Neutrosophic Sets and Systems, Vol. 31, 2020
University of New Mexico

Comment on "A Novel Method for Solving the Fully Neutrosophic Linear Programming Problems: Suggested Modifications"

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Abstract. Some clarifications of a previous paper with the same title are presented here to avoid any reading conflict [1]. Also, corrections of some typo errors are underlined. Each modification is explained with details for making the reader able to understand the main concept of the paper. Also, some suggested modifications advanced by Singh et al. [3] (Journal of Intelligent & Fuzzy Systems, 2019, DOI:10.3233/JIFS-181541) are discussed. It is observed that Singh et al. [3] have constructed their modifications on several mathematically incorrect assumptions. Consequently, the reader must consider only the modifications which are presented in this research.

1. Clarifications and Corrected Errors

In Section 5 and Step 3 of the proposed NLP method [1], the trapezoidal neutrosophic number was presented in the following form:

$\tilde{a}=\langle(a^l, a^{m1}, a^{m2}, a^u); T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}\rangle$,

where a^l, a^{m1}, a^{m2}, a^u are the lower bound, the first and second median values and the upper bound for trapezoidal neutrosophic number, respectively. Also, $T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}$ are the truth, indeterminacy and falsity degrees of the trapezoidal neutrosophic number. The ranking function for that trapezoidal neutrosophic number is as follows:

$$R(\tilde{a}) = \left|\left(\frac{-\frac{1}{3}(3a^l-9a^u)+2(a^{m1}-a^{m2})}{2}\right) \times (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})\right| \quad (8)$$

The previous ranking function is only for maximization problems.

But, if NLP problem is a minimization problem, then ranking function for that trapezoidal neutrosophic number is as follows:

$$R(\tilde{a}) = \left|\left(\frac{(a^l+a^u)-3(a^{m1}+a^{m2})}{-4}\right) \times (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})\right| \quad (9)$$

If reader deals with a symmetric trapezoidal neutrosophic number which has the following form:

$\tilde{a}=\langle(a^{m1}, a^{m2}); \alpha, \beta\rangle$,

where $\alpha = \beta,\ \alpha, \beta \geq 0$, then the ranking function for that number will be as follows:

$$R(\tilde{a}) = \left|\left(\frac{(a^{m1}+a^{m2})+2(\alpha+\beta)}{2}\right) \times (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})\right|. \quad (10)$$

We applied Eq. (10) directly in Example 1, but we did not illustrated it in the original work [1], and this caused a reading conflict. After handling typo errors in Example 1, the crisp model of the problem will be as follows:

Maximize $Z = 18x_1 + 19x_2 + 20x_3$

Subject to

$12x_1 + 13x_2 + 12x_3 \leq 502,$

$14x_1 + 13x_3 \leq 486,$

$12x_1 + 15x_2 \leq 490,$

$x_1, x_2, x_3 \geq 0.$

The initial simplex form will be as in Table 1.

Table 1 Initial simplex form

Basic variables	x_1	x_2	x_3	s_4	s_5	s_6	RHS
s_4	12	13	12	1	0	0	502
s_5	14	0	13	0	1	0	486
s_6	12	15	0	0	0	1	490
Z	-18	-19	-20	0	0	0	0

The optimal simplex form will be as in Table 2.

Table 2 Optimal form

Basic variables	x_1	x_2	x_3	s_4	s_5	s_6	RHS
x_2	-12/169	1	0	1/13	-12/169	0	694/169
x_3	14/13	0	1	0	1/13	0	486/13
s_6	2208/169	0	0	-15/13	180/169	1	72400/169
Z	370/169	0	0	19/13	32/169	0	139546/169

The obtained optimal solution is $x_1 = 0, x_2 = 4.11, x_3 = 37.38$.
The optimal value of the NLPP is $\tilde{z} \approx (13,15,2,2)x_1 + (12,14,3,3)x_2 + (15,17,2,2)x_3 = (13,15,2,2) * 0 + (12,14,3,3) * 4.11 + (15,17,2,2) * 37.38 =$
$(49.32,57.54,12.33,12.33) + (560.70,635.46,74.76,74.7) = (610.02,693,87.09,87.09)$.
$\tilde{z} \approx (610.02,693,87.09,87.09)$, which is in the symmetric trapezoidal neutrosophic number form. Since the traditional form of $\tilde{a} =\langle(a^{m1}, a^{m2}); \alpha, \beta\rangle$ is:
$\tilde{a} =\langle(a^{m1} - \alpha, a^{m1}, a^{m2}, a^{m2} + \beta)\rangle$,
where $a^{m1} - \alpha = a^{l}$, $a^{m2} + \beta = a^{u}$, then the optimal value of the NLPP can also be written as $\tilde{z} \approx (522.93,610.02,693,780.09)$.

