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A Study of Symbolic 2-Plithogenic Split-Complex Linear Diophantine Equations in Two Variables

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Abstract:

The equation $AX + BY = C$ is called symbolic 2-plithogenic linear Diophantine equation with two variables if A, B, X, Y, C are symbolic 2-plithogenic split-complex integers.

This paper aims to find an algebraic formula for solving the symbolic 2-plithogenic split-complex linear Diophantine equation with two variables with necessary and sufficient conditions for the solvability of this class. Also, some related examples will be illustrated.

Keywords: Split-complex, symbolic 2-plithogenic, linear Diophantine equation.

Introduction.

Diophantine equation is very interesting concept in Number theory, where they are considered as algebraic equations with integer solutions [1].

In the literature, we find many generalized kinds of Diophantine equations handled by many authors, see [2-5].

A classical linear Diophantine equation an equation with the following formula:

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$AX + BY = C$, where A, B, C, X, Y are integers.

Split-complex numbers were built over real numbers a generalization of them with a similar structure to the complex numbers, where a split-complex number is defined as follows:

$a + bJ$; $a, b \in R, J^2 = 1, J \neq \{-1,1\}$, and they are studied by many authors in [6-10].

If $a, b \in Z$, then $a + bJ$ is called a split-complex integer.

The concept of symbolic 2-plithogenic split-complex numbers was defined as an extension of symbolic 2-plithogenic numbers [12]. The generalizations of real numbers, especially the plithogenic numbers have many applications in many scientific fields, see [13-20].

In this work, we present an effective algorithm to find all solutions of the symbolic 2-plithogenic split-complex linear Diophantine equation with two variables.

Preliminaries

Main discussion.

Definition.

Let $AX + BY = C$ with:

$A = (a_0 + a_1P_1 + a_2P_2) + J(\acute{a}_0 + \acute{a}_1P_1 + \acute{a}_2P_2)$

$B = (b_0 + b_1P_1 + b_2P_2) + J(\acute{b}_0 + \acute{b}_1P_1 + \acute{b}_2P_2)$

$C = (c_0 + c_1P_1 + c_2P_2) + J(\acute{c}_0 + \acute{c}_1P_1 + \acute{c}_2P_2)$

$X = (x_0 + x_1P_1 + x_2P_2) + J(\acute{x}_0 + \acute{x}_1P_1 + \acute{x}_2P_2)$

$Y = (y_0 + y_1P_1 + y_2P_2) + J(\acute{y}_0 + \acute{y}_1P_1 + \acute{y}_2P_2)$

Where $x_i, y_i, a_i, b_i, c_i, \acute{x}_i, \acute{y}_i, \acute{a}_i, \acute{b}_i, \acute{c}_i \in Z$.

The previous equation is called symbolic 2-plithogenic split-complex Diophantine equation with two variables X and Y.

Example

$[(2 + P_1 + P_2) + J(1 + P_2)]X + [(1 - P_1 + 3P_2) + J(4 - 5P_1 + P_2)]Y = P_1 + P_2$

Is a symbolic 2-plithogenic split-complex Diophantine equation with two variables.

How can we find the solutions?

First, we must transform the equation to classical Diophantine equations.

For this goal, we must compute the products AX, BY.

$AX = (a_0 + a_1P_1 + a_2P_2)(x_0 + x_1P_1 + x_2P_2) + (\acute{a}_0 + \acute{a}_1P_1 + \acute{a}_2P_2)(\acute{x}_0 + \acute{x}_1P_1 + \acute{x}_2P_2) + J[(a_0 + a_1P_1 + a_2P_2)(\acute{x}_0 + \acute{x}_1P_1 + \acute{x}_2P_2) + (\acute{a}_0 + \acute{a}_1P_1 + \acute{a}_2P_2)(x_0 + x_1P_1 + x_2P_2)]$.

