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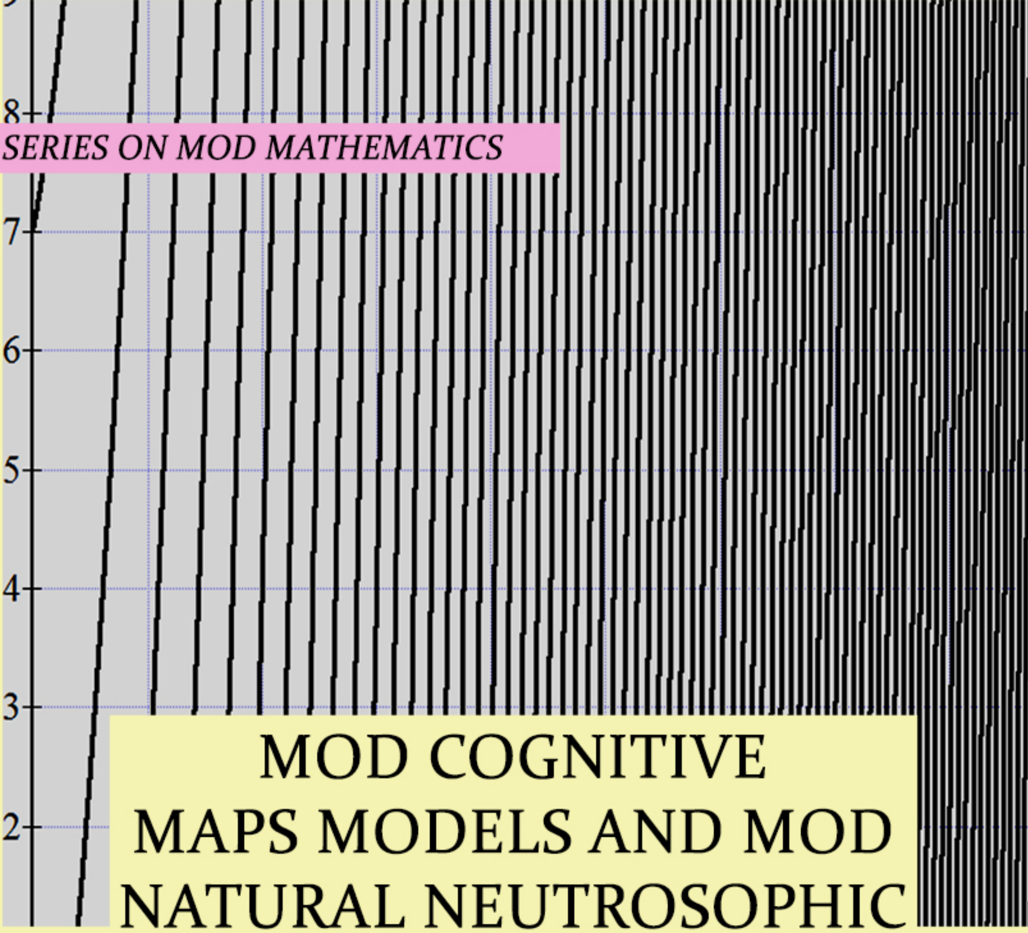

MOD COGNITIVE MAPS MODELS AND MOD NATURAL NEUTROSOPHIC COGNITIVE MAPS MODELS

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MOD Cognitive Maps Models and MOD Natural Neutrosophic Cognitive Maps Models

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PREFACE

Zadeh introduced the degree of membership/truth (t) in 1965 and defined the fuzzy set. Atanassov introduced the degree of non membership/falsehood (f) in 1986 and defined the intuitionistic fuzzy set. Smarandache introduced the degree of indeterminacy/neutrality (i) as independent component in 1995 (published in 1998) and defined the neutrosophic set on three components (t,i,f) = (truth, indeterminacy, falsehood).

The words "neutrosophy" and "neutrosophic" were coined/invented by F. Smarandache in his 1998 book. Etymologically, "neutro-sophy" (noun) [French *neutre* <Latin *neuter*, neutral, and Greek *sophia*, skill/wisdom] means knowledge of neutral thought. While "neutrosophic" (adjective), means having the nature of, or having the characteristic of Neutrosophy.

Neutrosophic Logic is a general framework for unification of many existing logics, such as fuzzy logic (especially intuitionistic fuzzy logic), paraconsistent logic, intuitionistic logic, etc. The main idea of NL is to characterize each logical statement in a 3D-Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of $]^-0, 1^+[$ with not necessarily any connection between them. For software engineering proposals the classical unit interval [0, 1] may be used. T, I, F are independent components, leaving room for incomplete information (when their superior sum < 1), paraconsistent and contradictory information (when the superior sum > 1), or complete information (sum of components = 1).

For software engineering proposals the classical unit interval [0, 1] is used. For single valued neutrosophic logic, the sum of the components is:

$0 \leq t+i+f \leq 3$ when all three components are independent;
$0 \leq t+i+f \leq 2$ when two components are dependent, while the third one is independent from them;
$0 \leq t+i+f \leq 1$ when all three components are dependent.

When three or two of the components T, I, F are independent, one leaves room for incomplete information (sum < 1), paraconsistent and contradictory information (sum > 1), or complete information (sum = 1). If all three components T, I, F are dependent, then similarly one leaves room for incomplete information (sum < 1), or complete information (sum = 1).

In 2013 Smarandache refined the neutrosophic set to n components: T_1, T_2, ...; I_1, I_2, ...; F_1, F_2, ... See http://fs.gallup.unm.edu/n-ValuedNeutrosophicLogic-PiP.pdf .

Neutrosophy <*philosophy*> (From Latin "neuter" - neutral, Greek "sophia" - skill/wisdom) A branch of philosophy, introduced by Florentin Smarandache in 1980, which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

Neutrosophy considers a proposition, theory, event, concept, or entity, "A" in relation to its opposite, "Anti-A" and that which is not A, "Non-A", and that which is neither "A" nor "Anti-A", denoted by "Neut-A".

Neutrosophy is the basis of neutrosophic logic, neutrosophic probability, neutrosophic set, and neutrosophic statistics. {From: *The Free Online Dictionary of Computing*, is edited by Denis Howe from England. Neutrosophy is an extension of the Dialectics.}

The most important books and papers in the development of neutrosophics

1995-1998 - Introduction of neutrosophic set/ logic/ probability/ statistics; generalization of dialectics to neutrosophy;
http://fs.gallup.unm.edu/eBook-neutrosophics4.pdf (4th edition)

2003 – Introduction of neutrosophic numbers (a+bI, where I = indeterminacy).
2003 – Introduction of I-neutrosophic algebraic structures.
2003 – *i*ntroduction to neutrosophic cognitive maps.
http://fs.gallup.unm.edu/NCMs.pdf

2005 - Introduction of interval neutrosophic set/logic.
http://fs.gallup.unm.edu/INSL.pdf

2009 – Introduction of N-norm and N-conorm
http://fs.gallup.unm.edu/N-normN-conorm.pdf

2013 - Development of neutrosophic probability (chance that an event occurs, indeterminate chance of occurrence, chance that the event does not occur)
http://fs.gallup.unm.edu/NeutrosophicMeasureIntegralProbability.pdf

2013 - Refinement of components (T_1, T_2, ...; I_1, I_2, ...; F_1, F_2, ...)
http://fs.gallup.unm.edu/n-ValuedNeutrosophicLogic.pdf

2014 – Introduction of the law of included multiple middle
(<A>; <neut1A>, <neut2A>, …; <antiA>)
http://fs.gallup.unm.edu/LawIncludedMultiple-Middle.pdf

2014 - Development of neutrosophic statistics (indeterminacy is introduced into classical statistics with respect to the sample/population, or with respect to the individuals that only partially belong to a sample/population)
http://fs.gallup.unm.edu/NeutrosophicStatistics.pdf

2015 - Introduction of neutrosophic precalculus and neutrosophic calculus
http://fs.gallup.unm.edu/NeutrosophicPrecalculusCalculus.pdf

2015 – Refined neutrosophic numbers ($a + b_1I_1 + b_2I_2 + \ldots + b_nI_n$), where $I_1, I_2, \ldots, I_n$ are subindeterminacies of indeterminacy I;
2015 – Neutrosophic graphs;
2015 - Thesis-Antithesis-Neutrothesis, and Neutrosynthesis, Neutrosophic Axiomatic System, neutrosophic dynamic systems, symbolic neutrosophic logic, (*t, i, f*)-Neutrosophic Structures, *I*-Neutrosophic Structures, Refined Literal Indeterminacy, Multiplication Law of Subindeterminacies:
http://fs.gallup.unm.edu/SymbolicNeutrosophicTheory.pdf

2015 – Introduction of the subindeterminacies of the form
$I_0^n = \frac{k}{0}$, for $k \in \{0, 1, 2, \ldots, n-1\}$, into the ring of modulo integers Z_n, are called *natural neutrosophic zeros*
http://fs.gallup.unm.edu/MODNeutrosophicNumbers.pdf

In this book authors for the first time introduce new mathematical models analogous to FCMs and NCMs. We in this book have constructed 12 types of MOD Cognitive Maps models using Z_n or $\langle Z_n \cup g\rangle$ or $\langle Z_n \cup I\rangle$ or $C(Z_n)$ or $\langle Z_n \cup h\rangle$ or $\langle Z_n \cup k\rangle$ which will be known us MOD Cognitive Maps model or MOD dual number Cognitive Maps model or MOD neutrosophic Cognitive Maps model or MOD finite complex number Cognitive Maps model or MOD special dual like number Cognitive Maps model or MOD special quasi dual number Cognitive Maps model respectively.

Apart from this we have defined MOD natural neutrosophic Cognitive Maps model, MOD natural neutrosophic-neutrosophic Cognitive Maps model, MOD natural neutrosophic dual number Cognitive Maps model, MOD natural neutrosophic special dual like number Cognitive Maps model, MOD natural neutrosophic special quasi dual number model and MOD natural neutrosophic finite complex number Cognitive Maps model using Z_n^I, $\langle Z_n \cup I\rangle_I$, $\langle Z_n \cup g\rangle_I$, $\langle Z_n \cup h\rangle_I$, $\langle Z_n \cup k\rangle_I$ or $C^I(Z_n)$ respectively.

These model certainly are more appropriate than the existing ones.

Finally we construct MOD interval Cognitive Maps model and describes it. The final chapter suggests a collection of problems for the interested reader.

We wish to acknowledge Dr. K Kandasamy for his sustained support and encouragement in the writing of this book.

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Chapter One

INTRODUCTION

In this chapter we mainly give the out line of the work given in this book. Further we indicate the references where one can get a proper background for understanding the later chapters of this book.

Fuzzy Cognitive Maps (FCMs) model have been developed defined and described by Kosko [2-5]. However NCMs analogous to FCMs have been introduced in [25].

Here we build MOD Cognitive Maps models using MOD integers. For MOD structures please refer [57-66].

Here we use these natural neutrosophic number I_t^n to build MOD natural neutrosophic Cognitive Maps model. Only based on this study all the following models were introduced.

Here MOD Cognitive Maps model, MOD finite complex modulo integer Cognitive Maps model, MOD neutrosophic Cognitive Maps model, MOD dual number Cognitive Maps model (MOD) special dual like number Cognitive Maps model and MOD special quasi dual number Cognitive Maps model are defined, described and developed for the first time in this book.

Secondly we see in case of NCMs or for that matter MOD Neutrosophic Cognitive Maps we have $I^2 = I$ that is we have an indeterminate which is an idempotent under product and addition of n times I is nI. But here we make use of new classes of natural neutrosophic numbers which are such that they are idempotents under sum and under product they can be nilpotents or idempotents or zero divisors. For instance consider Z_6 the MOD integer. Z_6^I is the collection of all natural neutrosophic numbers got by exerting the operation of division '/' in Z_6. The MOD natural neutrosophic numbers in Z_6 are I_0^6, I_2^6, I_3^6 and I_4^6 [64].

Clearly $I_3^6 \times I_3^6 = I_3^6$, $I_4^6 \times I_4^6 = I_4^6$, $I_3^6 \times I_2^6 = I_0^6$ thus we see the natural neutrosophic numbers are neutrosophic idempotents, neutrosophic zero divisors. We have a natural neutrosophic zero also denoted in Z_6 by I_0^6 in Z_n by I_0^n.

Consider Z_8^I we see I_4^8 is a natural neutrosophic nilpotent clearly $I_4^8 + I_4^8 = I_4^8$. Thus Z_8^I is a semiring we call Z_8^I as a strict semiring which may be considered also as a misnomer.

Now instead of using I we can use I_t^n where t is a zero divisor or idempotent or nilpotent element in Z_n.

When we use $\langle Z_n \cup g\rangle$ or $\langle Z_n \cup h\rangle$ or $\langle Z_n \cup k\rangle$ or $\langle Z_n \cup I\rangle$ or $C(Z_n)$ then we get more number of MOD natural neutrosophic numbers. For instance in $\langle Z_5 \cup g\rangle_I$ we have I_g^5, I_{2g}^5, I_{3g}^5, I_{4g}^5 are all natural neutrosophic dual numbers all of them are natural neutrosophic nilpotents and zero divisors. Similar type of analysis in case of $\langle Z_n \cup I\rangle$, $\langle Z_n \cup k\rangle$ and $\langle Z_n \cup h\rangle$ can be done. We have built MOD natural neutrosophic Cognitive Maps models using $\langle Z_n \cup I\rangle_I$, $\langle Z_n \cup g\rangle$ and so on.

Thus these 12 new types of MOD Cognitive Maps models will surely be a boon to any researcher.

Chapter Two

MOD COGNITIVE MAPS MODELS

In this chapter for the first time authors introduce the new type of model analogous to FCMs model using MOD integers Z_n; $2 \leq n < \infty$.

Here one is made to think why should we use of only 0 and 1 alone as in case of FCMs model. After all what we want is a resultant after a finite number of iterations or working.

We achieve this by working with modulo integers Z_n. Here the concepts are given at the initial state 1 or 0 that is on and off state respectively but the resultant can take any value in Z_n.

That value will signify the impact of that concept or node on the other concepts or nodes and its importance or otherwise using face value ordering in Z_n.

We will define this methodically. Before we proceed to define the model we just describe and develop the notion of matrix with entries from Z_n; $2 \leq n < \infty$ which we shortly call as MOD matrices.

In this chapter we develop only MOD square matrices as the new model analogous to FCMs model functions only on square matrices.

DEFINITION 2.1: *Let Z_n be the MOD integers $\{0, 1, 2, \ldots, n-1; (2 \leq n < \infty)\}$.*

Let $M = (m_{ij})_{m \times m}$ be the $m \times m$ square matrix with entries from Z_n ($2 \leq m < \infty$, m a positive integer).

M is defined as the MOD $m \times m$ square matrix with entries from Z_n.

We will illustrate this situation by some examples.

Example 2.1: Let

$$M = \begin{bmatrix} 3 & 0 & 4 & 2 \\ 5 & 1 & 0 & 3 \\ 1 & 2 & 2 & 5 \\ 4 & 0 & 1 & 0 \end{bmatrix}$$

be the MOD 4×4 square matrix with entries from Z_6.

Example 2.2: Let

$$P = \begin{bmatrix} 7 & 3 & 6 & 4 & 5 & 1 \\ 0 & 2 & 1 & 10 & 2 & 0 \\ 6 & 10 & 0 & 7 & 0 & 8 \\ 3 & 0 & 5 & 0 & 8 & 0 \\ 1 & 9 & 0 & 2 & 0 & 9 \\ 0 & 7 & 6 & 4 & 3 & 3 \end{bmatrix}$$

be the MOD 6×6 matrix with entries from Z_{11}.

We can have finitely many such matrices with entries from Z_{11}.

We define only one type of operation using them.

We will illustrate this by some examples.

Let X = $\{(a_1, \ldots, a_n) / a_i \in Z_m; 1 \leq i \leq n\}$ be the row matrix with entries from Z_m.

Elements of X are defined as MOD row matrices with entries from Z_m.

Example 2.3: Let

$$M = \begin{bmatrix} 3 & 2 & 0 & 4 & 5 \\ 0 & 1 & 2 & 0 & 6 \\ 2 & 0 & 1 & 5 & 0 \\ 6 & 5 & 0 & 1 & 2 \\ 4 & 3 & 6 & 0 & 1 \end{bmatrix}$$

be the MOD square matrix with entries from Z_7.

Let x = (1, 0, 0, 0, 0) be the MOD row matrix with entries from Z_7.

$xM = (3, 2, 0, 4, 5) = y_1$; $\quad y_1M = (4, 1, 2, 2, 5) = y_2$;
$y_2M = (6, 6, 6, 0, 0) = y_3$; $\quad y_3M = (2, 4, 4, 5, 3) = y_4$;
$y_4M = (1, 0, 2, 5, 5) = y_5$; $\quad y_5M = (1, 0, 4, 5, 6) = y_6$;
$y_6M = (6, 3, 5, 1, 0) = y_7$; $\quad y_7M = (6, 6, 4, 1, 1) = y_8$.

Certainly after finite number of iterations we will arrive at a MOD fixed point or a MOD limit cycle [ ].

It is to be noted if we start with 1 any of the coordinates of x till the end we maintain it as non zero.

If it becomes zero at some stage we replace that coordinate by 1.

Let x = (0, 0, 0, 0, 1) be a state vector.

We find the effect of x on M.

$xM = (4, 3, 6, 0, 1) = y_1$; $y_1M = (1, 0, 4, 4, 4) = y_2$;
$y_2M = (2, 7, 0, 0, 3) = y_3$; $y_3M = (4, 6, 4, 1, 6) = y_4$;
$y_4M = (1, 2, 3, 2, 1) = y_5$; $y_5M = (4, 3, 6, 0, 1) = y_6$;
$y_6M = (0, 0, 4, 4, 4) = y_7$; $y_7M = (6, 4, 0, 3, 5) = y_8$

and so on.

However after a finite number of iterations we are sure to arrive at a realized MOD fixed point or a realized MOD limit cycle.

Example 2.4: Let

$$M = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

be the MOD matrix with entries from Z_3.

Let x = (1, 0, 0) be the MOD row vector.

$xM = (1, 1, 2) = y_1$; $y_1M = (1, 1, 1) = y_2$;
$y_2M = (1, 1, 0) = y_3$; $y_3M = (1, 1, 2) = y_4$;

$$y_4M = (1, 1, 2) = y_5 \ (=y_4).$$

Thus the MOD resultant is a realized fixed point.

Let x = (0, 1, 0) be the MOD row vector.

To find the effect of x on M.

$xM = (1, 1, 0) = y_1$; $y_1M = (1, 1, 2) = y_2$;
$y_2M = (1, 1, 1) = y_3$; $y_3M = (1, 1, 0) = y_4$;
$y_4M = (1, 1, 2) = y_5$ $(=y_4)$.

So the resultant is a realized limit cycle.

Let x = (0, 0, 1) be the MOD row vector.

The effect of x on M is as follows.

$$xM = (0, 0, 1) = y_1.$$

Thus the resultant is a special classical fixed point.

Let x = (1, 1, 0) be the MOD row vector.

The effect of x on M.

$xM = (1, 1, 2) = y_1$; $y_1M = (1, 1, 1) = y_2$;
$y_2M = (1, 1, 0) = y_3$ $(= x)$.

Thus the resultant is a limit cycle.

Let x = (1, 0, 1) be the initial state vector.

To find the effect of x on M.

$xM = (1, 1, 1) = y_1$; $y_1M = (1, 1, 1) = y_2$ $(=y_1)$.

This is a realized fixed point and both iterations have to be updated to keep the on state on the vectors in x = (1, 0, 1).

Let x = (0, 1, 1) be the initial state vector.

$xM = (1, 1, 1) = y_1$;
$y_1M = (1, 1, 1) = y_2$ $(=y_1)$ is a realized fixed point.

Finally if x = (1, 1, 1) then xM = (1, 1, 1) after updating, so x = (1, 1, 1) can be considered as the classical fixed point.

Only this sort of operation is performed at each stage and this is the only operation which is meaningful for us to implement on the model.

We see the resultant is a classical fixed point or a realized fixed point or a realized limit cycle.

Further while working with the special type of multiplication we take care to see that at each time the on state of the vector in the initial state vector is always maintained to be in the on state.

Example 2.5: Let

$$B = \begin{bmatrix} 0 & 4 \\ 2 & 3 \end{bmatrix}$$

be the MOD matrix with entries from Z_5.

Let x = (1, 0) be the initial state vector. The effect of x on B;

$xB = (1, 4) = y_1$; $y_1B= (3, 1) = y_2$;
$y_2B = (2, 0) = y_3$; $y_3B = (1, 3) = y_4$;
$y_4B = (1, 3) = y_5$ $(=y_4)$.

Thus the resultant is a realized fixed point.

Let $x_1 = (0, 1)$ be the initial state vector.

To find the effect of x on B.

$x_1B = (2, 3) = y_1$; $y_1B = (1, 2) = y_2$;
$y_2B = (4, 1) = y_3$; $y_3B = (2, 4) = y_4$;
$y_4B = (3, 1) = y_5$; $y_5B = (2, 1) = y_6$;
$y_6B = (2, 1) = y_7$ $(=y_6)$.

Thus the resultant is a realized fixed point.

Let $x_2 = (1, 1)$; to find the effect of x on B.

$x_2B = (2, 2) = y_1$; $y_1B = (4, 4) = y_2$;
$y_2B = (3, 3) = y_3$; $y_3B = (1, 1) = y_4 (=x_2)$.

Thus the resultant is a realized fixed point.

Clearly $x + x_1 = (1, 0) + (0, 1) = (1, 1) = x_2$.

The resultant of x is (1, 3) I
The resultant of x_1 is (2, 1) II
The resultant of x_2 is (1, 1) III

But the resultant sum of x and x_1 is (3, 4) which is not III.

So $(x + x_1) B \neq xB + x_1B$ in general.

Example 2.6: Let

$$W = \begin{bmatrix} 1 & 0 & 2 & 4 & 0 \\ 6 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 3 & 0 \\ 7 & 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 2 & 0 \end{bmatrix}$$

be the MOD 5×5 matrix with entries from Z_8.

Let x = (1, 0, 0, 0, 0) be the initial state vector

$xW = (1, 0, 2, 4, 0) = y_1$; $y_1W = (5, 2, 2, 6, 4) = y_2$;
$y_2W = (3, 6, 0, 2, 2) = y_3$; $y_3W = (5, 2, 0, 0, 6) = y_4$;
$y_4W = (1, 6, 4, 0, 4) = y_5$; $y_5W = (5, 0, 0, 0, 4) = y_6$;
$y_6W = (5, 4, 2, 4, 0) = y_7$; $y_7W = (1, 2, 6, 2, 4) = y_8$;
$y_8W = (3, 2, 6, 6, 6) = y_9$; $y_9W = (1, 4, 4, 2, 2) = y_{10}$;
$y_{11}W = (2, 4, 2, 2, 0) = y_{12}$; $y_{12}W = (1, 2, 4, 6, 2) = y_{13}$;
$y_{13}W = (7, 6, 0, 4, 2) = y_{14}$; $y_{14}W = (7, 2, 4, 0, 0) = y_{15}$;

$y_{15}W = (3, 4, 0, 0, 4) = y_{16}$; $y_{16}W = (3, 4, 2, 4, 0) = y_{17}$;
$y_{17}W = (7, 2, 2, 2, 4) = y_{18}$; $y_{18}W = (1, 6, 4, 2, 6) = y_{19}$;
$y_{19}W = (3, 2, 4, 4, 6) = y_{20}$;

and so on and we are sure to arrive at a realized fixed point or a realized limit point.

Let x = (0, 0, 0, 0, 1) be the initial state vector.

To find the effect of x on W.

$xW = (0, 1, 0, 2, 1) = y_1$; $y_1W = (4, 1, 5, 2, 4) = y_2$;
$y_2W = (0, 1, 5, 7, 4) = y_3$; $y_3W = (7, 1, 7, 7, 1) = y_4$;
$y_4W = (6, 0, 5, 3, 1) = y_5$ and so on.

However after a finite number of iterations we will arrive at a realized fixed point or a realized limit cycle.

Example 2.7: Let

$$S = \begin{bmatrix} 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 4 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 5 \\ 1 & 0 & 0 & 3 & 0 \end{bmatrix}$$

be the MOD 5×5 matrix with entries from Z_5.

Let x = (1, 0, 0, 0, 0) be the given state vector.

To find the effect of x on S.

$xS \to (1, 1, 0, 0, 2) = y_1$; $y_1S \to (2, 1, 1, 0, 2) = y_2$;
$y_2S \to (1, 4, 1, 1, 5) = y_3$; $y_3S \to (1, 3, 5, 4, 2) = y_4$;
$y_4S = (4, 5, 1, 5, 3) = y_5$; $y_5S = (1, 0, 4, 4, 4) = y_6$;
$y_6S = (2, 3, 4, 4, 2) = y_7$; $y_7S = (1, 4, 1, 4, 4) = y_8$;
$y_8S = (2, 3, 2, 1, 4) = y_9$; $y_9S = (1, 0, 4, 2, 5) = y_{10}$;
$y_{10}S = (3, 3, 0, 1, 4) = y_{11}$; $y_{11}S = (4, 3, 4, 0, 5) = y_{12}$;

$y_{12}S = (3, 0, 3, 1, 0) = y_{13}$; $y_{13}S = (1, 3, 1, 3, 2) = y_{14}$

and so on and we will arrive at a realized fixed point or a realized limit cycle.

Let a = (0, 0, 1, 0, 0) be the initial state vector, to find the effect of a on S.

$aS \to (4, 2, 1, 1, 1) = y_1$; $y_1S = (5, 0, 3, 2, 2) = y_2$;
$y_2S = (2, 5, 4, 3, 3) = y_3$; $y_3S = (5, 4, 2, 1, 5) = y_4$;
$y_4S = (1, 3, 5, 5, 6) = y_5$; $y_5S = (1, 5, 2, 2, 2) = y_6$;
$y_6S = (4, 5, 1, 2, 2) = y_7$; $y_7S = (0, 0, 1, 1, 1) = y_8$;
$y_8S = (5, 2, 1, 4, 0) = y_9$; $y_9S = (4, 1, 1, 1, 1) = y_{10}$;
$y_{10}S = (5, 0, 2, 4, 2) = y_{11}$; $y_{11}S = (4, 3, 4, 2, 2) = y_{12}$;
$y_{12}S = (0, 0, 5, 4, 4) = y_{13}$; $y_{13}S = (0, 4, 4, 5, 1) = y_{14}$;
$y_{14}S = (5, 2, 3, 1, 5) = y_{15}$; $y_{15}S = (5, 5, 3, 0, 0) = y_{16}$;
$y_{16}S = (0, 5, 5, 3, 1) = y_{17}$; $y_{17}S = (3, 4, 2, 2, 2) = y_{18}$;
$y_{18}S = (4, 1, 1, 0, 0) = y_{19}$; $y_{19}S = (4, 0, 1, 1, 3) = y_{20}$;
$y_{20}S = (1, 0, 1, 4, 2) = y_{21}$; $y_{21}S = (0, 3, 4, 1, 5) = y_{22}$;

and so on.

Certainly after a finite number of iterations we are sure to arrive at a realized fixed point or a realized limit cycle.

Let b = (0, 1, 0, 0, 0) be the initial state vector.

To find the effect of b on S.

$bS \to (0, 1, 1, 0, 0) = y_1$; $y_1S = (4, 2, 1, 1,1) = y_2$;
$y_2S = (5, 1, 3, 4, 2) = y_3$; $y_3S = (2, 5, 5, 3, 3) = y_4$;
$y_4S = (5, 1, 2, 2, 0) = y_5$; $y_5S = (2, 3, 3, 2, 4) = y_6$;
$y_6S = (4, 2, 5, 3, 5) = y_7$; $y_7S = (1, 2, 5, 2, 4) = y_8$;
$y_8S = (0, 5, 4, 5, 5) = y_9$; $y_9S = (3, 2, 4, 1, 5) = y_{10}$;
$y_{10}S = (3, 5, 3, 1, 3) = y_{11}$; $y_{11}S = (3, 3, 0, 0, 2) = y_{12}$;
$y_{12}S = (2, 3, 3, 0, 0) = y_{13}$; $y_{13}S = (0, 2, 3, 3, 1) = y_{14}$;
$y_{14}S = (1, 1, 5, 0, 0) = y_{15}$; $y_{15}S = (2, 5, 1, 5, 1) = y_{16}$;
$y_{16}S = (4, 4, 4, 4, 0) = y_{17}$; $y_{17}S = (4, 1, 2, 4, 2) = y_{18}$;

and so on.

However we will arrive at a realized fixed point or a realized limit cycle.

Now we proceed onto study one more example before obtain results related with them.

Example 2.8: Let

$$B = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

be the MOD 3×3 matrix with entries from Z_3

Let x = (1, 0, 0) be the initial state vector.

To find the effect of x on B .

$xB \to (1, 1, 0) = y_1$; $y_1B = (2, 1, 1) = y_2$;
$y_2B = (2, 2, 0) = y_3$; $y_3B = (1, 2, 2) = y_4$;
$y_4B = (1, 1, 0) = y_5$ (= y_1).

Thus the resultant is a realized fixed point given by (1, 1, 0).

Consider x_1 = (0, 1, 0) to be a initial state vector.

To find the resultant of x_1 on B.

$x_1B = (2, 1, 1) = y_1$; $y_1B = (2, 2, 0) = y_2$;
$y_2B = (1, 1, 2) = y_3$; $y_3B = (2, 1, 2) = y_4$;
$y_4B = (2, 2, 2) = y_5$; $y_5B = (1, 2, 0) = y_6$;
$y_6B = (1, 1, 2) = y_7$ (= y_3).

Thus the resultant is a realized limit cycle (1, 1, 2).

Let x_2 = (0, 0, 1) be the given initial state vector.
To find the effect of x_2 on B.

$x_2B = (0, 0, 2) = y_1$; $y_1B = (0, 0, 1) = y_2 (=x_2)$;

thus the resultant realized limit cycle given by (0, 0, 2).

Let $x_3 = (1, 1, 0)$ be the given initial state vector.

Effect of x_3 on B.

$x_3B = (2, 1, 1) = y_1$; $y_1B = (2, 2, 0) = y_2$;
$y_2B = (1, 2, 2) = y_3$; $y_3B = (1, 1, 0) = y_4 (=x_3)$.

Thus the resultant is a realized limit cycle given by (1, 2, 2).

Now $x + x_1 = (1, 1, 0)$.
Resultant of x is (1, 1, 0) ; resultant of x_1 is (1, 1, 2).
Sum of the resultant is (2, 2, 2)

However resultant of $x + x_1 = (1, 1, 0)$ is (1, 2, 2).

Thus $(x + x_1) B \neq xB + x_1B$ in general.

In view of this we have the following theorem.

THEOREM 2.1: *Let M $(m_{ij})_{n \times n}$ be the $n \times n$ matrix with entries from Z_m, $(1 \leq i, j \leq n)$; $(2 \leq n < \infty)$.*

If x_1 and x_2 are initial state vectors; then $(x_1 + x_2)\ m \neq x_1M + x_2M$ in general.

Proof is direct and hence left as an exercise to the reader.

Next we proceed onto describe the MOD directed graphs where the edge weights are from Z_n.

Example 2.9: Let G be the MOD directed graph given by the following figure.

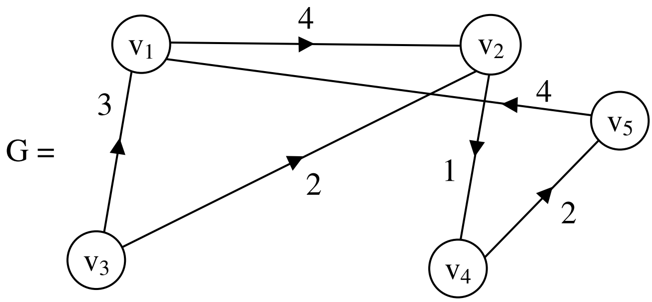


Figure 2.1

The directed graph G has the following adjacency matrix M.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 3 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 4 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Clearly M is a 5×5 MOD matrix with entries from Z_5.

Example 2.10: Let

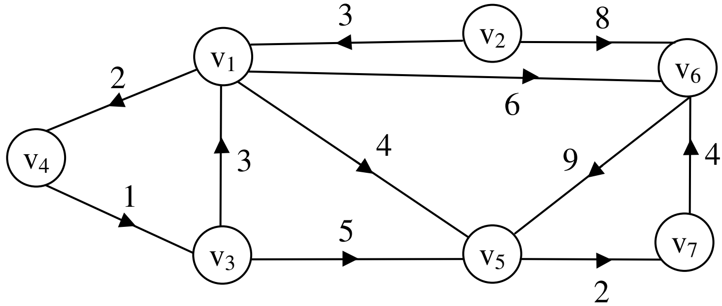


Figure 2.2

be the MOD directed graph with entries from Z_{10}.

The MOD matrix B associated with Figure 2.2 is as follows.

$$B = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 2 & 4 & 6 & 0 \\ 3 & 0 & 0 & 0 & 0 & 8 & 0 \\ 3 & 0 & 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{bmatrix} \end{array}$$

Thus for any given MOD graph we can always have a MOD matrix associated with it.

Example 2.11: Let G be the MOD graph with edge weights from Z_{12} given in Figure 2.3.

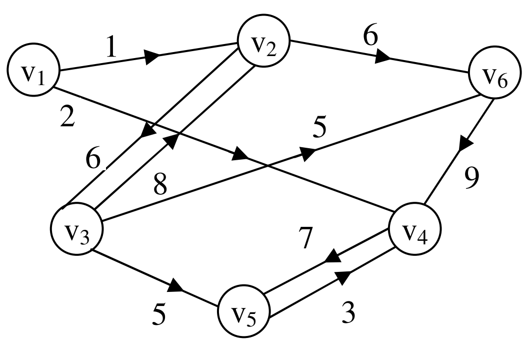


Figure 2.3

The MOD matrix M associated with G is as follows.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 & 6 \\ 0 & 8 & 0 & 0 & 5 & 5 \\ 0 & 0 & 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 9 & 0 & 0 \end{bmatrix} \end{array}.$$

M is a MOD 6×6 matrix with entries from Z_{12}.

Now we proceed onto define, describe and develop the notion of MOD Cognitive Maps (MOD-CMs) model.

DEFINITION 2.2: *Let $C_1, C_2, \ldots, C_m$ be m concepts / nodes. Let G be the MOD directed graph associated with these concepts with edge weights $e_{ij} \in Z_n$.*

Let E be the MOD matrix defined by $E = (e_{ij})$ where e_{ij} is the weight of the directed edge C_iC_j. E is called the adjacency MOD matrix of the MOD Cognitive Maps, where a MOD Cognitive Map is a MOD directed graph with concepts like policies, events etc., as nodes and causalities with weights from Z_n as edges.

It represents the weighted causal relationship between concepts.

DEFINITION 2.3: *Let $C_1, C_2, \ldots, C_m$ be the nodes of a MOD CM. $A = (a_1, \ldots, a_m)$ where $a_i \in \{0, 1\}$. A is called the initial or instantaneous state vector and it denotes the on-off state position of the node at an instant.*

$a_i = 0$ *if* a_i *is off*
and $a_i = 1$ *if* a_i *is on*
$i = 1, 2, \ldots, m.$

DEFINITION 2.4: *Let $C_1, C_2, \ldots, C_m$ be the nodes of a MODCM. Let $\overrightarrow{C_1C_2}, \ldots, \overrightarrow{C_iC_j}$ be the weighted edges of the MODCMs $(i \neq j)$ with weights from Z_n. Then the weighted edges form a directed cycle. A MODCMs is said to be MOD cyclic if it possess a directed MOD cycle.*

A MODCM is said to be MOD acyclic if it does not possess any directed cycle. A MODCM with MOD cycles is said to have feedback, when there is a feedback in a MODCM that is when the causal relations flow through a MOD cycle in a revolutionary way, the MODCM is called a MOD dynamical system.

DEFINITION 2.5:. *Let $\overrightarrow{C_1C_2}, \overrightarrow{C_2C_3}, \ldots, \overrightarrow{C_{m-1}C_n}$ be a MOD cycle when C_i is switched on and if the causality flows through the edges of a MOD cycle and if it again causes C_i we say the MOD dynamical system goes round and round. This is true for any node C_i; $i = 1, 2, \ldots, m$.*

The equilibrium state of the MOD dynamical system is called the MOD hidden pattern.

If the equilibrium state of a dynamical system is a unique MOD resultant vector then it is defined as the MOD fixed point. By MOD resultant we mean

$$R = \{(a_1, a_2, \ldots, a_m) / a_i \in Z_n;\ 1 \leq i \leq m\}.$$

Thus the major difference between the FCM and MODCM is that in the FCM, the resultant takes its value from

$A = \{(a_1, \ldots, a_m) / a_i \in \{0, 1\}, 1 \leq i \leq m\}$ but in case of MODCMs the MOD resultant takes its value from

$$R = \{(a_1, \ldots, a_m) / a_i \in Z_n;\ 1 \leq i \leq m\}.$$

This is one of the striking difference between the two models.

This will be represented by an example before we proceed onto describe MOD limit cycle of a MODCMs model.

Example 2.12: Let C_1, C_2, C_3, C_4, C_5 be the nodes of a MODCM dynamical system; M whose edge weights are taken from Z_3.

The MOD directed graph G associated with the MODCM is as follows.

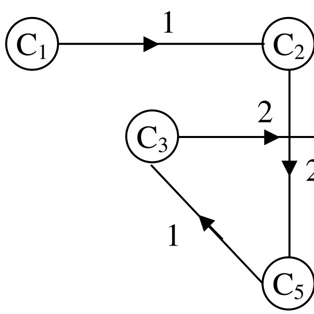


Figure 2.4

The MOD adjacency matrix (adjacency MOD matrix) M associated with G is as follows.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array}\begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}.$$

The MOD resultant vectors of the MOD dynamical system E is

$$R = \{(a_1, a_2, a_3, a_4, a_5) / a_i \in Z_3;\ 1 \le i \le 5\}.$$

Let x = (1, 0, 0, 0, 0) be the initial or instantaneous MOD vector.

The effect of x on the MOD dynamical system M is as follows;

$xM \to (1\ 1\ 0\ 0\ 0) = y_1$; $y_1M \to (1\ 1\ 0\ 0\ 2) = y_2$;
$y_2M \to (1, 1, 2, 0, 2) = y_3$; $y_3M \to (1, 1, 2, 1, 2) = y_4$;
$y_4M \to (1, 1, 2, 1, 2) = y_5\ (=y_4)$.

Thus the MOD resultant is a MOD fixed point given by (1, 1, 2, 1, 2).

($\to$ denotes at each stage the MOD resultant is updated while updating the vector we only replace 0 by 1).

Next consider $x_1 = (0, 0, 0, 0, 1)$ be the initial state MOD vector.

The effect of x_1 on M is as follows.

$x_1M \to (0, 0, 1, 0, 1) = y_1$; $y_1M \to (0, 0, 1, 2, 1) = y_2$;
$y_2M \to (0, 0, 1, 2, 1) = y_3\ (=y_2)$.

Thus the MOD resultant in this case is also a MOD fixed point.

Let $x_2 = (0, 1, 0, 0, 0)$ be the initial state vector.

To find the effect of x_2 on M,

$x_2M \to (0, 1, 0, 0, 2) = y_1$; $y_1M \to (0, 1, 2, 0, 2) = y_2$;
$y_2M \to (0, 1, 2, 1, 2) = y_3$; $y_3M \to (0, 1, 2, 1, 2) = y_4\ (=y_3)$.

Thus the resultant in this case also is a MOD fixed point.

Let $x_3 = (0, 0, 1, 0, 0)$ be the initial state vector.

To find the effect of x_3 on M.

$x_3M \to (0, 0, 1, 2, 0) = y_1$;
$y_1M \to (0, 0, 1, 2, 0) = y_2$ (= y_1).

The resultant is a MOD fixed point.

Consider
$x_1 + x_2 = (0, 0, 0, 0, 1) + (0, 1, 0, 0, 0) = (0, 1, 0, 0, 1) = x_5$

$x_5M \to (0, 1, 1, 0, 2) = y_1$; $y_1M \to (0, 1, 2, 2, 2) = y_2$;
$y_2M \to (0, 1, 2, 1, 2) = y_3$;
$y_3M \to (0, 1, 2, 2, 2) = y_4$ (=y_2).

Thus the MOD resultant is a MOD limit cycle.

Now the resultant of x_1 is a fixed point (0, 0, 1, 2, 1).

The resultant of x_2 is a fixed point (0, 1, 2, 1, 2).

Sum of the resultant of x_1 and x_2 is
(0, 0, 1, 2, 1) + (0, 1, 2, 1, 2) = (0 1 0 0 0); not in keeping with any of the basic properties or laws.

For the MOD resultant of the initial state vector
$x_1 + x_2 = (0, 1, 0, 0, 1) = x_5$ is a limit cycle.

Hence our claim.
We now define the notion of MOD limit cycle.

DEFINITION 2.6:. *If the MODCM settles down with a state vector repeating in the form*

$$x_1 \to x_2 \to \ldots \to x_I \to x_1$$

then the MOD equilibrium is a MOD limit cycle.

We saw in the examples the MOD resultant gave a MOD limit cycle.

Now we give one more example before we proceed onto define other related properties.

Example 2.13: Let C_1, C_2, C_3, C_4, C_5, C_6, C_7 be the 7 nodes / concepts and G be the MOD directed graph using these nodes C_1, C_2, …, C_7 with weights from Z_4.

The MOD directed graph is given in the following figure.

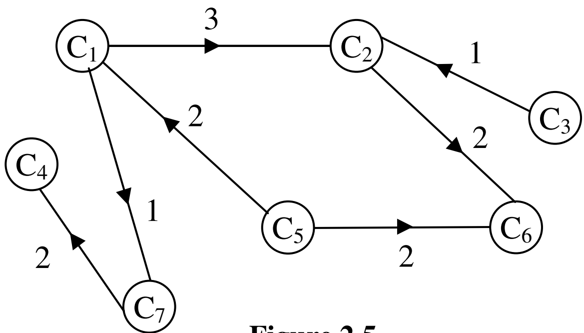


Figure 2.5

The MOD adjacency matrix B associated with G is as follows:

$$B = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 \end{array} \\ \begin{bmatrix} 0 & 3 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let x = (1, 0, 0, 0, 0, 0, 0) be the initial state vector.

To find the effect of x_1 on B.

$$x_1B \to (1, 3, 0, 0, 0, 0, 1) = y_1;$$
$$y_1B \to (1, 3, 0, 2, 0, 2, 1) = y_2;$$
$$y_2B \to (1, 3, 0, 2, 0, 2, 1) = y_3 \ (=y_2).$$

Thus the MOD resultant of x_1 is a MOD fixed point.

