Bi-level Linear Programming Problem with Neutrosophic Numbers

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Abstract. The paper presents a novel strategy for solving bi-level linear programming problem based on goal programming in neutrosophic numbers environment. Bi-level linear programming problem comprises of two levels namely upper or first level and lower or second level with one objective at each level. The objective function of each level decision maker and the system constraints are considered as linear functions with neutrosophic numbers of the form [p + q *I*], where p, q are real numbers and *I* represents indeterminacy. In the decision making situation, we convert neutrosophic numbers into interval numbers and the original problem transforms into bi-level interval linear programming problem. Using interval programming technique, the target interval of the objective function of each level is identified and the goal achieving function is developed. Since, the objectives of upper and lower level decision makers are generally conflicting in nature, a possible relaxation on the decision variables under the control of each level is taken into account for avoiding decision deadlock. Then, three novel goal programming models are presented in neutrosophic numbers environment. Finally, a numerical problem is solved to demonstrate the feasibility, applicability and novelty of the proposed strategy.

Keywords: Neutrosophic set, neutrosophic number, bi-level linear programming, goal programming, preference bounds.

1 Introduction

Bi-level programming [1, 2, 3, 4] consists of the objective of the upper level decision maker (UDM) at its upper or first level and that of the lower level decision maker (LDM) at the lower or second level where every decision maker (DM) independently controls a set of decision variables. Candler and Townsley [3] as well as Fortuny-Amat and McCarl [4] were credited to develop the traditional bi-level programming problem (BLPP) in crisp environment. Using Stackelberg solution concept, Anandalingam [5] proposed a new solution procedure for multi-level programming problem (MLPP) and extended the concept to decentralized BLPP (DBLPP). After the introduction of fuzzy sets by L. A. Zadeh [6], many important methodologies have been proposed for solving MLPPs, and DBLPPs such as satisfactory solution concept [7], solution procedure based on non-compensatory max-min aggregation operator [8] and compensatory fuzzy operator [9], interactive fuzzy programming [10, 11], fuzzy mathematical programming [12, 13], fuzzy goal programming (FGP) [14], etc.

Goal programming (GP) [15-21] is an significant and widely used mathematical apparatus for dealing with multi-objective mathematical programming problems with numerous and often conflicting objectives in computing optimal compromise solutions. In 1991, Inuguchi and Kume [22] introduced interval GP. GP in fuzzy setting is called fuzzy goal programming (FGP), where unity (one) is the maximum (highest) aspiration level. In 1980, Narasimhan [23] incorporated the concept of FGP by using deviational variables. Mohamed [24] established the relation between GP and FGP and applied the concept to multi-objective programming problems. After its inception, FGP received much attention to the researchers and has been applied to solve BLPPs [25, 26, 27], multi-objective BLPPs [28], multi-objective decentralized BLPPs [29, 30], MLPPs [14, 31], multi-objective MLPPs [32, 33], fractional BLPP [34], multi-objective fractional BLPPs [35-39], decentralized fractional BLPP [40], fractional MLPPs [41], quadratic BLPPs [42, 43], multi-objective quadratic BLPP [44, 45], water quality management [46], project network [47], transportation [48, 49], etc.

GP in intuitionistic fuzzy environment [50] is termed as an intuitionistic fuzzy GP (IFGP). IFGP has been employed to vector optimization [51], transportation [52], quality control [53], bi-level programming [54], multi-objective optimization problems [55-57], etc.

In 1998, Smarandache [58] incorporated a new set in mathematical philosophy called neutrosophic sets to cope with inconsistent, incomplete, indeterminate information where indeterminacy is an independent and important factor and it plays a pivotal role in decision making. In 2010, Wang et al. [59] defined single valued neutrosophic set (SVNS) by simplifying neutrosophic set for practical applications. SVNS has been widely employed to decision making problems [60-75].

Smarandache [76] incorporated the idea of neutrosophic number (NN) and proved its fundamental properties. In 2015, Smarandache [77] also defined neutrosophic interval function (thick function). Jiang and Ye [78] provided basic definition of NNs and NN functions for optimization model for solving optimal design of truss structures. Pramanik et al. [79] presented teacher selection strategy based on bidirectional projection measure in neutrosophic number environment. Mondal et al. [80] proposed score and accuracy functions of NNs for ranking. NNs. In the same study, Modal et al. [80] defined neutrosophic number harmonic mean operator (NNHMO); Neutrosophic number weighted harmonic mean operator (NNWHMO) and proved thier basic properties. Mondal et al.[80] also developed two multi-attribute group decision making (MAGDM) startegies in NN environment.

