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Entropy based Single Valued Neutrosophic Digraph and its applications

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Abstract: This paper introduces the single valued neutrosophic (i.e. SVN) digraph. The basic terminologies and operations of SVN digraphs have been defined. Later certain types of SVN digraphs are shown and some of the important properties of SVN digraphs are investigated. Finally SVN digraphs are applied in solving a multi-criterion decision making problems.

Keywords: SVN set, SVN digraph, Entropy, Similarity, Decision making.

1 Introduction

In 1995, neutrosophic logic and set theory was introduced by Smarandache [22, 23]. The neutrosophic sets are characterized by a truth membership function(t), a falsity membership function (f) and an indeterminacy membership function(i) respectively, which lies between the nonstandard unit interval $[0, 1]^*$. Unlike intuitionistic fuzzy sets, here the uncertainties present i.e. the indeterminacy factor, is independent of truth and falsity values. Hence Neutrosophic sets are more general than intuitionistic fuzzy set [6] and draw a special attraction to the researchers. Later on Wang et al. [25] introduced a special type of neutrosophic set say single valued neutrosophic set (SVNS). They also introduced the interval valued neutrosophic set (IVNS) in [26]. The SVN set is a generalization of classical set, fuzzy set [27], intuitionistic fuzzy set [6] etc. To see the practical application of the neutrosophic sets and SVN sets, one may see [1, 2, 3, 4, 7, 8] etc.

On the other hand, nowadays graphs and digraphs are widely used by the researchers to solve many pratical problems. The graphs are used as a tool for solving combinatorial problems in algebra, analysis, geometry etc. Many works on fuzzy graph theory, fuzzy digraph theory, intuitionistic fuzzy graphs, soft digraphs etc. are carried out by a number of researchers [12, 13, 15, 16, 17, 21]. Four main categories of neutrosophic graphs have been defined by Samarandache in the paper [24]. However the concept of single valued neutrosophic graphs was introduced by Broumi et al. [9, 10, 11].

In this paper we have introduced the notion of SVN digraphs for the first time. In section 2, some preliminaries regarding neutrosophic sets, graph theory, SVN sets etc. are discussed. In section 3, we have defined the SVN digraph and some terminologies regarding SVN digraphs with examples. We have solved a real life problem by using SVN digraph in Section 4. In Section 5, we have defined the volume of a SVN digraph and also the similarity measure between two SVN digraphs by using the volume of each SVN digraph. Finally in this section, we have computed the similarity measure of the digraphs of Section 4 and compared the results. Section 6 concludes the paper.

2 Preliminaries

In this section, we will discuss some definitions and terminologies regarding neutrosophic sets which will be used in the rest of the paper. However, for details on the neutrosophic sets, one can see [20].

Definition 1 *[20] Let X be a universal set. A neutrosophic set A on X is characterized by a truth membership function t_A, an indeterminacy membership function i_A and a falsity membership function f_A, where $t_A, i_A, f_A : X \to [0, 1]$, are functions and $\forall\ x \in X$, $x = x(t_A(x), i_A(x), f_A(x)) \in A$ is a single valued neutrosophic element of A.*

A single valued neutrosophic set (SVNS) A over a finite universe $X = \{x_1, x_2, \ldots, x_n\}$ is represented as below:

$$A = \sum_{i=1}^{n} \frac{x_i}{\langle t_A(x_i), i_A(x_i), f_A(x_i)\rangle}$$

Definition 2 *[20] The complement of a SVNS A is denoted by A^c and is defined by $t_{A^c}(x) = f_A(x), i_{A^c}(x) = 1 - i_A(x), f_{A^c}(x) = t_A(x)\ \forall\ x \in X$.*

Definition 3 *[20] A SVNS A is contained in the other SVNS B, denoted as $A \subseteq B$, if and only if $t_A(x) \leq t_B(x), i_A(x) \leq i_B(x)$ and $f_A(x) \geq f_B(x)\ \forall x \in X$. Two sets will be equal if $A \subseteq B$ and $B \subseteq A$.*

Definition 4 *[20] Suppose $N(X)$ be the collection of all SVN sets on X and $A, B \in N(X)$. A similarity measure between two SVN sets A and B is a function $S : N(X) \times N(X) \to [0, 1]$ which satisfies the following condition:*

(i) $0 \leq S(A, B) \leq 1$,

(ii) $S(A, B) = 1$ *if and only if* $A = B$.

iii) $S(A, B) = S(B, A)$

(iv) *If $A \subseteq B \subseteq C$, then $S(A, C) \le S(A, B)$ and $S(A, C) \le S(B, C)$ for all $A, B, C \in N(X)$.*

Note that here (i)-(iii) are essential for any similarity measure and (iv) is a desirable property although not mandatory.

