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Distance Measure Based MADM Strategy with Interval Trapezoidal Neutrosophic Numbers

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Abstract. In this paper, we introduce interval trapezoidal neutrosophic number and define some arithmetic operations of the proposed interval trapezoidal neutrosophic numbers. Then we consider a multiple attribute decision making (MADM) problem with interval trapezoidal neutrosophic numbers. The weight information of each attribute in the multi attribute decision making problem is expressed in terms of interval trapezoidal neutrosophic numbers. To develop distance measure based MADM strategy with interval trapezoidal neutrosophic numbers, we define normalised Hamming distance measure of the proposed numbers and develop an algorithm to determine the weight of the attributes. Using these weights, we aggregate the distance measures of preference values of each alternative with respect to ideal alternative. Then we determine the ranking order of all alternatives according to the aggregated weighted distance measures of all available alternatives. Finally, we provide an illustrative example to show the feasibility, applicability of the proposed MADM strategy with interval trapezoidal neutrosophic numbers.

Keywords: Interval trapezoidal neutrosophic number, Hamming distance measure, entropy, multi-attribute decision making.

1 Introduction

Neutrosophic set theory, pioneered by Smarandache [1], is an important tool for dealing with imprecise, incomplete, indeterminate, and inconsistent information occurred in decision making process. Neutrosophic set has three independent components: truth membership degree, indeterminate membership degree, and falsity membership degree lying in a non-standard unit interval $]^{-}0,1^{+}[$. Wang et al. [2] introduced single valued neutrosophic set which has three membership degrees and the value of each membership degree lies in [0,1]. Wang et al. [3] proposed interval neutrosophic set (INS) in which the values of its truth membership degree, indeterminacy membership degree, and falsity membership degree are intervals rather than crisp numbers. Therefore, INSs allow us flexibility in presenting neutrosophic information existing in modern decision making problem. Recently, many researchers have shown interest on possible application of INSs in the field of multi-attribute decision making (MADM) and multi-attribute group decision making (MAGDM).
Chi and Liu [4] extended technique for order preference by similarity to ideal solution (TOPSIS) strategy for MADM with INSs. Pramanik and Mondal [5] combined grey relational analysis (GRA) with MADM strategy for interval neutrosophic information and presented a novel MADM strategy in interval neutrosophic environment. Dey et al. [6] defined weighted projection measure and developed an MADM strategy for interval neutrosophic information. In the same study, Dey et al. [6] also developed an alternative strategy to solve MADM problems based on the combination of angle cosine and projection measure. Ye [7] defined some similarity measures of INSs and employed these measures in multi-criteria decision making (MCDM) problem. Pramanik et al. [8] proposed hybrid vector similarity measures of single valued neutrosophic sets as well as interval neutrosophic sets and developed two MADM strategies to solve MADM problems. Peng et al. [9] developed some aggregation operators of simplified neutrosophic sets to solve multi-criteria group decision making problem. Dey et al. [10] extended grey relational analysis strategy for solving weaver selection problem in interval neutrosophic environment. Zhang et al. [11] proposed an outranking strategy for MCDM problem with neutrosophic sets. Dalapati et al. [12] proposed cross entropy measure of INSs and employed the measure in solving MADGM problem.

However, the domain of single valued and interval neutrosophic set considered is a discrete set. Ye [13], and Şubaş [14] introduced single valued trapezoidal neutrosophic number (SVTrNN), where each element is expressed by trapezoidal numbers that has a truth membership degree, an indeterminate membership degree, and a falsity membership degree. Biswas et al. [15] also introduced SVTrNN in which each membership degree is charactrized by normalized trapezoidal fuzzy number. In the same study, Biswas et al. [15] proposed value and ambiguity based ranking strategyand applied this strategy to MADM problem. Deli and Şubaş [16] also proposed a ranking strategy of single valued neutrosophic number and utilized this strategy in MADM problems. Deli and Şubaş [17] developed some weighted geometric operators of triangular neutrosophic numbers to solve MADM problem. Ye [13] proposed two weighted aggregation operators of trapezoidal neutrosophic numbers and applied them to MADM problem. Liu and Zhang [18] presented some Maclaurin symmetric mean operators for single-valued trapezoidal neutrosophic numbers and discussed their applications to group decision making. Liang et al. [19] utilized preference relationon to solve MCDM strategy with SVTrNN. Basset et al. [20] intregated the analytical heirarchy process into Delphi framework based group decision making model with trapezoidal neutrosophic numbers.

However due to complexity of decision making problem, decision makers may face difficulties to express their opinion with the single valued truth membership degree, indeterminacy membership degree, and the falsity membership in neutrosophic environment. Then, it is easy to express their opinion in terms of three membership degrees with an interval number rather than exact real number. Therefore, we have an opportunity to investigate a trapezoidal neutrosophic number that has a three membership degrees represented in interval form. We call this new number as interval trapezoidal neutrosophic number (ITrNN). The proposed number permits us to deal with more neutrosophic information than SVTrNN. Hence, we need to develop some decision making strategies with the ITrNNs. At present no studies have been reported in the literature for MADM with ITrNNs.

