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On The Symbolic 2-Plithogenic Number Theory and Integers

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Abstract:

The objective of this paper is to study for the first time the foundational concepts of number theory in 2-plithogenic rings of integers, where concepts such as symbolic 2-plithogenic congruencies, division, semi primes, and greatest common divisors.

In addition, many elementary properties will be discussed in details through many theorems and examples.

Keywords: Symbolic 2-plithogenic integer, symbolic 2-plithogenic divison, symbolic 2-plithogenic semi prime.

Introduction and basic concepts

The concept of symbolic plithogenic sets was defined by Smarandache in [13-17 ,30], and he suggested an algebraic approach of these sets. Laterally, the concept of symbolic 2-plithogenic rings [31]. In general, we can say that symbolic plithogenic structures are very close to neutrosophic algebraic structures with many differences in the definition of multiplication operation [1-10].

Let R be a ring, the symbolic 2-plithogenic ring is defined as follows:

$2-SP_R=\{a_0+a_1P_1+a_2P_2;\ a_i\in R, {P_j}^2=P_j, P_1\times P_2=P_{max(1,2)}=P_2\}$.

Smarandache has defined algebraic operations on $2-SP_R$ as follows:

Addition:

$[a_0 + a_1P_1 + a_2P_2] + [b_0 + b_1P_1 + b_2P_2] = (a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2$.

Multiplication:

$[a_0 + a_1P_1 + a_2P_2].[b_0 + b_1P_1 + b_2P_2] = a_0b_0 + a_0b_1P_1 + a_0b_2P_2 + a_1b_0{P_1}^2 + a_1b_2P_1P_2 + a_2b_0P_2 + a_2b_1P_1P_2 + a_2b_2{P_2}^2 + a_1b_1P_1P_1 = a_0b_0 + (a_0b_1 + a_1b_0 + a_1b_1)P_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)P_2$.

It is clear that $(2 - SP_R)$ is a ring.

Also, if R is commutative, then $2 - SP_R$ is commutative, and if R has a unity (1), than $2 - SP_R$ has the same unity (1).

If R is a field, then $2 - SP_R$ is called a symbolic 2-plithogenic field.

In this paper, we study the symbolic 2-plithogenicnumber theoretical concepts according to many points of view, where congruencies, Euclidean division, Euler's function, and gratest common divisors will be presented in terms of theorems. In addition, many examples will be illustrated to explain the novelty of these ideas. In addition, we suggest many future applications of symbolic 2-plithogenic integers in cryptography and public key neutrosophic cryptography.

Main Discussion

Definition.

Let $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2 \in 2 - SP_Z$, we say that $A \setminus B$ if and only if there exists $C \in 2 - SP_Z$ such that $A \times B = C$.

Definition.

Let $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2, C = c_0 + c_1P_1 + c_2P_2$ be three symbolic 2-plithogenic integers, then $A \equiv B(mod\ C)$ if and only if $C \setminus A - B$.

Also, $C = gcd(A, B)$ if and only if $C \setminus A$ and $C \setminus B$ and for any $D \setminus A, D \setminus B$, then $D \setminus C$.

Definition.

We say that $A \leq B$ if $a_0 \leq b_0, a_0 + a_1 \leq b_0 + b_1, a_0 + a_1 + a_2 \leq b_0 + b_1 + b_2$.

Theorem.

Let $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2, C = c_0 + c_1P_1 + c_2P_2 \in 2 - SP_Z$, then:

1). $(\leq)$ is a partial order relation.

2). $A \setminus B$ if and only if $a_0 \setminus b_0, a_0 + a_1 \setminus b_0 + b_1, a_0 + a_1 + a_2 \setminus b_0 + b_1 + b_2$.

3). $gcd(A,B)=C$ if and only if $gcd(a_0,b_0)=c_0, gcd(a_0+a_1,b_0+b_1)=c_0+c_1, gcd(a_0+a_1+a_2,b_0+b_1+b_2)=c_0+c_1+c_2$.

4). $A\equiv B(mod\ C)$ if and only if:

$$\begin{cases} a_0\equiv b_0(mod\ c_0) \\ a_0+a_1\equiv b_0+b_1(mod\ c_0+c_1) \\ a_0+a_1+a_2\equiv b_0+b_1+b_2(mod\ c_0+c_1+c_2) \end{cases}$$

Proof.

