\mu,\nu)\in I^3$ with $(\lambda,\mu,\nu)\leq A(e)$, (i.e.,) $\lambda\leq T_A(e), \mu\leq I_A(e), \nu\geq F(e)$, $G_A^{(\lambda,\mu,\gamma)}$ is a subgroup of G ,where e is the identity of G .

Proof:

Clearly, $G_A^{(\lambda,\mu,\gamma)}\neq\phi$. Let $x,y\in G_A^{(\lambda,\mu,\gamma)}$ .Then $A(x)\geq(\lambda,\mu,\nu)$ and $A(y)\geq(\lambda,\mu,\nu)$.(i.e.,)

$T_A(x)\geq\lambda, I_A(x)\geq\mu, F_A(x)\leq\nu$ and $T_A(y)\geq\lambda, I_A(y)\geq\mu, F_A(y)\leq\nu$ .Since $A\in FNSG(G)$,

$T_A(xy)\geq T_A(x)\wedge T_A(y)\geq\lambda$, $I_A(xy)\geq I_A(x)\wedge I_A(y)\geq\mu$, $F_A(xy)\leq F_A(x)\vee F_A(y)\leq\nu$ .Thus $A(xy)\geq(\lambda,\mu,\nu)$ .So $xy\in G_A^{(\lambda,\mu,\nu)}$ .On the other hand ,

$T_A(x^{-1})\geq T_A(x)\geq\lambda, I_A(x^{-1})\geq I_A(x)\geq\mu, F_A(x^{-1})\leq F_A(x)\leq\nu$ .Thus $A(x^{-1})\geq(\lambda,\mu,\nu)$ .So $x^{-1}\in G_A^{(\lambda,\mu,\nu)}$ .Hence $G_A^{(\lambda,\mu,\gamma)}$ is a subgroup of G .

Proposition 3.17:

Let A be a fuzzy neutrosophic set in a group G such that $G_A^{(\lambda,\mu,\gamma)}$ is a subgroup of G for each $(\lambda,\mu,\nu)\in I^3$ with $(\lambda,\mu,\nu)\leq A(e)$ .Then A is a $FNSG$ of a group G .

Proof:

For any $x, y \in G$,let $A(x) = (t_1, s_1, r_1)$ and let $A(y) = (t_2, s_2, r_2)$ .Then clearly, $x \in G_A^{(t_1,s_1,r_1)}$ and $y \in G_A^{(t_2,s_2,r_2)}$ .Suppose $t_1 < t_2$, $s_1 < s_2$ and $r_1 > r_2$ .Then $G_A^{(t_2,s_2,r_2)} \subset G_A^{(t_1,s_1,r_1)}$ .Thus $y \in G_A^{(t_1,s_1,r_1)}$.
Since $G_A^{(t_1,s_1,r_1)}$ is a subgroup of G , $xy \in G_A^{(t_1,s_1,r_1)}$ .Then $A(xy) = (t_1, s_1, r_1)$.
(i.e.,) $T_A(xy) \geq t_1, I_A(xy) \geq s_1, F_A(xy) \leq r_1$.
So $T_A(xy) \geq T_A(x) \wedge T_A(y), I_A(xy) \geq I_A(x) \wedge I_A(y)$, $F_A(xy) \leq F_A(x) \vee F_A(y)$ .For each $x \in G$, let $A(xy) = (\lambda, \mu, \nu)$ .Then $x \in G_A^{(\lambda,\mu,\gamma)}$ .Since $G_A^{(\lambda,\mu,\gamma)}$ is a subgroup of G , $x^{-1} \in G_A^{(\lambda,\mu,\gamma)}$ .So $A(x^{-1}) \geq (\lambda, \mu, \nu)$.(i.e.,) $T_A(x^{-1}) \geq T_A(x), I_A(x^{-1}) \geq I_A(x), F_A(x^{-1}) \leq F_A(x)$ .Hence A is a $FNSG$ of a group G .