Consider $x_2 = (0, 1, 0, 0, 0, 0, 0)$ to be the initial state vector.

To find the effect of x_2 on B.

$$x_2B \to (0, 1, 0, 0, 0, 2, 0) = y_1$$
$$y_1B \to (0, 1, 0, 0, 0, 2, 0) = y_2 \ (=y_1).$$

Thus the MOD resultant of x_2 is a MOD fixed point.

Let $x_3 = (0, 0, 1, 0, 0, 0, 0)$ be the initial MOD state vector.

The effect of x_3 on B is as follows.

$$x_3B \to (0, 1, 1, 0, 0, 0, 0) = y_1;$$
$$y_1B \to (0, 1, 1, 0, 0, 2, 0) = y_2;$$
$$y_2B \to (0, 1, 1, 0, 0, 2, 0) = y_3 \ (=y_2).$$

Thus the MOD resultant of x_3 is also a MOD fixed point.

Consider $x_4 = (0, 0, 0, 1, 0, 0, 0)$ be the initial state vector.

To find the effect of x_4 on B.

$$x_4B \to (0, 0, 0, 1, 0, 0, 0) = x_4;$$

So x_4 is a classical MOD fixed point of a special kind.

Let $x_5 = (0, 0, 0, 0, 1, 0, 0)$ be the initial state vector.

To find the effect of x_5 on B.

$x_5B \to (2, 0, 0, 0, 1, 0, 0) = y_1$;
$y_1B \to (2, 2, 0, 0, 1, 0, 2) = y_2$;
$y_2B \to (2, 2, 0, 0, 1, 0, 2) = y_3$ (= y_2).

Thus the MOD resultant is also a MOD fixed point.

Consider x_6 = (0, 0, 0, 0, 0, 1, 0) to be the initial state vector.

To find the effect of x_6 on B.

$x_6B \to (0, 0, 0, 0, 2, 1, 0) = y_1$;
$y_1B \to (0, 0, 0, 0, 2, 1, 0) = y_2 = (y_1)$.

Thus the MOD resultant the MOD fixed point.

Consider x_7 = (0, 0, 0, 0, 0, 0, 1) be the initial state vector.

To find the effect of x_7 on B.

$x_7B \to (0, 0, 0, 2, 0, 0, 1) = y_1$;
$y_2B \to (0, 0, 0, 2, 0, 0, 1) = y_2$ (=y_1).

Thus the MOD resultant is a MOD fixed point.

It is important to keep on record these examples are just only illustrated not related with any real world problem.

Let x = (1, 0, 0, 0, 0, 0, 1) be the initial state vector.

To find the effect of x on B.

$xB \to (1, 3, 0, 2, 0, 0, 1) = y_1$;
$y_1B \to (1, 3, 0, 2, 0, 2, 1) = y_2$;
$y_2B \to (1, 3, 0, 2, 0, 2, 1) = y_3$ (=y_2).

Thus the MOD resultant is a MOD fixed point.

We see the classical way of defining the mode of combining a finite number of MODCMs to get the joint effect of all MODCMs is not very feasible for some entries say

$$m_{ij} + m_{ks} = 0 \text{ (mod n)}.$$

So to over come this problem we make the following stipulations which is first illustrated by some examples and then by proper definition.

Example 2.14: Let C_1, C_2, C_3 and C_4 be four nodes associated with some problem.

Let three experts give the MOD directed graphs by taking edge weights from Z_4.

The MOD directed graphs given by them are as follows. The MOD directed graph given by the first expert;

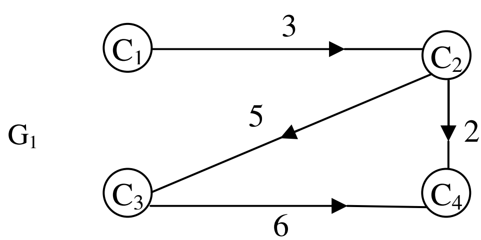


Figure 2.6

The MOD directed graph given by the second expert using edge weight from Z_7 is as follows.

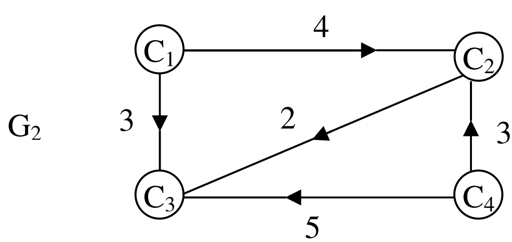


Figure 2.7

The MOD directed graph given by the third expert using the nodes C_1, C_2, C_3, C_4 and edge weights from Z_7 is as follows.

G_3

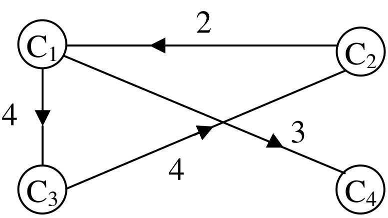


Figure 2.8

The corresponding MOD connection matrices associated with G_1, G_2 and G_3 be M_1, M_2 and M_3 respectively which is as follows.

$$M_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 0 & 3 & 0 & 0 \\ 0 & 0 & 5 & 2 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

$$M_2 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 0 & 4 & 3 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 5 & 0 \end{bmatrix} \end{array} \text{ and}$$

$$M_3 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 3 \\ 2 & 0 & 0 & 2 \\ 0 & 4 & 0 & 6 \\ 0 & 3 & 5 & 0 \end{bmatrix} \end{array}.$$

We now find $M_1 + M_2 + M_3$

$$= \begin{bmatrix} 0 & 3 & 0 & 0 \\ 0 & 0 & 5 & 2 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 4 & 3 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 5 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 4 & 3 \\ 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 3 \\ 2 & 0 & 0 & 2 \\ 0 & 4 & 0 & 6 \\ 0 & 3 & 5 & 0 \end{bmatrix} \end{array}.$$

However as $3 + 4 \equiv 0 \pmod 7$ and $6 = 2 \equiv 0 \pmod 7$ two of the effects has cancelled out.

This fact is misleading so we feel that the combined MOD dynamical system may not represent, the real situation of the problem or the experts opinion as this cancellation is wrong.

Further we cannot as in case of FCMs or NCMs go for simple ones where the edge weights are restrained to {0, 1} or {0, 1, I} respectively.

We define the new notion of special combination of MODCMs in the following.

DEFINITION 2.7: *Let s number of experts work with m nodes $C_1, C_2, \ldots, C_m$ and use for the MOD directed graphs $G_1, \ldots, G_s$ weights from Z_n. Let $M_1, \ldots, M_s$ be the collection of the MOD connection matrices associated with $G_1, G_2, \ldots, G_n$ respectively.*

$$\text{Let } M_t = (m_{ij}^t);\ 1 \le t \le s.$$

$$Let\ N_1 = M_1 + M_2 = (m_{ij}^1) + (m_{ij}^2)$$

$= (m_{ij}^1 + m_{ij}^2)$ *is taken as* $\left(\frac{m_{ij}^1 + m_{ij}^2}{2}\right)$ *if n is even and* $\frac{m_{ij}^1 + m_{ij}^2 + 1}{2}$ *if n is odd only, when ever* $m_{ij}^1 + m_{ij}^2 \equiv 0 \ (mod\ n)$ $(m_{ij}^1 \neq 0$ *and* $m_{ij}^2 \neq 0)$.

Now add $N_1 + M_3$ *use the same procedure as that used for* M_1 *and* M_2.

Repeat till all the s of the MOD matrices are used.

This will not cancel out however; how best it is suited is to be researched.

This final N_{s-1} *MOD matrix will be defined as the special combined MOD CM matrix or combined MODCM special dynamical system.*

We will illustrate this situation by an example.

Example 2.15: Let 5 experts work on a problem using the nodes $C_1, C_2, \ldots, C_6$ and the MOD directed graphs taking edge weights from Z_5. Let the corresponding MOD matrices of the five experts be M_1, M_2, M_3, M_4 and M_5 alone is given in the following:

M_1 be the MOD matrix given by expert one.

$$M_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & 3 & 0 & 0 & 2 & 1 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

M_2 be the MOD matrix given by the second expert

$$M_2 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & 2 & 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let M_3 be the MOD matrix given by the third expert

$$M_3 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 2 & 3 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let M_4 be the MOD matrix given by the fourth expert.

$$M_4 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & 3 & 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 & 0 & 3 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 \\ 1 & 3 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let M_5 be the MOD matrix given by the fifth expert

$$M_5 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & 4 & 0 & 0 & 0 & 2 \\ 0 & 0 & 4 & 0 & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 4 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

We find $N_1 = M_1 + M_2$

$$= \begin{bmatrix} 0 & 3 & 0 & 0 & 2 & 1 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 & 0 & 0 & 1 & 1 \\ 0 & 0 & 3 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \end{bmatrix}$$

using $\frac{1+m_{ij}^1+m_{ij}^2}{2}$ if $m_{ij}^1+m_{ij}^2 = 0$ (mod 5).

Next we find $N_1 + M_3 = N_2$.

$$N_1 + M_3 = N_2 = \begin{bmatrix} 0 & 3 & 0 & 0 & 1 & 1 \\ 0 & 0 & 3 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 2 & 3 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 4 & 0 & 0 & 2 & 3 \\ 0 & 0 & 4 & 0 & 0 & 3 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 3 & 0 & 0 & 0 & 0 & 4 \\ 0 & 3 & 3 & 0 & 0 & 0 \end{bmatrix}.$$

Now we find $N_3 = N_2 + M_4$.

$$= \begin{bmatrix} 0 & 4 & 0 & 0 & 2 & 3 \\ 0 & 0 & 4 & 0 & 0 & 3 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 3 & 0 & 0 & 0 & 0 & 4 \\ 0 & 3 & 3 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 3 & 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 & 0 & 3 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 \\ 1 & 3 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 0 & 0 & 4 & 3 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 & 2 & 0 \\ 3 & 0 & 0 & 0 & 0 & 3 \\ 1 & 1 & 3 & 0 & 0 & 0 \end{bmatrix} = N_3.$$

Next we find $N_4 = N_3 + M_5$

$$= \begin{bmatrix} 0 & 2 & 0 & 0 & 4 & 3 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 & 2 & 0 \\ 3 & 0 & 0 & 0 & 0 & 3 \\ 1 & 1 & 3 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 4 & 0 & 0 & 0 & 2 \\ 0 & 0 & 4 & 0 & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 4 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 & 4 & 3 \\ 0 & 0 & 3 & 0 & 0 & 3 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 2 & 0 & 0 & 2 & 2 \\ 3 & 0 & 0 & 0 & 0 & 1 \\ 1 & 3 & 3 & 0 & 0 & 0 \end{bmatrix}.$$

Thus N_4 is the special combined MOD matrix operator or the MOD special combined dynamical system.

We will test the effects of initial state vectors using M_1, M_2, M_3, M_4, M_5 and N_4.

Let x = (1, 0, 0, 0, 0, 0) be the initial state vector.

We find the effect of x on M_1, M_2, M_3, M_4, M_5 and N_4.

$$\begin{aligned}
xM_1 &\to (1, 3, 0, 0, 2, 1) = y_1;\\
y_1M_1 &= (1, 0, 2, 0, 2, 1) = y_2;\\
y_2M_1 &= (1, 2, 0, 1, 2, 1) = y_3;\\
y_3M_1 &= (1, 0, 3, 0, 2, 1) = y_4;\\
y_4M_1 &= (1, 0, 0, 4, 2, 1) = y_5;\\
y_5M_1 &= (1, 0, 0, 0, 2, 1) = y_6;\\
y_6M_1 &= (1, 0, 0, 0, 2, 1) = y_7\ (=y_6).
\end{aligned}$$

Effect of x on M_1 is a MOD fixed point given by

(1, 0, 0, 0, 2) ---- I

Next we find the effect of x on M.

$$\begin{aligned}
xM_2 &\to (1, 2, 0, 0, 4, 0) = y_1;\\
y_1M_2 &= (2, 2, 2, 0, 4, 2) = y_2;\\
y_2M_2 &= (2, 2, 2, 4, 3, 2) = y_3;\\
y_3M_2 &= (4, 1, 2, 4, 3, 2) = y_4;\\
y_4M_2 &= (4, 0, 1, 4, 1, 1) = y_5;\\
y_5M_2 &= (3, 1, 0, 2, 1, 0) = y_6;\\
y_6M_2 &= (3, 3, 1, 0, 2, 1) = y_7;\\
y_7M_2 &= (1, 0, 3, 2, 2, 3) = y_8;\\
y_8M_2 &= (1, 1, 0, 1, 4, 0) = y_9;\\
y_9M_2 &= (2, 3, 1, 0, 4, 1) = y_{10};\\
y_{10}M_2 &= (2, 3, 3, 2, 3, 3) = y_{11};\\
y_{11}M_2 &= (4, 3, 3, 1, 3, 3) = y_{12};\\
y_{12}M_2 &= (4, 1, 3, 1, 1, 3) = y_{13};\\
y_{13}M_2 &= (3, 1, 1, 1, 1, 1) = y_{14};\\
y_{14}M_2 &= (3, 1, 1, 2, 2, 1) = y_{15};
\end{aligned}$$

$y_{15}M_2 = (1, 2, 1, 2, 2, 1) = y_{16}$;
$y_{16}M_2 = (1, 3, 2, 2, 4, 2) = y_{17}$;
$y_{17}M_2 = (2, 2, 3, 4, 4, 3) = y_{18}$;
$y_{18}M_2 = (2, 0, 2, 1, 3, 2) = y_{19}$;
$y_{19}M_2 = (4, 3, 0, 4, 3, 0) = y_{20}$;
$y_{20}M_2 = (4, 2, 3, 0, 1, 3) = y_{21}$;
$y_{21}M_2 = (3, 0, 2, 1, 1, 2) = y_{22}$;
$y_{22}M_2 = (3, 0, 0, 4, 2, 0) = y_{23}$;
$y_{23}M_2 = (1, 0, 0, 0, 2, 0) = y_{24}$;
$y_{24}M_2 = (1, 2, 0, 0, 4, 0) = y_{25}$ ($=y_1$).

Thus the resultant is a MOD limit cycle given by
(1, 2, 0, 0, 4, 0) ---- II

Let us now find the effect of x = (1, 0, 0, 0, 0, 0) on the MOD matrix M_3.

$xM_3 \to (1, 1, 0, 0, 1, 2) = y_1$;
$y_1M_3 \to (1, 0, 2, 0, 1, 0) = y_2$;
$y_2M_3 \to (1, 1, 0, 2, 1, 1) = y_3$;
$y_3M_3 \to (1, 3, 4, 0, 0, 0) = y_4$;
$y_4M_3 \to (1, 1, 3, 4, 1, 4) = y_5$;
$y_5M_3 \to (1, 4, 3, 3, 4, 0) = y_6$;
$y_6M_3 \to (1, 1, 4, 3, 2, 4) = y_7$;
$y_7M_3 \to (1, 4, 3, 4, 2, 4) = y_8$;
$y_8M_3 \to (1, 4, 1, 3, 4, 1) = y_9$;
$y_9M_3 \to (1, 3, 2, 1, 2, 2) = y_{10}$;
$y_{10}M_3 \to (1, 0, 4, 2, 3, 2) = y_{11}$;
$y_{11}M_3 \to (1, 0, 1, 4, 0, 4) = y_{12}$;
$y_{12}M_3 \to (1, 4, 2, 1, 4, 2) = y_{13}$;
$y_{13}M_3 \to (1, 0, 0, 2, 3, 4) = y_{14}$;
$y_{14}M_3 \to (1, 4, 2, 0, 0, 4) = y_{15}$;
$y_{15}M_3 \to (1, 4, 1, 2, 1, 3) = y_{16}$;
$y_{16}M_3 \to (1, 2, 3, 1, 0, 2) = y_{17}$;
$y_{17}M_3 \to (1, 0, 3, 3, 3, 0) = y_{18}$;
$y_{18}M_3 \to (1, 1, 0, 3, 2, 4) = y_{19}$;
$y_{19}M_3 \to (1, 4, 3, 0, 2, 4) = y_{20}$;
$y_{20}M_3 \to (1, 4, 1, 3, 1, 1) = y_{21}$;

$$y_{21}M_3 \to (1, 3, 2, 1, 2, 2) = y_{22}\ (=y_{10}).$$

Thus the MOD resultant x on the MOD operator M is a MOD realized limit cycle given by (1, 3, 2, 1, 2, 2) --- III

Now consider the effect of x = (1, 0, 0, 0, 0, 0) on M_4;

$$
\begin{aligned}
&xM_4 \to (1, 3, 0, 0, 2, 0) = y_1;\\
&y_1M_4 = (4, 3, 1, 0, 2, 1) = y_2;\\
&y_2M_4 = (1, 0, 1, 3, 3, 1) = y_3;\\
&y_3M_4 = (1, 3, 0, 3, 2, 3) = y_4;\\
&y_4M_4 = (2, 4, 1, 0, 2, 1) = y_5;\\
&y_5M_4 = (1, 4, 3, 3, 4, 4) = y_6;\\
&y_6M_4 = (2, 2, 3, 4, 2, 1) = y_7;\\
&y_7M_4 = (1, 0, 4, 4, 4, 0) = y_8;\\
&y_8M_4 = (3, 4, 0, 2, 2, 4) = y_9;\\
&y_9M_4 = (3, 4, 3, 0, 4, 4) = y_{10};\\
&y_{10}M_4 = (2, 1, 3, 4, 1, 1) = y_{11};\\
&y_{11}M_4 = (3, 1, 2, 4, 4, 4) = y_{12};\\
&y_{12}M_4 = (2, 2, 2, 1, 1, 2) = y_{13};\\
&y_{13}M_4 = (4, 1, 4, 1, 4, 2) = y_{14};\\
&y_{14}M_4 = (1, 2, 2, 2, 3, 2) = y_{15};\\
&y_{15}M_4 = (3, 2, 4, 1, 2, 4) = y_{16};\\
&y_{16}M_4 = (3, 4, 4, 2, 1, 3) = y_{17};\\
&y_{17}M_4 = (1, 1, 3, 2, 1, 3) = y_{18};\\
&y_{18}M_4 = (1, 1, 2, 4, 2, 4) = y_{19};\\
&y_{19}M_4 = (3, 1, 2, 1, 2, 0) = y_{20};\\
&y_{20}M_4 = (4, 3, 2, 1, 1, 0) = y_{21};\\
&y_{21}M_4 = (2, 1, 1, 1, 3, 0) = y_{22};\\
&y_{22}M_4 = (1, 0, 2, 3, 4, 1) = y_{23};\\
&y_{23}M_4 = (4, 3, 0, 1, 2, 4) = y_{24};\\
&y_{24}M_4 = (3, 3, 1, 0, 3, 1) = y_{25};\\
&y_{25}M_4 = (2, 2, 1, 3, 1, 2) = y_{26};\\
&y_{26}M_4 = (4, 2, 4, 3, 4, 2) = y_{27};\\
&y_{27}M_4 = (1, 0, 4, 2, 3, 0) = y_{28};\\
&y_{28}M_4 = (1, 1, 0, 2, 2, 3) = y_{29};\\
&y_{29}M_4 = (2, 0, 2, 0, 2, 0) = y_{30};\\
&y_{30}M_4 = (4, 1, 0, 1, 4, 2) = y_{31};\\
&y_{31}M_4 = (1, 2, 2, 0, 3, 2) = y_{32};
\end{aligned}
$$

$y_{32}M_4 = (3, 4, 4, 4, 2, 4) = y_{33}$;
$y_{33}M_4 = (3, 0, 3, 2, 1, 4) = y_{34}$;
$y_{34}M_4 = (1, 4, 0, 4, 1, 1) = y_{35}$;
$y_{35}M_4 = (3, 2, 3, 0, 2, 3) = y_{36}$;
$y_{36}M_4 = (2, 3, 4, 4, 1, 3) = y_{37}$;
$y_{37}M_4 = (1, 1, 1, 2, 4, 0) = y_{38}$;
$y_{38}M_4 = (3, 1, 2, 3, 2, 2) = y_{39}$;
$y_{39}M_4 = (1, 2, 2, 1, 1, 0) = y_{40}$;
$y_{40}M_4 = (2, 2, 4, 1, 2, 2) = y_{41}$;
$y_{41}M_4 = (1, 1, 4, 2, 4, 3) = y_{42}$;
$y_{42}M_4 = (1, 0, 2, 2, 2, 2) = y_{43}$;
$y_{43}M_4 = (1, 2, 0, 1, 2, 2) = y_{44}$;
$y_{44}M_4 = (1, 3, 4, 0, 2, 3) = y_{45}$;
$y_{45}M_4 = (2, 2, 1, 2, 2, 1) = y_{46}$;
$y_{46}M_4 = (1, 2, 4, 3, 4, 3) = y_{47}$;
$y_{47}M_4 = (1, 4, 4, 2, 2, 0) = y_{48}$;
$y_{48}M_4 = (1, 4, 3, 2, 2, 4) = y_{49}$;
$y_{49}M_4 = (2, 2, 2, 4, 3, 0) = y_{50}$;
$y_{50}M_4 = (1, 2, 4, 1, 4, 4) = y_{51}$;
$y_{51}M_4 = (2, 4, 4, 2, 2, 0) = y_{52}$;
$y_{52}M_4 = (4, 4, 3, 2, 4, 4) = y_{53}$;
$y_{53}M_4 = (2, 2, 3, 4, 3, 1) = y_{54}$;
$y_{54}M_4 = (2, 0, 4, 4, 4, 4) = y_{55}$;
$y_{55}M_4 = (2, 4, 0, 2, 4, 4) = y_{56}$;
$y_{56}M_4 = (2, 1, 3, 0, 4, 1) = y_{57}$;
$y_{57}M_4 = (4, 4, 2, 4, 4, 2) = y_{58}$;
$y_{58}M_4 = (1, 4, 3, 1, 3, 1) = y_{59}$;
$y_{59}M_4 = (2, 0, 3, 4, 2, 0) = y_{60}$;
$y_{60}M_4 = (4, 2, 0, 4, 4, 2) = y_{61}$;
$y_{61}M_4 = (1, 4, 4, 0, 3, 0) = y_{62}$;
$y_{62}M_4 = (1, 3, 3, 2, 2, 0) = y_{63}$;
$y_{63}M_4 = (4, 1, 1, 4, 2, 1) = y_{64}$;
$y_{64}M_4 = (1, 1, 2, 3, 3, 0) = y_{65}$;
$y_{65}M_4 = (1, 0, 2, 1, 2, 1) = y_{66}$;
$y_{66}M_4 = (1, 0, 0, 1, 2, 2) = y_{67}$;
$y_{67}M_4 = (1, 3, 0, 0, 2, 2) = y_{68}$;
$y_{68}M_4 = (1, 4, 1, 0, 2, 1) = y_{69}$;
$y_{69}M_4 = (1, 1, 3, 3, 2, 0) = y_{70}$;
$y_{70}M_4 = (4, 0, 2, 4, 2, 0) = y_{71}$;

$y_{71}M_4 = (4, 3, 0, 1, 3, 2) = y_{72}$;
$y_{72}M_4 = (3, 2, 1, 0, 3, 2) = y_{73}$;
$y_{73}M_4 = (3, 0, 4, 3, 1, 4) = y_{74}$;
$y_{74}M_4 = (1, 3, 0, 2, 1, 1) = y_{75}$;
$y_{75}M_4 = (3, 4, 1, 0, 2, 0) = y_{76}$
and so on.

We are guaranteed of getting at a realized fixed point or a realized limit cycle. But we see this can go on to a large number of iterations.

Next we find the effect of x = (1, 0, 0, 0, 0, 0) on M_5.

$xM_5 = (1, 4, 0, 0, 0, 2) = y_1$;
$y_1M_5 \to (1, 2, 1, 0, 0, 0) = y_2$;
$y_2M_5 \to (1, 4, 3, 2, 0, 3) = y_3$;
$y_3M_5 \to (1, 4, 1, 1, 0, 4) = y_4$;
$y_4M_5 \to (1, 4, 1, 2, 0, 2) = y_5$;
$y_5M_4 \to (1, 0, 1, 2, 0, 4) = y_6$;
$y_6M_4 \to (1, 3, 0, 2, 0, 4) = y_7$;
$y_7M_4 \to (1, 3, 2, 0, 0, 2) = y_8$;
$y_8M_4 \to (1, 2, 2, 4, 0, 3) = y_9$;
$y_9M_4 \to (1, 2, 3, 4, 0, 4) = y_{10}$;
$y_{10}M_4 \to (1, 1, 3, 1, 0, 1) = y_{11}$;
$y_{11}M_4 \to (1, 2, 4, 1, 0, 1) = y_{12}$;
$y_{12}M_4 \to (1, 2, 3, 3, 0, 3) = y_{13}$;
$y_{13}M_4 \to (1, 3, 3, 1, 0, 2) = y_{14}$;
$y_{14}M_4 \to (1, 1, 2, 1, 0, 0) = y_{15}$;
$y_{15}M_4 \to (1, 3, 4, 4, 0, 3) = y_{16}$;
$y_{16}M_4 \to (1, 2, 2, 2, 0, 1) = y_{17}$;
$y_{17}M_4 \to (1, 1, 3, 4, 0, 0) = y_{18}$;
$y_{18}M_4 \to (1, 0, 4, 1, 0, 2) = y_{19}$;
$y_{19}M_4 \to (1, 1, 0, 3, 0, 4) = y_{20}$;
$y_{20}M_4 \to (1, 1, 4, 0, 0, 4) = y_{21}$;
$y_{21}M_4 \to (1, 0, 4, 3, 0, 4) = y_{22}$;
$y_{22}M_4 \to (1, 2, 0, 3, 0, 3) = y_{23}$;
$y_{23}M_4 \to (1, 3, 3, 0, 0, 2) = y_{24}$;
$y_{24}M_4 \to (1, 2, 2, 1, 0, 3) = y_{25}$;

$$y_{25}M_4 \to (1, 0, 3, 4, 0, 3) = y_{26};$$
$$y_{26}M_4 \to (1, 2, 0, 1, 0, 0) = y_{27};$$
$$y_{27}M_4 \to (1, 3, 3, 0, 0, 3) = y_{28};$$
$$y_{28}M_4 \to (1, 1, 2, 1, 0, 3) = y_{29};$$
$$y_{29}M_4 \to (1, 0, 4, 4, 0, 1) = y_{30};$$
$$y_{30}M_4 \to (1, 4, 0, 3, 0, 3) = y_{31};$$
$$y_{31}M_4 \to (1, 3, 1, 0, 0, 1) = y_{32};$$
$$y_{32}M_4 \to (1, 1, 2, 4, 0, 2) = y_{33};$$
$$y_{33}M_4 \to (1, 3, 4, 4, 0, 2) = y_{34};$$
$$y_{34}M_4 \to (1, 3, 2, 3, 0, 1) = y_{35};$$
$$y_{35}M_4 \to (1, 0, 2, 4, 0, 4) = y_{36};$$
$$y_{36}M_4 \to (1, 1, 0, 4, 0, 0) = y_{37};$$
$$y_{37}M_4 \to (1, 0, 4, 0, 0, 2) = y_{38};$$
$$y_{38}M_4 \to (1, 2, 0, 3, 0, 2) = y_{39};$$
$$y_{39}M_4 \to (1, 4, 3, 0, 0, 2) = y_{40};$$
$$y_{40}M_4 \to (1, 2, 3, 1, 0, 0) = y_{41};$$
$$y_{41}M_4 \to (1, 1, 3, 1, 0, 3) = y_{42};$$
$$y_{42}M_4 \to (1, 0, 4, 1, 0, 1) = y_{43};$$
$$y_{43}M_4 \to (1, 2, 0, 3, 0, 4) = y_{44};$$

and so on.

We find effect of x on N_4.

$$xN_4 \to (1, 1, 0, 0, 4, 3) = y_1;$$
$$y_1N_4 = (1, 0, 0, 0, 4, 0) = y_2;$$
$$y_2N_4 = (2, 1, 0, 0, 4, 2) = y_3;$$
$$y_3N_4 = (4, 3, 4, 0, 3, 3) = y_4;$$
$$y_4N_4 = (2, 3, 3, 1, 1, 4) = y_5;$$
$$y_5N_4 = (2, 1, 1, 2, 0, 3) = y_6;$$
$$y_6N_4 = (3, 0, 2, 4, 2, 3) = y_7 \text{ and so on.}$$

Now interested reader can compare the resultants of each of the models using Kosko distance [67].

Also one can take the combined value of the resultants using the procedure used in finding the combined MODCMs dynamical system (or MOD connection matrices of the MODCMs).

However it is an interesting work to write programs for MODCMs resultants.

Next we proceed onto describe the notion of a special type of combined MODCM analogous to that in FCMs given in [ ].

We will describe this by the following example.

Example 2.16: Let C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8 and C_9 be the attributes related with the problem. Let four experts work on this problem. MODCMs model using the edge weights from Z_6.

Let the first expert work on the problem using the nodes C_1, C_2, C_5 and C_6. The directed MOD graph G_1 given by him is as follows.

G_1

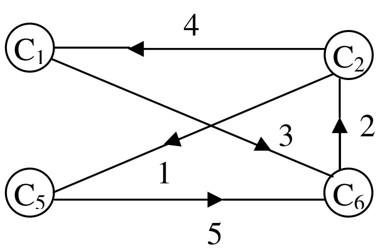


Figure 2.9

Let the MOD connection matrix M associated with G_1 is as follows.

$$M_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_5 \\ C_6 \end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 3 \\ 4 & 0 & 1 & 0 \\ 0 & 0 & 0 & 5 \\ 0 & 2 & 0 & 0 \end{bmatrix} \end{array}.$$

The second expert wishes to work with the nodes C_1, C_4, C_7, C_8 and C_9.

The MOD directed graph G_2 given by the second expert is as follows.

$G_2 =$

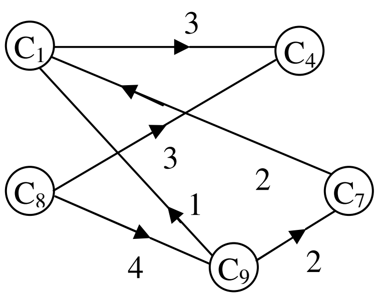


Figure 2.10

The MOD connection matrix M_2 associated with the MOD directed graph G_2 is given in the following:

$$M_2 = \begin{array}{c} \\ C_1 \\ C_4 \\ C_7 \\ C_8 \\ C_9 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_4 & C_7 & C_8 & C_9 \end{array} \\ \begin{bmatrix} 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 4 \\ 1 & 0 & 2 & 0 & 0 \end{bmatrix} \end{array}.$$

The third expert wishes to work with the nodes C_2, C_3, C_5, C_4 and C_8.

The MOD directed graph G_3 given by the third expert is as follows.

$G_3 =$

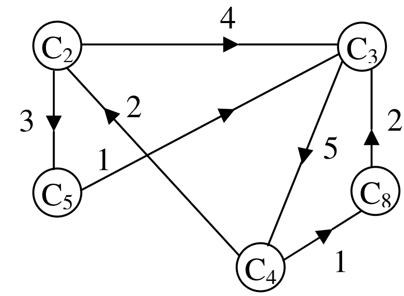


Figure 2.11

The MOD connection matrix M_3 associated withG_3 is as follows.

$$M_3 = \begin{array}{c} \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_8 \end{array} \begin{array}{c} \begin{array}{ccccc} C_2 & C_3 & C_4 & C_5 & C_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Now we finally give the MOD connection matrix and the MOD directed graph G_4 given by the fourth expert.

$G_4=$

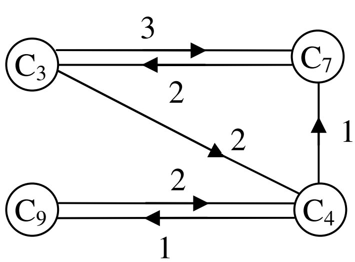


Figure 2.12

The MOD connection matrix M_4 associated with G_4 is as follows.

$$M_4 = \begin{array}{c} \\ C_3 \\ C_4 \\ C_7 \\ C_9 \end{array} \begin{array}{c} \begin{array}{cccc} C_3 & C_4 & C_7 & C_9 \end{array} \\ \begin{bmatrix} 0 & 2 & 3 & 0 \\ 0 & 0 & 1 & 1 \\ 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \end{array}.$$

Now using the four experts opinion we get the MOD combined special connection matrix M and the MOD combined special directed graph associated with M in the following.

$$\begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \end{array} \begin{array}{c} \begin{array}{ccccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 3 & 0 & 3 & 0 & 0 & 0 \\ 4 & 0 & 4 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 3 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 \end{bmatrix} \end{array}.$$

The MOD directed graph associated with M is as follows.

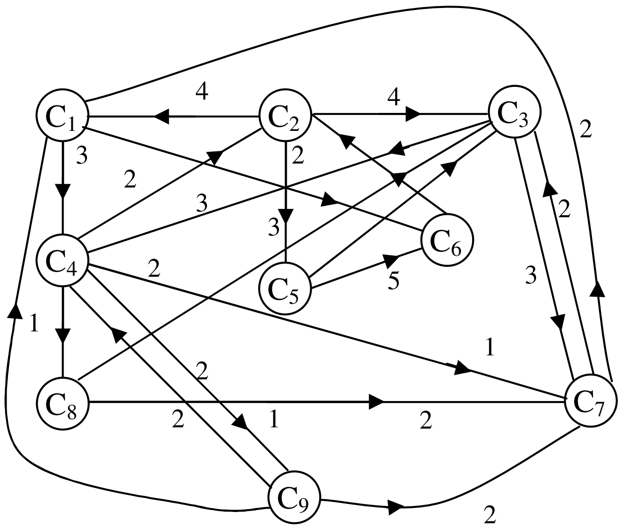


Figure 2.13

Here we believe any model technically defining and abstractly describing is not easy to understand by most of the researchers who are not mathematicians.

So we in almost all cases have tried to give examples or illustrative models which can be easily understood by socio scientists and other non mathematicians.

Thus we have two types of combined MODCMs, one type is that all the experts agree to work with all the m attributes C_1, C_2, …, C_m and they use for their MOD directed graphs only weighted elements of the graph from Z_n.

Then the first type of combined MODCMs model has the associated connection MOD matrix which is given by the method explained. It will be a MOD $m \times m$ matrix with entries from Z_n.

If there are some s experts who work with the problem but all of them do not work with all the attributes $C_1, C_2, \ldots, C_m$ but work with some subset of the attributes taken $C_1, C_2, \ldots, C_m$ only. Further all of them agree to work with the edge weights of the MOD directed only Z_n.

Thus if $G_1, G_2, \ldots, G_s$ are the MOD directed graphs given by them then we find the related MOD connection matrices and combine them in the special way as explained in the example 2.15 and obtain special combined MOD CM dynamical system.

Hence both the ways are distinct as the very method of approach of the experts who work with the problem are different. Thus the interested reader can construct both types of MOD CMs model and analyse the problem.

Next we proceed onto describe the merits or advantages of using MODCMs in place of FCMs and NCMs.

The first and foremost advantage is in the resultant we get the values of the MOD resultant vectors to be $(a_1, \ldots, a_m)$ where $a_i \in Z_m$ by which it gives not only the on-off state in case of FCMs but give a value of the state from Z_n when any of the C_i state is just on.

So this is significantly important as the value in Z_n is big then it is a most MOD influential node for the particular C_i.

As C_i's changes the MOD influential node also changes in general,

We will first describe this situation by an example or two.

Example 2.17: Let us consider the MODCM's connection matrix M which serves as the MOD dynamical system for a particular problem with entries from Z_{10}.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & 2 & 0 & 0 & 0 & 6 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 5 & 0 & 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 6 & 0 & 0 & 8 \\ 0 & 5 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

be the MODCMs dynamical system.

We find the MOD resultant of $x_1 = (1, 0, 0, 0, 0, 0)$ on M

$x_1M \to (1, 2, 0, 0, 0, 6) = y_1$;
$y_1M \to (1, 2, 8, 0, 0, 6) = y_2$;
$y_2M \to (1, 2, 8, 6, 0, 6) = y_3$;
$y_3M \to (1, 2, 8, 6, 0, 6) = y_4$; $(=y_3)$.

Thus the resultant is a MOD fixed point. The effect of C_1 which was in the on state remains the same. However C_3 happens to get the maximum values C_4 and C_6 happens to get the next great value as 6. C_2 gets the value to be 2. C_5 has no influence on C_1 directly or indirectly.

Let x_2 be the MOD initial state vector.

To find the MOD resultant of $x_2 = (0, 1, 0, 0, 0, 0)$ on M.

$x_2M \to (0, 1, 4, 0, 0, 0) = y_1$;
$y_1M \to (0, 1, 4, 8, 0, 0) = y_2$;
$y_2M \to (0, 1, 4, 8, 0, 0) = y_3$ $(=y_2)$.

Thus the MOD resultant is a fixed point given by
$(0, 1, 4, 8, 0, 0)$.

We see the node C_2 has the maximum influence on node C_4 and C_3 has also some medium effect over C_2.

However the nodes C_1, C_5 and C_6 are unaffected by C_2.

Consider the initial state vector $x_3 = (0, 0, 1, 0, 0, 0)$

To find the effect of x_3 on M;

$$x_3M \to (5, 0, 1, 7, 0, 0) = y_1;$$
$$y_1M \to (5, 0, 1, 7, 5, 0) = y_2;$$
$$y_2M \to (5, 0, 1, 7, 5, 0) = y_3\ (= y_2).$$

Thus the on state of C_3 has the maximum influences on C_4 followed by C_1, C_5 for C_1 and C_5 both has value 5.

However on state of C_3 node has no effect on C_2 and C_6.

Now we find the effect of
$x_4 = (0, 0, 0, 1, 0, 0)$ on M.

$$x_4M \to (0, 0, 0, 1, 5, 0) = y_1$$
$$y_1M \to (0, 0, 0, 1, 5, 0) = y_3\ (=y_1).$$

We see the on state of node C_4 has maximum and only influence on C_5 and nothing more all other states are unaffected by the on state of the node C_4.

Consider $x_5 = (0, 0, 0, 0, 1, 0)$ to be the initial state vector.

Effect of x_5 on M is

$$x_5M \to (0, 0, 6, 0, 1, 8) = y_1;$$
$$y_1M \to (0, 0, 6, 2, 1, 8) = y_2;$$
$$y_2M \to (0, 0, 6, 2, 1, 8) = y_3\ (=y_2).$$

The MOD resultant is a fixed point given by
$$(0, 0, 6, 2, 1, 8).$$

Clearly C_5 has the maximum effect of node C_6 as its value 8 following by the node C_3 whose value is 6 and the least affected node is C_4 whose value is 2 and the unaffected nodes are C_1 and C_2.

Finally let us study the effect of $x_6 = (0, 0, 0, 0, 0, 1)$ on M.

$$x_6M \to (0, 5, 0, 0, 0, 1) = y_1;$$
$$y_1M \to (0, 5, 0, 0, 0, 1) = y_2\ (=y_1).$$

Thus the MOD resultant is a MOD fixed point.

The on state of node C_6 influences only the node C_2 and all other nodes are left unaffected.

Thus by this MOD CM model from the MOD resultant we can get the nodes which are maximum influenced by the on state of the nodes in the initial state vector x.

Let a = (1, 0, 1, 0, 0, 0) be the initial state vector in which the nodes C_1 and C_3 alone are in the on state we study the effect.

$$aM \to (5, 2, 1, 7, 0, 6) = y_1;$$
$$y_1M \to (5, 0, 1, 7, 5, 0) = y_2;$$
$$y_2M \to (5, 0, 1, 7, 5, 0) = y_3\ (=y_2).$$

Thus the MOD resultant is a MOD fixed point.

The on state of node C_1 and C_3 makes C_4 the maximal influential nodes with value 7 however C_5 is also influenced with its value 5. The node C_3 influences C_1 and C_1 also is influenced to get a value 5.

But C_2 and C_6 remain unaffected by C_1 and C_3.

Here, we also study other possibilities of the on state of nodes and its influence on other. This method gives the influence exercised on other nodes for one is not also interested to find the impact that is the on state of a node that makes other nodes to on state but the amount of influence it exerts on the other state.

FCMs cannot cater to this sort of influence, only MODCMs models can give that sort of influence.

We also wish to study the following set up analogous to NCMs [ ]. In case of NCMs we see we can have in the resultant only values 0, 1 and I.

However if one of the concept is partly real and partly neutrosophic we will not be in a position to describe the situation.

Maximum adjustments, we can make is 1 + I which implies 50% real and 50% neutrosophic however in truth such is a very rare occurrence.

In no situation one has the 50 – 50 possibility. In order to over come all these short comings we define MOD Neutrosophic Cognitive Maps (MODNCMs) model using mod neutrosophic number $\{\langle Z_n \cup I\rangle\} = \{a + bI / a, b \in Z_n, I^2 = I\}$.

So in the resultant nodes can take real, pure neutrosophic and a + bI where a may not be equal to b or 0.

This is possible only in MODNCMs.

We now proceed onto define the notion of MODNCMs.

DEFINITION 2.8: *Let*

$$M = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ a_{21} & \dots & a_{2n} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix}$$

where $a_{ij} \in \langle Z_m \cup I\rangle = \{a + bI / a, b \in Z_m; I^2 = I\}$; $1 \leq i \leq n$ be the MOD neutrosophic $n \times n$ square matrix with entries from $\langle Z_m \cup I\rangle$.

We will illustrate this situation by some examples.

Example 2.18: Let

$$M = \begin{bmatrix} 3+2I & I & 0 \\ 4I & 0 & 3I \\ 5I+2 & 3 & 7+I \end{bmatrix}$$

be the MOD 3 × 3 neutrosophic matrix with entries from $\langle Z_8 \cup I\rangle = \{a + bI / a, b \in Z_8, I^2 = I$.

Example 2.19: Let

$$W = \begin{bmatrix} 8+4I & 6I & 0 & 3I \\ 0 & 7+I & 5 & 6I \\ 3+7I & 0 & 4+9I & 0 \\ 6 & 2+8I & 0 & 6+8I \end{bmatrix}$$

be the MOD 4 × 4 square neutrosophic matrix with entries from $\langle Z_{10} \cup I\rangle = \{a + bI / a, b \in Z_{10}, I^2 = I\}$.

Example 2.20: Let

$$S = \begin{bmatrix} 0 & 2I+1 & 3 & 5I & 0 \\ 6 & 0 & 4+2I & 0 & 7+3I \\ 5+I & 7I & 0 & 8+I & 0 \\ 0 & 6+5I & 8I & 0 & 3+I \\ 4+I & 5I & 0 & 6I & 7 \end{bmatrix}$$

be the MOD 5 × 5 square neutrosophic matrix with entries from $\langle Z_9 \cup I\rangle = \{a + bI / a, b \in Z_9, I^2 = I\}$.