1). $A\le A$ that is because $a_0\le a_0, a_0+a_1\le a_0+a_1, a_0+a_1+a_2\le a_0+a_1+a_2$.

If $A\le B$ and $B\le A$, then:

$$\begin{cases} a_0\le b_0, b_0\le a_0, thus\ a_0=b_0 \\ a_0+a_1\le b_0+b_1, b_0+b_1\le a_0+a_1, thus\ a_0+a_1=b_0+b_1, hence\ a_1=b_1 \\ a_0+a_1+a_2\le b_0+b_1+b_2, b_0+b_1+b_2\le a_0+a_1+a_2, thus\ a_0+a_1+a_2=b_0+b_1+b_2, hence\ a_2=b_2 \end{cases}$$

Hence $A=B$.

If $A\le B$ and $B\le C$, then $a_0\le b_0\le c_0$, $a_0+a_1\le b_0+b_1\le c_0+c_1$, $a_0+a_1+a_2\le b_0+b_1+b_2\le c_0+c_1+c_2$, thus $A\le C$.

2). If $A\setminus B$, then there exists C such that $A.C=B$. This equivalents:

$a_0c_0+P_1(a_0c_1+a_1c_0+a_1c_1)+P_2(a_0c_2+a_2c_0+a_2c_2+a_1c_2+a_2c_1)=b_0+b_1P_1+b_2P_2$,

there for:

$$\begin{cases} a_0c_0=b_0 \ldots (1) \\ a_0c_1+a_1c_0+a_1c_1=b_1 \ldots (2) \\ a_0c_2+a_2c_0+a_2c_2+a_1c_2+a_2c_1=b_2 \ldots (3) \end{cases}$$

We add (1) to (2) and (1) to (2) to (3), to get:

$$\begin{cases} a_0c_0=b_0 \\ (a_0+a_1)(c_0+c_1)=b_0+b_1 \\ (a_0+a_1+a_2)(c_0+c_1+c_2)=b_0+b_1+b_2 \end{cases}$$

Thus $a_0\setminus b_0, a_0+a_1\setminus b_0+b_1, a_0+a_1+a_2\setminus b_0+b_1+b_2$.

3). Assume that $gcd(A,B)=C$, then for any $D=d_0+d_1P_1+d_2P_2\in 2-SP_Z$ such that $D\setminus A, D\setminus B$ implies $D\setminus C$.

According to (2), we get $d_0\setminus c_0, d_0+d_1\setminus c_0+c_1, d_0+d_1+d_2\setminus c_0+c_1+c_2$, so that $gcd(a_0,b_0)=c_0, gcd(a_0+a_1,b_0+b_1)=c_0+c_1, gcd(a_0+a_1+a_2,b_0+b_1+b_2)=c_0+c_1+c_2$.

This implies that $gcd(A,B)=gcd(a_0,b_0)+P_1[gcd(a_0+a_1,b_0+b_1)-gcd(a_0,b_0)]+P_2[gcd(a_0+a_1+a_2,b_0+b_1+b_2)-gcd(a_0+a_1,b_0+b_1)]$.

4). $A\equiv B(mod\ C)$ if and only if $C\setminus A-B$, thus:

$c_0\setminus a_0-b_0, c_0+c_1\setminus (a_0+a_1)-(b_0+b_1), c_0+c_1+c_2\setminus (a_0+a_1+a_2)-(b_0+b_1+b_2)$

So that:

$$\begin{cases} a_0 \equiv b_0 (mod\ c_0) \\ a_0 + a_1 \equiv b_0 + b_1 (mod\ c_0 + c_1) \\ a_0 + a_1 + a_2 \equiv b_0 + b_1 + b_2 (mod\ c_0 + c_1 + c_2) \end{cases}$$

Theorem.

Let $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2 \in 2 - SP_Z$, then $gcd(A, B) = 1$ if and only if $gcd(a_0, b_0) = 1, gcd(a_0 + a_1, b_0 + b_1) = 1, gcd(a_0 + a_1 + a_2, b_0 + b_1 + b_2) = 1$.

The proof is clear.

Theorem.

