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Multi-Criteria Assignment Techniques in Multi-Dimensional Neutrosophic Soft Set Theory

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Abstract: In this paper, we have introduced a new concept of multi-dimensional neutrosophic soft sets together with various operations, properties and theorems on them. Then we have proposed an algorithm named $2-DNS$ based on our proposed two-dimensional neutrosophic soft set for solving neutrosophic multi-criteria assignment problems with multiple decision makers. At last, we have applied the $2-DNS$ Algorithm for solving neutrosophic multi-criteria assignment problem in medical science to evaluate the effectiveness of different modalities of treatment of a disease.

Keywords: Assignment, Neutrosophic Multi-Criteria, Multi-Dimensional Neutrosophic Soft Set, $2-DNS$ Algorithm, Application.

1 Introduction

Most of the recent mathematical methods meant for formal modeling,reasoning and computing are crisp, accurate and deterministic in nature. But in ground reality, crisp data is not always the part and parcel of the problems encountered in different fields like economics, engineering, social science, medical science, environment etc. As a consequence various theories viz. theory of probability, theory of fuzzy sets introduced by Zadeh [1], theory of intuitionistic fuzzy sets by Atanassov[2], theory of vague sets by Gau[3], theory of interval mathematics by Gorzalczany[4], theory of rough sets by Pawlak[5] have been evolved in process. But difficulties present in all these theories have been shown by Molodtsov [6]. The cause of these problems is possibly related to the inadequacy of the parametrization tool of the theories. As a result Molodtsov proposed the concept of soft theory as a new mathematical tool for solving the uncertainties which is free from the above difficulties. Maji et al. [7, 8] have further done various research works on soft set theory. For presence of vagueness Maji et al.[9, 10] have introduced the concept of Fuzzy Soft Set. Then Mitra Basu et al. [14] proposed the mean potentiality approach to get a balanced solution of a fuzzy soft set based decision making problem.

But the intuitionistic fuzzy sets can only handle the incomplete information considering both the truth-membership (or simply membership) and falsity-membership (or non-membership) values. It does not handle the indeterminate and inconsistent information which exists in belief system. Smarandache [13] introduced the concept of **neutrosophic set(NS)** which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. He showed that NS is a generalization of the classical sets, conventional fuzzy sets, Intuitionistic Fuzzy Sets (IFS) and Interval Valued Fuzzy Sets (IVFS). Then considering the fact that the parameters or criteria (which are words or sentences) are mostly neutrosophic set, Maji [11, 12] has combined the concept of soft set and neutrosophic set to make the mathematical model **neutrosophic soft set** and also given an algorithm to solve a decision making problem. But till now there does not exist any method for solving neutrosophic soft set based assignment problem.

In several real life situations we are encountered with a type of problem which includes in assigning men to offices, jobs to machines, classes in a school to rooms, drivers to trucks, delivery trucks to different routs or problems to different research teams etc in which the assignees depend on some criteria which posses varying degree of efficiency, called cost or effectiveness. The basic assumption of this type of problem is that one person can perform one job at a time. An assignment plan is optimal if it is able to

optimize all criteria. Now if such problem contains only one criterion then it can be solved by well known Hungarian method introduced by Kuhn[15]. In case of such problems containing more than one criterion, i.e., for multi-criteria assignment problems De et al [16] have proposed a solution methodology. Kar et al[17] have proposed two different methods for solving a neutrosophic multi-criteria assignment problem.
Till date these all research work have concentrated on multiple criteria assignment problems containing only one decision maker, i.e., all the criteria matrices are determined or observed by only one decision maker. But there may be such type of multiple criteria assignment problems in which the criteria be neutrosophic in nature and the degree of efficiency of the criteria are determined by more than one decision makers according to their own opinions. There does not exist any procedure to solve neutrosophic multi-criteria assignment problem with multiple decision makers or in other words there is a demand to come a methodology to solve multi-criteria assignment problems in the parlance of neutrosophic soft set theory.

In this paper we have first introduced the concept of neutrosophic multi-criteria assignment problem(NMCAP) with multiple decision makers. Then we have proposed the new concept of multi-dimensional neutrosophic soft sets along with few operations, properties and theorems on them. Moreover an algorithm named $2-DNS$ has been developed based on two-dimensional neutrosophic soft set for solving NMCAP with more than one decision maker. At last we have applied the $2-DNS$ Algorithm for solving neutrosophic multi-criteria assignment problem in medical science to evaluate the effectiveness of different modalities of treatment of a disease.

2 Preliminaries

2.1 Definition: Soft Set [6]

Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the set of all subsets of U. Let $A \subseteq E$. Then a pair (F, A) is called a **soft set** over U, where F is a mapping given by, $F: A \to P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U.

2.2 Definition: NOT Set of a Set of Parameters [9]

Let $E = \{e_1, e_2, e_3, ..., e_n\}$ be a set of parameters.

The NOT set of E denoted by $\rceil E$ is defined by

$$\rceil E = \{\rceil e_1, \rceil e_2, \rceil e_3, ..., \rceil e_n\}, where \rceil e_i = not\, e_i, \forall i.$$

The operator not of an object, say k, is denoted by $\rceil k$ and is defined as the negation of the object; e.g., let we have the object k = beautiful, then $\rceil k$ i.e., not k means k is not beautiful.

