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Generalization of Neutrosophic Rings and Neutrosophic Fields

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Abstract. In this paper we present the generalization of neutrosophic rings and neutrosophic fields. We also extend the neutrosophic ideal to neutrosophic biideal and neutrosophic N-ideal. We also find some new type of notions which are related to the strong or pure part of neutrosophy. We have given sufficient amount of examples to illustrate the theory of neutrosophic birings, neutrosophic N-rings with neutrosophic bifields and neutrosophic N-fields and display many properties of them in this paper.

Keywords: Neutrosophic ring, neutrosophic field, neutrosophic biring, neutrosophic N-ring, neutrosophic bifield neutrosophic N-field.

1 Introduction

Neutrosophy is a new branch of philosophy which studies the origin and features of neutralities in the nature. Florentin Smarandache in 1980 firstly introduced the concept of neutrosophic logic where each proposition in neutrosophic logic is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F so that this neutrosophic logic is called an extension of fuzzy logic. In fact neutrosophic set is the generalization of classical sets, conventional fuzzy set $[1]$, intuitionistic fuzzy set $[2]$ and interval valued fuzzy set $[3]$. This mathematical tool is used to handle problems like imprecise, indeterminacy and inconsistent data etc. By utilizing neutrosophic theory, Vasantha Kandasamy and Florentin Smarandache dig out neutrosophic algebraic structures in $[11]$. Some of them are neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loop, neutrosophic groupoids, and neutrosophic bigroupoids and so on.

In this paper we have tried to develop the the generalization of neutrosophic ring and neutrosophic field in a logical manner. Firstly, preliminaries and basic concepts are given for neutrosophic rings and neutrosophic fields. Then we presented the newly defined notions and results in neutrosophic birings and neutrosophic N-rings, neutrosophic bifields and neutosophic N-fields. Various types of neutrosophic biideals and neutrosophic N-ideal are defined and elaborated with the help of examples.

2 Fundamental Concepts

In this section, we give a brief description of neutrosophic rings and neutrosophic fields.

Definition: Let R be a ring. The neutrosophic ring $\langle R\cup I\rangle$ is also a ring generated by R and I under the operation of R, where I is called the neutrosophic element with property $I^2=I$. For an integer n, $n+I$ and nI are neutrosophic elements and $0.I=0$. I^{-1}, the inverse of I is not defined and hence does not exist.

Definition: Let $\langle R\cup I\rangle$ be a neutrosophic ring. A proper subset P of $\langle R\cup I\rangle$ is called a neutosophic subring if P itself a neutrosophic ring under the operation of $\langle R\cup I\rangle$.

Definition: Let T be a non-empty set with two binary operations $*$ and $\circ$. T is said to be a pseudo neutrosophic ring if

1. T contains element of the form $a+bI$ $(a,b$ are reals and $b\neq 0$ for atleast one value$)$.

2. $(T,*)$ is an abelian group.
3. $(T,\circ)$ is a semigroup.

Definition: Let $\langle R\cup I\rangle$ be a neutrosophic ring. A non-empty set P of $\langle R\cup I\rangle$ is called a neutrosophic ideal of $\langle R\cup I\rangle$ if the following conditions are satisfied.

1. P is a neutrosophic subring of $\langle R\cup I\rangle$, and
2. For every $p\in P$ and $r\in\langle R\cup I\rangle$, pr and $rp\in P$.

Definition: Let K be a field. The neutrosophic field generated by $\langle K\cup I\rangle$ which is denoted by $K(I)=\langle K\cup I\rangle$.

Definition: Let $K(I)$ be a neutrosophic field. A proper subset P of $K(I)$ is called a neutrosophic sufield if P itself a neutrosophic field.

3 **Neutrosophic Biring**

Definition **. Let $(BN(\mathrm{R}),*,\circ)$ be a non-empty set with two binary operations $*$ and $\circ$. $(BN(\mathrm{R}),*,\circ)$ is said to be a neutrosophic biring if $BN(\mathrm{Rs})=R_1\cup R_2$ where atleast one of $(\mathrm{R}_1,*,\circ)$ or $(\mathrm{R}_2,*,\circ)$ is a neutrosophic ring and other is just a ring. R_1 and R_2 are proper subsets of $BN(\mathrm{R})$.

