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A Neutrosophic Binomial Factorial Theorem with their Refrains

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Abstract. The Neutrosophic Precalculus and the Neutrosophic Calculus can be developed in many ways, depending on the types of indeterminacy one has and on the method used to deal with such indeterminacy. This article is innovative since the form of neutrosophic binomial factorial theorem was constructed in addition to its refrains. Two other important theorems were proven with their corollaries, and numerical examples as well. As a conjecture, we use ten (indeterminate) forms in neutrosophic calculus taking an important role in limits. To serve article's aim, some important questions had been answered.

Keyword: Neutrosophic Calculus, Binomial Factorial Theorem, Neutrosophic Limits, Indeterminate forms in Neutrosophic Logic, Indeterminate forms in Classical Logic.

1 Introduction (Important questions)

Q 1 *What are the types of indeterminacy?*

There exist two types of indeterminacy

a. Literal indeterminacy (I).

As example:

$$2 + 3I \tag{1}$$

b. Numerical indeterminacy.

As example:

$$x(0.6,0.3,0.4) \in A, \tag{2}$$

meaning that the indeterminacy membership = 0.3.

Other examples for the indeterminacy component can be seen in functions: $f(0) = 7 \text{ or } 9$ or $f(0 \text{ or } 1) = 5$ or $f(x) = [0.2,\ 0.3]\, x^2$ … etc.

Q 2 *What is the values of I to the rational power?*

1. Let

$$\sqrt{I} = x + y\, I$$
$$0 + I = x^2 + (2xy + y^2)I$$
$$x = 0, y = \pm 1. \tag{3}$$

In general,

$$\sqrt[2k]{I} = \pm I \tag{4}$$

where $k \in z^+ = \{1,2,3, \dots\}$.

2. Let

$$\sqrt[3]{I} = x + y\, I$$
$$0 + I = x^3 + 3x^2 y\, I + 3xy^2\, I^2 + y^3 I^3$$
$$0 + I = x^3 + (3x^2 y + 3xy^2 + y^3)I$$
$$x = 0, y = 1 \rightarrow \sqrt[3]{I} = I. \tag{5}$$

In general,

$$\sqrt[2k+1]{I} = I, \tag{6}$$

where $k \in z^+ = \{1,2,3, \dots\}$.

Basic Notes

1. A component I to the zero power is undefined value, (i.e. I^0 is undefined), since $I^0 = I^{1+(-1)} = I^1 * I^{-1} = \frac{I}{I}$ which is impossible case (avoid to divide by I).
2. The value of I to the negative power is undefined value (i.e. I^{-n} , $n > 0$ is undefined).

Q 3 *What are the indeterminacy forms in neutrosophic calculus?*

In classical calculus, the indeterminate forms are [4]:

$$\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, \infty^0, 0^0, 1^\infty, \infty - \infty. \tag{7}$$

The form 0 to the power I (i.e. 0^I) is an indeterminate form in Neutrosophic calculus; it is tempting to argue that an indeterminate form of type 0^I has zero value since "zero to any power is zero". However, this is fallacious, since 0^I is not a power of number, but rather a statement about limits.

Q 4 *What about the form 1^I?*

The base "one" pushes the form 1^I to one while the power I pushes the form 1^I to I, so 1^I is an indeterminate form in neutrosophic calculus. Indeed, the form a^I, $a \in R$ is always an indeterminate form.

Q 5 *What is the value of a^I , where $a \in R$?*

Let $y_1 = 2^x , x \in R , y_2 = 2^I$; it is obvious that $\lim_{x\to\infty} 2^x = \infty$, $\lim_{x\to-\infty} 2^x = 0$, $\lim_{x\to 0} 2^x = 1$; while we cannot determine if $2^I \to \infty \; or \; 0 \; or \; 1$, therefore we can say that $y_2 = 2^I$ indeterminate form in Neutrosophic calculus. The same for a^I, where $a \in R$ [2].