Ye [81] proposed a neutrosophic number linear programming method for solving neutrosophic number optimization. Recently, Ye et al. [82] introduced some basic operations of NNs and concepts of NN nonlinear functions and inequalities and formulated a NN- nonlinear programming method.

Pramanik and Banerjee and [83] suggested a goal programming strategy for single-objective linear programming problem involving neutrosophic coefficients where the coefficients of objective functions and the system constraints are neutrosophic numbers of the form p + q I , p, q are real numbers and I denotes indeterminacy. Pramanik and Banerjee [84] extended the concept of Pramanik and Banerjee [83] to develop goal programming strategy for multi-objective linear programming problem in neutrosophic number environment.

Research gap:

GP strategy for BLPP with neutrosophic numbers.

In order to fill the gap, we propose a novel strategy for BLPP through GP with neutrosophic numbers.

At the beginning, we convert the BLPP with neutrosophic numbersinto interval BLPP by interval programming technique. Then, the goal achieving function is developed by defining target interval of the objective function of each level. A possible relaxation on the decision variables is considered for both level DMs to find the compromise optimal solution of the bi-level system. Then, three novel GP models are developed for BLPP in indeterminate environments. Finally, a BLPP is solved to demonstrate applicability and effectiveness of the developed strategy.

The remainder of the article is organized as follows: Section 2 presents some basic concepts regarding interval numbers, neutrosophic numbers. Section 3 provides the formulation of BLPP with neutrosophic numbers. GP strategy for BLPP with neutrosophic numbers is described in section 4. A numerical example is solved in the next section to show the proposed procedure. Finally, conclusions are given in the last section.

2 Preliminaries

In this section, we present several basic discussions concerning interval numbers and neutrosophic numbers

2.1 Interval number [85]

An interval number is represented by $S = [S^L, S^U] = \{s: S^L \le s \le S^U, s \in \Re\}$, where S^L, S^U are left and right limit of the interval S on the real line $\Re$.

Definition 2.1: Suppose $m(S)$ and $w(S)$ be the midpoint and the width of an interval number, respectively.

Then, $m(S) = \frac{1}{2}[S^L + S^U]$ and $w(S) = [S^U - S^L]$

The scalar multiplication of S by α is represented as follows:

$$\alpha S = \begin{cases} [\alpha S^L, \alpha S^U], \alpha \ge 0, \\ [\alpha S^U, \alpha S^L], \alpha \le 0 \end{cases}$$

The absolute value of S is defined as follows:

$$|S| = \begin{cases} [S^L, S^U], S^L \ge 0, \\ [0, \max\{-S^L, S^U\}], S^L < 0 < S^U \\ [-S^U, -S^L], S^U \le 0 \end{cases}$$

The binary operation * between $S_1 = [S_1^L, S_1^U]$ and $S_2 = [S_2^L, S_2^U]$ is presented as given below.

$S_1 * S_2 = \{s_1 * s_2: S_1^L \le s_1 \le S_1^U, S_2^L \le s_2 \le S_2^U, s_1, s_2 \in \Re\}$.

2.2 Neutrosophic number [76]

A neutrosophic number is represented by N = p + q I, where p, q are real numbers where p is determinate part and q I is indeterminate part and $I \in [I^L, I^U]$ denotes indeterminacy.

Therefore, $N = [p + q I^L, p + q I^U] = [N^L, N^U]$, (say)

Example: Suppose a neutrosophic number N = 1+ 2I, where 1 is determinate part and 2 I is indeterminate part. Here, we consider $I \in [0.3, 0.5]$. Then, N becomes an interval number of the form N = [1.6, 2].