Example 2. Let $BN(\mathrm{R})=(\mathrm{R}_1,*,\circ)\cup(\mathrm{R}_2,*,\circ)$ where $(\mathrm{R}_1,*,\circ)=(\langle\mathbb{Z}\cup I\rangle,+,\times)$ and $(\mathrm{R}_2,*,\circ)=(\mathbb{Q},+,\times)$. Clearly $(\mathrm{R}_1,*,\circ)$ is a neutrosophic ring under addition and multiplication. $(\mathrm{R}_2,*,\circ)$ is just a ring. Thus $(BN(\mathrm{R}),*,\circ)$ is a neutrosophic biring.

Theorem: Every neutrosophic biring contains a corresponding biring.

Definition: Let $BN(\mathrm{R})=(\mathrm{R}_1,*,\circ)\cup(\mathrm{R}_2,*,\circ)$ be a neutrosophic biring. Then $BN(\mathrm{R})$ is called a commutative neutrosophic biring if each $(\mathrm{R}_1,*,\circ)$ and $(\mathrm{R}_2,*,\circ)$ is a commutative neutrosophic ring.

Example 2. Let $BN(\mathrm{R})=(\mathrm{R}_1,*,\circ)\cup(\mathrm{R}_2,*,\circ)$ where $(\mathrm{R}_1,*,\circ)=(\langle\mathbb{Z}\cup I\rangle,+,\times)$ and $(\mathrm{R}_2,*,\circ)=(\mathbb{Q},+,\times)$. Clearly $(\mathrm{R}_1,*,\circ)$ is a commutative neutrosophic ring and $(\mathrm{R}_2,*,\circ)$ is also a commutative ring. Thus $(BN(\mathrm{R}),*,\circ)$ is a commutative neutrosophic biring.

Definition: Let $BN(\mathrm{R})=(\mathrm{R}_1,*,\circ)\cup(\mathrm{R}_2,*,\circ)$ be a neutrosophic biring. Then $BN(\mathrm{R})$ is called a pseudo neutrosophic biring if each $(\mathrm{R}_1,*,\circ)$ and $(\mathrm{R}_2,*,\circ)$ is a pseudo neutrosophic ring.

Example 2. Let $BN(\mathrm{R})=(\mathrm{R}_1,+,\times)\cup(\mathrm{R}_2,+,\circ)$ where $(\mathrm{R}_1,+,\times)=\{0,I,2I,3I\}$ is a pseudo neutrosophic ring under addition and multiplication modulo 4 and $(\mathrm{R}_2,+,\times)=\{0,\pm1\mathrm{I},\pm2\,\mathrm{I},\pm3\mathrm{I},\ldots\}$ is another pseudo neutrosophic ring. Thus $(BN(\mathrm{R}),+,\times)$ is a pseudo neutrosophic biring.

Theorem: Every pseudo neutrosophic biring is trivially a neutrosophic biring but the converse may not be true.

Definition 8. Let $(BN(\mathrm{R})=R_1\cup R_2;*,\circ)$ be a neutrosophic biring. A proper subset $(T,*,\circ)$ is said to be a neutrosophic subbiring of $BN(\mathrm{R})$ if

1) $T=T_1\cup T_2$ where $T_1=R_1\cap T$ and $T_2=R_2\cap T$ and
2) At least one of $(T_1,\circ)$ or $(T_2,*)$ is a neutrosophic ring.

Example: Let $BN(\mathrm{R})=(\mathrm{R}_1,*,\circ)\cup(\mathrm{R}_2,*,\circ)$ where $(\mathrm{R}_1,*,\circ)=(\langle\mathbb{R}\cup I\rangle,+,\times)$ and $(\mathrm{R}_2,*,\circ)=(\mathbb{C},+,\times)$. Let $P=P_1\cup P_2$ be a proper subset of $BN(R)$, where $P_1=(\mathbb{Q},+,\times)$ and $P_2=(\mathbb{R},+,\times)$. Clearly $(P,+,\times)$ is a neutrosophic subbiring of $BN(R)$.