2 Indeterminate forms in Neutrosophic Logic

It is obvious that there are seven types of indeterminate forms in classical calculus [4],

$$\frac{0}{0}, \frac{\infty}{\infty}, 0.\infty, 0^0, \infty^0, 1^\infty, \infty-\infty.$$

As a conjecture, we can say that there are ten forms of the indeterminate forms in Neutrosophic calculus

$$I^0 , 0^I, \frac{I}{0}, I\cdot\infty, \frac{\infty}{I} , \infty^I, I^\infty, I^I,$$
$$a^I (a \in R), \infty \pm a\cdot I \; .$$

Note that:

$$\frac{I}{0} = I\cdot\frac{1}{0} = I\cdot\infty = \infty\cdot I.$$

3 Various Examples

Numerical examples on neutrosophic limits would be necessary to demonstrate the aims of this work.

Example (3.1) [1], [3]

The neutrosophic (numerical indeterminate) values can be seen in the following function:

Find $\lim_{x\to 0} f(x)$, where $f(x) = x^{[2.1,2.5]}$.

Solution:

Let $y = x^{[2.1,2.5]} \;\to \ln y = [2.1, 2.5] \;\ln x$

$$\therefore \lim_{x\to 0}\ln y = \lim_{x\to 0}\frac{[2.1,2.5]}{\frac{1}{\ln x}} = \frac{[2.1,2.5]}{\frac{1}{\ln 0}}$$
$$= \frac{[2.1,2.5]}{\frac{1}{-\infty}} = \frac{[2.1,2.5]}{-0}$$
$$= \left[\frac{2.1}{-0}, \frac{2.5}{-0}\right] = (-\infty, -\infty)$$
$$= -\infty$$

Hence $y = e^{-\infty} = 0$

OR it can be solved briefly by

$y = x^{[2.1,2.5]} = [0^{2.1}, 0^{2.5}] = [0,0] = 0.$

Example (3.2)

$\lim_{x\to[9,11]}[3.5,5.9]x^{[1,2]} = \; [3.5,5.9] \; [9,11]^{[1,2]} =$
$[3.5,5.9] \; [9^1, 11^2] = \; [(3.5)(9), (5.9)(121)] =$
$[31.5,713.9]$.

Example (3.3)

$$\lim_{x\to\infty}[3.5,5.9] \; x^{[1,2]} = [3.5,5.9] \;\; \infty^{[1,2]}$$
$$= [3.5,5.9] \; [\infty^1, \infty^2]$$
$$= \; [3.5\cdot(\infty) \; ,5.9\cdot(\infty)]$$
$$= (\infty, \infty) = \infty.$$

Example (3.4)

Find the following limit using more than one technique $\lim_{x\to 0}\frac{\sqrt{[4,5]\cdot x+1}-1}{x}$.

Solution:

The above limit will be solved firstly by using the L'Hôpital's rule and secondly by using the rationalizing technique.

<u>*Using L'Hôpital's rule*</u>

$$\lim_{x\to 0}\frac{1}{2}([4,5]\cdot x+1)^{-1/2} \; [4,5]$$
$$= \lim_{x\to 0}\frac{[4,5]}{2\sqrt{([4,5]\cdot x+1)}}$$
$$= \frac{[4,5]}{2} = \left[\frac{4}{2}, \frac{5}{2}\right] = \; [2,2.5]$$

<u>*Rationalizing technique [3]*</u>

$$\lim_{x\to 0}\frac{\sqrt{[4,5]\cdot x+1}-1}{x} = \frac{\sqrt{[4,5]\cdot 0+1}-1}{0}$$
$$= \frac{\sqrt{[4\cdot 0, 5\cdot 0]+1}-1}{0} = \frac{\sqrt{[0,0]+1}-1}{0}$$
$$= \frac{\sqrt{0+1}-1}{0} = \frac{0}{0}$$
$$= \text{undefined}.$$