Now, we present some properties of neutrosophic numbers as follows:

Consider, $N_1 = [p_1 + q_1 I_1] = [p_1 + q_1 I_1^L, p_1 + q_1 I_1^U] = [N_1^L, N_1^U]$ and $N_2 = [p_2 + q_2 I_2] = [p_2 + q_2 I_2^L, p_2 + q_2 I_2^U] = [N_2^L, N_2^U]$ be two neutrosophic numberswhere $I_1 \in [I_1^L, I_1^U]$, $I_2 \in [I_2^L, I_2^U]$, then

(i). $N_1 + N_2 = [N_1^L + N_2^L, N_1^U + N_2^U]$,

(ii). $N_1 - N_2 = [N_1^L - N_2^U, N_1^U - N_2^L]$,

(iii). $N_1 \times N_2$ = [Min $\{N_1^L \times N_2^L, N_1^L \times N_2^U, N_1^U \times N_2^L, N_1^U \times N_2^U\}$, Max $\{N_1^L \times N_2^L, N_1^L \times N_2^U, N_1^U \times N_2^L, N_1^U \times N_2^U\}$]

(iv). N_1 / N_2 = [Min $\{N_1^L / N_2^L, N_1^L / N_2^U, N_1^U / N_2^L, N_1^U / N_2^U\}$, Max $\{N_1^L / N_2^L, N_1^L / N_2^U, N_1^U / N_2^L, N_1^U / N_2^U\}$], if $0 \notin N_2$..

3 Formulation of BLPP for minimization-type objective function with neutrosophic numbers

We consider a BLPP for minimization-type objective function at each level. Mathematically, a BLPP with neutrosophic numbers can be presented as follows:

UDM: $\underset{x_1}{Min} f_1(x) = [C_{11} + D_{11} I_{11}] x_1 + [C_{12} + D_{12} I_{12}] x_2 + [E_1 + F_1 I_{13}]$ (1)

LDM: $\underset{x_2}{Min} f_2(x) = [C_{21} + D_{21} I_{21}] x_1 + [C_{22} + D_{22} I_{22}] x_2 + [E_2 + F_2 I_{23}]$ (2)

Subject to

$x \in X = \{x = (x_1, x_2) \in R^N \,|[A_1 + B_1 I_1] x_1 + [A_2 + B_2 I_2] x_2 \le \mu + \nu I_3, x \ge 0\}$. (3)

Here, $x_1 = (x_{11}, x_{12}, ..., x_{1N_1})^T$: Decision vector under the control of UDM,

$x_2 = (x_{21}, x_{22}, ..., x_{2N_2})^T$: Decision vector under the control of LDM

C_{i1}, D_{i1} (i = 1, 2) are N_1- dimension row vectors; C_{i2}, D_{i2} (i = 1, 2) are N_2 - dimension row vectors where N = $N_1 + N_2$; and E_i, F_i (i = 1, 2) are constants. A_i, B_i (i = 1, 2) are $M \times N_i$ (i = 1, 2) constant matrix and μ, ν are M dimensional constant column matrix. X ($\ne \Phi$) is considered compact and convex in R^N. Also, we have $I_{ij} \in [I_{ij}^L, I_{ij}^U]$, i = 1, 2, 3; j = 1, 2 and $I_i \in [I_i^L, I_i^U]$, i = 1, 2, 3. Representation of a BLPP is shown in Fig. 1.

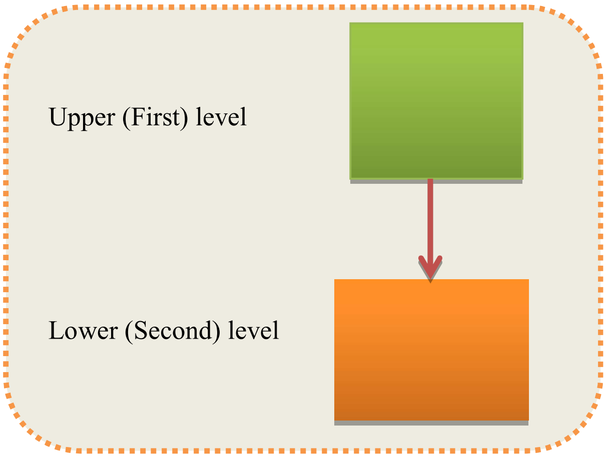

Fig. 1. Depiction of a BLPP

4 Goal programming formulation for BLPP with neutrosophic numbers

The objective functions of both level DMs of the problem defined in section 3 can be written as:

UDM:

$\underset{x_1}{Min} f_1(x) = [C_{11} + D_{11}I_{11}]\, x_1 + [C_{12} + D_{12}\, I_{12}]\, x_2 + [E_1 + F_1 I_{13}] = \{[C_{11} + D_{11}\, I_{11}^L]\, x_1 + [C_{12} + D_{12}\, I_{12}^L]\, x_2 + [E_1 + F_1\, I_{13}^L],\, [C_{11} + D_{11}\, I_{11}^U]\, x_1 + [C_{12} + D_{12}\, I_{12}^U]\, x_2 + [E_1 + F_1\, I_{13}^U]\} = [Y_1^L(x), Y_1^U(x)]$ (say); (4)