We have:

$(a_0 + a_1P_1 + a_2P_2)(x_0 + x_1P_1 + x_2P_2) = a_0x_0 + P_1[(a_0 + a_1)(x_0 + x_1) - a_0x_0] + P_2[(a_0 + a_1 + a_2)(x_0 + x_1 + x_2) - (a_0 + a_1)(x_0 + x_1)]$,

$(\acute{a}_0 + \acute{a}_1P_1 + \acute{a}_2P_2)(\acute{x}_0 + \acute{x}_1P_1 + \acute{x}_2P_2) = \acute{a}_0\acute{x}_0 + P_1[(\acute{a}_0 + \acute{a}_1)(\acute{x}_0 + \acute{x}_1) - \acute{a}_0\acute{x}_0] + P_2[(\acute{a}_0 + \acute{a}_1 + \acute{a}_2)(\acute{x}_0 + \acute{x}_1 + \acute{x}_2) - (\acute{a}_0 + \acute{a}_1)(\acute{x}_0 + \acute{x}_1)]$,

$(a_0 + a_1P_1 + a_2P_2)(\acute{x}_0 + \acute{x}_1P_1 + \acute{x}_2P_2) = a_0\acute{x}_0 + P_1[(a_0 + a_1)(\acute{x}_0 + \acute{x}_1) - a_0\acute{x}_0] + P_2[(a_0 + a_1 + a_2)(\acute{x}_0 + \acute{x}_1 + \acute{x}_2) - (a_0 + a_1)(\acute{x}_0 + \acute{x}_1)]$,

$(\acute{a}_0 + \acute{a}_1P_1 + \acute{a}_2P_2)(\acute{x}_0 + \acute{x}_1P_1 + \acute{x}_2P_2) = \acute{a}_0x_0 + P_1[(\acute{a}_0 + \acute{a}_1)(x_0 + x_1) - \acute{a}_0x_0] + P_2[(\acute{a}_0 + \acute{a}_1 + \acute{a}_2)(x_0 + x_1 + x_2) - (\acute{a}_0 + \acute{a}_1)(x_0 + x_1)]$,

So that.

$$\begin{aligned} AX = (a_0x_0 + \acute{a}_0\acute{x}_0) &+ P_1[(a_0 + a_1)(x_0 + x_1) + (\acute{a}_0 + \acute{a}_1)(\acute{x}_0 + \acute{x}_1) - a_0x_0 - \acute{a}_0\acute{x}_0] \\ &+ P_2[(a_0 + a_1 + a_2)(x_0 + x_1 + x_2) + (\acute{a}_0 + \acute{a}_1 + \acute{a}_2)(\acute{x}_0 + \acute{x}_1 + \acute{x}_2) \\ &- (a_0 + a_1)(x_0 + x_1) - (\acute{a}_0 + \acute{a}_1)(\acute{x}_0 + \acute{x}_1)] \\ &+ J\Big[(a_0\acute{x}_0 + \acute{a}_0x_0) \\ &+ P_1[(a_0 + a_1)(\acute{x}_0 + \acute{x}_1) + (\acute{a}_0 + \acute{a}_1)(x_0 + x_1) - a_0\acute{x}_0 - \acute{a}_0x_0] \\ &+ P_2[(a_0 + a_1 + a_2)(\acute{x}_0 + \acute{x}_1 + \acute{x}_2) + (\acute{a}_0 + \acute{a}_1 + \acute{a}_2)(x_0 + x_1 + x_2) \\ &- (a_0 + a_1)(\acute{x}_0 + \acute{x}_1) - (\acute{a}_0 + \acute{a}_1)(x_0 + x_1)]\Big] \end{aligned}$$

By a similar argument, we can write:

$$\begin{aligned} BY = \left(b_0y_0 + \acute{b}_0\acute{y}_0\right) &+ P_1\left[(b_0 + b_1)(y_0 + y_1) + \left(\acute{b}_0 + \acute{b}_1\right)(\acute{y}_0 + \acute{y}_1) - b_0y_0 - \acute{b}_0\acute{y}_0\right] \\ &+ P_2\left[(b_0 + b_1 + b_2)(y_0 + y_1 + y_2) + \left(\acute{b}_0 + \acute{b}_1 + \acute{b}_2\right)(\acute{y}_0 + \acute{y}_1 + \acute{y}_2) \right. \\ &\left. - (b_0 + b_1)(y_0 + y_1) - \left(\acute{b}_0 + \acute{b}_1\right)(\acute{y}_0 + \acute{y}_1)\right] \\ &+ J\left[\left(b_0\acute{y}_0 + \acute{b}_0y_0\right) \right. \\ &+ P_1\left[(b_0 + b_1)(\acute{y}_0 + \acute{y}_1) + \left(\acute{b}_0 + \acute{b}_1\right)(y_0 + y_1) - b_0\acute{y}_0 - \acute{b}_0y_0\right] \\ &+ P_2\left[(b_0 + b_1 + b_2)(\acute{y}_0 + \acute{y}_1 + \acute{y}_2) + \left(\acute{b}_0 + \acute{b}_1 + \acute{b}_2\right)(y_0 + y_1 + y_2) \right. \\ &\left.\left. - (b_0 + b_1)(\acute{y}_0 + \acute{y}_1) - \left(\acute{b}_0 + \acute{b}_1\right)(y_0 + y_1)\right]\right] \end{aligned}$$