2.3 Definition: Neutrosophic Set [13]

A **neutrosophic set** A on the universe of discourse X is defined as

$$A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\},$$
$$where\ T, I, F : X \to]^-0, 1^+[$$

and $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$; T, I, F are called neutrosophic components.
"Neutrosophic" etymologically comes from "neutro-sophy" (French *neutre* < Latin *neuter*, neutral and Greek *sophia*, skill/wisdom) which means knowledge of neutral thought.
From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. The non-standard finite numbers $1^+ = 1 + \delta$, where 1 is the standard part and δ is the non-standard part and $^-0 = 0\delta$, where 0 is its standard part and δ is non-standard part. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0, 1^+[$. Hence we consider the neutrosophic set which takes the value from the subset of $[0,1]$.
Any element neutrosophically belongs to any set, due to the percentages of truth/indeterminacy/falsity involved, which varies between 0 and 1 or even less than 0 or greater than 1.
Thus $x(0.5, 0.2, 0.3)$ belongs to A (which means, with a probability of 50 percent x is in A, with a probability of 30 percent x is not in A and the rest is undecidable); or $y(0,0,1)$ belongs to A (which normally means y is not for sure in A); or $z(0,1,0)$ belongs to A (which means one does know absolutely nothing about z 's affiliation with A); here $0.5 + 0.2 + 0.3 = 1$; thus A **is a NS and an IFS too**.
The subsets representing the appurtenance, indeterminacy and falsity may overlap, say the element $z(0.30, 0.51, 0.28)$ and in this case $0.30 + 0.51 + 0.28 > 1$; then **B is a NS but is not**

an IFS; we can call it paraconsistent set (from paraconsistent logic, which deals with paraconsistent information).

Or, another example, say the element $z(0.1,0.3,0.4)$ belongs to the set C, and here $0.1+0.3+0.4<1$; then B **is a NS but is not an IFS; we can call it intuitionistic set** (from intuitionistic logic, which deals with incomplete information).

Remarkably, in a NS one can have elements which have paraconsistent information (sum of components >1), or incomplete information (sum of components <1), or consistent information (in the case when the sum of components $=1$).

2.4 Definition: Complement of a Neutrosophic Set [18]

The complement of a neutrosophic set S is denoted by $c(S)$ and is defined by $T_{c(S)}(x)=F_S(x), I_{c(S)}(x)=1-I_S(x), F_{c(S)}(x)=T_S(x) \forall x \in X$

2.5 Definition: Neutrosophic Soft Set [12]

Let U be an initial universe set and E be a set of parameters. Consider $A \subseteq E$. Let $P(U)$ denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the **neutrosophic soft set** over U, where F is a mapping given by $F: A \rightarrow P(U)$.

2.6 Traditional Assignment Problems [15]

Sometimes we are faced with a type of problem which consists in assigning men to offices, jobs to machines, classes in a school to rooms, drivers to trucks, delivery trucks to different routs or problems to different research teams etc in which the assignees posses varying degree of efficiency, called cost or effectiveness. The basic assumption of this type of problem is that one person can perform one job at a time with respect to one criterion. An assignment plan is optimal if it optimizes the total effectiveness of performing all the jobs.

Example 2.1

Let us consider the assignment problem represented by the following cost matrix (Table-1) in which the elements represent the cost in lacs required by a machine to perform the corresponding job. The problem is to allocate the jobs to the machines so as to minimize the total cost.

Table-1: Cost Matrix

MACHINES

JOBS	M_1	M_2	M_3	M_4
A	7	25	16	10
B	12	27	3	25
C	37	18	17	14
D	18	25	23	9

3 Neutrosophic Multi-Criteria Assignment Problems With Multiple Decision Makers

Normally in traditional assignment problems one person is assigned for one job with respect to a single criterion but in real life there are different problems in which one person can be assigned for one job with respect to more than one criteria. Such type of problems is known as **Multi-Criteria Assignment Problem(MCAP)**. Moreover in such MCAP if atleast one criterion be neutrosophic in nature then the problems will be called **Neutrosophic Multi-Criteria Assignment Problem(NMCAP)**. Now there may be such type of NMCAP in which the criteria matrices are determined by more than one decision makers according to their own opinions. In such type of problems there may be more than one matrices associated with a single criterion as the criteria are determined by multiple decision makers. Now we will discuss these new type of NMCAP with more than one decision makers and develop an algorithm to solve such type of problems.

3.1 General Formulation of a Neutrosophic Multi-Criteria Assignment Problem With Multiple Decision Makers

Let m jobs have to be performed by m number of machines depending on p number of criteria (each criterion is neutrosophic in nature) according to q number of decision makers. Now suppose that to perform j-th job by i-th machine it will take the degree of efficiency ξ_q^k for the k-th criterion according to the q-th decision maker. Then the k-th ($k=1,2,....,p$) criteria matrix according to q-th decision maker will be as given in Table-2.

Table-2: criteria matrix of k -th criterion for q -th decision maker

MACHINES

JOBS

	M_1	M_2	M_3	$\cdots$	M_m
J_1	ξ^k_{q11}	ξ^k_{q12}	ξ^k_{q13}	$\cdots$	ξ^k_{q1m}
J_3	ξ^k_{q31}	ξ^k_{q32}	ξ^k_{q33}	$\cdots$	ξ^k_{q3m}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
J_m	ξ^k_{qm1}	ξ^k_{qm2}	ξ^k_{qm3}	$\cdots$	ξ^k_{qmm}

If the number of jobs and machines be equal in a criteria matrix then it is called a balanced criteria matrix otherwise it is known as unbalanced criteria matrix. Now the problem is to assign each machine with a unique job in such a way that the total degree of efficiency for an allocation will be optimized for all criteria which is illustrated in the following example.