Throughout this book we perform only a special type of operations on these MOD square neutrosophic matrices.

$X = \{(a_1, \ldots, a_n) / a_i \in \{0, 1, I\}; 1 \leq i \leq n\}$ denotes the collection of MOD row vectors also known as MOD initial state vectors.

Any way the vector can be mentioned and by all means by the context the situation would be clear.

We will just illustrate how this special type of operation is performed on MOD square neutrosophic matrices.

Example 2.21: Let

$$W = \begin{bmatrix} 6+I & 0 & 2I+5 & 0 & 1 \\ 0 & I & 0 & 2I & 1+3I \\ 4I+1 & 0 & 5 & 6I+2 & 0 \\ 0 & 3+4I & 0 & 2 & 4I \\ 5 & 0 & 3I & 0 & 0 \end{bmatrix}$$

Be the MOD 5×5 square neutrosophic matrix with entries from $\langle Z_7 \cup I\rangle$.

$$\text{Let } X = \{(a_1, a_2, a_3, a_4, a_5) / a_i \in \{0, I, 1\}; 1 \leq i \leq 5\}.$$

$$\text{Let } x = (1, 0, 0, 0, 0) \in X.$$

To find the effect of x on W;

$$xW = (6+I, 0, 2I+5, 0, 1) = y_1$$
$$y_1W = (5+6I, 0, 6+4I, I + 3, 6 + I) = y_2$$
$$y_2W = (3+2I, 2+5I, 6+2I, 4, 5+I) = y_3;$$

y_3W can be found.

We will certainly after a finite number of iterations arrive at a MOD fixed point or a MOD limit cycle. Clearly the elements are mixed neutrosophic.

Let x = (0, 0, 0, 0, 1) be the MOD initial state vector from X.

The effect of x on W is as follows.

$$
\begin{aligned}
xW &\to (5, 0, 3I, 0, 1) = y_1; \\
y_1W &= (6I, 0, 4, 3I, 5) = y_2; \\
y_2W &= (4 + 6I, 0, I + 6, 2I + 1, 4I) = y_3; \\
y_3W &= (2 + 4I, 3 + 4I, 1 + 4I, I, 4 + 4I) = y_4; \\
y_4W &= (4I + 5, 0, 1 + 4I, 2, 5I + 5) = y_5; \\
y_5W &= (5I, 6 + I, 3 + 3I, 6 + 3I, 5 + 5I) = y_6; \\
y_6W &= (4I, 4 + 3I, 1 + 3I, 4 + 6I, 6 +I) = y_7;
\end{aligned}
$$

and so on.

However we are guaranteed that after a finite number of iterations certainly we will arrive at a MOD fixed point or a MOD limit cycle.

Example 2.22: Let

$$
M = \begin{bmatrix} 0 & 2I+1 & 3 \\ 2 & 0 & 1+I \\ 3+I & I & 0 \end{bmatrix}
$$

be the MOD 3×3 neutrosophic square matrix with entries from $\langle Z_4 \cup I\rangle$.

Let $X = \{(a_1, a_2, a_3) / a_i \in \{0, 1, I\}; 1 \leq i \leq 3\}$ be the MOD initial state vector.

$x = (1, 0, 0) \in X$. To find the effect of x on M.

$$
\begin{aligned}
xM &\to (1, 2I + 1, 3) = y_1; \\
y_1M &= (3I + 3, 1 + I, I) = y_2; \\
y_2M &= (2 + 2I, 3, 2) = y_3;
\end{aligned}
$$

$y_3M = (2I, 2, 1 + I) = y_4;$
$y_4M = (3 + I, 0, 2) = y_5;$
$y_5M = (2 + 2I, 3 + 3I, 1 + I) = y_6;$
$y_6M = (1 + 3I, 2, 1 + 3I) = y_7;$
$y_7M = (3 + I, 1 + 3I, 1 + 3I) = y_8;$
$y_8M = (1 + 3I, 3 + I, 2I + 2) = y_9;$
$y_9M = (0, 1 + 3I, 2 + 2I) = y_{10};$
$y_{10}M = (1, 0, 1 + 3I) = y_{11};$
$y_{11}M = (3 + I, 2I + 1, 3) = y_{12};$
$y_{12}M = (3 +3I, 3, 2) = y_{13};$
$y_{13}M = (2I, 3 + I, 0) = y_{14};$
$y_{14}M = (2 + 2I, 2I, 3 + I) = y_{15};$
$y_{15}M = (1 + 3I, 2+2I, 2 + 2I) = y_{16};$
$y_{16}M = (2+2I, 1 + 3I, 1 + 3I) = y_{17};$
$y_{17}M = (1+3I, 2 + 2I, 3 + I) = y_{18};$
$y_{18}M = (1 + 3I, 1 + 3I, 1 + 3I) = y_{19};$
$y_{19}M = (1+3I, 3I+1, 0) = y_{20};$
$y_{20}M = (2I + 2, 1 + 3I, 0) = y_{21};$
$y_{21}M = (2 + 2I, 2 + 2I, 3 + I) = y_{22};$
$y_{22}M = (1 + 3I, 2 + 2I, 0) = y_{23};$
$y_{23}M = (1, 1 + 3I, 1 + 3I) = y_{24}\ (=y_{11}).$

The MOD resultant is a limit cycle.

This is got after 23 iterations.

Consider $x_2 = (0, 1, 0) \in X$.

To find the effect of x_2 on M.

$x_2M \to (2, 1, 1 + I) = y_1;$
$y_1M = (1 + I, 2 + 2I, 3 + I) = y_2;$
$y_2M = (1 + 3I, 1 + I, 1 + I) = y_3;$
$y_3M = (1 + 3I, 1 + I, 0) = y_4;$
$y_4M = (2 + 2I, 1 + 3I, 0) = y_5;$
$y_5M = (2 + 2I, 2 + 2I, 3 + I) = y_6;$
$y_6M = (1 + 3I, 2 + 2I, 0) = y_7;$
$y_7M = (0, 1 + 3I, 1 + 3I) = y_8;$

$y_8M = (3I + 1, 1, 1 + 3I) = y_9;$
$y_9M = (1 + I, 3I + 1, 2I) = y_{10};$
$y_{10}M = (2I +2, 1 + 3I, 2I) = y_{11};$
$y_{11}M = (2 + 2I, 2, 3 + I) = y_{12};$
$y_{12}M = (1 + 3I, 2 + 2I, 0) = y_{13}$ $(=y_7)$.

Thus the MOD resultant is a MOD limit cycle.

Let $x_3 = (0, 0, 1) \in X$.

To find the effect of x_3 on M.

$x_3M \to (3 + I, I, 1) = y_1;$
$y_1M = (3 + 3I, 3 + 2I, 1 + I) = y_2;$
$y_2M = (1 + I, 3 + I, 1) = y_3;$
$y_3M = (1+3I, 1 + 2I, 2) = y_4;$
$y_4M = (2I, 1 + I, 2I) = y_5;$
$y_5M = (2 + 2I, 0, 1 + I) = y_6;$
$y_6M = (3 + I, 2, 2 + 2I) = y_7;$
$y_7M = (2 + 2I, 3 + I, 3 + I) = y_8;$
$y_8M = (3 + I, 2 + 2I, 1 + 3I) = y_9;$
$y_9M = (3 + I, 3 + I, 3 + I) = y_{10};$
$y_{10}M = (3 + 3I, 3 + 2I, 2I) = y_{11};$

and so on.

Certainly after a finite number of iterations we will arrive at a MOD fixed point or MOD limit cycle.

Now having seen how only a special type of operation is needed in our study we proceed onto describe by examples the notion of MOD directed neutrosophic graphs.

Example 2.23:

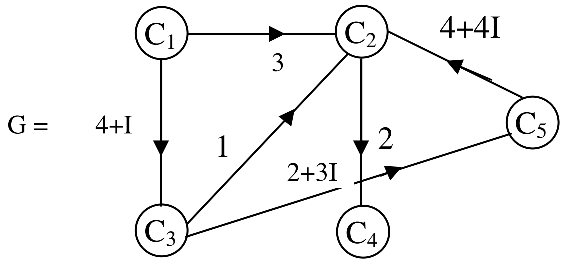


Figure 2.14

be the MOD directed graph G with edge weights from $\langle Z_5 \cup I \rangle = \{a + bI / a, b \in Z_5, I^2 = I\}$.

The MOD connection matrix M associated with G is as follows.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 0 & 3 & 4+1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 & 2+3I \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 4+4I & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

It is pertinent to keep on record that we are not using the dotted lines to show the edges are pure neutrosophic or mixed neutrosophic as the edge weights themselves will denote that we give yet another example of the MOD neutrosophic directed graph.

Example 2.24: Let G_1 be the MOD neutrosophic directed graph given by the following figure whose edge weights are taken from $\langle Z_{12} \cup I \rangle = \{a + bI / a, b \in Z_{12}, I^2 = I\}$.

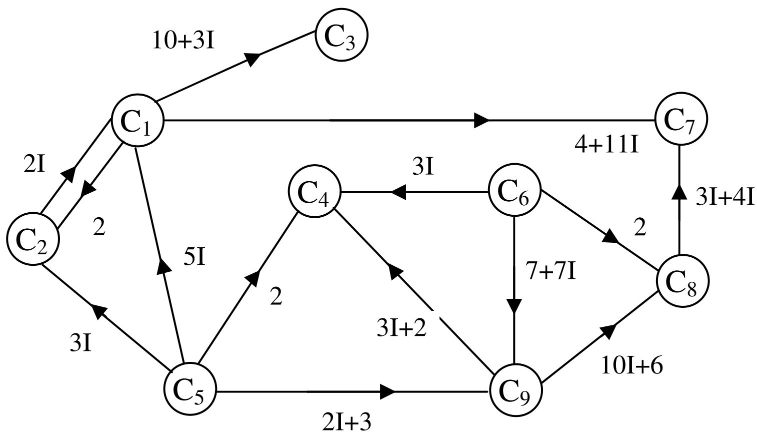


Figure 2.15

The MOD connection matrix associated with the MOD neutrosophic graph is as follows.

$$
M_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \end{array}
\begin{array}{c}
\begin{array}{ccccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 \end{array} \\
\left[\begin{array}{ccccccccc}
0 & 2 & 10+3I & 0 & 0 & 0 & 4+11I & 0 & 0 \\
2I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5I & 3I & 0 & 2 & 0 & 0 & 0 & 0 & 2I+3 \\
0 & 0 & 0 & 3I & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 3I+4 & 0 & 0 \\
0 & 0 & 0 & 3I+2 & 0 & 7+7I & 0 & 10I+6 & 0
\end{array}\right]
\end{array}.
$$

This is the way the MOD neutrosophic connection matrix M_1 is constructed using the MOD neutrosophic directed graph G_1.

Next we proceed onto describe the MOD Neutrosophic Cognitive Maps (MODNCMs) model. Suppose there are n-nodes say $C_1, C_2, \ldots, C_n$ associated with a problem P.

The expert wishes to take the edge weights from $\langle Z_m \cup I\rangle = \{a + bI / a, b \in Z_m, I^2 = I\}$ as the expert feels that some of the edges can be real or pure neutrosophic or mixed neutrosophic.

Let $X = \{(x_1, x_2, \ldots, x_n) / x_i \in \{0, 1, I\}; 1 \leq i \leq n\}$ be the collection of all MOD instantaneous / initial state vectors.

The expert gives the MOD directed neutrosophic graph G with edge weights from $\langle Z_m \cup I\rangle$.

Let M be the MOD connection neutrosophic matrix associated with MOD directed neutrosophic graph G.

M serves as the MOD neutrosophic cognitive maps model dynamical system, analogous to the MOD cognitive maps model dynamical system defined earlier in this chapter.

For more about the related concepts refer [ ].

We will illustrate this situation by an example or two.

Example 2.25: Let P be a problem and C_1, C_2, C_3, C_4 and C_5 be the five nodes associated with the problem.

The expert wishes to work with edge weights from
$\langle Z_3 \cup I\rangle = \{a + bI / a, b \in Z_3; I^2 = I\}$.

Let G be the MOD neutrosophic directed graph with edge weights from $\langle Z_3 \cup I\rangle$ given in the following figure.

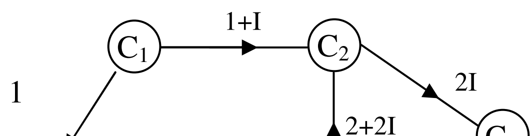


Figure 2.16

Let M be the MOD connection matrix associated with the MOD neutrosophic directed graph G.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 0 & 1+I & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2I \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 2+2I & 0 & 0 & I \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let $X = \{(a_1, a_2, a_3, a_4, a_5) / a_i \in \{0, 1, I\}; 1 \leq i \leq 5\}$ be the collection of MOD neutrosophic initial state vectors of the MOD dynamical neutrosophic system M.

Let $x_1 = (1, 0, 0, 0, 0) \in X$.

To find the effect of x on M;

$$\begin{aligned} x_1M &\to (1, 1 + I, 1, 0, 0) = y_1; \\ y_1M &\to (1, 1 + I, 1, 2, I) = y_2; \\ y_2M &\to (1, 2 + 2I, 1, 2, 0) = y_3; \\ y_3M &\to (1, 2 + 2I, 1, 2, 0) = y_4 (= y_3). \end{aligned}$$

Thus the MOD resultant is a fixed point.

Clearly the on state of the mode C_1 makes on only the three nodes C_2, C_3 and C_4. It has no effect on C_5.

However C_3 just become on but C_2 get the greatest mixed neutrosophic value or the neutrosophic value to be 2 + 2I. The node C_4 gets the greatest real value.

Thus by this new MOD neutrosophic cognitive maps model we see the nodes on or off state alone is not given but also the status of the node due to influence of the on state of the node in the initial state vector is also given.

By this method we can get the following C_3 and C_4 nodes are real but C_4 get the maximum real value and C_2 node gets the maximum neutrosophic value viz. 2 + 2I.

Let $x_2 = (0, 1, 0, 0, 0) \in X$ be the MOD initial state vector.

The effect of x_2 on M is as follows.

$$x_2M \to (0, 1, 0, 0, 2I) = y_1;$$
$$y_1M \to (0, 1, 0, 0, 2I) = y_2 (=y_1).$$

The MOD resultant is a fixed point and the on state of the node C_2 has no influence on the nodes C_1, C_3 and C_4, however it makes the node C_5 pure neutrosophic with maximum value 2I.

Let $x_3 = (0, 0, 1, 0, 0) \in X$ be the given initial state vector. To find the effect of x_3 on M.

$$x_3M \to (0, 0, 1, 2, 0) = y_1;$$
$$y_1M \to (0, 1 + I, 1, 2, 0) = y_2;$$
$$y_2M \to (0, 1 + I, 1, 2, 0) = y_3 (=y_2).$$

Thus the resultant is a MOD fixed point.

Clearly on state of the node C_3 has no effect on C_1 and C_5, but has greatest real effect on C_4 for the node C_4 value given by the fixed point is 2 and node C_2 takes the value 1+ I for it is 50% real and 50% neutrosophic when the node C_3 is the on state.

Consider the initial state vector $x_4 = (0, 0, 0, 1, 0) \in X$.

We find the effect of x_4 on M.

$$x_4M \to (0, 2 + 2I, 0, 1, I) = y_1;$$
$$y_1M \to (0, 2 + 2I, 0, 1, 0) = y_2;$$
$$y_2M \to (0, 2 + 2I, 0, 1, 0) = y_3 \ (=y_2).$$

Thus the resultant is a MOD fixed point given by $(0, 2 + 2I, 0, 1, 0)$.

We see when x_4 is in on state it has no effect or any impact on the nodes C_1, C_3 and C_5. It has impact only on the node C_2 and its value is $2 + 2I$ that is the greatest neutrosophic value in $\langle Z_3 \cup I\rangle$.

Finally we study the on state of the node C_5.

Let $x_5 = (0, 0, 0, 0, 1) \in X$.

The effect of x_5 on M is as follows.

$x_5M \to (0, 0, 0, 0, 1)$ is the MOD classical fixed point which is the resultant the on state of the node C_5 has no impact on any other nodes of the dynamical system.

Let $a_1 = (I, 0, 0, 0, 0) \in X$ be the initial state vector.

The effect of a on M is as follows.

$$a_1M \to (I, 2I, I, 0, 0) = y_1;$$
$$y_1M \to (I, I, I, 2I, I) = y_2;$$
$$y_2M \to (I, I, I, 2I, I) = y_3 \ (=y_2).$$

Thus the resultant is a MOD fixed point all coordinates come to pure neutrosophic state.

That is if C_1 is taken to be in the indeterminate state then C_2, C_3, C_4 and C_5 also come only to indeterminate state with C_4 taking the maximum indeterminate value.

None of the coordinates take real values.

In case of indeterminate state all other state becomes indeterminate.

Let $a_2 = (0, I, 0, 0, 0) \in X$; be the initial state vector.

To find the effect of a_2 on M.

$$a_2M \to (0, I, 0, 0, 2I) = y_1;$$
$$y_1M \to (0, I, 0, 0, 2I) = y_2\ (=y_1).$$

Thus the MOD resultant is a fixed point which makes only the node C_5 to on state. C_5 is also a maximum pure neutrosophic value.

Let $a_3 = (0, 0, I, 0, 0) \in X$ be the initial state vector.

To find the effect of a_3 on M;

$$a_3M \to (0, 0, I, 2I, 0) = y_1;$$
$$y_1M \to (0, 2I, I, 2I, 2I) = y_2;$$
$$y_2M \to (0, 2I, I, 2I, 0) = y_3;$$
$$y_3M \to (0, 2I, I, 2I, 0) = y_4.$$

Thus the resultant is a MOD fixed point.

The indeterminate state of the node C_3 makes the nodes C_2 and C_4 to be maximum pure neutrosophic as they take the value 2I and this C_3 has no effect on C_1 and C_5.

Let $a_4 = (0, 0, 0, I, 0) \in X$; to find the effect of a_4 on M.

$$a_4M \to (0, I, 0, I, I) = y_1;$$
$$y_1M \to (0, I, 0, I, I) = y_2\ (=y_1).$$

Thus the MOD resultant is a MOD fixed point.

The on state of C_4 as I has no effect on C_3 and C_1 however the nodes C_2 and C_5 have the pure neutrosophic state given by I.

Let $a_5 = (0, 0, 0, 0, I) \in X$ be the initial state vector.

Effect of a_5 on M gives $a_5M \to (0, 0, 0, 0, I)$.

Thus the MOD resultant is a MOD special classical fixed point.

The on state of C_5 has no effect on any of the nodes C_1, C_2, C_3 and C_4.

Example 2.26: Let P_1 be a problem at hand and let C_1, C_2, C_3, C_4 be the four nodes associated with the problem.

The edge weight of the MOD directed neutrosophic graph are taken from $\langle Z_6 \cup I\rangle = \{a + bI / a, b \in Z_6, I^2 = I\}$.

Let G_1 be the MOD neutrosophic directed graph given by the expert which is as follows.

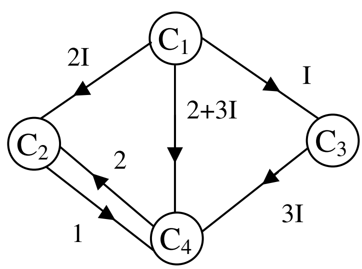


Figure 2.17

The MOD neutrosophic connection matrix N associated with the MOD neutrosophic directed graph G_1 is as follows.

$$N = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 0 & 2I & 2+3I & I \\ 0 & 0 & 0 & I \\ 0 & 0 & 0 & 3I \\ 0 & 2 & 0 & 0 \end{bmatrix} \end{array}$$

Let $x_1 = (1, 0, 0, 0)$ be the MOD initial state vector from $X = \{(a_1, a_2, a_3, a_4) / a_i \in \{0, 1, I\}, 1 \leq i \leq 4\}$.

The effect of x_1 on N

$x_1N \to (1, 2I, 2 + 3I, I) = y_1$;
$y_{1N} \to (1, 4I, 2 + 3I, 0) = y_2$;
$y_2N \to (1, 2I, 2 + 3I, I) = y_3$ $(=y_1)$.

Thus the MOD resultant is a MOD limit cycle. The on state of C_1 makes all the other nodes C_2, C_3 and C_4 to on state C_2 and C_2 are pure neutrosophic values whereas C_3 is a mixed neutrosophic value $2 + 3I$.

Let $x_2 = (0, 1, 0, 0) \in X$.

To find the effect of x_2 on the MOD neutrosophic dynamical system N.

$x_2N \to (0, 1, 0, 1) = y_1$; $y_1N = (0, 2, 0, 1) = y_2$;
$y_2N = (0, 2, 0, 2) = y_3$; $y_3N = (0, 4, 0, 4) = y_4$;
$y_4\, N_3 \to (0, 2, 0, 2)$.

Thus the MOD resultant in y_5 $(=y_3)$ this case is a MOD limit cycle. On state of C_2 only makes on the node C_4 to a value 1.

All the other nodes remain in the off state.

Let $x_3 = (0, 0, 1, 0) \in X$.

To find the effect of x_3 on N

$$x_3N \to (0, 0, 1, 3I) = y_1;$$
$$y_1N \to (0, 0, 1, 3I) = y_2 \ (=y_1).$$

Thus the MOD resultant of x_3 is a MOD fixed point. The on state of the node C_3 has no effect on x_1 and x_2 it has effect on C_4 and C_4 takes the pure neutrosophic value 3I.

Let $x_4 = (0, 0, 0, 1) \in x$.

$x_4N \to (0, 2, 0, 1) = y_1;$ $\quad$ $y_1N \to (0, 2, 0, 2) = y_2;$
$y_2N \to (0, 4, 0, 2) = y_3;$ $\quad$ $y_3N \to (0, 4, 0, 4) = y_4;$
$y_4N \to (0, 2, 0, 4) = y_5$
$y_5N \to (0, 2, 0, 2) = y_6 \ (=y_2).$

Thus the MOD resultant is a MOD limit cycle. The on state of the node C_4 makes only the node C_2 to on state and all other nodes remain only in the off state.

Further we can find all properties related with MOD Neutrosophic Cognitive Maps model as in case of MOD cognitive maps model with some appropriate changes.

This task is left as an exercise to the reader.

Next we will describe MOD complex cognitive maps model.

It may so happen that an expert / researcher may have to work with a problem which may have imaginary values associated with it in such cases this model can be adopted.

Infact the nodes can also be complex or mixed complex value.

However the MOD initial state vector

$X = (\{a_1, a_2, \ldots, a_m) / a_i \in \{0, 1, i_F\};\ i_F^2 = n - 1;\ 1 \leq i \leq m\}$. To this end we first define describe and develop the notion of MOD directed complex graph.

DEFINITION 2.9: *Let G be a directed graph with edge weights from $C(Z_n) = \{a + bi_F / a, b \in Z_m;\ i_F^2 = m - 1\}$. Then G is defined as the MOD directed finite complex integer graph.*

We will illustrate this situation by some examples.

Example 2.27: Let G_1 be the MOD directed finite complex number graph with edge weights from $C(Z_5)$ given in the following figure.

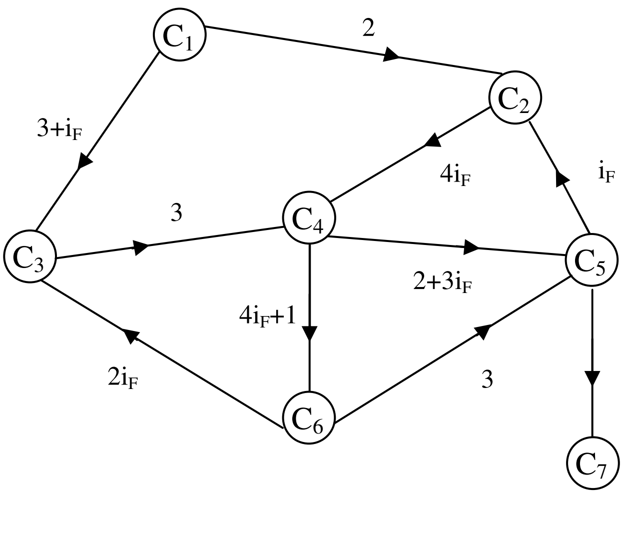


Figure 2.18

The MOD connection complex matrix or adjacency complex matrix, M_1 associated with the MOD-directed complex graph G_1 is as follows.

$$M_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \end{array}\begin{array}{c} \begin{array}{ccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 \end{array} \\ \begin{bmatrix} 0 & 2 & 3+i_F & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4i_F & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+3i_F & 1+4i_F & 0 \\ 0 & i_F & 0 & 0 & 0 & 0 & I \\ 0 & 0 & 2i_F & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

We will call any matrix with entries from $C(Z_n)$ as MOD complex matrix.

Example 2.28: Let G_2 be the MOD directed finite complex modulo integer graph given by the following figure with entries from $C(Z_{10})$.

G =

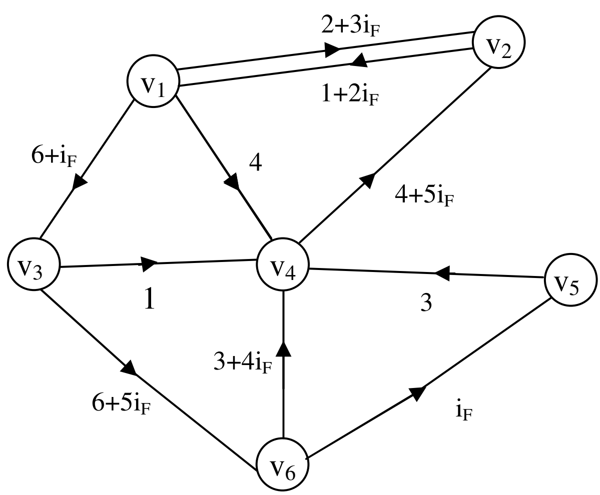


Figure 2.19

Example 2.29: Let G_3 be the MOD directed finite complex modulo integer graph with entries from $C(Z_7)$ given by the following figure.

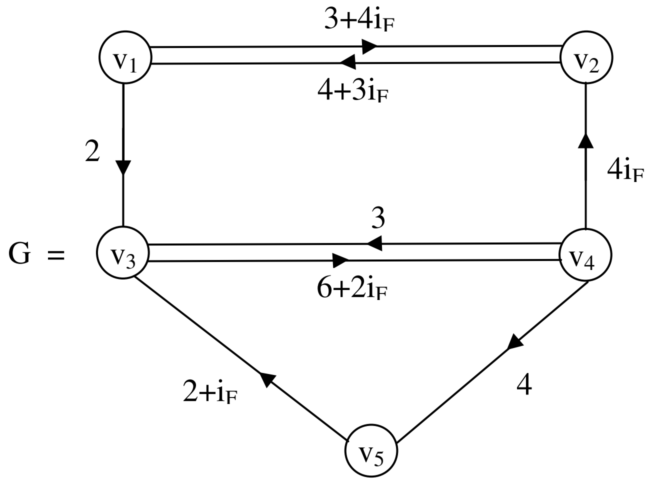


Figure 2.20

Now in the following we give a few examples of MOD finite complex modulo integer square matrices with entries from $C(Z_n)$.

We define and describe only a special type of operation on them.

Infact for more about these refer [ ].

Example 2.30: Let

$$M_1 = \begin{bmatrix} 3i_F & 1 & 0 \\ 0 & 2+4i_F & 3 \\ 4+i_F & 0 & 2+3i_F \end{bmatrix}$$

be the MOD 3 × 3 finite complex modulo integer matrix with entries from $C(Z_5)$.

Let $X = \{(a_1, a_2, a_3) / a_i \in \{0, 1, i_F); 1 \leq i \leq 3\}$ be the collection of MOD finite complex modulo integer special vectors.

We define special type of operation using elements of $\times$ and M_1 which is described in the following.

Let $x = (1, 0, 0) \in X$.
To find the effect of x on M_1;

$xM_1 \to (3i_F, 1, 0) = y_1$; $y_1M_1 = (1, 2 + 2i_F, 3) = y_2$;
$y_2M_1 = (i_F + 2, 4 + 2i_F, 2) = y_3$;
$y_3M_1 = (i_F, 2 + i_F, 1 + 2i_F) = y_4$;
$y_4M_1 = (4+4i_F, i_F, 2) = y_5$;
$y_5M_1 = (4i_F, i_F, 4 + 4i_F) = y_6$;
$y_6M_1 = (1, 1+i_F, 1 + 3i_F) = y_7$;
$y_7M_1 = (1 + i_F, 4 + i_F, 1 + 2i_F) = y_8$;
$y_8M_1 = (4+2i_F, 4i_F, 3) = y_9$;
$y_9M_1 = (1, 3, 1 + i_F) = y_{10}$;
$y_{10}M_1 = (3 + 3i_F, 2 + 2i_F, 3) = y_{11}$;

and so on.

However we are sure after a finite number of iterations we will arrive at a MOD fixed point or a MOD limit cycle.

Let $x_2 = (0, 1, 0) \in X$; to find the effect of x_2 on M_1.

$x_2M_1 = (0, 2 + 4i_F, 3) = y_1$;
$y_1M_1 = (2 + 3i_F, 3+ i_F, 2 + i_F) = y_2$;
$y_2M_1 = (3 + 2i_F, 4 + 2i_F, i_F) = y_3$;
$y_3M_1 = (3i_F + 3, 3 + 2i_F, 4 + 3i_F) = y_4$;
$y_4M_1 = (4, 1 + 4i_F, 3 + 4i_F) = y_5$;
$y_5M_1 = (3 + i_F, 2i_F, 2 + 4i_F) = y_6$;

and so on.

We are sure after a finite number of steps we will arrive at a MOD fixed point or a limit cycle.

Let $x_3 = (0, 0, 1) \in X$.

To find the effect of x_3 on M_1;

$x_3M_1 = (4 + i_F, 0, 2 + 3i_F) = y_1$;
$y_1M_1 = (2 + i_F, 4 + i_F, 2i_F) = y_2$;
$y_2M_1 = (2 + 4i_F, 1 + 4i_F, 1 + 2i_F) = y_3$;
$y_3M_1 = (2 + 4i_F, 1 + 2i_F, 4 + 4i_F) = y_4$;
$y_4M_1 = (i_F, 1 + 2i_F, 4 + 1i_F) = y_5$;
$y_5M_1 = (2 + 3i_F, 4 + 4i_F, 3) = y_6$;
$y_6M_1 = (4i_F + 3, 4 + 3i_F, 3+i_F) = y_7$;
y_7M_1 and so on.

We are sure to arrive at a MOD fixed point or a MOD limit cycle.

Example 2.31: Let M_2 be the MOD complex finite modulo integer matrix with entries from $C(Z_4)$.

$$M_2 = \begin{bmatrix} 0 & 2+i_F & 0 & 0 & 0 \\ 3i_F & 0 & 1 & 0 & 1+i_F \\ 0 & 2 & 0 & i_F & 0 \\ 1 & 0 & 2+i_F & 0 & 5 \\ 0 & i_F & 0 & 1+i_F & 0 \end{bmatrix}.$$

Let $X = \{(a_1, a_2, a_3, a_4, a_5) / a_i \in \{0, 1, i_F\}; 1 \leq i \leq 5\}$ be the special MOD row vectors.

Let $x_1 = (1, 0, 0, 0, 0) \in X$; to find the effect of x_1 on M_2;

$x_1M_2 \to (1, 2 + i_F, 0, 0 ,0) = y_1$;
$y_1M_2 = (2i_F + 1, 2 + i_F, 2 + i_F, 0, 1 + 3i_F) = y_2$;
$y_2M_2 = (2 + i_F, 1, 2 + i_F, i_F + 3, 1 + 3i_F) = y_3$;
$y_3M_2 = (3, 2 + 3i_F, 1 + i_F, 1 + 2i_F, 2) = y_4$;
$y_4M_2 = (3i_F, 3i_F, 2, 1 + 3i_F, 3i_F) = y_5$;
$y_5M_1 = (3i_F, 2 + 2i_F, 3, 1 + i_F, 2 + 2i_F)$ and so on.

Certainly after a finite number of iterations we will arrive at a MOD fixed point or a MOD limit cycle.

Let $x_4 = (0, 0, 0, 1, 0) \in X$.

To find the effect of x_4 on M_2;

$x_4M_2 \to (1, 0, 2 + i_F, 1, 3) = y_1$;
$y_1M_2 = (1, 2 + 2i_F, 2 + i_F, 2i_F + 3, 3) = y_2$;
$y_2M_2 = (1, 2 + 2i_F, 3i_F, 2 + i_F, 1 + 2i_F)$; and so on.

After a finite number of iterations we arrive at a MOD fixed point or a MOD fixed cycle.

Now we proceed onto describe a few MOD directed complex finite modulo integer graphs and then related MOD complex finite modulo integer matrix.

Example 2.32: Let H be the MOD directed complex number graph with edge weights from $C(Z_{10})$.

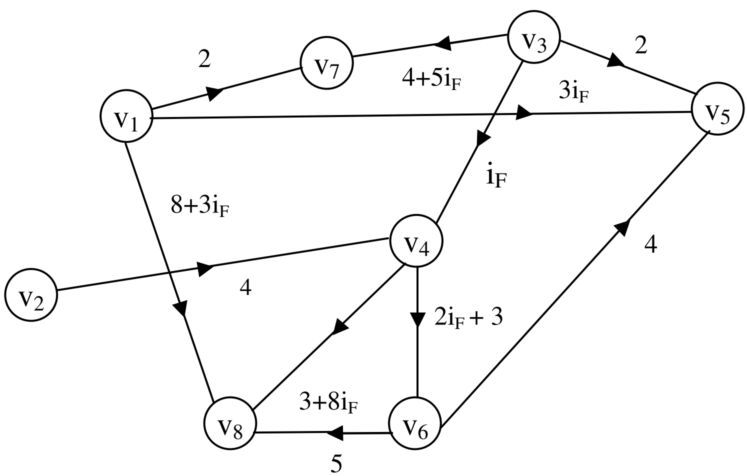
H =


Figure 2.21

Let P be the MOD matrix associated with the MOD directed complex modulo integer graph.

$$P = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\ \left[\begin{array}{cccccccc} 0 & 0 & 0 & 0 & 3i_F & 0 & 2 & 8+3i_F \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i_F & 2 & 0 & 4+5i_F & 0 \\ 0 & 0 & 0 & 0 & 0 & 2i_F+3 & 0 & 3+8i_F \\ 0 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}.$$

Having seen the notion of MOD directed finite complex modulo integer graphs and MOD finite complex modulo integer square matrices, we proceed on to describe the MOD finite complex modulo integer cognitive maps model or in short MOD Complex Cognitive Maps (MODCCM) model.

Let P be a problem in hand $C_1, C_2, \ldots, C_t$ be the t nodes / attributes associated with the problem P.

Suppose an expert wishes to work with this problem and takes for values of directed edges from $C(Z_n) = \{a + bi_F / a, b \in Z_n;\ i_F^2 = (n-1)\}$.

That is this system works with MOD complex integer $t \times t$ matrix with entries from $C(Z_n)$.

Clearly the model given by him is a MOD complex cognitive maps model.

We will illustrate this by an example or two.

Example 2.33: Let P be a problem which has C_1, C_2, C_3, C_4, C_5, C_6 to be 6 nodes. The expert gives the MOD directed graph G with edge weights from $C(Z_5)$ which is as follows.

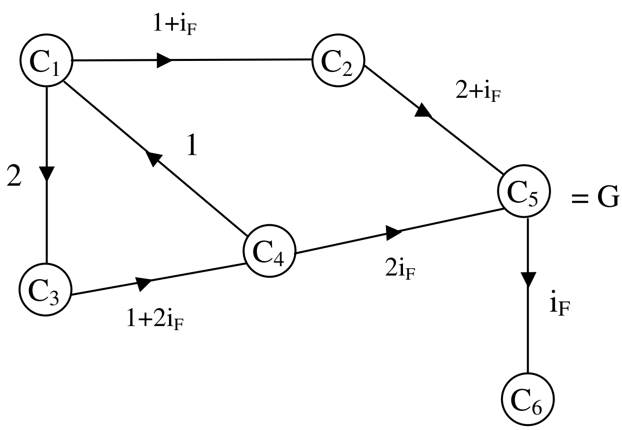


Figure 2.22

The MOD connection matrix M associated with graph G is as follows.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array}\begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & 1+i_F & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+i_F & 0 \\ 0 & 0 & 0 & 1+2i_F & 0 & 0 \\ 1 & 0 & 0 & 0 & 2i_F & 0 \\ 0 & 0 & 0 & 0 & 0 & i_F \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let $X = \{(a_1, a_2, a_3, a_4, a_5, a_6) / a_i \in \{0, 1, i_F\}; 1 \leq i \leq 6\}$ be the MOD initial special state vector associated with the MOD complex matrix M which serves as the MODCCMs dynamical system.

Let $x_1 = (1, 0, 0, 0, 0, 0) \in X$.

To find the effect of x_1 on M;

$x_1M \to (1, 1 + i_F, 2, 0, 0, 0) = y_1$;
$y_1M \to (1, 1 + i_F, 2, 2 + i_F, 1, 0) = y_2$;
$y_2M = (2 + i_F, 1 + i_F, 2, 2 + i_F, 2 + i_F, i_F) = y_3$;
$y_3M = (2 + i_F, 1, 1 + 2i_F, 2 + i_F, 2 + i_F, 2i_F + 2) = y_4$;
$y_4M = (2 + i_F, 1, 1 + 2i_F, i_F, 2i_F, 2i_F + 2) = y_5$ and so on.

We will certainly arrive at a MOD fixed point or MOD limit cycle.

Let $a = (0, 0, 0, 0, 1, 0) \in X$.

To find the effect of a on M.

$aM \to (0, 0, 0, 0, 1, i_F) = y_1$;
$y_1M \to (0, 0, 0, 0, 1, i_F) = y_2$ $(=y_1)$.

Thus the resultant is a MOD fixed point given by $(0, 0, 0, 0, 1, i_F)$.

Let $b = (0, 0, 0, 0, 0, 1) \in X$.

To find the effect of b on M.

$bM \to (0, 0, 0, 0, 0, 1)$.

Thus the resultant is a MOD is a special classical fixed point.

Let $c = (0, 0, 0, 1, 0, 0) \in X$.

To find the effect of c on M.

$cM \to (1, 0, 0, 1, 2i_F, 0) = y_1$;
$y_1M \to (1, 1 + i_F, 2, 1, 2i_F, 1) = y_2$;
$y_2M = (1, 1 + i_F, 2, 2 + i_F, 1 + 2i_F, 1) = y_3$;
$y_3M = (2 + i_F, 1 + i_F, 2, 2 + i_F, i_F, 1 + i_F) = y_4$

and so on.

Certainly after a finite number of iterations we will arrive at a MOD fixed point or a MOD limit cycle.

Let us consider $x_3 = (0, 0, 1, 0, 0, 0)$ be the initial state vector.

To find the effect of x_3 on M.

$x_3M \to (0, 0, 1, 1 + 2i_F, 0, 0) = y_1$;
$y_1M \to (1 + 2i_F, 0, 1, 1 + 2i_F, 2i_F + 2, 0) = y_2$;
$y_2M \to (1 + 2i_F, 0, 2 + i_F, 1 + 2i_F, 2i_F + 2, 2i_F + 1) = y_3$;
$y_3M = (2 + i_F, 0, 1 + 2i_F, 2i_F, 2i_F + 2, 2i_F + 1) = y_4$;
$y_4M = (2i_F, 1, 1 + 2i_F, i_F, 2, 2i_F + 1) = y_5$;
$y_5M = (i_F, 2i_F + 1, i_F, i_F, 2, 1 + i_F) = y_6$;
$y_6M = (i_F, i_F + 2, 2i_F, i_F + 1, i_F, 2i_F) = y_7$;
$y_7M = (1 + i_F, i_F + 2, 2i_F, 2 + 2i_F, i_F + 1, 2)$ and so on.

We will yet on more example in which we show that the nodes can be or 1 or pure complex or mixed complex of the form $a + bi_F$, $a \neq 0$, $b \neq 0$ depending on the on state of the node of the MOD finite complex cognitive maps model.

Example 2.34: Let P be the problem for which the following MOD directed finite complex number graph G with edge weights from $C(Z_6)$ is given in the following

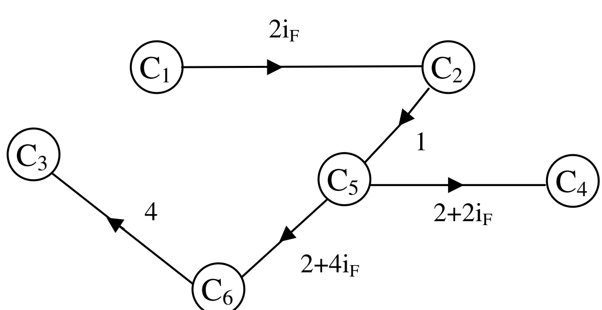


Figure 2.23

Let S be the MOD connection complex integer matrix associated with the MOD finite complex number directed graph which serves as the MOD complex cognitive maps model dynamical system.

Let $X = \{(a_1, a_2, \ldots, a_6) / a_i \in \{0, 1, i_F\}, 1 \leq i \leq 6\}$ be the special MOD initial complex vectors associated with the MOD CC maps model.

$$S = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & 2i_F & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2i_F & 0 & 2+4i_F \\ 0 & 0 & 4 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let $x_1 = (1, 0, 0, 0, 0, 0) \in X$.

The effect of x_1 on S.

$x_1S \to (1, 2i_F, 0, 0, 0, 0) = y_1;$
$y_1S \to (1, 2i_F, 0, 0, 2i_F, 0) = y_2;$
$y_2S \to (1, 2i_F, 0, 4i_F + 2, 2i_F, 4i_F + 4) = y_3;$
$y_3S \to (1, 2i_F, 4 + 4i_F, 4i_F + 4, 2i_F, 4) = y_4;$
$y_4S \to (1, 2i_F, 4, 4i_F + 2, 2i_F, 4i_F + 4) = y_5;$
$y_5S \to (1, 2i_F, 4 + 4i_F, 4i_F + 2, 2i_F, 4i_F + 4) = y_7$ $(=y_6)$.

We see the on state of the node C_1 makes all the other nodes to on state.

The node C_2 is purely complex. Node C_3 is mixed complex so is C_4. The node C_5 is pure complex.

The node C_6 is again a mixed complex number.

Thus we see on state of a node can give different types of states on other nodes however in FCMs its effect can give only on or off state of the other nodes whereas in case of NCMs the effect can be on or off or I the indeterminate state.

Let $x_2 = (0, 1, 0, 0, 0, 0) \in X$, to find the effect of x_2 on S.