LDM:

$\underset{x_2}{Min} f_2(x) = [C_{21} + D_{21}\, I_{21}]\, x_1 + [C_{22} + D_{22}\, I_{22}]\, x_2 + [E_2 + F_2 I_{23}] = \{[C_{21} + D_{21}\, I_{21}^L]\, x_1 + [C_{22} + D_{22}\, I_{22}^L]\, x_2 + [E_2 + F_2\, I_{23}^L],\, [C_{21} + D_{21}\, I_{21}^U]\, x_1 + [C_{22} + D_{22}\, I_{22}^U]\, x_2 + [E_2 + F_2\, I_{23}^U]\} = [Y_2^L(x), Y_2^U(x)]$ (say); (5)

and the system constrains reduce to

$[A_1 + B_1\, I_1]\, x_1 + [A_2 + B_2\, I_2]\, x_2 \geq \mu + \nu\, I_3$

$\Rightarrow \{[A_1 + B_1\, I_1^L]\, x_1 + [A_2 + B_2\, I_2^L]\, x_2,\, [A_1 + B_1\, I_1^U]\, x_1 + [A_2 + B_2\, I_2^U]\, x_2\} \geq [\mu + \nu\, I_3^L, \mu + \nu\, I_3^U] = [g^L, g^U]$ (say)

$\Rightarrow [Z^L(x), Z^U(x)] \geq [g^L, g^U]$. (6)

Proposition 6 1. [86]

Suppose $\sum_{j=1}^{n} [e_1^j, e_2^j] z_j \geq [f_1, f_2]$, then $\sum_{j=1}^{n} [e_2^j] z_j \geq f_1$, $\sum_{j=1}^{n} [e_1^j] z_j \geq f_2$ are the maximum and minimum value range inequalities for the constraint condition, respectively.

Now, from the proposition 1 due to Shaocheng [86], the interval inequality of the system constraints (6) reduce to the following inequalities as given below.

$[A_1 + B_1\, I_1^L]\, x_1 + [A_2 + B_2\, I_2^L]\, x_2 \geq g^U$, $[A_1 + B_1\, I_1^U]\, x_1 + [A_2 + B_2\, I_2^U]\, x_2 \geq g^L$, $x_i \geq 0$, i = 1, 2,

i.e. $Z^L(x) \geq g^U$, $Z^U(x) \geq g^L$, $x \geq 0$.

The minimization-type BLPP can be re-stated as follows:

UDM: $\underset{x_1}{Min} f_1(x) = [Y_1^L(x), Y_1^U(x)]$,

LDM: $\underset{x_2}{Min} f_2(x) = [Y_2^L(x), Y_2^U(x)]$

Subject to

$[Z^L(x), Z^U(x)] \geq [g^L, g^U]$, $x \geq 0$.

For obtaining the best optimal solution of f_i, (i = 1, 2), we solve the following problem due to Ramadan [87] as follows:

$\underset{x \in X}{Min} f_i(x) = Y_i^L(x)$, i = 1, 2

$Z^U(x) \geq g^L$, $x \geq 0$, i = 1, 2.

Suppose $x_i^b = (x_{i1}^b, x_{i2}^b, ..., x_{iN_1}^b, x_{iN_{1+1}}^b, ..., x_{iN}^b)$, (i = 1, 2) be the individual best solution of i-th level DM subject to the given constraints and $Y_i^L(x_i^b)$, (i = 1, 2) be the individual best objective value of i-th level DM.

Now for determining the worst optimal solution of f_i, (i = 1, 2), we solve the following problem due to Ramadan [85] as given below.

$\underset{x \in X}{Min} f_i(x) = Y_i^U(x)$, i = 1, 2

$Z^L(x) \geq g^U$, $x \geq 0$.

Let $x_i^w = (\mathrm{x}_{i1}^w, \mathrm{x}_{i2}^w, ..., \mathrm{x}_{iN_1}^w, \mathrm{x}_{iN_{1+1}}^w, ..., \mathrm{x}_{iN}^w)$, (i = 1, 2) be the individual best solution of i-th level DM subject to the given constraints and $Y_i^U(x_i^w)$, (i = 1, 2) be the individual best objective value of i-th level DM.