Example 3.1

Let us consider a NMCAP represented by the following cost matrices and time matrix in which the criteria are neutrosophic in nature and the elements of the matrices are representing the degree of cost and time required by a machine to perform the corresponding job according to two decision makers Mr. X and Mr.Y.

Table-3: Cost Matrix by Mr.X

MACHINES

JOBS

	M_1	M_2	M_3
	(0.8,0.2,0.6)	(0.2,0.5,0.9)	(0.6,0.4,0.4)
	(0.2,0.6,0.8)	(0.7,0.2,0.5)	(0.6,0.3,0.5)
	(0.6,0.3,0.5)	(0.6,0.2,0.7)	(0.6,0.1,0.5)

Table-4: Cost Matrix by Mr.Y

MACHINES

JOBS

	M_1	M_2	M_3
J_1	(0.7,0.4,0.3)	(0.2,0.5,0.9)	(0.5,0.4,0.6)
J_2	(0.3,0.6,0.8)	(0.7,0.2,0.4)	(0.6,0.4,0.3)
J_3	(0.5,0.3,0.6)	(0.6,0.3,0.5)	(0.5,0.2,0.7)

Table-5:Time Matrix by Mr.X and Mr.Y

MACHINES

JOBS

	M_1	M_2	M_3
	(0.3,0.5,0.8)	(0.7,0.2,0.4)	(0.5,0.2,0.6)
J_2	(0.8,0.3,0.3)	(0.2,0.5,0.9)	(0.5,0.3,0.7)
J_3	(0.5,0.3,0.6)	(0.5,0.4,0.5)	(0.4,0.3,0.7)

The problem is to allocate the jobs J_1, J_2, J_3 to the machines M_1, M_2, M_3 so as to minimize the total cost and time collectively and simultaneously.

4 The Concept of Multi-Dimensional Neutrosophic Soft Set

4.1 Definition: Multi-Dimensional Neutrosophic Soft Set

Let $U_1, U_2, ..., U_n$ be n non-null finite sets of n different type of objects such that, $U_1 = \{O_1, O_2, ..., O_{m1}\}, U_2 = \{O'_1, O'_2, ..., O'_{m2}\},. ...., U_n = \{O_1^{(n-1)'}, O_2^{(n-1)'}, ..., O_{mn}^{(n-1)'}\};$ where $m1, m2, ..., mn$ respectively be the cardinalities of $U_1, U_2, ..., U_n$ and let $U = U_1 \times U_2 \times .... \times U_n$. Now let E be the set of parameters clarifying all types of objects $O_{i1}, O'_{i2}, ..., O_{in}^{(n-1)'};$ $i1 = 1,2,..., m1; i2 = 1,2,..., m2; ....; in = 1,2,..., mn$ and each parameter is a neutrosophic word or neutrosophic sentence involving neutrosophic words and $A \subseteq E$. Suppose that N^U denotes all neutrosophic sets of U . Now a mapping F is defined from the parameter set A to N^U , i.e., $F : A \to \mathrm{N}^U$, then the algebraic structure (F, A) is said to be a n **-Dimensional neutrosophic soft set** over U .

Now n may be finite or, infinite. If $n = 1$ then (F, A) will be the conventional neutrosophic soft set, if $n = 2$ then (F, A) is said to be a two-dimensional neutrosophic soft set, if $n = 3$ then

(F, A) is said to be a three-dimensional neutrosophic soft set and so on.

4.2 The Features of Multi-Dimensional Neutrosophic Soft Set Compared to Neutrosophic Soft Set

Neutrosophic soft set is just a special type of multi-dimensional neutrosophic soft set where the dimension i.e., the number of the set of objects is one.

A neutrosophic soft set indicates that how a single set of objects is involved with a single set of parameters (or, criteria) where as a n -dimensional neutrosophic soft set(n may be any positive integer) reveals the involvement of n number of sets of different types of objects with a single set of parameters(or, criteria). **So from the perspective of application, multi-dimensional neutrosophic soft set has more vast scope than the conventional neutrosophic soft set.** Now we will discuss the example, operations and properties of two-dimensional neutrosophic soft set and for the higher dimensional neutrosophic soft set they can also be established in the identical manner.

Example 4.1: Let U_1 be the set of three jobs, say, $U_1 = \{J_1, J_2, J_3\}$ and let U_2 be the set of four machines, say, $U_2 = \{M_1, M_2, M_3, M_4\}$. Now let

$$E = \{\ cost\ requirement, time\ requirement, troublesome\ due\ to\ transportation\ \} = \{e_1, e_2, e_3\}(\text{say}).$$

Let $A = \{e_1, e_2\}$

Now let $U = U_1 \times U_2$ and $F : A \rightarrow \mathrm{N}^U$, s.t.,

$$F(cost\ requirement) = \{(J_1, M_1)/(.8,0.3,0.4), (J_1, M_2)/(.3,.2,.8), (J_1, M_3)/(.5,.4,.6), (J_1, M_4)/(.7,.2,.3), (J_2, M_1)/(.2,.3,.9), (J_2, M_2)/(.7,.3,.4)(J_2, M_3)/(.5,.5,.6), (J_2, M_4)/(.3,.2,.8), (J_3, M_1)/(.6,.4,.6), (J_3, M_2)/(.4,.2,.6), (J_3, M_3)/(.3,.4,.8), (J_3, M_4)/(.7,.2,.5)\}$$

and

$$F(time\ requirement) = \{(J_1, M_1)/(.2,.3,.9), (J_1, M_2)/(.6,.3,.5), (J_1, M_3)/(.5,.3,.7), (J_1, M_4)/(.4,.5,.8), (J_2, M_1)/(.7,.2,.5), (J_2, M_2)/(.2,.3,.9), (J_2, M_3)/(.6,.3,.5), (J_2, M_4)/(.6,.2,.7), (J_3, M_1)/(.4,.3,.7), (J_3, M_2)/(.5,.6,.7), (J_3, M_3)/(.6,.3,.5), (J_3, M_4)/(.3,.4,.8)\}$$