The equation $AX+BY=C$ is equivalent to the following system of Diophantine equations:

Equation (1):

$a_0x_0+\acute{a}_0\acute{x}_0+b_0y_0+\acute{b}_0\acute{y}_0=c_0$

Equation (2):

$(a_0+a_1)(x_0+x_1)+(\acute{a}_0+\acute{a}_1)(\acute{x}_0+\acute{x}_1)+(b_0+b_1)(y_0+y_1)+(\acute{b}_0+\acute{b}_1)(\acute{y}_0+\acute{y}_1)-a_0x_0-\acute{a}_0\acute{x}_0-b_0y_0-\acute{b}_0\acute{y}_0=c_1$, thus:

$$(a_0+a_1)(x_0+x_1)+(\acute{a}_0+\acute{a}_1)(\acute{x}_0+\acute{x}_1)+(b_0+b_1)(y_0+y_1)+(\acute{b}_0+\acute{b}_1)(\acute{y}_0+\acute{y}_1)=c_0+c_1$$

Equation (3):

$(a_0+a_1+a_2)(x_0+x_1+x_2)+(\acute{a}_0+\acute{a}_1+\acute{a}_2)(\acute{x}_0+\acute{x}_1+\acute{x}_2)+(b_0+b_1+b_2)(y_0+y_1+y_2)+(\acute{b}_0+\acute{b}_1+\acute{b}_2)(\acute{y}_0+\acute{y}_1+\acute{y}_2)-(a_0+a_1)(x_0+x_1)-(\acute{a}_0+\acute{a}_1)(\acute{x}_0+\acute{x}_1)-(b_0+b_1)(y_0+y_1)-(\acute{b}_0+\acute{b}_1)(\acute{y}_0+\acute{y}_1)=c_2$, thus:

$$(a_0+a_1+a_2)(x_0+x_1+x_2)+(\acute{a}_0+\acute{a}_1+\acute{a}_2)(\acute{x}_0+\acute{x}_1+\acute{x}_2)+(b_0+b_1+b_2)(y_0+y_1+y_2)+(\acute{b}_0+\acute{b}_1+\acute{b}_2)(\acute{y}_0+\acute{y}_1+\acute{y}_2)=c_0+c_1+c_2$$

Equation (4):

$a_0\acute{x}_0+\acute{a}_0x_0+b_0\acute{y}_0+\acute{b}_0y_0=\acute{c}_0$

Equation (5):

$$(a_0+a_1)(\acute{x}_0+\acute{x}_1)+(\acute{a}_0+\acute{a}_1)(x_0+x_1)+(b_0+b_1)(\acute{y}_0+\acute{y}_1)+(\acute{b}_0+\acute{b}_1)(y_0+y_1)=\acute{c}_0+\acute{c}_1$$

Equation (6):

$$(a_0+a_1+a_2)(\acute{x}_0+\acute{x}_1+\acute{x}_2)+(\acute{a}_0+\acute{a}_1+\acute{a}_2)(x_0+x_1+x_2)+(b_0+b_1+b_2)(\acute{y}_0+\acute{y}_1+\acute{y}_2)+(\acute{b}_0+\acute{b}_1+\acute{b}_2)(y_0+y_1+y_2)=\acute{c}_0+\acute{c}_1+\acute{c}_2$$

By now, we have six linear Diophantine equation with four variables.

We will transform them into easier forms.

We add equation (1) to (4), we get:

$(a_0+\acute{a}_0)(x_0+\acute{x}_0)+(b_0+\acute{b}_0)(y_0+\acute{y}_0)=c_0+\acute{c}_0 \quad (I)$

We add equation (2) to (5), we get:

$$(a_0 + a_1 + \acute{a}_0 + \acute{a}_1)(x_0 + x_1 + \acute{x}_0 + \acute{x}_1) + \left(b_0 + b_1 + \acute{b}_0 + \acute{b}_1\right)(y_0 + y_1 + \acute{y}_0 + \acute{y}_1)$$
$$= c_0 + c_1 + \acute{c}_0 + \acute{c}_1 \quad (II)$$