$x_2S \to (0, 1, 0, 0, 1, 0) = y_1$;
$y_1S \to (0, 1, 0, 2 + 2i_F, 1, 4i_F + 2) = y_2$;
$y_2S \to (0, 1, 2 + 4i_F, 2 + 2i_F, 1, 2 + 4i_F) = y_3$;
$y_3S \to (0, 1, 2 + 4i_F, 2 + 2i_F, 1, 2 + 4i_F) = y_4 (=y_3)$.

Thus the MOD resultant is a MOD fixed point.

The on state of C_2 makes on the nodes C_3, C_4, C_5 and C_6.

However C_1 remains unaffected by the on state of C_2. The nodes C_4 and C_3 becomes mixed a complex number states. The node C_5 just has become on.

The node C_6 has become a mixed complex number state. Thus this is the main importance of the MODCCMs model.

All other properties like combined and special combined MODCCMs model can be derived. This is considered as a matter of routine so left as an exercise to the reader.

Now we can work with MOD dual number Cognitive Maps model using the MOD dual number
$\langle Z_n \cup g\rangle = \{a + bg / a, b \in Z_n, g^2 = 0\}$.

Let P be a problem at times one may have to work with dual numbers. Then can we have some way to achieve this. The answer is yes. The MOD dual number Cognitive Maps model can serve the purpose.

In order to have this model we need to have the notion of MOD directed dual number graph and MOD dual number matrices.

We will describe them with examples.

Example 2.35: Let G be the directed graph with dual weights from $\langle Z_4 \cup g\rangle = \{a + bg / a, b \in Z_4; g^2 = 0\}$.

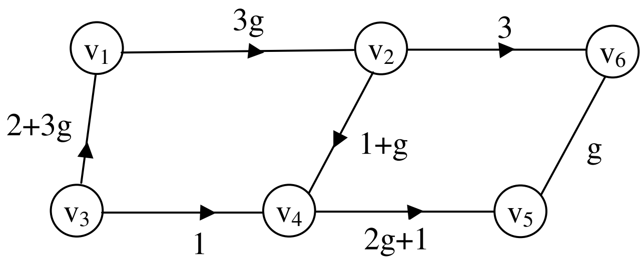


Figure 2.24

This graph will be known as the MOD dual number directed graph.

Example 2.36: Let G_1 be the directed dual number graph with edge weights from $\langle Z_7 \cup g\rangle = \{a + bg / a, b \in Z_7, g^2 = 0\}$ given by the following figure.

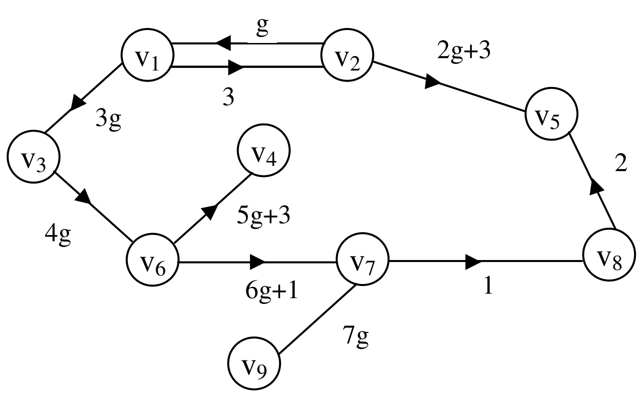


Figure 2.25

Now we define the notion of MOD dual number directed graph or MOD directed dual number graph.

DEFINITION 2.10: *Let G be a directed graph with vertices v_1, …, v_n and if the edge weights are taken from $\langle Z_m \cup g\rangle = \{a + bg / a, b \in Z_m, g^2 = 0\}$ then we define G to be the MOD dual number directed graph or MOD directed dual number graph.*

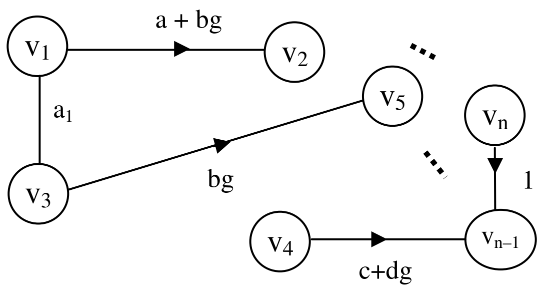


Figure 2.26

where a, b, c, d, a, b_1, $1 \in Z_m \setminus \{0\}$ $(2 \leq n < \infty)$.

We first describe MOD dual number square matrices.

Example 2.37: Let

$$M = \begin{bmatrix} 9+2g & 0 & 3 & 4g \\ 0 & 5+2g & 0 & 9g+3 \\ 3g & 0 & 2+g & 0 \\ 1 & 3 & 5g+1 & 2+9g \end{bmatrix}$$

be a 4×4 square matrix with entries from $\langle Z_{10} \cup g\rangle = \{a + bg / a, b \in Z_{10}, g^2 = 0\}$.

M is a MOD 4×4 square dual number matrix with entries from $\langle Z_{10} \cup g\rangle$.

Example 2.38: Let

$$P = \begin{bmatrix} 10+2g & 0 & 5g & 4+3g & 3 & 0 \\ 0 & 5g+5 & 0 & 2 & 9g+1 & 4g \\ 4g & 0 & 8g+1 & 0 & 6g & 7g+1 \\ 0 & 9g+9 & 0 & 3+4g & 0 & 2 \\ 10g & 0 & 7 & 6g & 1 & 0 \\ 0 & 4g & 1+g & 0 & 1+5g & 2g+2 \end{bmatrix}$$

be the MOD dual number square matrix with entries from $\langle Z_{11} \cup g\rangle = \{a + bg / a, b \in Z_{11}, g^2 = 0\}$.

DEFINITION 2.11: *Let $S = (s_{ij})$ be a $n \times n$ matrix with entries from $\langle Z_m \cup g\rangle = \{a + bg / a, b \in Z_m, g^2 = 0\}$. S is defined as the MOD dual number square matrix or MOD square dual number matrix.*

We have given examples of them.

Now we proceed onto describe a MOD dual number directed graph and the adjacency matrix associated with them.

Example 2.39: Let G_1 be the MOD directed dual number graph with entries from $\langle Z_7 \cup g\rangle = \{a + bg / a, b \in Z_7, g^2 = 0\}$ given by the following figure.

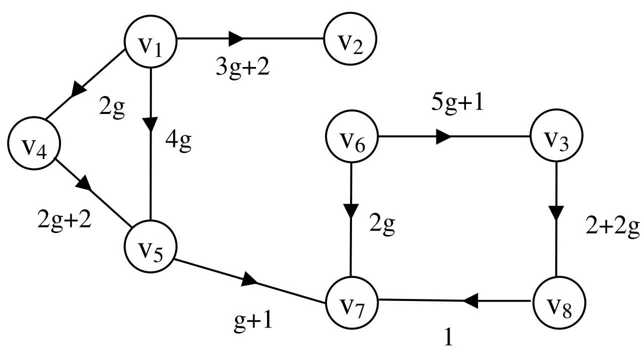


Figure 2.27

Let M be the connection MOD dual number matrix associated with M.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array}\begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\ \begin{bmatrix} 0 & 3g+2 & 0 & 2g & 4g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2+2g \\ 0 & 0 & 0 & 0 & 2g+2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & g+1 & 0 \\ 0 & 0 & 5g+1 & 0 & 0 & 0 & 2g & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}.$$

This MOD dual number square matrix has all its diagonal entries to be zero.

Example 2.40: Let G_2 be the MOD directed dual number with entries from $\langle Z_{16} \cup g\rangle = \{a + bg / a,b \in Z_{16}; g^2 = 0\}$.

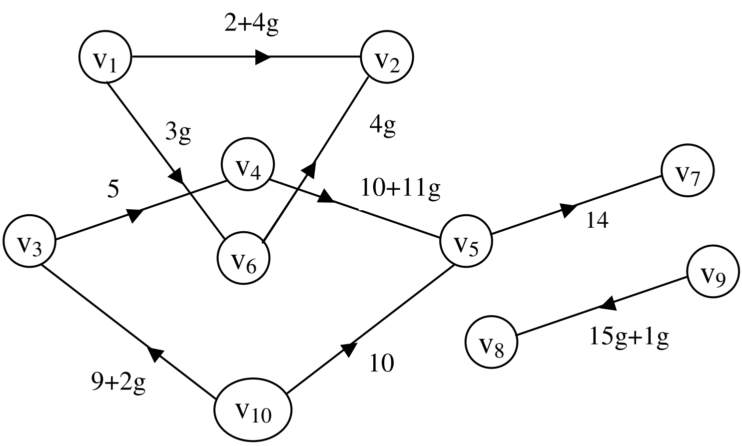


Figure 2.28

Let N be the MOD square dual number matrix given in the following

$$N = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \end{array} \begin{array}{c} \begin{array}{cccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} \end{array} \\ \begin{bmatrix} 0 & 2+4g & 0 & 0 & 0 & 3g & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10+11g & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 14 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 15g+1 & 0 & 0 \\ 0 & 0 & 9+2g & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Now we describe only a special type of operation defined on them MOD dual number square matrices.

Example 2.41: Let

$$M = \begin{bmatrix} 0 & 2g & 3g & 1 & 0 \\ 4+g & 0 & 0 & 2g & 4 \\ 0 & 6g+1 & 6 & 0 & 2+g \\ 2+3g & 0 & g+1 & 3 & 0 \\ 0 & 4 & 0 & 2g+1 & 0 \end{bmatrix}$$

be the MOD 5×5 dual number square matrix with entries from

$$\langle Z_7 \cup g \rangle = \{a + bg / a, b \in Z_7, g^2 = 0\}.$$

$X = \{(a_1, a_2, a_3, a_4, a_5) / a_i \in \{0, 1, g\}\ 1 \leq i \leq 5\}$ be the MOD special initial state vector.

Let $x = (1, 0, 0, 0, 0) \in X$.

To find the effect of x on M.

$xM \to (1, 2g, 3g, 1, 0) = y_1$;

$\to$ symbol denotes the resultant state vector has be updated.

$$\begin{aligned}
y_1M &= (2 + 4g, 5g, g + 1, 4, 0) = y_2; \\
y_2M &= (1 + 4g, 4g + 1, 3 + 2g, 4g, 2 + 2g) = y_3; \\
y_3M &= (4g + 4, 2g + 4, 4, 3 + 3g, 3 + 2g) = y_4;
\end{aligned}$$

and so on.

Certainly after a finite number of iterations, we will arrive at a MOD fixed point o r a MOD limit cycle.

Let $x_3 = (0, 0, g, 0, 0) \in X$.

The effect of x_3 on M,

$$\begin{aligned}
x_3M &= (0, g, 6g, 0, 2g) = y_1; \\
y_1M &= (4g, 0, g, 2g, 2g) = y_2; \\
y_2M &= (4g, 2g, g, 5g, 2g) = y_3; \\
y_3M &= (5g, 2g, 6g, 0, 3g) = y_4; \\
y_4M &= (g, 4g, g, g, 6g) = y_5; \\
y_5M &\to (4g, 4g, g, 3g, 4g) = y_6; \\
y_6M &= (g, 3g, 2g, 3g, 4g) = y_7; \\
y_7M &= (4g, 4g, g, 3g, 2g) = y_8; \\
y_8M &= (g, 2g, 2g, 6g, 4g) = y_9; \\
y_9M &= (6g, 4g, 4g, 5g, 5g) = y_{10}; \\
y_{10}M &= (5g, 3g, 2g, 5g, 3g) = y_{11}; \\
y_{11}M &= (g, 0, 5g, 6g, 2g) = y_{12}; \\
y_{12}M &= (3g, 4g, 3g, 0, 6g) = y_{13}; \\
y_{13}M &= (2g, 6g, 4g, 2g, g) \text{ and so on.}
\end{aligned}$$

However we are sure after a finite numbers of iterations we will arrive at a MOD fixed point or a MOD limit cycle.

Example 2.42: Let

$$M = \begin{bmatrix} 3g+1 & 2g & 0 \\ g & 0 & 2+2g \\ 0 & 1 & 0 \end{bmatrix}$$

be the MOD square dual number matrix with entries from
$$\langle Z_4 \cup g\rangle = \{a + bg / a, b \in Z_4; g^2 = 0\}.$$

$X = \{(a_1, a_2, a_3) / a_i \in \{0, 1, g\}, 1 \leq i \leq 3\}$ be the MOD special dual number initial state vectors related with the MOD square matrix M.

Let $x = (1, 0, 0) \in X$.

The effect of x on M.

$xM = (3g + 1, 2g, 0) = y_1$; $y_1M = (2g + 1, 2g, 0) = y_2$;
$y_2M = (1 + g, 2g, 0) = y_3$; $y_3M = (1, 2g, 0) = y_4$;
$y_4M = (3g + 1, 2g, 0) = y_5$ $(=y_1)$.

Thus the MOD resultant is a MOD limit cycle given by
$$(2g + 1, 2g, 0).$$

Let $x = (g, 0, 0) \in X$.

To find the effect of x on M.

$xM = (g, 0, 0) = y_1$ $(=x)$.

Thus the MOD resultant is a MOD classical fixed point.

Next we consider the effect of $x_1 = (0, 1, 0) \in X$.

$x_1M \to (g, 1, 2 + 2g) = y_1$;
$y_1M = (2g, 2 + 2g, 2 + 2g) = y_2$;
$y_2M = (0, 2 + 2g, 0) = y_3$;
$y_3M \to (2g, 1, 0) = y_4$;

$y_4M \to (3g, 1, 2 + 2g) = y_5;$
$y_5M = (0, 2 + 2g, 2 + 2g) = y_6;$
$y_6M = (2g, 2 + 2g, 0) = y_7;$
$y_7M \to (0, 1, 0) = y_8\ (=x_1).$

Thus the MOD resultant is a MOD limit cycle leading to the MOD classical limit point $x_1 = (0, 1, 0)$.

Consider $a = (0, g, 0) \in X$.

To find the effect of a on M

$aM \to (0, g, 2g) = y_1;$ $\quad y_1M = (0, 2g, 0) = y_2;$
$y_2M \to (0, g, 0) = y_3\ (=a).$

Thus the MOD resultant is a MOD limit cycle which is again a MOD classical limit cycle.

Let $x_3 = (0, 0, 1) \in X$.

To find the effect of x_3 on M.

$x_3M \to (0, 1, 1) = y_1$

$y_1M = (g, 1, 2 + 2g) = y_2;$ $\quad y_2M = (2g, 2 + 2g, 2 + 2g) = y_3;$
$y_3M \to (0, 2 + 2g, 1) = y_4;$ $\quad y_4M \to (2g, 1, 1) = y_5;$
$y_5M = (3g, 1, 2) = y_6;$ $\quad y_6M = (0, 2, 2 + 2g) = y_7;$
$y_7M \to (2g, 2 + 2g, 1) = y_8;$ $\quad y_8M \to (0, 1, 1) = y_9 = y_1.$

Thus the MOD resultant is a MOD limit cycle which is not MOD classical limit cycle.

Let $b = (0, 0, g) \in X$.

To find the effect of b on M.

$bM \to (0, g, g) = y_1;$ $\quad y_1M = (0, g, 2g) = y_2;$
$y_2M = (0, 2g, 2g) = y_3;$ $\quad y_3M \to (0, 2g, 1) = y_4;$

$y_4M \to (0, 1, 1) = y_5$; $y_5M = (g, 1, 2 + 2g) = y_6$;
$y_6M = (2g, 2 + 2g, 2 + 2g) = y_7$;
$y_7M = (0, 2 + 2g, 1) = y_8$; $y_8M = (2g, 1, 1) = y_9$;
$y_9M = (3g, 1, 2 + 2g) = y_{10}$;
$y_{10}M = (0, 2 + 2g, 2 + 2g) = y_{11}$;
$y_{11}M \to (2g, 2 + 2g, 1) = y_{12}$;
$y_{12}M \to (2g, 1, 1) = y_{12}$ $(=y_9)$.

Thus the MOD resultant is a MOD limit cycle.

Now we in the following first describe the MOD dual number cognitive maps model.

Suppose there is a problem in which the researcher / expert wishes to work with dual set up that is there are nodes and edges which deploys the property of dual numbers then this new MOD dual number Cognitive Maps (MODDNCMs) model or equivalently we can have MOD Dual Number Cognitive Maps model.

Let P be a problem with $C_1, C_2, \ldots, C_n$ be the n distinct nodes / concepts associated with P.

Suppose the edge weights are taken from $\langle Z_m \cup g\rangle = \{a + bg / a, b \in Z_m, g^2 = 0\}$.

Let G be the MOD directed graph given by the expert with edge weights from $\langle Z_m \cup g\rangle$.

Let M be the MOD connection $n \times n$ dual number matrix associated with G.

Then we can have two types of MOD special initial state vector $X = \{(a_1, \ldots, a_n) / a_1 = \{0, 1\}, 1 \leq i \leq n\}$ or

$X_s = \{(a_1, \ldots, a_n) / a_i \in \{0, 1, g\}; 1 \leq i \leq n\}$ the MOD dual number special initial state vectors.

Study can be carried out for both.

However in both cases we agree upon to state MOD resultant can for a particular time give the nodes the mixed dual number MOD values or pure number or pure dual number and so on.

We will explain this by an example or two in the following.

Example 2.43: Let P, be a problem with $C_1, C_2, \ldots, C_6$ the associated nodes / concepts.

Suppose the expert wishes to work using the MOD DN CM model taking edge weights of the MOD directed dual graph G from $\langle Z_3 \cup g\rangle = \{bg + a / a, b \in Z_3, g^2 = 0\}$.

Let M_1 be the MOD 6×6 dual number connection matrix which acts as the MOD dual number dynamical system.

The MOD directed dual number graph G_1 is as follows.

$G_1 =$

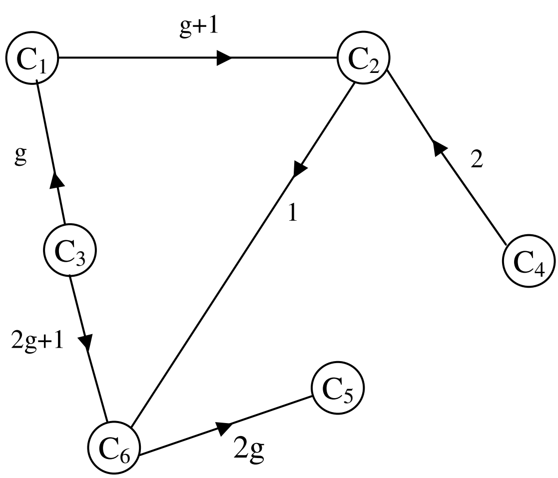


Figure 2.29

Let M_1 be the MOD dual number square matrix associated with G_1, which serves as the dynamical system for MODDN CMs model.

$$M_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & g+1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ g & 0 & 0 & 0 & 0 & 2g+1 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2g & 0 \end{bmatrix} \end{array}.$$

Let $X = \{(a_1, \ldots, a_6) / a_i \in \{0, 1\}; 1 \leq i \leq 6\}$ and

$X_s = \{(a_1, a_2, \ldots, a_6) / a_i \in \{0, 1, g\}; 1 \leq i \leq 6\}$ be the MOD initial state vector of both types associated with M_1.

Let $x_1 = (1, 0, 0, 0, 0, 0) \in X$.

To find the effect of x on M_1;

$x_1M_1 \to (1, g + 1, 0, 0, 0, 0) = y_1$;
$y_1M_1 \to (1, g + 1, 0, 0, 0, g + 1) = y_2$;
$y_2M \to (1, g + 1, 0, 0, 2g, g + 1) = y_3$;
$y_3M \to (1, g + 1, 0, 0, 2g, 1 + g) = y_4$ $(=y_3)$.

Thus the MOD resultant is a MOD fixed point given by
$(1, g +1, 0, 0, 2g, 1 + g)$.

Let $x_2 = (0, 1, 0, 0, 0, 0) \in X$.

To find the effect of x_2 on M_1

$x_2M_1 \to (0, 1, 0, 0, 0, 1) = y_1$;
$y_1M_1 \to (0, 1, 0, 0, 2g, 1) = y_2$;
$y_2M_1 \to (0, 1, 0, 0, 2g, 1) = y_3$ $(=y_2)$.

Thus the MOD resultant is again a MOD fixed point given by $(0, 1, 0, 0, 2g, 1)$.

Let $x_3 = (0, 0, 1, 0, 0, 0) \in X$;

To find the effect of x_3 on M_1;

$x_3M_1 \to (g, 0, 1, 0, 0, 2g + 1) = y_1$ (say);
$y_1M_1 \to (g, 0, 1, 0, 2g, 2g + 1) = y_2$;
$y_2M_1 \to (g, 0, 1, 0, 2g, 2g + 1) = y_3\ (=y_2)$.

Thus the MOD resultant is again MOD fixed point given by $(g, 0, 1, 0, 2g, 2g + 1)$.

Let $x_4 = (0, 0, 0, 1, 0, 0)$ be the initial state vector.

The effect of x_4 on M_1 is as follows.

$x_4M_1 \to (0, 2, 0, 1, 0, 0) = y_1$;
$y_1M_1 \to (0, 2, 0, 1, 0, 2) = y_2$;
$y_2M_1 \to (0, 2, 0, 1, g, 2) = y_3$;
$y_3M_1 \to (0, 2, 0, 0, g, 2) = y_4\ (=y_3)$.

Thus the MOD resultant of x_4 is $(0, 2, 0, 0, g, 2)$.

Let $x_5 = (0, 0, 0, 0, 1, 0) \in X$.

To find the effect of x_5 on M_1;

$x_5M_1 \to (0, 0, 0, 0, 1, 0) = y_1\ (=x_5)$.

Thus the MOD resultant is a MOD special classical fixed point given by $x_5 = (0, 0, 0, 0, 1, 0)$ which is also the MOD classical fixed point.

Let $x_6 = (0, 0, 0, 0, 0, 1) \in X$; to find the effect of x_6 on M_1;

$x_6M_1 \to (0, 0, 0, 0, 2g, 1) = y_1$;

$y_1M_1 \to (0, 0, 0, 0, 2g, 1) = y_2$.

Thus the MOD resultant is a MOD fixed point given by

$$(0, 0, 0, 0, 2g, 1).$$

Let $x_1 = (g, 0, 0, 0, 0, 0) \in X_S$.

To find the effect of x_1 on M_1;

$$x_1M_1 \to (g, g, 0, 0, 0, 0) = y_1;$$
$$y_1M_1 \to (g, g, 0, 0, 0, g) = y_2;$$
$$y_2M_1 \to (g, g, 0, 0, 0, g) = y_3 \ (= y_2).$$

Thus the MOD resultant is a MOD fixed point.

Let $x_2 = (0, g, 0, 0, 0, 0) \in X_S$.

To find the effect of x_2 on M_1;

$$x_2M_1 \to (0, g, 0, 0, 0, g) = y_1;$$
$$y_1M_1 \to (0, g, 0, 0, 0, g) = y_2 \ (=y_1).$$

Thus the MOD resultant is a MOD fixed point.

Let $x_3 = (0, 0, g, 0, 0, 0) \in X_S$.

To find the effect of x_3 on M_1;

$$x_3M_1 \to (0, 0, g, 0, 0, g) = y_1;$$
$$y_1M_1 \to (0, 0, g, 0, 0, g) = y_2 \ (=y_1).$$

Thus the MOD resultant is a MOD fixed point given by $(0, 0, g, 0, 0, g)$.

Let $x_4 = (0, 0, 0, g, 0, 0) \in X_S$.

To find the effect of x_4 on M_1;

$$x_4M_1 \rightarrow (0, 2g, 0, g, 0, 0) = y_1;$$
$$y_1M_1 \rightarrow (0, 2g, 0, g, 0, 0) = y_2 \ (=y_1).$$

Thus the MOD resultant is a MOD fixed point given by (0, 2g, 0, g, 0, 0).

Let $x_5 = (0, 0, 0, 0, g, 0) \in X_S$.

$$x_5M_1 \rightarrow (0, 0, 0, 0, g, 0) = y_1 \ (=x_5).$$

Thus the MOD resultant is a MOD special classical fixed point.

Let $x_6 = (0, 0, 0, 0, 0, g) \in X_S$.

To find the effect of x_6 on M_1.

$$x_6M \rightarrow (0, 0, 0, 0, 0, g) = y_1 \ (=x_6).$$

Thus the MOD resultant is a MOD classical fixed point. Next we give one more example.

Example 2.44: Let P_2 be a problem which is associated with the nodes / concepts C_1, C_2, C_3 and C_4 the expert wishes to adopt the MOD dual number cognitive maps model with edge weights from $\langle Z_4 \cup g\rangle = \{a + bg / a, b \in Z_4, g^2 = 0\}$.

Let G_2 be the MOD directed dual number graph given by him

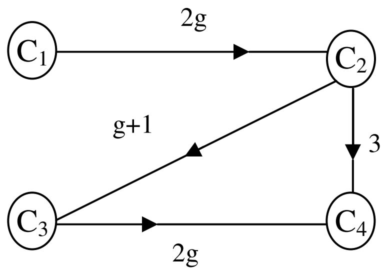


Figure 2.30

Let M_2 be the MOD dual number connection matrix associated with the MOD dual number graph G_2.

$$M_2 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 0 & 2g & 0 & 0 \\ 0 & 0 & g+1 & 3 \\ 0 & 0 & 0 & 2g \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let $X = \{(a_1, a_2, a_3, a_4) / a_i \in \{0, 1\}; 1 \leq i \leq 4\}$ and

$X_S = \{(a_1, a_2, a_3, a_4) / a_i \in \{0, 1, g\}\ 1 \leq i \leq 4\}$ be the MOD dual number initial state vectors and MOD special dual number initial state vectors respectively.

Let $x_1 = (1, 0, 0, 0) \in X$, the effect of x_1 on M_2 is as follows.

$x_1M_2 \to (1, 2g, 0, 0) = y_1;$
$y_1M_2 \to (1, 2g, 2g, 2g) = y_2;$
$y_2M_2 \to (1, 2g, 2g, 2g) = y_3 = (=y_2).$

Thus the MOD resultant is a MOD fixed point given by
$(1, 2g, 2g, 2g)$.

We see if C_1 is on then all the other three coordinates C_2, C_3 and C_4 are pure dual number.

That the on state of C_1 makes all other nodes to pure dual number states.

Let $x_2 = (0, 1, 0, 0) \in X$; the effect of x_2 on M_2;

$x_2M_2 \to (0, 1, g + 1, 3) = y_1;$
$y_1M_2 \to (0, 1, g + 1, 2g + 3) = y_2;$
$y_2M_2 \to (0, 1, g + 1, 2g + 3) = y_3\ (=y_5).$

Thus the MOD resultant is a MOD fixed point given by $(0, 1, g + 1, 2g + 3)$

That is the on state of C_2 has no effect on C_1, but both C_3 and C_4 get the mixed dual number states as $g + 1$ and $2g + 3$ respectively.

Let $x_3 = (0, 0, 1, 0) \in X$.

To find the effect of x_3 on M_2.

$x_3M_2 \to (0, 0, 1, 2g) = y_1$
$y_1M_2 \to (0, 0, 1, 2g) = y_2$ $(=y_1)$.

Thus the MOD resultant is a MOD fixed point given by $(0, 0, 1, 2g)$.

We see the on state of C_3 make on only the node C_4 and C_4 get the pure dual number value $2g$.

However C_1 and C_2 are unaffected by the on state of the node C_3.

Let $x_4 = (0, 0, 0, 1) \in X$.

The effect of x_4 on M_2; $x_4M_2 \to (0, 0, 0, 1) = y_1$ $(=x_4)$.

Thus the MOD resultant is the MOD special classical fixed point given by $(0, 0, 0, 1)$.

Thus on state of the node C_4 keeps all other nodes C_1, C_2 and C_3 to off state or has no effect on them.

Let $a_1 = (g, 0, 0, 0) \in \times_s$; to find the effect of a_1 on M_2.

$a_1M_2 \to (g, 0, 0, 0) = y_1$ $(=a_1)$.

Thus the MOD resultant is a MOD special classical fixed point $(g, 0, 0, 0)$.

The dual state of C_1 has no effect on C_2, C_3 and C_4.

Let $a_2 = (0, g, 0, 0) \in X_s$.

The effect of a_2 on M_2;

$a_2M_2 \to (0, g, g, 3g) = y_1$
$y_1M_2 \to (0, g, g, 3g) = y_2$ $(=y_1)$.

Thus the MOD resultant of the a_2 is MOD fixed point given by (0, g, g, 3g) where the dual state of C_2 has no effect on C_1 however both C_3 and C_4 comes to dual state.

Let $a_3 = (0, 0, g, 0) \in X_S$.

The effect of a_3 on M_2.

$a_3M_2 \to (0, 0, g, 0) = y_1$ $(=a_3)$.

Thus the dual state of node C_3 has no effect on other nodes.

In fact the MOD resultant is a MOD classical fixed point given by $(0, 0, g, 0) = a_3$.

Let $a_4 = (0, 0, 0, g) \in X_S$.

The effect of a_4 on M_2 is

$$a_4M_2 \to (0, 0, 0, g) = y_1 \ (=a_4).$$

Thus the MOD resultant in case of a_4 is also a MOD special classical fixed point given by $(0, 0, 0, g) = a_4$.

Let $b_1 = (1, 0, g, 0) \in X_S$.

To find the effect of b_1 on M_2;

$b_1M_2 \rightarrow (1, 2g, g, 0) = y_1$;
$y_1M_2 \rightarrow (1, 2g, 2g, 2g) = y_2$;
$y_2M_2 \rightarrow (1, 2g, 2g, 2g) = y_3$ ($=y_2$).

Thus the MOD resultant of b_1 is a MOD fixed point given by (1, 2g, 2g, 2g).

We see on state of C_1 and dual stage of C_3 makes on the nodes C_2 and C_4 however they get only the state to be pure dual numbers.

Next we proceed onto describe MOD Special Dual Like Number Cognitive Maps (MOD SDL NCMs) model.

For this we first need the notion of MOD special dual like number directed graph and MOD special dual like number matrix.

We will illustrate these situations only by examples as the definition of them is considered as a matter of routine.

Let G be a directed graph with edge weights from $\langle Z_n \cup h\rangle = \{a + bh / a, \; b \in Z_n, h^2 = h\}$; the special dual like numbers.

We call a directed graph with edge weights from the special dual like number $\langle Z_n \cup h\rangle = \{a + bh / a, b \in Z_n, h^2 = h\}$ as the MOD directed special dual like number graphs.

We will give examples of MOD directed special dual like number graphs.

Example 2.45: Let G_1 be the MOD directed special dual like number graph with edge weights from

$$\langle Z_6 \cup h\rangle = \{a + bh / a, b \in Z_6, h^2 = h\}.$$

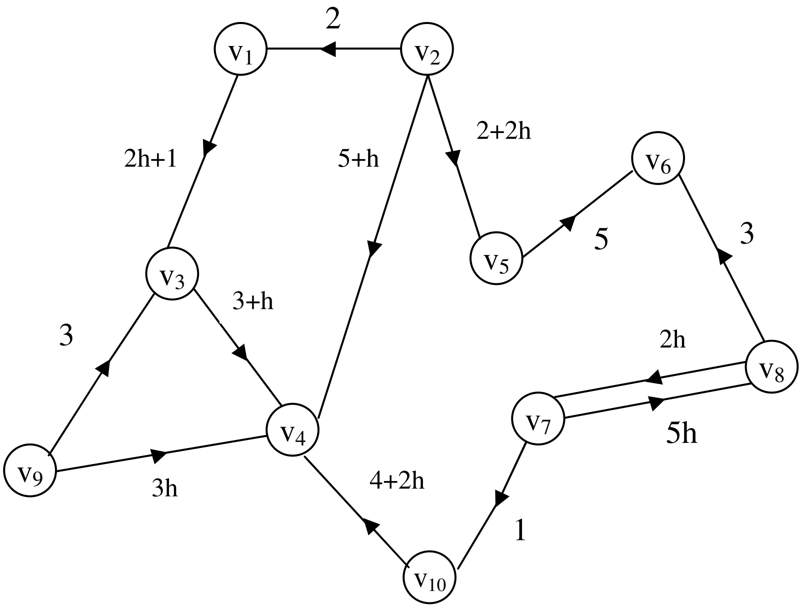


Figure 2.31

Example 2.46: Let G_2 be the MOD directed special dual like number graph with edge weights from
$\langle Z_7 \cup h\rangle = \{a + bh / a, b \in Z_7, h^2 = y\}$.

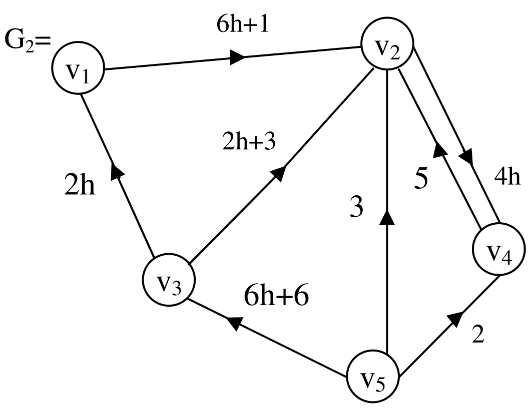


Figure 2.32

Next we proceed on to give examples of MOD special dual like number square matrices.

Example 2.47: Let

$$M = \begin{bmatrix} 3h & 0 & 2h+4 \\ 0 & 4h+2 & 3 \\ 2h+1 & 0 & 5h \end{bmatrix}$$

be the MOD special dual like number 3×3 matrix with entries from $\langle Z_6 \cup h\rangle = \{a + bh / a, b \in Z_6, h^2 = h\}$.

Example 2.48: Let $\langle Z_{12} \cup h\rangle = \{a + bh / a, b \in Z_{12}, h^2 = h\} = P$.

$$L = \begin{bmatrix} 5h+10 & 1 & 2h+11 & 0 & 2h+2 \\ 6 & 0 & 6h & 4h+8 & 0 \\ 11h & 7h+1 & 0 & 6 & 5h+8 \\ 10h+9 & 0 & 7h+10 & 0 & 6h+3 \\ 0 & 2h & 0 & 5h & 0 \end{bmatrix}$$

be the MOD special dual like number matrix with entries from P.

Example 2.49: Let

$$W = \begin{bmatrix} 0 & 2h+1 & 0 & 5h+3 \\ 7 & 0 & 2+5h & 0 \\ 0 & 4h & 0 & 5 \\ 6h+1 & 0 & 7h & 0 \end{bmatrix}$$

be the MOD special dual like number matrix with entries from $\langle Z_8 \cup h\rangle = \{a + b \in Z_8, h^2 = h\}$.

Let $M = (m_{ij})$ be a $n \times n$ matrix with entries from

$\langle Z_m \cup h\rangle = \{a + bh / a, b \in Z_m, h^2 = h\}$.

M is defined as the MOD special dual like number square matrix.

We have already given examples of them.

Let $X = \{(a_1, a_2, \ldots, a_n) / a_i \in \{0,1\}, 1 \leq i \leq n\}$, X is defined as the MOD initial state vectors associated with M.

$X_s = \{(a_1, a_2, \ldots, a_n) / a_i \in \{0, h, 1\}; 1 \leq i \leq n\}$ is defined as the MOD special initial state special dual like number vectors.

Here we describe only the special type of operations using the MOD special dual like number square matrices and the MOD special state vectors X and X_s.

Example 2.50: Let

$$M = \begin{bmatrix} 0 & 3h & 4h+2 \\ 1 & 0 & 2h+3 \\ 0 & h+2 & 0 \end{bmatrix}$$

be the MOD special dual like number matrix with entries from

$$\langle Z_5 \cup h\rangle = \{a + bh / a, b \in Z_5, h^2 = h\}.$$

Let $X = \{(a_1, a_2, a_3) / a_i \in \{0, 1\}, 1 \leq i \leq 3\}$ and

$X_s = \{(a_1, a_2, a_3) / a_i \in \{0, 1, h\}; 1 \leq i \leq 3\}$ be the MOD initial state vectors MOD special initial state vectors respectively.

Let $x_1 = (1, 0, 0) \in X$.

To find the effect of x_1 on M.

$$x_1M \to (1, 3h, 4h +2) = y_1;$$
$$y_1M = (3h, 2h + 4, 4h + 2) = y_2;$$

$y_2M = (2h + 4, 3h + 2, h + 2) = y_3$;
$y_3M = (3h + 2, 3h + 2, 2h + 4) = y_4$;
$y_4M = (3h + 2, 3, 0) = y_5$;
$y_5M = (3, 0, 2h + 3) = y_6$;
$y_6M = (1, 3h + 1, 2h + 1) = y_7$;
$y_7M = (3h + 1, 2, h) = y_8$;
$y_8M = (2, 0, h + 3) = y_9$ and so on.

However after a finite set of iterations we will arrive at a MOD fixed point or a MOD limit cycle.

Let $x_2 = (0, 1, 0) \in X$.

To find the effect of x_2 on M.

$x_2M \to (1, 1, 2h + 3) = y_1$; $y_1M = (1, 1, h) = y_2$;
$y_2M = (1, h, h) = y_3$; $y_3M = (1, h, h) = y_4$ ($=y_3$).

Thus the MOD resultant is a MOD fixed point given by (1, h, h).

Let $x_3 = (0, 0, 1) \in X$.

To find the effect of x_3 on M

$x_3M \to (0, h + 2, 1) = y_1$;
$y_1M = (h + 2, h + 2, 4h + 1) = y_2$;
$y_2M = (h + 2, 3h + 2, 3h) = y_3$;
$y_3M = (3h+2, 3h, 3h) = y_4$;
$y_4M = (3h, 4h, h + 4) = y_5$;
$y_5M = (4h, 4h + 3, 3h) = y_6$;
$y_6M = (4h + 3, h, 4) = y_7$;
$y_7M = (h, 3, h + 1) = y_8$;
$y_8M = (3, 2h + 2, 2h + 4)$;

and so on.

Certainly after a finite number of iterations we will arrive at a MOD fixed point or a MOD limit cycle.

Let $a_1 = (h, 0, 0) \in X_S$; to find the effect of a_1 on M.

$a_1M \to (h, 3h, h) = y_1$; $y_1M = (3h, h, h) = y_2$;
$y_2M = (h, 2h, 3h) = y_3$; $y_3M = (2h, 2h, h) = y_4$;
$y_4M = (2h, 4h, 2h) = y_5$; $y_5M = (4h, 2h, 2h) = y_6$;
$y_6M = (2h, 3h, 4h) = y_7$; $y_7M = (3h, 3h, 2h) = y_8$;
$y_8M = (3h, 0, 3h) = y_9$; $y_9M = (h, 3h, 3h) = y_{10}$;
$y_{10}M = (3h, 2h, h) = y_{11}$; $y_{11}M = (2h, 2h, 3h) = y_{12}$;
$y_{12}M = (2h, 0, 2h) = y_{13}$; $y_{13}M = (h, 2h, 2h) = y_{14}$;
$y_{14}M = (2h, 4h, h) = y_{15}$; $y_{15}M = (4h, 4h, 2h) = y_{16}$.

However after a finite number of iterations we will arrive at a MOD fixed point or a MOD limit cycle.

Let $a_2 = (0, h, 0) \in X_s$.

To find the effect of a_2 on M.

$a_2M \to (h, h, 0) = y_1$; $y_1M = (h, 3h, h) = y_2$;
$y_2M = (3h, h, h) = y_3$; $y_3M = (h, 2h, 3h) = y_4$;
$y_4M = (2h, 2h, h) = y_5$; $y_5M = (2h, 4h, 2h) = y_6$;
$y_6M = (4h, 2h, 2h) = y_7$; $y_7M = (2h, 3h, 4h) = y_8$;
$y_8M = (3h, 3h, 2h) = y_9$; $y_9M = (3h, h, 3h) = y_{10}$;
$y_{10}M = (h, 3h, 3h) = y_{11}$; $y_{11}M = (3h, 2h, h) = y_{12}$;
$y_{12}M = (2h, 2h, 3h) = y_{13}$; $y_{13}M = (2h, h, 2h) = y_{14}$;
$y_{14}M = (h, 2h, 2h) = y_{15}$; $y_{15}M = (2h, 4h, h) = y_{16}$;
$y_{16}M = (4h, 4h, 2h) = y_{17}$; $y_{17}M = (4h, 4h, 4h) = y_{18}$;
$y_{18}M = (4h, 4h, 4h) = y_{19}$ $(=y_{18})$.

Thus the MOD resultant is a MOD fixed point given by (4h, 4h, 4h).

All nodes come to on state.

When the initial on state node is a special dual like number all the other states which come to on state are also special dual like number.

Let $a_3 = (0, 0, h) \in X_s$.

To effect of a_3 on M is as follows.

$a_3M \to (0, 3h, h) = y_1$;
$y_1M \to (3h, 3h, h) = y_2$;
$y_2M \to (3h, 2h, 3h) = y_3$;
$y_3M \to (2h, 3h, 3h) = y_4$;
$y_4M = (3h, 0, 2h) = y_5$;
$y_5M = (0, 0, 3h) = y_6$;
$y_6M = (0, 4h, h) = y_7$;
$y_7M \to (4h, 3h, h) = y_8$;
$y_8M = (3h, 0, 4h) = y_9$;
$y_9M = (0, h, 3h) = y_{10}$;
$y_{10}M = (h, 4h, h) = y_{11}$

and so on.

However we are sure to get the MOD resultant after a finite number of iteration.

By this new MODSDLNCM model we are in a position to get the state in the resultant vector to be real, or pure special dual like number and special dual like number.

Next we proceed onto build the new notion of MOD directed special quasi dual number graphs, MOD special quasi dual number matrices and using these two notions the MOD Special Quasi Dual Number Cognitive Maps (MOD SQDNCMs) model in the following.

First we describe the MOD special quasi dual number directed graphs.

Example 2.51: Let G_1 be the directed graph with edge weights from the special quasi dual numbers

$\langle Z_9 \cup k\rangle = \{a+bk \ / \ k^2 = 8k, a, b \in Z_9\}$ given in the following figure.

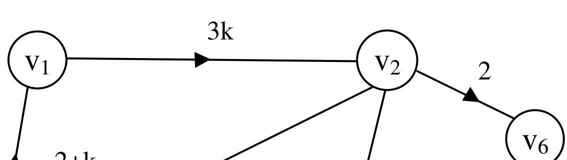


Figure 2.33

Example 2.52: Let G_2 be the directed graph with edge weights from $\langle Z_{11} \cup k\rangle = \{a + bk / a, b \in Z_{11}, k^2 = 10k\}$ which is given in the following.