The main objectives of the study are:

- To introduce ITrNN and present some of its operational rules.
- To define normalized Hamming distance measure of ITrNNs.
- To develop a novel strategy for solving MADM problem with ITrNNs.

The remainder of the paper is outlined as follows: Section 2reviews some basics on single valued neutrosophic sets, interval neutrosophic sets. Section 3 introduces ITrNNs and defines some arithmetical operations. Section 4 presents a novel strategyfor solving MADM with interval trapezoidal neutrosophic numbers. Section 5 provides an illustrative example to illustrate the proposed strategy. Finally, Section 6 draws some concluding remarks with future research directions.

2 Preliminaries

Definition 1. [2] Assume that$X$ be a universe of discourse. A single-valued neutrosophic set A in X is given by

$$A = \{x, < T_A(x), I_A(x), F_A(x) > | \, x \in X\} \qquad (1)$$

where $T_A(x): X \to [0,1]$, $I_A(x): X \to [0,1]$ and $F_A(x): X \to [0,1]$, with the condition

$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ for all $x \in X$.

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ represent, respectively, the truth membership function, the indeterminacy function and the falsity membership function of the element x to the set A.

Definition 2. [3] Let X be a universe of discourse and $D[0,1]$ be the set of all closed sub-intervals. An interval neutrosophic set $\tilde{A}$ in X is given by

$$\tilde{A} = \{x, < T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x) > | \, x \in X\} \qquad (2)$$

where $\tilde{T}_{\tilde{A}}(x): X \to D[0,1], \tilde{I}_{\tilde{A}}(x): X \to D[0,1]$ and $\tilde{F}_{\tilde{A}}(x): X \to D[0,1]$, with the condition

$0 \le sup\tilde{T}_{\tilde{A}}(x) + sup\tilde{I}_{\tilde{A}}(x) + sup\tilde{F}_{\tilde{A}}(x) \le 3$ for all $x \in X$.

The intervals $\tilde{T}_{\tilde{A}}(x)$, $\tilde{I}_{\tilde{A}}(x)$ and $\tilde{F}_{\tilde{A}}(x)$ represent the truth membership degree, the indeterminacy membership degree and the falsity membership degree of the element x to the set $\tilde{A}$, respectively.

3. Interval trapezoidal neutrosophic numbers (ITrNNs)

In this section, we present the concept of interval trapezoidal neutrosophic number and define its basic operations.

Definition 3. Let $\tilde{a}$ is a trapezoidal neutrosophic number in the set of real numbers, its truth membership function is

$$T_{\tilde{a}}(x) = \begin{cases} \dfrac{(x-a)t_{\tilde{a}}}{b-a} & a \le x < b; \\ t_{\tilde{a}} & b \le x \le c; \\ \dfrac{(d-x)t_{\tilde{a}}}{d-c} & c < x \le d; \\ 0 & otherwise, \end{cases}$$

Its indeterminacy membership function is

$$I_{\tilde{a}}(x)=\begin{cases}\dfrac{b-x+(x-a)i_{\tilde{a}}}{b-a} & a\le x<b;\\ i_{\tilde{a}} & b\le x\le c;\\ \dfrac{x-c+(d-x)i_{\tilde{a}}}{d-c} & c<x\le d;\\ 0 & otherwise,\end{cases}$$

and its falsity membership function is

$$F_{\tilde{a}}(x)=\begin{cases}\dfrac{b-x+(x-a)f_{\tilde{a}}}{b-a} & a\le x<b;\\ f_{\tilde{a}} & b\le x\le c;\\ \dfrac{x-c+(d-x)f_{\tilde{a}}}{d-c} & c<x\le d;\\ 0 & otherwise,\end{cases}$$

where $t_{\tilde{a}} \subset [0,1]$, $i_{\tilde{a}} \subset [0,1]$, and $f_{\tilde{a}} \subset [0,1]$ are interval numbers and $0 \le \sup(t_{\tilde{a}}) + \sup(i_{\tilde{a}}) + \sup(f_{\tilde{a}}) \le 3$.

Then $\tilde{a}$ is called an interval trapezoidal neutrosophic number and it is denoted by $\tilde{a} = ([a,\, b,\, c,\, d]; t_{\tilde{a}}, i_{\tilde{a}}, f_{\tilde{a}})$. For convenience we can take $t_{\tilde{a}} = [\underline{t},\, \overline{t}]$, $i_{\tilde{a}} = [\underline{i},\, \overline{i}]$, and $f_{\tilde{a}} = [\underline{f},\, \overline{f}]$. Then the number $\tilde{a}$ can be denoted by $\tilde{a} = ([a,\, b,\, c,\, d]; [\underline{t},\, \overline{t}],\, [\underline{i},\, \overline{i}], [\underline{f},\, \overline{f}])$.