Now the two-dimensional neutrosophic soft set (F, A) describing the requirements for the objects is given by,

$$(F, A) = \{cost\ requirement = \{(J_1, M_1)/(.8,0.3,0.4), (J_1, M_2)/(.3,.2,.8), (J_1, M_3)/(.5,.4,.6), (J_1, M_4)/(.7,.2,.3), (J_2, M_1)/(.2,.3,.9), (J_2, M_2)/(.7,.3,.4) (J_2, M_3)/(.5,.5,.6), (J_2, M_4)/(.3,.2,.8), (J_3, M_1)/(.6,.4,.6), (J_3, M_2)/(.4,.2,.6), (J_3, M_3)/(.3,.4,.8), (J_3, M_4)/(.7,.2,.5)\},$$

$$time\ requirement = \{(J_1, M_1)/(.2,.3,.9), (J_1, M_2)/(.6,.3,.5), (J_1, M_3)/(.5,.3,.7), (J_1, M_4)/(.4,.5,.8), (J_2, M_1)/(.7,.2,.5), (J_2, M_2)/(.2,.3,.9), (J_2, M_3)/(.6,.3,.5), (J_2, M_4)/(.6,.2,.7), (J_3, M_1)/(.4,.3,.7), (J_3, M_2)/(.5,.6,.7), (J_3, M_3)/(.6,.3,.5), (J_3, M_4)/(.3,.4,.8)\}$$

The Tabular Representation of the two-dimensional neutrosophic soft set (F, A) is as follows:

Table-6

Tabular Representation of (F, A)

	e_1	e_2
(J_1, M_1)	(.8,0.3,0.4)	(.2,.3,.9)
(J_1, M_2)	(.3,.2,.8)	(.6,.3,.5)
(J_1, M_3)	(.5,.4,.6)	(.5,.3,.7)
(J_1, M_4)	(.7,.2,.3)	(.4,.5,.8)
(J_2, M_1)	(.2,.3,.9)	(.7,.2,.5)
(J_2, M_2)	(.7,.3,.4)	(.2,.3,.9)
(J_2, M_3)	(.5,.5,.6)	(.6,.3,.5)
(J_2, M_4)	(.3,.2,.8)	(.6,.2,.7)
(J_3, M_1)	(.6,.4,.6)	(.4,.3,.7)
(J_3, M_2)	(.4,.2,.6)	(.5,.6,.7)
(J_3, M_3)	(.3,.4,.8)	(.6,.3,.5)
(J_3, M_4)	(.7,.2,.5)	(.3,.4,.8)

4.3 Definition: Choice Value:

According to a decision making problem the parameters of a decision maker's choice or requirement which forms a subset of the whole

parameter set of that problem are known as **choice parameters**.
Choice value of an object is the sum of the true-membership values of that object corresponding to all the choice parameters associated with a decision making problem.

4.4 Definition: Rejection Value:
Rejection value of an object is the sum of the falsity-membership values of that object corresponding to all the choice parameters associated with a decision making problem.

4.5 Definition: Confusion Value:
Confusion value of an object is the sum of the indeterminacy-membership values of that object corresponding to all the choice parameters associated with a decision making problem.

4.6 Definition: Null Two-dimensional Neutrosophic Soft Set:
Let $U_1 \times U_2$ be the initial universe set, E be the universe set of parameters and $A \subset E$. Then a two-dimensional neutrosophic soft set (F, A) is said to be a **null two-dimensional neutrosophic soft set (** ϕ_A) with respect to the parameter set A if for each $e \in A$

$$F(e) = \{(O_i, O_j^{'})/0.0\} \forall (O_i, O_j^{'}) \in U_1 \times U_2\}$$

4.7 Definition: Universal Two-dimensional Neutrosophic Soft Set:
Let $U_1 \times U_2$ be the initial universe set, E be the universe set of parameters and $A \subset E$. Then a two-dimensional neutrosophic soft set (F, A) is said to be a **universal two-dimensional neutrosophic soft set (**U_A) with respect to the parameter set A if for each $e \in A$

$$F(e) = \{(O_i, O_j^{'})/1.0\} \forall (O_i, O_j^{'}) \in U_1 \times U_2\}$$

4.8 Definition: Complement of a Two-dimensional Neutrosophic Soft Set
The **complement** of a two-dimensional neutrosophic soft set (F, A) over the universe U where $U = U_1 \times U_2; U_1 = \{O_1, O_2, ..., O_i\}$, $U_2 = \{O_1^{'}, O_2^{'}, ..., O_j^{'}\}; i, j \in N$ over the parameter set E (where each parameter is a neutrosophic word or neutrosophic sentence involving neutrosophic words)is denoted by $(F, A)^C$ and is defined by $(F, A)^C = (F^C, \rceil A)$ where $F^C : \rceil A \to \mathrm{N}^U$ where $\rceil A$ is the NOT set of the parameter set A.