The reader must also note that one can transform the symmetric trapezoidal neutrosophic numbers from Example 1 in [1] to its traditional form, and use Eq. (8) for solving the problem, obtaining the same result. By comparing the result with other existing models mentioned in the original research [1], the proposed model is the best.
By using Eq. (8) and solving Example 2 in [1], the crisp model will be as follows:
Maximize $Z = 25x_1 + 48x_2$
Subject to

$13x_1 + 28x_2 \leq 31559$,
$26x_1 + 9x_3 \leq 16835$,
$21x_1 + 15x_2 \leq 19624$,
$x_1, x_2 \geq 0$.

The initial simplex form will be as in Table 3.

Table 3 Initial simplex form

Basic variables	x_1	x_2	s_3	s_4	s_5	RHS
s_3	13	28	1	0	0	31559
s_4	26	9	0	1	0	16835
s_5	21	15	0	0	1	19624
Z	-25	-48	0	0	0	0

The optimal simplex form will be as in Table 4.

Table 4 Optimal simplex form

Basic variables	x_1	x_2	s_3	s_4	s_5	RHS
x_2	0	1	7/131	0	-13/393	407627/393
s_4	0	0	67/131	1	-611/393	969250/393
x_1	1	0	-5/131	0	28/393	76087/393
Z	0	0	211/131	0	76/393	21468271/393

The optimal value of objective function is 54627.
By using Eq. (9) and solving Example 3 in [1], the crisp model will be as follows:
Minimize $Z = 6x_1 + 10x_2$
Subject to
$2x_1 + 5x_2 \geq 6$,
$3x_1 + 4x_2 \geq 3$,
$x_1, x_2 \geq 0$.
The optimal simplex form will be as in Table 5.

Table 5 Optimal simplex form

Basic variables	x_1	x_2	s_3	s_4	RHS
s_4	-7/5	0	-4/5	1	0
x_2	2/5	1	-1/5	0	10
Z	-2	0	-2	0	12

Hence, the optimal solution has the value of variables:
$x_1 = 0, x_2 = 1.2$, Z = 12.
The obtained result is better than Saati et al. [2] method.
By correcting typo errors which percolated in the Case study in [1], the problem formulation model will be as follows:
Maximize $\tilde{Z} = \tilde{9}x_1 + \widetilde{12}x_2 + \widetilde{15}x_3 + \widetilde{11}x_4$
Subject to

$0.5x_1 + 1.5x_2 + 1.5x_3 + x_4 \leq \widetilde{1500}$,
$3x_1 + x_2 + 2x_3 + 3x_4 \leq \widetilde{2350}$,
$2x_1 + 4x_2 + x_3 + 2x_4 \leq \widetilde{2600}$,
$0.5x_1 + 1x_2 + 0.5x_3 + 0.5x_4 \leq \widetilde{1200}$,
$x_1 \leq \widetilde{150}$,
$x_2 \leq \widetilde{100}$,
$x_3 \leq \widetilde{300}$,
$x_4 \leq \widetilde{400}$,
$x_1, x_2, x_3, x_4 \geq 0$.

The values of each trapezoidal neutrosophic number remain the same [1].
By using Eq. (8) and solving the Case study, the crisp model will be as follows:
Maximize $\tilde{Z} = 10x_1 + 10x_2 + 12x_3 + 9x_4$
Subject to

$0.5x_1 + 1.5x_2 + 1.5x_3 + x_4 \leq 1225$,
$3x_1 + x_2 + 2x_3 + 3x_4 \leq 1680$,
$2x_1 + 4x_2 + x_3 + 2x_4 \leq 2030$,
$0.5x_1 + 1x_2 + 0.5x_3 + 0.5x_4 \leq 945$,
$x_1 \leq 122$,
$x_2 \leq 87$,
$x_3 \leq 227$,
$x_4 \leq 297$,
$x_1, x_2, x_3, x_4 \geq 0$.

By solving the previous model using simplex approach, the results are as follows:
$x_1 = 122, x_2 = 87, \ x_3 = 227, x_4 = \frac{773}{3}, \ Z = \ 7133.$

2. A Note on the modifications suggested by Singh et al. [3]

This part illustrates how Singh et al. [3] constructed their modifications of Abdel-Basset et al.'s method [1] on wrong concepts. The errors in Singh et al.'s [3] modifications reflects the misunderstanding of Abdel-Basset et al.'s method [1].

