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A New Model based on Subjective Logic and Neutrosophic Measure for Legal Reasoning

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Abstract. Legal sciences are the theoretical body of law. This branch of knowledge studies the rules and principles that govern the correct functioning of society. The proper administration of justice is essential for the satisfaction of the subjective and objective needs of citizens. It ensures that members of society fulfill their duties and can satisfy their rights before their families and other citizens. The purpose of this paper is the presentation of a neutrosophy-based model for representing decision-making within a trial, specifically concerning both, the sufficient proof and weighing of pieces of evidence. Concepts based on the neutrosophic measure are used to enrich an earlier model that used subjective logic. We follow the principle that neutrosophic theory allows for greater precision in legal reasoning because it makes it possible to explicitly differentiate and evaluate which parts are determined and known and which parts are indeterminate and unknown. Keeping in mind that a trial is plagued with unknown, imprecise, confusing, contradictory, and paradoxical elements; and these are the ones that must be clarified with proofs and pieces of evidence. This model can be the basis of a Decision Support System or an Expert System

Keywords: Legal reasoning, neutrosophic measure, neutrosophic probabilistic measure, neutrosophic belief function, subjective logic.

1 Introduction

Legal Sciences, also called Sciences of Law, are those that carry out the complex and constant study of the legal system and its application in society [1, 2]. Legal sciences make interpretations of the norm and it is through social phenomena, it is determined whether these functions are adequate or need to be reformed. The foundation of these sciences is the problem among humans. In a community of people, humans interact with each other and establish relationships, to set up the parameters on which these relationships are based, the laws must be fully complied with, otherwise, those who defend justice must act with discipline to enforce it.

The Legal Sciences advance along with society's advances, always trying to maintain a step forward to maintain control of the relationship between the people of the community and the foreigners with the inhabitants of the population. The history of Roman law shows us how was the life of that individual who wanted to conquer, dominate and expand his/her power throughout a region. The different stages of the Roman government (monarchy, republic, and empire) show us an interesting feature of the legal sciences in antiquity and when compared with what is understood today by law, it gives us to understand the relevance of the facts that were generated at that time [3, 4].

The greatest responsibility of the sciences of law is to integrate all humans into a rational system of laws that, although rooted in common law, must be maintained in conjunction with a standard of principles and values such as morality, equity, and justice. To maintain in society a balance between objective law (the established norm) and subjective law (the capacity of man/woman to decide his/her destiny) can be considered an art, it is a profession that is studied every day, as man/woman faces new situations. The Legal Sciences are studied by mankind in different ways, what gives so many nuances to the study of law are the cultures, customs, and traditions that man/woman carries with him/her in the community.

The object of Legal Science is the positive, perishable, and criminal law. That is to say, the validity in a given community and at a given time. The central nucleus of legal science is the norm or the set of norms that form the legal system, which is a datum for the legal scientist, aware that this positive law is situated in history and therefore is founded and evolves as a product of culture, which is a historical product.

As indicated above, matching what individuals think is right and moral with what the laws dictate is a challenge for the legal sciences in all modern societies. An example that shows us the complexity of this is the legalization of abortion. This is a thorny issue, since when abortion is performed, it is putting an end to a future life, yet some countries consider it legal. In some countries with strong religious traditions, it is considered legal but immoral. Some individuals consider the act of abortion to be immoral, although legal, and would not resort to these methods of termination of pregnancy, even if they had the best legal and medical guarantees that this would have no consequences. In other countries, due to specific circumstances, women wish to have an abortion, but the laws of their country prevent them from doing so and they resort to illegal mechanisms with few health guarantees, which can cost them their lives. This is why Deontology, or the Science of Morality ([5]), does not always coincide with what is permitted, which constitutes a challenge for the Legal Sciences.

Making a decision based on the law by a judge or jury to declare a defendant innocent or guilty is a great responsibility, since making one decision or another can in some cases change the life of a person and his/her family or society. In cases such as a simple brawl or driving a motor vehicle without a license can be resolved with a fine or community service. However, when that quarrel or the driving of the vehicle causes the death of one or more persons, it becomes a case for easy justice, only if it is serious enough, for example, if the one who provoked the quarrel or the one who was driving the car intended premeditatedly to kill the other person. However, when the event occurred under certain unclear circumstances, where the individual cannot be blamed 100% for what he or she did, the question arises as to how to categorize the event from a criminal point of view.