Multiply with the conjugate of the numerator:

$$\lim_{x\to 0}\frac{\sqrt{[4,5]x+1}-1}{x}\cdot\frac{\sqrt{[4,5]x+1}+1}{\sqrt{[4,5]x+1}+1}$$
$$=\lim_{x\to 0}\frac{\left(\sqrt{[4,5]x+1}\right)^2-(1)^2}{x\left(\sqrt{[4,5]x+1}+1\right)}$$
$$=\lim_{x\to 0}\frac{[4,5]\cdot x\cancel{+1-1}}{x\cdot\left(\sqrt{[4,5]x+1}+1\right)}$$
$$=\lim_{x\to 0}\frac{[4,5]\cancel{\cdot x}}{\cancel{x\cdot}\left(\sqrt{[4,5]x+1}+1\right)}$$
$$=\lim_{x\to 0}\frac{[4,5]}{\left(\sqrt{[4,5]x+1}+1\right)}$$
$$=\frac{[4,5]}{\left(\sqrt{[4,5]\cdot 0+1}+1\right)}=\frac{[4,5]}{\sqrt{1}+1}$$
$$=\frac{[4,5]}{2}=\left[\frac{4}{2},\frac{5}{2}\right]=[2,2.5].$$

Identical results.

Example (3.5)

Find the value of the following neutrosophic limit $\lim_{x\to -3}\frac{x^2+3x-[1,2]x-[3,6]}{x+3}$ using more than one technique .

Analytical technique [1], [3]

$\lim_{x\to -3}\frac{x^2+3x-[1,2]x-[3,6]}{x+3}$

By substituting x= -3 ,

$$\lim_{x\to -3}\frac{(-3)^2+3\cdot(-3)-[1,2]\cdot(-3)-[3,6]}{-3+3}$$
$$=\frac{9-9-[1\cdot(-3),2\cdot(-3)]-[3,6]}{0}$$
$$=\frac{0-[-6,-3]-[3,6]}{0}=\frac{[3,6]-[3,6]}{0}$$
$$=\frac{[3-6,6-3]}{0}=\frac{[-3,3]}{0},$$

which has undefined operation $\frac{0}{0}$, since $0\in[-3,3]$. Then we factor out the numerator, and simplify:

$$\lim_{x\to -3}\frac{x^2+3x-[1,2]x-[3,6]}{x+3}=$$
$$\lim_{x\to -3}\frac{(x-[1,2])\cdot(x+3)}{(x+3)}=\lim_{x\to -3}(x-[1,2])$$
$$=-3-[1,2]=[-3,-3]-[1,2]$$
$$=-([3,3]+[1,2])=[-5,-4].$$

Again, Solving by using L'Hôpital's rule

$$\lim_{x\to -3}\frac{x^2+3x-[1,2]x-[3,6]}{x+3}$$
$$=\lim_{x\to -3}\frac{2x+3-[1,2]}{1}$$
$$=\lim_{x\to -3}\frac{2(-3)+3-[1,2]}{1}$$
$$=-6+3-[1,2]$$
$$=-3-[1,2]$$
$$=[-3-1,-3-2]$$
$$=[-5,-4]$$

The above two methods are identical in results.

4 New Theorems in Neutrosophic Limits

Theorem (4.1) (Binomial Factorial)

$\lim_{x\to\infty}(I+\frac{1}{x})^x=Ie$; I is the literal indeterminacy, e = 2.7182828

Proof

$$\left(I+\frac{1}{x}\right)^x=\binom{x}{0}I^X\left(\frac{1}{x}\right)^0+\binom{x}{1}I^{X-1}\left(\frac{1}{x}\right)^1$$
$$+\binom{x}{2}I^{X-2}\left(\frac{1}{x}\right)^2+\binom{x}{3}I^{X-3}\left(\frac{1}{x}\right)^3$$
$$+\binom{x}{4}I^{X-4}\left(\frac{1}{x}\right)^4+\cdots$$
$$=I+x.I.\frac{1}{x}+\frac{I}{2!}\left(1-\frac{1}{x}\right)$$
$$+\frac{I}{3!}\left(1-\frac{1}{x}\right)\left(1-\frac{2}{x}\right)+\frac{I}{4!}\left(1-\frac{1}{x}\right)\left(1-\frac{2}{x}\right)$$
$$\left(1-\frac{3}{x}\right)+\cdots$$

It is clear that $\frac{1}{x}\to 0$ as $x\to\infty$

$\therefore\lim_{x\to\infty}(I-\frac{1}{x})^x=I+I+\frac{I}{2!}+\frac{I}{3!}+\frac{I}{4!}+\cdots=I+\sum_{n=1}^{\infty}\frac{I^n}{n!}$

$\therefore\lim_{x\to\infty}(I+\frac{1}{x})^x=Ie$, where $e=1+\sum_{n=1}^{\infty}\frac{1}{n!}$, I is the literal indeterminacy.