Definition 5 *[20] The entropy of SVNS A is defined as a function $E : N(X) \to [0, 1]$ which satisfies the following axioms:*

(i) *$E(A) = 0$ if A is a crisp set.*

(ii) *$E(A) = 1$ if $(t_A(x), i_A(x), f_A(x)) = (0.5, 0.5, 0.5)$ $\forall x \in X$.*

(iii) *$E(A) \ge E(B)$ if A is more uncertain than B i.e. $t_A(x) + f_A(x) \le t_B(x) + f_B(x)$ and $|i_A(x) - i_{A^c}(x)| \le |i_B(x) - i_{B^c}(x)|$ $\forall\, x \in X \;\; A, B \in X$.*

(iv) *$E(A) = E(A^c)$ $\forall A \in N(X)$, where $N(X)$ is the collection of all SVNS over X.*

Example 6 *An entropy measure of an element x_1 of a SVNS A can be calculated as follows:*

$$E_1(x_1) = 1 - (t_A(x_1) + f_A(x_1)) \times |i_A(x_1) - i_{A^c}(x_1)|.$$

Graph and digraphs played an important role in many applications of mathematics like Chinese post-man problems, shortest path problems etc. For graph theoretic terminologies, one can see any standard reference, e.g. [14] or [19].

3 SVN Digraph

In this section, we will define SVN digraph D for the first time corresponding to a SVNS $V_D = \{(v_i, \langle t_{V_D}(v_i), i_{V_D}(v_i), f_{V_D}(v_i)\rangle), i = 1, \ldots, n\}$ over a finite universal set X. For sake of implicity henceforth we will denote V_D by $V_D = \{v_1, v_2, \ldots, v_n\}$ in the rest of paper.

Definition 7 *A SVN digraph D is of the form $D = (V_D, A_D)$ where,*

(i) *$V_D = \{v_1, v_2, v_3, \ldots, v_n\}$ such that the functions $t_{V_D} : V_D \to [0, 1]$, $i_{V_D} : V_D \to [0, 1]$, $f_{V_D} : V_D \to [0, 1]$ denote the truth-membership function, a indeterminacy-membership function and falsity-membership function of the element $v_i \in V_D$ respectively and $0 \le t_{V_D}(v_i) + i_{V_D}(v_i) + f_{V_D}(v_i) \le 3$, $\forall v_i \in V_D, i = 1, 2, \ldots, n$.*

(ii) *$A_D = \{(v_i, v_j); (v_i, v_j) \in V_D \times V_D\}$ provided $0 < E(v_i) - E(v_j) \le 0.5$ and the functions $t_{A_D} : A_D \to [0, 1], i_{A_D} : A_D \to [0, 1], f_{A_D} : A_D \to [0, 1]$ are defined by*

$$t_{A_D}(\{v_i, v_j\}) \le min[t_{V_D}(v_i), t_{V_D}(v_j)],$$
$$i_{A_D}(\{v_i, v_j\}) \ge max[i_{V_D}(v_i), i_{V_D}(v_j)],$$
$$f_{A_D}(\{v_i, v_j\}) \ge max[f_{V_D}(v_i), f_{V_D}(v_j)]$$

where $t_{A_D}, i_{A_D}, f_{A_D}$ denotes the truth-membership function, a indeterminacy-membership function and falsity-membership function of the arc $(v_i, v_j) \in A_D$ respectively where $0 \le t_{A_D}(v_i, v_j) + i_{A_D}(v_i, v_j) + f_{A_D}(v_i, v_j) \le 3$, $\forall (v_i, v_j) \in A_D,\ i, j \in \{1, 2, \ldots n\}$.

We call V_D as the vertex set of D, A_D as the arc set of D where $E(v)$ is the entropy of the vertex v. Please note that if $E(v_i) = E(v_j)$, then $\{(v_i, v_j), (v_j, v_i)\} \in A_D$. Since for a vertex $v \in V_D$ of a SVN digraph D we have $E(v) = E(v)$, thus every vertex of a SVN digraph D contains a loop (v, v) at v. On the other hand, if $E(v_i) - E(v_j) > 0.5$, we define that there exists no arc between the vertices v_i and v_j. A SVN digraph $D = (V_D, A_D)$ is said to be symmetric if $(u, v) \in A_D$ implies $(v, u) \in A_D$. On the other hand, D is asymmetric if $(u, v) \in A_D$ implies $(v, u) \notin A_D$.