Definition 4. Let $\tilde{a} = ([a,\, b,\, c,\, d]; [\underline{t},\, \overline{t}],\, [\underline{i},\, \overline{i}], [\underline{f},\, \overline{f}])$ be an ITrNN. If $a \ge 0$ and one of the four values $of\ a,\, b,\, c$ and d is not equal to zero, then the ITrNN $\tilde{a}$ is called positive ITrNN.

3.1. Some arithmetic operations on ITrNNs

Definition 5. Let $\tilde{a}_1 = ([a_1,\, b_1,\, c_1,\, d_1]; [\underline{t_1},\, \overline{t_1}],\, [\underline{i_1},\, \overline{i_1}], [\underline{f_1},\, \overline{f_1}])$ and

$\tilde{a}_2 = ([a_2,\, b_2,\, c_2,\, d_2]; [\underline{t_2},\, \overline{t_2}],\, [\underline{i_2},\, \overline{i_2}], [\underline{f_2},\, \overline{f_2}])$ be two INTrNs and $\lambda \ge 0$. Then the following operations are valid.

1. $$\tilde{a}_1 \oplus \tilde{a}_2 = \begin{pmatrix}[a_1+a_2,\, b_1+b_2,\, c_1+c_2,\, d_1+d_2];\\ \left[\underline{t_1}+\underline{t_2}-\underline{t_1}\,\underline{t_2},\ \overline{t_1}+\overline{t_2}-\overline{t_1}\,\overline{t_2}\right],\\ \left[\underline{i_1}\,\underline{i_2},\ \overline{i_1}\,\overline{i_2}\right],\ \left[\underline{f_1}\,\underline{f_2},\ \overline{f_1}\,\overline{f_2}\right]\end{pmatrix}$$

2. $$\tilde{a}_1 \otimes \tilde{a}_2 = \begin{pmatrix}[a_1+a_2,\, b_1+b_2,\, c_1+c_2,\, d_1+d_2]; \left[\underline{t_1}\,\underline{t_2},\ \overline{t_1}\,\overline{t_2}\right],\\ \left[\underline{i_1}+\underline{i_2}-\underline{i_1}\,\underline{i_2},\ \overline{i_1}+\overline{i_2}-\overline{i}\,\overline{i_2}\right],\\ \left[\underline{f_1}+\underline{f_2}-\underline{f_1}\,\underline{f_2},\ \overline{f_1}+\overline{f_2}-\overline{f_1}\,\overline{f_2}\right]\end{pmatrix};$$

3. $$\lambda\,\tilde{a}_1 = \begin{pmatrix}[\lambda\, a_1,\, \lambda\, b_1,\, \lambda\, c_1,\, \lambda\, d_1];\\ \left[1-\left(1-\underline{t_1}\right)^{\lambda},\, 1-(1-\overline{t_1})^{\lambda}\right],\\ \left[\left(\underline{i_1}\right)^{\lambda},\, \left(\overline{i_1}\right)^{\lambda}\right],\ \left[\left(\underline{f_1}\right)^{\lambda},\, \left(\overline{f_1}\right)^{\lambda}\right]\end{pmatrix}, \lambda > 0$$

4. $$(\tilde{a}_1)^{\lambda} = \begin{pmatrix}[a_1^{\lambda}, b_1^{\lambda}, c_1^{\lambda}, d_1^{\lambda}]; \left[\left(\underline{t_1}\right)^{\lambda},\, \left(\overline{t_1}\right)^{\lambda}\right],\\ \left[1-\left(1-\underline{i_1}\right)^{\lambda},\, 1-\left(1-\overline{i_1}\right)^{\lambda}\right],\\ \left[1-\left(1-\underline{f_1}\right)^{\lambda},\, 1-\left(1-\overline{f_1}\right)^{\lambda}\right]\end{pmatrix}, \lambda > 0.$$

Definition 6. The ideal choice of interval neutrosophic trapezoidal number is

$$\tilde{I}^{+} = ([1,\, 1,\, 1,\, 1]; [1,\, 1],\, [0,0], [0,0]). \tag{3}$$

3.2. Hamming distance between two ITrNNs.

Let $\tilde{a}_1 = ([a_1,\, b_1,\, c_1,\, d_1]; [\underline{t_1},\, \overline{t_1}],\, [\underline{i_1},\, \overline{i_1}], [\underline{f_1},\, \overline{f_1}])$ and $\tilde{a}_2 = ([a_2,\, b_2,\, c_2,\, d_2]; [\underline{t_2},\, \overline{t_2}],\, [\underline{i_2},\, \overline{i_2}], [\underline{f_2},\, \overline{f_2}])$ be any two INTrNs, then the normalized Hamming distance between $\tilde{a}_1$ and $\tilde{a}_2$ is defined as follows:

$d(\tilde{a}_1,\, \tilde{a}_2) =$

$$\frac{1}{24}\left(\begin{array}{l}\left|\begin{array}{l}a_1\left(2+\underline{t_1}-\underline{i_1}-\underline{f_1}\right)+a_1\left(2+\overline{t_1}-\overline{i_1}-\overline{f_1}\right)\\ -a_2\left(2+\underline{t_2}-\underline{i_2}-\underline{f_2}\right)-a_2\left(2+\overline{t_2}-\overline{i_2}-\overline{f_2}\right)\end{array}\right|\\ +\left|\begin{array}{l}b_1\left(2+\underline{t_1}-\underline{i_1}-\underline{f_1}\right)+b_1\left(2+\overline{t_1}-\overline{i_1}-\overline{f_1}\right)\\ -b_2\left(2+\underline{t_2}-\underline{i_2}-\underline{f_2}\right)-b_2\left(2+\overline{t_2}-\overline{i_2}-\overline{f_2}\right)\end{array}\right|\\ +\left|\begin{array}{l}c_1\left(2+\underline{t_1}-\underline{i_1}-\underline{f_1}\right)+c_1\left(2+\overline{t_1}-\overline{i_1}-\overline{f_1}\right)\\ -c_2\left(2+\underline{t_2}-\underline{i_2}-\underline{f_2}\right)-c_2\left(2+\overline{t_2}-\overline{i_2}-\overline{f_2}\right)\end{array}\right|\\ +\left|\begin{array}{l}d_1\left(2+\underline{t_1}-\underline{i_1}-\underline{f_1}\right)+d_1\left(2+\overline{t_1}-\overline{i_1}-\overline{f_1}\right)\\ -d_2\left(2+\underline{t_2}-\underline{i_2}-\underline{f_2}\right)-d_2\left(2+\overline{t_2}-\overline{i_2}-\overline{f_2}\right)\end{array}\right|\end{array}\right) \tag{4}$$

Theorem 1. The normalized Hamming distance measure $d(.)$ between $\tilde{a}_1$ and $\tilde{a}_2$ obeys the following properties:

i. $d(\tilde{a}_1, \tilde{a}_2) \ge 0$,
ii. $d(\tilde{a}_1, \tilde{a}_2) = d(\tilde{a}_2, \tilde{a}_1)$,
iii. $d(\tilde{a}_1, \tilde{a}_3) \le d(\tilde{a}_1, \tilde{a}_2) + d(\tilde{a}_2, \tilde{a}_3)$, where $\tilde{a}_3 = ([a_3,\, b_3,\, c_3,\, d_3]; [\underline{t_3},\, \overline{t_3}],\, [\underline{i_3},\, \overline{i_3}], [\underline{f_3},\, \overline{f_3}])$is an ITrNN.

Proof.

i. The distance measure $d(\tilde{a}_1, \tilde{a}_2) > 0$holds for any two $\tilde{a}_1$ and $\tilde{a}_2$. If $\tilde{a}_1 \approx \tilde{a}_2$ that is for $a_1 = a_2, b_1 =$

$b_2, c_1 = c_2, d_1 = d_2, \underline{t}_1 = \underline{t}_2$, $\bar{t}_1 = \bar{t}_2$, $\underline{i}_1 = \underline{i}_2$, $\bar{\imath}_1 = \bar{\imath}_2, \underline{f}_1 = \underline{f}_2, \bar{f}_1 = \bar{f}_2$, then we have $d(\tilde{a}_1, \tilde{a}_1) = 0$ and consequently, $d(\tilde{a}_1, \tilde{a}_2) \geq 0.$

ii. The proof is obvious.

iii. The normalized Hamming distance between $\tilde{a}_1$ and $\tilde{a}_3$ is taken as follows:

$$d(\tilde{a}_1, \tilde{a}_3) = \frac{1}{24}\left(\begin{array}{l}\left|\begin{array}{l} a_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+a_1\left(2+\bar{t}_1-\bar{\imath}_1-\bar{f}_1\right) \\ -a_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-a_3\left(2+\bar{t}_3-\bar{\imath}_3-\bar{f}_3\right)\end{array}\right| \\ +\left|\begin{array}{l} b_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+b_1\left(2+\bar{t}_1-\bar{\imath}_1-\bar{f}_1\right) \\ -b_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-b_3\left(2+\bar{t}_3-\bar{\imath}_3-\bar{f}_3\right)\end{array}\right| \\ +\left|\begin{array}{l} c_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+c_1\left(2+\bar{t}_1-\bar{\imath}_1-\bar{f}_1\right) \\ -c_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-c_3\left(2+\bar{t}_3-\bar{\imath}_3-\bar{f}_3\right)\end{array}\right| \\ +\left|\begin{array}{l} d_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+d_1\left(2+\bar{t}_1-\bar{\imath}_1-\bar{f}_1\right) \\ -d_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-d_3\left(2+\bar{t}_3-\bar{\imath}_3-\bar{f}_3\right)\end{array}\right|\end{array}\right)$$