The guilt of the accused can be decided by a judge or jury in a trial that should be impartial, although on some occasions impartiality is a challenge for those judging because of the brutality of the act, or because there was a high degree of cruelty, or because the victim was a child, etc. Another challenge is the consideration of sufficient proof, which is when evidence is presented and it constitutes a key to clarify the circumstances in which the facts occurred and considerably diminishes the doubt that could have been had about the case.

For its part, taking into account that the trial is related to evidence, the weighing of pieces of evidence is considered crucial to admit that the presented facts are admissible and bring light on the case. However, not everything in the courts is clear. The course of the trial can be plagued by uncertainty, doubts, lack of knowledge, contradictions, inconsistencies, etc. The defendant may consciously or unconsciously try to manipulate the jury or the judge, or there may be key facts that are unclear and need to be clarified. It is assumed that during the trial a consensus will be reached as to what actually happened and the degree of guilt of the defendant as realistic as possible.

For all these reasons, we consider neutrosophy to be the logical-mathematical theory that can best model legal decision-making. This theory allows us representing more clearly than fuzzy logic theories and their generalizations the possible states of information or knowledge. According to neutrosophy, a concept, theory, idea, phenomenon, etc. denoted by A, could be separated into three components, which are <A> itself, <AntiA> which is what is opposed to A, and <NeutA> which is neither A nor AntiA, [6, 7].

This paper aims to generalize within the neutrosophic framework a model based on subjective logic, which is based on logic and probabilities [8]. The proposed model has the advantage that hybridizing it with neutrosophy will allow us to explicitly model what is indeterminate. This differentiation of what is indeterminate from what is known is crucial, because it allows those who judge to differentiate what is to be clarified and thus what is to be insisted upon in the judgment, and if a proof or piece of evidence sheds light on that indeterminate part, then that constitutes sufficient proof or a burden piece of evidence.

Mathematically speaking, this hybridization will be based on the neutrosophic measure theory, the neutrosophic probability measure, and the neutrosophic belief function, [6, 9]. Which extend the definitions of measure, probability measure, and belief function in the neutrosophic framework, respectively. It is not the first time that neutrosophy is used to model problems within the legal sciences. Some approaches can be found in [10-16]. However, none of them creates a new neutrosophic model for legal sciences, but only solves specific problems with the help of neutrosophic tools. One idea that seems interesting is the logical modeling of legal sciences with the use of deontic logic [17-24]. In some scientific articles, the problems of legal sciences are modeled with the help of fuzzy logic, [20, 25].

The present article has the following structure; section 2 is devoted to exposing the basic concepts of neutrosophic measure. Section 3 contains the details of the proposed model and one example. Finally, section 4 shows the conclusions of the paper.

2 Neutrosophic measure

This section contains the basic notions of neutrosophic measure, which is a necessary concept in the approach we propose in this paper, [6, 9, 26]. Let <A> be an item. <A> can be a notion, an attribute, an idea, a proposition, a theorem, a theory, etc. And let <antiA> be the opposite of <A>; while <neutA> be neither <A> nor <antiA> but the neutral (or indeterminacy, unknown) related to <A>. Let X be a neutrosophic space, and Σ be a σ-neutrosophic algebra over X. A *neutrosophic measure* ν is defined for neutrosophic set $A \in \Sigma$ by $\nu: X \to R^3$, such that:

$$\nu(A) = (m(A), m(neutA), m(antiA)) \quad (1)$$

with antiA := the opposite of A, and neutA := the neutral (indeterminacy) neither A nor antiA.

For any $A \subseteq X$ and $A \in \Sigma$:

1. $m(A)$ means *measure of the determinate* part of A;
2. $m(neutA)$ means *measure of indeterminate* part of A; and
3. $m(antiA)$ means *measure of the determinate part* of antiA.