4.9 Definition: Union
The **union** of two two-dimensional neutrosophic soft sets (F, A) and (G, B) over the same universe U (where $U = U_1 \times U_2; U_1 = \{O_1, O_2, ..., O_i\}$, $U_2 = \{O_1^{'}, O_2^{'}, ..., O_j^{'}\}; i, j \in N$) and over the parameter set E (where $A, B \subseteq E$ and each parameter is a neutrosophic word or neutrosophic sentence involving neutrosophic words)is denoted by $(F, A) \tilde{\cup} (G, B)$ and is defined by $(F, A) \tilde{\cup} (G, B) = (H, C)$ where

$$H(e) = \begin{cases} F(e), & if\ e \in (A-B) \\ G(e), & if\ e \in (B-A) \\ \{(O_i, O_j^{'})/max\{\mu_{F(e)}(O_i, O_j^{'}), \mu_{G(e)}(O_i, O_j^{'})\} \forall (O_i, O_j^{'}) \in U_1 \times U_2\}, & if\ e \in A \cap B \end{cases}$$

where $\mu_{F(e)}(O_i, O_j^{'})$ and $\mu_{G(e)}(O_i, O_j^{'})$ denote the membership values of $(O_i, O_j^{'})$ w.r.t the functions F and G respectively associated with the parameter e.

4.10 Definition: Intersection
The **intersection** of two two-dimensional neutrosophic soft sets (F, A) and (G, B) over the same universe U where $U = U_1 \times U_2; U_1 = \{O_1, O_2, ..., O_i\}$, $U_2 = \{O_1^{'}, O_2^{'}, ..., O_j^{'}\}; i, j \in N$ and over the parameter set E (where $A, B \subseteq E$ and each parameter is a neutrosophic word or neutrosophic sentence involving neutrosophic words)is denoted by $(F, A) \tilde{\cap} (G, B)$ and is defined by $(F, A) \tilde{\cap} (G, B) = (H, C)$ where

$$H(e) = \begin{cases} F(e), & if\ e \in (A-B) \\ G(e), & if\ e \in (B-A) \\ \{(O_i, O_j^{'})/min\{\mu_{F(e)}(O_i, O_j^{'}), \mu_{G(e)}(O_i, O_j^{'})\} \forall (O_i, O_j^{'}) \in U_1 \times U_2\}, & if\ e \in A \cap B \end{cases}$$

where $\mu_{F(e)}(O_i, O_j^{'})$ and $\mu_{G(e)}(O_i, O_j^{'})$ denote the membership values of $(O_i, O_j^{'})$ w.r.t the functions F and G respectively associated with the parameter e.

4.11 Properties:

Let $(F,A),(G,B)$ and (H,C) be three two-dimensional neutrosophic soft sets over the same universe U and parameter set E. Then we have,

$(i)(F,A)\,\tilde{\cup}((G,B)\,\tilde{\cup}(H,C)) = ((F,A)\,\tilde{\cup}(G,B))\,\tilde{\cup}(H,C)$

$(ii)(F,A)\,\tilde{\cup}(G,B) = (G,B)\,\tilde{\cup}(F,A)$

$(iii)((F,A)^C)^C = (F,A)$

$(iv)(F,A)\,\tilde{\cup}(F,A) = (F,A)$

$(v)(F,A)\,\tilde{\cap}(F,A) = (F,A)$

$(vi)(F,A)\,\tilde{\cup}\phi_A = (F,A)$, where ϕ_A is the null two-dimensional neutrosophic soft set with respect to the parameter set A.

$(vii)(F,A)\,\tilde{\cap}\phi_A = \phi_A$

$(viii)(F,A)\,\tilde{\cup}U_A = U_A$, where U_A is the universal two-dimensional neutrosophic soft set with respect to the parameter set A.

$(ix)(F,A)\,\tilde{\cap}U_A = (F,A)$

4.12 De Morgan's laws in two-dimensional neutrosophic soft set theory:

The well known De Morgan's type of results hold in two-dimensional neutrosophic soft set theory for the newly defined operations: complement, union and intersection.

Theorem 4.1

Let (F,A) and (G,B) be two two-dimensional neutrosophic soft sets over a common universe U and parameter set E. Then

$i)((F,A)\,\tilde{\cup}(G,B))^C = (F,A)^C\,\tilde{\cap}(G,B)^C$

$ii)((F,A)\,\tilde{\cap}(G,B))^C = (F,A)^C\,\tilde{\cup}(G,B)^C$

5 The Methodology Based On Two-Dimensional Neutrosophic Soft Set For Solving Neutrosophic Multi-Criteria Assignment Problems With Multiple Decision Makers

In many real life problems we have to assign each object of a set of objects to another object in a different set of objects such as assigning men to offices, jobs to machines, classes in a school to rooms, drivers to trucks, delivery trucks to different routs or problems to different research teams etc. in which the assignees posses varying degree of efficiency, depending on neutrosophic multiple criteria such as cost, time etc. The basic assumption of this type of problem is that one person can perform one job at a time. To solve such type of problems our aim is to make such assignment that optimize the criteria i.e., minimize the degree of cost and time or maximizes the degree of profit. **Since in such type of problems the degrees of each criterion (or, parameter) of a set of criteria (or, parameter set) are evaluated with respect to two different types of objects, to solve such problems we can apply two-dimensional neutrosophic soft set and their various operations.**

The stepwise procedure to solve such type of problems is given below.