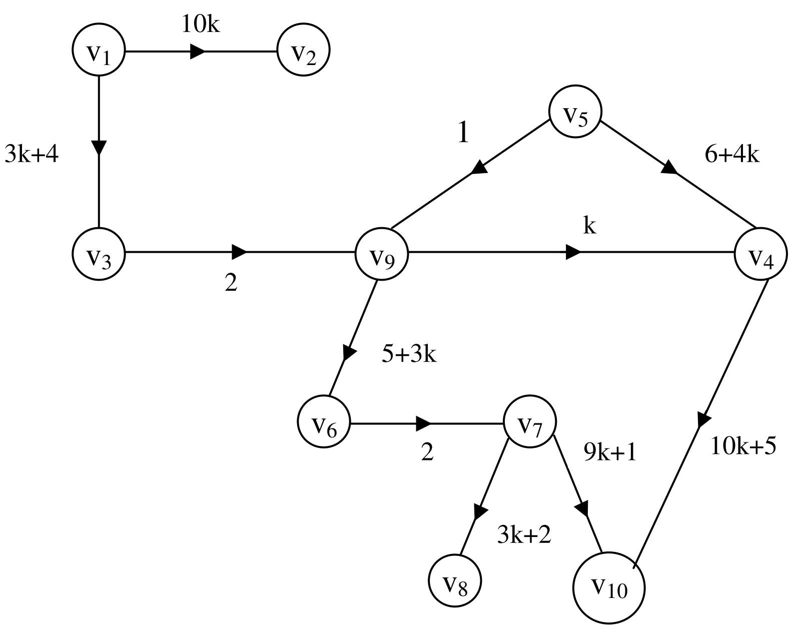


Figure 2.34

Now we define the notion of MOD Special Quasi Dual Number directed graph.

Let G be any directed graph if the edge weights are taken from $\langle Z_n \cup k\rangle = \{a + bk / a, b \in Z_n, k^2 = (n - 1) k\}$ then we define G to be a MOD special quasi dual number directed graph.

We have given examples of them.

Now we proceed onto describe the MOD special quasi dual number square matrices by some examples.

Example 2.53: Let

$$M = \begin{bmatrix} 3k & 0 & 4k+2 \\ 0 & 5k+1 & 0 \\ 4 & 2k & 5 \end{bmatrix}$$

be a matrix with entries from

$$\langle Z_6 \cup k\rangle = \{a + bk / a, b \in Z_6, k^2 = 5k\}.$$

We call M as the MOD special quasi dual number matrix.

Example 2.54: Let

$$W = \begin{bmatrix} 0 & 5k & 2 & 3+4k \\ 5 & 0 & 1+k & 0 \\ 3k+1 & 2 & 0 & 4 \\ 0 & 7k+2 & 5k & 0 \end{bmatrix}$$

be a matrix with entries from

$$\langle Z_8 \cup k\rangle = \{a + bk / a, b \in Z_8, k^2 = 7k\}.$$

We call W as the MOD special quasi dual number matrix.

Thus if $M = (m_{ij})$ is a $n \times n$ square matrix with entries from
$$\langle Z_m \cup k\rangle = \{a + bk / a, b \in Z_m, k^2 = (m - 1)k\}.$$

Then we define M to be the MOD special quasi dual number square matrix.

We would be using these two notions to built the MOD Special Quasi Dual Number Cognitive Maps model.

Example 2.55: Let P be a problem C_1, C_2, C_3, C_4, C_5 be 5 of the concepts / nodes associated with the problem.

The experts has given the directed graph with edge weights from $\langle Z_4 \cup k\rangle = \{a + bk / a, b \in Z_4, k^2 = 3k\}$.

That is the directed graph given by him is the MOD special quasi dual number directed graph G_1 which is as follows.

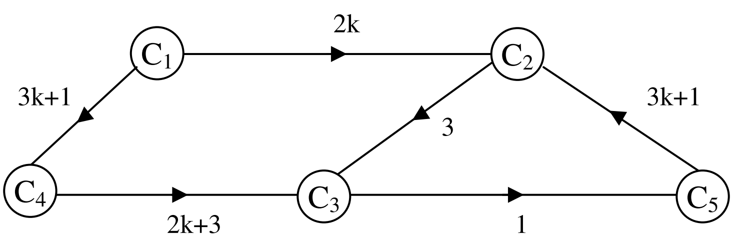


Figure 2.35

The MOD connection special quasi dual number matrix M_1 associated with G_1 is as follows.

$$M_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 0 & 2k & 0 & 3k+1 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2k+3 & 0 & 0 \\ 0 & 3k+1 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Now let $X = \{(a_1, a_2, \ldots, a_5) / a_i \in \{0, 1\}, 1 \leq i \leq 5\}$ and

$X_S = \{(a_1, a_2, \ldots, a_5) / a_i \in \{0, 1, k\}, 1 \leq i \leq 5\}$

be the MOD initial state vector and MOD special initial state vector.

Let $x_1 = (1, 0, 0, 0, 0) \in X$.

To find the effect of x_1 on M_1;

$x_1M_1 \to (1, 2k, 0, 3k + 1, 0) = y_1$;
$y_1M_1 \to (1, 2k, 3k + 3, 3k + 1, 0) = y_2$;
$y_2M_1 \to (1, 2k, 3k + 1, 3k + 1, 3k + 2) = y_3$;
$y_3M_1 \to (1, k + 3, 3k + 3, 3k + 1, 3k + 1) = y_4$;
$y_4M_1 \to (1, 3k + 1, 0, 3k + 1, 3k + 3) = y_5$;
$y_5M_1 \to (1, 3, 2k + 2, 3k + 1, 0) = y_6$;
$y_6M_1 \to (1, 2k, k, 3k + 1, 2k + 2) = y_7$;
$y_7M_1 \to (1, 2, 3k + 3, 3k + 1, k) = y_8$;
$y_8M_1 \to (1, 0, k + 1, 3k + 1, 3k + 3) = y_9$;
$y_9M_1 \to (1, k + 3, k + 3, 3k + 1, k + 1)$

and so on.

Certainly after a finite number of iterations.

We will arrive at a MOD fixed point or a MOD limit cycle.

Let $x_2 = (0, 1, 0, 0, 0) \in X$.

To find the effect of x_2 on M_1;

$x_2M_1 \to (0, 1, 3, 0, 0) = y_1$;
$y_1M_1 \to (0, 1, 3, 0, 3) = y_2$;
$y_2M_1 \to (0, 3 + k, 3, 0, 3) = y_3$;
$y_3M_1 \to (0, 1 + 3k, 1 + 3k, 0, 3) = y_4$;
$y_4M_1 \to (0, 3 + k, 3 + k, 0, 1 + 3k) = y_5$;
$y_5M_1 = (0, 1, 1 + 3k, 0, 3 + k) = y_6$;
$y_6M_1 = (0, 3k + 3, 3, 0, 1 + 3k) = y_7$;
$y_7M_1 = (0, 1, 1 + k, 0, 3k + 3) = y_8$;
$y_8M_1 = (0, 2k + 3, 3, 0, 1 + k) = y_9$;
$y_9M_1 = (0, 1, 2k + 3, 0, 3) = y_{10}$;
$y_{10}M_1 = (0, k + 3, 3, 0, 2k+3) = y_{11}$;
$y_{11}M_1 = (0, k + 3, 3k + 1, 0, 3) = y_{12}$;
$y_{12}M_1 = (0, 3 + k, 1 + 3k, 0, 3k + 1) = y_{13}$;
$y_{13}M_1 = (0, 1, 1 + 3k, 0, 1 + 3k) = y_{14}$;
$y_{14}M_1 = (0, 1, 3 + k, 0, 1 + 3k) = y_{15}$;

and so on.

We are sure after a finite number of iterations we will arrive at a MOD fixed point or a MOD limit cycle.

Let $x_5 = (0, 0, 0, 0, 1) \in X$.

To find the effect of x_5 on M_1;

$x_5M_1 \to (0, 3k + 1, 0, 0, 1) = y_1$;
$y_1M_1 \to (0, 3k + 1, 3 + k, 0, 1) = y_2$;
$y_2M_1 \to (0, 3k + 1, 3k + 1, 0, 3 + k)$;

and so on.

Certainly after a finite set of iterations we arrive at a resultant.

We will arrive at a MOD fixed point or a MOD limit cycle.

Let $a_1 = (k, 0, 0, 0, 0) \in X_s$.

To find the effect of a_1 on M_1;

$$a_1M_1 \to (k, 2k, 0, 2k, 0) = y_1;$$
$$y_1M_1 \to (k, 2k, 0, 2k, 0) = y_2\ (y_1).$$

Thus the MOD resultant is a MOD fixed point.

Let $b_1 = (0, 0, k, 0, 1) \in x_s$.

To find the effect of b_1 on M_1;

$$b_1M_1 \to (0, 3k + 1, k, 0, k) = y_1;$$
$$y_1M_1 \to (0, 2k, 3 + 3k, 0, k) = y_2;$$
$$y_2M_1 \to (0, 2k, 2k, 0, 3 + 3k) = y_3;$$
$$y_3M_1 = (0, 3 + 3k, 2k, 0, 2k) = y_4;$$
$$y_4M_1 = (0, 0, 1 + k, 0, 2k) = y_5;$$
$$y_5M_1 = (0, 0, k, 0, 1 + k) = y_6;$$
$$y_6M_1 \to (0, k + 1, k, 0, k) = y_7;$$
$$y_7M_1 = (0, 0, 3k + 3, 0, k) = y_8;$$
$$y_8M_1 = (0, 0, k, 0, 3k+3) = y_9;$$
$$y_9M_1 = (0, k + 3, k, 0, k) = y_{10};$$
$$y_{10}M_1 = (0, 0, 1 + 3k, 0, k) = y_{11};$$
$$y_{12}M_1 = (0, 0, 3k, 0, 1 + 3k) = y_{13};$$
$$y_{13}M_1 = (0, 0, k+1, 0, k) = y_{14}$$

and so on however we after a finite number of iterations will arrive a MOD fixed point or a MOD limit cycle.

Thus we can work with MOD SQDNCMs model and arrive at several interesting results.

However we will give one more example of this situation.

Example 2.56: Let P be the problem at hand and an expert works with the nodes C_1, C_2, C_3, C_4 using the edge weights of the directed graphs from

$$\langle Z_3 \cup k \rangle = \{a + bk / k^2 = 2k, a, b \in Z_3\}.$$

Let G_2 be the MOD special quasi dual number directed graph which is as follows.

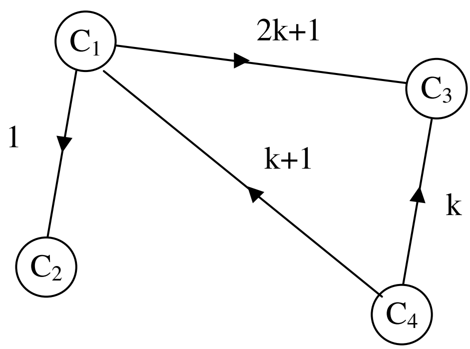


Figure 2.36

Let M_2 be the MOD connection matrix associated with G_2.

$$M_2 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 2k+1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ k+1 & 0 & k & 0 \end{bmatrix} \end{array}.$$

Let $X = \{(a_1, a_2, a_3, a_4) / a_i \in \{0, 1\}; 1 \leq i \leq 4\}$ be the MOD initial state vectors.

Let $X_s = \{(a_1, a_2, a_3, a_4) / a_i \in \{0, 1, k\}; 1 \leq i \leq 4\}$ be the MOD special quasi dual number initial state vectors.

Let $x = (1, 0, 0, 0) \in X$.

To find the effect of x on M_2;

$$xM_2 \to (1, 1, 2k + 1, 0) = y_1;$$
$$y_1M_2 \to (1, 1, 2k + 1, 0) = y_2\ (=y_1).$$

Thus the MOD resultant is a MOD fixed point given by $(1, 1, 2k + 1, 0)$ where the on state of C_1 has no effect of C_4 and makes on C_2 and gives the value $2k + 1$ for the node C_3.

Let $x_1 = (k, 0, 0, 0) \in X_s$.

To find the effect of x_1 on M_2;

$$x_1M_1 \to (k, k, 2k, 0) = y_1;$$
$$y_1M_2 \to (k, k, 2k, 0) = y_2\ (=y_1).$$

Thus the MOD resultant is a MOD fixed point given by $(k, k, 2k, 0)$.

Clearly when the node C_1 is the special quasi dual number k then it has no effect of C_4 but C_3 and C_2 get only the values which are again special quasi dual numbers.

Let $a = (0, 1, 0, 0) \in X$.

To find the effect of a on M_2.

$$aM_2 \to (0, 1, 0, 0) = a.$$

Thus the MOD resultant is a MOD special classical fixed point.

Consider $a_1 = (0, k, 0, 0) \in X_s$.

To effect of a_1 on M_2

$$a_1M_2 \rightarrow (0, k, 0, 0) = a_1;$$

which is again a MOD classical special fixed point.

Let $b = (0, 0, 0, 1) \in X$.

The effect of b on M_2.

$$bM_2 \rightarrow (k + 1, 0, k, 1) = y_1;$$
$$y_1M_2 \rightarrow (k + 1, k + 1, 2k + 1, 1) = y_2;$$
$$y_2M_2 \rightarrow (k + 1, k + 1, 2k + 1, 1) = y_3 \; (=y_2).$$

Thus the MOD resultant is a MOD fixed point given by $(k + 1, k + 1, 2k + 1, 1)$.

The on state of the node C_4 makes on the nodes C_1, C_2 and C_3 all of them are not real but special quasi dual number only.

The interested reader can work with more examples.

The author wish to mention the following fact.

The new MOD Cognitive Maps model will be a boon to any socio scientist, engineers, a scientist, and so on.

For these days with advancement of computers and other related technological development a state of the node need not be always on or off as defined and developed by the FCM or NCMs.

The state can be a real value some integers or can be a dual number $a + bg$, $g^2 = 0$, or a special dual like number $a + bh$,

$h^2 = h$ or can be a special quasi dual number $a + bk$, $k^2 = (n - 1)k$ where in all these cases $a, b \in Z_n$.

Apart from this the edges values also can be $a + bi_F$, $a, b \in Z_n$, $i_F^2 = n - 1$ that the MOD finite complex numbers or MOD finite neutrosophic number

$$a + bI,\ I^2 = I \text{ and } a, b \in Z_n\ (2 \leq n < \infty).$$

So these newly built models will be special boon to all researchers in all emerging fields.

Infact these new MOD models can be realized as a special type of generalization of FCMs and NCMs.

Chapter Three

MOD-NATURAL NEUTROSOPHIC COGNITIVE MAPS (MOD NCMS) MODEL AND MOD INTERVAL COGNITIVE MAPS MODEL

In this chapter we for the first time define the notion of MOD natural Neutrosophic Cognitive Maps model.

In chapter two MOD cognitive maps of different types were analysed, defined and described.

Here we proceed onto describe with examples and then the routine definition is made.

Recall $Z_n^I = \{Z_n, I_0^n, I_t^n$ / t is a zero divisor or a unit or an idempotent or a nilpotent in $Z_n\}$ + and $\times$ operations are defined on them Z_n^I [60].

Example 3.1: Let $\{Z_3^I, +, \times\} = \{0, 1, 2, I_0^3, 1 + I_0^3, 2 + I_0^3\}$ be the MOD natural neutrosophic semiring of order 6.

Example 3.2: Let $\langle Z_6^I, +, \times\rangle, = \{Z_6, a + I_0^6, a + I_2^6, I_3^6 + a, a + I_4^6, a + I_0^6 + I_2^6, a + I_0^6 + I_3^6, a + I_0^6 + I_4^6, a + I_2^6 + I_3^6, a + I_2^6 + I_4^6, a + I_3^6 + I_4^6, a + I_0^6 + I_2^6 + I_3^6, a + I_0^6 + I_2^6 + I_4^6, a + I_0^6 + I_3^6 + I_4^6, a + I_2^6 + I_4^6 + I_3^6, a + I_0^6 + I_3^6 + I_2^6 + I_4^6, a \in Z_6; +, \times\}$ be a semiring of finite order.

Now we will be building MOD natural neutrosophic Cognitive Maps model first we will describe this tools essential for it by some illustrations.

Example 3.3: Let

$$M = \begin{bmatrix} 0 & 3+I_2^4 & I_0^4 & 2 \\ I_2^4 & 0 & 2+I_0^4 & 0 \\ 0 & 3 & 0 & I_0^4+I_2^4 \\ I_0^4 & 0 & I_2^4 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic matrix with entries from Z_4^I.

Example 3.4: Let

$$N = \begin{bmatrix} 3+I_0^6 & 0 & 4+I_3^6+I_2^6 & 0 & 3 \\ 0 & 5+I_4^6 & 0 & I_2^6 & 0 \\ 2+I_3^6 & 0 & I_2^6+I_4^6 & 0 & I_4^6 \\ 0 & 5 & 0 & 3+I_0^6 & 1 \\ 1+I_4^6 & 3 & I_3^6 & I_0^6+I_2^6 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic square matrix with entries from $Z_6^I = \langle 0, 1, 2, \ldots, 5, I_0^6, I_2^6, I_4^6, I_3^6\rangle$ under + and $\times$.

Example 3.5: Let

$$S = \begin{bmatrix} 2+I_2^8 & 0 & I_2^8+I_4^8 & I_0^8+I_6^8+I_4^8 & 0 & 6 \\ 0 & I_2^8+I_6^8 & 0 & 3 & I_4^8 & I_2^8 \\ 1+I_4^8 & 0 & 5 & I_2^8+I_4^8 & 0 & 7 \\ 4 & I_4^8+I_0^8 & I_0^8 & I_4^8 & I_0^8 & 0 \\ 0 & 0 & 6+I_0^8+I_2^8 & 0 & 0 & I_4^8+4 \\ I_2^8+I_4^8+I_0^8 & 5 & 7 & I_0^8+I_4^8 & I_2^8 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic 6×6 matrix with entries from $\langle Z_8^I, +, \times\rangle$. We now define MOD natural neutrosophic matrix.

Let M = (m_{ij}) be a $n \times n$ matrix with entries from Z_m^I; the MOD natural neutrosophic matrix. We have seen examples of them.

We will illustrate the notion of MOD natural neutrosophic directed graphs.

Example 3.6: Let G be a directed graph with edge weights from Z_9^I. G is the MOD natural neutrosophic directed graph which is as follows.

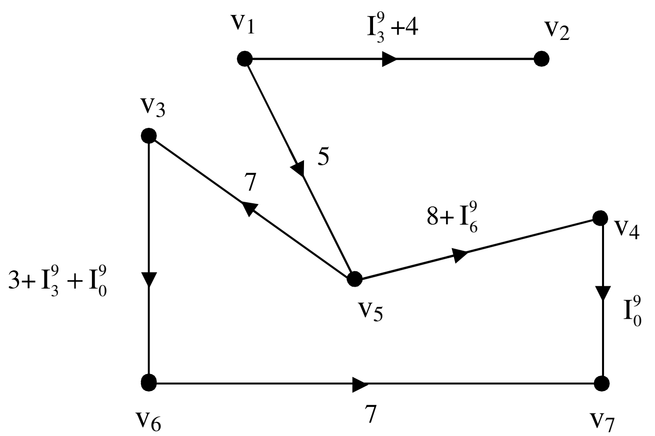


Figure 3.1

Example 3.7: Let G_1 be the MOD directed natural neutrosophic integer graph with edge weights from Z_{10}^{I}.

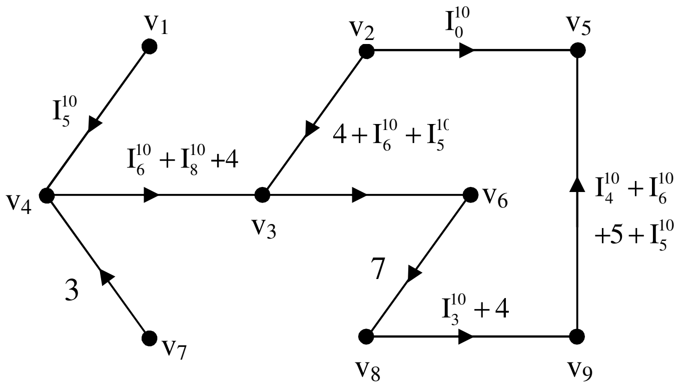


Figure 3.2

Example 3.8: Let G_2 be the directed graph with edge weights from Z_{12}^{I}, The MOD directed natural neutrosophic graph G_2 is as follows.

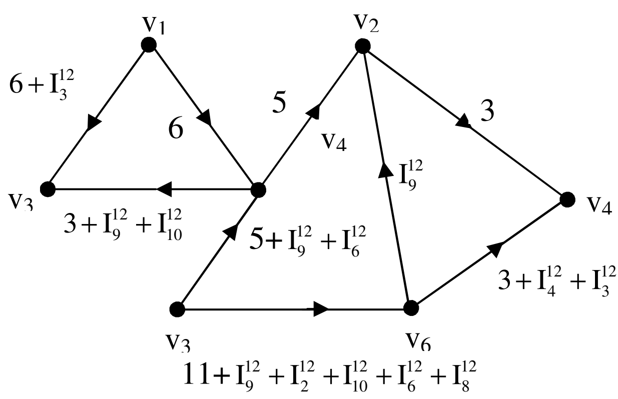


Figure 3.3

Now we define the MOD natural neutrosophic directed graph G to be a directed graph whose edge weights are from

Z_n^I ; $2 \leq n < \infty$.

We have already seen examples of them.

Next we proceed onto describe the notion of MOD natural Neutrosophic Cognitive Naps (MOD NCMs) model.

Example 3.9: Let P be a problem at hand. Suppose $C_1, C_2, \ldots, C_6$ be the nodes with which the expert works and uses the edge weights of the directed graph G from the set Z_4^I. The MOD natural neutrosophic graph is as follows.

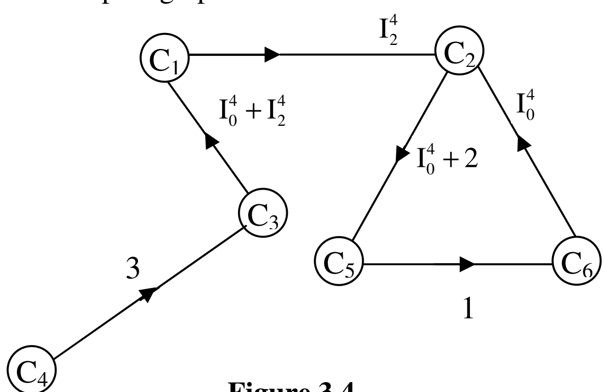


Figure 3.4

The MOD natural neutrosophic connection matrix M associated with the graph G is as follows.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & I_2^4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_0^4+2 & 0 \\ I_0^4+I_2^4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & I_0^4 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Example 3.10: Let P_1 be the problem with which is associated the nodes $C_1, C_2, \ldots, C_7, C_8$, using edge weights from Z_6^I. Let H be the MOD natural neutrosophic directed graph given by the except which is as follows.

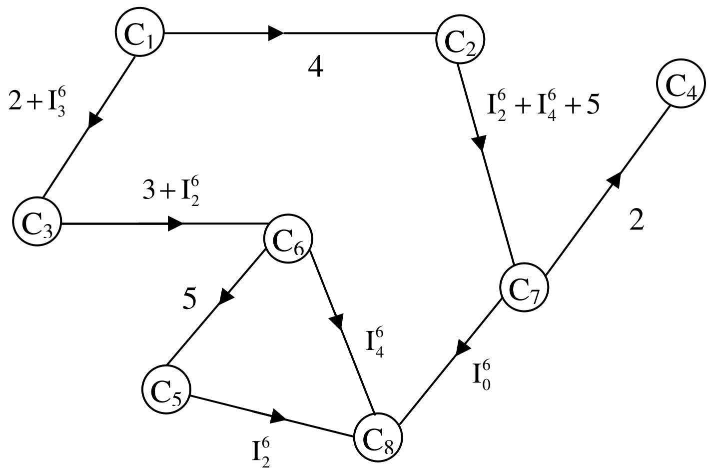


Figure 3.5

Let M_1 be the MOD natural neutrosophic connection matrix associated with H.

$$M_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \end{array}\begin{array}{c} \begin{array}{cccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \end{array} \\ \begin{bmatrix} 0 & 4 & 2+I_3^6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5+I_2^6+I_4^6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3+I_6^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_2^6 \\ 0 & 0 & 0 & 0 & 5 & 0 & 0 & I_4^6 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & I_0^6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Now we proceed onto describe the special operations we need on these M [ ].

First we will illustrate this yet by an example.

Example 3.11: Let P be a problem and let C_1, C_2, C_3 and C_4 be the nodes associated with the problem P.

Let G be the MOD natural neutrosophic direct graph given by the expert with edge weights from Z_6^I in the following.

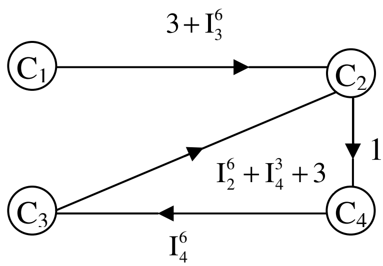


Figure 3.6

Let M be the MOD natural neutrosophic connection matrix associated with G.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 0 & 3+I_3^6 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ I_2^6+I_4^6+3 & 0 & 0 & 0 \\ 0 & 0 & I_4^6 & 0 \end{bmatrix} \end{array}.$$

Let $X = \{(a_1, a_2, a_3, a_4) \mid a_i \in \{0, 1\}; 1 \leq i \leq 4\}$ and

$X_s = \{(a_1, a_2, a_3, a_4) \mid a_i \in \{0, 1, I_0^6, I_2^6, I_2^6, I_4^6\}, 1 \leq i \leq 4\}$ be the MOD initial state vector and MOD special initial state vector associated with the MOD N Cognitive Maps model.

Let $x_1 = (1, 0, 0, 0) \in X$; to find the effect of x_1 on M.

$$x_1M \to (1, 3 + I_3^6, 0, 0) = y_1$$
$$y_1M \to (1, 3 + I_3^6, 0, 3 + I_3^6,) = y_2$$
$$y_2M \to (1, 3 + I_3^6, I_4^6 + I_0^6, 3 + I_3^6) = y_3$$
$$y_3M = (I_2^6 + I_0^6 + I_4^5, 3 + I_3^6, I_4^6 + I_0^6, 3 + I_3^6) = y_4$$
$$y_4M = (I_2^6 + I_0^6 + I_4^6, I_2^6 + I_0^6 + I_4^6, I_4^6 + I_0^6, 3+ I_3^6) = y_5$$
$$y_5M = (I_2^6 + I_3^6 + I_4^6, I_2^6 + I_0^6 + I_4^6, I_4^6 + I_0^6, I_2^6 + I_0^6 + I_4^6) = y_6$$
$$y_6M = (I_2^6 + I_3^6 + I_4^6, I_2^6 + I_0^6 + I_4^6, I_4^6 + I_0^6, I_2^6 + I_0^6 + I_4^6) = y_7$$
$$y_7M = (I_2^6 + I_3^6 + I_4^6, I_2^6 + I_0^6 + I_4^6, I_4^6 + I_0^6, I_2^6 + I_0^6 + I_4^6) = y_8$$
$$(= y_7).$$

Thus the MOD resultant is a MOD fixed point.

In this way we can always find the MOD resultant to be a MOD fixed point or the MOD limit cycle.

Now we just describe the working and terminology of the MOD natural neutrosophic cognitive maps model used in this book.

Suppose we have a problem P in hand. The expert wishes to work using fuzzy cognitive maps model but has the edge weights from Z_n^I, $(2 \leq n < \infty)$.

Let $C_1, C_2, \ldots, C_m$ be the nodes with which the expert works.

Let G be the MOD natural neutrosophic directed graph G with edge weights from Z_n^I.

Let M be the MOD natural neutrosophic connection matrix associated with G.

Let $X = \{(a_1, a_2, \ldots, a_m) \mid a_i \in \{0, 1); 1 \leq i \leq m\}$ be the MOD initial state of vectors.

$X_s = \{(a_1, a_2, \ldots, a_m) \mid a_i \in \{0, 1, I_t^n$; t = 0 and all zero divisors, idempotents and nilpotents of Z_n, $1 \leq i \leq m\}$ be the MOD initial special state vector associated M.

Suppose $x = (a_1, \ldots, a_m) \in X$; the effect x on M.

$xM \to y_1$; $y_1M \to y_2$ and so on we will arrive after a finite number of steps the MOD fixed point or MOD limit cycle.

Suppose y is the end result we call y as the MOD hidden pattern of the MOD NCMs dynamical system.

Suppose $x_s \in X_s$ in a way we arrive at y_s the MOD fixed point or the MOD limit cycle as the MOD hidden pattern.

Next we see whether X is on or X_s is on we arrive at a MOD resultant and the nodes of this MOD NCMs model can be real, 0 or 1 or natural neutrosophic or mixed natural neutrosophic.

This is the main specialty between NCMs and FCMs and MOD NCMs model.

Only in this case we see the nodes can take any of the values 0, 1, I_t^n ; t is a zero divisor or nilpotent or an idempotent Z_n.

We will illustrate this situation by an example.

Example 3.12: Let P be a problem in hand.

Suppose C_1, C_2, C_3, C_4 be the nodes associated with the problem.

Let G be the MOD natural neutrosophic directed graph given by the following figure with edge weight from Z_4^I.

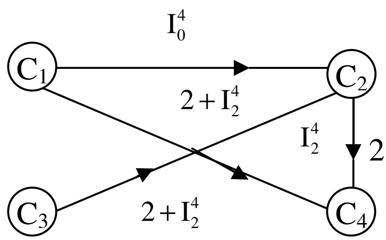


Figure 3.7

Let M be the MOD connection matrix with entries from Z_4^I

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 0 & I_0^4 & 0 & 2+I_2^4 \\ 0 & 0 & 0 & 2 \\ 0 & 2+I_2^4 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let X = $\{(a_1, a_2, a_3, a_4) \mid a_i \in \{0, 1\}; 1 \leq i \leq 4\}$ and

$X_s = \{(a_1, a_2, a_3, a_4) \mid a_i \in \{0, 1, I_0^4, I_2^4\}; 1 \leq i \leq 4\}$ be the MOD initial state vectors and MOD special initial state vectors respectively.

Let $x_1 = (1, 0, 0, 0) \in X$; to find the effect of x_1 on M.

$x_1M \to (1, I_0^4, 0, 2 + I_2^4) = y_1$
$y_1M \to (1, I_0^4, 0, 2 + I_2^4 + I_0^4) = y_2$
$y_2M \to (1, I_0^4 + I_2^4, 0, 2 + I_2^4 + I_0^4) = y_3$
$y_3M \to (1, I_0^4 + I_2^4, 0, 2 + I_2^4 + I_0^4) = y_4 (= y_3)$.

Thus the MOD resultant is MOD fixed point.

Let $x_2 = (0, 1, 0, 0) \in X$ the effect of x_2 on M is as follows.

$x_2M_2 \to (0, 1, 0, 2) = y_1$

$y_1M_2 \to (0, 1, 0, 2) = y_2$ $(= y_1)$.

Thus the MOD resultant is a MOD fixed point given by (0, 1, 0, 2).

Let $x_3 = (0, 0, 1, 0) \in X$; the effects of x_3 on M.

$x_3M \to (0, 2 + I_2^4, 1, 0) = y_1$
$y_1M \to (0, 2 + I_2^4, 1, I_2^4) = y_2$
$y_2M \to (0, 2 + I_2^4, 1, I_2^4) = y_3$ $(=y_2)$.

Thus the MOD resultant is a MOD fixed point given by $(0, 2 + I_2^4, 1, I_2^4)$.

Let $x_4 = (0, 0, 0, 1) \in X$; to find the effect of x_4 on M

$x_4M \to (0, 0, 0, 1) = x_4$ is the MOD classical special fixed point.

Let $x_5 = (I_0^4, 0, 0, 0) \in X_s$. To find the effect of x_5 on M.

$x_5M \to (I_0^4, I_0^4, 0, I_0^4) = y_1$
$y_1M \to (I_0^4, I_0^4, 0, I_0^4) = y_2(=y_1)$.

Thus the resultant is a MOD fixed point $(I_0^4, I_0^4, 0, I_0^4)$ --- I

The nodes C_2 and C_4 are just the natural neutrosophic zero where as the on state of C_1 has no effect on C_3.

Let $x_6 = (I_2^4, 0, 0, 0) \in X_s$.

The effect of x_6 on M;

$x_6M \to (I_2^4, I_0^4, 0, I_2^4 + I_0^4) = y_1$
$y_1M \to (I_2^4, I_0^4, 0, I_2^4 + I_0^4) = y_2$ $(= y_1)$.

Thus the MOD resultant is a MOD fixed point given by $(I_2^4, I_0^4, 0, I_2^4 + I_0^4)$

We see this is different from I.

Let $x_7 = (0, 1, I_0^4, 0) \in X_s$.

To find the effect of x_7 on M.

$$x_7M \to (0, I_0^4, I_0^4, 2) = y_1$$
$$y_1M \to (0, I_0^4, I_0^4, I_0^4) = y_2$$
$$y_2M \to (0, I_0^4, I_0^4, I_0^4) = y_3 \ (= y_2).$$

Thus the MOD resultant is a MOD fixed point.

Interested reader can work in this direction.

Once again while working with combined MOD NCMs model we need to make some simple modifications.

However, we wish to give two special MOD NCMs model examples in which one takes all edge weights for the MOD natural neutrosophic directed graph only from Z_n another takes values only from $Z_n^I \setminus Z_n$.

Example 3.13: Let P be a problem in hand and $C_1, C_2, \ldots, C_6$ be the nodes associated with the problem.

Suppose the edge weights are taken from $Z_8 \subseteq Z_8^I$ for the MOD directed graph G is as follows.

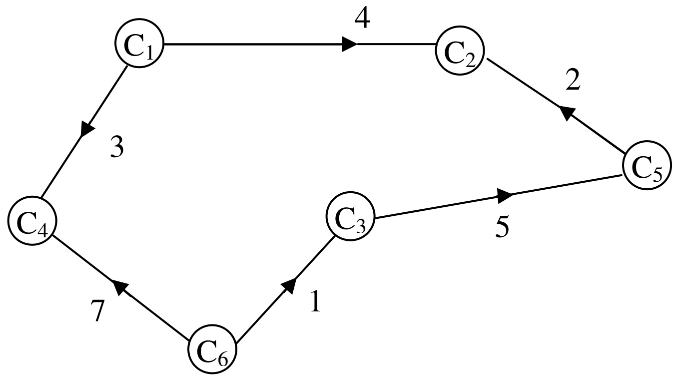


Figure 3.8

Let M the MOD natural neutrosophic connection matrix associated with G.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & 4 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 7 & 0 & 0 \end{bmatrix} \end{array}.$$

Let $X = \{(a_1, a_2, \ldots, a_6\} \mid a_i \in \{0, 1\}; 1 \leq i \leq 6$ and

$X_s = \{(a_1, a_2, \ldots, a_6) \mid a_i \in \{0, 1, I_0^8, I_2^8, I_4^8, I_6^8\}; 1 \leq i \leq 6\}$ be the MOD initial state vector and the MOD initial special state vector associated with the MOD N Cognitive Maps model.

Let $x_1 = (1, 0, 0, 0, 0, 0) \in X$ to find the effect of x_1 on M

$x_1M \to (1, 4, 0, 3, 0, 0) = y_1$
$y_1M \to (1, 4, 0, 3, 0, 0) = y_2 \ (= y_1)$.

Thus the MOD resultant is a MOD fixed point given by (1, 4, 0, 3, 0, 0).

We see the MOD resultant is real if the MOD NCMs matrix which represents the MOD NCMs dynamical matrix M has only real entries.

Let $x_2 = (0, 1, 0, 0, 0, 0) \in X$, to find the effect of x_2 on M

$x_2M \to (0, 1, 0, 0, 0, 0) = y_1$ $(=x_2)$.

Thus the MOD resultant is a MOD special classical fixed point.

Let $x_6 = (0, 0, 0, 0, 0, 1) \in X$; to find the effect of x_6 on M

$$
\begin{aligned}
x_6M &\to (0, 0, 1, 7, 0, 1) = y_1 \\
y_1M &\to (0, 0, 1, 7, 5, 1) = y_2 \\
y_2M &\to (0, 2, 1, 7, 5, 1) = y_3 \\
y_3M &\to (0, 2, 1, 7, 5, 1) = y_4 \ (=y_3).
\end{aligned}
$$

The MOD resultant is not the MOD special classical fixed point only a MOD fixed point given by (0, 2, 1, 7, 5, 1) which is real.

Let $a_1 = (I_2^8, 0, 0, 0, 0, 0) \in X_s$ to find the effect of a_1 on M.

$$
\begin{aligned}
a_1M &\to (I_2^8, I_2^8, 0, I_2^8, 0, 0) = y_1 \\
y_1M &\to (I_2^8, I_2^8, 0, I_2^8, 0, 0) = y_2 \ (= y_1).
\end{aligned}
$$

This is the MOD resultant is the MOD fixed point given by $(I_2^8, I_2^8, 0, I_2^8, 0, 0)$ which is pure natural neutrosophic.

That is if the MOD initial state vector is from $X_s \setminus X$ then the MOD resultant is pure natural neutrosophic only.

Let $a_1 = (0, 1, 0, 0, I_6^8, 0) \in X_s$; to find the effect of a_1 on M.

$a_1M \to (0, I_6^8, 0, 0, I_6^8, 0) = y_1$
$y_1M \to (0, I_6^8, 0, 0, I_6^8, 0) = y\ (=y_1)$.

Thus the MOD resultant in this case is also a MOD fixed point in which all the nodes are pure natural neutrosophic only.

Let $a_2 = (0, I_4^8, 0, 0, 0, 1) \in X_s$, to find the effect of a_2 on M.

$a_2M \to (0, I_4^8, 1, 7, 0, 1) = y_1$
$y_1M \to (0, I_4^8, 1, 7, 0, 1) = y_2\ (= y_1)$.

Thus the MOD resultant is a MOD fixed point given by $(0, I_4^8, 1, 7, 0, 1)$ which is such that some nodes are pure natural neutrosophic and some are real however the MOD special initial state vector has both real and natural neutrosophic coordinates to be in the on state.

In view of this we have the following theorem.

THEOREM 3.1: *Let P be a problem; C_1, …, C_m be the nodes associated with P.*

Let M be the MOD NCMs model associated with P with entries from $Z_n \subseteq Z_n^I$.

Let $X = \{(a_1, …, a_n) / a_i \in \{0, 1\}, 1 \leq i \leq m\}$ and

$X_s = \{(a_1, a_2, …, a_n) / a_i \in \{0, 1, I_t^n$ where t is a zero divisor or nilpotent or idempotent or 0 of $Z_n\}$. $1 \leq i \leq m\}$ be the MOD initial state vectors and MOD special initial state vector respectively associated with this MOD NCMs model M.

i) *If $x \in X$, the MOD resultant of x on M is always MOD fixed point or a MOD limit cycle which has the node values from Z_n.*

ii) If $x_1 \in X_s \setminus X$ the MOD resultant of x_1 on M has the nodes to be pure natural neutrosophic only.

iii) If $x_2 \in X_s$ (some nodes are pure natural neutrosophic and some or 1) then the MOD resultant of x_2 on M can be such that the nodes associated with them can be pure natural neutrosophic and real or pure natural neutrosophic but it cannot be all real.

Proof is direct and hence left as an exercise to the reader.

Next we proceed onto give examples of MOD NCM model which has all the edge weights associated with the directed graphs to be pure neutrosophic.

Example 3.14: Let P be a problem in hand and C_1, C_2, C_3, C_4, C_5 be the nodes associated with P. The expert wishes to work with MOD NCMs model using entries from Z_{10}^{I}.

Let G be the MOD direct graph given by the expert is as follows.

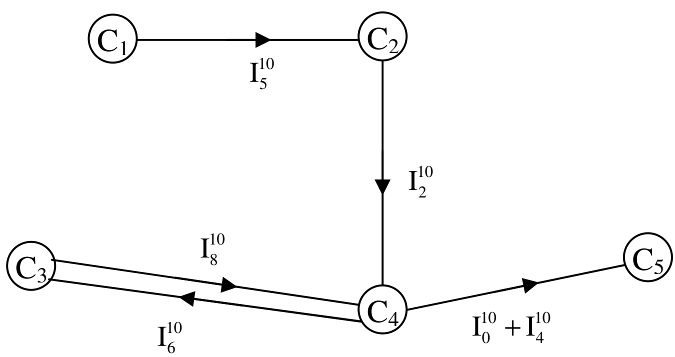


Figure 3.9

Let M_1 be the MOD n connection matrix associated with G.

$$M_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 0 & I_5^{10} & 0 & 0 & 0 \\ 0 & 0 & 0 & I_2^{10} & 0 \\ 0 & 0 & 0 & I_8^{10} & 0 \\ 0 & 0 & I_6^{10} & 0 & I_0^{10} + I_4^{10} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let X = $\{(a_1, a_2, a_3, a_4, a_5) / a_i \in \{0, 1\}, 1 \leq i \leq 5\}$ and

$X_s = \{(a_1, a_2, a_3, a_4, a_5) / a_i \in \{0, 1, I_2^{10}, I_4^{10}, I_0^{10}, I_6^{10}, I_8^{10}, I_5^{10}\}, 1 \leq i \leq 5\}$ be the MOD initial state vectors and MOD special initial state vectors respectively associated with M_1.

Let $x_1 = (1, 0, 0, 0, 0) \in X$; to find the effect of x_1 on M_1.

$x_1M_1 \to (1, I_5^{10}, 0, 0, 0) = y_1$

$y_1M_1 \to (1, I_5^{10}, 0, I_0^{10}, 0) = y_2$

$y_2M_1 \to (1, I_5^{10}, I_0^{10}, I_0^{10}, I_0^{10}) = y_3$

$y_3M_1 \to (1, I_5^{10}, I_0^{10}, I_0^{10}, I_0^{10}) = y_4\ (=y_3)$.

Thus the MOD resultant is a MOD fixed point given by $(1, I_5^{10}, I_0^{10}, I_0^{10}, I_0^{10})$, that is the on state of C_1 makes on all the four nodes C_2, C_3, C_4 and C_5 and node C_2 is a natural neutrosophic number I_5^{10} and other three are just natural neutrosophic zeros I_0^{10}.

Let $x_2 = (0, 1, 0, 0, 0) \in X$; the effect of x_2 on M_1

$x_2M_1 \to (0, 1, 0, I_2^{10}, 0) = y_1$

$y_1M_1 \to (0, 1, I_2^{10}, I_2^{10}, I_0^{10} + I_8^{10}) = y_2$

$y_2M_1 \to (0, 1, I_2^{10}, I_2^{10} + I_6^{10}, I_2^{10} + I_8^{10}) = y_3$

$y_3M_1 \to (0, 1, I_2^{10} + I_6^{10}, I_2^{10} + I_6^{10} + I_8^{10}, I_0^{10} + I_8^{10} + I_4^{10}) = y_4$ and so on.

