



# A New Approach to Multi-spaces Through the Application of Soft Sets

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**Abstract.** Multi-space is the notion combining different fields in to a unifying field, which is more applicable in our daily life. In this paper, we introduced the notion of multi-soft space which is the approximated collection of

the multi-subspaces of a multi-space . Further, we defined some basic operations such as union, intersection, AND, OR etc. We also investigated some properties of multi-soft spaces.

**Keywords:** Multi-space, soft set, multi-soft space.

## 1. Introduction

Multi-spaces [24] were introduced by Smarandache in 1969 under the idea of hybrid structures: combining different fields into a unifying field [23] that are very effective in our real life. This idea has a wide range of acceptance in the world of sciences. In any domain of knowledge a Smarandache multi-space is the union of  $n$  different spaces with some different for an integer  $n \geq 2$  . Smarandache multi-space is a qualitative notion as it is too huge which include both metric and non-metric spaces. This multi-space can be used for both discrete or connected spaces specially in spacetimes and geometries in theoretical physics. Multi-space theory has applied in physics successfully in the Unified Field Theory which unite the gravitational, electromagnetic, weak and strong interactions or in the parallel quantum computing or in the mu-bit theory etc. Several multi-algebraic structures have been introduced such as multi-groups, multi-rings, multi-vector spaces, multi-metric spaces etc. Literature on multi-algebraic structures can be found in [17].

Molodtsov [20] proposed the theory of soft sets. This mathematical framework is free from parameterization inadequacy, syndrome of fuzzy

set theory, rough set theory, probability theory and so on. Soft set theory has been applied successfully in many areas such as smoothness of functions, game theory, operation research, Riemann integration, Perron integration, and probability theory. Soft sets gained much attention of the researchers recently from its appearance and some literature on soft sets can be seen in [1]-[16]. Some other properties and algebras may be found in [18,19,20]. Some other concepts together with fuzzy set and rough set were shown in [21,22,23].

In section 2, we review some basic concepts and notions on multi-spaces and soft sets. In section 3, we define multi-subspac. Then multi-soft spaces has been introduced in the current section. Multi-soft space is a parameterized collection of multi-subspaces. We also investigated some properties and other notions of multi-soft spaces.

## 2. Basic Concepts

In this section, we review some basic material of multi-spaces and soft sets.

**Definition 2.1 [24].** For any integer  $i, 1 \leq i \leq n$ , let  $M_i$  be a set with ensemble of law  $L_i$ , and the intersection of  $k$  sets  $M_{i_1}, M_{i_2}, \dots, M_{i_k}$  of them constrains the law  $I M_{i_1}, M_{i_2}, \dots, M_{i_k}$ . Then the union of  $M_i, 1 \leq i \leq n$

$$M = \bigcup_{i=1}^n M_i$$

is called a multi-space.

Let  $U$  be an initial universe,  $E$  is a set of parameters,  $P(U)$  is the power set of  $U$ , and  $A, B \subset E$ . Molodtsov defined the soft set in the following manner:

**Definition 2.2 [20].** A pair  $(F, A)$  is called a soft set over  $U$ , where  $F$  is a mapping given by  $F : A \rightarrow P(U)$ . In other words, a soft set over  $U$  is a parameterized family of subsets of the universe  $U$ . For  $a \in A$ ,  $F a$  may be considered as the set of  $a$ -elements of the soft set  $(F, A)$ , or as the set of  $a$ -approximate elements of the soft set.

**Example 2.3.** Suppose that  $U$  is the set of shops.  $E$  is the set of parameters and each parameter is a word or sentence. Let

$$E = \left\{ \begin{array}{l} \text{high rent, normal rent,} \\ \text{in good condition, in bad condition} \end{array} \right\}$$

Let us consider a soft set  $(F, A)$  which describes the attractiveness of shops that Mr.  $Z$  is taking on rent. Suppose that there are five houses in the universe

$U = \{s_1, s_2, s_3, s_4, s_5\}$  under consideration, and that

$A = \{a_1, a_2, a_3\}$  be the set of parameters where

$a_1$  stands for the parameter 'high rent,

$a_2$  stands for the parameter 'normal rent,

$a_3$  stands for the parameter 'in good condition.