$2-DNS$ Algorithm:

Step 1: Convert each unbalanced criteria matrix to balanced by adding a fictitious job or machine with zero cost of efficiency.

Step 2: From these balanced criteria matrices construct a two-dimensional neutrosophic soft set (F_i,E_i) according to each decision maker $d_i; i=1,2,...,q;$ q be the number of decision makers.

Step 3: Combining the opinions of all the decision makers about the criteria, take the union of all these two-dimensional neutrosophic soft sets $(F_i,E_i); i=1,2,...,q$ as follows

$$(F,E) = \tilde{\cup}_{i=1}^{q}(F_i,E_i)$$

Step 4: Then compute the complement $(F,E)^C$ of the two-dimensional neutrosophic soft set (F,E) if our aim be to minimize the criteria (such as cost, time etc.).

Step 5: Construct the tabular representation of (F,E) or, $(F,E)^C$ according to maximization or minimization problem with row wise sum of parametric values which is known as choice value ($C_{(J_i,M_j)}$).

Step 6: Now for i-th job, consider the choice values $C_{(J_i,M_j)}, \forall j$ and point out the maximum choice value $C^{max}_{(J_i,M_j)}$ with a $*$.

Step 7: If $C^{max}_{(J_i,M_j)}$ holds for all distinct j's then assign M_j machine for J_i job and put a tick mark(√) beside the choice values corresponding to M_j to indicate that already M_j machine has been assigned.

Step 8: If for more than one i, $C^{max}_{(J_i,M_j)}$ hold for the same j, ie., if there is a tie for the assignment of M_j machine in more than one job then we have to consider the difference value ($V_{d_{(J_i,M_j)}}$) between the maximum and the next to maximum choice values(corresponding to those machines which are not yet assigned). If $V_{d_{(J_{i_1},M_j)}} < V_{d_{(J_{i_2},M_j)}}$ then M_j machine will be assigned for the job J_{i_2}. Now if the difference values also be same, i.e., $V_{d_{(J_{i_1},M_j)}} = V_{d_{(J_{i_2},M_j)}}$ then go to the next step.

Step 9: Now for i -th job, consider the rejection values $R_{(J_i,M_j)}, \forall j$ and point out the minimum rejection value $R^{min}_{(J_i,M_j)}$ with a $*$.

Step 10: If for more than one i, $R^{min}_{(J_i,M_j)}$ hold for the same j, consider the difference value ($V_{dR_{(J_i,M_j)}}$) between the minimum and the next to minimum rejection values(corresponding to those machines which are not yet assigned). If $V_{dR_{(J_{i_1},M_j)}} < V_{dR_{(J_{i_2},M_j)}}$ then M_j machine will be assigned for the job J_{i_2}. Now if the difference values also be same then go to the final step.

Step 11: Now for i -th job, consider the confusion values $\zeta_{(J_i,M_j)}, \forall j$ and point out the minimum confusion value $\zeta^{min}_{(J_i,M_j)}$ with a $*$.

Step 12: If for more than one i, $\zeta^{min}_{(J_i,M_j)}$ hold for the same j, consider the difference value ($V_{d\zeta_{(J_i,M_j)}}$) between the minimum and the next to minimum confusion values(corresponding to those machines which are not yet assigned). If $V_{d\zeta_{(J_{i_1},M_j)}} < V_{d\zeta_{(J_{i_2},M_j)}}$ then M_j machine will be assigned for the job J_{i_2}. Now if the difference values also be same i.e., $V_{d\zeta_{(J_{i_1},M_j)}} = V_{d\zeta_{(J_{i_2},M_j)}}$ then M_j machine may be assigned to any one of the jobs J_{i_1} or J_{i_2}.

6 Application of $2-DNS$ Algorithm For Solving Neutrosophic Multi-Criteria Assignment Problems in Medical Science

In medical science there also exist neutrosophic multi-criteria assignment problems and we may apply the $2-DNS$ Algorithm for solving those problems. Now we will discuss a such type of problem with its solution.

Problem 1: In medical science[19] there are different types of diseases and various modalities of treatments in respect to them. On the basis of different aspects of the treatment procedure (such as degree of pain relief, cost and time requirements for treatment etc.) we may measure the degree of effectiveness of the treatment for the disease. Here we consider three common diseases of oral cavity such as dental caries, gum disease and oral ulcer. Now medicinal treatment, extraction and scaling that are commonly executed, have more or less impacts on the treatment of these three diseases. According to the statistics, (true-membership value, indeterminacy-membership value, falsity-membership value) of pain relief in case of medicinal treatment on the basis of pain score for dental caries, gum disease, oral ulcer are $(0.7,0.7,0.5),(0.6,0.8,0.5)$ and $(0.9,0.5,0.2)$ respectively; by extraction the degrees of pain relief for dental caries, gum disease and oral ulcer are $(0.8,0.5,0.3)$, $(0.8,0.7,0.4)$ and $(0.5,0.7,0.6)$ respectively and by scaling the degrees of pain relief for dental caries, gum disease and oral ulcer are $(0.3,0.8,0.8)$, $(0.9,0.4,0.2)$ and $(0.6,0.7,0.5)$ respectively. Now the degree of cost to avail the medicinal treatment, extraction and scaling for both the diseases dental caries, gum disease are $(0.4,0.3,0.8),(0.3,0.2,0.7)$ and $(0.5,0.4,0.6)$ respectively and that for oral ulcer are $(0.3,0.2,0.8),(0.2,0.3,0.9)$ and $(0.4,0.4,0.7)$ respectively. Moreover the degree of time taken to the medicinal treatment, extraction and scaling for gum disease are $(0.6,0.3,0.5),(0.4,0.2,0.8),(0.5,0.5,0.6)$ and for oral ulcer are $(0.6,0.4,0.7),(0.4,0.3,0.8),(0.5,0.5,0.5)$ respectively and that of for dental caries are $(0.6,0.2,0.3),(0.5,0.4,0.7)$ and $(0.3,0.2,0.9)$ respectively. **Now the problem is to assign a treatment for each disease so that to maximize the pain relief and minimize the cost and time simultaneously as much as possible.**