Where ν is a function that satisfies the following two properties:

a) Null empty set: $\nu(\emptyset) = (0,0,0)$.
b) Countable additivity (or σ -additivity): For all countable collections $\{A_n\}_{n\in L}$ of disjoint neutrosophic sets in Σ, we have:

$$\nu(\cup_{n\in L} A_n) = \left(\sum_{n\in L} m(A_n), \sum_{n\in L} m(neutA_n), \sum_{n\in L} m(antiA_n) - (n-1)m(X)\right) \quad (2)$$

Where X is the whole neutrosophic space, and

$$\sum_{n\in L} m(antiA_n) - (n-1)m(X) = m(X) - \sum_{n\in L} m(A_n) = m\left((\cap_{n\in L} antiA_n)\right) \quad (3)$$

A *neutrosophic measure space* is a triplet (X, Σ, ν).

A *neutrosophic normalized measure* is $NN = (m(X), m(neutX), m(antiX))$, where $m(X), m(neutX), m(antiX) \geq 0$ and $m(X) + m(neutX) + m(antiX) = 1$.

Where X is the whole neutrosophic measure space.

A neutrosophic measure space (X, Σ, ν) is called *finite* if $\nu(X) = (a, b, c)$ such that all a, b, and c are finite (rather than infinite). A neutrosophic measure is called *σ-finite* if X can be decomposed into a countable union of neutrosophic measurable sets of fine neutrosophic measure. Analogously, a set A in X is said to have a *σ-finite neutrosophic measure* if it is a countable union of sets with finite neutrosophic measure.

The neutrosophic measure ν satisfies the *axiom of non-negativity*, if: $\forall A \in \Sigma$,

$$\nu(A) = (a_1, a_2, a_3) \geq 0 \quad (4)$$

If $a_1, a_2, a_3 \geq 0$.

While a neutrosophic measure ν, that satisfies only the null empty set and countable additivity axioms (hence not the non-negativity axiom), takes on at most one of the $\pm\infty$ values. The members of Σ are called *measurable neutrosophic sets*, while (X, Σ) is called a *measurable neutrosophic space*.

A function $f: (X, \Sigma_X) \to (Y, \Sigma_Y)$, mapping two measurable neutrosophic spaces, is called *neutrosophic measurable function* $\forall B \in \Sigma_Y, f^{-1}(B) \in \Sigma_X$ (the inverse image of a neutrosophic Y-measurable set is a neutrosophic X-measurable set). The properties of Neutrosophic measures are the following:

a) Monotonicity:

If A_1 and A_2 are neutrosophic measurable, with $A_1 \subseteq A_2$, where $\nu(A_1) = (m(A_1), m(neutA_1), m(antiA_1))$ and $\nu(A_2) = (m(A_2), m(neutA_2), m(antiA_2))$, then $m(A_1) \leq m(A_2)$, $m(neutA_1) \leq m(neutA_2)$, $m(antiA_1) \geq m(antiA_2)$.

b) Additivity:

If $A_1 \cap A_2 = \emptyset$, then $\nu(A_1 \cup A_2) = \nu(A_1) + \nu(A_2)$,

Where the sum of two measures is defined as follows:

$$(a_1, a_2, a_3) + (b_1, b_2, b_3) = \left(a_1 + b_1, a_2 + b_2, a_3 + b_3 - m(X)\right) \quad (5)$$

Where X is *the* whole neutrosophic space, and $a_3 + b_3 - m(X) = m(X) - m(A) - m(B) = m(X) - a_1 - a_2 = m(antiA \cap antiB)$.

The *neutrosophic probability measure* is a mapping:

$$NP: X \to [0,1]^3 \quad (6)$$

Where X is a neutrosophic sample space (i.e. X contains some indeterminacy),

$$NP(A) = \left(ch(A), ch(indetermA), ch(\underline{A})\right) \quad (7)$$

That is to say, it is decomposed into three components, the chance that A occurs, the chance that A is indeterminate, and the chance that A does not occur. By using another notation we have:

$$NP(A) = (ch(A), ch(NeutA), ch(AntiA)) \quad (8)$$

Which satisfies the condition, $-0 \leq ch(A), ch(indetermA), ch(\underline{A}) \leq 3^+$, that is to say, there exist probabilities such that $ch(A) + ch(indetermA) + ch(\underline{A})$ are equal to 1, <1 or >1.