Let $A, B, C, D, E \in 2 - SP_Z$, where:

$A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2, C = c_0 + c_1P_1 + c_2P_2, D = d_0 + d_1P_1 + d_2P_2, E = e_0 + e_1P_1 + e_2P_2;\ c_i, a_i, b_i, e_i, d_i \in Z$, then:

1). If $A \equiv B(mod\ C), D \equiv E(mod\ C)$, then $A + D \equiv B + E(mod\ C)$, $A - D \equiv B - E(mod\ C)$.

2). $A.D \equiv B.E(mod\ C)$.

3). If $gcd(A, B) = 1$, then:

$$A^{-1}(mod\ B) = a_0{}^{-1}(mod\ b_0) + P_1[(a_0 + a_1)^{-1}(mod\ b_0 + b_1) - a_0{}^{-1}(mod\ b_0)] + P_2[(a_0 + a_1 + a_2)^{-1}(mod\ b_0 + b_1 + b_2) - (a_0 + a_1)^{-1}(mod\ b_0 + b_1)]$$

Proof.

1). Assume that $A \equiv B(mod\ C), D \equiv E(mod\ C)$, thus:

$$\begin{cases} a_0 \equiv b_0 (mod\ c_0) \\ a_0 + a_1 \equiv b_0 + b_1 (mod\ c_0 + c_1) \\ a_0 + a_1 + a_2 \equiv b_0 + b_1 + b_2 (mod\ c_0 + c_1 + c_2) \end{cases}$$

And

$$\begin{cases} d_0 \equiv e_0 (mod\ c_0) \\ d_0 + d_1 \equiv e_0 + e_1 (mod\ c_0 + c_1) \\ d_0 + d_1 + d_2 \equiv e_0 + e_1 + e_2 (mod\ c_0 + c_1 + c_2) \end{cases}$$

This implies:

$$\begin{cases} a_0 + d_0 \equiv b_0 + e_0 (mod\ c_0) \\ a_0 + a_1 + d_0 + d_1 \equiv b_0 + b_1 + e_0 + e_1 (mod\ c_0 + c_1) \\ a_0 + a_1 + a_2 + d_0 + d_1 + d_2 \equiv b_0 + b_1 + b_2 + e_0 + e_1 + e_2 (mod\ c_0 + c_1 + c_2) \end{cases}$$

So that $A + D \equiv B + E(mod\ C)$.

We can prove that $A - D \equiv B - E(mod\ C)$ by a similar.

2). By using a similar discussion, we can write:

$$\begin{cases} a_0d_0 \equiv b_0e_0 (mod\ c_0) \\ (a_0 + a_1)(d_0 + d_1) \equiv (b_0 + b_1)(e_0 + e_1)(mod\ c_0 + c_1) \\ (a_0 + a_1 + a_2)(d_0 + d_1 + d_2) \equiv (b_0 + b_1 + b_2)(e_0 + e_1 + e_2)(mod\ c_0 + c_1 + c_2) \end{cases}$$

Thus $A.D \equiv B.E(mod\ C)$.

3). Suppose that $gcd(A,B)=1$, then $gcd(a_0,b_0)=gcd(a_0+a_1,b_0+b_1)=gcd(a_0+a_1+a_2,b_0+b_1+b_2)=1$.

We put

$$T=a_0^{-1}(mod\ b_0)+P_1[(a_0+a_1)^{-1}(mod\ b_0+b_1)-a_0^{-1}(mod\ b_0)]$$
$$+P_2[(a_0+a_1+a_2)^{-1}(mod\ b_0+b_1+b_2)-(a_0+a_1)^{-1}(mod\ b_0+b_1)]$$
$$A.T=a_0a_0^{-1}(mod\ b_0)+P_1[(a_0+a_1)(a_0+a_1)^{-1}(mod\ b_0+b_1)-a_0a_0^{-1}(mod\ b_0)]$$
$$+P_2[(a_0+a_1+a_2)(a_0+a_1+a_2)^{-1}(mod\ b_0+b_1+b_2)$$
$$-(a_0+a_1)(a_0+a_1)^{-1}(mod\ b_0+b_1)]=1+P_1(1-1)+P_2(1-1)=1$$

Thus $T=A^{-1}$.

Example:

Consider $A=5+4P_1+2P_2, B=2+P_1+P_2, C=3+4P_2$, we have:

$5\equiv 2(mod\ 3), 5+4=9\equiv(2+1)(mod\ 3+0), 5+4+2=11\equiv(2+1+1)(mod\ 3+0+4)$, thus $A\equiv B(mod\ C)$.