Proposition 3.18:

Let A be a fuzzy neutrosophic set in X and let $(\lambda_1, \mu_1, \nu_1), (\lambda_2, \mu_2, \nu_2) \in \mathrm{Im}(A)$ .If $\lambda_1 < \lambda_2, \mu_1 < \mu_2, \nu_1 > \nu_2$ then $A^{(\lambda_1,\mu_1,\nu_1)} \supset A^{(\lambda_2,\mu_2,\nu_2)}$.

Result 3.19: Let A be a fuzzy neutrosophic set in a group G .Then A is a $FNSG$ of G if and only if $A^{(\lambda,\mu,\nu)}$ is a subgroup of G for each $(\lambda, \mu, \nu) \in \mathrm{Im}(A)$.

Definition 3.20:

Let A be a $FNSG$ of group G and let $(\lambda, \mu, \nu) \in \mathrm{Im}(A)$ .Then the subgroup $A^{(\lambda,\mu,\nu)}$ is called a $(\lambda, \mu, \nu) -$ level subgroup of A .

Lemma 3.21:

Let A be any fuzzy neutrosophic set in X.Then $T_A(x) = \vee\{\lambda : x \in A^{(\lambda,\mu,\nu)}\}$, $I_A(x) = \vee\{\mu : x \in A^{(\lambda,\mu,\nu)}\}, F_A(x) = \wedge\{\nu : x \in A^{(\lambda,\mu,\nu)}\}$ where $x \in X$ and $(\lambda, \mu, \nu) \in I^3$ with $\lambda + \mu + \nu \leq 3$.

Proof: Let $\alpha = \vee\{\lambda : x \in A^{(\lambda,\mu,\nu)}\}, \beta = \vee\{\mu : x \in A^{(\lambda,\mu,\nu)}\}, \gamma = \wedge\{\nu : x \in A^{(\lambda,\mu,\nu)}\}$ and let $\varepsilon > 0$ be arbitrary. Then $\alpha - \varepsilon < \vee\{\lambda : x \in A^{(\lambda,\mu,\nu)}\}, \beta - \varepsilon < \vee\{\mu : x \in A^{(\lambda,\mu,\nu)}\}, \gamma + \varepsilon > \wedge\{\nu : x \in A^{(\lambda,\mu,\nu)}\}$ .Thus there exist $\lambda, \mu, \nu \in I$ with $\lambda + \mu + \nu \leq 3$ such that $x \in A^{(\lambda,\mu,\nu)}$, $\alpha - \varepsilon < \lambda, \beta - \varepsilon < \mu, \gamma + \varepsilon > \nu$ .Since $x \in A^{(\lambda,\mu,\nu)}$, $T_A(x) \geq \lambda, I_A(x) \geq \mu, F_A(x) \leq \nu$ .Thus $T_A(x) > \alpha - \varepsilon, I_A(x) > \beta - \varepsilon, F_A(x) < \gamma + \varepsilon$ .Since $\varepsilon > 0$ is arbitrary , $T_A(x) \geq \alpha, I_A(x) \geq \beta, F_A(x) \leq \gamma$.

We now show that $T_A(x) \leq \alpha, I_A(x) \leq \beta, F_A(x) \geq \gamma$ .Suppose $T_A(x) = t_1, I_A(x) = t_2, F_A(x) = t_3$ .Then $t_1 + t_2 + t_3 \leq 3$.Thus $x \in A^{(t_1,t_2,t_3)}$ .So $t_1 \in \{\lambda : x \in A^{(\lambda,\mu,\nu)}\}$, $t_2 \in \{\mu : x \in A^{(\lambda,\mu,\nu)}\}$, $t_3 \in \{\nu : x \in A^{(\lambda,\mu,\nu)}\}$.Thus, $t_1 \leq \vee\{\lambda : x \in A^{(\lambda,\mu,\nu)}\} t_2 \leq \vee\{\mu : x \in A^{(\lambda,\mu,\nu)}\}, t_3 \geq \wedge\{\nu : x \in A^{(\lambda,\mu,\nu)}\}$.(i.e.,) $T_A(x) \leq \alpha, I_A(x) \leq \beta, F_A(x) \geq \gamma$.