The extension of the Kolmogorov axioms to the neutrosophic space is the following:

Let $(N\Omega, NF, NP)$ be a neutrosophic probability space, where $N\Omega$ is a neutrosophic sample space, NF is a neutrosophic event space, and NP is a neutrosophic probability measure.

1. The neutrosophic probability of event A is non-negative.

2. The neutrosophic probability of the sample space is between -0 and 3^{+}.
3. Neutrosophic σ-additivity:

$$NP(A_1 \cup A_2 \cup \cdots) = \left(\sum_{j=1}^{\infty} ch(A_j), ch(indeterm A_1 \cup A_2 \cup \cdots), ch\left(\underline{A_1 \cup A_2 \cup \cdots}\right)\right) \quad (9)$$

Where $A_1, A_2, \cdots$ is a countable sequence of disjoint (or mutually exclusive) neutrosophic events.
If we relax the third axiom we get a *neutrosophic quasi-probability* distribution.

3 The model

This section contains the concepts of the proposed model, which is the generalization of the model in [8] to the neutrosophic framework. Firstly, the precedent model uses the term *frame of discernment* from the Dempster-Shafer belief model, [27]. This concept refers to a set of possible states of a given system. They choose the term "state" instead of "set" in the definition of frame of discernment in legal sciences, [8].

Definition 1 (*Neutrosophic mass assignment*) ([28]): A *neutrosophic mass assignment* is $m(\cdot) = \left(m_t(\cdot), m_i(\cdot), m_f(\cdot)\right)$; $m_t(\cdot), m_i(\cdot), m_f(\cdot): 2^{\Theta} \to]-0, 1^{+}[^3$ satisfying the following axioms for each dimension of the neutrosophic space:

$$\sum_{A \subset \Theta} sup\left(m_t(A)\right) \geq 1 \quad (10)$$

$$\sum_{A \subset \Theta} inf\left(m_f(A)\right) \geq |\Theta| - 1 \quad (11)$$

Where $|\Theta|$ represents the cardinality of the frame of discernment Θ.

Definition 2([28]): A *neutrosophic belief function* for all $A \subset \Theta$, $Bel(\cdot) = \left(Bel_T(\cdot), Bel_I(\cdot), Bel_F(\cdot)\right)$ is defined as:

$$Bel_T(A) = \sum_{B \subset A} m_t(B) \quad (12)$$
$$Bel_I(A) = \sum_{B \subset A} m_i(B)$$
$$Bel_F(A) = \sum_{B \subset A} m_f(B)$$

Definition 3: A *neutrosophic disbelief function* for all $A \subset \Theta$, $d(\cdot) = \left(d_T(\cdot), d_I(\cdot), d_F(\cdot)\right)$ is defined as:

$$d_T(A) = \sum_{A \cap B = \emptyset} m_t(B) \quad (13)$$
$$d_I(A) = \sum_{A \cap B = \emptyset} m_i(B)$$
$$d_F(A) = \sum_{A \cap B = \emptyset} m_f(B)$$

Definition 4: A *neutrosophic uncertainty function* for all $A \subset \Theta$, $u(\cdot) = \left(u_T(\cdot), u_I(\cdot), u_F(\cdot)\right)$ is defined as:

$$u_T(A) = \sum_{A \cap B \neq \emptyset\ B \not\subseteq A} m_t(B) \quad (14)$$
$$u_I(A) = \sum_{A \cap B \neq \emptyset\ B \not\subseteq A} m_i(B)$$
$$u_F(A) = \sum_{A \cap B \neq \emptyset\ B \not\subseteq A} m_f(B)$$

Definition 5: Let Θ be a frame of discernment and let $A, B \in 2^{\Theta}$. Then the *relative atomicity* of A to B is the function $a: 2^{\Theta} \to]-0, 1^{+}[$ defined by:

$$a(A/B) = \frac{|A \cap B|}{|B|} \quad (15)$$

$A, B \in 2^{\Theta}$.

Let us observe that $A \cap B = \emptyset$ implies $a(A/B) = 0$, whereas $B \subseteq A$ implies $a(A/B) = 1$. $a(\cdot)$ measures the degree of overlap between A and B.