Certainly after a finite number of iterations we will arrive a MOD fixed or a MOD limit cycle.

Further all the nodes which come to on state will be pure natural neutrosophic however the on state of the that node may be 1 in the MOD resultant.

Let $x_5 = (0, 0, 0, 0, 1) \in X_s$ to find the effect of x_5 on M_1 is as follows.

$x_5M_1 \to (0, 0, 0, 0, 1) = x_5.$

Thus the MOD resultant is a MOD special classical fixed point.

Let $x_4 = (0, 0, 0, 1, 0) \in X$, the effect of x_4 on M_1 is as follows.

$x_4M_1 \to (0, 0, I_6^{10}, 1, I_0^{10} + I_4^{10}) = y_1$
$y_1M_1 \to (0, 0, I_6^{10}, I_8^{10}, I_0^{10} + I_4^{10}) = y_2$
$y_2M_1 \to (0, 0, I_8^{10}, I_8^{10}, I_0^{10} + I_2^{10}) = y_3$
$y_3M_1 \to (0, 0, I_8^{10}, I_4^{10}, I_0^{10} + I_6^{10}) = y_4$
$y_4M_1 \to (0, 0, I_4^{10}, I_4^{10}, I_0^{10} + I_4^{10}) = y_5$
$y_5M_1 \to (0, 0, I_4^{10}, I_2^{10}, I_0^{10} + I_6^{10}) = y_6$
$y_6M_1 \to (0, 0, I_2^{10}, I_2^{10}, I_0^{10} + I_8^{10}) = y_7$
$y_7M_1 \to (0, 0, I_2^{10}, I_6^{10}, I_0^{10} + I_6^{10}) = y_8$
$y_8M_1 \to (0, 0, I_6^{10}, I_6^{10}, I_0^{10} + I_4^{10})$ and so on.

We will arrive at a MOD resultant after a finite number of iterations which may be a MOD fixed point or a MOD limit cycle.

But whatever be the case the on state of the node has no effect on nodes C_2 and C_1 only the node C_3 and C_6 come to on state with pure natural neutrosophic values.

Let $a_1 = (I_6^{10}, 0, 0, 0, 0) \in X_s$, to find the effect of a_1 on M_1

$a_1M_1 \to (I_6^{10}, I_0^{10}, 0, 0, 0) = y_1$
$y_1M_1 \to (I_6^{10}, I_0^{10}, 0, I_0^{10}, 0) = y_2$
$y_2M_1 \to (I_6^{10}, I_0^{10}, I_0^{10}, I_0^{10}, I_0^{10}) = y_3$
$y_3M_1 \to (I_6^{10}, I_0^{10}, I_0^{10}, I_0^{10}, I_0^{10}) = y_4\ (=y_3).$

Thus the MOD resultant is a MOD mid point and on state of the node with natural neutrosophic number I_6^{10} makes on all the other nodes to on state with the natural neutrosophic zero value.

Let $b_1 = (1, 0, I_4^{10}, 0, 0) \in X_s$.

To find the effect of b_1 on M_1

$b_1M_1 \to (1, I_5^{10}, I_4^{10}, I_2^{10}, 0) = y_1$
$y_1M_1 \to (1, I_5^{10}, I_2^{10}, I_0^{10} + I_2^{10}, I_0^{10} + I_8^{10}) = y_2$
$y_2M \to (1, I_5^{10}, I_0^{10} + I_2^{10}, I_0^{10} + I_6^{10}, I_0^{10} + I_8^{10}) = y_3$
$y_3M \to (1, I_5^{10}, I_0^{10} + I_6^{10}, I_0^{10} + I_6^{10}, I_0^{10} + I_4^{10}) = y_4$
$y_4M \to (1, I_5^{10}, I_0^{10} + I_6^{10}, I_0^{10} + I_8^{10}, I_0^{10} + I_4^{10})$ and so on.

All nodes come to on state C_1 remains in the same state as 1, however all other states are MOD natural neutrosophic numbers.

Example 3.15: Let P be a problem in hand. C_1, C_2, C_3 and C_4 are the nodes. The expert works, with the MOD N C Maps model entries from Z_{12}^{I}.

The directed graph G_2 given by him is as follows.

$G_2 =$

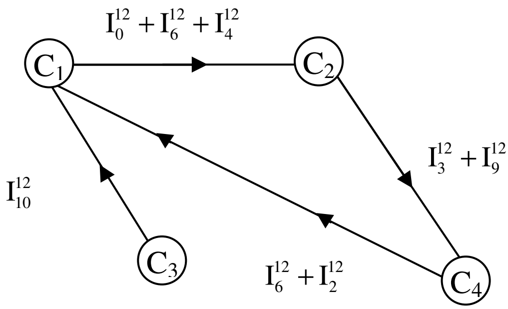


Figure 3.10

Let M_2 be the MOD connection matrix associated with G_2.

$$M_2 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & \quad\quad C_2 \quad\quad & C_3 & C_4 \end{array} \\ \begin{bmatrix} 0 & I_0^{12} + I_6^{12} + I_4^{12} & 0 & 0 \\ 0 & 0 & 0 & I_3^{12} + I_9^{12} \\ I_{10}^{12} & 0 & 0 & 0 \\ I_6^{12} + I_2^{12} & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let $X = \{(a_1, a_2, a_3, a_4) \mid a_i \in \{0, 1\}; 1 \leq i \leq 4\}$ and

$X_s = \{(a_1, a_2, a_3, a_4) \mid a_i \in \{0, 1, I_0^{12}, I_2^{12}, I_4^{12}, I_6^{12}, I_8^{12}, I_{10}^{12}, I_3^{12}, I_9^{12}\}; 1 \leq i \leq 4\}$ be the MOD initial state vector and MOD special natural neutrosophic initial state vectors respectively associated with M_2.

Clearly $X \subseteq X_s$.

Let $x_1 = (1, 0, 0, 0) \in X$, to find the effect of x_1 on M_2

$x_1 M_2 \to (1, I_0^{12} + I_6^{12} + I_4^{12}, 0, 0) = y_1$

$$y_1M_2 \rightarrow (1,\ I_0^{12} + I_6^{12} + I_4^{12},\ 0,\ I_0^{12} + I_6^{12} + I_8^{12}) = y_2$$
$$y_2M_2 \rightarrow (I_0^{12} + I_4^{12},\ I_0^{12} + I_6^{12} + I_4^{12},\ 0,\ I_0^{12} + I_6^{12}) = y_3$$
$$y_3M_2 \rightarrow (I_0^{12},\ I_0^{12} + I_4^{12},\ 0,\ I_0^{12} + I_6^{12}) = y_4$$
$$y_4M_2 \rightarrow (I_0^{12},\ I_0^{12} + I_4^{12},\ 0,\ I_0^{12}) = y_5$$
$$y_5M_2 \rightarrow (I_0^{12},\ I_0^{12} + I_4^{12},\ 0,\ I_0^{12}) = y_6\ (=y_5).$$

Thus the MOD resultant is a MOD fixed point, all the nodes which has come to on state has become natural neutrosophic zero or a natural neutrosophic number. However C_3 remain unaffected by the on state of C_1.

Interested author can with different real world models.

We prove the following theorem.

THEOREM 3.2: *Let P be a problem $C_1, C_2, \ldots, C_n$ be the nodes associated with the problem. M be the $n \times n$ MOD natural neutrosophic matrix with entries from $Z_m^I \setminus Z_m$ serves as the dynamical system of the MOD natural neutrosophic-neutrosophic Cognitive Maps model.*

Let $X = \{(a_1, a_2, \ldots, a_n) / a_i \in \{0, 1\}, 1 \leq i \leq n\}$ and

$X_s = \{(a_1, a_2, \ldots, a_n) / a_i \in \{0, 1, I_t^n$, t a zero divisor or idempotent or a nilpotent element of $Z_m\}; 1 \leq i \leq n\}$ be the MOD initial state vectors and MOD special natural neutrosophic initial state vectors.

i) If $x \in X$ then the MOD resultant of x on M is always a MOD fixed point or a MOD limit cycle with entries from $Z_m^I \setminus Z_m$ or 1 where 1 is the initial state vector in x.

ii) If $x_s \in X_s \setminus X$ then the MOD resultant of x_s on M is always a vector whose entries are from $X_s \setminus \{1\}$.

Proof is direct and hence left as an exercise to the reader.

Working for more results in this direction is left as an exercise to the reader as it is considered as a matter of routine.

Next we proceed onto define describe and develop the notion of MOD natural neutrosophic complex Cognitive Maps model.

$C^I(Z_n) = \{a + bi_F / a, b \in Z_n, i_F^2 = (n - 1)\} \cup \{ I_t^C$ where t is a zero divisor or unit or nilpotent of $C(Z_n)\}$.

We first define MOD natural neutrosophic complex number directed graphs.

Example 3.16: Let G be a directed graph with edge weight from $C^I(Z_4)$ given by the following figure.

G is defined as the MOD natural neutrosophic complex directed graph.

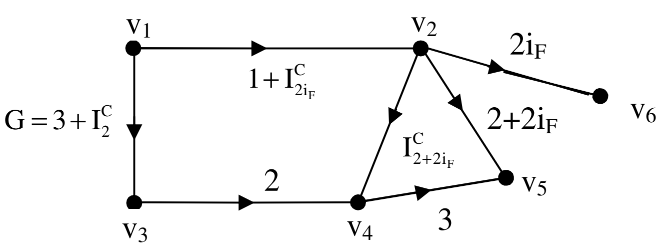


Figure 3.11

Example 3.17: Let G, be the MOD natural neutrosophic number complex directed graph given in Figure 3.12 with edge weights from $C^I(Z_6)$.

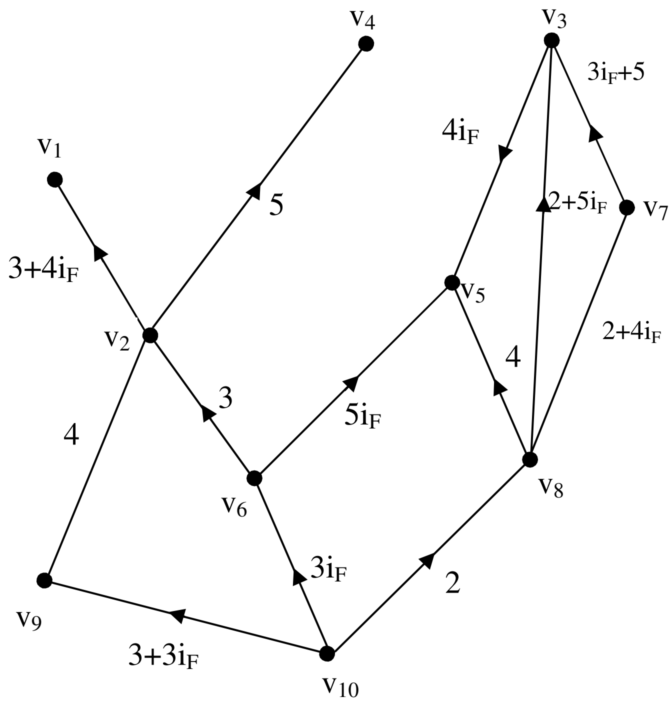


Figure 3.12

Now we proceed onto describe the notion of MOD natural neutrosophic complex finite number directed graph in the following.

Let $C^I(Z_n)$ be the MOD finite complex number natural neutrosophic elements as they are got using modulo integers we choose to call them as MOD finite complex natural neutrosophic number elements instead of finite complex natural neutrosophic numbers in [ ].

Let G be the directed graph of the edge weights are taken from $C^I(Z_n)$ then we define G to be the MOD finite complex natural neutrosophic directed graph.

We have already given examples of them.

Now we proceed onto describe by examples MOD finite complex natural neutrosophic number square matrices.

Example 3.18: Let

$$M = \begin{bmatrix} 0 & i_F & 2+3i_F & 0 & 4 & 2i_F \\ 2 & 0 & 4 & i_F+3 & 0 & i_F \\ 0 & 4+i_F & 0 & 0 & 2+i_F & 0 \\ i_F & 0 & 4i_F+3 & I_3^C & 0 & 2+3i_F \\ 0 & I_{2i_F}^C & 2 & 0 & 4 & 0 \\ 2+4i_F & 0 & 0 & 4+i_F & 0 & 3+i_F \end{bmatrix}$$

be the MOD finite complex natural neutrosophic number square matrix with entries from $C^I(Z_6)$.

Example 3.19: Let

$$M = \begin{bmatrix} 0 & 4+I_{4i_F}^C & 1 & 3+I_{2i_F}^C \\ I_0^C & 0 & I_2^C+I_4^C & 0 \\ 2 & 5 & 0 & 3+I_6^C \\ 0 & 4+I_{6+6i_F}^C & 3 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic finite complex number square matrix with entries from $C^I(Z_8)$.

Thus if $M = (m_{ij})$ is a $n \times n$ square matrix with entries from $C^I(Z_m)$ then we define M to be a MOD natural neutrosophic finite complex number square matrix.

We have already gives examples of them we need to define only a new type of special operations using them.

Let $X = \{(a_1, a_2, \ldots, a_n) / a_i \in \{0, 1\}, 1 \leq i \leq n\}$ and

$X_s = \{(a_1, a_2, \ldots, a_n) \mid a_i \in \{0, 1, I_t^C$, where t is a zero divisor or nilpotent or an idempotent in $C^I(Z_m)\}, 1 \leq i \leq n\}$ be the MOD initial state vectors and MOD special finite complex natural neutrosophic integer initial state vectors respectively associated with the MOD natural neutrosophic finite complex number matrix M.

The special operations will be illustrated by the following example.

Example 3.20: Let

$$M = \begin{bmatrix} 0 & 3 & 4+2i_F & 0 & I_0^C \\ I_2^C & 0 & i_{2i_F}^C + 1 & 2 & 0 \\ 0 & 3 + I_{2+4i_F}^C & 0 & I_4^C & 2 \\ 1 & 0 & 1 & 0 & i_F \\ 0 & 4+i_F & 0 & 0 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic finite complex number matrix with entries from $C^I(Z_6)$.

Let $X = \{(a_1, a_2, a_3, a_4, a_5) / a_i \in \{0, 1\}; 1 \leq i \leq 5\}$ be the initial state of vector and

$X_s = \{(a_1, a_2, \ldots, a_5) / a_i \in \{0, 1, I_t^C$ where t is a zero divisor or idempotent or nilpotent element of $C^I(Z_6)\}; 1 \leq i \leq 5\}$ be the MOD special natural neutrosophic complex number initial state vectors related with the MOD matrix M.

Let $x = (1, 0, 0, 0, 0) \in X$; to find the effect of x on M.

$xM \to (1, 3, 4 + 2i_F, 0, I_0^C) = y_1$

$y_1M \to (I_2^C, 3 + I_0^C + I_{2+4i_F}^C, 1 + 2i_F + I_{2i_F}^C, I_4^C, I_0^C + 2 + 4i_F)$

$= y_2$

$y_2M \to (I_0^C + I_2^C + I_4^C, I_{4+4i_F}^C + I_{2i_F}^C + I_{2+4i_F}^C + 3, 3 + I_2^C + I_0^C + I_4^C + I_{2i_F}^C + I_{2+4i_F}^C, I_0^C + I_4^C + I_{2i_F}^C, I_0^C + 2 + 4i_F + I_{2i_F}^C + I_4^C) = y_3$ and so on however after a finite number of iterations we are sure to arrive at a MOD fixed point or a MOD limit cycle.

Let $x_2 = (0, 1, 0, 0, 0) \in X$, to find the effect of x_2 on M.

$x_2M \to (I_2^C, 1, 1 + I_{2i_F}^C, 2, 0) = y_1$

$$y_1M = (2 + I_2^C, 3 + I_{2i_F}^C, I_2^C + I_{4+4i_F}^C, 3 + I_2^C + I_{2i_F}^C, 2 + I_4^C + I_{2i_F}^C, 2 + 2i_F + I_0^C + I_{2i_F}^C) = y_2 \text{ and so on.}$$

Thus certainly after a finite number of iterations we arrive at a MOD fixed point or a MOD limit cycle.

Let $a_1 = (I_0^C, 0, 0, 1, 0) \in X_s$; to find the effect of a_1 on M.

$a_1M \to (I_0^C + 1, I_0^C, 1 + I_0^C, 1, I_0^C + i_F) = y_1$

$$y_1M \to (I_0^C + 1, I_0^C + I_{2+4i_F}^C, 5 + 2i_F + I_0^C, I_0^C + I_4^C, 2 + i_F + I_0^C) = y_2$$

$y_2M \to (I_0^C + I_4^C, I_0^C + I_{2+4i_F}^C, 4 + 2i_F + I_0^C + I_4^C + I_{4+4i_F}^C + I_{2+4i_F}^C, I_0^C + I_4^C + I_{2+4i_F}^C + 2i_F, 4 + I_0^C + 4iF + I_4^C) = y_3$ and so on.

We are sure after a finite number of iterations we will arrive at a MOD fixed point or a MOD limit cycle.

Interested reader can construct more examples of such models.

Example 3.21: Let

$$M = \begin{bmatrix} 0 & 2i_F & I^C_{5i_F} & 0 & 0 \\ 0 & 0 & 1 & I^C_0 & 0 \\ 0 & 0 & 0 & 1 & 4 \\ 2 & 0 & 0 & 0 & i_F \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic finite complex number matrix with entries from $C^I(Z_{10})$.

Let $X = \{(a_1, a_2, \ldots, a_5) / a_i \in \{0, 1\}, 1 \leq i \leq 5\}$ and

$X_S = \{(a_1, a_2, \ldots, a_5) / a_i \in \{0, 1, I^C_0, I^C_2, I^C_4, I^C_6, I^C_8, I^C_{2i_F}, I^C_{4i_F}, I^C_5, I^C_{5i_F}, \ldots, I^C_{2+8i_F}, \ldots, I^C_{8+8i_F}\}, 1 \leq i \leq 5\}$ be the initial state vector MOD special initial state of vectors associates with M.

Let $x_1 = (1, 0, 0, 0, 0) \in X$; the effect of x_1 on M.

$x_1M \to (1, 2i_F, I^C_{5i_F}, 0, 0) = y_1$

$y_1M \to (1, 2i_F, I^C_{5i_F} + 2i_F, I^C_{5i_F}, 0) = y_2$

$y_2M \to (I^C_{5i_F}, 2i_F, I^C_{5i_F} + 2i_F, I^C_0 + I^C_{5i_F} + 2i_F, I^C_{5i_F})$ and so on.

After a finite number of iteration we will arrive at a MOD fixed point or MOD limit cycle.

Let $x_2 = (0, 1, 0, 0, 0) \in X$.

$x_2M \to (0, 1, 1, I^C_0, 0) = y_1$

$y_1M \to (I^C_0, 1, 1, I^C_0, 4 + I^C_0) = y_2$

$y_2M \to (4 + I^C_0, I^C_0, I^C_0 + 5, 1 + I^C_0, 4 + I^C_0) = y_3$

$$y_3M = (6 + I_0^C, 8i_F + I_0^C, \; I_{5i_F}^C + I_0^C + 4, \; I_0^C + 5, 4 + i_F + I_0^C) = y_4$$

Thus the MOD resultant will be arrived after a finite number of iterations which may be a MOD fixed point or a MOD limit cycle.

Let $a_3 = (I_5^C, 0, 0, 0, 0) \in X_5$.

The effect of a_3 on M

$$a_3M \to (I_5^C, I_5^C, I_{5i_F}^C, 0, 0) = y_1$$

$$y_1M \to (I_5^C, I_5^C, I_{5i_F}^C + I_5^C, \; I_0^C + I_{5i_F}^C, \; I_{5i_F}^C) = y_2$$

$$y_2M \to (I_0^C + I_{5i_F}^C, \; I_5^C, \; I_{5i_F}^C + I_5^C, \; I_0^C + I_{5i_F}^C + I_5^C, \; I_0^C + I_{5i_F}^C + I_5^C) = y_3$$ and so on.

Certainly after a finite number of iteration we will arrive at a MOD fixed point or a MOD limit cycle.

Let $p_1 = (1, 0, 0, 0, I_2^C) \in X_s$, to find the effect of p_1 on M.

$$p_1M \to (I_2^C, 2i_F, \; I_{5i_F}^C + I_2^C, 0, \; I_2^C) = y_1$$

$$y_1M \to (I_2^C, I_2^C, I_0^C + 2i_F + I_2^C, \; I_0^C + I_{5i_F}^C + I_2^C, \; I_2^C + I_{5i_F}^C) = y_2$$

and so on.

Certainly after a finite number of iterations we will arrive at a MOD fixed point or a MOD limit cycle.

Interested reader can construct more such model for real world problem and analyse them.

Example 3.22: Let

$$M = \begin{bmatrix} 0 & I_2^C & 0 & 1 + I_{2i_F}^C \\ 0 & 0 & 1 & 2 \\ 1 & 0 & 0 & 1 \\ 3 & 0 & 2 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic finite complex modulo integer matrix with entries from $C^I(Z_4)$.

Let $X = \{(a_1, a_2, a_3, a_4) \mid a_i \in \{0, 1\}, 1 \leq i \leq 4\}$ be the initial state vector.

Let $x = (1, 0, 0, 0) \in X$, for find the effect of x on M.

$xM \to (1, I_2^C, 0, I_{2i_F}^C + 1) = y_1$

$y_1M \to (3 + I_{2i_F}^C, I_2^C, I_2^C + 2 + I_{2i_F}^C, 1 + I_{2i_F}^C + I_2^C) = y_2$

$y_2M \to (1 + I_2^C + I_{2i_F}^C, I_0^C + I_2^C, 2 + I_2^C + I_{2i_F}^C, 1 + I_{2i_F}^C + I_2^C)$
$= y_3$

$y_3M \to (1 + I_2^C + I_{2i_F}^C, I_2^C + I_0^C, I_0^C + I_2^C + 2 + I_{2i_F}^C, 3 + I_2^C +$
$I_{2i_F}^C + I_0^C) = y_4$ and so on.

Certainly after a finite number of iterations we will arrive at a MOD fixed point or MOD limit cycle.

Let $x_2 = (0, 1, 0, 0) \in X$; to find the effect of x_2 on M.

$x_2M \to (0, 1, 1, 2) = y_1$

$y_1M \to (3, 1, 1, 3) = y_2$

$y_2M = (2, I_2^C, 3, 2 + I_{2i_F}^C) = y_3$

$y_3M = (1 + I_{2i_F}^C, I_2^C, I_2^C + I_{2i_F}^C, 2 + I_2^C + I_{2i_F}^C) = y_4$ and so on.

We will after a finite number of iterations arrive at a MOD fixed point or a MOD limit cycle.

Let $x = (0, 0, 1, 1) \in X$. To find the effect of x on M.

$xM = (0, 0, 2, 1) = y_1$

$y_1M = (1, 0, 2, 2) = y_2$

$y_2M = (0, 0, 1, 3 + I_{2i_F}^C + 0) = y_3$

$y_3M = (2 + I_{2i_F}^C, 0, 2 + I_{2i_F}^C, 1) = y_4$

$y_4M = (1 + I_{2i_F}^C, I_2^C + I_0^C, 2, I_{2i_F}^C + I_0^C) = y_5$ and so on.

We see that the MOD resultant is a MOD fixed point or a MOD limit cycle but the on state will give the resultant nodes to be 1 or 0 or real or complex or mixed complex number or natural complex neutrosophic.

We see the on state of a node C_i can make other nodes complex or just 1 or just 0 and so on.

This is one of the main advantages of using these new MOD natural neutrosophic complex cognitive maps model very distinct from FCMs and NCMs which is commonly used by researchers.

Second is these models can be very powerful for the human mind usually thinks at times a nodes as not a completely real quantity or a completely complex quantity or a completely a natural neutrosophic quantity but a mixture of all also.

Hence this model works more like our brains functioning.

Next we proceed onto describe the notion of MOD natural neutrosophic dual number cognitive maps model.

For to introduce this new model we have to first describe MOD dual number directed graphs and MOD natural neutrosophic dual number square matrices and special operations on them.

Example 3.23: Let G be the MOD natural neutrosophic dual number directed graph with edge weights from

$$\langle Z_5 \cup g\rangle_I = \{a + bg,\ I_g^g,\ I_{2g}^g,\ I_0^g,\ I_g^g + I_0^g,\ I_g^g + 2,\ 1 + I_g^g,\ \ldots\}.$$

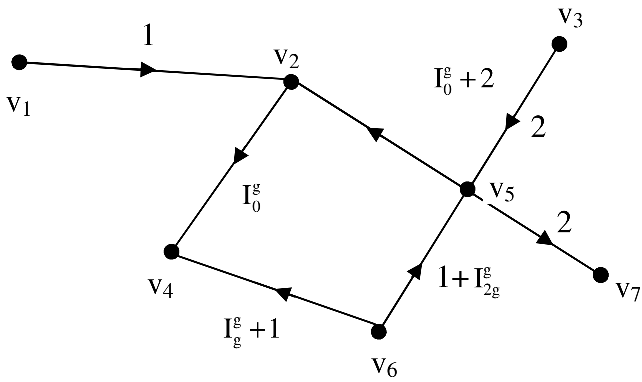


Figure 3.13

Example 3.24: Let H be the MOD natural neutrosophic dual number directed graph with edge weights from $\langle Z_6 \cup g \rangle_I$ given in Figure 3.14.

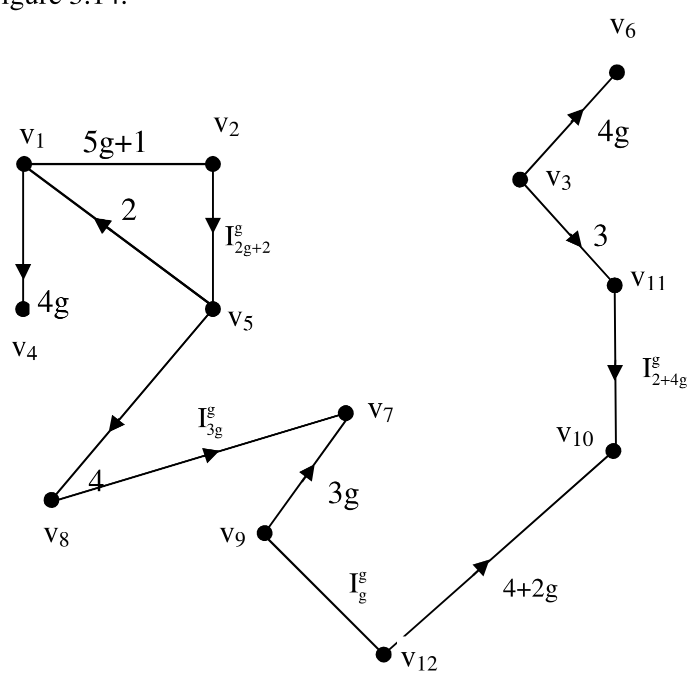


Figure 3.14

Now we define the MOD natural neutrosophic dual number directed graph in the following.

Let G be the directed graph with edge weights from $\langle Z_n \cup g\rangle_I$; then we define G to be a MOD natural neutrosophic dual number directed graph.

We have given examples of them.

Next we describe MOD natural neutrosophic dual number matrix by some examples.

Example 3.25: Let

$$M = \begin{bmatrix} 3g+2 & 0 & I^g_{2g} & I^g_{4g}+4 & 2 \\ 0 & 0 & 6g & 0 & 2g+4 \\ 4 & 2+I^g_{4g+2} & 0 & 7g & 0 \\ 0 & I^g_{2g}+I^g_4 & 2g & I^g_0 & 1 \\ 1 & 0 & 2 & 0 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic dual number 5×5 matrix with entries from $\langle Z_8 \cup g\rangle_I$.

Example 3.26: Let

$$P = \begin{bmatrix} 0 & 2g+4 & 5 \\ 6g & 0 & I^g_0 \\ I^g_g & 6+I^g_{6g} & 0 \end{bmatrix}$$

be the MOD natural neutrosophic dual number square matrix with entries from $\langle Z_7 \cup g\rangle_I$.

Now having seen examples of MOD natural neutrosophic dual number square matrix we will define them in the following.

Let $M = (m_{ij})$ be a $n \times n$ matrix with entries from $\langle Z_m \cup g\rangle_I$ then we define M to be a MOD natural neutrosophic dual number square matrix.

Now we define special type of operations on them.

Let $X = \{(a_1, \ldots, a_n) / a_i \in \{0, 1\}; 1 \leq i \leq n\}$ be the set of MOD initial state vectors and

$X_s = \{(a_1, a_2, \ldots, a_n) / a_i \in \{0, 1, I_t^g / t$ is a nilpotent or idempotent or a zero divisors in $\langle Z_7 \cup g\rangle_I\}; 1 \leq i \leq n\}$ be the set of MOD special natural neutrosophic dual number state vectors.

We find xM; $x \in X$ or X_s and update at each stage since the collection $\langle Z_m \cup g\rangle_I$ is finite we will arrive at a MOD fixed point or a MOD limit cycle after a finite number of iterations.

We will illustrate this situation by some examples.

Example 3.27: Let

$$M = \begin{bmatrix} 0 & 3+g & I_2^g & 0 \\ I_{3g}^g & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ g & 0 & 1+g & 0 \end{bmatrix}$$

be the MOD natural neutrosophic dual number matrix with entries from $\langle Z_4 \cup g\rangle_I$.

Let $X = \{(a_1, a_2, a_3, a_4) / a_i \in \{0, 1\}; 1 \leq i \leq 4\}$ and

$X_s = \{(a_1, a_2, a_3, a_4) / a_i \in \{0, 1, I_t^g$; t is a zero divisors or nilpotents or idempotents from $\langle Z_4 \cup g\rangle_I\}; 1 \leq i \leq 4\}$ be the MOD initial state vector and MOD special initial state vector respectively.

Let $x = (1, 0, 0, 0) \in X$; the effect of x on M,

$$xM \to (1, 3 + g, I_2^g, 0) = y_1$$
$$y_1M \to (I_{3g}^g, 3 + g, I_2^g, I_2^g) = y_2$$
$$y_2M \to (I_{3g}^g + I_2^g, I_{3g}^g, I_{2g}^g, I_2^g) = y_3$$
$$y_3M \to (I_0^g + I_2^g, I_{3g}^g + I_2^g, I_{2g}^g, I_{2g}^g) = y_4$$
$$y_4M \to (I_0^g + I_{2g}^g, I_0^g + I_2^g, I_0^g + I_{2g}^g, I_{2g}^g) = y_5$$
$$y_5M \to (I_0^g + I_{2g}^g, I_0^g + I_{2g}^g, I_0^g + I_{2g}^g, I_0^g + I_{2g}^g) = y_6$$
$$y_6M \to (I_0^g + I_{2g}^g, I_0^g + I_{2g}^g, I_0^g + I_{2g}^g, I_0^g + I_{2g}^g) = y_7 (= y_6).$$

The MOD resultant is a MOD fixed point given by $(I_0^g + I_{2g}^g, I_0^g + I_{2g}^g, I_0^g + I_{2g}^g, I_0^g + I_{2g}^g)$.

We see the on state of the node C_1 makes all the nodes to on natural neutrosophic state $I_0^g + I_{2g}^g$ including C_1.

So every node has the same effect when the node C_1 is on with initial value 1.

Let $x_4 = (0, 0, 0, 1) \in X$. To find the effect of x_4 on M.

$$x_4M \to (g, 0, 1 + g, 1) = y_1$$
$$y_1M = (g, 3g, I_2^g + 1 + g, 1 + g) = y_2$$
$$y_2M = (I_{3g}^g + g, 3g, I_2^g + 1 + 2g, I_{2g}^g + 1 + g) = y_3$$
$$y_3M = (I_{3g}^g + I_2^g + g + I_{2g}^g, I_{3g}^g + 3g, I_{2g}^g + I_2^g + 1 + 2g, I_2^g + 1 + 2g) = y_4 \text{ and so on.}$$

We see on state of C_4 has after four iterations made on all the other nodes and they get different values.

However it is left as an exercise for the reader to find the MOD resultant of x_4 on M.

This is the way the special operations are performed on M.

We give yet one more example before we define and describe the MOD natural neutrosophic dual number Cognitive Maps model (MOD NCMs model).

Example 3.28: Let

$$M = \begin{bmatrix} 0 & g & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & I_g^g & 0 \end{bmatrix}$$

be the MOD natural neutrosophic dual number square matrix with entries from $\langle Z_6 \cup g\rangle_I$.

Let $X = \{(a_1, a_2, a_3, a_4, a_5) \mid a_i \in \{0, 1\}, 1 \le i \le 5\}$ and

$X_s = \{(a_1, a_2, \ldots, a_5) \mid a_i \in \{0, 1, I_t^g;$ t a zero divisor or nilpotent or idempotent in $\langle Z_6 \cup g\rangle_I$; $1 \le i \le 5\}$, with usual notations.

$x_1 = (1, 0, 0, 0, 0) \in X$, the effect of x on M ;

$x_1M \to (1, g, 0, 0, 0) = y_1$
$y_1M \to (1, g, g, 0, 0) = y_2$
$y_2M \to (2g, g, g, 0, 0) = y_3$
$y_3M = (2g, 0, g, 0, 0) = y_4$
$y_4M = (2t, 0, 0, 0, 0) = y_5$
$y_5M \to (1, 0, 0, 0, 0) = y_6 (=x_1)$.

Thus the MOD resultant of x_1 is a MOD limit cycle leading to the same x_1.

Let $x_2 = (0, 1, 0, 0, 0) \in X$. To effect of x_2 on M.

$x_2M = (0, 1, 1, 0, 0) = y_1$
$y_1M \to (2, 1, 1, 0, 0) = y_2$

$y_2M \to (2, 2g, 1, 0, 0) = y_3$
$y_3M = (2g, 2g, 2g, 0, 0) = y_4$
$y_4M = (4g, 2g, 2g, 0, 0) = y_5$
$y_5M \to (4g, 1, 2g, 0, 0) = y_6$
$y_6M = (4g, 1, 1, 0, 0) = y_7$
$y_7M \to (2, 1, 1, 0, 0) = y_8$ ($=y_2$).

Thus the MOD resultant is a MOD limit cycle given by (2, 1, 1, 0, 0) the on state of C_2 makes on the nodes C_1 and C_3 taking values 2 and 1 respectively.

However the nodes C_4 and C_5 remain in the zero state.

Let $x_5 = (0, 0, 0, 0, 1) \in X$; to find the effect of x_5 on M.

$x_5M \to (0, 0, 0, I_g^g, 1) = y_1$

$y_1M \to (0, 0, I_g^g, I_g^g, 1) = y_2$

$y_2M \to (I_g^g, I_g^g, I_g^g, I_g^g, 1) = y_3$

$y_3M \to (I_g^g, I_g^g, I_g^g, I_g^g, 1) = y_4$ ($= y_3$).

Thus the MOD resultant is a MOD fixed point given by $(I_g^g, I_g^g, I_g^g, I_g^g, 1)$. That is on state of the node C_5 with 1 has impact on all the nodes.

All the nodes C_1, C_2, C_3 and C_4 take equal values I_g^g the natural neutrosophic number.

So the real state of C_5 has only same or equal natural neutrosophic state on all other nodes of the system.

Let $a_4 = (0, 0, 0, I_{2g+4}^g, 0) \in X_s$.
To find the effect of a_4 on M ,

$a_4M \to (0, 0, I_{2g+4}^g, I_{2g+4}^g, 0) = y_1$

$y_1M \to (I_{2g+4}^g, 0, I_{2g+4}^g, I_{2g+4}^g, 0) = y_2$

$y_2M \to (I^g_{2g+4}, I^g_{2g+4}, I^g_{2g+4}, I^g_{2g+4}, 0) = y_3$

$y_3M \to (I^g_{2g+4}, I^g_{2g+4}, I^g_{2g+4}, I^g_{2g+4}, 0) = y_4$ $(=y_3)$.

Thus the on state of C_4 with node value I^g_{2g+4} makes on the other nodes C_1, C_2, and C_3 to take the same value as I^g_{2g+4} and the node C_5 has no effect as its value is 0.

So the natural neutrosophic dual number value of C_4 makes the nodes C_1, C_2 and C_3 also to take the same value.

Thus we see each on state of the node has a varied effect on other nodes seen from this example.

Interested can work with any of the nodes on state using initial state vectors from X or X_s.

Now we proceed onto describe the new MOD neutral neutrosophic dual number Cognitive Maps (MOD NCMs) model.

Let P be the problem in hand with $C_1, C_2, \ldots, C_n$ n-nodes associated with P. Let G be directed graph given by the expert who uses the edge weights from $\langle Z_m \cup g\rangle_I$. So G is the MOD natural neutrosophic dual number directed graph.

Let $M = (m_{ij})$ be the $n \times n$ connection matrix associated with G. m is defined as the MOD natural neutrosophic dual number cognitive maps model's dynamical system M functions in a similar way as that of FCMs or NCM with only difference being that the initial state vectors can be natural neutrosophic numbers also.

We will illustrate this model with one more example. It is pertinent to keep on record that this model also functions like MOD natural neutrosophic numbers Cognitive Maps model or MOD natural neutrosophic finite complex numbers Cognitive Maps model.

Example 3.29: Let P be a problem in hand. $C_1, C_2, \ldots, C_5$ be the nodes associated with the problem P.

The expert uses for the directed graph G with edge weights from $\langle Z_{10} \cup g\rangle_I$.

The graph G is a MOD natural neutrosophic dual number directed graph given in the following.

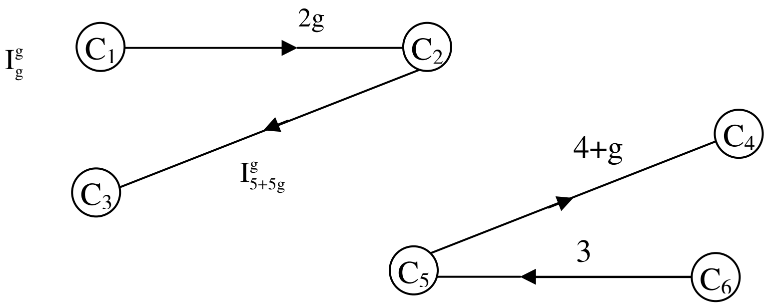


Figure 3.15

Let M be the MOD natural neutrosophic dual number connection matrix associated with G the MOD natural neutrosophic directed graph G.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & 2g & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{5g+5g}^g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4+g & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \end{bmatrix} \end{array}.$$

Let $X = \{(a_1, a_2, \ldots, a_6) \mid a_i \in \{0, 1\}; 1 \leq i \leq 6\}$ be the MOD initial state vector.

$X_s = \{(a_1, a_2, \ldots, a_6) / a_i \in \{0, 1, I_t^g$; t a nilpotent, idempotent, or zero divisor in $\langle Z_{10} \cup g\rangle_I$; $1 \leq i \leq 6\}$ be the MOD special initial state vectors associated with M.

Let $x_1 = (1, 0, 0, 0, 0, 0) \in X$, to find the effect of x_1 on M.

$x_1M \to (1, 2g, 0, 0, 0, 0) = y_1$

$y_1M \to (1, 2g, I_{5+5g}^g, 0, 0, 0) = y_2$

$y_2M \to (1, 2g, I_{5+5g}^g, 0, 0, 0) = y_3 (= y_2)$.

Thus the MOD resultant is a MOD fixed point given by $(1, 2g, I_{5+5g}^g, 0, 0, 0)$. Thus we see the on state of the node C_1 has no effect on the nodes C_4, C_5 and C_6.

However for the node C_2 we get the value 2g the pure dual number and C_3 node get the natural neutrosophic dual number I_{5+5g}^g.

Let $x_2 = (0, 1, 0, 0, 0, 0) \in X$; to find the effect of x_2 on M;

$x_2M \to (0, 1, I_{5+5g}^g, 0, 0, 0) = y_1$

$y_1m \to (0, 1, I_{5+5g}^g, 0, 0, 0) = y_2 (= y_1)$.

Thus we see the on state of the node C_2 has no effect on the nodes C_1, C_4, C_5 and C_6; however the node C_3 comes to on state with the value I_{5+5g}^g a natural neutrosophic dual number.

Let $x_3 = (0, 0, 1, 0, 0, 0) \in X$.

To find the effect of x_3 on M;

$x_3M \to (0, 0, 1, 0, 0, 0) = y_1 (= x_3)$.

Thus the MOD resultant is a MOD special classical fixed point given by (0, 0, 1, 0, 0, 0) and has no effect on all the other nodes.

Let x_4 = (0, 0, 0, 1, 0, 0) ∈ X; to find the effect of x_4 on M,

$x_4M \to (0, 0, 0, 1, 0, 0) = y_1$ $(=x_4)$.

Thus the MOD resultant again is a MOD special classical fixed point (0, 0, 0, 1, 0, 0) has no effect on other nodes.

It is to be noted that the nodes C_3 and C_4 behave in the same way their on states gives the MOD resultant as a MOD special classical fixed point.

Let x_5 = (0, 0, 0, 0, 1, 0) ∈ X; to find the effect of x_5 on M.

$x_5M \to (0, 0, 0, 4 + g, 1, 0) = y_1$
$y_1M \to (0, 0, 0, 4 + g, 1, 0) = y_2$ $(=y_1)$.

Thus the MOD resultant of x_5 is a MOD fixed point given by (0, 0, 0, 4 + g, 1, 0).

We see the nodes C_1, C_2, C_3 and C_5 remain in the off state where as the node C_4 gets the value 4 + g a mixed dual number. However the on state of the node C_4 has no effect on any of the nodes.

Let x_6 = (0, 0, 0, 0, 0, 1) ∈ X; to find the effect of x_6 on M.

$x_6M \to (0, 0, 0, 0, 3, 1) = y_1$
$y_1M \to (0, 0, 0, 2 + 3g, 3, 1) = y_2$
$y_2M \to (0, 0, 0, 2 + 3g, 3, 1) = y_3$ $(= y_2)$

Thus the MOD resultant is a MOD fixed point. The on state of the node C_6 has no impact of the nodes C_1, C_2 and C_3.

However the nodes C_4 and C_5 come to on state.

The node C_4 takes the mixed dual number value 2 + 3g and the node C_5 is real with the node value 3.

Consider

$a_1 = (I_{3g}^{g}, 0, 0, 0, 0, 0) \in X_s$, to find the effect of a_1 on M.

$a_1M \to (I_{3g}^{g}, I_{3g}^{g}, 0, 0, 0, 0) = y_1$

$y_1M \to (I_{3g}^{g}, I_{3g}^{g}, I_{5g}^{g}, 0, 0, 0) = y_2$

$y_2M \to (I_{3g}^{g}, I_{3g}^{g}, I_{5g}^{g}, 0, 0, 0) = y_3 (=y_2)$.