We add equation (3) to (6), we get:

$$(a_0 + a_1 + a_2 + \acute{a}_0 + \acute{a}_1 + \acute{a}_2)(x_0 + x_1 + x_2 + \acute{x}_0 + \acute{x}_1 + \acute{x}_2)$$
$$+ \left(b_0 + b_1 + b_2 + \acute{b}_0 + \acute{b}_1 + \acute{b}_2\right)(y_0 + y_1 + y_2 + \acute{y}_0 + \acute{y}_1 + \acute{y}_2)$$
$$= c_0 + c_1 + c_2 + \acute{c}_0 + \acute{c}_1 + \acute{c}_2 \quad (III)$$

We subtract equation (3) from (1), we get:

$$(a_0 - \acute{a}_0)(x_0 - \acute{x}_0) + \left(b_0 - \acute{b}_0\right)(y_0 - \acute{y}_0) = c_0 - \acute{c}_0 \quad (IV)$$

We subtract equation (5) to (2), we get:

$$(a_0 + a_1 - \acute{a}_0 - \acute{a}_1)(x_0 + x_1 - \acute{x}_0 - \acute{x}_1) + \left(b_0 + b_1 - \acute{b}_0 - \acute{b}_1\right)(y_0 + y_1 - \acute{y}_0 - \acute{y}_1)$$
$$= c_0 + c_1 - \acute{c}_0 - \acute{c}_1 \quad (IIV)$$

We subtract equation (6) from (3), we get:

$$(a_0 + a_1 + a_2 - \acute{a}_0 - \acute{a}_1 - \acute{a}_2)(x_0 + x_1 + x_2 - \acute{x}_0 - \acute{x}_1 - \acute{x}_2)$$
$$+ \left(b_0 + b_1 + b_2 - \acute{b}_0 - \acute{b}_1 - \acute{b}_2\right)(y_0 + y_1 + y_2 - \acute{y}_0 - \acute{y}_1 - \acute{y}_2)$$
$$= c_0 + c_1 + c_2 - \acute{c}_0 - \acute{c}_1 - \acute{c}_2 \quad (IIIV)$$

We change the variables by the following:

$$\begin{cases} x_0 + \acute{x}_0 = t_0, x_0 - \acute{x}_0 = \acute{t}_0 \\ x_0 + x_1 + \acute{x}_0 + \acute{x}_1 = t_1, x_0 + x_1 - \acute{x}_0 - \acute{x}_1 = \acute{t}_1 \\ x_0 + x_1 + x_2 + \acute{x}_0 + \acute{x}_1 + \acute{x}_2 = t_2, x_0 + x_1 + x_2 - \acute{x}_0 - \acute{x}_1 - \acute{x}_2 = \acute{t}_2 \\ y_0 + \acute{y}_0 = s_0, y_0 + \acute{y}_0 = \acute{s}_0 \\ y_0 + y_1 + \acute{y}_0 + \acute{y}_1 = s_1, y_0 + y_1 - \acute{y}_0 - \acute{y}_1 = \acute{s}_1 \\ y_0 + y_1 + y_2 + \acute{y}_0 + \acute{y}_1 + \acute{y}_2 = s_2, y_0 + y_1 + y_2 - \acute{y}_0 - \acute{y}_1 - \acute{y}_2 = \acute{s}_2 \end{cases}$$

The equation can be written as follows:

$$(a_0 + \acute{a}_0)(t_0) + \left(b_0 + \acute{b}_0\right)(s_0) = c_0 + \acute{c}_0 \quad (I)$$

$$(a_0 + a_1 + \acute{a}_0 + \acute{a}_1)(t_1) + \left(b_0 + b_1 + \acute{b}_0 + \acute{b}_1\right)(s_1) = c_0 + c_1 + \acute{c}_0 + \acute{c}_1 \quad (II)$$

$$(a_0 + a_1 + a_2 + \acute{a}_0 + \acute{a}_1 + \acute{a}_2)(t_2) + \left(b_0 + b_1 + b_2 + \acute{b}_0 + \acute{b}_1 + \acute{b}_2\right)(s_2)$$
$$= c_0 + c_1 + c_2 + \acute{c}_0 + \acute{c}_1 + \acute{c}_2 \quad (III)$$

$$(a_0 - \acute{a}_0)(\acute{t}_0) + \left(b_0 - \acute{b}_0\right)(\acute{s}_0) = c_0 - \acute{c}_0 \quad (IV)$$

$$(a_0 + a_1 - \acute{a}_0 - \acute{a}_1)(\acute{t}_1) + \left(b_0 + b_1 - \acute{b}_0 - \acute{b}_1\right)(\acute{s}_1) = c_0 + c_1 - \acute{c}_0 - \acute{c}_1 \quad (IIV)$$

$$(a_0 + a_1 + a_2 - \acute{a}_0 - \acute{a}_1 - \acute{a}_2)(\acute{t}_2) + \left(b_0 + b_1 + b_2 - \acute{b}_0 - \acute{b}_1 - \acute{b}_2\right)(\acute{s}_2)$$
$$= c_0 + c_1 + c_2 - \acute{c}_0 - \acute{c}_1 - \acute{c}_2 \quad (IIIV)$$