$gcd(A,B)=gcd(5,2)+P_1[gcd(9,3)-gcd(5,2)]+P_2[gcd(11,4)-gcd(9,3)]=1+P_1(3-1)+P_2(1-3)=1+2P_1-2P_2$.

Example.

Consider $A=2+P_1+P_2, B=3+P_1+P_2$, it is clear that $gcd(A,B)=1$.

$A^{-1}(mod\ B)=2^{-1}(mod\ 3)+P_1[3^{-1}(mod\ 4)-2^{-1}(mod\ 3)]+P_2[4^{-1}(mod\ 5)-3^{-1}(mod\ 4)]=2+P_1(3-2)+P_2(4-3)=2+P_1+P_2$.

Definition.

Let $A=a_0+a_1P_1+a_2P_2>0$ be a symbolic 2-plithogenic integer, we define $\varphi_S: 2-SP_Z \to 2-SP_Z$ such that:

$\varphi_S(A)=\varphi(a_0)+P_1[\varphi(a_0+a_1)-\varphi(a_0)]+P_2[\varphi(a_0+a_1+a_2)-\varphi(a_0+a_1)]$.

Where φ is the classical phi-Euler's function.

Example.

Take $A=3+5P_1-P_2,\ a_0=3, a_1=5, a_2=-1$. We have:

$a_0=3>0, a_0+a_1=8>0, a_0+a_1+a_2=7>0$, so that $A>0$.

$\varphi(a_0)=2, \varphi(a_0+a_1)=4, \varphi(a_0+a_1+a_2)=6$, hence:

$\varphi_S(A)=2+P_1[4-2]+P_2[6-4]=2+2P_1+2P_2$.

Theorem.

Let $A=a_0+a_1P_1+a_2P_2, M=m_0+m_1P_1+m_2P_2\in 2-SP_Z$ such that $gcd(A,M)=1$, then

$A^{\varphi_S(M)} \equiv 1(mod\ M)$.

Proof.

According to [ ]:

$$A^{\varphi_S(M)} = a_0{}^{\varphi(m_0)} + P_1\left[(a_0 + a_1)^{\varphi(m_0+m_1)} - a_0{}^{\varphi(m_0)}\right] + P_2\left[(a_0 + a_1 + a_2)^{\varphi(m_0+m_1+m_2)} - (a_0 + a_1)^{\varphi(m_0+m_1)}\right]$$

Since $gcd(A, M) = 1$, then $gcd(a_0, m_0) = gcd(a_0 + a_1, m_0 + m_1) = gcd(a_0 + a_1 + a_2, m_0 + m_1 + m_2) = 1$, so that:

$$\begin{cases} a_0{}^{\varphi(m_0)} \equiv 1(mod\ m_0) \\ (a_0 + a_1)^{\varphi(m_0+m_1)} \equiv 1(mod\ m_0 + m_1) \\ (a_0 + a_1 + a_2)^{\varphi(m_0+m_1+m_2)} \equiv 1(mod\ m_0 + m_1 + m_2) \end{cases}$$

Thus $A^{\varphi_S(M)} \equiv 1 + P_1(1 - 1) + P_2(1 - 1)(\ mod\ M) \equiv 1(mod\ M)$

Example.

Take $A = 2 + 3P_1 - 2P_2, M = 3 + 4P_1 + 4P_2$, we have $gcd(A, M) = 1$.

$\varphi_S(M) = 2 + P_1(6 - 2) + P_2(10 - 6) = 2 + 4P_1 + 4P_2$

$A^{\varphi_S(M)} = 2^2 + P_1[5^6 - 2^2] + P_2[3^{10} - 5^6]$

$2^2 \equiv 1(\ mod\ 3), 5^6 \equiv 1(\ mod\ 7), 3^{10} \equiv 1(\ mod\ 11)$, thus $A^{\varphi_S(M)} \equiv 1(mod\ M)$

Theorem.

Let $C = gcd(A, B) \in 2 - SP_Z$, then there exists $M, N \in 2 - SP_Z$ such that $C = MA + NB$.

Proof.