Remark 8 *Here, we are trying to represent a SVN set by a SVN digraph. For this reason, we have taken a SVN set V_D and have considered the set V_D as the vertex set of the SVN Digraph D. Thus we are only considering the entropy of the vertex set V_D of the SVN digraph D. However we have seen that the arc set A_D of D forms a new SVN set and we have the following corollary.*

Corollary 9 *The arc set A_D of a SVN digraph $D = (V_D, A_D)$ forms a neutrosophic set on $X \times X$.*

Example 10 *Consider the SVN digraph $D_1 = (V_{D_1}, A_{D_1})$ in Figure 1 with vertex set $V_{D_1} = \{v_1, v_2, v_3, v_4\}$ and arc set $A_{D_1} = \{(v_2, v_1), (v_1, v_3), (v_2, v_3), (v_2, v_4), (v_3, v_4), (v_4, v_1)\}$ with one loop at each vertex as follows:*

$$\begin{bmatrix} & v_1 & v_2 & v_3 & v_4 \\ t_{V_D} & 0.4 & 0.4 & 0.5 & 0.2 \\ i_{V_D} & 0.1 & 0.3 & 0.2 & 0.5 \\ f_{V_D} & 0.2 & 0.1 & 0.5 & 0.3 \\ E & 0.52 & 0.8 & 0.4 & 1 \end{bmatrix},$$

$$\begin{bmatrix} & (v_2, v_1) & (v_2, v_3) & (v_4, v_1) & (v_1, v_3) & (v_4, v_2) \\ t_{A_D} & 0.3 & 0.2 & 0.1 & 0.4 & 0.2 \\ i_{A_D} & 0.4 & 0.3 & 0.5 & 0.3 & 0.5 \\ f_{A_D} & 0.2 & 0.6 & 0.3 & 0.4 & 0.4 \end{bmatrix}.$$

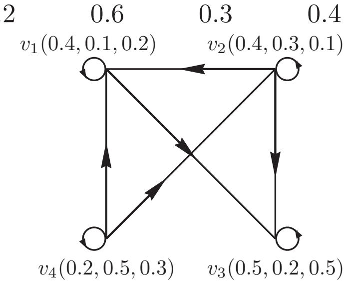

Figure 1: The SVN Digraph D_1

Remark 11 (i) In a SVN digraph D, if $x = (v_i, v_j)$ is an arc, we say that x is *incident* with v_i and v_j; v_i is *adjacent to*

v_j; and v_j is *adjacent from* v_i. It is customary to represent a digraph by a diagram with nodes representing the vertices and directed line segments (arcs) representing the arcs of the digraph. Every vertex of any SVN digraph contains loops by definition. For the sake of simplicity, we define $(t_{A_D}(v,v), i_{A_D}(v,v), f_{A_D}(v,v)) = (0,1,1)$ for each arc $(v,v) \in A_D$ of a SVN digraph D.

(ii) The *order* of D, denoted by $|D|$, is the number of vertices of D. The *size* of a SVN digraph D, is the number of arcs of D i.e. $|A_D|$. For example, the order and size of the digraph D_1 in Figure 1 is 4 and 9 respectively.

Definition 12 *A SVN-subdigraph $H = (V_H, A_H)$ of a SVN-digraph $D = (V_D, A_D)$ is a SVN-digraph such that*

(i) $V_H \subseteq V_D$ *where* $t_{V_H}(v_i) \leq t_{V_D}(v_i)$, $i_{V_H}(v_i) \leq i_{V_D}(v_i)$, $f_{V_H}(v_i) \geq f_{V_D}(v_i)$ $\forall v_i \in V_H$.

(ii) $A_H \subseteq A_D$ *where* $t_{A_H}(v_i,v_j) \leq t_{A_D}(v_i,v_j)$, $i_{A_H}(v_i,v_j) \leq i_{A_D}(v_i,v_j)$, $f_{A_H}(v_i,v_j) \geq f_{A_D}(v_i,v_j)$ $\forall (v_i,v_j) \in A_H$.

Example 13 *Consider the SVN digraph $D_2 = (V_{D_2}, A_{D_2})$ in Figure 2 with vertex set $V_{D_2} = \{v_1, v_2, v_3\}$ and arc set $A_{D_2} = \{(v_2,v_1), (v_1,v_3), (v_2,v_3)\}$ with one loop at each vertex as follows:*

$$\begin{array}{cccc} & v_1 & v_2 & v_3 \\ t_{V_D} & 0.4 & 0.4 & 0.5 \\ i_{V_D} & 0.1 & 0.3 & 0.2 \\ f_{V_D} & 0.2 & 0.1 & 0.5 \\ E & 0.52 & 0.8 & 0.4 \end{array}$$

,

$$\begin{array}{cccc} & (v_2,v_1) & (v_2,v_3) & (v_1,v_3) \\ t_{A_D} & 0.3 & 0.2 & 0.4 \\ i_{A_D} & 0.4 & 0.3 & 0.3 \\ f_{A_D} & 0.2 & 0.6 & 0.4 \end{array}.$$

It is clear that the SVN digraph D_2 in Figure 2 is a SVN subdigraph of D_1 in Figure 1.