According to the previous argument, we can see that the symbolic 2-plithogenic split-complex Diophantine equation $AX + BY = C$ is solvable if and only if the equations $(I, II, III, IV, IIV, IIIV)$ are solvable, which is equivalent to:

$$\begin{cases} gcd(a_0 + \acute{a}_0, b_0 + \acute{b}_0) \setminus c_0 + \acute{c}_0 \\ gcd(a_0 - \acute{a}_0, b_0 - \acute{b}_0) \setminus c_0 + \acute{c}_0 \\ gcd(a_0 + a_1 + \acute{a}_0 + \acute{a}_1, b_0 + b_1 + \acute{b}_0 + \acute{b}_1) \setminus c_0 + c_1 + \acute{c}_0 + \acute{c}_1 \\ gcd(a_0 + a_1 - \acute{a}_0 - \acute{a}_1, b_0 + b_1 - \acute{b}_0 - \acute{b}_1) \setminus c_0 + c_1 - \acute{c}_0 - \acute{c}_1 \\ gcd(a_0 + a_1 + a_2 + \acute{a}_0 + \acute{a}_1 + \acute{a}_2, b_0 + b_1 + b_2 + \acute{b}_0 + \acute{b}_1 + \acute{b}_2) \setminus c_0 + c_1 + c_2 + \acute{c}_0 + \acute{c}_1 + \acute{c}_2 \\ gcd(a_0 + a_1 + a_2 - \acute{a}_0 - \acute{a}_1 - \acute{a}_2, b_0 + b_1 + b_2 - \acute{b}_0 - \acute{b}_1 - \acute{b}_2) \setminus c_0 + c_1 + c_2 - \acute{c}_0 - \acute{c}_1 - \acute{c}_2 \end{cases}$$

The algorithm for solution:

To solve $AX + BY = C$; A, X, B, Y, C are symbolic 2-plithogenic split-complex integers, we follow these steps:

Step (1).

We transform $AX + BY = C$ to the equivalent system of classical Diophantine equations $(I) \to (IIIV)$.

Step (2).

We check if equations $(I) \to (IIIV)$ are solvable in Z.

If there exists one equation which is not solvable, then $AX + BY = C$ I not solvable.

Step (3).

We solve the system $(I) \to (IIIV)$.

Step (4).

$$x_0 = \frac{1}{2}(t_0 + \acute{t}_0), y_0 = \frac{1}{2}(s_0 + \acute{s}_0), \acute{x}_0 = \frac{1}{2}(t_0 - \acute{t}_0), \acute{y}_0 = \frac{1}{2}(s_0 - \acute{s}_0)$$

$$x_1 = \frac{1}{2}(t_1 + \acute{t}_1) - \frac{1}{2}(t_0 + \acute{t}_0), y_1 = \frac{1}{2}(s_1 + \acute{s}_1) - \frac{1}{2}(s_0 + \acute{s}_0)$$

$$\acute{x}_1 = \frac{1}{2}(t_1 - \acute{t}_1) - \frac{1}{2}(t_0 - \acute{t}_0), \acute{y}_1 = \frac{1}{2}(s_1 - \acute{s}_1) - \frac{1}{2}(s_0 - \acute{s}_0)$$

$$x_2 = \frac{1}{2}(t_2 + \acute{t}_2) - \frac{1}{2}(t_1 + \acute{t}_1), y_2 = \frac{1}{2}(s_2 + \acute{s}_2) - \frac{1}{2}(s_1 + \acute{s}_1)$$

$$\acute{x}_2 = \frac{1}{2}(t_2 - \acute{t}_2) - \frac{1}{2}(t_1 - \acute{t}_1), \acute{y}_2 = \frac{1}{2}(s_2 - \acute{s}_2) - \frac{1}{2}(s_1 - \acute{s}_1)$$

Remark.