Definition 6: (*Neutrosophic Probability Expectation*) Let Θ be a frame of discernment with $m(\cdot)$ be the neutrosophic mass assignment, then the *neutrosophic probability expectation function* corresponding with $m(\cdot)$ is the function $E: 2^{\Theta} \to]-0, 1^{+}[^3$ defined by:

$$E_T(A) = \sum_B m_t(B) a(A/B) \quad (16)$$
$$E_I(A) = \sum_B m_i(B) a(A/B)(1 - a(A/B))$$
$$E_F(A) = \sum_B m_f(B)(1 - a(A/B))$$

$A, B \in 2^{\Theta}$.

Theorem 1: Given a frame of discernment Θ with $m(\cdot)$ be the neutrosophic mass assignment, the probability expectation function $E(\cdot)$ with domain 2^{Θ} satisfies:

1. $E(A) \geq 0$ for all $A \in 2^{\Theta}$,
2. If $A_1, A_2, \cdots, A_n \in 2^{\Theta}$ are pairwise disjoint then $E(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} E(A_i)$.

Proof.

1. It is a consequence of $m(\cdot)$ is non-negative.
2. Because of $A_1, A_2, \cdots, A_n \in 2^{\Theta}$ are pairwise disjoint, then we have $a(A/B) \neq 0$ only if $A = B$ and $a(A/A) = 1$, so the formula is true.□

Definition 7 (Opinion): Let Θ be a binary frame of discernment with 2 atomic states A and A, and let $m(\cdot)$ be a Neutrosophic mass assignment on Θ where $b(A)$, $d(A)$, $u(A)$, and $a(A)$ (i.e., $B = \Theta$ in $a(A) = a(A/\Theta)$) represent the belief, disbelief, uncertainty, and relative atomicity functions on A in Θ, respectively.

Then the opinion about A, denoted by ω_A, is the quadruple defined by:

$$\omega_A \equiv (b(A), d(A), u(A), a(A)) \quad (17)$$

The expectation of the opinion ω_A is defined by using the following Equations:

$$E_T(\omega_A) = b_T(A) + u_T(A)a(A) \quad (18)$$
$$E_I(\omega_A) = b_I(A) + u_I(A)a(A)(1 - a(A))$$
$$E_F(\omega_A) = b_F(A) + u_F(A)(1 - a(A))$$

Definition 8 (Ordering of Opinions)([8]): Let ω_A and ω_B be two opinions. They can be ordered according to the following criteria by priority:

1. The greatest probability expectation gives the greatest opinion.
2. The least uncertainty gives the greatest opinion.
3. The least relative atomicity gives the greatest opinion.

Let us note that the order we referred to above is the neutrosophic order.

Definition 9: Let Θ_A and Θ_B be two distinct binary frames of discernment and let A and B be propositions about states in Θ_A and Θ_B, respectively. Let $\omega_A = (b(A), d(A), u(A), a(A))$ and $\omega_B = (b(B), d(B), u(B), a(B))$ be an agent's opinions about A and B, respectively. Let $\omega_{A\wedge B} = (b(A \wedge B), d(A \wedge B), u(A \wedge B), a(A \wedge B))$ be the opinion such that:

1. $b(A \wedge B) = b(A) \wedge_N b(B)$,
2. $d(A \wedge B) = d(A) \vee_N d(B)$,
3. $u(A \wedge B) = u(A) \vee_N u(B)$,
4. $a(A \wedge B) = a(A)a(B)$.

Where $\wedge_N$ is a neutrosophic norm or n-norm, and $\vee_N$ is a neutrosophic conorm or n-conorm, [29]. This is called the *Propositional Conjunction*.

Definition 10: Let Θ_A and Θ_B be two distinct binary frames of discernment and let A and B be propositions about states in Θ_A and Θ_B, respectively. Let $\omega_A = (b(A), d(A), u(A), a(A))$ and $\omega_B = (b(B), d(B), u(B), a(B))$ be an agent's opinions about A and B, respectively. Let $\omega_{A\vee B} = (b(A \vee B), d(A \vee B), u(A \vee B), a(A \vee B))$ be the opinion such that:

1. $b(A \vee B) = b(A) \vee_N b(B)$,
2. $d(A \vee B) = d(A) \wedge_N d(B)$,
3. $u(A \vee B) = u(A) \wedge_N u(B)$,
4. $a(A \vee B) = a(A) + a(B) - a(A) \cdot a(B)$.