$$= \frac{1}{24}\left(\begin{array}{l}\left|\begin{array}{l} a_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+a_1\left(2+\bar{t}_1-\bar{\imath}_1-\bar{f}_1\right) \\ +a_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)+a_2\left(2+\bar{t}_2-\bar{\imath}_2-\bar{f}_2\right) \\ -a_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)-a_2\left(2+\bar{t}_2-\bar{\imath}_2-\bar{f}_2\right) \\ -a_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-a_2\left(2+\bar{t}_3-\bar{\imath}_3-\bar{f}_3\right)\end{array}\right| \\ +\left|\begin{array}{l} b_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+b_1\left(2+\bar{t}_1-\bar{\imath}_1-\bar{f}_1\right) \\ +b_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)+b_2\left(2+\bar{t}_2-\bar{\imath}_2-\bar{f}_2\right) \\ -b_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)-b_2\left(2+\bar{t}_2-\bar{\imath}_2-\bar{f}_2\right) \\ -b_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-b_2\left(2+\bar{t}_3-\bar{\imath}_3-\bar{f}_3\right)\end{array}\right| \\ +\left|\begin{array}{l} c_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+c_1\left(2+\bar{t}_1-\bar{\imath}_1-\bar{f}_1\right) \\ +c_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)+c_2\left(2+\bar{t}_2-\bar{\imath}_2-\bar{f}_2\right) \\ -c_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)-c_2\left(2+\bar{t}_2-\bar{\imath}_2-\bar{f}_2\right) \\ -c_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-c_3\left(2+\bar{t}_3-\bar{\imath}_3-\bar{f}_3\right)\end{array}\right| \\ +\left|\begin{array}{l} d_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+d_1\left(2+\bar{t}_1-\bar{\imath}_1-\bar{f}_1\right) \\ +d_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)+d_2\left(2+\bar{t}_2-\bar{\imath}_2-\bar{f}_2\right) \\ -d_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)-d_2\left(2+\bar{t}_2-\bar{\imath}_2-\bar{f}_2\right) \\ -d_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-d_3\left(2+\bar{t}_3-\bar{\imath}_3-\bar{f}_3\right)\end{array}\right|\end{array}\right)$$

$$\leq \frac{1}{24}\left(\begin{array}{l}\left|\begin{array}{l} a_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+a_1\left(2+\bar{t}_1-\bar{\imath}_1-\bar{f}_1\right) \\ -a_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)-a_2\left(2+\bar{t}_2-\bar{\imath}_2-\bar{f}_2\right)\end{array}\right| \\ +\left|\begin{array}{l} b_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+b_1\left(2+\bar{t}_1-\bar{\imath}_1-\bar{f}_1\right) \\ -b_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)-b_2\left(2+\bar{t}_2-\bar{\imath}_2-\bar{f}_2\right)\end{array}\right| \\ +\left|\begin{array}{l} c_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+c_1\left(2+\bar{t}_1-\bar{\imath}_1-\bar{f}_1\right) \\ -c_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)-c_2\left(2+\bar{t}_2-\bar{\imath}_2-\bar{f}_2\right)\end{array}\right| \\ +\left|\begin{array}{l} d_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+d_1\left(2+\bar{t}_1-\bar{\imath}_1-\bar{f}_1\right) \\ -d_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)-d_2\left(2+\bar{t}_2-\bar{\imath}_2-\bar{f}_2\right)\end{array}\right|\end{array}\right)$$

$$+ \frac{1}{24}\left(\begin{array}{l}\left|\begin{array}{l} a_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)+a_2\left(2+\bar{t}_2-\bar{\imath}_2-\bar{f}_2\right) \\ -a_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-a_2\left(2+\bar{t}_3-\bar{\imath}_3-\bar{f}_3\right)\end{array}\right| \\ +\left|\begin{array}{l} b_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)+b_2\left(2+\bar{t}_2-\bar{\imath}_2-\bar{f}_2\right) \\ -b_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-b_2\left(2+\bar{t}_3-\bar{\imath}_3-\bar{f}_3\right)\end{array}\right| \\ +\left|\begin{array}{l} c_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)+c_2\left(2+\bar{t}_2-\bar{\imath}_2-\bar{f}_2\right) \\ -c_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-c_3\left(2+\bar{t}_3-\bar{\imath}_3-\bar{f}_3\right)\end{array}\right| \\ +\left|\begin{array}{l} d_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)+d_2\left(2+\bar{t}_2-\bar{\imath}_2-\bar{f}_2\right) \\ -d_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-d_3\left(2+\bar{t}_3-\bar{\imath}_3-\bar{f}_3\right)\end{array}\right|\end{array}\right)$$

$\leq d(\tilde{a}_1, \tilde{a}_2) + d(\tilde{a}_2, \tilde{a}_3)$. □

4 MADM strategy with interval trapezoidal neutrosophic numbers

In this section we put forward a framework for determining the attribute weights and the ranking orders for all the alternatives with incomplete weight information under interval trapezoidal neutrosophic number environment.