This completes the proof.

We shall denote by (A) the $FNSG$ generated by the fuzzy neutrosophic set A in G .We shall use the same notation $(A^{(\lambda,\mu,\nu)})$ for the ordinary subgroup of the group generated by the level subset $A^{(\lambda,\mu,\nu)}$.

Theorem 3.22:

Let G be a group and let $A \in FNS(G)$ .Let $A^* \in FNS(G)$ be defined as follows :for each $x \in G$, $T_{A^*}(x) = \vee\{\lambda : x \in A^{(\lambda,\mu,\nu)}\} I_{A^*}(x) = \vee\{\mu : x \in A^{(\lambda,\mu,\nu)}\}, F_{A^*}(x) = \wedge\{\nu : x \in A^{(\lambda,\mu,\nu)}\}$,where

$\lambda, \mu, \nu \in I$ with $\lambda + \mu + \nu \leq 3$.Then A^* is a $FNSG$ of G such that $A^* = \bigcap\{B \in FNSG(G) : A \subset B\}$.In this case , A^* is called the fuzzy neutrosophic subgroup generated by A in G and denoted by (A).

Proof:

Let $(t_1, t_2, t_3) \in \text{Im}(A^*)$ and let $\alpha = t_1 - \frac{1}{n}, \beta = t_2 - \frac{1}{n}, \gamma = t_3 + \frac{1}{n}$ where n is any sufficiently large positive number .Let $x \in G$ .Suppose $x \in A^{*(t_1.t_2,t_3)}$ .Then $T_{A^*}(x) \geq t_1, I_{A^*}(x) \geq t_2, F_{A^*}(x) \leq t_3$ .Thus there exist $\lambda, \mu, \nu \in I$ with $\lambda + \mu + \nu \leq 3$ such that $\lambda > \alpha, \mu > \beta, \nu < \gamma$ and $x \in A^{(\lambda,\mu,\nu)}$. $(\alpha, \beta, \gamma) < (\lambda, \mu, \nu)$ and $\alpha + \beta + \gamma \leq 3, A^{(\lambda,\mu,\nu)} \subset A^{(\alpha,\beta,\gamma)}$.So $x \in A^{(\alpha,\beta,\gamma)}$.(i.e.,) $x \in \left(A^{(\alpha,\beta,\gamma)}\right)$ .Now suppose $x \in \left(A^{(\alpha,\beta,\gamma)}\right)$.Then

$\alpha \in \left\{\lambda : x \in \left(A^{(\lambda,\mu,\nu)}\right)\right\}, \beta \in \left\{\mu : x \in \left(A^{(\lambda,\mu,\nu)}\right)\right\}, \gamma \in \left\{\nu : x \in \left(A^{(\lambda,\mu,\nu)}\right)\right\}$.Thus $\alpha \leq \vee\left\{\lambda : x \in \left(A^{(\lambda,\mu,\nu)}\right)\right\}$,

$\beta \leq \vee\left\{\mu : x \in \left(A^{(\lambda,\mu,\nu)}\right)\right\}, \gamma \geq \wedge\left\{\nu : x \in \left(A^{(\lambda,\mu,\nu)}\right)\right\}$.So

$t_1 - \frac{1}{n} \leq T_{A^*}(x), t_2 - \frac{1}{n} \leq I_{A^*}(x), t_3 + \frac{1}{n} \geq F_{A^*}(x)$.

(i.e.,) $t_1 \leq T_{A^*}(x), t_2 \leq I_{A^*}(x), t_3 \geq F_{A^*}(x)$.