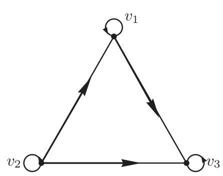

Figure 2: The SVN Digraph D_2

Definition 14 *A SVN-digraph $K = (V_K, A_K)$ is a spanning SVN-subdigraph of a SVN-digraph $D = (V_D, A_D)$ if*

(i) $V_K = V_D$ *where* $t_{V_K}(v_i) = t_{V_D}(v_i)$, $i_{V_K}(v_i) = i_{V_D}(v_i)$, $f_{V_K}(v_i) = f_{V_D}(v_i)$ $\forall v_i \in V_K$.

(ii) $A_K \subseteq A_D$ *where* $t_{A_K}(v_i,v_j) = t_{A_D}(v_i,v_j), i_{A_K}(v_i,v_j) = i_{A_D}(v_i,v_j), f_{A_K}(v_i,v_j) = f_{A_D}(v_i,v_j)$ $\forall (v_i,v_j) \in A_K$.

Definition 15 *For vertices u, v in a SVN digraph D, a u-v SVN path $P = (V_P, A_P)$ is a SVN subdigraph of D whose distinct vertices and arcs can be written in an alternating sequence:*

$$v_1(v_1,v_2)v_2(v_2,v_3)v_3 \cdots v_{k-1}(v_{k-1},v_k)v_k,$$

where $v_1 = u, v_k = v$ and $t_{A_D}(v_i,v_{i+1}) > 0, i_A(v_i,v_{i+1}) > 0, f_A(v_i,v_{i+1}) > 0, 0 < E(v_i) - E(v_{i+1}) \leq 0.5,\ \forall 1 \leq i \leq k$. Further, if (v,u) is an arc in D, then the subdigraph P together with (v,u) is a SVN cycle of length k or a k-SVN cycle in D. For convenience, we denote the cycle as $C = [v_1, v_2, \ldots v_k]$. A SVN digraph having no cycle of length greater than 1 is said to be acyclic. A 1-cycle consists of a vertex v and a loop at v.

Definition 16 *A SVN digraph D is said to be connected if the simple SVN graph G associated to D (i.e., the graph with vertex set V_D and edge set $\{\{u,v\} : (u,v) \in A_D, u \neq v\}$) is connected. The SVN digraph D is said to be strongly connected if for every pair (u,v) of vertices, D contains a u-v SVN path and a v-u SVN path both. The maximal connected (resp. strongly connected) SVN subdigraphs of D are called components (resp. strong components) of SVN D.*

Definition 17 *Let $D = (V_D, A_D)$ be a SVN digraph. Then the outdegree (resp. indegree) of any vertex v is sum of degree of truth-membership, sum of degree of indeterminacy-membership and sum of degree of falsity-membership of all those arcs which are adjacent from (resp. to) vertex v denoted by $O_d(v) = O(d_t(v), d_i(v), d_f(v))$ and $I_d(v) = I(d_t(v), d_i(v), d_f(v))$, where,*

$$\begin{aligned} d_t(v) &= \frac{1}{n}\sum t_{A_D}(u,v) \text{ denotes the degree of truth membership} \\ d_i(v) &= \frac{1}{n}\sum i_{A_D}(u,v) \text{ denotes the degree of indeterminacy members} \\ d_f(v) &= \frac{1}{n}\sum f_{A_D}(u,v) \text{ denotes the degree of falsity membership,} \end{aligned}$$

where n is the number of arcs adjacent from (resp. to) vertex v.

Example 18 *Consider the SVN digraph D_1 in Figure 1. Here we have, the outdegree and indegree of each vertex as follows:*

$$\begin{aligned} O_d(v_1) = (0.8, 0.86, 0.88), O_d(v_2) = (0.05, 0.73, 0.7), \\ O_d(v_3) = (0,1,1), O_d(v_4) = (0.05, 0.83, 0.78), \\ I_d(v_1) = (0.07, 0.82, 0.75), I_d(v_2) = (0.04, 0.90, 0.88), \\ I_d(v_3) = (0.1, 0.76, 0.83), I_d(v_4) = (0,1,1). \end{aligned}$$

Remark 19 *The set of in-degrees (out-degrees) of the vertices of an SVN digraph forms a neutrosophic set on V_D (i.e. on X).*

Definition 20 *A SVN digraph $D = (V_D, A_D)$ is called k-regular SVN digraph if the sum of outdegree and indegree of each vertex v is k. That is, $d(v) = \sum\{O_d(v) + I_d(v)\} = (k,k,k)$ for all $v \in V_D$.*