The available solutions are under the conditions

$t_0+\acute{t_0}, t_0-\acute{t_0}, t_1+\acute{t_1}, t_1-\acute{t_1}, t_2+\acute{t_2}, t_2-\acute{t_2} \in 2Z$

$s_0+\acute{s_0}, s_0-\acute{s_0}, s_1+\acute{s_1}, s_1-\acute{s_1}, s_2+\acute{s_2}, s_2-\acute{s_2} \in 2Z$

Example.

Take the symbolic 2-plithogenic split-complex Diophantine equation with two variable:

$$[(1+P_1-P_2)+J(2+2P_1+3P_2)]X+[(3+P_1+2P_2)+J(1-3P_1+P_2)]Y$$
$$=(11+7P_1+8P_2)+J(6+6P_1+6P_2)$$

We have:

$$\begin{cases} a_0=1, a_1=1, a_2=-1 \\ \acute{a_0}=2, \acute{a_1}=2, \acute{a_2}=3 \\ b_0=3, b_1=1, b_3=2 \\ \acute{b_0}=1, \acute{b_1}=-3, \acute{b_2}=1 \\ c_0=11, c_1=7, c_3=8 \\ \acute{c_0}=6, \acute{c_1}=6, \acute{b_2}=6 \end{cases}$$

The equivalent system is:

$$\begin{cases} 3t_0+4s_0=17 \quad (I) \\ 6t_1+2s_1=30 \quad (II) \\ 8t_2+5s_2=44 \quad (III) \\ -\acute{t_0}+2\acute{s_0}=5 \quad (IV) \\ -2\acute{t_1}+6\acute{s_1}=6 \quad (IIV) \\ -6\acute{t_2}+7\acute{s_2}=8s \quad (IIIV) \end{cases}$$

All equation $(I) \to (IIIV)$ are solvable, that is because:

$gcd(3,4)=1 \setminus 17, gcd(6,2)=2 \setminus 30, gcd(8,5)=1 \setminus 44, gcd(-1,2)=1 \setminus 5, gcd(-2,6)=2 \setminus 6, gcd(-6,7)=1 \setminus 8$

We will take one solution for each equation:

$t_0=3, s_0=2$ is a solution of (I).

$\acute{t_0}=-1, \acute{s_0}=2$ is a solution of (IV).

$t_1=5, s_1=0$ is a solution of (II).

$\acute{t_1}=-3, \acute{s_1}=2$ is a solution of (IIV).

$t_2=3, s_2=4$ is a solution of (III).

$\acute{t_2}=1, \acute{s_2}=2$ is a solution of $(IIIV)$.

$$x_0=\frac{1}{2}(t_0+\acute{t_0})=1, y_0=\frac{1}{2}(s_0+\acute{s_0})=2, \acute{x_0}=\frac{1}{2}(t_0-\acute{t_0})=2, \acute{y_0}=\frac{1}{2}(s_0-\acute{s_0})=0$$

$x_1 = \frac{1}{2}(t_1 + t_1') - \frac{1}{2}(t_0 + t_0') = 0, y_1 = \frac{1}{2}(s_1 + s_1') - \frac{1}{2}(s_0 + s_0') = -2$

$x_1' = \frac{1}{2}(t_1 - t_1') - \frac{1}{2}(t_0 - t_0') = 2, y_1' = \frac{1}{2}(s_1 - s_1') - \frac{1}{2}(s_0 - s_0') = 0$

$x_2 = \frac{1}{2}(t_2 + t_2') - \frac{1}{2}(t_1 + t_1') = 1, y_2 = \frac{1}{2}(s_2 + s_2') - \frac{1}{2}(s_1 + s_1') = 3$

$x_2' = \frac{1}{2}(t_2 - t_2') - \frac{1}{2}(t_1 - t_1') = -3, y_2' = \frac{1}{2}(s_2 - s_2') - \frac{1}{2}(s_1 - s_1') = 1$

Thus $X = (1 + P_2) + J(2 + 2P_1 - 3P_2), Y = (2 - 2P_1 + 3P_2) + J(8P_2)$ is a solution of the original equation.

Conclusion

In this paper, we have presented an effective algorithm to solve a symbolic 2-plithogenic split-complex linear Diophantine equation with two variables. Also, we have illustrated a related example to clarify the strength of the presented algorithm.

In the future, we aim to study other Diophantine equations with symbolic 2-plithogenic and 3-plithogenic split-complex linear and non-linear Diophantine equations.

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