Consider a MADM problem, where $A = \{A_1, A_2, \ldots, A_m\}$ is a set of m alternatives and $C = \{C_1, C_2, \ldots, C_n\}$ is a set of n attributes.The attribute value of alternative $A_i (i = 1,2, \ldots, m)$ over the attribute $C_j (j = 1,2, \ldots, n)$ is expressed in terms of ITrNNs $\tilde{a}_{ij} = \left(\left[a_{ij}, b_{ij}, c_{ij}, d_{ij}\right]; \tilde{t}_{ij}, \tilde{\imath}_{ij}, \tilde{f}_{ij}\right)$, where, $0 \leq \tilde{t}_{ij} \leq 1, 0 \leq \tilde{\imath}_{ij} \leq 1$, $0 \leq \tilde{f}_{ij} \leq 1$, and $0 \leq \tilde{t}_{ij} + \tilde{\imath}_{ij} + \tilde{f}_{ij} \leq 3$ for $i = 1,2, \ldots, m$ and $j = 1,2, \ldots, n$. Here, $\tilde{t}_{ij}$ denotes the interval truth membership degree, $\tilde{\imath}_{ij}$ denotes the interval indeterminate membership degree, and $\tilde{f}_{ij}$ denotes the interval falsity membership degree to consider the trapezoidal number $\left[a_{ij}, b_{ij}, c_{ij}, d_{ij}\right]$ as the rating values of A_i with respect to the attribute C_j.

We consider an MADM problem in the decision matrix form where each entry represents the rating of alternatives with respect to the corresponding attribute. Thus we obtain the following neutrosophic decision matrix (see Equation 5):

$$D = (\tilde{a}_{ij})_{m\times n} = \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \dots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \dots & \tilde{a}_{mn} \end{pmatrix}. \qquad (5)$$

We assume that the attributes have different weights. The weight vector of the attributes is prescribed as $W = (\tilde{w}_1, \tilde{w}_2, .., \tilde{w}_n)$, where $\tilde{w}_j$ is the weight of the attribute $C_j (j = 1,2, \dots, n)$ and expressed in the form of ITrNNs.

Using the following steps, we present MADM strategy under ITrNN environment.

Step-1. Determine the weight of attributes

Weight measure plays an important role in MADM problems and has a direct relationship with the distance measure between two rating values. To deal with decision information with ITrNNs, we use normalized Hamming distance between two ITrNNs.

We assume that the attribute weight $\tilde{w}_j$ is expressed by-ITrNNs as:

$\tilde{w}_j = ([w_j^1, w_j^2, w_j^3, w_j^4]; [\underline{t_j}, \overline{t_j}], [\underline{i_j}, \overline{i_j}], [\underline{f_j}, \overline{f_j}])$ for $j = 1,2, .., n$.

If $\Delta(\tilde{w}_j, \tilde{I}^+)$ is a distance between weight $\tilde{w}_j$ and $\tilde{I}^+$, then the distance vector is given by

$$\Lambda(\mathrm{W}) = (\Delta(\tilde{w}_1, \tilde{I}^+), \Delta(\tilde{w}_2, \tilde{I}^+), \dots, \Delta(\tilde{w}_n, \tilde{I}^+)), \qquad (6)$$

where

$$\Delta(\tilde{w}_j, \tilde{I}^+) = \frac{1}{24}\begin{pmatrix} \left|w_j^1\left(2 + \underline{t_1} - \underline{i_1} - \underline{f_1}\right) + w_j^1(2 + \overline{t_1} - \overline{i_1} - \overline{f_1}) - 6\right| \\ + \left|w_j^2\left(2 + \underline{t_1} - \underline{i_1} - \underline{f_1}\right) + w_j^2(2 + \overline{t_1} - \overline{i_1} - \overline{f_1}) - 6\right| \\ + \left|w_j^3\left(2 + \underline{t_1} - \underline{i_1} - \underline{f_1}\right) + w_j^3(2 + \overline{t_1} - \overline{i_1} - \overline{f_1}) - 6\right| \\ + \left|w_j^4\left(2 + \underline{t_1} - \underline{i_1} - \underline{f_1}\right) + w_j^4(2 + \overline{t_1} - \overline{i_1} - \overline{f_1}) - 6\right| \end{pmatrix} \qquad (7)$$

The corresponding normalized distance vector is given by

$$\bar{\Lambda} = (\bar{\Delta}(\tilde{w}_1, \tilde{I}^+), \bar{\Delta}(\tilde{w}_2, \tilde{I}^+), \dots, \bar{\Delta}(\tilde{w}_n, \tilde{I}^+)) \qquad (8)$$

where, $\bar{\Delta}(\tilde{w}_j, \tilde{I}^+) = \left[\frac{\Delta(\tilde{w}_j, \tilde{I}^+)}{\max_j \Delta(\tilde{w}_j, \tilde{I}^+)}\right]$ for $j = 1,2, .., n$.