Definition 21 *A SVN digraph $D = (V_D, A_D)$ is called strong SVN digraph if*

$$t_{A_D}(v_i, v_j) = min[t_{V_D}(v_i), t_{V_D}(v_j)],$$
$$i_{A_D}(v_i, v_j) = max[i_{V_D}(v_i), i_{V_D}(v_j)],$$
$$f_{A_D}(v_i, v_j) = max[f_{V_D}(v_i), f_{V_D}(v_j)]$$

for all $(v_i, v_j) \in A_D$

Example 22 *Consider the SVN digraph $D_3 = (V_{D_3}, A_{D_3})$ in Figure 3 with vertex set $V_{D_3} = \{v_1, v_2, v_3, v_4\}$ and arc set $A_{D_3} = \{(v_2, v_1), (v_1, v_3), (v_2, v_3), (v_2, v_4), (v_3, v_4), (v_4, v_1)\}$ with one loop at each vertex as follows:*

$$\begin{array}{c} \\ t_{V_D} \\ i_{V_D} \\ f_{V_D} \\ E \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0.9 & 0.1 & 0.5 & 0.2 \\ 0.3 & 0.6 & 0.1 & 0.6 \\ 0.8 & 0.1 & 0.5 & 0.1 \\ 0.3 & 0.6 & 0.2 & 0.4 \end{bmatrix} \end{array}$$

,

$$\begin{array}{c} \begin{array}{cccccc} (v_2, v_1) & (v_2, v_3) & (v_4, v_3) & (v_4, v_1) & (v_1, v_3) & (v_2, v_4) \end{array} \\ \begin{bmatrix} 0.1 & 0.1 & 0.2 & 0.2 & 0.5 & 0.1 \\ 0.6 & 0.6 & 0.6 & 0.6 & 0.3 & 0.6 \\ 0.8 & 0.5 & 0.5 & 0.8 & 0.8 & 0.1 \end{bmatrix} \end{array}.$$

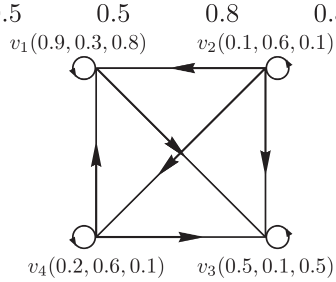

Figure 3: The SVN Digraph D_3

Definition 23 *A SVN digraph $D = (V_D, A_D)$ corresponding to a SVNS V_D is called complete if the following holds:*

(i) $V_D = \{v_1, v_2, v_3, \ldots, v_n\}$.

(ii) $A_D = \{(v_i, v_j); v_i, v_j \in V_D\}$, *provided* $E(v_i) = E(v_j)\ \forall v_i \in V_D$.

Example 24 *Consider the SVN digraph $D_4 = (V_{D_4}, A_{D_4})$ in Figure 4 with vertex set $V_{D_4} = \{v_1, v_2, v_3\}$ and arc set $A_{D_4} = \{(v_2, v_1), (v_1, v_3), (v_2, v_3), (v_1, v_2), (v_3, v_1), (v_3, v_2)\}$ with one loop at each vertex as follows:*

$$\begin{array}{c} \\ t_{V_D} \\ i_{V_D} \\ f_{V_D} \\ E(v_i) \end{array} \begin{array}{c} \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \\ \begin{bmatrix} 0.5 & 0.4 & 0.7 \\ 0.2 & 0.2 & 0.2 \\ 0.4 & 0.5 & 0.5 \\ 0.46 & 0.46 & 0.46 \end{bmatrix} \end{array}.$$

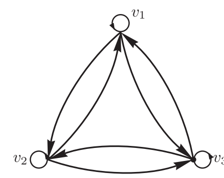

Figure 4: The SVN Digraph D_4

In D_4, we consider that each non-loop arc has neutrosophic value as $(0.4, 0.2, 0.5)$. It is clear that the SVN digraph D_4 is a complete digraph and also a k-regular digraph.

Remark 25 (i) *There does not exist any asymmetric SVN digraph with a cycle of length ≥ 3. Suppose an asymmetric cyclic SVN digraph $D = (V_D, A_D)$ has vertex set $V_D = \{v_1, v_2, v_3, \ldots, v_n\}$. Without loss of generality, let D has a cycle of length 3 say $\langle v_1, v_2, v_3 \rangle$. Then we have $E(v_1) > E(v_2) > E(v_3) > E(v_1)$- which is impossible. Hence D does not have a cycle of length ≥ 3.*

(ii) *Every SVN digraph D is self-complementary. For any SVN digraph having vertex set V_D and its complement set V_D^C, each vertex have same entropy. Hence the result follows.*

Definition 26 *Suppose $D = (V_D, A_D)$ and $H = (V_H, A_H)$ be two SVN digraphs with $|V_D| = |V_H| = n$ corresponding to the SVNS V_D and V_H over an universal set X. Then the union of two SVN digraphs D and H is defined as a SVN digraph $C = (V_C, A_C)$ in which the following holds;*