Definition: If both $(\mathrm{R}_1,*)$ and $(\mathrm{R}_2,\circ)$ in the above definition ** are neutrosophic rings then we call

$(BN(\mathrm{R}),*,\circ)$ to be a strong neutrosophic biring.

Example 2. Let $BN(\mathrm{R})=(\mathrm{R}_1,*,\circ)\cup(\mathrm{R}_2,*,\circ)$ where $(\mathrm{R}_1,*,\circ)=(\langle\mathbb{Z}\cup I\rangle,+,\times)$ and $(\mathrm{R}_2,*,\circ)=(\langle\mathbb{Q}\cup I\rangle,+,\times)$. Clearly R_1 and R_2 are neutrosophic rings under addition and multiplication. Thus $(BN(\mathrm{R}),*,\circ)$ is a strong neutrosophic biring.

Theorem. All strong neutrosophic birings are trivially neutrosophic birings but the converse is not true in general.

To see the converse, we take the following Example.

Example 2. Let $BN(\mathrm{R})=(\mathrm{R}_1,*,\circ)\cup(\mathrm{R}_2,*,\circ)$ where $(\mathrm{R}_1,*,\circ)=(\langle\mathbb{Z}\cup I\rangle,+,\times)$ and $(\mathrm{R}_2,*,\circ)=(\mathbb{Q},+,\times)$. Clearly $(\mathrm{R}_1,*,\circ)$ is a neutrosophic ring under addition and multiplication. $(\mathrm{R}_2,*,\circ)$ is just a ring. Thus $(BN(\mathrm{R}),*,\circ)$ is a neutrosophic biring but not a strong neutrosophic biring.

Remark: A neutrosophic biring can have subbirings, neutrosophic subbirings, strong neutrosophic subbirings and pseudo neutrosohic subbirings.

Definition 8. Let $(BN(\mathrm{R})=R_1\cup R_2;*,\circ)$ be a neutrosophic biring and let $(T,*,\circ)$ is a neutrosophic subbiring of $BN(\mathrm{R})$. Then $(T,*,\circ)$ is called a neutrosophic biideal of $BN(R)$ if

1) $T=T_1\cup T_2$ where $T_1=R_1\cap T$ and $T_2=R_2\cap T$ and
2) At least one of $(T_1,*,\circ)$ or $(T_2,*,\circ)$ is a neutrosophic ideal.

If both $(T_1,*,\circ)$ and $(T_2,*,\circ)$ in the above definition are neutrosophic ideals, then we call $(T,*,\circ)$ to be a strong neutrosophic biideal of $BN(R)$.

Example: Let $BN(\mathrm{R})=(\mathrm{R}_1,*,\circ)\cup(\mathrm{R}_2,*,\circ)$ where $(\mathrm{R}_1,*,\circ)=(\langle\mathbb{Z}_{12}\cup I\rangle,+,\times)$ and $(\mathrm{R}_2,*,\circ)=(\mathbb{Z}_{16},+,\times)$. Let $P=P_1\cup P_2$ be a neutrosophic subbiring of $BN(R)$, where $P_1=\{0,6,2I,4I,6I,8I,10I,6+2I,...,6+10I\}$ and $P_2=\{02I,4I,6I,8I,10I,12I,14I\}$. Clearly $(P,+,\times)$ is a neutrosophic biideal of $BN(R)$.

Theorem: Every neutrosophic biideal is trivially a neutrosophic subbiring but the converse may not be true.

Theorem: Every strong neutrosophic biideal is trivially a neutrosophic biideal but the converse may not be true.

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Definition 8. Let $(BN(\mathrm{R})=R_1\cup R_2;*,\circ)$ be a neutrosophic biring and let $(T,*,\circ)$ is a neutrosophic subbiring of $BN(\mathrm{R})$. Then $(T,*,\circ)$ is called a pseudo neutrosophic biideal of $BN(R)$ if

1. $T=T_1\cup T_2$ where $T_1=R_1\cap T$ and $T_2=R_2\cap T$ and
2. $(T_1,*,\circ)$ and $(T_2,*,\circ)$ are pseudo neutrosophic ideals.