In the second paragraph of the introductory section, Singh et al. [3] assert that "in Abdel-Basset et al.'s method [1], firstly, a neutrosophic linear programming problem (NLPP) is transformed into a crisp linear programming problem (LPP) by replacing each parameter of the NLPP, represented by a trapezoidal neutrosophic number with its equivalent defuzzified crisp value". However, this is not true, since the neutrosophic linear programming problem (NLPP) is transformed into a crisp linear programming problem (LPP) by replacing each parameter of the NLPP, represented by a trapezoidal neutrosophic number with its equivalent deneutrosophic crisp value. The deneutrosophication process means transforming a neutrosophic value to its equivalent crisp value. In Section 2, Step 1 Singh et al. [3] alleged that Abdel-Basset et al.'s method [1] for comparing two trapezoidal neutrosophic numbers is based on maximization and minimization of problem, which is again not true.
In Section 3 and Definition 4, Abdel-Basset et al. [1] illustrated that the method for comparing two trapezoidal neutrosophic numbers is as follows:

1. If $R(\tilde{A}) > R(\tilde{B})$ then $\tilde{A} > \tilde{B}$,
2. If $R(\tilde{A}) < R(\tilde{B})$ then $\tilde{A} < \tilde{B}$,
3. If $R(\tilde{A}) = R(\tilde{B})$ then $\tilde{A} = \tilde{B}$.

There is well known that if $a^l = a^{m1} = a^{m2} = a^u$, then the trapezoidal number $\tilde{a}=\langle (a^l, a^{m1}, a^{m2}, a^u); 1, 0, 0\rangle$ will be transformed into a real number $a = \langle (a, a, a, a); 1, 0, 0\rangle$, and hence in this case $R(a) = a$. We presented this fact to illustrate a great error in the suggested modifications of Singh et al. [3]
In the Suggested modifications section [3], the authors claimed that:

$$R\left(\sum_{i=1}^{m} \langle a_i^l, a_i^{m1}, a_i^{m2}, a_i^u, T_{\tilde{a}_i}, I_{\tilde{a}_i}, F_{\tilde{a}_i}\rangle\right) = \sum_{i=1}^{m} R\,\langle a_i^l, a_i^{m1}, a_i^{m2}, a_i^u, T_{\tilde{a}_i}, I_{\tilde{a}_i}, F_{\tilde{a}_i}\rangle - \sum_{i=1}^{m} T_{\tilde{a}_i}$$
$$+ \sum_{i=1}^{m} I_{\tilde{a}_i} + \sum_{i=1}^{m} F_{\tilde{a}_i} + min_{1\le j\le n}\{T_{\tilde{c}_i}\} - max_{1\le j\le n}\{I_{\tilde{c}_i}\} - max_{1\le j\le n}\{F_{\tilde{c}_i}\} \quad (11)$$

instead of ,
$R\left(\sum_{i=1}^{m} \langle a_i^l, a_i^{m1}, a_i^{m2}, a_i^u, T_{\tilde{a}_i}, I_{\tilde{a}_i}, F_{\tilde{a}_i}\rangle\right) = \sum_{i=1}^{m} R\,\langle a_i^l, a_i^{m1}, a_i^{m2}, a_i^u, T_{\tilde{a}_i}, I_{\tilde{a}_i}, F_{\tilde{a}_i}\rangle$.

Let us consider the following example for proving the error in this suggestion [3]
Let $m = 3$, which are three trapezoidal neutrosophic numbers $\tilde{a}_1, \tilde{a}_2, \tilde{a}_3$; since $\tilde{a}_1=\langle (1, 1,1,1); 1, 0, 0\rangle$, $\tilde{a}_2 = \langle (2, 2,2,2); 1, 0, 0\rangle$, $\tilde{a}_3 = \langle (3, 3,3,3); 1, 0, 0\rangle$, then,
$R\left(\sum_{i=1}^{m} \langle a_i^l, a_i^{m1}, a_i^{m2}, a_i^u, T_{\tilde{a}_i}, I_{\tilde{a}_i}, F_{\tilde{a}_i}\rangle\right) = R(\langle (1, 1,1,1); 1, 0, 0\rangle + \langle (2, 2,2,2); 1, 0, 0\rangle + \langle (3, 3,3,3); 1, 0, 0\rangle)$ $= R(\langle (6, 6,6,6); 1, 0, 0\rangle)$, and according to the previously determined fact "if $a^l = a^{m1} = a^{m2} = a^u$ then the trapezoidal number $\tilde{a} = \langle (a^l, a^{m1}, a^{m2}, a^u); 1, 0, 0\rangle$ will be transformed into a real number $a = \langle (a, a, a, a); 1, 0, 0\rangle$ and hence in this case $R(a) = a$ ", the value of $R(\langle (6, 6,6,6); 1, 0, 0\rangle) = 6$.