(i) $V_C = V_D \cup V_H$,

(ii) $t_{V_C}(v) = max(t_{V_D}(v), t_{V_H}(v))$;
$i_{V_C}(v) = max(i_{V_D}(v), i_{V_H}(v))$;
$f_{V_C}(v) = min(f_{V_D}(v), f_{V_H}(v)); \forall\, v \in V_D \cup V_H$ *and,*

(iii) $A_C = \{(v_i, v_j); (v_i, v_j) \in V_C \times V_C\}$ *provided* $0 < E(v_i) - E(v_j) \leq 0.5$,

Definition 27 *Suppose $D = (V_D, A_D)$ and $H = (V_H, A_H)$ be two SVN digraphs with $|V_D| = |V_H|$ corresponding to the SVNS V_D and V_H over an universal set X. Then the intersection of two SVN digraphs D and H is defined as a SVN digraph $C = (V_C, A_C)$ in which the following holds:*

(i) $V_C = V_D \cap V_H$,

(ii) $t_{V_C}(v) = min(t_{V_D}(v), t_{V_H}(v))$;
$i_{V_C}(v) = min(i_{V_D}(v), i_{V_H}(v))$;
$f_{V_C}(v) = max(f_{V_D}(v), f_{V_H}(v)); \forall\, v \in V_D \cap V_H$ *and,*

(iii) $A_C = \{(v_i, v_j); (v_i, v_j) \in V_C \times V_C\}$ *provided* $0 < E(v_i) - E(v_j) \leq 0.5$,

Definition 28 *Suppose $D = (V_D, A_D)$ and $H = (V_H, A_H)$ be two SVN digraphs with $|V_D| = |V_H|$ corresponding to the SVNS V_D and V_H over an universal set X. Consider $V_D = V_H =$*

$\{v_1, v_2, v_3, \ldots, v_n\}$. *The similarity measure between the neutrosophic digraphs D and H can be evaluated by the function,*

$$S(D, H) = \frac{|A_D \cap A_H|}{|A_D \cup A_H|},$$

where $|A_D \cap A_H|, |A_D \cup A_H|$ denotes the number of arcs in $A_D \cap A_H$ and $A_D \cup A_H$ respectively with a set theoretic point of view.

It is clear that $S(D, H)$ satisfies the properties of the Definition 4.

4 An application using SVN Digraph

In this section, we have applied our SVN digraph to solve a multi-criteria decision-making problem. To find out the best alternative decision set, we will use the idea of model set. For this purpose now, we define a model set $M = \{(.1, 0, 0.05), (.1, 0, 0.04), (.1, 0, .03), (.1, 0, 0.02)\}$.

The problem is based on a similar problem discussed in [13]. Now we assume that there exists a set of suppliers $S = \{S_1, S_2, S_3, S_4\}$ whose performances are examined w.r.t the following criteria (T_1, T_2, T_3, T_4), where T_1 the adaptation of new technology , T_2 performance in supply , T_3 the ability of controlling man-power and T_4 quality of service. We will use our proposed decision making technique to select the best supplier. The evaluation of an supplier $S_i, i = 1, 2, 3, 4$ with respect to a criterion $T_j; j = 1, 2, 3, 4$, it based on the knowledge of a domain expert. For example, the opinion of an expert about a supplier S_1 with respect to a criterion T_1 , is as follows: the statement is good is 0.4 and the statement is poor is 0.3 and the degree of not sure is 0.4. For a neutrosophic point of view, it can be expressed as neutrosophic element $v_{11} = \{0.4, 0.3, 0.4\}$. Thus for the alternative S_1, the neutrosophic set is $M_1 = \{(0.4, 0.3, 0.40), (0.6, 0.3, 0.30), (0.2, 0.2, 0.5), (0.5, 0.3, 0.2)\}$. Similarly the other neutrosophic sets for the alternatives S_2, S_3, S_4 respectively are

$$\begin{aligned} M_2 = \{&(0.4, 0.4, 0.2), (0.5, 0.4, 0.30), (0.5, 0.20, 0.40), \\ &(0.5, 0.3, 0.1)\}, \\ M_3 = \{&(0.6, 0.2, 0.2), (0.6, 0.3, 0.40), (0.5, 0.4, 0.10), \\ &(0.3, 0.2, 0.40)\}, \\ M_4 = \{&(0.6, 0.2, 0.20), (0.1, 0.4, 0.50), (0.4, 0.2, 0.60), \\ &(0.4, 0.1, 0.3). \end{aligned}$$

Now we first draw the SVN digraph D_M for the model set M as follows:

Now we draw simultaneously the digraphs $D_{M_1}, D_{M_2}, D_{M_3}, D_{M_4}$ in the Figure 6, Figure 7 respectively. In each case we now calculate the similarity measure of each of the digraphs $D_{M_i};\ i = 1, 2, 3, 4$.