Solution By $2-DNS$ Algorithm

The set of universe $U = U_1 \times U_2$ where

$U_1 = \{dental\,caries, gum\,disease, oral\,ulcer\}$
$= \{d_1, d_2, d_3\}$,
$U_2 = \{medicinal\,treatment, extraction, scaling\}$
$= \{t_1, t_2, t_3\}$

and the set of parameters
$E = \{$ *pain score, cost requirement, time requirement*
$= \{e_1, e_2, e_3\}$(say)
Now from the given data we have the following criteria matrices:

Table-7(Pain Score Matrix)

TREATMENTS

DISEASES	t_1	t_2	t_3
d_1	(0.5,0.2,0.6)	(0.4,0.3,0.8)	(0.2,0.6,0.9)
d_2	(0.2,0.5,0.9)	(0.6,0.3,0.5)	(0.5,0.3,0.6)
d_3	(0.2,0.5,0.9)	(0.6,0.3,0.5)	(0.5,0.3,0.6)

Table-8(Cost Matrix)

TREATMENTS

DISEASES	t_1	t_2	t_3
d_1	(0.4,0.3,0.8)	(0.3,0.2,0.7)	(0.5,0.4,0.6)
d_2	(0.4,0.2,0.7)	(0.3,0.3,0.8)	(0.5,0.4,0.6)
d_3	(0.3,0.2,0.8)	(0.2,0.3,0.9)	(0.4,0.4,0.7)

Table-9(Time Matrix)

TREATMENTS

	t_1	t_2	t_3
d_1	(0.6,0.2,0.3)	(0.5,0.4,0.7)	(0.3,0.2,0.9)
d_2	(0.6,0.3,0.5)	(0.4,0.2,0.8)	(0.5,0.5,0.6)
d_3	(0.6,0.4,0.7)	(0.4,0.3,0.8)	(0.5,0.5,0.5)

To solve this problem by $2-DNS$ algorithm at first we have to form the two-dimensional neutrosophic soft set (F,E) describing the impact of the treatments for the diseases from the given criteria matrices as:

$(F,E) = \{$ *degree of pain score* $= \{(d_1,t_1)/(0.5,0.3,0.7),$
$(d_1,t_2)/(0.3,0.2,0.7), (d_1,t_3)/(0.8,0.2,0.3),$
$(d_2,t_1)/(0.5,0.2,0.6), (d_2,t_2)/(0.4,0.3,0.8),$
$(d_2,t_3)/(0.2,0.6,0.9), (d_3,t_1)/(0.3,0.2,0.8),$
$(d_3,t_2)/(0.6,0.3,0.5), (d_3,t_3)/(0.5,0.3,0.6)\},$
degree of cost requirement $= \{(d_1,t_1)/(0.4,0.3,0.8),$
$(d_1,t_2)/(0.3,0.2,0.7), (d_1,t_3)/(0.5,0.4,0.6),$
$(d_2,t_1)/(0.4,0.2,0.7), (d_2,t_2)/(0.3,0.3,0.8),$
$(d_2,t_3)/(0.5,0.4,0.6), (d_3,t_1)/(0.3,0.2,0.8),$
$(d_3,t_2)/(0.2,0.3,0.9), (d_3,t_3)/(0.4,0.4,0.7)\},$
degree of time requirement $= \{(d_1,t_1)/(0.6,0.2,0.3),$
$(d_1,t_2)/(0.5,0.4,0.7), (d_1,t_3)/(0.3,0.2,0.9),$
$(d_2,t_1)/(0.6,0.3,0.5), (d_2,t_2)/(0.4,0.2,0.8),$
$(d_2,t_3)/(0.5,0.5,0.6), (d_3,t_1)/(0.6,0.4,0.7),$
$(d_3,t_2)/(0.4,0.3,0.8), (d_3,t_3)/(0.5,0.5,0.5)\}\}$

Here,
$|E = \{$ *pain relief, not requirement of cost, not requirement of time* $\} = \{\bar{e}_1, \bar{e}_2, \bar{e}_3\},$
then