And by calculating the right hand side of Eq. (11), which is $\sum_{i=1}^{m} R\,\langle a_i^l, a_i^{m1}, a_i^{m2}, a_i^u, T_{\tilde{a}_i}, I_{\tilde{a}_i}, F_{\tilde{a}_i}\rangle - \sum_{i=1}^{m} T_{\tilde{a}_i} + \sum_{i=1}^{m} I_{\tilde{a}_i} + \sum_{i=1}^{m} F_{\tilde{a}_i} + min_{1\le j\le n}\{T_{\tilde{c}_i}\} - max_{1\le j\le n}\{I_{\tilde{c}_i}\} - max_{1\le j\le n}\{F_{\tilde{c}_i}\}$, we note that,
$R\langle (1, 1,1,1); 1, 0, 0\rangle + R\langle (2, 2,2,2); 1, 0, 0\rangle + R\langle (3, 3,3,3); 1, 0, 0\rangle - 3 + 0 + 0 + 1 - 0 - 0 = \quad 1 + 2 + 3 - 3 + 0 + 0 + 1 - 0 - 0 = 4.$

And then, the left hand side of Eq. (11) does not equal the right hand side, i.e. $6 \neq 4$.
Consequently, the authors [3] built their suggestions on a wrong concept.

Beside Eq. (11), the authors [3] used the expressions $R(a) = 3a + 1$ for maximization problems, and $R(a) = -2a + 1$ for minimization problems, and this shows peremptorily that their assumptions are scientifically incorrect.

There is also a repeated error in all corrected solutions suggested by Singh et al. [3] which contradicts with the basic operations of trapezoidal neutrosophic numbers. This error is iterated in Section 7, as in Example 1, in Step 6. Singh et al. [3] illustrated that the optimal value of the NLPP is calculated using the optimal solution obtained in Step 5 as follows:
$(11,13,15,17)x_1 + (9,12,14,17)x_2 + (13,15,17,19)x_3 = (11,13,15,17) * 0 + (9,12,14,17) * 0 + (13,15,17,19) * (\frac{245}{18}) = 13\,(\frac{245}{18}) + 15\,(\frac{245}{18}) + 17\,(\frac{245}{18}) + 19\,(\frac{245}{18}) = \frac{7840}{9}$, and because the basic operation of multiplying trapezoidal neutrosophic number by a constant value is as follows:
$\gamma\tilde{a} = \begin{cases} \langle(\gamma a_1, \gamma a_2, \gamma a_3, \gamma a_4); \mathrm{T}_{\tilde{a}}, \mathrm{I}_{\tilde{a}}, \mathrm{F}_{\tilde{a}}\rangle \; if\,(\gamma \geq 0) \\ \langle(\gamma a_4, \gamma a_3, \gamma a_2, \gamma a_1); \mathrm{T}_{\tilde{a}}, \mathrm{I}_{\tilde{a}}, \mathrm{F}_{\tilde{a}}\rangle \; if\,(\gamma < 0) \end{cases}$, then the value of $(11,13,15,17) * 0 + (9,12,14,17) * 0 + (13,15,17,19) * (\frac{245}{18}) = \left(\frac{3185}{18}, \frac{1225}{6}, \frac{4165}{18}, \frac{4655}{18}; 1,0,0\right)$.Then the optimal value of the NLPP is$\tilde{z} \approx = \left(\frac{3185}{18}, \frac{1225}{6}, \frac{4165}{18}, \frac{4655}{18}\right)$.

The same error appears in Example 4, where the optimal value of the NLPP is calculated by Singh et al. [3] using the optimal solution obtained in Step 5 as follows:
$(6,8,9,12)x_1(9,10,12,14)x_2 + (12,13,15,17)x_3 + (8,9,11,13)x_4 = (6,8,9,12)(\frac{3700}{21}) + (9,10,12,14)(0) + (12,13,15,17)(\frac{6200}{7}) + (8,9,11,13)(0) = 6(\frac{3700}{21}) + 8\left(\frac{3700}{21}\right) + 9\left(\frac{3700}{21}\right) + 12(\frac{3700}{21}) + 12\left(\frac{6200}{7}\right) + 13\,(\frac{6200}{7}) + 15(\frac{6200}{7}) + 17(\frac{6200}{7}) = \frac{1189700}{21}$, which is scientifically incorrect and reflects only the weak background of the authors in the neutrosophic field.
Therefore, we concluded that it is scientifically incorrect to use Singh et al.'s modifications [3].

3. Conclusions

Clarifications and corrections of some typo errors are presented here to avoid any reading conflict. Also, the correct results of NLPPs are presented. By using three modified functions for ranking process which were presented by Abdel-Basset et al. [1], the reader will be able to solve all types of linear programming problems with trapezoidal and symmetric trapezoidal neutrosophic numbers. Also, the mathematically incorrect assumptions used by Singh et al. [3] are discussed and rejected.

Conflict of interest

We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

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Received: November 7, 2019. **Accepted**: February 3, 2020