Corollary (4.1.1)

$\lim_{x\to 0}(I+x)^{\frac{1}{x}}=Ie$

Proof:-

Put $y=\frac{1}{x}$

It is obvious that $y\to\infty$, as $x\to 0$

$\therefore\lim_{x\to 0}(I+x)^{\frac{1}{x}}=\lim_{y\to\infty}(I+\frac{1}{y})^y=Ie$

(using Th. 4.1)

Corollary (4.1.2)

$\lim_{x\to\infty}(I+\frac{k}{x})^x=Ie^k$, where k > 0 & $k\neq 0$, I is the literal indeterminacy.

Proof

$$\lim_{x\to\infty}(I+\frac{k}{x})^x = \lim_{x\to\infty}\left[(I+\frac{k}{x})^{\frac{x}{k}}\right]^k$$

Put $y=\frac{k}{x}\to xy=k\to x=\frac{k}{y}$

Note that $y\to 0\ as\ x\to\infty$

$$\therefore \lim_{x\to\infty}\left(I+\frac{k}{x}\right)^x = \lim_{y\to 0}\left[(I+y)^{\frac{1}{y}}\right]^k$$

(using corollary 4.1.1).

$$=\left[\lim_{y\to 0}(I+y)^{\frac{1}{y}}\right]^k = (Ie)^k = I^k e^k = Ie^k$$

Corollary (4.1.3)

$\lim_{x\to 0}(I+\frac{x}{k})^{\frac{1}{x}} = (Ie)^{\frac{1}{k}} = \sqrt[k]{Ie}$,

where $k\neq 1$ & $k>0$.

Proof

The immediate substitution of the value of x in the above limit gives indeterminate form I^∞,

i.e. $\lim_{x\to 0}(I+\frac{x}{k})^{\frac{1}{x}} = \lim_{x\to 0}(I+\frac{0}{k})^{\frac{1}{0}} = I^\infty$

So we need to treat this value as follow:-

$$\lim_{x\to 0}(I+\frac{x}{k})^{\frac{1}{x}} = \lim_{x\to 0}\left[(I+\frac{x}{k})^{\frac{k}{x}}\right]^{\frac{1}{k}} = \left[\lim_{x\to 0}(I+\frac{x}{k})^{\frac{k}{x}}\right]^{\frac{1}{k}}$$

put $y=\frac{x}{k}\to x=ky\to\frac{1}{x}=\frac{1}{ky}$

As $x\to 0$, $y\to 0$

$$\lim_{x\to 0}\left(I+\frac{x}{k}\right)^{\frac{1}{x}} = \lim_{y\to 0}\left[(I+y)^{\frac{1}{y}}\right]^{\frac{1}{k}}$$

$$=\left[\lim_{y\to 0}(I+y)^{\frac{1}{y}}\right]^{\frac{1}{k}}$$

Using corollary (4.1.1)

$$=(Ie)^{\frac{I}{k}} = \sqrt[k]{Ie}$$

Theorem (4.2)

$$\lim_{x\to 0}\frac{(lna)[Ia^x-I]}{xlna+lnI} = \frac{lna}{1+lnI}$$

Where $a>0, a\neq 1$

Note that $\lim_{x\to 0}\frac{(lna)[Ia^x-I]}{xlna+lnI} = \lim_{x\to 0}\frac{Ia^x-I}{x+\frac{lnI}{lna}}$