We assume that $C = gcd(A, B)$, then:

$$\begin{cases} c_0 = gcd(a_0, b_0) \\ c_1 = gcd(a_0 + a_1, b_0 + b_1) - gcd(a_0, b_0) \\ c_2 = gcd(a_0 + a_1 + a_2, b_0 + b_1 + b_2) - gcd(a_0 + a_1, b_0 + b_1) \end{cases}$$

So there exists $m_0, n_0, m_1, n_1, m_2, n_2 \in Z$ such that:

$$\begin{cases} c_0 = m_0a_0 + n_0b_0 \\ c_0 + c_1 = m_1(a_0 + a_1) + n_1(n_0 + n_1) \\ c_0 + c_1 + c_2 = m_2(a_0 + a_1 + a_2) + n_2(b_0 + b_1 + b_2) \end{cases}$$

We put $M = m_0 + (m_1 - m_0)P_1 + (m_2 - m_1)P_2, N = n_0 + (n_1 - n_0)P_1 + (n_2 - n_1)P_2$, now let us compute:

$$M.A = [m_0 + (m_1 - m_0)P_1 + (m_2 - m_1)P_2][a_0 + a_1P_1 + a_2P_2]$$

$$M.A = m_0a_0 + P_1(m_0a_1 + m_1a_0 - m_0a_0 + m_1a_1 - m_0a_1) + P_2(m_0a_2 + m_2a_0 - m_1a_0 + m_2a_1 - m_1a_1 + m_1a_2 - m_0a_2 + m_2a_2 - m_1a_2)$$

$$M.A = m_0a_0 + P_1(m_1a_0 + m_1a_1 - m_0a_0) + P_2(m_2a_0 - m_1a_0 + m_2a_1 - m_1a_1 + m_1a_2 + m_2a_2)$$

$$N.B = n_0b_0 + P_1(n_1b_0 + n_1b_1 - n_0b_0) + P_2(n_2b_0 - n_1b_0 + n_2b_1 - n_1b + n_1b_2 + n_2b_2)$$

$$MA + NB = (m_0a_0 + n_0b_0) + P_1[m_1(a_0 + a_1) + n_1(b_0 + b_1) - n_0b_0 - m_0a_0] + P_2[m_2(a_0 + a_1 + a_2) + n_2(b_0 + b_1 + b_2) - m_1(a_0 + a_1) - n_1(b_0 + b_1)] = c_0 + c_1P_1 + c_2P_2 = C$$

Example.

Consider $A = 3 + 2P_1 + P_2, B = 3 + P_1 + 3P_2$, we have:

$a_0 = 3, a_1 = 2, a_2 = 1, b_0 = 3, b_1 = 1, b_2 = 3.$

$$gcd(a_0, b_0) = 3, gcd(a_0 + a_1, b_0 + b_1) = gcd(5,4) = 1, gcd(a_0 + a_1 + a_2, b_0 + b_1 + b_2) = gcd(6,7) = 1$$

Thus $gcd(A, B) = 3 + (1 - 3)P_1 + (1 - 1)P_2 = 3 - 2P_1$.

On the other hand, we have:

$$\begin{cases} 3 = 1.3 + 0.3 \; hence \; m_0 = 1, n_0 = 0 \\ 1 = 1.5 - 1.4 \; hence \; m_1 = 1, n_1 = -1 \\ 1 = -1.6 + 1.7 \; hence \; m_2 = -1, n_2 = 1 \end{cases}$$

Thus $M = 1 + (1 - 3)P_1 + (-1 - 1)P_2 = 1 - 2P_2$, $N = 0 + (-1 - 0)P_1 + (1 + 1)P_2 = -P_1 + 2P_2$

We can see that:

$$MA + NB = (1 - 2P_2)(3 + 2P_1 + P_2) + (-P_1 + 2P_2)(3 + P_1 + 3P_2) = 3 + 2P_1 + P_2 - 6P_2 - 4P_2 - 2P_2 - 3P_1 - P_1 - 3P_1 + 6P_2 + 2P_2 + 6P_2 = 3 - 2P_1 = C = gcd(A, B)$$

Definition.

Let $S = s_0 + s_1P_1 + s_2P_2 \in 2 - SP_Z$, we say that S is a 2-plithogenic semi prime if $s_0, s_0 + s_1, s_0 + s_1 + s_2$ are primes.

Example.

The 2-plithogenic integer $S = 2 + P_1 + 2P_2$ is a semi prime, that is because $s_0 = 2, s_0 + s_1 = 3, s_0 + s_1 + s_2 = 5$ are primes.