Thus the MOD resultant is a MOD fixed point given by

$$(I_{3g}^{g}, I_{3g}^{g}, I_{5g}^{g}, 0, 0, 0).$$

The nodes C_4, C_5 and C_6 takes values 0 where as the other two nodes C_2 and C_3 get the natural neutrosophic dual number.

$$a_2 = (0, I_{2g}^{g}, 0, 0, 0, 0) \in X_s.$$

To find the effect of a_2 on M;

$a_2M \to (0, I_{2g}^{g}, I_{0}^{g}, 0, 0, 0) = y_1$

$y_1M \to (0, I_{2g}^{g}, I_{0}^{g}, 0, 0, 0) = y_2(=y_1)$.

Thus the MOD resultant is a MOD fixed point given by

$$(0, I_{2g}^{g}, I_{0}^{g}, 0, 0, 0).$$

That is the natural neutrosophic dual number state of C_2 has no effect on C_1, C_4, C_5 and C_6 but make the node C_3 into the zero natural neutrosophic dual number.

Let $x_5 = (0, 0, 0, 0, I_{8g+2}^{g}, 0) \in X_s$ to find the effect of x_5 on M.

$x_5M \to (0, 0, 0, 6 + 4g + I_{8g+2}^{g}, I_{8g+2}^{g}, 0) = y_1$

$y_1M \to (0, 0, 0, 4 + 2g + I_{8g+2}^{g}, I_{8g+2}^{g} + 4, 0) = y_2$

$y_2M \to (0, 0, 0, 6 + 2g + I_{8g+2}^{g}, I_{8g+2}^{g} + 4, 0) = y_3$

$y_3M \to (0, 0, 0, 4 + 4g, + I_{8g+2}^{g}, I_{8g+2}^{g} + 4, 0) = y_4$

$y_4M \to (0, 0 , 0, 6 + I_{8g+2}^{g}, 4 + I_{8g+2}^{g}, 0) = y_5$

$y_5M \to (0, 0, 0, 4 + 6\, g + I_{8g+2}^{g}, 4 + I_{8g+2}^{g}, 0) = y_6$

$y_6M \to (0, 0, 0, 4 + I_{8g+2}^{g} + 2g, I_{8g+2}^{g} + 4, 0) = y_7 (= y_2).$

Thus the MOD resultant is a MOD limit cycle given by $(0, 0, 0, 4 + 2g + I_{8g+2}^{g} + 4, 0)$.

Thus the on state of node C_5 with value $4 + I_{8g+2}^{g}$ has no effect on the nodes C_1, C_2, C_3 and C_6 however the node C_4 takes the value $4 + 2g + I_{8g+2}^{g}$.

Interested reader can work with the on state of other nodes.

We now proceed onto study MOD natural neutrosophic special dual like cognitive maps models.

We will first illustrate by examples the MOD natural neutrosophic special dual like number directed graphs.

Example 3.30: Let G be a directed graph with edge weights from $\langle Z_5 \cup h\rangle_I$ given by the following figure.

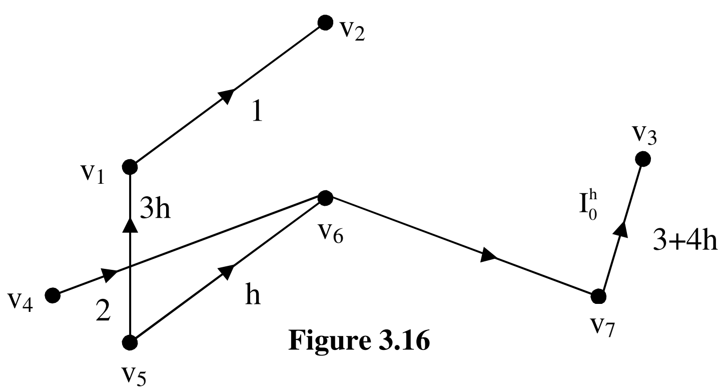


Figure 3.16

G is a MOD natural neutrosophic special dual like number directed graph.

Example 3.31: Let H be the MOD natural neutrosophic special dual like number directed graph with entries $\langle Z_9 \cup h\rangle_I$ given in Figure 3.17.

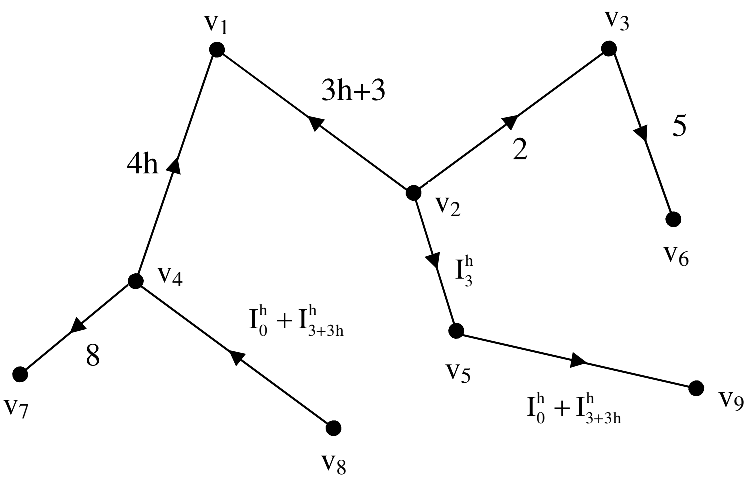


Figure 3.17

Thus in a directed graph G if the edge weights are taken by $\langle Z_n \cup h\rangle_I$ where $h^2 = h$ is the collection of MOD natural neutrosophic special dual like numbers then we define G to be a MOD natural neutrosophic special dual like numbers directed graph.

We have already given examples of them.

Example 3.32: Let G be the MOD natural neutrosophic special dual like number directed graph with edge weights from $\langle Z_{10} \cup h\rangle_I$ given by the following figure.

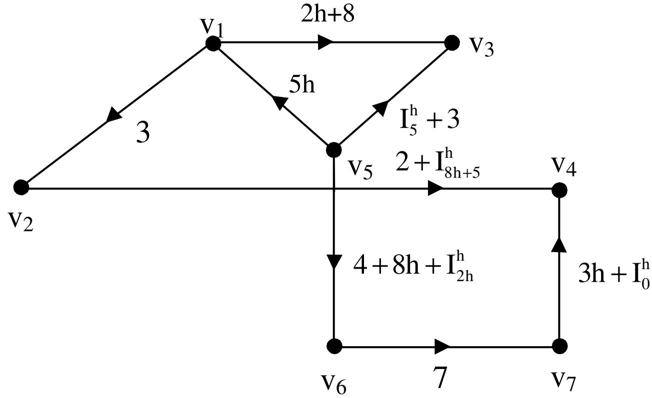


Figure 3.18

The adjacency or connection matrix M associated with G is given in the following.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array}\begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2h+g & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+I_{8h+5}^h & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5h & 0 & 3+I_5^h & 0 & 0 & 4+8h+I_{2h}^h & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 3h+I_0^h & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

M is defined as the MOD natural neutrosophic special dual like number square matrix.

We will give some more examples of MOD natural neutrosophic special dual like number square matrices.

Example 3.33: Let

$$S = \begin{bmatrix} 0 & I_{3h}^{h} & I_{0}^{h}+2 & 1 \\ 3 & 0 & 2h & 0 \\ I_{6h+1}^{h} & 2h & 0 & 4h+I_{2h}^{h} \\ 0 & I_{0}^{h}+I_{4h}^{h} & 2 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic special dual like number square matrix with entries from $\langle Z_8 \cup h\rangle_I$.

Example 3.34: Let

$$P = \begin{bmatrix} 0 & 2h & 0 & 1 & I_{6h}^{h} & 2 \\ 5 & 0 & 6h & 0 & 1 & I_{0}^{h} \\ 0 & I_{0}^{h}+I_{2h}^{h} & 0 & 3h & 0 & 2h \\ 1 & 0 & 7 & 0 & 6h+1 & 0 \\ I_{4h}^{h}+I_{10}^{h} & 2 & 0 & I_{10h}^{h} & 0 & 3 \\ 0 & 0 & 3 & 0 & 1 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic special dual like number square matrix with entries from $\langle Z_{12} \cup h\rangle_I$.

Example 3.35: Let

$$M = \begin{bmatrix} 0 & I_{3h}^{h}+I_{0}^{h} & 3 & 2h \\ 3h & 0 & 2+I_{6h}^{h} & 0 \\ 4 & I_{0}^{h}+h & 0 & I_{3h}^{h} \\ I_{7h}^{h}+I_{0}^{h} & 3 & 4h & 0 \end{bmatrix}$$

be the MOD natural neutrosophic special dual like number square matrix with entries from $\langle Z_9 \cup h\rangle_I$.

Now we define the MOD natural neutrosophic special dual like number square matrix.

Let $M = (m_{ij})$ be a $n \times n$ square matrix where the entries m_{ij} are from $\langle Z_m \cup h\rangle_I$.

We call or define M to be a MOD natural neutrosophic special dual like number square matrix.

We have seen examples of them.

Now we proceed onto describe a special type of operations on them which are essential to construct MOD natural neutrosophic special dual like number cognitive maps model.

Let $X = \{(a_1, a_2, \ldots, a_n) \mid a_i \in \{0, 1\}; 1 \leq i \leq n\}$ and

$X_s = \{(a_1, a_2, \ldots, a_n) \mid a_i \in \{0, 1, I_t^h$ where t is a zero divisor or idempotent or nilpotent element of $\langle Z_m \cup h\rangle_I\}, 1 \leq i \leq n\}$ be the MOD initial state vectors or MOD special initial state vectors associated with a $n \times n$ MOD natural neutrosophic special dual like number matrix.

We will illustrate these operations by some examples.

Example 3.36: Let

$$S = \begin{bmatrix} 0 & I_2^h + 3h & 0 & 4 & 0 & 7h \\ 2h & 0 & I_0^h + I_{9h}^h & 0 & 8 & 0 \\ 0 & 3 & 0 & 3 + I_{7h}^h & 0 & 9 + h \\ 1 & h + 11 & 3 + I_{8h+4}^h & 0 & 6h + I_4^h & 0 \\ 0 & 0 & 0 & 7 + I_6^h & 0 & 4 \\ 8h & 6h + I_{3h}^h & 7h + I_4^h & 0 & 7 & I_6^h + 3 \end{bmatrix}$$

be the MOD natural neutrosophic special dual like number 6×6 square matrix with entries from $\langle Z_{12} \cup h\rangle_I$, the MOD natural neutrosophic special dual like number set.

Let X = {(a_1, a_2, …, a_6) | $a_i \in$ {0, 1), $1 \leq i \leq 6$} be the MOD initial state vectors associated with S.

X_s = {(a_1, a_2, …, a_6) | $a_i \in$ {0, 1, I_t^h }, where t is the nilpotent element or idempotent element or a zero divisor in $\langle Z_{12} \cup h\rangle_I$}; $1 \leq i \leq 6$} be the MOD special initial state vector associated with S.

Let x_1 = (1, 0, 0, 0, 0, 0) $\in$ X, the effect of x_1 on S is as follows.

$$x_1S \to (1,\ I_2^h + 3h,\ 0,\ 4,\ 0,\ 7h) = y_1$$

$$y_1S \to (4 + I_2^h,\ I_2^h,\ h + 8 + I_2^h + I_{3h}^h,\ I_0^h + I_{6h}^h + I_{9h}^h + I_{8h+4}^h + I_4^h + h,\ 4,\ h + I_2^h + I_4^h,\ 4h + I_6^h) = y_2 \text{ and so on.}$$

We can find after a finite number of iterations the MOD resultant to be a MOD fixed point or a MOD limit cycle.

Let x_2 = (0, 1, 0, 0, 0, 0) $\in$ X; to find the effect of x_2 on S.

$$x_2S \to (2h,\ 0,\ I_0^h + I_{9h}^h,\ 0,\ 8,\ 0) = y_1$$

$$y_1S = (0,\ 6h + I_2^h + I_0^h + I_{9h}^h,\ 0,\ 8 + 8h + I_0^h + I_{9h}^h + I_{3h}^h + I_6^h,\ 0,\ 8 + 2h + I_0^h + I_{9h}^h) = y_2$$

y_2S = $(8 + 4h + I_2^h + I_0^h + I_{9h}^h + I_6^h + I_{3h}^h,\ 4 + 8h + I_0^h + I_6^h + I_{9h}^h + I_{3h}^h,\ I_0^h + I_6^h + I_{3h}^h + I_{9h}^h + I_4^h + I_{8h+4}^h + 10h,\ 0,\ 8 + 2h + I_0^h + I_2^h + I_{9h}^h + I_{3h}^h + I_6^h + I_4^h,\ 6h + I_6^h + I_0^h + I_{9h}^h + I_6^h) = y_3$ and so on.

Thus we see after a finite number of iterations we will arrive at a MOD resultant which may be a MOD fixed point or a MOD limit cycle.

Let x_3 = (0, 0, 1, 0, 0, 0) $\in$ X, to find the effect of x_3 on S.

$$x_3S \to (0,\ 3,\ 1,\ 3 + I_{7h}^h,\ 0,\ 9 + h) = y_1$$

$y_1S \to (2h + 3 + I^h_{7h}, 3h + I^h_{7h} + I^h_{3h}, 9 + 10h + I^h_0 + I^h_{9h} + I^h_{7h} + I^h_4 + I^h_{8h+4}, 3 + I^h_{7h}, 3 + h + I^h_4 + I^h_{7h} + I^h_{4h}, 4h + I^h_6) = y_2$ and so on.

After a finite number of iterations we are sure to arrive at a MOD resultant which may be a MOD fixed point or a MOD limit cycle.

Let $x_5 = (0, 0, 0, 0, 1, 0) \in X$ to find the effect of x_5 on S.

$$x_5S \to (0, 0, 0, 7 + I^h_6, 1, 4) = y_1$$
$$y_1S \to (8h + 7 + I^h_6, 15 + 7h + I^h_6 + I^h_{3h}, 9 + I^h_6 + I^h_{8h+4} + 4h + I^h_4, 7 + I^h_6, 6 + I^h_6 + I^h_4, 4 + I^h_6) = y_2$$

Interested reader can find the MOD resultant of x_5 which after a finite number of iterations will arrive at a MOD fixed point or a MOD limit cycle.

Example 3.37: Let

$$P = \begin{bmatrix} 0 & 2 & 1 \\ I^h_{2h} & 0 & 0 \\ 0 & h & 0 \end{bmatrix}$$

be the MOD natural neutrosophic special dual like number matrix with entries from $\langle Z_3 \cup h\rangle_I$.

Let $X = \{(a_1, a_2, a_3) \mid a_i \in \{0, 1\}; 1 \leq i \leq 3\}$ and

$X_s = \{(a_1, a_2, a_3) / a_i \in \{0, 1, I^h_0, I^h_h, I^h_{2h}\}; 1 \leq i \leq 3\}$ be the MOD initial state vector and MOD special initial state vector respectively.

$x_1 = (1, 0, 0) \in X$, to find the effect of x_1 on P;

$$x_1P \to (1, 2, 1) = y_1$$
$$y_1P = (I_{2h}^h, 2 + h, 1) = y_2$$
$$y_2P = (I_{2h}^h, I_{2h}^h + h, I_{2h}^h) = y_3$$
$$y_3P = (I_{2h}^h, I_{2h}^h, I_{2h}^h) = y_4$$
$$y_4P = (I_{2h}^h, I_{2h}^h, I_{2h}^h) = y_5 \ (=y_4).$$

Thus the MOD resultant is a MOD fixed point given by $(I_{2h}^h, I_{2h}^h, I_{2h}^h)$ so all the nodes come to state and all of them are same a natural neutrosophic value I_{2h}^h.

Let $x_2 = (0, 1, 0) \in X$, to find the effect of x_2 on P is as follows.

$$x_2P \to (I_{2h}^h, 1, 0) = y_1$$
$$y_1P \to (I_{2h}^h, I_{2h}^h, I_{2h}^h) = y_2$$
$$y_2P \to (I_{2h}^h, I_{2h}^h, I_{2h}^h) = y_3 \ (= y_2).$$

Thus the MOD resultant is a MOD fixed point given by

$$(I_{2h}^h, I_{2h}^h, I_{2h}^h).$$

Interested reader can work with such models.

Next we proceed onto build the MOD natural neutrosophic special quasi dual number cognitive maps model using the MOD natural neutrosophic special quasi numbers set

$\langle Z_n \cup k\rangle_I = \{Z_n, k^2 = k, I_t^h$, t a zero divisor or idempotent or nilpotent elements of $\langle Z_n \cup k\rangle\}$.

We will illustrate this situation by some examples.

Example 3.38: Let G be a director graph where edge weight are from the $\langle Z_6 \cup k\rangle_I$ given in the following figure.

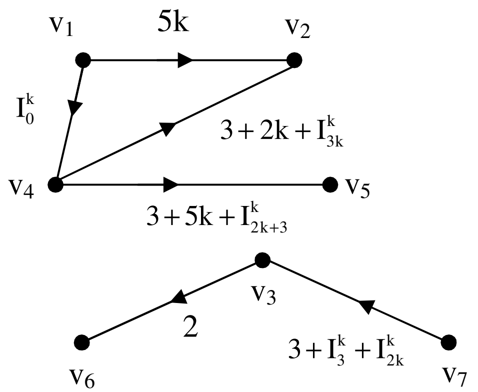


Figure 3.19

Example 3.39: Let H be the MOD natural neutrosophic special quasi dual number directed graph with entries from $\langle Z_7 \cup k\rangle_I$.

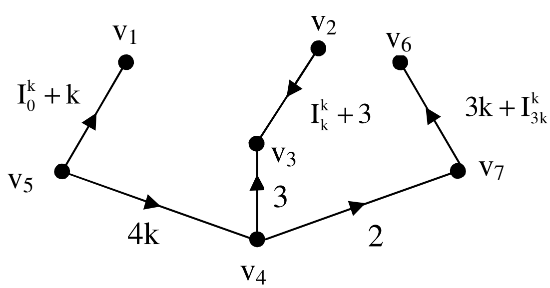


Figure 3.20

Thus we define a MOD natural neutrosophic special quasi dual number directed graph in the following.

Let G be a directed graph with edge weights from $\langle Z_n \cup k\rangle_I$.

We define G to be a MOD natural neutrosophic special quasi dual number directed graph as its edge weights are from $\langle Z_n \cup k\rangle_I$

We now proceed onto describe the notion of MOD natural neutrosophic special quasi dual number square matrices by examples.

Example 3.40: Let

$$S = \begin{bmatrix} 0 & I^k_{3+4k}+2 & 3k & 0 & 1 & I^k_0 & 8k+4 \\ I^k_{4k} & 0 & 0 & 4+I^k_{2k} & 0 & 0 & 0 \\ 0 & 4+7k & k+I^k_5 & 0 & I^k_5 & 7 & I^k_{5k}+1 \\ 8+4k & 0 & 0 & 2+I^k_{8k} & 0 & 0 & 0 \\ 0 & 9k & I^k_0 & 0 & 1 & I^k_4 & I^k_{2k}+k \\ 9+I^k_8 & 0 & 0 & 1+I^k_4 & 0 & 0 & 0 \\ 0 & 3k+2 & I^k_4+3 & 0 & I^k_0 & 9k & 0 \end{bmatrix}$$

be the MOD natural neutrosophic special quasi dual number square matrix with entries from $\langle Z_{10} \cup k\rangle_I$

Example 3.41: Let

$$M = \begin{bmatrix} 0 & 2k & 0 & I^k_{2k}+4 \\ 4+I^k_0 & 0 & 1 & 0 \\ 0 & 3+I^k_2 & 0 & I^k_0 \\ 1 & 0 & 2 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic special quasi dual number matrix with entries from $\langle Z_5 \cup k\rangle_I$.

So we define a MOD square matrix with entries $\langle Z_n \cup k\rangle_I$ to be a MOD natural neutrosophic special quasi dual number square matrix.

We describe special operations using them in the following by examples.

Example 3.42: Let

$$M = \begin{bmatrix} 0 & 3k+1 & I_0^k & 0 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ I_4^k & 0 & 1 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic special quasi dual number matrix with entries from $\langle Z_6 \cup k\rangle_I$.

Let X = $\{(a_1, a_2, a_3, a_4) \mid a_i \in \{0, 1\}, 1 \leq i \leq 4\}$ be the MOD initial state vectors.

We define only special type of operations using them.

Let $x_1 = (1, 0, 0, 0) \in X$, the effect o f x on M is analysed in the following.

$x_1M \to (1, 3k + 1, I_0^k, 0) = y_1$

$y_1M \to (3k+1, 3k+1, I_0^k, 2 + I_0^k) = y_2$

$y_2M \to (3k+1 + I_4^k + I_0^k, 3k+1, I_0^k, 2 + I_0^k) = y_3$

$y_3M \to (3k+1 + I_4^k + I_0^k, I_4^k + I_0^k + 1 + 3k, I_0^k + 2, 2 + I_0^k)$ and so on.

Thus after a finite number of iterations one is sure to get at a MOD resultant which is a MOD fixed point or a MOD limit cycle.

Let $x_2 = (0, 1, 0, 0) \in X$, to find the effect of x_2 on M.

$x_2M \to (1, 1, 0, 2) = y_1$

$y_1M \to (1 + I_4^k, 3k + 1, 2 + I_0^k, 2) = y_2$

$y_2M \to (3k+1 + I_4^k, 3k+1 + I_4^k, I_0^k + 2, 4 + I_0^k) = y_3$ and so on.

After a finite number of iterations we get the MOD resultant to be a MOD fixed point or a MOD limit cycle.

Let $x_3 = (0, 0, 1, 0) \in X$, to find the effect of x_3 on M.

$x_3M \to (0, 0, 1, 1) = y_1$
$y_1M = (I_4^k, 0, 1, 1) = y_2$
$y_2M = (I_4^k, I_4^k, 1 + I_4^k, 1) = y_3$
$y_2M = (I_4^k, I_4^k, I_4^k + 1, 1 + I_4^k) = y_4$
$y_4M = (I_4^k, I_4^k, I_4^k + 1, I_4^k + 1) = y_5 (= y_4)$.

Thus the MOD resultant is a MOD fixed point given by

$$(I_4^k, I_4^k, 1 + I_4^k, 1 + I_4^k).$$

Now we define for any $n \times n$ MOD natural neutrosophic special quasi dual number matrix M with entries from $\langle Z_m \cup k\rangle_I$

$X = \{(a_1, a_2, \ldots, a_n) / a_i \in \{0, 1\}, 1 \leq i \leq n\}$ and

$X_s = \{(a_1, a_2, \ldots, a_n) / a_i \in \{0, 1, I_t^k$ where t is a nilpotent element or idempotent element or a zero divisor in $\langle Z_m \cup k\rangle\}$; $1 \leq i \leq n\}$ be the MOD initial state vector or MOD special initial state vector associated with the matrix M.

We shall describe special operations related with them by some examples.

Example 3.43: Let

$$M = \begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & I_{3k}^k \\ 2k & 1 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic special dual quasi dual number matrix with entries from $\langle Z_4 \cup k\rangle_I$.

Let $X = \{(a_1, a_2, a_3) / a_i \in \{0, 1\}; 1 \leq i \leq 3\}$ and

$X_s = \{(a_1, a_2, a_3) / a_i \in \{0, 1, I_t^k$ where t is a nilpotent or zero divisor or idempotent in $\langle Z_4 \cup k\rangle\}; 1 \leq i \leq 3\}$ be the MOD initial state vectors and MOD special initial state vectors associated with M.

Let $x_1 = (1, 0, 0) \in X$; to find the effect of x_1 on M.

$x_1M \to (1, 3, 0) = y_1$

$y_1M \to (1, 3, I_{3k}^k) = y_2$

$y_2M = (I_{3k}^k, 3 + I_{3k}^k, I_{3k}^k) = y_3$

$y_3M = (I_{3k}^k, I_{3k}^k, I_{3k}^k) = y_4$

$y_4M = (I_{3k}^k, I_{3k}^k, I_{3k}^k) = y_5\ (=y_4)$.

Thus the MOD resultant is MOD fixed points given by

$$(I_{3k}^k, I_{3k}^k, I_{3k}^k).$$

When the C_1 is on with 1 in the MOD resultant all the nodes C_1, C_2 and C_3 take the same node value I_{3k}^k that is they are natural neutrosophic.

Let $x_2 = (0, 1, 0) \in X$, to find the effect of x_2 on M.

$x_2M \to (0, 1, I_{3k}^k) = y_1$

$y_1m = (I_{3k}^k, I_{3k}^k, I_{3k}^k) = y_2$

$y_2M = (I_{3k}^k, I_{3k}^k, I_{3k}^k) = y_3\ (=y_2)$.

Thus the MOD resultant is a MOD fixed point given by

$$(I_{3k}^k, I_{3k}^k, I_{3k}^k).$$

Hence when node C_2 is on with value 1 the MOD resultant makes all nodes take the same value I_{3k}^k, a natural neutrosophic value.

Let $b_1 = (I_2^k, 0, 0) \in X_s$; to find the effect of b_1 on M.

$b_1M \to (I_2^k, I_2^k, 0) = y_1$
$y_1M \to (I_2^k, I_2^k, I_{2k}^k) = y_2$
$y_2M \to (I_{2k}^k, I_{2k}^k + I_2^k, I_{2k}^k) = y_3$
$y_2M \to (I_2^k, I_2^k + I_{2k}^k, I_{2k}^k) = y_4$ $(=y_3)$.

Thus the MOD resultant is a MOD fixed point given by $(I_2^k, I_2^k + I_{2k}^k, I_2^k)$.

Interested reader can work any of the MOD initial state vectors from X or X_s.

Now we proceed onto briefly describe the MOD natural neutrosophic special quasi dual number cognitive maps model.

Let P be a problem at hand in which an expert is interested.

Let $C_1, C_2, \ldots, C_n$ be the nodes / concepts associated with P.

Let G be the MOD directed natural neutrosophic special quasi dual number graph given by the expert with edge weights from $\langle Z_m \cup k\rangle_I$.

Let M be the MOD natural neutrosophic special quasi dual number connection matrix associated with the MOD directed graph G.

Let $X = \{(a_1, a_2, \ldots, a_n) / a_i \in \{0, 1\}; 1 \leq i \leq n\}$ be the MOD initial state vector and

$X_s = \{(a_1, a_2, \ldots, a_n) / a_i \in \{0, 1, I_t^k$; t a nilpotent element or idempotent or zero divisor in $\langle Z_m \cup k\rangle_I$, $1 \leq i \leq n\}$ be the MOD initial special state vector associated with M.

We call M as the MOD natural neutrosophic special quasi dual number Cognitive Maps model dynamical system.

We can work with this in the usual way as other MOD models.

Next we proceed onto study the notion of MOD natural neutrosophic-neutrosophic Cognitive Maps model built using $\langle Z_n \cup I\rangle_I$ by some examples.

Example 3.44: Let G be the directed graph with edge weights from $\langle Z_6 \cup I\rangle_I$ given by the following figure.

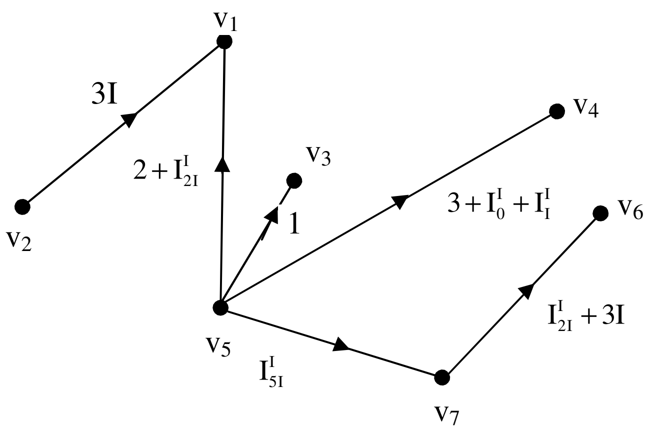


Figure 3.21

We call this G as the MOD natural neutrosophic number neutrosophic directed graph.

Example 3.45: Let G be the MOD natural neutrosophic number neutrosophic directed graph with edge weights from $\langle Z_3 \cup I\rangle_I$ given by the following figure.

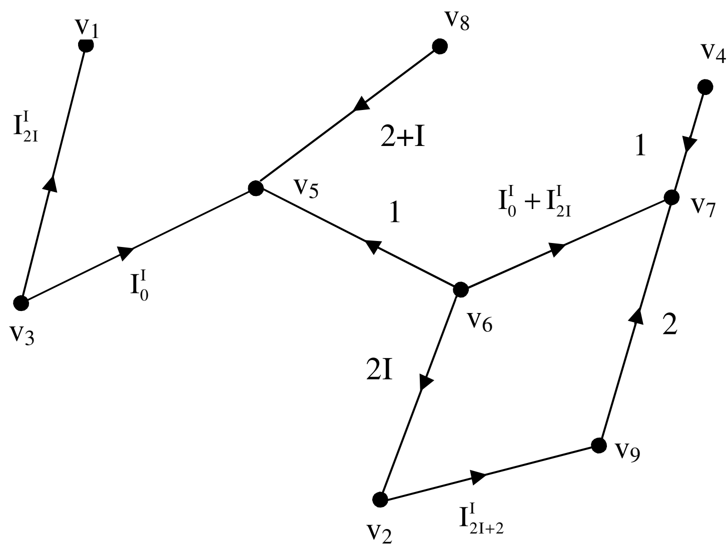


Figure 3.22

Hence we define the MOD natural neutrosophic number neutrosophic directed graph G as follows.

Let G be a directed graph with edge weights from $\langle Z_n \cup I\rangle_I$; then we define G to be a MOD natural neutrosophic number neutrosophic directed graph.

We have seen examples of them.

Let us now give some examples of MOD natural neutrosophic number neutrosophic matrix.

Example 3.46: Let

$$S = \begin{bmatrix} 0 & I_{3I}^{I} & 3+2I & 0 & I_0^I + I_I^I \\ 2 & 0 & I_{5I}^I + 3 & 7I & 0 \\ 0 & 5I + I_{6I+4}^I & 0 & I_{3I}^I + I_{2I}^I & 6 \\ 7I & 0 & 6I + I_{2I}^I & 0 & 7I+4 \\ 0 & 2I & 0 & 6+I & 0 \end{bmatrix}$$

be the MOD natural neutrosophic number neutrosophic matrix with entries from $\langle Z_8 \cup I\rangle_I$.

Example 3.47: Let

$$M = \begin{bmatrix} 0 & 3+6I & 0 \\ I_0^I & 0 & I_I^I + I_{6I}^I \\ 0 & 2 + I_{2I}^I & 0 \end{bmatrix}$$

be the MOD natural neutrosophic number neutrosophic square matrix.

We will define special operations using the MOD natural neutrosophic number neutrosophic square matrix.

In the first place a n × n matrix $P = (p_{ij}) = p_{ij} \in \langle Z_m \cup I\rangle_I$ is defined as the MOD natural neutrosophic element neutrosophic square matrix.

We have already seen examples of them.

Let $X = \{(a_1, a_2, \ldots, a_n) / a_i \in \{0, I, 1\}, 1 \leq i \leq n\}$ is defined as the MOD initial state row vector associated with P.

Let $X_s = \{(a_1, a_2, \ldots, a_n) / a_i \in \{0, 1, I, I_t^I$ where t is a zero divisor or an idempotent or a nilpotent in $\langle Z_m \cup I\rangle\}\}$ is defined as the MOD special initial state vectors associated with P.

We can define special type of operations using them which we describe by some examples.

Example 3.48: Let

$$M = \begin{bmatrix} 0 & 3I+4 & 0 & I_{2I}^{I}+1 & 0 \\ 2I & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & I & 3 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic number neutrosophic matrix with entries from $\langle Z_6 \cup I\rangle_I$.

Let X = $\{(a_1, a_2, a_3, a_4, a_5) / a_i \in \{0, 1, I\}; 1 \leq i \leq 5\}$ be the MOD initial state vectors associated with M.

$X_s = \{(a_1, a_2, \ldots, a_5) / a_i \in \{I, 0, 1, I_t^I$, to a zero divisor or idempotent or nilpotent element of $\langle Z_6 \cup I\rangle\}; 1 \leq i \leq 5\}$ be the MOD special initial state vectors associated with M.

We now describe the special type of operations on M using X and X_s.

Let $x_1 = (1, 0, 0, 0, 0) \in X$; to find the effect of x_1 on M;

$x_1M \to (1, 3I + 4, 0, I_{2I}^{I} + 1, 0) = y_1$
$y_1M \to (2I + 1 + I_{2I}^{I}, 3I + I_{2I}^{I}, 3I + 4, I_{2I}^{I} + 1, 0) = y_2$
$y_2M = (1 + I_{2I}^{I}, 2I + 2 + I_{2I}^{I}, 3I + 4, 2I + 1 + I_{2I}^{I}, 3I) = y_3$
$y_3M = (4I + 1 + I_{2I}^{I}, I_{2I}^{I} + I, 3I + 4, I_{2I}^{I} + 5I + 5, 3I) = y_4$

and so on.

After a finite number of iterations we will arrive at a MOD fixed point or MOD limit cycle.

Let $x_2 = (0, 1, 0, 0, 0) \in X$, to find the effect of x_2 on M.

$x_2M \to (2I, 1, 1, 0, 0) = y_1$
$y_1M = (2I, 2I, 1, I_{2I}^{I} + 3I, 3) = y_2$
$y_2M = (1 + I_{2I}^{I}, 2I + I_{2I}^{I}, 2I, 3I + I_{2I}^{I}, 3) = y_3$
$y_3M = (1 + I_{2I}^{I}, I + I_{2I}^{I}, 2I + I_{2I}^{I}, I_{2I}^{I} + 3I, 0) = y_4$
$y_4M = (5I + I_{2I}^{I}, I + I_{2I}^{I}, I + I_{2I}^{I}, 3I + I_{2I}^{I}, I_{2I}^{I}) = y_5$
$y_5M = (5I + I_{2I}^{I}, 5I + I_{2I}^{I}, I + I_{2I}^{I}, I_{2I}^{I}, 3I + I_{2I}^{I}) = y_6$

and so on.

Certainly after a finite number of iterations we will arrive at a MOD fixed point or a MOD limit cycle.

Let $x_3 = (0, 0, 1, 0, 0) \in X$, to find the effect of x_3 on M

$x_3M \to (0, 0, 1, I, 3) = y_1$
$y_1M \to (I, 2I, I, I, 3) = y_2$
$y_2M = (5I, 3I, 2I, 2I + I_{2I}^{I}, 3I) = y_3$
$y_3M = (2I + I_{2I}^{I}, I + I_{2I}^{I}, 3I, I + I_{2I}^{I}, 0) = y_4$
$y_4M = (5I + I_{2I}^{I}, 4I + I_{2I}^{I}, I + I_{2I}^{I}, 5 + I_{2I}^{I}, 3I) = y_5$ and so on.

Certainly after a finite number of iterations are will arrive at a MOD fixed point or a MOD limit cycle.

Let $x_5 = (0, 0, 0, 0, 1) \in X$, to find the effect of x_5 on M.

$x_5M \to (0, 0, 0, 0, 1) = x_5$.

Thus the MOD resultant is a MOD special classical fixed point.

Interested reader can work with different set of state vectors from X or X_s.

Example 3.49: Let

$$S = \begin{bmatrix} 0 & 2I+2 & 0 \\ I_2^I & 0 & 2 \\ 0 & 3 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic number neutrosophic matrix with entries from $\langle Z_4 \cup I\rangle_I$.

Let X = $\{(a_1, a_2, a_3) \mid a_i \in \{0, 1, t\}, 1 \le i \le 3\}$ and

$X_s = \{(a_1, a_2, a_3) \mid a_i \in \{0, 1, I, I_0^I, I_2^I, I_{2I}^I, \ldots, \} 1 \le i \le 3\}$

be the MOD initial state vectors and MOD special initial state vectors respectively.

Let $x_1 = (1, 0, 0) \in X$, to find the effect of x_1 on S.

$x_1S \to (1, 2I + 2, 0) = y_1$
$y_1S \to (I_2^2, 2I + 2, 0) = y_2$
$y_2S = (I_2^2, I_2^2, 0) = y_3$
$y_3S = (I_2^2, I_2^2, I_2^2) = y_4$
$y_4S = (I_2^2, I_2^2, I_2^2) = y_5\ (= y_4)$.

Thus the MOD resultant is a MOD fixed point. All nodes indeterminate nodes viz., I_2^I and they are natural neutrosophic nilpotent element of order two.

Let $x_2 = (0, 1, 0) \in X$, the effect of x_2 on S.

$x_2S \to (I_2^I, 1, 2) = y_1$
$y_1S \to (I_2^I, I_2^I + 2, 2) = y_2$
$y_2S \to (I_2^I, I_2^I + 2, I_2^I) = y_3$
$y_3S \to (I_2^I, I_2^I, I_2^I) = y_4$
$y_4S \to y_5\ (= y_4)$.

Thus the MOD resultant is a MOD fixed point and all nodes are natural neutrosophic nilpotent of order two.

One can have several such examples and work for the MOD resultant.

We now define the MOD natural neutrosophic-neutrosophic cognitive maps model.

Let P be a problem in hand, $C_1, C_2, \ldots, C_n$ be n nodes / concepts associated with this problem.

Suppose the expert wishes to work with elements from MOD natural neutrosophic-neutrosophic integers $\langle Z_m \cup I\rangle_I$; that is the directed graph G given by the expert has edge weights from $\langle Z_m \cup I\rangle_I$.

Then the MOD connect matrix M associated with G severs as the dynamical system for the MOD-natural neutrosophic-neutrosophic Cognitive Maps model.

The functioning of it is akin to the MOD Cognitive Maps models dealt in this chapter as well as in chapter II of this book.

We will illustrate this situation by some examples.

Example 3.50: Let P be a problem with associated nodes $C_1, C_2, \ldots, C_6$.

Let G be the directed graph given by the expert with edge weights from $\langle Z_6 \cup I\rangle_I$ is given in the following

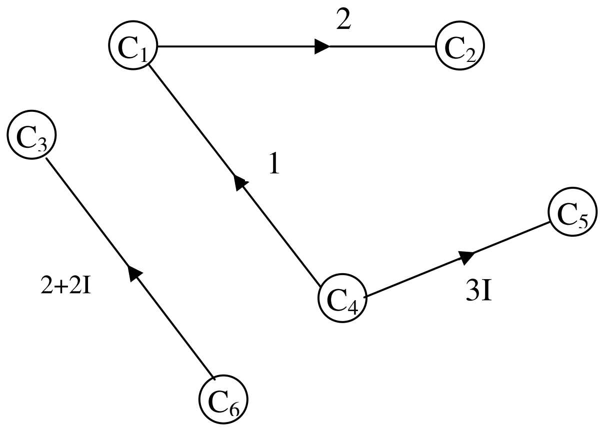


Figure 3.23

G is the MOD-natural neutrosophic-neutrosophic directed graph whose MOD-natural neutrosophic-neutrosophic connection matrix M serves as the MOD natural neutrosophic-neutrosophic Cognitive Maps model dynamical system which is as follows.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 3I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2+2I & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let $X = \{(a_1, a_2, \ldots, a_6) \mid a_i \in \{0, 1\}; 1 \leq i \leq 6\}$ be the MOD initial state vectors and

$X_s = \{(a_1, a_2, \ldots, a_6) \mid a_i \in \{0, 1, I, I_t^I$; t is a zero divisor or an idempotent or nilpotent of $\langle Z_6 \cup I\rangle\}; 1 \leq i \leq 6\}$ be the MOD special state vectors associated with the MOD-natural neutrosophic-neutrosophic cognitive maps model dynamical system M.

Let $x_1 = (1, 0, 0, 0, 0, 0) \in X$, to find the effect of x_1 on M.

$x_1M \to (1, 2, 0, 0, 0, 0) = y_1$
$y_1M \to (1, 2, 0, 0, 0, 0) = y_2$ $(=y_2)$.

Thus the MOD resultant is a MOD fixed point given by (1, 2, 0, 0, 0, 0). The on state of C_1 has no impact on C_3, C_4, C_5 and C_6. Only the node C_2 comes to on state.

Let $x_2 = (0, 0, 0, 1, 0, 0) \in X$; to find the effect of x_2 on M.

$x_2M \to (I, 0, 0, 1, 3I, 0) = y_1$
$y_1M \to (I, 2I, 0, 1, 3I, 0) = y_2$ $(= y_1)$.

Thus the MOD resultant is a MOD fixed point given by (I, 2I, 0, 1, 3I, 0).

On state of C_4 has no effect on the nodes C_3 and C_6.

However the nodes C_1, C_2 and C_5 come to on state with neutrosophic value I, 2I and 3I respectively.

Let $a = (0, 0, I_{2I}^{I}, 0, 0, 0) \in X_s$, to find the effect of a on M.

$aM \to (0, 0, I_{2I}^{I}, 0, 0, 0) = a$.

Thus the MOD resultant of a is a MOD special classical fixed point a itself.

This is the way operations are performed on MOD natural neutrosophic-neutrosophic cognitive maps model.

Interested reader can construct more such models.

Now we proceed onto define MOD interesting directed graph and MOD interval matrices.

Using these two concepts we build the MOD interval cognitive maps model.

We will describe them by their examples.

Example 3.51: Let G be a directed graph with edge weights from [0, 5).

We call G to be the MOD interval directed graph which is given in the following figure.

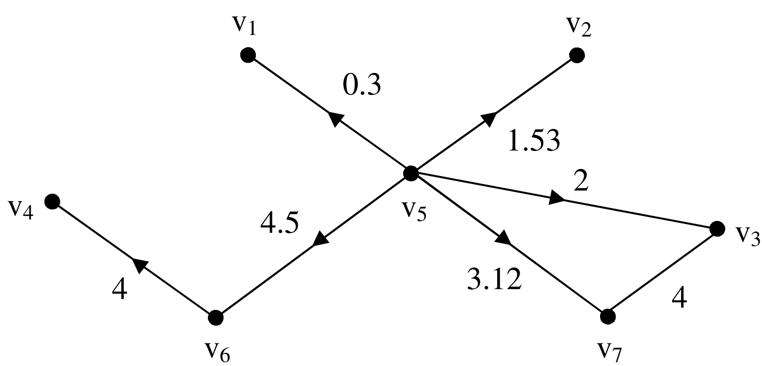


Figure 3.24

Example 3.52: Let G be the MOD interval directed graph with edge weights from [0, 20) given by the following figure.