Therefore, $[Y_i^L(x_i^b), Y_i^U(x_i^w)]$ be the optimal value of i-th level DM in the interval form.

Suppose that $[Y_i^*, Y_i^+]$ be the target interval of i-th objective functions set by level DMs.

Now the target level of i-th objective function can be written as follows:

$Y_i^U(x) \geq Y_i^*$, (i = 1, 2)

$Y_i^L(x) \leq Y_i^+$, (i = 1, 2).

Hence, the goal achievement functions are presented in the following form:

$-Y_i^U(x) + D_i^U = -Y_i^*$, (i = 1, 2)

$Y_i^L(x) + D_i^L = Y_i^+$, (i = 1, 2)

where D_i^U, D_i^L, (i = 1, 2) are deviational variables.

However, since the individual best solutions of the level DMs are not same, cooperation between the two level DMs is necessary to arrive at a compromise optimal solution. For more details see [27, 30, 31, 36, 37, 42, 44, 45, 55, 88].

Let, $x_i^b = (\mathrm{x}_{i1}^b, \mathrm{x}_{i2}^b, ..., \mathrm{x}_{iN_i}^b, \mathrm{x}_{iN_i+1}^b, ..., \mathrm{x}_{iN}^b)$, (i = 1, 2) be the individual best solution of i-th level DM. Suppose $(\mathrm{x}_{1i}^b - l_{1i})$ and $(\mathrm{x}_{1i}^b + u_{1i})$, (i = 1, 2, ..., N_1) be the lower and upper bounds of decision vector provided by UDM where l_{1i} and u_{1i}, (i = 1, 2, ..., N_1) are the negative and positive tolerance variables which are not essentially same. Also, suppose that $(\mathrm{x}_{2i}^b - l_{2i})$ and $(\mathrm{x}_{2i}^b + u_{2i})$, (i = 1, 2, ..., N_2) be the lower and upper bounds of decision vector provided by LDM where l_{2i} and u_{2i}, (i = 1, 2, ..., N_2) are the negative and positive tolerance variables which are not same in general. Therefore, we can write

$(\mathrm{x}_{1i}^b - l_{1i}) \leq \mathrm{x}_{1i} \leq (\mathrm{x}_{1i}^b + u_{1i})$, (i = 1, 2, ..., N_1)

$(\mathrm{x}_{2i}^b - l_{2i}) \leq \mathrm{x}_{2i} \leq (\mathrm{x}_{2i}^b + u_{2i})$, (i = 1, 2, ..., N_2)

Finally, we develop three new GP models (see the flowchart of GP model in Fig.2) for solving BLPP with neutrosophic numbers as follows:

GP Model I.

Min $\sum_{i=1}^{2} (D_i^U + D_i^L)$

Subject to

$-Y_i^U(x) + D_i^U = -Y_i^*$, (i = 1, 2)

$Y_i^L(x) + D_i^L = Y_i^+$, (i = 1, 2)

$Z^L(x) \geq g^U$, $Z^U(x) \geq g^L$,

$(\mathrm{x}_{1i}^b - l_{1i}) \leq \mathrm{x}_{1i} \leq (\mathrm{x}_{1i}^b + u_{1i})$, (i = 1, 2, ..., N_1)

$(\mathrm{x}_{2i}^b - l_{2i}) \leq \mathrm{x}_{2i} \leq (\mathrm{x}_{2i}^b + u_{2i})$, (i = 1, 2, ..., N_2)

$D_i^L, D_i^U, x \geq 0$, (i = 1, 2).

GP Model II.

Min $\sum_{i=1}^{2} (w_i^U D_i^U + w_i^L D_i^L)$

Subject to

$-Y_i^U(x) + D_i^U = -Y_i^*$, (i = 1, 2)

$Y_i^L(x) + D_i^L = Y_i^+$, (i = 1, 2)

$Z^L(x) \geq g^U$, $Z^U(x) \geq g^L$,

$(\mathrm{x}_{1i}^b - l_{1i}) \leq \mathrm{x}_{1i} \leq (\mathrm{x}_{1i}^b + u_{1i})$, (i = 1, 2, ..., N_1)

$(\mathrm{x}_{2i}^b - l_{2i}) \leq \mathrm{x}_{2i} \leq (\mathrm{x}_{2i}^b + u_{2i})$, (i = 1, 2, ..., N_2)

$w_i^U \geq 0$, $w_i^L \geq 0$, (i = 1, 2), $D_i^L, D_i^U, x \geq 0$, (i = 1, 2).