Proof

Let $y=Ia^x-I\ \to y+I=Ia^x\ \to\ \ln(y+I)=\ln I+\ln a^x$

$\to\ln(y+I)=\ln I+xlna\to$

$$x=\frac{\ln(y+I)-lnI}{lna}$$

$$\frac{(\ln a)(Ia^x-I)}{xlna+lnI} = \frac{(Ia^x-I)}{x+\frac{lnI}{lna}}$$

$$=\frac{y}{\frac{\ln(y+I)-lnI}{lna}+\frac{lnI}{lna}}$$

$$=lna.\frac{y}{\ln(y+I)} = lna.\frac{1}{\frac{1}{y}\ln(y+I)}$$

$$=lna.\frac{1}{\ln(y+I)^{\frac{1}{y}}}$$

$$\therefore\lim_{x\to 0}\frac{(\ln a)(Ia^x-I)}{xlna+lnI} = lna\ \frac{1}{\lim_{y\to 0}ln(y+I)^{\frac{1}{y}}}$$

$$=lna\,.\frac{1}{ln\lim_{y\to 0}(y+I)^{\frac{1}{y}}}$$

$=lna\ \frac{1}{ln(Ie)}$ using corollary (4.1.1)

$$=\frac{lna}{lnI+\ lne} = \frac{lna}{lnI+1}$$

Corollary (4.2.1)

$$\lim_{x\to 0}\frac{Ia^{kx}-I}{x+\frac{lnI}{lna^k}} = \frac{k\ lna}{1+lnI}$$

Proof

Put $y=kx\to x=\frac{y}{k}$

$y\to 0\ \ as\ x\to 0$

$$\lim_{x\to 0}\frac{Ia^{kx}-I}{x+\frac{lnI}{lna^k}} = \lim_{y\to 0}\frac{Ia^y-I}{\frac{y}{k}+\frac{lnI}{k\,lna}} = k.\lim_{y\to 0}\frac{Ia^y-I}{y+\frac{lnI}{lna}}$$

using Th. (4.2)

$$=k.(\frac{lna}{1+lnI})$$

Corollary (4.2.2)

$$\lim_{x\to 0}\frac{Ie^x-I}{x+lnI} = \frac{1}{1+lnI}$$

Proof

Let $y=\ Ie^x-I\ \ ,y\to 0\ \ as\ x\to 0$

$y+I=\ Ie^x\to\ln(y+I)=lnI+x\ lne$

$x=\ln(y+I)-lnI$

$$\therefore\ \frac{Ie^x-I}{x+\ lnI} = \frac{y}{\ln(y+I)-lnI+lnI}$$

$$=\frac{1}{\frac{1}{y}\ln(y+I)}$$

$$=\frac{1}{\ln(y+I)^{\frac{1}{y}}}$$

$$\therefore\lim_{x\to 0}\frac{Ie^x-I}{x+lnI} = \lim_{y\to 0}\frac{1}{\ln(y+I)^{\frac{1}{y}}}$$

$$=\frac{1}{\ln\ \lim_{y\to 0}(y+I)^{\frac{1}{y}}}$$

using corollary (4.1.1)

$$\frac{1}{\ln(Ie)} = \frac{1}{lnI+lne} = \frac{1}{lnI+1}$$

Corollary (4.2.3)

$$\lim_{x\to 0}\frac{Ie^{kx}-I}{x+\frac{lnI}{k}}=\frac{k}{1+lnI}$$

Proof

let $y=kx \to x=\frac{y}{k}$

$y\to 0\ \ as\, x\to 0$

$\lim_{x\to 0}\frac{Ie^{kx}-I}{x+\frac{lnI}{k}}=\lim_{y\to 0}\frac{Ie^{y}-I}{\frac{y}{k}+\frac{lnI}{k}}=k.\lim_{y\to 0}\frac{Ie^{y}-I}{y+lnI}$ using Corollary (4.2.2) to get

$$=k.\left(\frac{1}{1+lnI}\right)=\frac{k}{1+lnI}$$

Theorem (4.3)