Suppose that

$$F(a_1) = \{s_1, s_4\},$$

$$F(a_2) = \{s_2, s_5\},$$

$$F(a_3) = \{s_3\}.$$

The soft set  $(F, A)$  is an approximated family  $\{F(a_i), i = 1, 2, 3\}$  of subsets of the set  $U$  which gives us a collection of approximate description of an object. Then  $(F, A)$  is a soft set as a collection of approximations over  $U$ , where

$$F(a_1) = \text{high rent} = \{s_1, s_2\},$$

$$F(a_2) = \text{normal rent} = \{s_2, s_5\},$$

$$F(a_3) = \text{in good condition} = \{s_3\}.$$

**Definition 2.4 [19].** For two soft sets  $(F, A)$  and  $(H, B)$  over  $U$ ,  $(F, A)$  is called a soft subset of  $(H, B)$  if

1.  $A \subseteq B$  and
2.  $F(a) \subseteq H(a)$ , for all  $x \in A$ .

This relationship is denoted by  $(F, A) \subset (H, B)$ . Similarly  $(F, A)$  is called a soft superset of  $(H, B)$  if  $(H, B)$  is a soft subset of  $(F, A)$  which is denoted by  $(F, A) \supset (H, B)$ .

**Definition 2.5 [19].** Two soft sets  $(F, A)$  and  $(H, B)$  over  $U$  are called soft equal if  $(F, A)$  is a soft subset of  $(H, B)$  and  $(H, B)$  is a soft subset of  $(F, A)$ .

**Definition 2.6 [19].** Let  $(F, A)$  and  $(G, B)$  be two soft sets over a common universe  $U$  such that  $A \cap B \neq \phi$ . Then their restricted intersection is denoted by  $(F, A) \cap_R (G, B) = (H, C)$  where  $(H, C)$  is defined as  $H(c) = F(c) \cap G(c)$  for all  $c \in C = A \cap B$ .

**Definition 2.7 [12].** The extended intersection of two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  is the soft set  $(H, C)$ , where  $C = A \cup B$ , and for all  $c \in C$ ,  $H(c)$  is defined as

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - B, \\ G(c) & \text{if } c \in B - A, \\ F(c) \cap G(c) & \text{if } c \in A \cap B. \end{cases}$$

We write  $(F, A) \cap_{\varepsilon} (G, B) = (H, C)$ .

**Definition 2.8 [19].** The restricted union of two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  is the soft set  $(H, B)$ , where  $C = A \cup B$ , and for all  $c \in C$ ,  $H(c)$  is defined as  $H(c) = F(c) \cup G(c)$  for all  $c \in C$ . We write it as  $(F, A) \cup_R (G, B) = (H, C)$ .

**Definition 2.9 [12].** The extended union of two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  is the soft set  $(H, B)$ , where  $C = A \cup B$ , and for all  $c \in C$ ,  $H(c)$  is defined as

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - B \\ G(c) & \text{if } c \in B - A, \\ F(c) \cup G(c) & \text{if } c \in A \cap B. \end{cases}$$

We write  $(F, A) \cup_{\varepsilon} (G, B) = (H, C)$ .

In the next section, we introduced multi-soft spaces.

### 3. Multi-Soft Space and Its Properties

In this section, first we introduced the definition of multi-subspace. Further, we introduced multi-soft spaces and their core properties.

**Definition 3.1.** Let  $M$  be a multi-space and  $M' \subseteq M$ . Then  $M'$  is called a multi-subspace if  $M'$  is a multi-space under the operations and constraints of  $M$ .

**Definition 3.2.** Let  $A_1 = \{a_j : j \in J\}$ ,  $A_2 = \{a_k : k \in K\}$ , ...,  $A_n = \{a_n : n \in L\}$  be n-set of parameters. Let  $(F_1, A_1), (F_2, A_2), \dots, (F_n, A_n)$  are soft set over the distinct universes  $M_1, M_2, \dots, M_n$  respectively. Then  $(H, C)$  is called a multi-soft space over  $M = M_1 \cup M_2 \cup \dots \cup M_n$ , where  $(H, C) = (F_1, A_1) \cup_E (F_2, A_2) \cup_E \dots \cup_E (F_n, A_n)$  such that  $C = A_1 \cup A_2 \cup \dots \cup A_n$  and for all  $c \in C$ ,  $H(c)$  is defined by

$$H(c) = F_{i_1}(c) \cup F_{i_2}(c) \cup \dots \cup F_{i_k}(c)$$

if  $c \in (A_{i_1} \cap A_{i_2} \cap \dots \cap A_{i_k}) - (A_{i_{k+1}} \cup A_{i_{k+2}} \cup \dots \cup A_{i_n})$ ,

where  $(i_1, i_2, \dots, i_k, i_{k+1}, \dots, i_n)$  are all possible permutations of the indexes  $(1, 2, \dots, n)$   $k = 1, 2, \dots, n$ . There are  $2^{n-1}$  pieces of the piece-wise function  $(H, C)$ .