(i) $S_1(D_M, D_{M_1}) = \frac{7}{13} = 0.538$

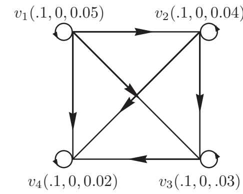

Figure 5: The SVN Digraph D_M

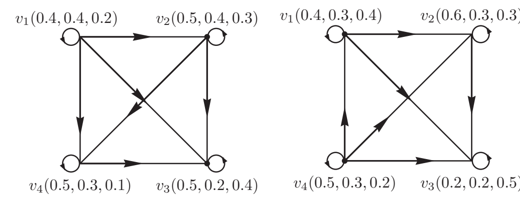

Figure 6: The SVN Digraphs $D_{M_1},\ D_{M_2}$

(ii) $S_2(D_M, D_{M_2}) = \frac{9}{11} = 0.81$

(iii) $S_3(D_M, D_{M_3}) = \frac{6}{14} = 0.42$

(iv) $S_4(D_M, D_{M_4}) = \frac{8}{12} = 0.666$

Therefore as per the similarity measures, the ranking order of the four suppliers is $S_2 > S_4 > S_1 > S_3$. Hence, the best supplier is S_2. From the above example, we can observe that the proposed single valued neutrosophic multi-criteria decision-making method can be handled easily with the help of SVN digraphs. Ashraf et al. [5] have studied SVN graph where as we have tried SVN Digraph. In SVN graph there is no edge direction. Thus it is quite familiar that between any two vertices of SVN graph there is always an arc satisfying some required condition. In SVN digraph theory, between any two vertices there may not be an arc. Here arcs are present depending on the entropy difference of the vertices. Thus these two notions of SVN graphs and digraphs are completely different. Also Ashraf et al. [5] have studied regular SVN graphs which has equal degree of each vertices. But in SVN digraphs all vertices may or may not have same degrees. In future one may study the SVN regular digraphs which may be a completely new idea.

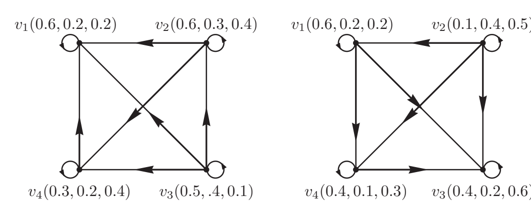

Figure 7: The SVN Digraphs $D_{M_3},\ D_{M_4}$

5 Similarity Measure using the Volume of a SVN Digraph

In this section we introduce some new definition which are given below:

Definition 29 *Suppose A and B be two SVNS over X. Then A is said to be more uncertain than B, denoted by $A \leq B$,*

(i) $t_A(x) + f_A(x) \leq t_B(x) + f_B(x)$

(ii) $|i_A(x) - i_{A^C}(x)| \leq |i_B(x) - i_{B^C}(x)|, \forall \in X$.

Example 30 *Suppose*

$$A = \{(0.4, 0.3, 0.3), (0.3, 0.5, 0.2), (0.5, 0.4, 0.1), (0.2, 0.2, 0.3)\},$$

$$B = \{(0.5, 0.1, 0.6), (0.4, 0.3, 0.4), (0.4, 0.2, 0.6), (0.4, 0.1, 0.5)\}$$

be two SVNS over an universal set $X = \{x_1, \ldots, x_4\}$. Then for all $x_i \in X$, the above definition $A \leq B$ is satisfied.

Remark 31 *From Definition 5 and Example 6, it follows that $E(A) \geq E(B)$ for any two SVNS $A, B \in X$ with $A \leq B$.*

Definition 32 *Suppose A and B be two SVNS over X. Then their maximum sum $C = A \bigoplus B$, denoted by $Max(A, B)$ is again an SVNS on X which is defined as follows:*

(i) $t_c(x) = max\{t_A(x), t_B(x)\}$,

(ii) $f_c(x) = max\{f_A(x), f_B(x)\}$,

(iii) $i_c(x) = max\{|i_A(x) - i_{A^C}(x)|, |i_B(x) - i_{B^C}(x)|\}$, $\forall x \in X$.

Example 33 *From Example 30, it follows that the SVNS*

$$C = \{(0.5, 0.8, 0.6), (0.4, 0.4, 0.4), (0.5, 0.6, 0.6), (0.4, 0.8, 0.5)\}$$

is the maximum sum of two SVNS A, B over X.