$$\lim_{x\to 0}\frac{\ln(I+kx)}{x}=k(1+lnI)$$

Proof

$$\lim_{x\to 0}\frac{\ln(I+kx)}{x}=\lim_{x\to 0}\frac{\ln(I+kx)-lnI+lnI}{x}$$

Let $y=\ln(I+kx)-lnI \to y+lnI=\ln(I+kx)$

$$e^{y+lnI}=I+kx\to x=\frac{e^{y}e^{lnI}-I}{k}=\frac{I\,e^{y}-I}{k}$$

$y\to 0\ \ as\, x\to 0$

$$\lim_{x\to 0}\frac{\ln(I+kx)-lnI+lnI}{x}$$

$$=\lim_{y\to 0}\frac{y+lnI}{\frac{I\,e^{y}-I}{k}}$$

$$\lim_{y\to 0}\frac{k}{\frac{I\,e^{y}-I}{y+lnI}}=\frac{k}{\lim_{y\to 0}(\frac{I\,e^{y}-I}{y+lnI})}$$

using corollary (4.2.2) to get the result

$$=\frac{k}{\frac{1}{1+lnI}}=k(1+lnI)$$

Theorem (4.4)

Prove that, for any two real numbers a,b

$\lim_{x\to 0}\frac{Ia^{x}-I}{Ib^{x}-I}=1$, where $a,b>0$ & $a,b\neq 1$

Proof

The direct substitution of the value x in the above limit conclude that $\frac{0}{0}$,so we need to treat it as follow:

$$\lim_{x\to 0}\frac{Ia^{x}-I}{Ib^{x}-I}=\lim_{x\to 0}\frac{\frac{lna[Ia^{x}-I]}{xlna+lnI}*\frac{xlna+lnI}{lna}}{\frac{lnb[Ib^{x}-I]}{xlnb+lnI}*\frac{xlnb+lnI}{lnb}}$$

$$=\frac{\lim_{x\to x}\frac{lna[Ia^{x}-I]}{xlna+lnI}}{\lim_{x\to x}\frac{lnb[Ib^{x}-I]}{xlnb+lnI}}*\frac{\lim_{x\to 0}(xlna+lnI)}{\lim_{x\to 0}(xlnb+lnI)}*\frac{lnb}{lna}$$

(using Th.(4.2) twice (first in numerator second in denominator))

$$=\frac{\frac{lna}{1+lnI}}{\frac{lnb}{1+lnI}}*\frac{lnI}{lnI}*\frac{lnb}{lna}=1.$$

5 Numerical Examples

Example (5.1)

Evaluate the limit $\lim_{x\to 0}\frac{I5^{4x}-I}{x+\frac{lnI}{ln5^{4}}}$

Solution

$\lim_{x\to 0}\frac{I5^{4x}-I}{x+\frac{lnI}{ln5^{4}}}=\frac{4ln5}{1+lnI}$ (using corollary 4. 2.1)

Example (5.2)

Evaluate the limit $\lim_{x\to 0}\frac{Ie^{4x}-I}{I3^{2x}-I}$

Solution

$$\lim_{x\to 0}\frac{Ie^{4x}-I}{I3^{2x}-I}=\lim_{x\to 0}\frac{\frac{ln3[Ie^{4x}-I]}{(x+\frac{lnI}{4})}*(x+\frac{lnI}{4})}{\frac{ln3[I3^{2x}-I]}{(x+\frac{lnI}{ln3^{2}})}*(x+\frac{lnI}{ln3^{2}})}$$

$$=\frac{\lim_{x\to 0}\frac{ln3[Ie^{4x}-I]}{\left(x+\frac{lnI}{4}\right)}}{\lim_{x\to 0}\frac{ln3[I3^{2x}-I]}{\left(x+\frac{lnI}{ln3^{2}}\right)}}*\frac{\lim_{x\to 0}\left(x+\frac{lnI}{4}\right)}{\lim_{x\to 0}\left(x+\frac{lnI}{ln3^{2}}\right)}$$

(using corollary (4.2.3) on numerator & corollary (4.2.1) on denominator)

$$=\frac{\frac{4}{1+lnI}}{\frac{2ln3}{1+lnI}}*\frac{\frac{lnI}{4}}{\frac{lnI}{ln3^{2}}}=1.$$

5 Conclusion

In this article, we introduced for the first time a new version of binomial factorial theorem containing the literal indeterminacy *(I).* This theorem enhances three corollaries. As a conjecture for indeterminate forms in classical calculus, ten of new indeterminate forms in Neutrosophic calculus had been constructed. Finally, various examples had been solved.

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