**Proposition 3.3.** Let  $M$  be a universe of discourse and  $(F, A)$  is a soft set over  $M$ . Then  $(F, A)$  is a multi-soft space over  $M$  if and only if  $M$  is a multi-space.

**Proof:** Suppose that  $M$  is a multi-space and  $F : A \rightarrow P(M)$  be a mapping. Then clearly for each  $a \in A$ , then  $F(a)$  is a subset of  $M$  which is a multi-subspace. Thus each  $F(a)$  is a multi-subspace of  $M$  and so the soft set  $(F, A)$  is the parameterized collection of multi-subspaces of  $M$ . Hence  $(F, A)$  is a multi-soft space over  $M$ .

For converse, suppose that  $(F, A)$  is a multi-soft space over  $M$ . This implies that  $F(a)$  is a multi-subspace of  $M$  for all  $a \in A$ . Therefore,  $M$  is a multi-space.

This situation can be illustrated in the following Example.

**Example 3.4.** Let  $M = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$  be an initial universe such that  $M$  is a multi-space. Let  $A_1 = \{a_1, a_2, a_3, a_8\}$ ,  $A_2 = \{a_2, a_4, a_5, a_6, a_8\}$  and  $A_3 = \{a_5, a_7, a_8\}$  are set of parameters. Let  $(F_1, A_1), (F_2, A_2)$  and  $(F_3, A_3)$  respectively be the soft sets over  $M$  as following:

$$F_1(a_1) = \{m_1, m_2, m_3\},$$

$$F_1(a_2) = \{m_4, m_5\},$$

$$F_1(a_3) = \{m_1, m_4, m_6, m_7\},$$

$$F_1(a_8) = \{m_2, m_4, m_6, m_7\}.$$

and

$$F_2(a_2) = \{m_1, m_2, m_3, m_6, m_7\},$$

$$F_2(a_4) = \{m_3, m_4, m_5, m_6\},$$

$$F_2(a_5) = \{m_2, m_4, m_5\},$$

$$F_2(a_6) = \{m_1, m_7\},$$

$$F_2(a_8) = \{m_1, m_3, m_5, m_7\}.$$

Also

$$F_3(a_5) = \{m_1, m_2, m_3, m_4, m_5\},$$

$$F_3(a_7) = \{m_4, m_5, m_7\},$$

$$F_3(a_8) = \{m_2\}.$$

Let  $A = A_1 \cup A_2 \cup A_3 = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$ .

Then the multi-soft space of  $(F_1, A_1), (F_2, A_2)$  and  $(F_3, A_3)$  is  $(F, A)$ , where  $(F, A) = (F_1, A_1) \cup_E (F_2, A_2) \cup_E (F_3, A_3)$  such that

$$F(a_1) = F_1(a_1) = \{m_1, m_2, m_3\}, \text{ as } a_1 \in A_1 - A_2 \cup A_3,$$

$$F(a_2) = F_1(a_2) \cup F_2(a_2) = \{m_1, m_2, m_3, m_4, m_5, m_6, m_7\}, \text{ as } a_2 \in A_1 \cap A_2 - A_3,$$

$$F(a_3) = F_1(a_3) = \{m_1, m_4, m_6, m_7\} \text{ as } a_3 \in A_1 - A_2 \cup A_3,$$

$$F(a_4) = F_2(a_4) = \{m_3, m_4, m_5, m_6\}, \text{ as } a_4 \in A_2 - A_1 \cup A_3,$$

$$F(a_5) = F_2(a_5) \cup F_3(a_5) = \{m_1, m_2, m_3, m_4, m_5\}, \text{ as } a_5 \in A_2 \cap A_3 - A_1,$$

$$F(a_6) = F_2(a_6) = \{m_1, m_7\} \text{ as } a_6 \in A_2 - A_1 \cup A_3,$$

$$F(a_7) = F_3(a_7) = \{m_4, m_5, m_7\} \text{ as } a_7 \in A_3 - A_1 \cup A_2,$$

$$F(a_8) = F_1(a_8) \cup F_2(a_8) \cup F_3(a_8) = \{m_1, m_2, m_3, m_4, m_5, m_6, m_7\} \text{ as } a_8 \in A_1 \cap A_2 \cap A_3.$$

**Definition 3.5.** Let  $(F, A)$  and  $(H, B)$  be two multi-soft spaces over  $M_1 \cup M_2 \cup \dots \cup M_n$ . Then  $(F, A)$  is called a multi-soft subspace of  $(H, B)$  if

1.  $A \subseteq B$  and
2.  $F(a) \subseteq H(a)$ , for all  $a \in A$ .