Remark 34 *It can be easily proved that $A, B \leq A \bigoplus B$. So, $E(A) \geq E(A \bigoplus B)$ and $E(B) \geq E(A \bigoplus B)$. Therefore*

$$2E(A \bigoplus B) \leq E(A) + E(B).$$

Definition 35 *Suppose V_D and V_H are two SVNS over an universal set X such that $V_D = V_H = \{v_1, v_2, \ldots, v_n\}$. Consider $D = (V_D, A_D)$ and $H = (V_H, A_H)$ be two SVN digraph corresponding to the SVN set D and H respectively. Then the maximum sum digraph C of two digraphs D, H is a SVN digraph $C = (V_C, A_C)$ where $V_C = V_D \bigoplus V_H$ and $A_C = \{(v_i, v_j); (v_i, v_j) \in V_C \times V_C\}$ provided $0 < E(v_i) - E(v_j) \leq 0.5$.*

Definition 36 *Suppose $D_1 = (V_{D_1}, A_{D_1})$ be a SVN digraph over a set X. Then the volume of a SVN digraph D_1 is defined as*

$$Vol(G) = \sum_{v \in V_{D_1}} E(v) = E(V_{D_1}).$$

Example 37 *Consider the SVN digraph D_M in Figure 5 in Section 4. Then the volume of the digraph D_M is*

$$Vol(D_M) = \sum\{E(v_1) + E(v_2) + E(v_3) + E(v_4)\}$$

$= 0.85 + 0.84 + 0.83 + 0.82 = 3.34.$

Definition 38 *Now we define the similarity measure between two SVN digraph D and H as follows:*

$$S(V, H) = \frac{2Vol(V_D \bigoplus V_H)}{Vol(V_D) + Vol(V_H)}.$$

It can be noted that the Definition 38 satisfies the first three postulates of the Definition 4. Now we consider the SVN digraphs $D_M, D_{M_i}, i = 1 \ldots, 4$ in Figure 5,6,7 in Section 4 and calculate the similarity measure between them by using the Definition 38. Here we the following result obtained by the Definition 38:

(i) $S_1(D_M, D_{M_1}) = \frac{1.8}{5.96} = 0.30$

(ii) $S_2(D_M, D_{M_2}) = \frac{2.2}{6.46} = 0.34$

(iii) $S_3(D_M, D_{M_3}) = \frac{1.8}{5.68} = 0.31$

(iv) $S_4(D_M, D_{M_4}) = \frac{1.8}{5.55} = 0.32$

According to the similarity measure followed by the Definition 38 we have obtained the order $S_2 > S_4 > S_3 > S_1$. Hence the best supplier is S_2.

Thus it can be seen that using both the techniques described in Section 4 and 5, we have got a similar result using SVN digraphs.

6 Conclusion

Although Fuzzy digraph theory is very successful in handling uncertainties arising from vagueness or partial belongingness of an element in a set, it cannot model all sorts of uncertainties prevailing in different real physical problems such as problems involving incomplete information. Hence further generalizations of fuzzy and intuitionistic fuzzy digraphs are required. So there are also scopes of evolution of new theories which will have more powers of handling different kinds of uncertainties. Unlike in intuitionistic fuzzy digraphs, where the incorporated uncertainty is dependent of the degree of belongingness and degree of nonbelongingness, here the uncertainty present, i.e. indeterminacy factor, is independent of truth and falsity values. Single valued neutrosophic digraphs were motivated from the practical point of view and that can be used in real scientific and engineering applications. The single valued neutrosophic digraph theory is a generalization of fuzzy digraph theory, intuitionistic digraph theory.

SVN digraph theory is based on the entropy differences of the vertices of SVN digraph. It represents a SVN neutrosophic Set which is used as the vertices of the SVN digraph. In fuzzy digraph theory or Intuitionistic Fuzzy digraph theory, arcs of these digraphs arises depending on the binary relations of the vertices. In SVN digraphs, every vertices have loop it it where as in other theories it may not be possible. Thus the recently proposed notion of SVN digraph theory is a general formal framework for studying uncertainties arising due to 'indeterminacy' factors. Also single valued neutrosophic digraph theory can be used in modeling real scientific and engineering problems. It is also possible to combine neutrosophic digraphs with other digraphs such as soft digraphs etc. to generate different hybrid graphical structure. Therefore the study of neutrosophic digraph theory and its properties have a considerable significance in the sense of applications as well as in understanding the fundamentals of uncertainty. This new topic is very sophisticated and it has immense possibilities which are to be explored.

Smarandache gave the idea of a neutrosophic set to deal with uncertain, incomplete, and inconsistent information that exist in real world. It has been seen that the neutrosophic set draws a special attraction to the researchers than classical set, fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set, interval valued intuitionistic fuzzy set etc. simply because all these sets can be obtained from a neutrosophic set as special cases. In this paper we have developed the SVN digraph theory and studied some of its important properties and shown its application in solving multi-criteria decision making problem. In future, one may further study the deeper properties of SVN digraphs and may apply it in solving many real life problems which involves uncertainty.

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