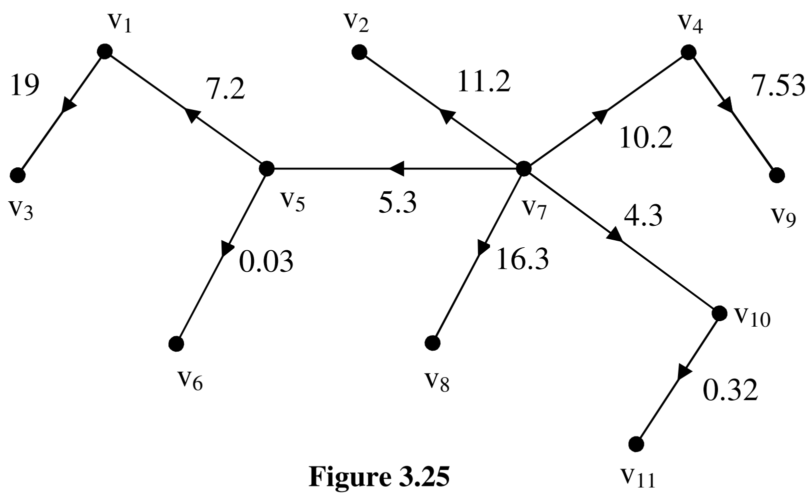


Figure 3.25

Now just we recall by examples the MOD interval square matrices with entries from [0, n), $2 \leq n < \infty$.

Example 3.53: Let

$$S = \begin{bmatrix} 0.3 & 4.2 & 0 \\ 1.7 & 0 & 6.3 \\ 7.5 & 8.11 & 0 \end{bmatrix}$$

be a MOD interval matrix with entries from [0, 9).

Example 3.54: Let

$$P = \begin{bmatrix} 3 & 0 & 0.1 & 0 & 4.1 & 0 \\ 0 & 0.7 & 0 & 6.1 & 0 & 0.2 \\ 1.1 & 0 & 1.5 & 0.3 & 4.2 & 0 \\ 0 & 4.2 & 0 & 2.4 & 0 & 6.1 \\ 2.4 & 0 & 3.2 & 0 & 3.1 & 0 \\ 0 & 1.3 & 0 & 5.1 & 0 & 4.1 \end{bmatrix}$$

be the MOD interval matrix with entries from the MOD interval [0, 7).

We proceed only special type of operations on MOD interval matrices.

We consider G a directed graph with edge weights from interval [0, n); then we define G to be a MOD interval directed graph. We have already seen examples of them.

Let M = (m_{ij}) be a n × n matrix with entries from the interval [0, m); $2 \leq m < \infty$.

We define M to be the MOD interval matrix. We have seen examples them.

Let X = $\{(a_1, a_2, \ldots, a_n) \mid a_i \in \{0, 1\}; 1 \leq i \leq n\}$ be the MOD interval initial state vector. We can perform operations using them.

This will be explained by some examples.

Example 3.55: Let

$$H = \begin{bmatrix} 0 & 3.1 & 0 & 1.3 & 0 & 1 \\ 1.2 & 0 & 1.2 & 0 & 2 & 0 \\ 0 & 1.1 & 0 & 1 & 0 & 2 \\ 1 & 0 & 1.3 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 \\ 1.2 & 0 & 1 & 0 & 2 & 0 \end{bmatrix}$$

MOD interval matrix with entries from [0, 4).

Let $X = \{(a_1, a_2, \ldots, a_6) \mid a_i \in \{1, 0\}; 1 \leq i \leq 6\}$ be the MOD interval initial state vector associated with H.

Let $x_1 = (1, 0, 0, 0, 0, 0) \in X$; to find the effect of x_1 on H.

$$x_1H \to (1, 3.1, 0, 1.3, 0, 1) = y_1$$

$$y_1H \to (1, 1, 0, 0, 0, 0) = y_2$$

The thresholding takes place in this form

$$x_i = 1 \text{ if } x_i \geq 2$$

$$= 0 \text{ if } x_i < 2; 1 \leq i \leq 6.$$

$y_2H = (1.2, 3.1, 1.2, 1.3, 2, 1) \to (1, 1, 0, 0, 1, 0) = y_3$
$y_3H = (1.2, 4.1, 1.2, 3.3, 2, 2) \to (1, 1, 0, 1, 1, 1) = y_4$
$y_4H = (3.2, 4.1, 3.5, 3.3, 4.5, 2) \to (1, 1, 1, 1, 1,1) = y_5$
$y_5H = (3.4, 5.2, 3.5, 4.3, 4.5, 4) \to (1, 1, 1, 1, 1, 1) = y_6$
$(=y_5)$.

Thus the MOD interval resultant is a MOD fixed point.

We see certainly after a finite number of steps we arrive at a MOD fixed point or a MOD limit cycle.

Let $x_2 = (0, 1, 0, 0, 0, 0) \in X$, to find the effect of x_2 on H.

$x_2H = (1.2, 0, 1.2, 0, 2, 0) \to (0, 1, 0, 0, 1, 0) = y_1$
$y_1H = (1.2, 1, 1.2, 2, 2, 1) \to (0, 1, 0, 1, 1, 0) = y_2$
$y_2H = (2.2, 1, 2.5, 2, 2.5, 1) \to (1, 1, 1, 1, 1, 0) = y_3$

$$y_3H = (2.2, 5.1, 2.5, 4.3, 2, 0) = (2.2, 1.1, 2.5, 0.3, 2, 0)$$
$$\to (1, 1, 1, 0, 1, 0) = y_4$$

$y_4H = (2.2, 1.2, 1.2, 0.3, 2, 0) \to (1, 1, 0, 0, 1, 0) = y_5$
$y_5H = (1.2, 0.1, 1.2, 3.3, 2, 2) \to (0, 1, 0, 1, 1, 0) = y_6$
$(=y_2)$.

Thus the MOD interval resultant is a MOD interval limit cycle given by (0, 1, 0, 1, 1, 0).

That is the on state of C_2 makes on the nodes C_4 and C_5.

Let $x_3 = (0, 0, 1, 0, 0, 0) \in X$, to find the effect of x_3 on H.

$x_3H = (0, 1.1, 0, 1, 0, 2) \to (0, 0, 1, 0, 0, 1) = y_1$.
$y_1H = (1.2, 1.1, 1, 1, 2, 2) \to (0, 0, 1, 0, 1, 1) = y_2$
$y_2H = (1.2, 2.2, 1, 3, 2, 3) \to (0, 1, 1, 1, 1, 1) = y_3$
$y_3H = (3.4, 2.2, 3.5, 3, 0.5, 3) \to (1, 1, 1, 1, 0, 1) = y_4$
$y_4H = (3.4, 0.2, 3.5, 2.3, 0.5, 0) \to (1, 0, 1, 1, 0, 0) = y_5$
$y_5H = (1, 0.2, 1.3, 2.3, 0.5, 3) \to (0, 0, 1, 1, 0, 1) = y_6$
$y_6H = (2.2, 1.1, 2.3, 1, 2.5, 2) \to (1, 0, 1, 0, 1, 1) = y_7$
$y_7H = (1.2, 1.2, 1, 0.3, 2, 0) \to (0, 0, 1, 0, 1, 0) = y_8$
$y_8H = (0, 2.2, 0, 3, 0, 3) \to (0, 1, 1, 1, 0, 1) = y_9$
$y_9H = (3.4, 1.1, 3.5, 1, 0.5, 2) \to (1, 0, 1, 0, 0, 1) = y_{10}$
$y_{10}H = (1.2, 0.2, 1, 2.3, 2, 0) \to (0, 0, 1, 1, 1, 0) = y_{11}$
$y_{11}H = (1, 2.1, 1.3, 3, 0.5, 3) \to (0, 1, 1, 1, 0, 1) = y_{12}$
$(= y_9)$.

Thus the MOD interval resultant is a MOD interval limit cycle given by (0, 1, 1, 1, 0, 1) that is on state of C_3 makes on C_2, C_4 and C_6 and has no effect on C_1 and C_5.

Let $x_6 = (0, 0, 0, 0, 0, 1) \in X$; to find the effect of x_6 on H.

$x_6H = (1.2, 0, 1, 0, 2, 0) \to (0, 0, 0, 0, 1, 1) = y_1$
$y_1H = (1.2, 1, 1, 2, 2, 1) \to (0, 0, 0, 1, 1, 1) = y_2$

$$y_2H = (2.2, 1, 2.3, 2, 2.5, 1) \to (1, 0, 1, 1, 1, 1) = y_3$$
$$y_3H = (2.2, 1.2, 2.3, 0.3, 2.5, 0) \to (1, 0, 1, 0, 1, 1) = y_4$$
$$y_4H = (1.2, 1.2, 1, 0.3, 2, 0) \to (0, 0, 0, 0, 1, 1) = y_5 \; (=y_1).$$

Thus the MOD interval resultant is a MOD interval limit cycle given by (0, 0, 0, 0, 1, 1).

The on state of the node C_6 has no impact on the nodes C_1, C_2, C_3 and C_4 only the node C_5 comes to on state.

Let x = (0, 1, 0, 0, 0, 1) $\in$ X, to find the effect of x on H.

$$xH = (2.4, 0, 2.2, 0, 0, 0) \to (1, 0, 1, 0, 0, 1) = y_1$$
$$y_1H = (1.2, 0.2, 1, 2.3, 2, 3) \to (0, 1, 0, 1, 1, 1) = y_2$$
$$y_2H = (3.4, 1, 3.5, 2, 0.5, 1) \to (1, 1, 1, 1, 0, 1) = y_3$$

$$y_3H = (3.4, 0.2, 3.5, 2.3, 0.5, 3) \to (1, 1, 1, 1, 0, 1) = y_4 \; (=y_3).$$

Thus the MOD interval resultant is a MOD interval fixed point given by (1, 1, 1, 1, 0, 1).

That is on state of the nodes C_2 and C_6 makes on the nodes C_1, C_3 and C_4, however the node C_5 remain unaffected.

This is the way the MOD interval matrix operator special operations are carried out.

We supply one more example of the same.

Example 3.56: Let

$$M = \begin{bmatrix} 0 & 2.1 & 0 & 1 & 0 & 2.2 & 0 & 1 \\ 0 & 0 & 1.2 & 0 & 1 & 0 & 2.1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0.4 & 0 & 0.6 & 0 & 0 & 0.7 & 0 & 0 \\ 0 & 1.2 & 0 & 0 & 1 & 0 & 0.1 & 0.2 \\ 0 & 0 & 0.1 & 0 & 0.3 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0.1 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

be the MOD interval matrix with entries from [0, 3).

Let $X = \{(x_1, x_2, \ldots, x_8) / x_i \in \{0, 1\}; 1 \leq i \leq 8\}$ be the MOD initial state vectors associated with M.

$x_1 = (1, 0, 0, \ldots, 0) \in X$; to find the effect of x_1 on M.

$$x_1M = (0, 2.1, 0, 1, 0, 2.2, 0, 1) \to (1, 1, 0, 0, 0, 1, 0, 0) = y_1$$

$$y_1M = (0, 2.1, 1.3, 1, 1.3, 2.2, 0.1, 1) \to (1, 1, 0, 0, 0, 1, 0, 0) = y_2\ (=y_1).$$

Thus the MOD interval resultant is a MOD interval fixed point given by (1, 1, 0, 0, 0, 1, 0, 0).

The on state of C_1 makes C_2 and C_6 to on state and all the other nodes C_3, C_4, C_5, C_7 and C_8 are unaffected by the on state of node C_1.

Let $x_2 = (0, 1, 0, 0, 0, 0, 0, 0) \in X$ the effect of x_2 on M is as follows.

$x_2M = (0, 0, 1.2, 0, 1, 0, 2.1, 0) \rightarrow$
$(0, 1, 0, 0, 0, 0, 1, 0) = y_1$

$y_1M = (1, 0, 1.2, 2, 1, 1, 2.1, 1) \rightarrow$
$(0, 1, 0, 1, 0, 0, 1, 0) = y_2$

$y_2M = (1.4, 0, 1.8, 1, 1, 1.7, 2.1, 1) \rightarrow$
$(0, 1, 1, 0, 0, 1, 1, 0) = y_3$

$y_3M = (2, 0, 1.3, 2, 1.3, 1, 2.1, 2) \rightarrow$
$(1, 0, 0, 1, 0, 0, 1, 1) = y_4$ and so on.

Certainly after a finite number of iterations we will arrive at a MOD interval fixed point or a MOD interval realized limit cycle.

Let $x_8 = (0, 0, \ldots, 0, 1) \in X$; to find the effect of x_8 on M.

$x_8M = (0, 0.1, 0, 0, 1, 0, 1, 0) \rightarrow$
$(0, 0, 0, 0, 0, 0, 0, 1) = (x_8)$.

Thus the MOD interval resultant is a MOD interval special classical fixed point.

Let $a = (0, 0, 1, 0, 0, 0, 1, 0) \in X$; to find the effect of a on M.

$aM = (2, 0, 0, 2, 0, 1, 0, 2) \rightarrow (1, 0, 1, 1, 0, 0, 1, 1) = y_1$

$y_1M = (2.4, 2.2, 0.6, 0, 1, 0.9, 1, 0) \rightarrow$
$(1, 1, 1, 0, 0, 0, 1, 0) = y_2$

$y_2M = (2, 2.1, 1.2, 0, 1, 0.2, 2.1, 0) \rightarrow$
$(1, 1, 1, 0, 0, 0, 1, 0) = y_3\ (=y_2)$.

Thus the resultant is a MOD interval fixed point.

The on state of the node C_3 and C_7 has no impact on C_4, C_5, C_6 and C_8.

Only the nodes C_1 and C_2 come to on state.

Interested reader can work with any such MOD interval matrices with this special type of product of them.

Now we technically define the MOD Interval Cognitive Maps (MOD ICMs) model.

Let P be a problem in hand.

Let C_1, C_2, …, C_n be the n-nodes / concepts which an experts wishes to work with.

Let G be the MOD interval directed graph with edge weights from $[0, m)$; $2 \leq m < \infty$.

Let M be the MOD interval connection matrix associated with M which will serve as the dynamical system of MOD interval Cognitive Maps model; construct analogous to FCMs or NCMs.

We will illustrate this situation by an example.

Example 3.57: Let P be a problem in hand C_1, C_2, C_3, C_4 and C_5 be the 5 nodes associated with problem P.

Edge weights are taken from the interval $[0, 6)$.

Let G be the MOD interval directed graph associated with this problem given by the following figure.

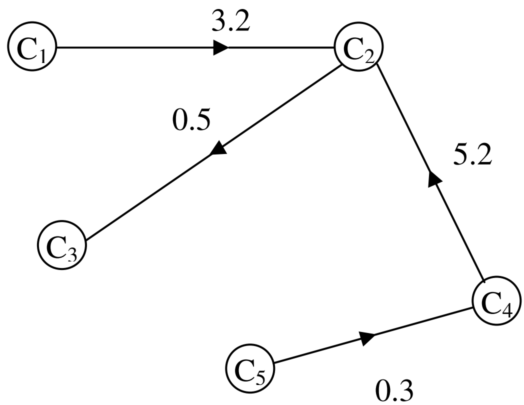


Figure 3.26

Let M be the MOD interval connection matrix associated with G.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 0 & 3.2 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 5.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0 \end{bmatrix} \end{array}.$$

Let $X = \{(a_1, a_2, a_3, a_4, a_5) \mid a_i \in \{0, 1\};\ 1 \leq i \leq 5\}$ be the MOD interval initial state vector.

Let $x_1 = (1, 0, 0, 0, 0) \in X$; to find x_1 on M.

$x_1M = (0, 3.2, 0, 0, 0) \to (1, 1, 0, 0, 0) = y_1$
$y_1M = (0, 3.2, 0.5, 0, 0) \to (1, 1, 0, 0, 0) = y_2$
$y_2M = (0, 3.2, 0.5, 0, 0) \to (1, 1, 0, 0, 0) = y_3$ $(=y_1)$.

Thus the MOD interval fixed point given by (1, 1, 0, 0, 0). So on state of C_1 makes only C_2 to on state all other nodes C_3, C_4 and C_5 are unaffected by the on state of C_1.

Let $x_4 = (0, 0, 0, 1, 0) \in X$ to find the effect of x_4 on M.

$x_4M = (0, 5.2, 0, 0, 0) \to (0, 1, 0, 1, 0) = y_1$
$y_1M = (0, 5.2, 0.5, 0, 0) \to (0, 1, 0, 1, 0) = y_2$ $(=y_1)$.

Thus on state of C_4 makes only C_2 to on state and all other nodes are in the off state.

Let $x_2 = (0, 1, 0, 0, 0) \in X$; to find the effect of x_2 on M

$x_2M = (0, 0, 0.5, 0, 0) \to (0, 1, 0, 0, 0) = x_2$.

Thus the MOD interval resultant is the MOD interval special classical fixed point.

Let $x_3 = (0, 0, 1, 0, 0) \in X$, to find the effect of x_3 on M.

$$x_3M = (0, 0, 0, 0, 0) \to (0, 0, 1, 0, 0) = y_1 \ (=x_3)$$

Thus the MOD interval resultant is a MOD interval special classical fixed point.

Let $x_5 = (0, 0, 0, 0, 1) \in X$, to find the effect of x_5 on M

$x_5M = (0, 0, 0, 0.3, 0) \to (0, 0, 0, 0, 1) = y_1$ (= x_5).

Thus the MOD interval resultant is the MOD interval special classical fixed point.

Let x = (1, 0, 0, 1, 0) ∈ X; to find the effect of x on M

$xM = (0, 2.4, 0, 0, 0) \to (1, 1, 0, 1, 0) = y_1$
$y_1M = (0, 2.4, 0.5, 0, 0) \to (1, 1, 0, 1, 0) = y_2$ (= y_1).

Thus the MOD interval resultant of x is the MOD interval fixed point given by (1, 1, 0, 1, 0).

That is the on state of C_1 and C_4 makes the node C_2 to the on state and C_3 and C_5 are left unaffected by them. Interested reader can work with more such model.

The main advantage of using this model is that the expert can give any of the value for the edge weights from the interval [0, n).

However we are sure to find the resultant after a finite number of steps.

So this new model will be helpful to researches who wishes to have different weights for the edge weights.

Chapter Four

SUGGESTED PROBLEMS

In this chapter we suggest problems for the reader. Most of them are only simple exercise only a few of them are really difficult.

These problems will motivate the researcher in using these new mathematical models.

1. Obtain all the special features enjoyed by MOD Cognitive Maps model.

2. Find a suitable programme to find MOD fixed point and MOD limit cycle for any $n \times n$ MOD Cognitive Maps model matrix with entries from Z_m.

3. Let M =

$$\begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 \end{array} \\ \begin{pmatrix} 0 & 3 & 2 & 5 & 0 & 0 & 4 \\ 0 & 0 & 3 & 0 & 2 & 1 & 0 \\ 4 & 0 & 0 & 2 & 4 & 0 & 1 \\ 3 & 6 & 2 & 0 & 0 & 6 & 4 \\ 0 & 0 & 0 & 4 & 0 & 3 & 0 \\ 2 & 0 & 1 & 0 & 1 & 0 & 3 \\ 4 & 1 & 0 & 2 & 0 & 5 & 0 \end{pmatrix} \end{array}$$

be the MOD Cognitive Maps model with entries from Z_7.

i) Find the number of MOD fixed elements associated with M.
ii) Find the number of MOD limit points associated with M.
iii) If x and y are any two initial state MOD vectors when will the resultant xM and yM be such that
xM + yM = (x + y) M.
iv) Find the maximum number of iterations needed to arrive at the MOD fixed point or a MOD limit cycle using this M.
v) List all the special features of MOD Cognitive Maps model and compare it with Fuzzy Cognitive Maps and Neutrosophic Cognitive Maps models.

4. Let

$$N = \begin{bmatrix} 0 & 2 & 0 & 1 & 5 & 5 & 4 \\ 0 & 0 & 4 & 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 & 1 \\ 4 & 0 & 2 & 0 & 0 & 6 & 0 \\ 9 & 2 & 0 & 8 & 0 & 8 & 6 \\ 0 & 0 & 7 & 0 & 2 & 0 & 0 \\ -1 & 3 & 0 & 6 & 0 & 1 & 5 \end{bmatrix}$$

be the MOD connection matrix with entries from Z_{10}.

Study question (i) to (v) of problem (3) for this N.

5. Let W = $\begin{bmatrix} 0 & 2 & 0 & 3 & 0 & 4 & 0 & 3 & 1 & 6 \\ 1 & 0 & 2 & 0 & 1 & 0 & 10 & 0 & 12 & 0 \\ 0 & 12 & 0 & 2 & 0 & 7 & 0 & 1 & 0 & 9 \\ 6 & 0 & 13 & 0 & 15 & 0 & 2 & 7 & 12 & 0 \\ 0 & 7 & 0 & 21 & 0 & 9 & 0 & 7 & 0 & 18 \\ 3 & 0 & 6 & 0 & 1 & 0 & 4 & 0 & 2 & 0 \\ 0 & 13 & 0 & 12 & 0 & 6 & 0 & 7 & 0 & 4 \\ 2 & 0 & 2 & 0 & 6 & 0 & 0 & 0 & 6 & 0 \\ 11 & 1 & 0 & 7 & 0 & 3 & 6 & 0 & 0 & 2 \\ 0 & 17 & 15 & 0 & 19 & 0 & 18 & 17 & 2 & 0 \end{bmatrix}$ be the

MOD connection matrix with entries from Z_{23}.

Study questions (i) to (v) of problem (3) for this W.

6. Find all special features enjoyed by MOD finite complex number $C(Z_n)$ Cognitive Maps model.

   (i) Compare this model with MOD Cognitive Maps models using Z_n.

   (ii) Can we say when n is prime the MOD Cognitive Maps Models are perfect?

7. Let P be the problem where M be the MOD complex Cognitive Maps model associated with the problem with entries from $C(Z_{12})$.

$$P = \begin{bmatrix}
0 & 3+i_F & 2 & 0 & 4 & 0 & 5 & 0 & 8 & 9 & 0 \\
1+i_F & 0 & 6 & 1+i_F & 2 & 6 & 0 & 2 & 0 & 0 & 1+5i_F \\
0 & 0 & 0 & 5 & 4 & 0 & 2 & 7 & 0 & 8 & 0 \\
0 & 6 & 0 & 0 & 2 & 1 & 0 & 0 & 7 & 0 & 2i_F \\
10 & 0 & 1+i_F & 1 & 0 & 5 & 11 & 0 & 11+i_F & 1 & 0 \\
0 & 11+3i_F & 0 & 0 & 1+5i_F & 0 & 0 & 3 & 0 & 4 & 2+i_F \\
1+i_F & 0 & 1 & 3 & 0 & 7 & 0 & 0 & 1 & 1+6i_F & 0 \\
0 & 1 & 2+7i_F & 0 & 1 & 2i_F & 0 & 0 & 0 & 0 & 2 \\
5+6i_F & 0 & 3i_F & 1 & 4i_F & 0 & i_F & 0 & 0 & i_F & 0 \\
0 & 7+8i_F & 0 & i_F & 0 & 1+5i_F & 0 & 1 & 3i_F & 0 & 2 \\
6i_F & 0 & 7 & 0 & 6i_F & 0 & 2+i_F & 0 & 0 & 3 & 0
\end{bmatrix}.$$

i) Study questions (i) to (v) of problem (3) for this P.
ii) Distinguish the MOD complex Cognitive Maps model from that of MOD Cognitive Maps model, FCMs model and NCM model.

8. Let S be a problem which has the following MOD complex number matrix;

$$S = \begin{bmatrix} 0 & 2+i_F & 3 & 0 & 1 & 4 \\ i_F & 0 & 2i_F & 1 & 0 & 5 \\ 0 & 1 & 0 & 1+i_F & 2 & 0 \\ 3+i_F & 0 & 2+i_F & 0 & 0 & i_F \\ 0 & 1 & i_F & 0 & 0 & 1 \\ 2 & 0 & 0 & i_F & 4+i_F & 0 \end{bmatrix}$$

with entries from $C(Z_6)$ acting as the associated MOD complex number dynamical system of the problem.

Study questions (i) to (v) of problem (3) for this S.

9. Let S_1 be the MOD dual number dynamical system associated with the MOD dual number Cognitive Maps model with entries from $\langle Z_n \cup g\rangle$.

$$S_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & 2g & 0 & 2+g & 0 & 7g \\ 2 & 0 & g & 3g & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+4g \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 5 & 2+7g & 0 & 4 & 0 & 3 \\ 0 & 0 & 7g & 0 & 4+3g & 0 \end{bmatrix} \end{array}.$$

Study questions (i) to (v) of problem (3) for this S_1

10. Obtain all special features associated with MOD dual number Cognitive Maps model.

11. What are the advantages of using MOD dual number Cognitive Maps model in the place of FCMs and NCMs?

12. Compare FCMs model with MOD complex modulo integer Cognitive Maps model.

13. Compare MOD complex modulo integer Cognitive Maps model with MOD dual number Cognitive Maps model built using $C(Z_{15})$ and $\langle Z_{15} \cup g\rangle$ respectively for a same problem.

14. Give some real world applications of the MOD finite complex modulo integer Cognitive Maps model.

15. Find the problems in which MOD dual number Cognitive Maps model would be better than FCMs and NCMs.

16. What are the advantages of using MOD special quasi dual number Cognitive Maps model?

17. Compare MOD special quasi dual number Cognitive Maps model with FCMs and NCMs.

18. Distinguish the MOD special quasi dual number Cognitive Maps model from the MOD complex modulo integer Cognitive Maps model.

19. Let P be the MOD special quasi dual number connection matrix special associated with the MOD special quasi dual number Cognitive Maps model given below;

$$P = \begin{bmatrix} 0 & 6+2k & 0 & 4 & 0 & 5k \\ 1 & 0 & 2k & 0 & 1 & 0 \\ k & 0 & 0 & 1+k & 0 & 2k+1 \\ 1+k & 0 & 0 & 0 & k & 0 \\ 0 & 1 & 1+k & k & 0 & 0 \\ 0 & 0 & 0 & 0 & 1+k & 0 \end{bmatrix}$$

is the dynamical system with entries from $\langle Z_n \cup k\rangle$; $k^2 = 6k$.

a) Study questions (i) to (v) of problem (3) for this P.

b) Obtain any other special feature one can associate with this MOD special quasi dual number Cognitive Maps model.

20. Let P be a MOD connection matrix of the MOD special dual like number Cognitive Maps model with entries from $\langle Z_4 \cup h\rangle$, $h^2 = h$ given below;

$$P = \begin{bmatrix} 0 & 1 & 2 & h & 3h & 0 \\ 0 & 0 & h & 0 & 1+h & 2 \\ 0 & 0 & 0 & 3h+1 & 0 & 0 \\ 2 & 0 & 1+h & 0 & 0 & h \\ 0 & 2+3h & 0 & 0 & 0 & 1+h \\ 0 & h & 3h & 0 & 2 & 0 \end{bmatrix}.$$

Study questions (i) to (v) of problem (3) for this P.

21. What are the advantages of using MOD special dual like number Cognitive Maps model?

22. Compare MOD special quasi dual number Cognitive Maps model with MOD special dual like number Cognitive Maps model.

23. Compare this new MOD NCMs model with FCMs and NCMs.

24. Let S be the MOD neutrosophic Cognitive Maps model associated dynamical system; S takes its entries from $\langle Z_9 \cup I\rangle$ and S is as follows.

$$S = \begin{bmatrix} 0 & 3+I & 1 & 0 & 2 & I & 3I+3 \\ 1 & 0 & 0 & 2I & 1 & 0 & 0 \\ 4+I & 0 & 0 & 0 & 0 & 2 & I \\ 0 & 1 & 0 & 0 & 0 & 0 & 4I+3 \\ 0 & 0 & 5I & 1 & 0 & 0 & 0 \\ 1 & 7I & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 2+I & 1 & 0 & 0 & 0 \end{bmatrix}.$$

Study questions (i) to (v) of problem (3) for this S.

25. Compare a MOD neutrosophic Cognitive Maps model with NCMs model.

26. Enumerate the advantages of using MOD neutrosophic Cognitive Maps model in the place of NCMs model.

27. What are the special features associated with MOD directed graphs?

28. Compare MOD directed graphs with usual directed graphs.

29. Compare the MOD complex modulo integer directed graph with edge weights from $C(Z_{12})$ with usual directed graphs.

30. Compare the MOD complex modulo integer directed graph with MOD dual number directed graphs.

31. Let G be the MOD dual number directed graph with edge weights from $\langle Z_5 \cup g\rangle$.

    Compare this graph G with the MOD directed graph built using Z_5.

32. What are the special features associated with MOD natural neutrosophic directed graphs G?

33. Can we claim it is advantageous to use MOD natural neutrosophic directed graphs in mathematical models?

34. Let M be the connection matrix of a MOD natural neutrosophic model with entries from Z_4^I given in the following;

$$M = \begin{bmatrix} 0 & 2+I_2^4 & 0 & I_0^4+1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 & 0 & I_4^0 \\ 1 & I_2^4 & 0 & 0 & 1 & 0 \\ 0 & 0 & I_0^4 & 0 & 0 & 3 \\ I_2^4+I_0^4 & 0 & 0 & 2 & I_2^4 & 0 \end{bmatrix}.$$

i) Find all MOD natural neutrosophic special classical fixed points associated with M.

ii) How many of the MOD resultants associated with M are MOD natural neutrosophic fixed points?

iii) What is the highest number of iterations needed to arrive at a MOD fixed point?

iv) How many of the MOD resultants associated with M are MOD natural neutrosophic limit cycles?

v) What is the highest number of iterations needed to arrive at a MOD natural neutrosophic limit cycle?

vi) Enumerate all special features associated with this MOD natural neutrosophic Cognitive Maps models.

35. Let
$$P = \begin{bmatrix} 0 & I_0^4 & 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 3 & 0 & 2 & 0 & 0 & 6 & 0 \\ 1 & 0 & 4 & 0 & 6 & 0 & 0 & 3 \\ 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic Cognitive Maps model connection matrix with entries from Z_7^I.

Study questions (i) to (vi) of problem (34) for this P.

36. Let
$$T = \begin{bmatrix} 0 & 3 & 1+I_5^{10} & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 1 & 0 & I_0^{10} & 0 & 6 & 2 \\ 4 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 2 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

be the connection MOD matrix of a MOD natural neutrosophic Cognitive Maps model with entries from Z_{10}^I.

Study questions (i) to (vi) of problem (34) for this T.

37. Let

$$W = \begin{bmatrix} 0 & 1 & 2 & 0 & 0 & 5 \\ 0 & 0 & 0 & 2 & 1 & 0 \\ 4 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 5 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 6 & 0 & 0 \end{bmatrix}$$

be the MOD natural neutrosophic connection matrix associated with a MOD natural neutrosophic Cognitive Maps model with entries from Z_6^I.

Let $X = \{(a_1, a_2, \ldots, a_6) / a_i \in \{0, 1\}; 1 \leq i \leq 6\}$ and

$X_S = \{(a_1, a_2, \ldots, a_6) / a_i \in \{0, 1, I_0^6, I_2^6, I_4^6, I_3^6\}; 1 \leq i \leq 6\}$ be the MOD natural neutrosophic initial state vectors and MOD natural neutrosophic special initial state vectors respectively associated with W.

i) Study questions (i) to (vi) of problem (34) for this W.

38. Compare MOD natural neutrosophic Cognitive Maps model with NCMs model.

39. Derive all special and distinct features enjoyed by MOD natural neutrosophic finite complete modulo integer Cognitive Maps model.

40. Let P = $\begin{bmatrix} 0 & i_F+2 & 0 & 3i_F+1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & I^C_{2+2i_F} \\ i_F & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix}$

be MOD natural neutrosophic finite complete modulo integer connection matrix associated with a MOD natural neutrosophic finite complex modulo integer Cognitive Maps model with entries from $C^I(Z_{24})$.

i) Obtain the special and distinct features enjoyed by MOD natural neutrosophic finite complex modulo integer Cognitive Maps model.

ii) Compare this model with MOD complex modulo integer Cognitive Maps model built using $C(Z_4)$.

41. Let M = $\begin{bmatrix} 0 & 3+5i_F & 0 & 0 & I^C_{3i_F} & 0 \\ 0 & 0 & 1 & 0 & 0 & 5 \\ 2 & 0 & 0 & 3i_F & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ i_F & 0 & 0 & 0 & 0 & 2i_F \\ 0 & 0 & 1+i_F & 0 & 0 & 0 \end{bmatrix}$

be the MOD natural neutrosophic finite complex modulo integer connection matrix associated with a MOD NCM model, with entries from $C^I(Z_9)$.

Study questions (i) to (vi) of problem (34) for this M.

42. Let P be a MOD natural neutrosophic dual number Cognitive Maps model with entries from $\langle Z_3 \cup g\rangle_I$.

Derive all special features enjoyed by this P.

43. Let S = $\begin{bmatrix} 0 & 2 & 0 & 4g & 0 & 0 & I_2^g \\ 0 & 0 & 1+g & 0 & 1 & 2g & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 3g & 0 & 0 & 0 & 0 & 0 & g \\ 0 & 1+3g & 0 & 0 & 0 & 4+g & 0 \\ 0 & 0 & g & 0 & 5+g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

be the MOD natural neutrosophic dual number Cognitive Maps model connection matrix with entries from $\langle Z_6 \cup g\rangle_I$.

Study questions (i) to (vi) problem (34) for this S.

44. Let W = $\begin{bmatrix} 0 & g & 0 & 0 & 1+g & 0 \\ 0 & 0 & 1+5g & 9g & 0 & 0 \\ 3g & 0 & 0 & 0 & 0 & I_4^g \\ 0 & 10 & 11g & 0 & 0 & 0 \\ 1+g & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+5g & 0 & 0 \end{bmatrix}$ be the

MOD natural neutrosophic dual number connection matrix associated with the MOD natural neutrosophic dual number Cognitive Maps model with entries from $\langle Z_{12} \cup g\rangle_I$.

i) Study questions (i) to (vi) of problem (34) for this W.
ii) Let $x_1 = (I_{6+6g}^g, 0, 0, 0, 0, 0)$ be the initial state vector find the MOD resultant of x_1 on W.
iii) Let $x_2 = (0, 0, 0, g, 0, 0)$ be the initial state vector find the MOD resultant of x_2.

iv) Let $x_3 = (0, 0, 0, 0, 0, I_0^g)$ be the initial state vector find the MOD resultant of x_3 on W.

v) Let $x_4 = (0, 0, 1, 0, 0, 0)$ be the initial state vector find the MOD resultant of x_4 on W.

vi) Compare the MOD resultants given by the initial state vector x_1, x_2, x_3 and x_4 on W.

45. Compare the MOD natural neutrosophic finite complex modulo integer Cognitive Maps model with MOD natural neutrosophic dual number Cognitive Maps model built using $C^I(Z_{18})$ and $\langle Z_{18} \cup g\rangle_I$ respectively.

46. Suppose B be the MOD natural neutrosophic special dual like number Cognitive Maps models dynamical system given in the following built using $\langle Z_{15} \cup h\rangle_I$;

$$B = \begin{bmatrix} 0 & 2h & 0 & 4h+1 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{5h+10}^h & 0 & 0 & 0 & h & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 1+h & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6+h & 0 & 10h & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 14+6h & 0 & 0 \end{bmatrix}.$$

Study questions (i) to (vi) of problem (34) for this B.

47. Derive all special features associated with MOD natural neutrosophic special dual like number Cognitive Maps model.

48. Compare MOD natural neutrosophic special dual like number Cognitive Maps model with MOD special dual like number Cognitive Maps model.

49. What are the benefits of using MOD natural neutrosophic dual like number Cognitive Maps model in the place of FCMs or NCMs?

50. Let

$$W_1 = \begin{bmatrix} 0 & h+1 & 0 & 0 & 0 & I_{2h}^h \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3h+2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2h \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

be the MOD special dual like number Cognitive Maps models connection matrix with entries from $\langle Z_6 \cup h\rangle_I$.

i) Study questions (i) to (vi) of problem (34) for this W_1

ii) If $x_1 = (I_{3h}^h, 0, 0, 0, 0, 0)$ be the initial state vector find the MOD resultant of x_1 on W_1.

iii) If $y_1 = (0, 0, 0, 3h, 0, 0)$ be the initial state vector find the MOD resultant of y_1 on W_1.

iv) Let $x_3 = (0, 0, 0, I_2^h, 1, 0)$ be the initial state vector find the MOD resultant of x_3 on W_1.

51. Let V be the MOD natural neutrosophic special quasi dual number connection matrix associated with MOD NCMs model given in the following; with entries from $\langle Z_{16} \cup k\rangle_I$.

$$V = \begin{bmatrix} 0 & k & 0 & 0 & I_2^k & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2+4k \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I_{4k}^k & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

i) Study questions (i) to (vi) problem (34) for this V.

ii) If $x_1 = (3 + I_{8k}^k + I_2^k, 0\ \ 0\ 0, \ldots, 0)$ be the initial state vector find the MOD resultant of x_1 on V.

iii) Let $x = (0, 0, 0, 0, \ldots, 0\ 2\ k + 4)$ be the initial state vector find the MOD resultant of x on V.

iv) Let $x_2 = (1, 0, \ldots, 0)$ be the initial state vector find the MOD resultant of x_2 on V.

v) Compare the MOD resultants of x_1 and x_2 on V in questions (ii) and (iv).

vi) Let $x_3 = (0, 0, \ldots, 0, I_{4k}^k)$ be the initial state vector. Find the MOD resultant of x_3 on V.

vii) Compare the MOD resultant of x and x_3 on V in questions (iii) and (vi) respectively.

viii) Let $y = (0, 1, \ldots, I_{4k}^k)$ be the initial state vector. Find the MOD resultant of y on V.

ix) Compare the MOD resultants of x_2, x_3 and y on V in questions (iv), (vi) and (viii) respectively.

x) Obtain any other special or striking feature associated with V.

52. Bring out all special features associated with MOD natural neutrosophic special quasi dual numbers cognitive maps model.

53. Compare the model mentioned in problem (52) with that of FCMs and NCMs models.

54. What are the main differences between MOD natural neutrosophic special dual like numbers model and MOD special quasi dual number model.

    Justify your claim.

55. Let $$S = \begin{bmatrix} 0 & 1+I_3^I & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & I_2^I & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

    be the MOD natural neutrosophic-neutrosophic square matrix which serves as the connection matrix or MOD dynamical system of the MOD natural neutrosophic-neutrosophic Cognitive Maps model with entries from $\langle Z_6 \cup I \rangle_I$.

    i) Study questions (i) to (vi) of problem (34) for this S.

    ii) If x = (I, 0, 0, 0, 0) is the initial state vector, find the MOD resultant of x on S.

    iii) Let $x_1 = (I_{3I}^I, 0, 0, 0, 0)$ be the MOD initial special state vector, find the MOD resultant of x_1 on S.

iv) Compare x and x_1 of questions (ii) and (iii) by comparing the MOD resultants.

v) Obtain any special feature associated with MOD natural neutrosophic-neutrosophic Cognitive Maps model.

vi) Compare this MOD n.n. neutrosophic cognitive maps model with NCMs and MOD neutrosophic Cognitive Maps model.

vii) Let $y_1 = (1, 0, 0, 0, 0)$ be the initial state vector, find the MOD resultant of y_1 on S.

viii) Compare the MOD resultants initial state vectors given in (ii), (iii) and (vii).

ix) Let $z = (0, 0, 0, 1, I_0^I)$ be the initial state vector, find the MOD resultant of z on S.

x) Find all initial state vector x_i's such that the MOD resultants of their sum is the sum of the MOD resultants.

56. Compare the MOD natural neutrosophic-neutrosophic Cognitive Maps model with MOD natural neutrosophic dual number Cognitive Maps model.

57. Which of the MOD Cognitive Maps model is powerful to study social problem?

58. Which of the MOD natural neutrosophic Cognitive Maps model is best suited to study scientific or technological problems?

59. What are the advantages and disadvantages of these MOD neutrosophic Cognitive Maps model?

60. Can we say MOD neutrosophic Cognitive Maps model is more powerful than NCMs model?

61. Can we claim different types of indeterminate in MOD natural neutrosophic Cognitive Maps model is useful than NCMs models?

62. Adopt MOD natural neutrosophic complex modulo integer Cognitive Maps model is a social or scientific problem and show it gives better result than MOD finite complex Cognitive Maps model.

63. Describe the MOD interval Cognitive Maps model.

64. Compare the MOD interval Cognitive Maps model with MOD integers Cognitive Maps model.

65. Apply the MOD interval cognitive maps model in some real world problem.

Let $M = \begin{bmatrix} 0 & 3.1 & 0.112 & 0 & 0 & 1 \\ 0 & 0 & 1.22 & 0.1 & 0.4 & 0 \\ 1.3 & 0 & 0 & 0 & 0 & 4.1 \\ 0 & 1.32 & 0.1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$ be the MOD

interval Cognitive Maps model dynamical system.

i) Study questions (i) to (v) of problem (3) for this M.
ii) How many MOD interval initial state vectors lead to MOD interval special classical fixed points?
iii) How many MOD interval initial state vectors yield the MOD interval resultant as MOD interval limit cycle?

66. What are the advantages of using MOD interval Cognitive Maps model in the place of FCMs?

67. Mention the application of MOD interval Cognitive Maps model.

68. Let P be a problem with C_1, c_2, C_3, C_4 as its nodes / concepts. The expert has worked the problem P using MOD interval Cognitive Maps model using the interval [0, 9).

Let M_1 be the MOD interval connection matrix which serves as the MOD dynamical system for this problem P.

$$M_1 = \begin{bmatrix} 0 & 8.01 & 0.1 & 2 \\ 0 & 0 & 6.3 & 0 \\ 3.2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

i) Study questions (i) to (iv) of problem (3) for this M_1.

ii) Let $x_1 = (1, 0, 0, 0)$ be the initial state vectors find the MOD interval resultant of x_1 on M.

iii) Let $x_3 = (0, 0, 1, 0)$ be the MOD interval initial state vector. Find the MOD interval resultant of x_3 on M.

iv) Let $y = (1, 0, 1, 0)$ be the MOD interval initial state vector. Compare the sum of the MOD resultants of x_1 and x_3 with that of the MOD resultant of y.

v) Can we replace the $X = \{(a_1, a_2, a_3, a_4) / a_i \in \{0, 1\}, 1 \leq i \leq 4\}$ by $X_m = \{(a_1, a_2, a_3, a_4) / a_i \in Z_9, 1 \leq i \leq 4\}$ and study the model?

vi) Prove using X_m will also yield the MOD resultant after a finite number of iterations.

FURTHER READING

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On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.

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In this book authors for the first time introduce new mathematical models analogous to Fuzzy Cognitive Maps (FCMs) and Neutrosophic Cognitive Maps (NCMs) models. Several types of MOD Cognitive Maps models are constructed in this book. They are MOD Cognitive Maps model, MOD dual number Cognitive Maps model, MOD neutrosophic Cognitive Maps model, MOD finite complex number Cognitive Maps model, MOD special dual like number Cognitive Maps model and MOD special quasi dual number Cognitive Maps model.

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