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Interval-Valued Neutrosophic Oversets, Neutrosophic Undersets, and Neutrosophic Offsets

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***Abstract*-**We have proposed since 1995 the existence of degrees of membership of an element with respect to a neutrosophic set to also be partially or totally above 1 (over-membership), and partially or totally below 0 (under-membership) in order to better describe our world problems [published in 2007].

Keywords*-interval neutrosophic overset, interval neutrosophic underset, interval neutrosophic offset, interval neutrosophic overlogic, interval neutrosophic underlogic, interval neutrosophic offlogic, interval neutrosophic overprobability, interval neutrosophic underprobability, interval neutrosophic offprobability, interval overmembership (interval membership degree partially or totally above 1), interval undermembership (interval membership degree partially or totally below 0), interval offmembership (interval membership degree off the interval [0, 1]).*

I. INTRODUCTION

"Neutrosophic" means based on three components *T (truth-membership), I (indeterminacy),* and *F (falsehood-non-membership)*. And "over" means above 1, "under" means below 0, while "offset" means behind/beside the set on both sides of the interval [0, 1], over and under, more and less, supra and below, out of, off the set. Similarly, for "offlogic", "offmeasure", "offprobability", "offstatistics" etc..

It is like a pot with boiling liquid, on a gas stove, when the liquid swells up and leaks out of pot. The pot (the interval [0, 1]) can no longer contain all liquid (i.e., all neutrosophic truth/indeterminate/falsehood values), and therefore some of them fall out of the pot (i.e., one gets neutrosophic truth/indeterminate/falsehood values which are > 1), or the pot cracks on the bottom and the liquid pours down (i.e., one gets neutrosophic truth/indeterminate/falsehood values which are < 0).

Mathematically, they mean getting values off the interval [0, 1].

The American aphorism "think outside the box" has a perfect resonance to the neutrosophic offset, where the box is the interval [0, 1], yet values outside of this interval are permitted.

II. EXAMPLE OF MEMBERSHIP ABOVE 1 AND MEMBERSHIP BELOW 0

Let's consider a spy agency $S = \{S_1, S_2, \ldots, S_{1000}\}$ of a country Atara against its enemy country Batara. Each agent S_j, $j \in \{1, 2, \ldots, 1000\}$, was required last week to accomplish 5 missions, which represent the full-time contribution/membership.

Last week agent S_{27} has successfully accomplished his 5 missions, so his membership was $T(S_{27}) = 5/5 = 1 = 100\%$ (full-time membership).

Agent S_{32} has accomplished only 3 missions, so his membership is $T(S_{32}) = 3/5 = 0.6 = 60\%$ (part-time membership).

Agent S_{41} was absent, without pay, due to his health problems; thus $T(S_{41}) = 0/5 = 0 = 0\%$ (null-membership).

Agent A_{53} has successfully accomplished his 5 required missions, plus an extra mission of another agent that was absent due to sickness, therefore $T(S_{53}) = (5+1)/5 = 6/5 = 1.2 > 1$ (therefore, he has membership above 1, called over-membership).

Yet, agent S_{75} is a double-agent, and he leaks highly confidential information about country Atara to the enemy country Batara, while simultaneously providing misleading information to the country Atara about the enemy country Batara. Therefore A_{75} is a negative agent with respect to his country Atara, since he produces damage to Atara, he was estimated to having intentionally done wrongly all his 5 missions, in addition of compromising a mission of another agent of country Atara, thus his membership $T(S_{75}) = -(5+1)/5 = -6/5 = -1.2 < 0$ (therefore, he has a membership below 0, called under-membership).

III. DEFINITION OF INTERVAL-VALUED NEUTROSOPHIC OVERSET

Let U be a universe of discourse and the neutrosophic set $A_1 \subset U$. Let T(x), I(x), F(x) be the functions that describe the degrees of membership, indeterminate-membership, and non-membership respectively, of a generic element $x \in U$, with respect to the neutrosophic set A_1:

$$T(x), I(x), F(x) : U \rightarrow P([0, \Omega]),$$

where $0 < 1 < \Omega$, and Ω is called over limit,

T(x), I(x), F(x) $\subseteq [0, \Omega]$, and P($[0, \Omega]$) is the set of all subsets of $[0, \Omega]$.

An Interval-Valued Neutrosophic Overset A_1 is defined as:

A_1 = {(x, <T(x), I(x), F(x)>), x ∈ U},
such that there exists at least one element in A_1 that has at least one neutrosophic component that is partially or totally above 1, and no element has neutrosophic components that is partially or totally below 0.

For *example*: A_1 = {(x_1, <(1, 1.4], 0.1, 0.2>), (x_2, <0.2, [0.9, 1.1], 0.2>)}, since T(x_1) = (1, 1.4] is totally above 1, I(x_2) = [0.9, 1.1] is partially above 1, and no neutrosophic component is partially or totally below 0.