Theorem: Every pseudo neutrosophic biideal is trivially a neutrosophic subbiring but the converse may not be true.

Theorem: Every pseudo neutrosophic biideal is trivially a strong neutrosophic subbiring but the converse may not be true.

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Theorem: Every pseudo neutrosophic biideal is trivially a strong neutrosophic biideal but the converse may not be true.

4 Neutrosophic N-ring

Definition*. Let $\{\mathrm{N}(\mathrm{R}),*_1,...,*_2,\circ_1,\circ_2,...,\circ_N\}$ be a non-empty set with two N-binary operations defined on it. We call $N(R)$ a neutrosophic N-ring (N a positive integer) if the following conditions are satisfied.

1) $\mathrm{N}(\mathrm{R})=R_1\cup R_2\cup...\cup R_N$ where each R_i is a proper subset of $\mathrm{N}(\mathrm{R})$ i.e. $R_i\not\subset R_j$ or $R_j\not\subset R_i$ if $i\neq j$.

2) $(R_i, *_i, \circ_i)$ is either a neutrosophic ring or a ring for $i = 1, 2, 3, ..., N$.

Example 2. Let $N(R) = (R_1, *, \circ) \cup (R_2, *, \circ) \cup (R_3, *, \circ)$ where $(R_1, *, \circ) = (\langle \mathbb{Z} \cup I \rangle, +, \times)$, $(R_2, *, \circ) = (\mathbb{Q}, +, \times)$ and $(R_3, *, \circ) = (Z_{12}, +, \times)$. Thus $(N(R), *, \circ)$ is a neutrosophic N-ring.

Theorem: Every neutrosophic N-ring contains a corresponding N-ring.

Definition: Let $N(R) = \{R_1 \cup R_2 \cup .... \cup R_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ be a neutrosophic N-ring. Then $N(R)$ is called a pseudo neutrosophic N-ring if each $(R_i, *_i)$ is a pseudo neutrosophic ring where $i = 1, 2, ..., N$.

Example 2. Let $N(R) = (R_1, +, \times) \cup (R_2, +, \times) \cup (R_3, +, \times)$ where $(R_1, +, \times) = \{0, I, 2I, 3I\}$ is a pseudo neutrosophic ring under addition and multiplication modulo 4, $(R_2, +, \times) = \{0, \pm 1I, \pm 2I, \pm 3I, ...\}$ is a pseudo neutrosophic ring and $(R_3, +, \times) = \{0, \pm 2I, \pm 4I, \pm 6I ...\}$. Thus $(N(R), +, \times)$ is a pseudo neutrosophic 3-ring.

Theorem: Every pseudo neutrosophic N-ring is trivially a neutrosophic N-ring but the converse may not be true.

Definition. If all the N-rings $(R_i, *_i)$ in definition * are neutrosophic rings (i.e. for $i = 1, 2, 3, ..., N$) then we call $N(R)$ to be a neutrosophic strong N-ring.

Example 2. Let $N(R) = (R_1, *, \circ) \cup (R_2, *, \circ) \cup (R_3, *, \circ)$ where $(R_1, *, \circ) = (\langle \mathbb{Z} \cup I \rangle, +, \times)$, $(R_2, *, \circ) = (\langle \mathbb{Q} \cup I \rangle, +, \times)$ and $(R_3, *, \circ) = (\langle \mathbb{Z}_{12} \cup I \rangle, +, \times)$. Thus $(N(R), *, \circ)$ is a strong neutrosophic N-ring.

Theorem: All strong neutrosophic N-rings are neutrosophic N-rings but the converse may not be true.

Definition 13. Let $N(R) = \{R_1 \cup R_2 \cup .... \cup R_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ be a neutrosophic N-ring. A proper subset $P = \{P_1 \cup P_2 \cup .... P_N, *_1, *_2, ..., *_N\}$ of $N(R)$ is said to be a neutrosophic N-subring if $P_i = P \cap R_i, i = 1, 2, ..., N$ are subrings of R_i in which atleast some of the subrings are neutrosophic subrings.