$(F,E)^c = \{$ *degree of pain relief* $= \{(d_1,t_1)/(0.7,0.7,0.5),$
$(d_1,t_2)/(0.7,0.8,0.3), (d_1,t_3)/(0.3,0.8,0.8),$
$(d_2,t_1)/(0.6,0.8,0.5), (d_2,t_2)/(0.8,0.2,0.4),$
$(d_2,t_3)/(0.9,0.4,0.2), (d_3,t_1)/(0.8,0.8,0.3),$
$(d_3,t_2)/(0.5,0.7,0.6), (d_3,t_3)/(0.6,0.7,0.5)\},$
degree of not requirement of cost $= \{(d_1,t_1)/(0.8,0.7,0.4),$
$(d_1,t_2)/(0.7,0.8,0.3), (d_1,t_3)/(0.6,0.6,0.5),$
$(d_2,t_1)/(0.7,0.8,0.4), (d_2,t_2)/(0.8,0.7,0.3),$
$(d_2,t_3)/(0.6,0.6,0.5), (d_3,t_1)/(0.8,0.8,0.3),$
$(d_3,t_2)/(0.9,0.7,0.2), (d_3,t_3)/(0.7,0.6,0.4)\},$
degree of not requirement of time $= \{(d_1,t_1)/(0.3,0.8,0.6),$
$(d_1,t_2)/(0.7,0.6,0.5), (d_1,t_3)/(0.9,0.8,0.3),$
$(d_2,t_1)/(0.5,0.7,0.6), (d_2,t_2)/(0.8,0.8,0.4),$
$(d_2,t_3)/(0.6,0.5,0.5), (d_3,t_1)/(0.7,0.6,0.6),$
$(d_3,t_2)/(0.8,0.7,0.4), (d_3,t_3)/(0.5,0.5,0.5)\}\}$

Therefore the tabular representation of $(F,E)^c$ is as follows:

Table-10

Tabular Representation of $(F,E)^c$ with choice rejection and confusion values $(C_{(d_i,t_j)}, R_{(d_i,t_j)}, \zeta_{(d_i,t_j)})$

	$\bar{e}_1$	$\bar{e}_2$	$\bar{e}_3$	$C_{(d_i,t_j)}$	$R_{(d_i,t_j)}$	$\zeta_{(d_i,t_j)}$
(d_1,t_1)	(0.7,0.7,0.5),	(0.8,0.7,0.4)	(0.3,0.8,0.6)	1.8	1.5	2.2
(d_1,t_2)	(0.7,0.8,0.3)	(0.7,0.8,0.3)	(0.7,0.6,0.5)	*2.1	1.1	2.2
(d_1,t_3)	(0.3,0.8,0.8)	(0.6,0.6,0.5)	(0.9,0.8,0.3)	1.8	1.6	2.2 √
(d_2,t_1)	(0.6,0.8,0.5)	(0.7,0.8,0.4)	(0.5,0.7,0.6)	1.8	1.5	2.3
(d_2,t_2)	(0.8,0.2,0.4)	(0.8,0.7,0.3)	(0.8,0.8,0.4)	*2.4	1.1	1.7 √
(d_2,t_3)	(0.9,0.4,0.2)	(0.6,0.6,0.5)	(0.6,0.5,0.5)	2.1	1.2	1.5
(d_3,t_1)	(0.8,0.8,0.3)	(0.8,0.8,0.3)	(0.7,0.6,0.6)	*2.3 √	1.2	2.2
(d_3,t_2)	(0.5,0.7,0.6)	(0.9,0.7,0.2)	(0.8,0.7,0.4)	2.2	1.2	2.1
(d_3,t_3)	(0.6,0.7,0.5)	(0.7,0.6,0.4)	(0.5,0.5,0.5)	1.8	1.4	1.8

Now among the choice values $C_{(d_3,t_j)}; j=1,2,3$, $C_{(d_3,t_1)}$ is maximum(2.3), which implies that t_1 **treatment has to be assigned for the disease** d_3.

But for both the diseases d_1 and d_2, $C_{(d_i,t_j)}; j=1,2,3$ take the maximum value at $j=2$, i.e., for the assignment of t_2 treatment there is a tie between the diseases d_1 and d_2. We have to consider the difference value $V_{d_{(d_i,t_j)}}; i=1,2; j=2,3$ between the maximum and the next to maximum choice values(corresponding to those treatments which are not yet assigned).

Now since $V_{d_{(d_1,t_j)}} = 0.3 = V_{d_{(d_2,t_j)}}$ for $j=2,3$; we have to consider the rejection values. But for both the diseases d_1 and d_2, $R_{(d_i,t_j)}; j=1,2,3$ take the minimum value at $j=2$, therefore we have to consider their confusion values. Now since $\zeta_{(d_2,t_j)}; j=2,3$ take the minimum value (1.7) at $j=2$, t_2 **treatment has to be assigned for the disease** d_2 **and the rest treatment** t_3 **is assigned for the disease** d_1.

Block Diagram of 2DNS-Algorithm to Assign a Treatment for a Disease

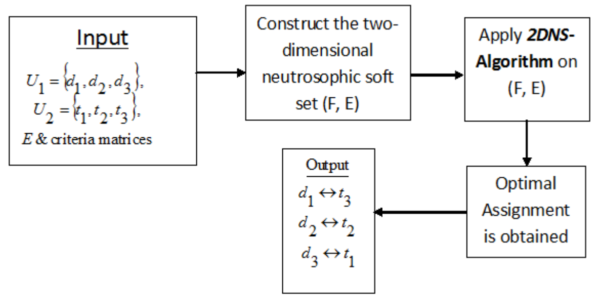

Figure 1: Block Diagram of $2DNS$ -Algorithm to Assign a Treatment for a Disease

7 Conclusion:

In this paper, we have introduced a new concept of multi-dimensional neutrosophic soft set. Using this new idea, an algorithm named $2-DNS$ has been proposed to solve neutrosophic multi-criteria assignment problems with multiple decision makers. Finally, our newly proposed $2-DNS$ algorithm has been applied to solve an assignment problem in medical science.

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Received: January 31, 2017. Accepted: February 17, 2017.