Hence $x \in A^{*(t_1.t_2,t_3)}$. (i.e.,) $\left(A^{(\alpha,\beta,\gamma)}\right) \subset A^{*(t_1.t_2,t_3)}$.

Hence $A^{*(t_1.t_2,t_3)} = \left(A^{(\alpha,\beta,\gamma)}\right)$.Since $\left(A^{(\alpha,\beta,\gamma)}\right)$ is a subgroup of G , $A^{*(t_1.t_2,t_3)}$ is a subgroup of G. By Result 3.19, A^* is a $FNSG$ of G .

Now , we show that $A \subset A^*$ .Let $x \in G$ .Then by Lemma 3.21, $T_A(x) = \vee\left\{\lambda : x \in \left(A^{(\lambda,\mu,\nu)}\right)\right\}$,

$I_A(x) = \vee\left\{\mu : x \in \left(A^{(\lambda,\mu,\nu)}\right)\right\}, F_A(x) = \wedge\left\{\nu : x \in \left(A^{(\lambda,\mu,\nu)}\right)\right\}$.Thus

$T_A(x) \leq \vee\left\{\lambda : x \in \left(A^{(\lambda,\mu,\nu)}\right)\right\}, I_A(x) \leq \vee\left\{\mu : x \in \left(A^{(\lambda,\mu,\nu)}\right)\right\}, F_A(x) \geq \wedge\left\{\nu : x \in \left(A^{(\lambda,\mu,\nu)}\right)\right\}$.So $A \subset A^*$.

Finally , let B be any $FNSG$ of G such that $A \subset B$ .We show that $A^* \subset B$. Let $x \in G$ and $A^*(x) = (t_1, t_2, t_3)$ .Then $A^{*(t_1.t_2,t_3)} = \left(A^{(\alpha,\beta,\gamma)}\right)$,where $\alpha = t_1 - \frac{1}{n}, \beta = t_2 - \frac{1}{n}, \gamma = t_3 + \frac{1}{n}$, and n is any sufficiently large positive integer. Thus $x \in \left(A^{(\alpha,\beta,\gamma)}\right)$.So $x = a_1 a_2 ......a_m$,where a_i or a_i^{-1} belongs to $A^{(\alpha,\beta,\gamma)}\ (i = 1,....m)$.

On the other hand, $T_B(x) = T_B(a_1 a_2 ......a_m)$

$\geq T_B(a_1) \wedge T_B(a_2) \wedge T_B(a_3)...... \wedge T_B(a_m)$

$\geq T_A(a_1) \wedge T_A(a_2) \wedge ..... \wedge T_A(a_m) \geq \alpha = t_1 - \frac{1}{n}$.

Similarly $I_B(x) = I_B(a_1 a_2 ......a_m)$

$\geq I_B(a_1) \wedge I_B(a_2) \wedge I_B(a_3)...... \wedge I_B(a_m)$

$\geq I_A(a_1) \wedge I_A(a_2) \wedge ..... \wedge I_A(a_m) \geq \beta = t_2 - \frac{1}{n}$ and

$F_B(x) = F_B(a_1 a_2 ......a_m)$

$\leq F_B(a_1) \vee F_B(a_2) \vee F_B(a_3)...... \vee F_B(a_m)$

$\leq F_A(a_1) \vee F_A(a_2) \vee ..... \vee F_A(a_m) \leq \gamma = t_3 + \frac{1}{n}$.

Since n is sufficiently large positive integer , $T_B(x) \geq t_1, I_B(x) \geq t_2, F_B(x) \leq t_3$ .So $A^* \subset B$. Hence $A^* = \bigcap\{B \in FNSG(G) : A \subset B\}$.This completes the proof.

Lemma 3.23:

Let G be a finite group.Suppose there exists a $FNSG$ A of G satisfying the following conditions:for any $x, y \in G$,

$(i) A(x) = A(y) \Rightarrow (x) = (y)$

$(ii) T_A(x) > T_A(y), I_A(x) > I_A(y), F_A(x) < F_A(y) \Rightarrow (x) \subset (y)$. Then G is cyclic.