This is called the *Propositional Conjunction*, which means the agent's opinion about A or B.

Definition 11: Let Θ_A and Θ_B be two distinct binary frames of discernment and let A and B be propositions about states in Θ_A and Θ_B, respectively. Let $\omega_A = (b(A), d(A), u(A), a(A))$ and $\omega_B = (b(B), d(B), u(B), a(B))$ be an agent's opinions about A and B, respectively. Let us define:

1. $E(\omega_{A\wedge B}) = E(\omega_A) \wedge_N E(\omega_B)$,
2. $E(\omega_{A\vee B}) = E(\omega_A) \vee_N E(\omega_B)$.

The negation of an opinion about proposition A represents the agent's opinion about A being false. It is defined as follows:

Definition 12: Let $\omega_A = (b(A), d(A), u(A), a(A))$ be an opinion about proposition A. Then, $\omega_A = (b(A), d(A), u(A), a(A))$ is the negation of A, defined as:

1. $b(A) = d(A)$,
2. $d(A) = b(A)$,
3. $u(A) = u(A)$,
4. $a(A) = 1 - a(A)$.

Definition 13: Let $\omega_A^\alpha = (b^\alpha(A), d^\alpha(A), u^\alpha(A), a^\alpha(A))$ and $\omega_A^\beta = \left(b^\beta(A), d^\beta(A), u^\beta(A), a^\beta(A)\right)$, ⟨`s and ®`s opinions about the same proposition A, respectively. $\omega_A^{\alpha,\beta} = \left(b^{\alpha,\beta}(A), d^{\alpha,\beta}(A), u^{\alpha,\beta}(A), a^{\alpha,\beta}(A)\right)$ is the conjoint opinion and it is defined as follows:

1. $b^{\alpha,\beta}(A) = b^\alpha(A) \wedge_N b^\beta(A)$,
2. $d^{\alpha,\beta}(A) = d^\alpha(A) \vee_N d^\beta(A)$,
3. $u^{\alpha,\beta}(A) = u^\alpha(A) \vee_N u^\beta(A)$,
4. $a^{\alpha,\beta}(A) = a^\alpha(A)a^\beta(B)$.

Let us illustrate the method with an example:

Example 1 (Adapted from [30]): Mr. Jones has been murdered, and we know that the murderer was one of three notorious assassins, Peter, Paul, and Mary, so we have a set of hypotheses, i.e., frame of discernment $\Theta = \{Peter, Paul, Mary\}$. The only evidence we have is that one person (let us denote him by W_1) who saw the killer leaving is 80% sure that it was a man, 1% unsure, and 1% sure there was not a man. i.e., $P_{W_1}(man) = (0.8, 0.1, 0.1)$. Thus, we have the following $m_1(\cdot)$ for witness 1:

$m_1(\{Peter, Paul\}) = (0.8, 0.1, 0.1)$,
$m_1(\{Peter, Paul, Mary\}) = (0.016667, 0.1, 0.15)$,
$m_1(\{Mary\}) = (0.001, 0.1, 0.15)$,
$m_1(\{Peter\}) = (0.016667, 0.1, 0.15)$,
$m_1(\{Paul\}) = (0.016667, 0.1, 0.15)$,
$m_1(\{Peter, Mary\}) = (0.016667, 0.1, 0.15)$,
$m_1(\{Paul, Mary\}) = (0.016667, 0.1, 0.15)$, and, $m_1(\emptyset) = (0, 0, 0)$.

On the other hand, there is a second witness such that $m_2(\cdot)$ is the following:

$m_2(\{Peter, Paul\}) = (0.8, 0.1, 0.1)$,
$m_2(\{Peter, Paul, Mary\}) = (0.016667, 0.1, 0.15)$,
$m_2(\{Mary\}) = (0.016667, 0.1, 0.15)$,
$m_2(\{Peter\}) = (0.016667, 0.1, 0.15)$,
$m_2(\{Paul\}) = (0.02, 0.1, 0.15)$,
$m_2(\{Peter, Mary\}) = (0.016667, 0.1, 0.15)$,
$m_2(\{Paul, Mary\}) = (0.016667, 0.1, 0.15)$, and, $m_2(\emptyset) = (0, 0, 0)$.