Application In Future Studies

Symbolic 2-plithogenic number theory as a new research direction maybe very useful branches of knowledge.

We suggest the following research points that symbolic 2-plithogenic integers may have a very big effect on it.

1-). How can we use symbolic 2-plithogenic integers in the improvement of crypto-systems [39-41], for example:

a). How can we build a 2-plithogenic version of RSA algorithm.

b). How can we build a 2-plithogenic version of Diffie-Hellman key exchange algorithm.

c). How can we build a 2-plithogenic version of EL-Gamal algorithm for cryptography.

2-). How can we a solve non-linear symbolic 2-plithogenic Diophantine equations and congruencies.

References

[1] Abobala, M., "AH-Subspaces in Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 6 , pp. 80-86. 2020.

[2] Abobala, M.,. "A Study of AH-Substructures in n-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science", Vol. 9, pp.74-85. 2020.

[3] Abobala, M., Hatip, A., Bal,M.," A Study Of Some Neutrosophic Clean Rings", International journal of neutrosophic science, 2022.

[4] Abobala, M., Bal, M., Aswad, M., "A Short Note On Some Novel Applications of Semi Module Homomorphisms", International journal of neutrosophic science, 2022.

[5] Celik, M., and Hatip, A., " On The Refined AH-Isometry And Its Applications In Refined Neutrosophic Surfaces", Galoitica Journal Of Mathematical Structures And Applications, 2022.

[6] Adeleke, E.O., Agboola, A.A.A.,and Smarandache, F., "Refined Neutrosophic Rings I", International Journal of Neutrosophic Science, Vol. 2(2), pp. 77-81. 2020.

[7] M. Ibrahim. A. Agboola, B.Badmus and S. Akinleye. On refined Neutrosophic Vector Spaces . International Journal of Neutrosophic Science, Vol. 7, 2020, pp. 97-109.

[8] Von Shtawzen, O., " On A Novel Group Derived From A Generalization Of Integer Exponents and Open Problems", Galoitica journal Of Mathematical Structures and Applications, Vol 1, 2022.

[9] Hatip, A., "An Introduction To Weak Fuzzy Complex Numbers ", Galoitica Journal Of Mathematical Structures and Applications, Vol.3, 2023.

[10] Merkepci, H., and Ahmad, K., " On The Conditions Of Imperfect Neutrosophic Duplets and Imperfect Neutrosophic Triplets", Galoitica Journal Of Mathematical Structures And Applications, Vol.2, 2022.

[11] Von Shtawzen, O., " Conjectures For Invertible Diophantine Equations Of 3-Cyclic and 4-Cyclic Refined Integers", Journal Of Neutrosophic And Fuzzy Systems, Vol.3, 2022.

[12] Abobala, M., and Zeina, M.B., " A Study Of Neutrosophic Real Analysis By Using One Dimensional Geometric AH-Isometry", Galoitica Journal Of Mathematical Structures And Applications, Vol.3, 2023.

[13] F. Smarandache, Neutrosophic Quadruple Numbers, Refined Neutrosophic Quadruple Numbers,

Absorbance Law, and the Multiplication of Neutrosophic Quadruple Numbers. In *Symbolic NeutrosophicTheory*, Chapter 7, pages 186-193, Europa Nova, Brussels, Belgium, 2015.

[14] F. Smarandache, Plithogeny, Plithogenic Set, Logic, Probability, and Statistics, 141 pages, Pons Editions,Brussels, Belgium, 2017. arXiv.org (Cornell University), Computer Science - Artificial Intelligence, 03Bxx:

[15] Florentin Smarandache, Physical Plithogenic Set, 71st Annual Gaseous Electronics Conference, Session LW1, Oregon Convention Center Room, Portland, Oregon, USA, November 5–9, 2018.

[16] Florentin Smarandache: Plithogenic Set, an Extension of Crisp, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Sets – Revisited, *Neutrosophic Sets and Systems*, vol. 21, 2018, pp. 153-166.

[17] Florentin Smarandache, Plithogenic Algebraic Structures. Chapter in "Nidus idearum Scilogs, V: joining the dots" (third version), Pons Publishing Brussels, pp. 123-125, 2019.

[18] P. K. Singh, Data with Turiyam Set for fourth dimension Quantum Information Processing. *Journal of*

Neutrosophic and Fuzzy Systems, Vol. 1, Issue 1, pp. 9-23, 2021.