Here, w_i^U and w_i^L are the negative deviational variables.

GP Model III.

Min α

Subject to

$- Y_i^U(x) + D_i^U = - Y_i^*$, (i = 1, 2)

$Y_i^L(x) + D_i^L = Y_i^+$, (i = 1, 2)

$Z^L(x) \geq g^U$, $Z^U(x) \geq g^L$,

$(x_{1i}^b - l_{1i}) \leq x_{1i} \leq (x_{1i}^b + u_{1i})$, (i = 1, 2, ..., N_1)

$(x_{2i}^b - l_{2i}) \leq x_{2i} \leq (x_{2i}^b + u_{2i})$, (i = 1, 2, ..., N_2)

$\alpha \geq D_i^U$, $\alpha \geq D_i^L$, (i = 1, 2), D_i^L, D_i^U, $x \geq 0$, (i = 1, 2).

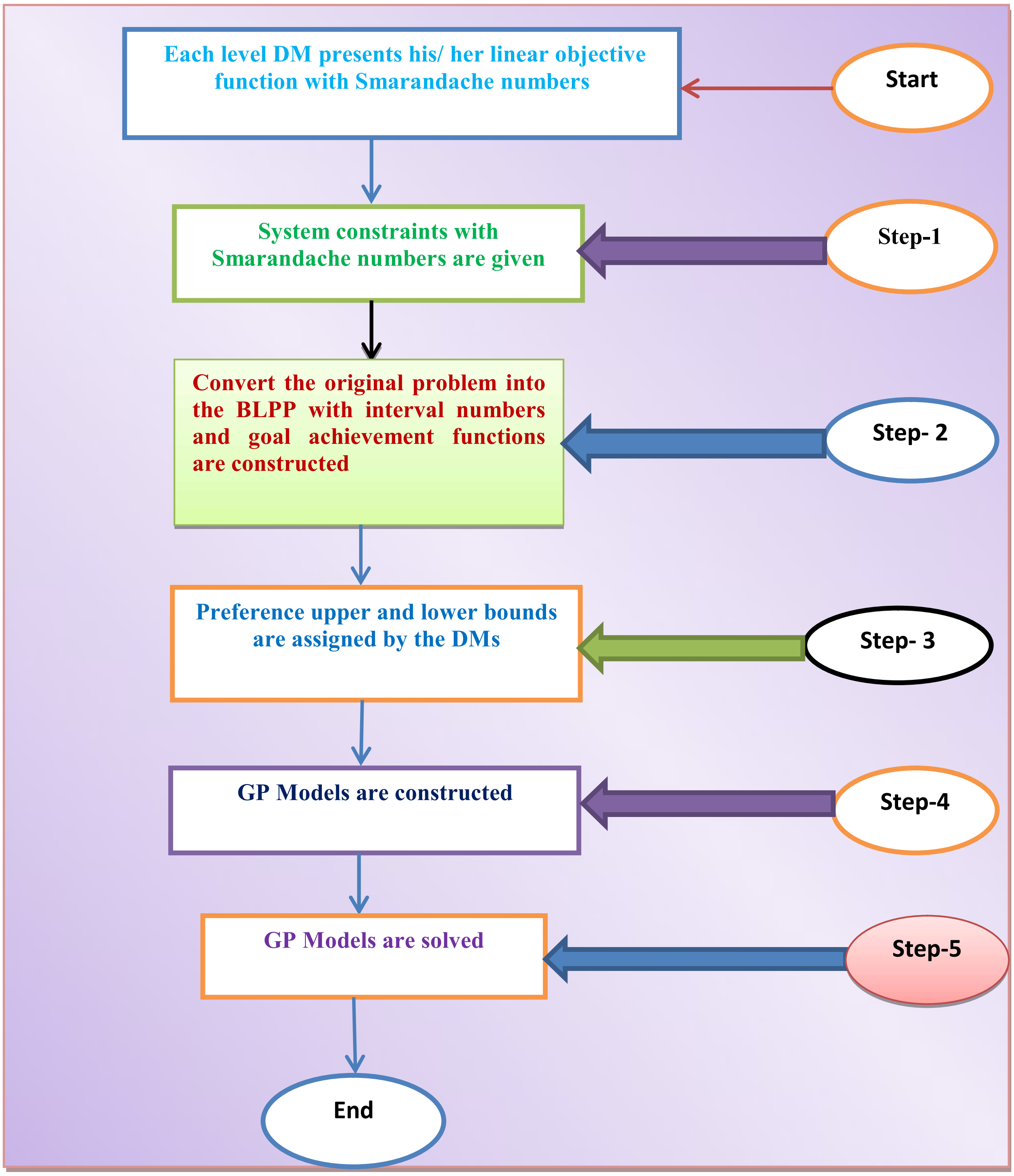

Fig. 2. Flowchart of the GP strategy for BLPP

5 Numerical Example

Consider the following BLPP with neutrosophic numbers to show the efficiency of the proposed strategy. We consider $I \in [0, 1]$.

UDM: $\underset{x_1}{Min} f_1(x) = [1 + 2I]\, x_1 + [4 + 5I]\, x_2 + [1 + 2I]$,

LDM: $\underset{x_1}{Min} f_1(x) = [3 + 4I]\, x_1 + [2 + 3I]\, x_2 + [3 + 2I]$,

Subject to

$[4 + 2I]\, x_1 + [3 + 7I]\, x_2 \geq [15 + 10I]$,

$[6 + I]\, x_1 + [-2 + 4I]\, x_2 \geq [5 + 3I]$,

$x_1, x_2 \geq 0$.

The transformed problem of UDM is shown Table 1.

Table 1. UDM's problem for best and worst solutions

UDM's problem to find best solution	UDM's problem to find worst solution
Min $Y_1^L(x) = x_1 + 4x_2 + 1$ Subject to $6x_1 + 10x_2 \geq 15$, $7x_1 + 2x_2 \geq 5$, $x_1, x_2 \geq 0$.	Min $Y_1^U(x) = 3x_1 + 9x_2 + 3$ Subject to $4x_1 + 3x_2 \geq 25$, $6x_1 - 2x_2 \geq 8$, $x_1, x_2 \geq 0$.

The best and worst solutions of UDM are computed as given below (see Table 2)

Table 2. UDM's best and worst solutions

The best solution of UDM	The worst solution of UDM