Proof:

Suppose A is constant on G .Then $A(x) = A(y)$ for any $x, y \in G$ .By the condition (i) , $(x) = (y)$ .So $G = (x)$ .Now suppose A is not constant on G .Let $\mathrm{Im}(A) = \{(t_0, s_0, r_0), (t_1, s_1, r_1), ...(t_n, s_n, r_n)\}$,where $t_0 > t_1 > ..... > t_n, s_0 > s_1 > ..... > s_n, r_0 < r_1 < ..... < r_n$ .Then by Proposition 3.18 and Result 3.19, we obtain the chain of level subgroups of $A : A^{(t_0,s_0,r_0)} \subset A^{(t_1,s_1,r_1)} \subset ..... \subset A^{(t_n,s_n,r_n)} = G$.

Let $x \in G - A^{(t_{n-1},s_{n-1},r_{n-1})}$ .We show that $G = (x)$ .Let $g \in G - A^{(t_{n-1},s_{n-1},r_{n-1})}$. Since $t_0 > t_1 > ..... > t_n, s_0 > s_1 > ..... > s_n, r_0 < r_1 < ..... < r_n$, $A(g) = A(x) = A^{(t_{n-1},s_{n-1},r_{n-1})}$. By the condition (i) , $(g) = (x)$ .Thus $G - A^{(t_{n-1},s_{n-1},r_{n-1})} \subset (x)$ .Now let $g \in A^{(t_{n-1},s_{n-1},r_{n-1})}$ .Then $T_A(g) \geq t_{n-1} > t_n = T_A(x), I_A(g) \geq s_{n-1} > s_n = I_A(x), F_A(g) \leq r_{n-1} < r_n = F_A(x)$ .By the condition (ii) , $(g) \subset (x)$ .Thus $A^{(t_{n-1},s_{n-1},r_{n-1})} \subset (x)$ .So $G = (x)$ .Hence in either case , G is cyclic.

Lemma 3.24:

Let G be a cyclic group of order p^n ,where p is prime .Then there exists a $FNSG$ A of G satisfying the following conditions: for any $x, y \in G$,

$(i) A(x) = A(y) \Rightarrow (x) = (y)$

$(ii) T_A(x) > T_A(y), I_A(x) > I_A(y), F_A(x) < F_A(y) \Rightarrow (x) \subset (y)$.

Proof:

Consider the following chain of subgroups of G :

$(e) = G_0 \subset G_1 \subset .... \subset G_{n-1} \subset G_n = G$,where G_i is the subgroup of G generated by an element of order $p^i, i = 0,1,...n$ and e is the identity of G . We define a complex mapping $A = (T_A, I_A, F_A) : G \to I^3$ as follows: for each $x \in G, A(e) = (t_0, s_0, r_0)$ and $A(x) = (t_i, s_i, r_i)$ *if* $x \in G_i - G_{i-1}$ for any $i = 1,2,...n,$ where $t_i, s_i, r_i \in I$ such that $t_i + s_i + r_i \leq 3,$ $t_0 > t_1 > ..... > t_n, s_0 > s_1 > ..... > s_n, r_0 < r_1 < ..... < r_n$ .Then we can easily check that A is a $FNSG$ of G satisfying the conditions (i) and (ii).

From Lemma 3.23 and Lemma 3.24 we obtain the following :

Theorem 3.25:

Let G be a group of order p^n . Then G is cyclic if and only if there exists a $FNSG$ A of G such that for any $x, y \in G$,

$(i) A(x) = A(y) \Rightarrow (x) = (y)$

$(ii) T_A(x) > T_A(y), I_A(x) > I_A(y), F_A(x) < F_A(y) \Rightarrow (x) \subset (y).$

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