Example: Let $N(R) = (R_1, *, \circ) \cup (R_2, *, \circ) \cup (R_3, *, \circ)$ where $(R_1, *, \circ) = (\langle \mathbb{R} \cup I \rangle, +, \times)$, $(R_2, *, \circ) = (\mathbb{C}, +, \times)$ and $(R_2, *, \circ) = (Z_{10}, +, \times)$ Let $P = P_1 \cup P_2 \cup P_3$ be a proper subset of $N(R)$, where $P_1 = (\mathbb{Q}, +, \times)$, $P_2 = (\mathbb{R}, +, \times)$ and $(R_3, *, \circ) = \{0, 2, 4, 6, 8, I, 2I, 4I, 6I, 8I\}$. Clearly $(P, +, \times)$ is a neutrosophic sub 3-ring of $N(R)$.

Definition 14. Let $N(R) = \{R_1 \cup R_2 \cup .... \cup R_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ be a neutrosophic N-ring. A proper subset $T = \{T_1 \cup T_2 \cup .... \cup T_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ of $N(R)$ is said to be a neutrosophic strong sub N-ring if each $(T_i, *_i)$ is a neutrosophic subring of $(R_i, *_i, \circ_i)$ for $i = 1, 2, ..., N$ where $T_i = R_i \cap T$.

Remark: A strong neutrosophic su N-ring is trivially a neutrosophic sub N-ring but the converse is not true.

Remark: A neutrosophic N-ring can have sub N-rings, neutrosophic sub N-rings, strong neutrosophic sub N-rings and pseudo neutrosohic sub N-rings.

Definition 16. Let $N(R) = \{R_1 \cup R_2 \cup .... \cup R_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ be a neutrosophic N-ring. A proper subset $P = \{P_1 \cup P_2 \cup .... \cup P_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ where $P_t = P \cap R_t$ for $t = 1, 2, ..., N$ is said to be a neutrosophic N-ideal of $N(R)$ if the following conditions are satisfied.

1) Each it is a neutrosophic subring of $R_t, t = 1, 2, ..., N$.

2) Each it is a two sided ideal of R_t for $t = 1, 2, ..., N$.

If $(P_i, *_i, \circ_i)$ in the above definition are neutrosophic ideals, then we call $(P_i, *_i, \circ_i)$ to be a strong neutrosophic N-ideal of $N(R)$.

Theorem: Every neutrosophic N-ideal is trivially a neutrosophic sub N-ring but the converse may not be true.

Theorem: Every strong neutrosophic N-ideal is trivially a neutrosophic N-ideal but the converse may not be true.

Theorem: Every strong neutrosophic N-ideal is trivially a neutrosophic sub N-ring but the converse may not be true.

Theorem: Every strong neutrosophic biideal is trivially a strong neutrosophic subbiring but the converse may not be true.

Definition 16. Let $N(R) = \{R_1 \cup R_2 \cup .... \cup R_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ be a neutrosophic N-ring. A proper subset $P = \{P_1 \cup P_2 \cup .... \cup P_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ where $P_t = P \cap R_t$ for $t = 1, 2, ..., N$ is said to be a pseudo neutrosophic N-ideal of $N(R)$ if the following conditions are satisfied.

1. Each it is a neutrosophic subring of $R_t, t = 1, 2, ..., N$.
2. Each $(P_i, *_i, \circ_i)$ is a pseudo neutrosophic ideal.

Theorem: Every pseudo neutrosophic N-ideal is trivially a neutrosophic sub N-ring but the converse may not be true.

Theorem: Every pseudo neutrosophic N-ideal is trivially a strong neutrosophic sub N-ring but the converse may not be true.

Theorem: Every pseudo neutrosophic N-ideal is trivially a neutrosophic N-ideal but the converse may not be true.

Theorem: Every pseudo neutrosophic N-ideal is trivially a strong neutrosophic N-ideal but the converse may not be true.