This can be denoted by  $(F, A) \subset (H, B)$ .

Similarly  $(F, A)$  is called a multi-soft superspace of  $(F, A)$  if  $(F, A)$  is a multi-soft subspace of  $(F, A)$  which is denoted by  $(F, A) \supset (H, B)$ .

**Definition 3.6.** Two multi-soft spaces  $(F, A)$  and  $(H, B)$  over  $M_1 \cup M_2 \cup \dots \cup M_n$  are called multi-soft multi-equal if  $(F, A)$  is a multi-soft subspace of  $(H, B)$  and  $(H, B)$  is a multi-soft subspace of  $(F, A)$ .

**Proposition 3.6.** Let  $(F, A)$  and  $(K, B)$  be two multi-soft spaces over  $M_1 \cup M_2 \cup \dots \cup M_n$  such that  $A \cap B \neq \phi$ . Then their restricted intersection  $(F, A) \cap_R (K, B) = (H, C)$  is also a multi-soft space over  $M_1 \cup M_2 \cup \dots \cup M_n$ .

**Proposition 3.7.** The extended intersection of two multi-soft multi-spaces  $(F, A)$  and  $(K, B)$  over  $M_1 \cup M_2 \cup \dots \cup M_n$  is again a multi-soft multi-space over  $M_1 \cup M_2 \cup \dots \cup M_n$ .

**Proposition 3.8.** Let  $(F, A)$  and  $(K, B)$  be two multi-soft multi-spaces over  $M_1 \cup M_2 \cup \dots \cup M_n$  such that  $A \cap B \neq \phi$ . Then their restricted union  $(F, A) \cup_R (K, B) = (H, C)$  is also a multi-soft multi-space over  $M_1 \cup M_2 \cup \dots \cup M_n$ .

**Proposition 3.9.** The extended union of two multi-soft multi-spaces  $(F, A)$  and  $(K, B)$  over  $M_1 \cup M_2 \cup \dots \cup M_n$  is again a multi-soft multi-space over  $M_1 \cup M_2 \cup \dots \cup M_n$ .

**Proposition 3.10.** The AND operation of two multi-soft multi-spaces  $(F, A)$  and  $(K, B)$  over

$M_1 \cup M_2 \cup \dots \cup M_n$  is again a multi-soft multi-space over  $M_1 \cup M_2 \cup \dots \cup M_n$ .

**Proposition 3.11.** The OR operation of two multi-soft multi-spaces  $(F, A)$  and  $(K, B)$  over

$M_1 \cup M_2 \cup \dots \cup M_n$  is again a multi-soft multi-space over  $M_1 \cup M_2 \cup \dots \cup M_n$ .

**Proposition 3.12.** The complement of a multi-soft space over a multi-space  $M$  is again a multi-soft space over  $M$ .

**Prof.** This is straightforward.

**Definition 3.13.** A multi-soft multi-space  $(F, A)$  over  $M_1 \cup M_2 \cup \dots \cup M_n$  is called absolute multi-soft multi-space if  $F(a) = M_1 \cup M_2 \cup \dots \cup M_n$  for all  $a \in A$ .

**Proposition 3.14.** Let  $(F, A)$ ,  $(G, B)$  and  $(H, C)$  are three multi-soft multi-spaces over  $M_1 \cup M_2 \cup \dots \cup M_n$ . Then

1.  $(F, A) \cup_E (G, B) \cup_E (H, C) = (F, A) \cup_E (G, B) \cup_E (H, C)$ ,
2.  $(F, A) \cap_R (G, B) \cap_R (H, C) = (F, A) \cap_R (G, B) \cap_R (H, C)$ .

**Proposition 3.15.** Let  $(F, A)$ ,  $(G, B)$  and  $(H, C)$  are three multi-soft multi-spaces over  $M_1 \cup M_2 \cup \dots \cup M_n$ . Then

1.  $(F, A) \wedge (G, B) \wedge (H, C) = (F, A) \wedge (G, B) \wedge (H, C)$ ,
2.  $(F, A) \vee (G, B) \vee (H, C) = (F, A) \vee (G, B) \vee (H, C)$ .

## Conclusion

In this paper, we introduced multi-soft spaces which is a first attempt to study the multi-spaces in the context of soft sets. Multi-soft spaces are more rich structure than the multi-spaces which represent different fields in an approximated unifying field. We also studied some properties of multi-soft spaces. A lot of further research can do in the future in this area. In the future, one can define the algebraic structures of multi-soft spaces.

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Received: Novemer 10, 2014. Accepted: January 6, 2015.