IV. Definition of Interval-Valued Neutrosophic Underset

Let U be a universe of discourse and the neutrosophic set $A_2 \subset U$. Let T(x), I(x), F(x) be the functions that describe the degrees of membership, indeterminate-membership, and nonmembership respectively, of a generic element x ∈ U, with respect to the neutrosophic set A_2:

T(x), I(x), F(x) : U → $[\Psi, 1]$,

where $\Psi < 0 < 1$, and Ψ is called underlimit,

T(x), I(x), F(x) $\subseteq [\Psi, 1]$, and P($[\Psi, 1]$) is the set of all subsets of $[\Psi, 1]$.

An Interval-Valued Neutrosophic Underset A_2 is defined as:

A_2 = {(x, <T(x), I(x), F(x)>), x ∈ U},

Such that there exists at least one element in A_2 that has at least one neutrosophic component that is partially or totally below 0, and no element has neutrosophic components that are partially or totally above 1.

For *example*: A_2 = {(x_1, <(-0.5,-0.4), 0.6, 0.3>), (x_2, <0.2, 0.5, [-0.2, 0.2]>)}, since T(x_1) = (-0.5, -0.4) is totally below 0, F(x_2) = [-0.2, 0.2] is partially below 0, and no neutrosophic component is partially or totally above 1.

V. Definition of Interval-Valued Neutrosophic Offset

Let U be a universe of discourse and the neutrosophic set $A_3 \subset U$. Let T(x), I(x), F(x) be the functions that describe the degrees of membership, indeterminate-membership, and nonmembership respectively, of a generic element x ∈ U, with respect to the set A_3:

T(x), I(x), F(x) : U → P($[\Psi, \Omega]$),

where $\Psi < 0 < 1 < \Omega$, and Ψ is called underlimit, while Ω is called overlimit,

T(x), I(x), F(x) $\subseteq [\Psi, \Omega]$, and P($[\Psi, \Omega]$) is the set of all subsets of $[\Psi, \Omega]$.

An Interval-Valued Neutrosophic Offset A_3 is defined as:

A_3 = {(x, <T(x), I(x), F(x)>), x ∈ U},

such that there exist some elements in A_3 that have at least one neutrosophic component that is partially or totally above 1, and at least another neutrosophic component that is partially or totally below 0.

For *examples*: A_3 = {(x_1, <[1.1, 1.2], 0.4, 0.1>), (x_2, <0.2, 0.3, (-0.7, -0.3)>)}, since T(x_1) = [1.1, 1.2] that is totally above 1, and F(x_2) = (-0.7, -0.3) that is totally below 0.

Also B_3 = {(a, <0.3, [-0.1, 0.1], [1.05, 1.10]>)}, since I(a) = [- 0.1, 0.1] that is partially below 0, and F(a) = [1.05, 1.10] that is totally above 1.

VI. Interval-Valued Neutrosophic Overset / Underset / Offset Operators

Let U be a universe of discourse and A = {(x, <T_A(x), I_A(x), F_A(x)>), x ∈ U} and B = {(x, <T_B(x), I_B(x), F_B(x)>), x ∈ U} be two interval-valued neutrosophic oversets / undersets / offsets.

T_A(x), I_A(x), F_A(x), T_B(x), I_B(x), F_B(x): U → P($[\Psi, \Omega]$),

where P($[\Psi, \Omega]$) means the set of all subsets of $[\Psi, \Omega]$,

and T_A(x), I_A(x), F_A(x), T_B(x), I_B(x), F_B(x) $\subseteq [\Psi, \Omega]$,

with $\Psi \le 0 < 1 \le \Omega$, and Ψ is called underlimit, while Ω is called overlimit.

We take the inequality sign ≤ instead of < on both extremes above, in order to comprise all three cases: overset {when Ψ = 0, and $1 < \Omega$ }, underset {when $\Psi < 0$, and $1 = \Omega$ }, and offset {when $\Psi < 0$, and $1 < \Omega$ }.

A. *Interval-Valued Neutrosophic Overset / Underset / Offset Union*

Then A∪B =
{(x, <[max{inf(T_A(x)), inf(T_B(x))}, max{sup(T_A(x)), sup(T_B(x)}],
[min{inf(I_A(x)), inf(I_B(x))}, min{sup(I_A(x)), sup(I_B(x)}],
[min{inf(F_A(x)), inf(F_B(x))}, min{sup(F_A(x)), sup(F_B(x)}]>, x ∈ U}.