The concept of entropy [21] has been extended in this paper and, the entropy measure of the jth attribute (C_j) for m available alternative can be obtained from

$$e_j = -\frac{1}{In(m)}\left[\frac{\bar{\Delta}(\tilde{w}_j, \tilde{I}^+)}{\sum_{j=1}^{n}\bar{\Delta}(\tilde{w}_j, \tilde{I}^+)} In\left(\frac{\bar{\Delta}(\tilde{w}_j, \tilde{I}^+)}{\sum_{j=1}^{n}\bar{\Delta}(\tilde{w}_j, \tilde{I}^+)}\right)\right]. \qquad (9)$$

Using Equation (9), we finally obtain the normalized weight of the jth attribute

$$w_j = \frac{1 - e_j}{\sum_{j=1}^{n}(1 - e_j)}. \qquad (10)$$

Consequently, we get the weight vector $w = (w_1, w_2, \dots, w_n)$, where $0 \le w_j \le 1$ for $j = 1,2, \dots, n$.

Step 2. Determine the aggregated weighted distances between ideal alternative and each alternative

The Hamming distance measure between the attribute value

$$(\tilde{a}_{ij}) = \left([a_{ij}, b_{ij}, c_{ij}, d_{ij}]; [\underline{t_{ij}}, \overline{t_{ij}}], [\underline{i_{ij}}, \overline{i_{ij}}], [\underline{f_{ij}}, \overline{f_{ij}}]\right)$$

and the ideal value $\tilde{I}^+ = ([1, 1, 1, 1]; [1, 1], [0,0], [0,0])$ is obtained as follows:

$$\Delta(\tilde{a}_{ij}, \tilde{I}^+) = \frac{1}{24}\begin{pmatrix} \left|a_{ij}\left(2 + \underline{t_{ij}} - \underline{i_{ij}} - \underline{f_{ij}}\right) + a_{ij}(2 + \overline{t_{ij}} - \overline{i_{ij}} - \overline{f_{ij}}) - 6\right| \\ + \left|b_{ij}\left(2 + \underline{t_{ij}} - \underline{i_{ij}} - \underline{f_{ij}}\right) + b_{ij}(2 + \overline{t_{ij}} - \overline{i_{ij}} - \overline{f_{ij}}) - 6\right| \\ + \left|c_{ij}(2 + \overline{t_{ij}} - \overline{i_{ij}} - \overline{f_{ij}}) + c_{ij}(2 + \overline{t_{ij}} - \overline{i_{ij}} - \overline{f_{ij}}) - 6\right| \\ + \left|d_{ij}(2 + \overline{t_{ij}} - \overline{i_{ij}} - \overline{f_{ij}}) + d_{ij}(2 + \overline{t_{ij}} - \overline{i_{ij}} - \overline{f_{ij}}) - 6\right| \end{pmatrix}. \qquad (11)$$

Therefore the distance vector for the alternative $A_i (i = 1,2, \dots, m)$ with respect to ideal value $\tilde{I}^+$ can be set as

$$\Lambda(\mathrm{A}_i) = (\Delta(\tilde{a}_{i1}, \tilde{I}^+), \Delta(\tilde{a}_{i2}, \tilde{I}^+), \dots, \Delta(\tilde{a}_{in}, \tilde{I}^+)) \qquad (12)$$

Using Equation (10), and Equation(11), we calculate the aggregated weighted distance $\Delta^{\mathrm{w}}(A_i, \tilde{I}^+)$ between the ideal point and the alternative A_i for $i = 1,2, \dots, m$ as

$$\Delta^{w}(A_i, \tilde{I}^+) = \sum_{j=1}^{n} w_j \, \Delta(\tilde{a}_{ij}, \tilde{I}^+) \text{ for } i = 1,2, \dots, m; j = 1,2, \dots, n. \qquad (13)$$

Step 3. Determine the rank of alternatives

Finally, the ranking of alternatives is performed using the values of the distances $\Delta^{w}(A_i, \tilde{I}^+)$ for $i = 1,2, \dots, m$. The basic idea of ranking the alternative is – smaller the value of $\Delta^{w}(A_i, \tilde{I}^+)$ better the performance/closeness of an alternative to ideal solution.

The schematic diagram of the proposed strategy is presented in the Figure 1.

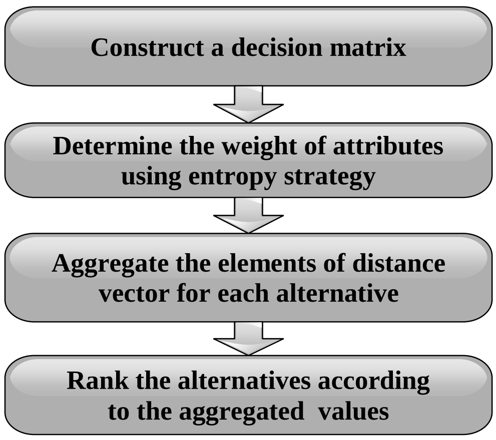

Figure 1. The schematic diagram of the proposed strategy

5 An illustrative numerical example of MADM

In this section, we consider an MADM problem which deals with the supplier selection in supply chain management.