[19] Khaldi, A., " A Study On Split-Complex Vector Spaces", Neoma Journal Of Mathematics and Computer Science, 2023.

[20] Ahmad, K., " On Some Split-Complex Diophantine Equations", Neoma Journal Of Mathematics and Computer Science, 2023.

[21]. Ali, R., " On The Weak Fuzzy Complex Inner Products On Weak Fuzzy Complex Vector Spaces", Neoma Journal Of Mathematics and Computer Science, 2023.

[22] A. Alrida Basher, Katy D. Ahmad, Rosina Ali, An Introduction to the Symbolic Turiyam Groups and AH-Substructures,,*Journal of Neutrosophic and Fuzzy Systems*, Vol. 03, No. 02, pp. 43-52, 2022.

[23] Agboola, A.A.A., "On Refined Neutrosophic Algebraic Structures," Neutrosophic Sets and Systems,Vol.10, pp. 99-101. 2015.

[24] T.Chalapathi and L. Madhavi,. "Neutrosophic Boolean Rings", Neutrosophic Sets and Systems, Vol. 33,pp. 57-66, 2020.

[25] G. Shahzadi, M. Akram and A. B. Saeid, "An Application of Single-Valued Neutrosophic Sets in Medical Diagnosis," *Neutrosophic Sets and Systems,* vol. 18, pp. 80-88, 2017.

[**26**] Sarkis, M., " On The Solutions Of Fermat's Diophantine Equation In 3-refined Neutrosophic Ring of Integers", Neoma Journal of Mathematics and Computer Science, 2023.

[27] J. Anuradha and V. S, "Neutrosophic Fuzzy Hierarchical Clustering for Dengue Analysis in Sri Lanka," *Neutrosophic Sets and Systems,* vol. 31, pp. 179-199, 2020.

[28] Celik, M., and Olgun, N., " An Introduction To Neutrosophic Real Banach And Hillbert Spaces", Galoitica Journal Of Mathematical Structures And Applications, 2022.

[29] Celik, M., and Olgun, N., " On The Classification Of Neutrosophic Complex Inner Product Spaces", Galoitica Journal Of Mathematical Structures And Applications, 2022.

[30] Smarandache, F., " Introduction to the Symbolic Plithogenic Algebraic Structures (revisited)", Neutrosophic Sets and Systems, vol. 53, 2023.

[31] Merkepci, H., and Abobala, M., " On The Symbolic 2-Plithogenic Rings", International Journal of Neutrosophic Science, 2023.

[32] Abobala, M., and Hatip, A., "An Algebraic Approach To Neutrosophic Euclidean Geometry", Neutrosophic Sets and Systems, Vol. 43, 2021.

[33] Abobala, M., "On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations", Mathematical Problems in Engineering, Hindawi, 2021

[34] Agboola, A.A.A., Akinola, A.D., and Oyebola, O.Y., " Neutrosophic Rings I" , International J.Mathcombin, Vol 4,pp 1-14. 2011

[35] Smarandache, F., and Ali, M., "Neutrosophic Triplet Group", Neural. Compute. Appl. 2019.

[36] Chellamani, P., and Ajay, D., "Pythagorean neutrosophic Fuzzy Graphs", International Journal of Neutrosophic Science, Vol. 11, 2021.

[37] Giorgio, N, Mehmood, A., and Broumi, S.," Single Valued neutrosophic Filter", International Journal of Neutrosophic Science, Vol. 6, 2020.

[38] Hatip, A., " On Intuitionistic Fuzzy Subgroups of (M-N) Type and Their Algebraic Properties", Galoitica Journal Of Mathematical Structures and Applications, Vol.4, 2023.

[39] Merkepci, M., Abobala, M., and Allouf, A., " The Applications of Fusion Neutrosophic Number Theory in Public Key Cryptography and the Improvement of RSA Algorithm ", Fusion: Practice and Applications, 2023.

[40] Merkepci, M., and Abobala, M., " Security Model for Encrypting Uncertain Rational Data Units Based on Refined Neutrosophic Integers Fusion and El Gamal Algorithm ", Fusion: Practice and Applications, 2023.

[41] Merkepci, M., Sarkis, M. An Application of Pythagorean Circles in Cryptography and Some Ideas for Future Non Classical Systems. Galoitica Journal of Mathematical Structures and Applications 2022.

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