$Y_1^* = 3.5$ at (2.5, 0)	$Y_1^+ = 21.75$ at (6.25, 0)

The transformed problem of LDM can be presented as follows (see Table 3).

Table 3. LDM's problem for best and worst solutions

LDM's problem to find best solution	LDM's problem to find worst solution
Min $Y_2^L(x) = 3x_1 + 2x_2 + 3$ Subject to $6x_1 + 10x_2 \geq 15$, $7x_1 + 2x_2 \geq 5$, $x_1, x_2 \geq 0$.	Min $Y_2^U(x) = 7x_1 + 5x_2 + 5$ Subject to $4x_1 + 3x_2 \geq 25$, $6x_1 - 2x_2 \geq 8$, $x_1, x_2 \geq 0$.

The best and worst solutions of LDM are determined as given below (see Table 4)

Table 4. LDM's best and worst solutions

The best solution of LDM	The worst solution of LDM
$Y_2^* = 6.621$ at (0.345, 1.293)	$Y_2^+ = 47.615$ at (2.846, 4.538)

The objective function of UDM with specified targets can be presented as given below.

$x_1 + 4x_2 + 1 \leq 21.5$, $3x_1 + 9x_2 + 3 \geq 4$,

The goal achievement functions of UDM with specified targets can be presented as

$x_1 + 4x_2 + 1 + D_1^L = 21.5$, $-3x_1 - 9x_2 - 3 + D_1^U = -4$,

The objective function of LDM with specified targets can be presented as given below.

$3x_1 + 2x_2 + 3 \leq 47$, $7x_1 + 5x_2 + 5 \geq 7$,

Also, the goal achievement functions of LDM with specified targets can be written as follows:

$3x_1 + 2x_2 + 3 + D_2^L = 47$, $-7x_1 - 5x_2 - 5 + D_2^U = -7$.

Suppose, the UDM provides preference bounds on the decision variable x_1 as 2.5 - 1.5 $\le x_1 \le$ 2.5 + 2 and the LDM offers preference bounds on the decision variable x_2 as 1.293 – 0.793 $\le x_2 \le$ 1.293 + 1.207 to reach optimal compromise solution.