Assume that the MADM problem consists of three suppliers A_1, A_2, A_3 and four attributes C_1, C_2, C_3, C_4.

The four attributes are
1. Product quality(C_1),
2. Service(C_2),
3. Delivery (C_3) and
4. Affordable price(C_4).

We also assume that the alternatives A_1, A_2, A_3 are to be assessed in terms of the interval neutrosophic trapezoidal numbers with respect to the four attributes C_1, C_2, C_3, C_4. The following decision matrix represents the assessment values of alternatives over the attributes:

Table 1. Rating values of alternatives

	C_1	C_2
A_1	([.3,.4,.5,.6]; [.6,.7],[.3,.4][.1,.3])	([.4,.5,.6,.7]; [.5,.6],[.4,.5][.2,.3])
A_2	([.7,.8,.9,.1.0]; [.5,.7],[.2,.3][.1,.2])	([.6,.7,.8,.9]; [.4,.6],[.2,.4][.2,.3])
A_3	([.2,.3,.5,.6]; [.5,.6],[.3,.4][.2,.3])	([.3,.4,.6,.7]; [.7,.8],[.1,.2][.1,.2])
	C_3	C_4
A_1	([.3,.4,.5,.6]; [.5,.6],[.3,.4][.2,.3])	([.6,.7,.8,.9]; [.7,.8],[.1,.2][.1,.2])
A_2	([.5,.6,.8,.9]; [.5,.7],[.1,.2][.1,.2])	([.6,.7,.8,.9]; [.6,.8],[.2,.3][.1,.2])
A_3	([.6,.7,.8,.9]; [.5,.6],[.3,.4][.2,.3])	([.4,.6,.7,.8]; [.4,.5],[.2,.3][.1,.2])

The importance of attributes $C_j (j = 1,2,3,4)$ are given by

$$W = \left\{ \begin{matrix} ([.2,.3,.4,.5];\ [.5,.6],[.3,.4][.2,.3]), \\ ([.1,.2,.3,.4];\ [.4,.5],[.1,.2][.1,.2]) \\ ([.3,.4,.5,.6];\ [.5,.6],[.3,.4][.2,.3]), \\ ([.4,.5,.6,.7];\ [.3,.5],[.2,.4][.1,.3]) \end{matrix} \right\}$$
$$= \{\widetilde{w}_1, \widetilde{w}_2, \widetilde{w}_3, \widetilde{w}_4\} \quad (12)$$

In order to solve the problem, we consider the following steps:

Step-1. Determine the weights of attributes

Using Equation (7) and Equation (8), we obtain the distance vector with respect to ideal interval neutrosophic trapezoidal number as:

$\Lambda = (0.7725,\ 0.8208,\ 0.7075,\ 0.6517))$.

Utilizing Equation (9) and Equation (10), we obtain the weight vector of the attributes:

$w = \{0.2485,\ 0.2468,\ 0.2509,\ 0.2538\}$.

Step 2. Determine the aggregated weighted distances for each alternative

Using Equation (13) and the weight vector$w$, we obtain the aggregated weighted distances of alternatives:

$\Delta(A_1, \tilde{I}^{+}) = 0.6092, \Delta(A_2, \tilde{I}^{+}) = 0.4512,$ and $\Delta(A_3, \tilde{I}^{+}) = 0.6039.$

Step 3. Rank the alternatives

Smaller value of distance indicates the better alternative. So the ranking of the alternatives appears as:

$$A_2 \succ A_3 \succ A_1.$$

The ranking order reflects that A_2 is the best supplier for the considered problem.

6 Conclusions

In this paper, we have introduced new neutrosophic number called interval neutrosophic trapezoidal number (INTrN) characterized by interval valued truth, indeterminacy, and falsity membership degrees. We have defined some arithmetic operations on INTrNs, and normalized Hamming distance between INTrNs. We have developed a new multi-attribute decision making strategy, where the rating values of alternatives over the attributes and the importance of weight of attributes assume the form of INTrNs. We have used entropy strategy to determine attribute weight and then used it to calculate aggregated weighted distance measure. We have determined ranking order of alternatives with the help of aggregated weighted distance measures. Finally, we have provided an illustrative example to show the feasibility, applicability and effectiveness of the proposed strategy. We hope that the proposed interval neutrosophic trapezoidal number as well as the proposed MADM strategy will be widely applicable in decision making science, especially, in brick selection [22, 23], logistics centre location selection [24, 25], school choice [26], teacher selection [27, 28], weaver selection [29], etc.

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