So, W_1's belief that Paul murdered Mr. Jones vs. Paul did not murder him is $b_1(\{Paul\}) = (0.016667, 0.1, 0.15)$ and $b_1(\{Mary, Peter\}) = (0.016667, 0.1, 0.15) + (0.001, 0.1, 0.15) + (0.016667, 0.1, 0.15) = (0.034334, 0.3, 0.45)$, respectively.

$d_1(\{Paul\}) = (0.016667, 0.1, 0.15) + (0.001, 0.1, 0.15) + (0.016667, 0.1, 0.15) = (0.034334, 0.3, 0.45)$,
$u_1(\{Paul\}) = (0.8, 0.1, 0.1) + (0.016667, 0.1, 0.15) + (0.016667, 0.1, 0.15) = (0.83333, 0.3, 0.4)$,
$a_1(\{Paul\}) = \frac{1}{3}$.

W_2's belief that Paul murdered Mr. Jones vs. Paul did not murder Mr. Jones is $b_2(\{Paul\}) = (0.02, 0.1, 0.15)$ and $b_2(\{Mary, Peter\}) = (0.016667, 0.1, 0.15) + (0.016667, 0.1, 0.15) + (0.016667, 0.1, 0.15) = (0.05, 0.300000, 0.45)$, respectively.

$d_2(\{Paul\}) = (0.016667, 0.1, 0.15) + (0.016667, 0.1, 0.15) + (0.016667, 0.1, 0.15) = (0.05, 0.300000, 0.45)$,
$u_2(\{Paul\}) = (0.8, 0.1, 0.1) + (0.016667, 0.1, 0.15) + (0.016667, 0.1, 0.15) = (0.83333, 0.3, 0.4)$,
$a_2(\{Paul\}) = \frac{1}{3}$.

Let us note that $b_1(\{Paul\}) \prec b_2(\{Paul\})$, because witness 2 is more sure about Paul's guiltiness. Also, see m_i, b_i, d_i, and u_i, can be considered neutrosophic probability measures, this is because they can be either additive, subadditive or superadditive.

To calculate the conjoint W_1 and W_2's opinion we use the formula in Definition 13, and the n-norm and n-conorm, $(a_1, a_2, a_3) \wedge_N (b_1, b_2, b_3) = (min\{a_1, b_1\}, max\{a_2, b_2\}, max\{a_3, b_3\})$ and $(a_1, a_2, a_3) \vee_N (b_1, b_2, b_3) = (max\{a_1, b_1\}, min\{a_2, b_2\}, min\{a_3, b_3\})$, respectively.

Then, the conjoint opinion of the two witnesses is formed by $b^{W_1, W_2}(\{Paul\}) = (0.016667, 0.1, 0.15)$, $d^{W_1, W_2}(\{Paul\}) = (0.05, 0.300000, 0.45)$, $u^{W_1, W_2}(\{Paul\}) = (0.83333, 0.3, 0.4)$, and $a^{W_1, W_2}(\{Paul\}) = \frac{1}{9}$.

Conclusion

This paper introduced a neutrosophic model for legal reasoning. For this purpose, concepts not yet sufficiently explored in neutrosophy were used, such as the neutrosophic belief function, from which the neutrosophic disbelief function was defined. The model is based on evidence to deal with the aspects of sufficient proof and weighing of pieces of evidence, which are basic in criminal or civil court trials. The novelty of this model based on another model that appeared in [8], which uses subjective logic, lies in the extension of the previous model to the neutrosophic framework.

It is known that in criminal and civil courts one deals with arguments, information, and knowledge that can become contradictory, confusing, incoherent, vague, uncertain, malicious, indeterminate, paradoxical, unknown, and so on. Therefore, rather than fuzzy logic or subjective logic, neutrosophic logic is better suited to deal with indetermination because the explicitness of the areas of indeterminacy allows the judge and/or jury to determine in which areas the facts and the defendant's guilt need to be clarified. Therefore, this model is feasible to use in Legal Decision Support Systems and Expert Systems.

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