Therefore, the GP models are developed as given below.

GP Model I.

Min ($D_1^L + D_1^U + D_2^L + D_2^U$)

Subject to

$x_1 + 4x_2 + 1 + D_1^L = 21.5,$

$-3x_1 - 9x_2 - 3 + D_1^U = -4,$

$3x_1 + 2x_2 + 3 + D_2^L = 47,$

$-7x_1 - 5x_2 - 5 + D_2^U = -7,$

$6x_1 + 10x_2 \ge 15,$

$7x_1 + 2x_2 \ge 5,$

$4x_1 + 3x_2 \ge 25,$

$6x_1 - 2x_2 \ge 8,$

$2.5 - 1.5 \le x_1 \le 2.5 + 2,$

$1.293 - 0.793 \le x_2 \le 1.293 + 1.207,$

$D_i^L, D_i^U \ge 0$, (i = 1, 2)

$x_1, x_2 \ge 0.$

GP Model II.

Min $\frac{1}{4}$ ($D_1^L + D_1^U + D_2^L + D_2^U$)

Subject to

$x_1 + 4x_2 + 1 + D_1^L = 21.5,$

$-3x_1 - 9x_2 - 3 + D_1^U = -4,$

$3x_1 + 2x_2 + 3 + D_2^L = 47,$

$-7x_1 - 5x_2 - 5 + D_2^U = -7,$

$6x_1 + 10x_2 \ge 15,$

$7x_1 + 2x_2 \ge 5,$

$4x_1 + 3x_2 \ge 25,$

$6x_1 - 2x_2 \ge 8,$

$2.5 - 1.5 \le x_1 \le 2.5 + 2,$

$1.293 - 0.793 \le x_2 \le 1.293 + 1.207,$

$x_1, x_2 \ge 0.$

GP Model III.

Min α

Subject to

$x_1 + 4x_2 + 1 + D_1^L = 21.5,$

$-3x_1 - 9x_2 - 3 + D_1^U = -4,$

$3x_1 + 2x_2 + 3 + D_2^L = 47,$

$-7x_1 - 5x_2 - 5 + D_2^U = -7,$

$6x_1 + 10x_2 \ge 15,$

$7x_1 + 2x_2 \ge 5,$

$4x_1 + 3x_2 \ge 25,$

$6x_1 - 2x_2 \ge 8,$

$2.5 - 1.5 \le x_1 \le 2.5 + 2,$

$1.293 - 0.793 \le x_2 \le 1.293 + 1.207,$

$\alpha \ge D_i^L, \alpha \ge D_i^U$, (i = 1, 2)

$D_i^L, D_i^U \geq 0$, (i = 1, 2)
$x_1, x_2 \geq 0.$

The solutions of the proposed GP models are shown in the Table 5 as given below.

Table 5. The solutions of the BLPP

	Solution point	Objective values of UDM	Objective values of LDM
GP Model I	(4.5, 2.333)	[14.832, 37.497]	[21.166, 37.497]
GP Model II	(4.5, 2.333)	[14.832, 37.497]	[21.166, 37.497]
GP Model III	(4.375, 2.5)	[15.375, 38.625]	[21.125, 48.125]

Conclusion

The paper presented three new goal programming models for bi-level linear programming problem where the objective functions of both level decision makers and the system constraints are linear functions with neutrosophic numbers. Using interval programming technique, we transform the bi-level linear programming problem into interval programming problem and calculated the best and the worst solutions for both level decision makers. Both decision makers assign preference upper and lower bounds on the decision variables under their control to obtain optimal compromise solution of the hierarchical organization. Finally, a new goal programming strategy has been developed to solve bi-level linear programming problem by minimizing deviational variables. We obtain the optimal compromise solution of the system in interval form which is more realistic. A numerical problem involving neutrosophic numbersis is solved to demonstrate the applicability and efficiency of the proposed procedure.

We hope that the bi-level linear programming technique in neutrosophic number environment will open up a new avenue of research for future neutrosophic researchers. Furthermore, we believe that the proposed strategy can be effective for dealing with multi-objective bi-level linear programming, multi-objective decentralized bi-level linear programming, multi-objective decentralized multi-level linear programming, priority based multi-objective linear programming problems, real world decision making problems such as agriculture, bio-fuel production, portfolio selection, transportation, etc. with neutrosophic numbers information.

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