B. *Interval-Valued Neutrosophic Overset / Underset / Offset Intersection*

Then $A \cap B =$

$$\{(x, <[\min\{\inf(T_A(x)), \inf(T_B(x))\}, \min\{\sup(T_A(x)), \sup(T_B(x)\}],$$
$$[\max\{\inf(I_A(x)), \inf(I_B(x))\}, \max\{\sup(I_A(x)), \sup(I_B(x)\}],$$
$$[\max\{\inf(F_A(x)), \inf(F_B(x))\}, \max\{\sup(F_A(x)), \sup(F_B(x)\}]>, x \in U\}.$$

C. *Interval-Valued Neutrosophic Overset / Underset / Offset Complement*

The complement of the neutrosophic set A is

$$C(A) = \{(x, <F_A(x), [\Psi + \Omega - \sup\{I_A(x)\}, \Psi + \Omega - \inf\{I_A(x)\}], T_A(x)>), x \in U\}.$$

VII. CONCLUSION

After designing the neutrosophic operators for single-valued neutrosophic overset/underset/offset, we extended them to interval-valued neutrosophic overset/underset/offset operators. We also presented another example of membership above 1 and membership below 0.

Of course, in many real world problems the neutrosophic union, neutrosophic intersection, and neutrosophic complement for interval-valued neutrosophic overset/underset/offset can be used. Future research will be focused on practical applications.

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Florentin Smarandache is professor of mathematics at the University of New Mexico and he published over 300 articles and books. He coined the words "neutrosophy" [(French neutre < Latin neuter, neutral, and Greek sophia, skill/wisdom) means knowledge of neutral thought] and its derivatives: neutrosophic, neutrosophication, neutrosophicator, deneutrosophication, deneutrosophicator, etc. He is the founder and developer of neutrosophic set / logic / probability / statistics etc. In 2006 he introduced the degree of dependence/independence between the neutrosophic components T, I, F.

In 2007 he extended the neutrosophic set to Neutrosophic Overset (when some neutrosophic component is > 1), and to Neutrosophic Underset (when some neutrosophic component is < 0), and to Neutrosophic Offset (when some neutrosophic components are off the interval [0, 1], i.e. some neutrosophic component > 1 and some neutrosophic component < 0). Then, similar extensions to respectively Neutrosophic Over/Under/Off Logic, Measure, Probability, Statistics etc.

Then, introduced the Neutrosophic Tripolar Set and Neutrosophic Multipolar Set, also the Neutrosophic Tripolar Graph and Neutrosophic Multipolar Graph.

He then generalized the Neutrosophic Logic/Set/Probability to Refined Neutrosophic Logic/Set/Probability [2013], where T can be split into subcomponents T1, T2, ..., Tp, and I into I1, I2, ..., Ir, and F into F1, F2, ..., Fs, where $p+r+s = n \geq 1$. Even more: T, I, and/or F (or any of their subcomponents Tj ,Ik, and/or Fl) could be countable or uncountable infinite sets.

In 2015 he refined the indeterminacy "I", within the neutrosophic algebraic structures, into different types of indeterminacies (depending on the problem to solve), such as I1, I2, , Ip with integer $p \geq 1$, and obtained the refined neutrosophic numbers of the form Np = a+b1I1+b2I2+ +bpIp where a, b1, b2, , bp are real or complex numbers, and a is called the determinate part of Np, while for each k in {1, 2, , p} Ik is called the k-th indeterminate part of Np.

Then consequently he extended the neutrosophic algebraic structures to Refined Neutrosophic Algebraic Structures [or Refined Neutrosophic I-Algebraic Structures] (2015), which are algebraic structures based on sets of the refined neutrosophic numbers a+b1I1+b2I2+ +bpIp.

He introduced the (T, I, F)-Neutrosophic Structures [2015]. In any field of knowledge, each structure is composed from two parts: a space, and a set of axioms (or laws) acting (governing) on it. If the space, or at least one of its axioms (laws), has some indeterminacy, that structure is a (T, I, F)-Neutrosophic Structure. And he extended them to the (T, I, F)-Neutrosophic I-Algebraic Structures [2015], i.e. algebraic structures based on neutrosophic numbers of the form a+bI, but also having indeterminacy related to the structure space (elements which only partially belong to the space, or elements we know nothing if they belong to the space or not) or indeterminacy related to at least an axiom (or law) acting on the structure space. Then he extended them to Refined (T, I, F)-Neutrosophic Refined I-Algebraic Structures.

Also, he proposed an extension of the classical probability and the imprecise probability to the 'neutrosophic probability' [1995], that he defined as a tridimensional vector whose components are real subsets of the non-standard interval [-0, 1+] introduced the neutrosophic measure and neutrosophic integral:
http://fs.gallup.unm.edu/NeutrosophicMeasureIntegralProbability.pdf
and also extended the classical statistics to neutrosophic statistics:
http://fs.gallup.unm.edu/NeutrosophicStatistics.pdf