5 Neutrosophic Bi-Fields and Neutrosophic N-Fields

Definition **. Let $(BN(F), *, \circ)$ be a non-empty set with two binary operations $*$ and $\circ$. $(BN(F), *, \circ)$ is said to be a neutrosophic bifiel if $BN(F) = F_1 \cup F_2$ where atleast one of $(F_1, *, \circ)$ or $(F_2, *, \circ)$ is a neutrosophic field and other is just a field. F_1 and F_2 are proper subsets of $BN(F)$.

If in the above definition both $(F_1, *, \circ)$ and $(F_2, *, \circ)$ are neutrosophic fields, then we call $(BN(F), *, \circ)$ to be a neutrosophic strong bifield.

Example 2. Let $BN(F) = (F_1, *, \circ) \cup (F_2, *, \circ)$ where $(F_1, *, \circ) = (\langle \mathbb{C} \cup I \rangle, +, \times)$ and $(F_2, *, \circ) = (\mathbb{Q}, +, \times)$. Clearly $(F_1, *, \circ)$ is a neutrosophic field and $(F_2, *, \circ)$ is just a field. Thus $(BN(F), *, \circ)$ is a neutrosophic bifield.

Theorem: All strong neutrosophic bifields are trivially neutrosophic bifields but the converse is not true.

Definition 8. Let $BN(F) = (F_1 \cup F_2, *, \circ)$ be a neutrosophic bifield. A proper subset $(T, *, \circ)$ is said to be a neutrosophic subbifield of $BN(F)$ if

3) $T = T_1 \cup T_2$ where $T_1 = F_1 \cap T$ and $T_2 = F_2 \cap T$ and

4) At least one of $(T_1, \circ)$ or $(T_2, *)$ is a neutrosophic field and the other is just a field.

Example: Let $BN(F) = (F_1, *, \circ) \cup (F_2, *, \circ)$ where $(F_1, *, \circ) = (\langle \mathbb{R} \cup I \rangle, +, \times)$ and $(F_2, *, \circ) = (\mathbb{C}, +, \times)$. Let $P = P_1 \cup P_2$ be a proper subset of $BN(F)$, where $P_1 = (\mathbb{Q}, +, \times)$ and $P_2 = (\mathbb{R}, +, \times)$. Clearly $(P, +, \times)$ is a neutrosophic subbifield of $BN(F)$.

Definition*. Let $\{N(F), *_1, ..., *_2, \circ_1, \circ_2, ..., \circ_N\}$ be a non-empty set with two N-binary operations defined on it. We call $N(R)$ a neutrosophic N-field (N a positive integer) if the following conditions are satisfied.

1. $N(F) = F_1 \cup F_2 \cup ... \cup F_N$ where each F_i is a proper subset of $N(F)$ i.e. $R_i \not\subset R_j$ or $R_j \not\subset R_i$ if $i \neq j$.
2. $(R_i, *_i, \circ_i)$ is either a neutrosophic field or just a field for $i = 1, 2, 3, ..., N$.

If in the above definition each $(R_i, *_i, \circ_i)$ is a neutrosophic field, then we call $N(R)$ to be a strong neutrosophic N-field.

Theorem: Every strong neutrosophic N-field is obviously a neutrosophic field but the converse is not true.

Definition 14. Let $N(F) = \{F_1 \cup F_2 \cup .... \cup F_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ be a neutrosophic N -field. A proper subset $T = \{T_1 \cup T_2 \cup .... \cup T_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ of $N(F)$ is said to be a neutrosophic N -subfield if each $(T_i, *_i)$ is a neutrosophic subfield of $(F_i, *_i, \circ_i)$ for $i = 1, 2, ..., N$ where $T_i = F_i \cap T$.

Conclusion

In this paper we extend neutrosophic ring and neutrosophic field to neutrosophic biring, neutrosophic N-ring and neutrosophic bifield and neutrosophic N-field. The neutrosophic ideal theory is extend to neutrosophic biideal and neutrosophic N-ideal. Some new type of neutrosophic ideals are discovered which is strongly neutrosophic or purely neutrosophic. Related examples are given to illustrate neutrosophic biring, neutrosophic N-ring, neutrosophic bifield and neutrosophic N-field and many theorems and properties are discussed.

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Received: May 30, 2014. Accepted: June 22, 2014 .